Proceedings

THE 22ND SEFI SPECIAL INTEREST GROUP IN MATHEMATICS SEMINAR

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The 22nd SEFI Special Interest Group in Mathematics Seminar

Innovations in Mathematical Education for Engineers: Bridging Past, Present, and Future

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Introduction – SEFI Mathematics SIG Seminar 2025 Proceedings

The Steering Committee of the SEFI Mathematics Special Interest Group is pleased to present the proceedings of the 22nd SEFI MSIG Seminar on Mathematics in Engineering Education. This seminar was locally organized by the Ostfalia University of Applied Sciences – Wolfenbüttel Germany, and has taken place from June 25th to 27th, 2025. Since its establishment in 1982, the SEFI Mathematics Working Group (MWG), now known as Mathematics Special Interest Group (MSIG), has consistently upheld its goals, which remain pertinent and applicable even after 43 years. These objectives include:

- Providing a platform for the exchange of perspectives and ideas among individuals interested in engineering Mathematics.
- Promoting a comprehensive understanding of the role of Mathematics in the Engineering curriculum and its significance in meeting industrial needs.
- Cultivating collaboration in the development of courses and supporting materials.
- Recognizing and advocating for the role of Mathematics in the ongoing education of engineers, in collaboration with the industry.

I express my heartfelt thanks to the local organising team: Kathrin Thiele, Gaby Bender, Nico Marten, Henrike Schulz, and Sebastian Wirthgen - whose generosity, professionalism, and warm welcome have made this event possible. I would also like to thank my friends from the steering committee: Daniela, Marie, Burkhard, Morten, Samuel and Petr, for their ongoing commitment and enthusiasm.

Hosting this year's seminar in the beautiful and historic town of Wolfenbüttel added something very special to our gathering. With its rich cultural heritage, academic tradition, and tranquil surroundings, Wolfenbüttel offered the perfect backdrop for reflection, dialogue, and shared inspiration.

It is with great pleasure and deep gratitude that I extend a heartfelt thank you to our distinguished keynote speakers: Professor Annoesjka Cabo from Delft University of Technology in the Netherlands, Professor Peter Riegler from Ostfalia University and the Bavarian Center for Innovative Teaching in Germany, and Dr Haiko von Rebenstock from Domino Printing Sciences PLC in the United Kingdom. Your presence at this seminar was truly an honour, and we are immensely grateful for your willingness to share your expertise, vision, and experience with us. The insights you brought from both academic and industrial perspectives enriched our discussions and broaden our understanding of mathematics education in the context of today's rapidly evolving challenges. Thank you for joining us and for contributing so meaningfully to this event.

Each year, our seminar allows us to reconnect, to share advances and challenges in mathematics education, and to reaffirm our role as educators striving to keep learning meaningful, rigorous, and human-centred. This year's seminar was centred around five key topics, all of which are highly relevant to the challenges and opportunities we face today:

- Artificial Intelligence in mathematics learning and teaching
- Project-Based Learning as a tool to assess competences
- Study techniques and metacognition for first-year students
- The role of mathematical theory in engineering education
- How to handle, and meet the needs of, diverse groups

These themes reflect not only pedagogical concerns but also a growing recognition of the broader transformations reshaping higher education. Among them, the topic of Artificial Intelligence stands out. This year, more than ever, we are called to think critically about how Artificial Intelligence is transforming the educational landscape. AI is no longer just a tool of the future; it is already embedded in the everyday practices of our students - and increasingly, in our own. AI has already begun to change how we teach, how students learn, and how we evaluate knowledge. But as educators, we must go beyond simply using AI as a tool, we must equip our students with the ability to use it critically and responsibly. This includes helping them to understand what AI can and cannot do, how it reaches its conclusions, and the biases that may be embedded in its models.

Among the advantages of AI, we can highlight the support it offers in adaptive learning, accessibility, and feedback at scale. It can help personalise learning pathways and make resources available to students with very diverse backgrounds and needs. Yet, there are also risks. These include over-reliance on AI-generated responses, the loss of students' sense of agency, potential erosion of critical thinking, and the ethical implications of opaque algorithms. We must guide students to see AI not as an oracle, but as a collaborator, one that must be questioned, validated, and understood.

As we explore these seminar themes over the next paper chapters, I invite you all to share your experiences, your questions, and your insights using the following email address deolinda.rasteiro@gmail.com . Let us learn from one another and work together to create engaging, inclusive, and rigorous learning environments, ones in which mathematics continues to play a foundational and transformative role.

We may explore many perspectives on these questions - drawing on the experience, imagination, and dedication of our diverse international community. And that, for me, is one of the greatest strengths of our SIG: the generosity with which we share, support, and grow together.

Let us keep pushing the boundaries of what is possible in mathematics education, while staying anchored in what is essential: critical thinking, curiosity, and care for our learners.

All accepted contributions are incorporated as full papers in the proceedings, which are freely accessible on the SEFI MSIG webpage, https://www.sefi.be/activities/special-interest-groups/mathematics/ . This initiative aims to offer a comprehensive overview of the seminar's topics and provide unrestricted access to the presented papers for all interested colleagues. The primary goals of the group are to uphold the ongoing process of collecting published materials and reports on all crucial topics identified in the mathematical education of engineers. This aims to construct a robust body of knowledge within this field. Lastly, the author expresses gratitude to all members of the SEFI Mathematics Special Interest Group Steering Committee, language editors, and local organizers for their efforts in conducting language checks and editing the proceedings. These contributions are intended to enhance the quality of the proceedings for the benefit of all potential readers.

In Coimbra, August 2025
Deolinda Dias Rasteiro
SEFI Mathematics SIG chair

Content

Comparing Performance of Freshmen and Sophomores in Mathematics

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Abstract

Two discrete mathematics courses with a significant overlap in contents were taught to freshmen and sophomores. Moreover, these courses differed in their emphasis on mathematical language. We will compare performance of students on comparable problems at the final exam. We focus both on solutions that follow algorithmic procedures and on solutions that require mathematical reasoning.

Introduction

There are many factors that influence learning outcomes of a course. Some of them are related to students:

- their background,
- their mathematical experience, in particular their familiarity with mathematical language,
- their motivation.

These factors are at least as important as the teacher-side considerations like course design, teaching approach and other pedagogical factors.

We used the opportunity to teach two groups that are in a way the exact opposites to compare their performance.

Basic Information on the Courses

We compare two courses on discrete mathematics with a significant subject overlap. For the common topics, these courses had equivalent questions on the final exam.

DMA(E):
- sample size: 32;
- students with one or more year of university experience;
- international students with varying backgrounds;
- exchange students, for many the course was not crucial, hence a lower motivation observed.

DMA:
- sample size: 37;
- recent high-school graduates, having passed a common state-guaranteed exit exam;
- freshmen, their first mathematics course;
- BSc computer science students for whom this course was obligatory for the program.

Results

Both courses included common topics with analogous final exam questions that are being compared here. We did not compare scores, but the actual answers and sorted them into groups. The numbers in tables refer to percentages with respect to the number of final exams written for each course.

Problem: Determine whether the four standard properties (reflexivity, symmetry, antisymmetry, transitivity) are valid for a given binary relation.

Comparison: We focus on the ability to apply a general definition to a particular case (the column "all defs correct" in Table 1 below), and the ability to form concise arguments supporting students' assertions. This can be found as columns "all correct" and "3 or 4 correct", referring to properties that were handled properly; the latter also includes the previous case (all correct).

Course:	all defs correct	all correct	3 or 4 correct
DMA(E)	88%	18%	52%
DMA	92%	46%	76%

Table 1: Results for problem on properties of relations

Observations:

1. The majority of students knew definitions well. When mistakes were made, they were common to both courses: in reflexivity they felt that an implication should be incorporated in one way or another; and in symmetry they used conjunction in place of implication.

2. The difference in performance was mainly due to the handling of reflexivity. Since this is a straightforward application, the cause of the higher failure rate for DMA(E) is probably general underestimation of this topic.

Problem: Prove an inequality by induction.

Comparison: We focus on how well students handle the difficult task of adjusting induction to an inequality, where a mere substitution of the induction hypothesis does not yield the desired outcome.

Failures were sorted by their cause, in Table 2 we find columns (the numbers in percentages again):
- correct: the proof was correct, perhaps with minor formal errors;
- struct.: structure of induction was wrong;
- backward: backward argument;
- algebra: incorrect handling of inequalities.

Course:	correct	struct.	backward	algebra	other fails
DMA(E)	39%	27%	15%	9%	10%
DMA	38%	13%	22%	27%	0%

Table 2: Results for problem on induction

Observations:

1. The success rate is similar. Students in both groups learned for the first time that induction can be used for other purposes than just proving simple equalities. Interestingly, DMA(E) shows a higher proportion of problems with the structure of the proof. This could be caused by the fact that in the DMA course, the induction was given an extra week of instruction focusing on general structure of induction to alleviate problems observed in previous years. Moreover, while many DMA(E) students claimed to have met induction before, many of DMA students practiced induction in high-school quite extensively, albeit in its weak form.

2. Proving an inequality presents an added challenge in that algebraic operations needed for completing the proof are not all equivalent. This was a bigger challenge for students from the DMA course, in particular they needed to overcome the faulty ``backward proof'' approach which is commonly taught at high schools in the country.

Problem: Solve a given non-homogeneous linear recurrence equation using the standard algorithm, including the undetermined coefficients method.

Comparison: The main focus is not on the result as such, because it is often influenced by numerical errors. Rather, we focus on ability to apply an algorithm that is not one straightforward procedure, but involves several steps, some of which require judgement.

Course:	correct	partially	fail
DMA(E)	52%	30%	18%
DMA	46%	30%	24%

Table 3: Results for problem on linear recurrence

Observations:

1. It seems that freshmen students were somewhat less ready to follow a less trivial procedure.

General conclusions

Typically, going through several mathematical courses at university has a cumulative effect on student's mathematical competencies. We therefore expected the students of the DMA(E) course to do better compared to freshmen, especially in less routine tasks. However, it did not happen, most likely due to difference in motivation.

Do We Listen to Our Students?

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Abstract

This article reports on findings of Erasmus+ project Pythagoras aimed at investigating the impact of active teaching/learning methods on the knowledge acquisition in basic courses of mathematics at the engineering study programmes. Presented analysis of the feedback received from the international groups of students who tested prepared learning materials reveals their opinion on the 5 tested learning scenarios: escape room activity, mini-PBL, eduScrum, gamification, and stack exercises. Comments from students serve a as source of inspiration for development of appropriate teaching/learning methods reflecting students´ primary needs and requirements.

Introduction

One of the main reasons for a high drop-out rate in the first year of engineering study programmes seems to be problems with passing exams in the basic courses of Mathematics. This is due to a lack of student engagement, various personal issues, and even high demands based on required deeper learning outcomes, see in Kayoko (2022). The Erasmus+ project, Pythagoras, strives to develop approaches that will make learning Mathematics more inclusive, efficient, enjoyable and real; connecting Mathematics teaching with real life cases linked to the students' fields of study. All project outcomes and activities are tailored to address the needs of the partner institutions in order to strengthen knowledge acquisition in the fundamental mathematics background. The results are checked from a variety of perspectives: mathematical content, mathematical processes, views about the nature of mathematics, and personal opinion of students regarding their learning preferences, as suggested in O´Leary (2023).

One of the project activities was a Summer School for Students, organised as Learning-Teaching-Activity lasting 5 days. Groups of 4 students from all 7 partner institutions attended the event and tested 5 different learning scenarios applied in teaching basic courses of mathematics in university bachelor study programmes. Learning activities were prepared by teams of mathematics teachers from the partner universities: Lucian Blaga University in Sibiu, Romania as project coordinator, Aalborg University in Denmark, The Hellenic Mediterranean University in Chania, Crete, Greece, The Porto Polytechnic in Oporto, Portugal, Slovak University of Technology in Bratislava, Slovakia, The University of La Laguna, Tenerife, Spain and Karlstad University, Sweden. A different didactic approach and active learning method were tested each day, while authors of each activity were responsible for the day's program and its evaluation. Students answered the prepared feedback questionnaires after each of the activities, where they stated their opinions on the presented didactic scenario. They were also asked to express their personal views on each particular method with respect to its impact on their knowledge acquisition, along with ideas for possible improvements from the learners' perspective.

Method of Investigation

Students worked in groups of 4 on solving particular tasks. Groups were sometimes mixed, formed of students from different countries, sometimes these groups represented a particular university and country. After completing the required tasks and delivering results, each activity was followed by an on-line questionnaire available for students to fill out individually. The feedback survey consisted of several general questions with a choice of answers, open questions to give opinions on the tested didactic method, and some questions about expected knowledge gain and efficiency of the learning process under this particular teaching/learning scenario. Questionnaires were anonymous, students were registered only by the country of their origin, and answers in all five surveys were analysed generally, with respect to the best and most favourite method from the student perspective. All questionnaires were prepared as on-line applications accessible using a unique QR code. Students were asked to express their opinions openly and honestly, with the declared intention that their views will be taken into account for development of new educational activities meeting their needs and expectations. The aim was to receive fresh ideas from those actors of educational process, who are mostly playing a passive role as recipients of new knowledge without the opportunity to comment on ineffective methods. Such methods are used in goodwill as a way to try to promote active learning, enhance curiosity in learning mathematics and support deeper conceptual understanding of students. The approach of all students during these activities was more positive than expected. They were keen on testing various ways of teaching/learning, eager to solve the assigned tasks, and very competitive, struggling to receive the best results in several scheduled games, quizzes and in team work on small projects that were carefully planned and prepared as computer applications, on-line games and spreadsheets or available as paper work in handed out packages. The teachers who authored the tested materials presented a short introduction at the beginning of lesson, with instructions on how to use a particular computer programme, application, ready-made applet, or provided paper sheets. They watched the engagement of students during the work, their interest and cooperation and social behaviour. All were ready to help students with possible problems that might occur during their work with presented materials due to different cultural, historical or other restrictions of particular country backgrounds and traditions in dealing with basic mathematical concepts and their symbolic representation. A summary of the feedback analysis is presented separately for each of the 5 active learning/teaching methods. Overall satisfaction with the event activities during all days is presented on the diagram from Pythagoras project report by Luntraru (2024) in Figure 1.

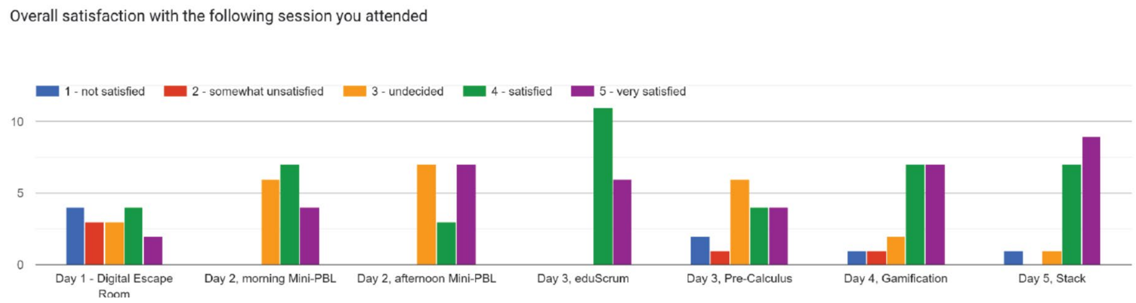

Figure 1. Overall satisfaction with the event activities

Digital Escape Room

The first day activity was work in small groups of 4 students, mixed or from the same university according to their own decision. Many students preferred to be in a mixed group in order to start international cooperation which they found as one of the added values and the most important benefit received from participation at this event. Groups received access to a newly designed computer application – an on-line digital escape room with special mathematical problems that had to be solved in order to get out. Partial solutions of small problems allowed players to continue through 3 rooms with additional pieces of information leading to final solution of the slightly more complex problem. The environment was designed to be a virtual 3D space with objects in the rooms, doors opened on inserting correct answers and codes, which was quite demanding on the quality of computers, necessitating longer processing time caused by many active players at the same time, and the response speed of the internet connection. Students were interested in using this game, but a bit disappointed by technical constraints. Some of the tasks were based on manual skills with playing games, e.g. quick fetching and moving some objects in the virtual scene, which were not related to any mathematical knowledge or task, but quite significantly influenced the score and results. Some students did not fulfil the final mathematical task in the third room, as they were not able to get access to this last room and stayed without reaching any mathematical problem, information or knowledge. Their disappointment was also reflected in the mostly negative attitude expressed in this day questionnaire answers. Dissatisfaction stemmed from design issues within the escape room application, which hindered its effective execution. Students articulated these concerns in their feedback, highlighting the technical challenges they encountered and how these problems detracted from their overall experience during that particular session. Some of the comments: "*The Escape room didn't have a way to escape :(* ". "*The first day's virtual escape room failed when it came to not crashing, not bugging, and enunciating the problems correctly. Otherwise, it would have been very interesting*".

Mini PBL

The next activity was adapted PBL - Problem Based Learning method, in the form of miniPBL. Groups of students were asked to work on solutions of small problems related to some environmental issues. The task was to understand the applied problem described in a short introduction, to develop mathematical model of this problem and to find its optimised solution, while the theoretical result had to be interpreted back to the practical solution of the applied problem. This approach was highly appreciated by students, who were interested in finding solutions of the real problems. They very much valued the fact that mathematics might help to model real life situations and to suggest their possible solutions with available optimisations. The feedback from students on this activity received in the evaluation questionnaire was overwhelmingly positive. A significant majority of participants reported that the sessions were well organized and met their expectations, reflecting a high level of satisfaction with the content delivered, as seen on diagram in Figure 2, from the Pythagoras project report by Luntraru (2024). A substantial portion of the respondents also indicated that the duration of the sessions was appropriate, the instructors effectively encouraged questions and fostered discussions among group members during the work on problems. This engagement is crucial for deepening understanding and enhancing the overall learning experience, suggesting that this activity supported the active participation and dialogue among students.

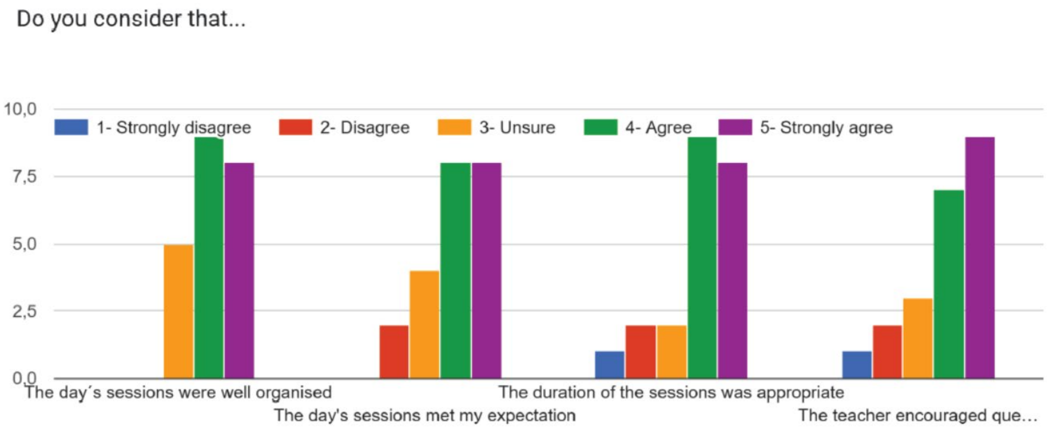

Figure 2. Analysis of answers in questionnaire on miniPBL activities.

EduScrum Method and Pre-Calculus Course

Two activities were planned for the third day. In the morning session the eduScrum activity was implemented, which was highly regarded and considered the highlight of the week. This method was marked by all students as the most suitable and relevant with respect to knowledge gain and satisfaction with the learning outcomes. Students worked in groups as national teams solving provided sprints that are sets of 5 problems related to Calculus I and II. Four problems were traditional tasks testing calculation skills and understanding of basic concepts from this topic, and the fifth one was an applied problem related to environmental issues to be solved by optimisation. Team members chose a leader – the scrum master, who guided the team work, distributed problems to team members and submitted collected solutions. Working achievements of all teams were assessed, and the best teams with the highest point score were awarded small prizes. All students were satisfied with the work in small teams and they appreciated the possibility to discuss problems together and share the workload. They also pointed to the social aspects of the team-work, working at their own pace, discussing misunderstandings among team members, and share responsibility for the sprint tasks as a team. Almost all students considered this learning/teaching method to be most rewarding with respect to their personal knowledge gain and depth of achieved understanding.

The feedback on the afternoon Pre-Calculus course session was less favourable. Students commented on the inconsistency in teaching approaches during the afternoon session, stating, *"If one teacher tells us that teachers shouldn't teach while facing the board, then why does the next teacher do that?*" This highlights a perceived contradiction in teaching methods and suggests that greater consistency might enhance student engagement. Overall, less than half of the students who responded to the questionnaire felt that the day's sessions were well organized and their duration was appropriate, the sessions met their expectations and that the teachers effectively encouraged their engagement.

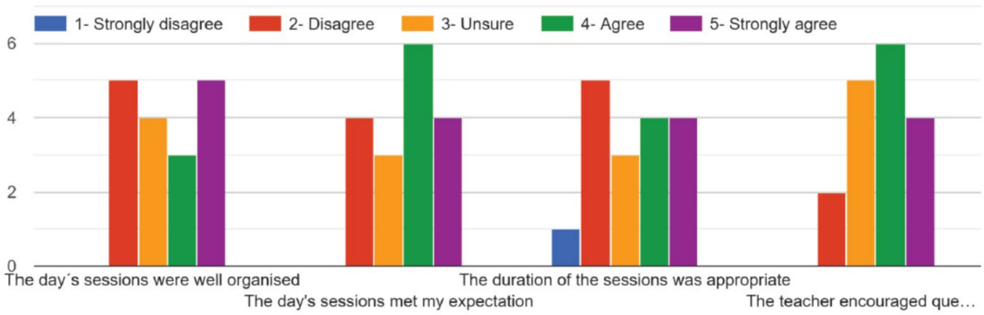

Figure 3. Analysis of answers in questionnaire on the third day activities.

Gamification

The activity for the day four was gamification, a learning method aiming to test acquired knowledge in basic mathematics. Students, in small 4-member teams, solved several problems presented as games. Points were awarded for correct answers, final points scores were declared after each game and winners received small prizes. This activity - learning, repeating and revising by playing, was regarded as interesting and efficient for fostering team cooperation and rewarding individual achievements in a pleasant, relaxing atmosphere. All students confirmed that the sessions were well organized and met their expectations. They also commended the teacher for his proactive attitude in encouraging questions and facilitating discussions. Students also gave positive remarks, as "*Very interesting topic, excellent presentation, engaging activities, and fun examples*." and "*The interaction with students and competitive games really encouraged teamwork*." This activity proved to be the most relaxing and valuable in terms of deepening knowledge and understanding, and particularly for the transfer of this knowledge to another context. Diagram with feedback results is in Figure 4.

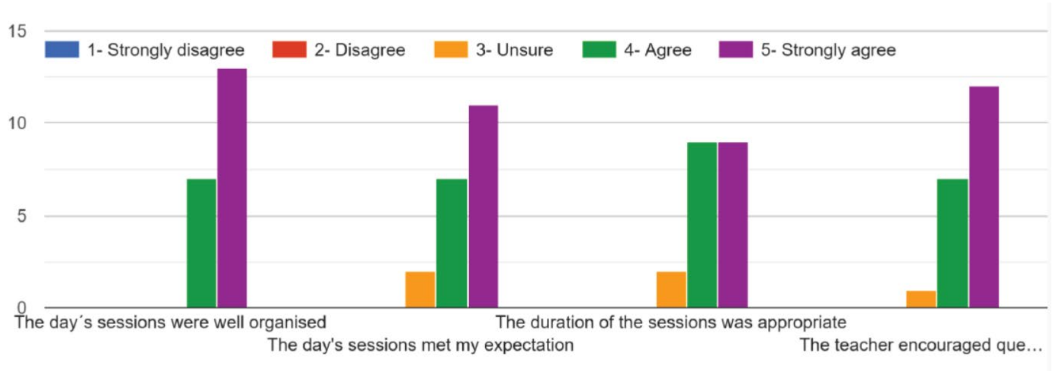

Figure 4. Analysis of answers in questionnaire on the gamification activity.

Stack Exercises

On the last day of the event, Stack exercises were introduced as the learning activity. The students´ task was to solve several problems presented in the Moodle environment as an exercise, in step-by-step mode. Comments were given to partial solution and any incomplete answers followed by hints that aimed to help solver to find the right procedure or to use the correct formula. References to textbooks or other digital study materials were provided to solver, who could find basic theoretical information in case if necessary, and repeat the topic. The opinion of students on this learning activity was very positive, most of them found this approach to be helpful and instructive. A few students did not like this approach. They criticised the fact that one could come to a correct solution without any knowledge gain, just following the given instructions. The majority of participants affirmed that the session met their expectations, highlighted its effectiveness in delivering the intended content, and the lead teachers' efforts in fostering an environment conducive to active participation. This feedback reflects a strong endorsement of the instructors' ability to engage students effectively and facilitate interaction throughout the educational session. The activity was regarded as the most suitable for individual study approach.

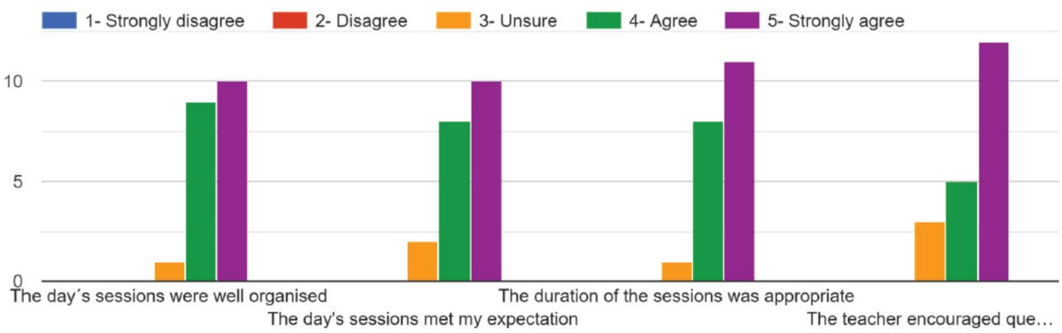

Figure 5. Analysis of answers in questionnaire on the Stack exercise activity.

Conclusions for Education

No teaching/learning method could be declared as the most effective one. Each method has its role and place in education and should be used carefully with a strategic plan taking seriously into consideration needs and expectations of students. EduScrum is the most appealing one to students, engaging their interest for the teamwork and share of responsibility, not neglecting the social aspects of this kind of cooperation.

We should listen to our students.

Acknowledgement

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Portfolio assessment in mathematics promotes active learning

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Abstract

This paper introduces a portfolio assessment in mathematics for first-year engineering students that was implemented at the University of Agder (UiA) in 2022. The portfolio consists of two key components, contributing 60% and 40% of the final grade, respectively:

1. Four computer-aided assessment (CAA) tests under supervision, providing immediate feedback on the students' performance. The students can take each test up to four times, with their best attempt counting toward their final grade.

2. A written assignment, where students apply mathematical concepts from the curriculum to solve an engineering problem.

We observe that the students take advantage of multiple test attempts, and that after each test, they organically form groups to discuss the problems with each other. Additionally, the university's math support center sees increased usage during test periods. Biggs et al. (2007) claim that learning is a result of what the student does and that active students foster deep learning. We argue that this assessment design encourages the students to become active in their own learning.

The written assignment also helps students to relate mathematical competence to realistic engineering problems, thus making the subject more relevant to their studies. We claim that this competence is equally important for engineers as it is to solve difficult calculus problems by hand.

Introduction

At the University of Agder (UiA), mathematics is a common course for all engineering programs, with approximately 400 students enrolled each year. The course has traditionally concluded with a high-stake final exam, leading to considerable stress and a failure rate of around 40%. Although there is mandatory coursework, it is completed with varying levels of commitment.

Several studies have highlighted the benefits of active learning at universities. For instance, Biggs et al. (2007) argue that all types of students benefit from active learning, emphasizing that it is the nature of their engagement—what students actually do during the learning process—that significantly impacts their learning. Similarly, Freeman et al. (2014) found that students in STEM fields achieve better outcomes in courses that incorporate active learning compared to traditional lecture-based instruction.

Our analysis of the traditional approach to teaching mathematics—which involves large lectures, problem-solving sessions, and a high-stakes, closed-book exam at the end of the semester—reveals that students often engage passively in their own learning (Zakariya et al., 2022). Students report receiving minimal feedback throughout the course, which contributes to a high failure rate. One suggestion from Zakariya et al. (2022) was to develop low stake testing to foster learning.

We believe that all students are capable of learning mathematics sufficiently as required to achieve an engineering degree. However, the amount of effort required varies from student to student. To support 400 students in successfully completing the course, it is essential to activate them at the beginning of the semester, helping them understand their current skill level early on and adjust their workload accordingly. With this mindset, we developed a new assessment system designed to motivate students to engage with mathematics earlier in the semester, and at the same time keep the academic quality of the student's mathematical education.

This paper presents the portfolio assessment model, including an example of a digital test exercise and learning outcomes. The data includes student grade outcomes and anecdotal feedback gathered through student quotes.

We developed a new assessment method during a one-week design sprint

The structure of the new assessment method was created through close collaboration between academic staff, university administration, and insights gathered from student interviews. By bringing all stakeholder together to focus on a shared problem, we were able to identify the motivations, perspectives, and requirements of each group involved in the process:

(1) Academic staff wanted to increase student engagement in the course and promote deeper learning of mathematics. They recognized that the assessment form is the biggest motivator for students' activities, and that mandatory home assignments were often done with different levels of effort. While frequent testing is an effective motivator, such tests typically assess only procedural skills. The staff also aimed to help students understand how to apply mathematics in engineering contexts and to foster more reflection. To address these goals, an individual written assignment was introduced to assess conceptual understanding and application. Lowering the difficulty of the final exam to increase pass rates was not considered a viable option.

(2) University administration focused on respecting students' rights in assessment. For written tests under supervision, students must be offered multiple opportunities—at least three—to pass. A shared concern with academic staff was the considerable time spent grading. With 400 students, repeated testing, and multiple attempts per student, it became clear that digital assessments with automatic grading was the only practical solution.

(3) Students supported the idea of multiple tests with multiple tries. However, they were clear about the need for deadlines between tests to avoid procrastination. Their experience with unsupervised home exams during the COVID-19 pandemic highlighted the necessity of supervision. Cheating was seen as unavoidable in

unsupervised settings—not only frustrating for instructors but also unfair to students who followed the rules.

The assessment consists of four digital tests and one written assignment

The design sprint resulted in a portfolio-based assessment model, consisting of four supervised computer-aided assessment (CAA) tests and one individual written assignment, as illustrated in Figure 1. Each test can be attempted up to four times, with testing opportunities spread across four two-week periods throughout the semester. Students can earn a maximum of 100 points, with 40 points required to pass the course. There is no minimum score required on any single component; only the total score determines whether a student passes.

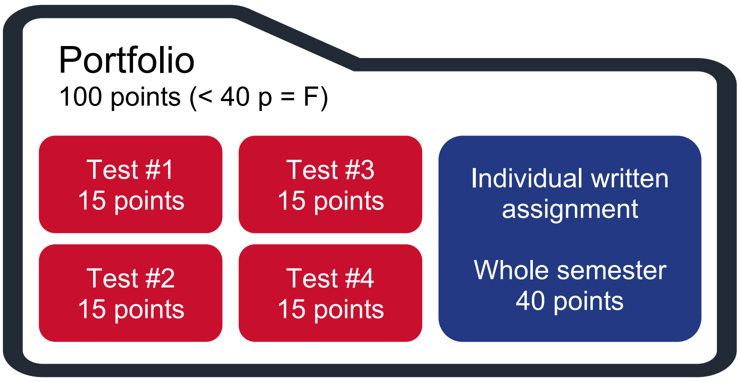

Figure 1. Overview of the portfolio-based assessment model for the first mathematics course for engineering students at UiA.

We developed our own customized CAA tests using the STACK assessment tool (*STACK*, 2025). Figure 2 shows an example of a typical test question. Questions are selected from a database, with randomized numerical values to prevent students from simply memorizing answers. With sufficiently large database of questions, we believe the most effective way for students to succeed on these tests is by genuinely learning the mathematical methods required to solve the problems.

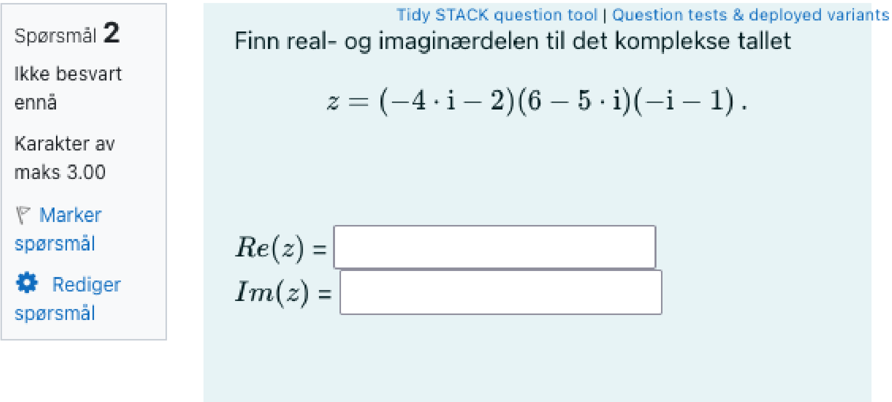

Figure 2. Example question from one of the four digital tests. Translated from Norwegian: *"Find the real and imaginary part of the complex number."*

The written assignment presents an engineering problem that requires students to apply the mathematical models taught in the course. Students are expected not only to solve the problem but also to reflect on how their solution connects to the mathematics curriculum. This helps them present their mathematical understanding within a context more familiar and relevant than abstract theory alone.

Initially, the written assignment was the same for all engineering students but has since been adapted to reflect the specific contexts of each engineering discipline—civil engineering, renewable energy, mechatronics, electrical engineering, and computer science. This tailored approach fosters a deeper understanding of mathematics within each field, an aspect that was often lacking when assessment was limited to a single final exam.

Mathematics in not purely about mastering procedures and formulas. Niss et al. (2011) argues that learning mathematics should develop a wider set of mathematical competencies that enable students to understand, apply, and reflect on mathematics in meaningful ways. These competencies cannot be fully assessed through a single evaluation method. The portfolio assessment addresses this limitation by allowing us to evaluate a wider range of competencies than was possible with a single end-of-semester exam.

The students' grades improved significantly

The portfolio assessment was introduced in 2022, resulting in a significant drop in failure rates—from approximately 40% to 10–20%, as shown in Figure 3. From 2014 to 2021, assessment was based on a single end-of-semester exam, with up to two retake opportunities. The figure presents the result from the students' first attempt. After the introduction of the portfolio assessment, the failure rate dropped significantly. The sharp decrease in 2020 was due to the use of a home exam format during the COVID-19 pandemic.

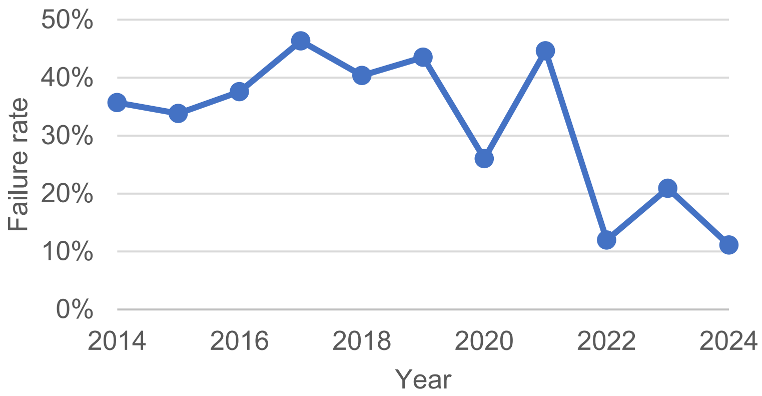

Figure 3. Failure rate in mathematics for first-year students. The portfolio assessment was introduced in 2022. The exam in 2020 was a home exam due to COVID-19.

Reflections on learning outcomes

There are many factors to consider when evaluating whether this form of assessment improves student learning outcomes. While this paper does not attempt to draw definitive conclusions, it offers several observations—primarily from *MatRIC* (2025), a student-driven center focused on supporting students in their studies.

The support center has reported an increase in students seeking help around test periods—not only beforehand, to prepare, but also afterward, to reflect on mistakes and deepen their understanding of the curriculum (Gjesteland, 2025). Immediate feedback and the opportunity for retakes encourage students to actively engage with the material and seek improvement. In contrast, traditional exams typically provide grades after a delay of up to three weeks—too late for most students to reflect meaningfully on their performance or identify areas for improvement.

In the first year of implementing the portfolio assessment, students tended to collect all available exam questions and focus exclusively on those, rather than studying the broader curriculum. At the time, the exercise database was relatively small, making this strategy effective. However, as the database has grown, this approach has become less viable. In any case, the increased number of tests has led to greater student engagement. As one student reflected: "*We think we're fooling you [professors] by collecting all the exam questions and only working on those—but in reality, you're fooling us into doing more math.*"

While students occasionally find shortcuts to solve problems, we believe that, overall, the system encourages them to learn the underlying mathematical methods rather than simply memorizing procedures.

The written assignment fosters a deeper level of mathematical understanding than standard exercises. As one student noted: "*I didn't really understand the relevance of the digital tests until I started working on the written assignment.*" We believe this higher level of understanding is achieved when students work independently—without relying on chatbots or similar aids. To ensure authenticity and assess true understanding, we are considering replacing the evaluation of written assignments with oral examinations.

Concluding remarks

We have developed a new assessment method that targets a broader range of mathematical competencies beyond simply mastering procedures and formulas. This approach has improved student grades and increases their engagement throughout the semester, particularly during testing periods. However, there is still work to be done. We need to continuously expand the task database to prevent memorization of exercises. Additionally, we must ensure that students work independently and reflect thoughtfully on their written assignments to develop the mathematical competencies expected of engineers.

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Using ChatGPT in a Mathematics Homework Exercise

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Abstract

This paper presents a comparison between a mathematics exercise which students solved with the help of an AI chatbot and one from a previous semester which students solved without a chatbot. Ideally, chatbots should improve student performance. But in this instance, the chatbot appeared to have had an equalising effect on the students' performance in that it helped weaker students while at the same time hindering stronger students. Thus, questions remain open with respect to how AI chatbots might be employed effectively in order to augment human abilities.

Introduction

The development of AI chatbots has been progressing rapidly over the last few years and has facilitated the emergence of new applications for learning and teaching (Heidt 2025). Ideally, employing AI tools should lead to what Engelbart (1962) calls the *augmentation of human intellect* which means enabling human achievements which would otherwise not be possible and inspiring further intellectual development. Brynjolfsson (2023), however, discusses negative consequences of AI, if instead of augmentation it results in *automation* and thus a complete replacement of human activities by AI tools. Fulbright and Morrison (2024) observe that if students employ AI chatbots in exercises, augmentation is not automatically achieved because chatbots might overwhelm students with unnecessary and misleading information. Rojas (2024) reports on a case that could be considered augmentation, in an assignment that integrates the use of chatbots for scientific writing together with instructor and peer feedback. Park and Manley (2024) argue that chatbots can assist students in writing mathematical proofs, resulting in improvements even though the final proofs may still not necessarily be complete and free of errors.

This paper presents a further example of how chatbots do not automatically improve student work. In particular, it can be observed in this example that weaker students profit from using chatbots whereas stronger students might be held back by them. This raises open questions such as: how to employ chatbots in a manner that facilitates augmentation of human skills; whether future, more sophisticated chatbots will make augmentation more seamless; or whether effective communication between humans and chatbots presents a challenge that requires novel, yet to be determined approaches.

Description of the Exercise

A homework exercise in an introductory mathematics class for first year computing students consists of assigning a simple set or number theoretical statement to each student to be mathematically proven and to be implemented in some examples in Python. Because logic, but not specifically proof techniques, are taught in the class, in previous semesters the students were not expected or required to solve the exercise completely by themselves but were encouraged to consult textbooks or the internet for help. Most students searched the web and basically nobody used textbooks, presumably because using a textbook would have required more effort.

Because the mathematical precision of AI chatbots is improving continuously, in the 2024 autumn semester students were required to use ChatGPT in the version OpenAI gpt-4o that was licensed by the university at the time. The interface is called Olaf by the university and shall be referred to under that name in the remainder of this paper. The students were supposed to consult Olaf for the exercise but to amend and correct Olaf's output if it was not perfect. The students were also expected to comment on how they employed Olaf and reflect on its usefulness. Apart from a very short introduction to the topic of AI chatbots the students received no further training on how to prompt a chatbot. In the second part of the exercise, each student was asked to review the anonymous submissions of three other students and describe any errors that they detected. It was hypothesised that, with the help of Olaf, the quality of the homework exercise would be superior to that of previous semesters. About 50 students were in the class in the semester Olaf was employed, and about 80 students were in the class the previous semester.

Olaf's Results

On the surface, what Olaf produced looked very good because the structure of the proofs and the code was usually perfect. But the answers from Olaf tended to be overly verbose and often contained redundant text which was somewhat more complicated than necessary. For example, Olaf appeared to not use complete induction even though that might have resulted in simpler proofs in some cases. It is unlikely that the lack of induction was a choice made by students while prompting Olaf because in previous semesters students did present solutions with induction. Quite often Olaf made at least small mistakes, such as inconsistent use of variables or proposing slightly incorrect definitions. Olaf also sometimes made basic logical mistakes such as not proving both directions of an equivalence, proving the wrong direction or returning False instead of True if the premise of an implication was false. In only one instance Olaf was completely wrong by asserting that a true statement was false. Because the false response could not be replicated afterwards, it is not known whether Olaf really responded incorrectly, whether the student made a mistake when prompting or paraphrasing Olaf's response, or maybe whether the student used a different, older version of the chatbot. None of the three students who reviewed that submission detected the mistake. In general, it appeared that students overlooked more errors while reviewing each other's solutions than in previous semesters, maybe because Olaf's answers always have a very professional structure and are asserted in a very confident manner.

The Python code that the students obtained from Olaf was always syntactically correct. Because a proof with Python code was not possible for most of the mathematical statements, the code was meant to indicate the plausibility of the statements with some examples. But Olaf often supplied insufficient examples, such as only a positive example and not a negative one or, again, supplied examples for the wrong direction of an implication. In summary, Olaf's most significant errors tended to be of a logical nature.

Unfortunately, several students did not detect such errors. Because such errors correspond to common misconceptions, it would be interesting to know whether the errors were caused by the chatbot learning incorrectly or whether the errors were already contained in the training data set if that was sourced from texts of dubious quality.

A further source of problems for the students appeared to be that Olaf did not know what the students had learned in class. Thus, Olaf sometimes employed definitions that differed from the ones provided in the textbook or used Python constructs that the students were unfamiliar with. The students were instructed to ask Olaf to use different constructs in such cases. Most students did prompt Olaf accordingly with respect to the Python code but not necessarily with respect to mathematical definitions. Most likely, encountering differences amongst mathematical definitions was very challenging for the students.

Marking the Assignment

In introductory mathematics courses, students mostly learn to replicate and apply previously taught standard methods. There is often little or no opportunity for students to experience what it is like to perform actual mathematical work, such as modelling and proving, apart from repeating standard proof methods such as induction. Because of its open nature, the homework exercise provides such an opportunity. All of the problems in the exercise are of a kind that can be proven directly using basic set or number theoretic arguments. Since proof skills are not taught systematically in the class, the students need to employ logical thinking and general problem solving skills. In a previous semester, about 40% of the students developed their own proofs without much help, evidenced by how the proofs were written, containing small imperfections and details on how the proof was conceived, which would be omitted in a textbook. Some of these proofs had what might be called a "wow factor" because it was clear that the students spent a significant amount of time on the exercise and produced individual solutions that demonstrated good and often creative mathematical problem solving but which were not so perfect that they seemed copied from somewhere else. Wow factor solutions were given full points even if they contained minor mistakes, which resulted in about 40% of the students achieving full points.

The marking scheme for the first part of the homework exercise is very simple and consists of up to 4 points that are given for correct aspects and subtracted for errors. Although the following analysis is based on this very simple marking scheme, it can be speculated that it highlights a general tendency of using AI chatbots. Figure 1 shows the points for a semester without Olaf as a black line with circles and with Olaf as a dashed line with diamonds. Without Olaf, the appearance of solutions with a wow factor explains why the distribution is not Gaussian but has a peak at the top mark. With Olaf, an expectation was that the quality of the students' submissions would be higher than in previous semesters. Thus, points were subtracted for any errors which resulted in a more Gaussian distribution as shown in the dashed line in Figure 1. With Olaf, only about 20% of the solutions had absolutely no mistakes and none of the solutions had a wow factor because all proofs followed standard patterns and none gave an impression of being particularly creative. The students had been told that they could modify and improve the output from Olaf or even replace it completely if they included a comment explaining why they changed it. But while some students did write their own Python code, there was no evidence of anybody replacing one of Olaf's proofs with an independently constructed proof.

It can be suspected that all students spent less time on the exercise and worked more superficially because of Olaf's good-looking answers. There were significantly more requests from the students than in previous semesters for further feedback after the exercise was finished. It appeared that some students were still convinced that Olaf's good-looking answers must be correct even if the received mark indicated problems, and instead of questioning Olaf's result they questioned the marking.

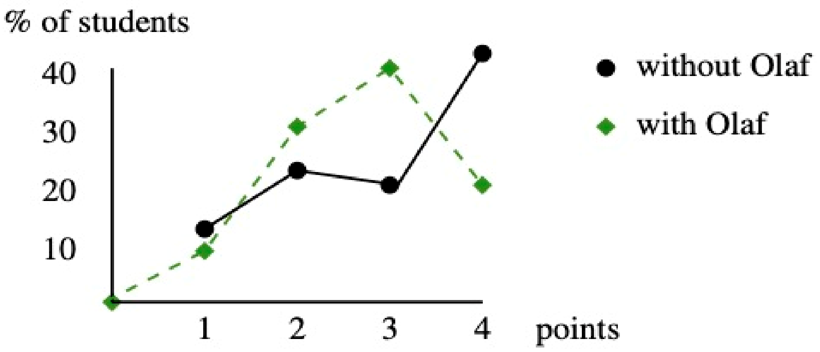

Figure 1: Points achieved with/without Olaf

Using Olaf had an equalising effect on the students' work with 70% of the students achieving a medium performance. Interestingly, there were still students who obtained 0 points because they invested so little time that they did not follow the instructions at all and some students who obtained only 1 point because of very obvious errors or omissions. Thus even if an exercise becomes presumably very easy because of a sophisticated tool such as Olaf, a small number of students still fail because of insufficient effort. Most likely students who would otherwise perform slightly below average benefitted most from using Olaf as demonstrated in Figure 1 because they were able to achieve an average mark. On the top end of the scale, it was much more difficult to obtain full points because of the general reasonably good quality of Olaf's results which led to a subtraction of 1 point for even just a single small mistake. In other words, if the quality of submissions is generally high for all students, it becomes more difficult to stand out. No student submitted a solution with a wow factor. It seemed as if Olaf had stopped students from thinking independently for themselves.

Other than the points mentioned above, marking the exercise did not feel different from previous semesters. Although Olaf's answers always followed a similar structure and Olaf mostly chose certain proof patterns (by contradiction and case distinction), the details of the answers were sufficiently different so that it did not raise any suspicions of students plagiarising each other's work.

Discussion and Conclusion

Obviously, a single homework exercise is insufficient for drawing long reaching conclusions. However, the student numbers (50-80) are reasonably substantial, and Park and Manley (2024) report a similar case of employing a chatbot in an exercise of mathematical proof writing which also led to improved but sometimes incomplete proofs that contained errors. It is likely that at some point in the near future, chatbots will be capable of producing flawless mathematical proofs. In some sense, however, mathematical reasoning is the simplest form of reasoning for a computer because of its formal structure. Thus, not all tasks may be achievable by chatbots as easily. Furthermore, it will most likely still be a concern in the future as to whether humans critically examine and enhance the chatbot outputs or simply accept them, representing augmentation in the former case and automation in the latter. It would be commendable if chatbots were to improve the work of weaker students by enabling them to study more effectively, but not if students only substitute their work with chatbot output.

An interesting question arising from the homework exercise is how to employ chatbots so that they do not stifle but boost the performance of stronger students. This question will be even more important if the chatbots present ever more impressive answers in the future. In a dystopian view, one could speculate whether human labour will simply become redundant. In a non-dystopian view, human and AI intelligence could be augmented leading to better results than either human or AI could achieve on their own. Because of the verbose and confident but human-like manner of the chatbot answers, one research angle would be to examine whether the style of communication between users and chatbots can be modified in order to strengthen critical thinking on the human part and facilitate augmentation.

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Integration of Academic Knowledge and Active Citizenship: Service-Learning in Engineering Training

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Abstract:

Service Learning is a pedagogical methodology that combines academic knowledge with training aiming at the development of active citizenship (Diaz Fernandez, 2021). This approach is implemented through a carefully planned project in which students engage with the real needs of a community, contributing to its improvement. Service Learning is based on civic education, aligned with active learning and integrated with other approaches such as cooperative learning and project-based learning. Recent studies indicate that Service Learning offers considerable benefits to university students, promoting improvements in the quality of learning. Among the main benefits are the strengthening of leadership skills, the encouragement of attitudes of generosity, empathy and solidarity, increased self-esteem, the development of social skills, and the acquisition of significant learning (Narong, 2024). Service Learning is not just an isolated practical experience, but rather a way of integrating academic learning and social engagement, where students participate in activities that help develop essential skills and abilities for their professional future while responding to the needs of the community or society (Resch, 2023). This study aims to analyse how Service Learning can serve as a powerful educational tool, enabling students to develop essential technical and socio-emotional skills by engaging in real-world projects that integrate academic knowledge with professional and community challenges, ultimately promoting their growth as socially responsible and well-prepared citizens.
The projects developed by Mechanical Engineering and Electromechanical Engineering students at the Coimbra Institute of Engineering allowed the application of theoretical knowledge of Statistical Methods in practical situations, which not only improved the understanding of the concepts but also showed how these concepts can be used in real-world situations promoting sustainable solutions. The real data collected sometimes brings unexpected challenges, and students need to adapt, adjust methodologies and correct any flaws in the data or approaches adopted. Can students identify relevant problems and apply statistics to solve them effectively? Are students able to apply appropriate statistical techniques in the context of the proposed projects? Do students develop a broader view of their role as engineers in the social and community context? By reflecting on these questions, it is possible to identify both the challenges and

opportunities that arise during the process, as long as the development of students' potential technical and socio-emotional skills. When developing solutions, students can reflect on how their analyses can contribute to more sustainable practices in different areas. At the end of the project, students also had the opportunity to think about what they learned and how to apply their knowledge in the future.

Keywords: Service-learning; active methodology; cooperative learning; project-based learning; competencies; skills

Introduction

In recent years, the integration of Service-Learning (SL) methodologies into engineering education has gained significant momentum, driven by the dual objective of enhancing students' academic competencies while fostering civic responsibility and social engagement. Originating from the experiential learning paradigm (Kolb, 1984), SL is a pedagogical approach that combines formal academic instruction with community service, structured reflection, and reciprocal partnerships (Jacoby, 2015). When applied to STEM fields, and particularly to Mathematics and Statistics, SL has demonstrated its potential to bridge the gap between abstract theoretical concepts and the complex, nuanced challenges found in real-world social contexts (Hadlock, 2005; Queiruga-Dios et al., 2021). This methodological shift coincides with broader transformations in higher education, where there is increasing emphasis on developing transversal competencies, such as teamwork, ethical judgement, and communication, alongside technical proficiency (Niss & Højgaard, 2019; Alpers et al., 2013). Mathematics, traditionally taught as a self-contained discipline, is being reimagined as a fundamental discipline for interpreting, modelling, and transforming social realities (Zeidmane, 2012; Puzi et al., 2022). Within this context, SL offers a unique opportunity to reposition mathematical knowledge as a technical skill and as a civic instrument capable of generating social value (Bowman, 2011; McNatt, 2020). Despite its growing relevance, the incorporation of SL into mathematics and statistics education remains limited. A bibliometric review conducted by Narong & Hallinger (2024) revealed that less than 1% of SL projects indexed in major academic databases are explicitly related to mathematics education, highlighting a persistent underrepresentation of this field in SL research. Moreover, Queiruga-Dios et al. (2021, 2023) confirmed that within engineering curricula, SL projects involving mathematical modelling and data analysis are still relatively rare, though highly impactful when implemented. These findings align with recent efforts to promote SL practices through interdisciplinary collaborations between mathematics educators, engineering departments, and local community organisations (Frutos-Bernal et al., 2024; Dias Rasteiro, D. L., et al., 2024). The benefits of SL in mathematical contexts are multi-layered. First, it enables students to develop key mathematical competencies, such as reasoning, modelling, problem-solving, and statistical interpretation, in authentic contexts (Niss & Højgaard, 2019; Rasteiro et al., 2023). Second, it fosters reflective thinking by requiring students to evaluate how their academic expertise can contribute meaningfully to communities, particularly in contexts marked by inequality, vulnerability, or lack of access to data-driven decision-making. Third, SL enhances motivation and engagement by allowing students to experience mathematics as a dynamic tool for diagnosing and acting upon real challenges (Muñoz-Medina et al., 2021).

Building on these principles, the present paper explores the integration of SL into the Statistical Methods curricular unit taught at the Coimbra Institute of Engineering (ISEC), Polytechnic University of Coimbra. During the 2024/2025 academic year, thirteen students engaged in this pedagogical experience, organised into three collaborative groups (two groups of four and one group of five). Guided by the SL framework, they were challenged to apply statistical concepts in response to real-world social and professional questions. The projects developed, analysing ethical consumption patterns, evaluating community service quality, and mapping financial literacy behaviours, serve as case studies for examining how mathematical competencies can be activated in contexts that also promote civic engagement and social reflection. Notably, one of the participating students had been diagnosed with Asperger syndrome, and his engagement in the group work and SL activities offers valuable insights into the inclusive potential of this pedagogical approach, which will be further discussed.

The structure of the paper is as follows: the present section provides a brief introduction to the use of Service-Learning methodology in educational contexts, highlighting its relevance for the development of mathematical and transversal competencies. In the following section the methodological framework, describing the participants involved in the study and detailing the SL procedures applied throughout the semester, is described. This is followed by the results and discussion section, where the main findings of the thirteen student-led projects are presented and critically analysed. The paper concludes with a synthesis of key outcomes and reflections, including considerations on inclusivity and directions for future research.

Methodology

This study was implemented within the framework of the curricular units Statistical Methods (from the Mechanical Engineering degree) and Linear Algebra (from the Electromechanical Engineering degree) at the Coimbra Institute of Engineering (ISEC), during the academic year 2024/2025. At the beginning of the semester, students were introduced to the concept of Service-Learning (SL) through a structured presentation prepared by the course coordinator, which outlined the pedagogical principles, societal relevance, and expected learning outcomes of the methodology. Following this session, students who voluntarily wished to engage in this approach registered via a Google Form. A total of 13 students applied: 7 from Statistical Methods and 6 from Linear Algebra.

In line with recognised SL frameworks (Rasteiro, D. M. L. D. et al, 2024; Jacoby, 2015; Queiruga-Dios et al., 2023), the experience followed seven structured phases: observation, preparation, action, reflection, data registration, evaluation, and recognition. During the observation phase, a collaborative brainstorming session allowed students to identify relevant social topics and possible contexts for intervention. The themes proposed reflected students' interests and were grounded in real-world challenges. All selected projects involved the design of a questionnaire, data collection in the field, and statistical data analysis. Regular meetings with the authors were held throughout the

semester to support survey development, validate research questions, and guide the application of analytical methods.
Figure 1 illustrates the interdisciplinary process conducted between Linear Algebra and Statistical Methods students. The flowchart summarises the sequence of activities, from problem identification to group analysis, presentation, and reflection. This visual representation reflects the cumulative and interconnected nature of the learning process.

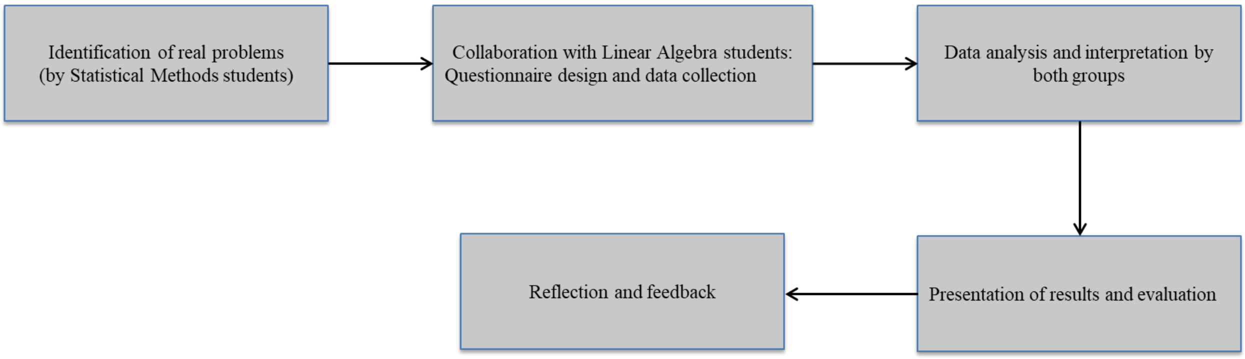

Figure 1. Structured workflow of the interdisciplinary Service-Learning activity, combining statistical modelling and linear algebra to address real-world problems.

Students were organised into three working groups (two groups of four students and one of five), mixing participants from the two-degree programmes to foster interdisciplinary collaboration. Importantly, one student participating in the experience had a diagnosis of Asperger syndrome.
One of the students participating in this Service-Learning experience was diagnosed with Asperger syndrome, a condition within the autism spectrum characterised by difficulties in social communication, sensory sensitivity, and the need for structured environments. In traditional mathematical learning settings, students with Asperger syndrome may face challenges related to group dynamics, ambiguity in instructions, or limited flexibility in assessment formats (Baron-Cohen et al., 2009; Hedley et al., 2018). To address these needs, differentiated pedagogical strategies were adopted, including the clear definition of roles within the group, the use of asynchronous tools for reflection, and increased instructor mediation in moments of uncertainty. The structured and purpose-driven nature of SL, especially when tied to real societal needs, offered a more predictable and meaningful learning context. Inclusive support strategies were adopted, especially in managing group dynamics and promoting meaningful engagement in all SL phases. His successful participation added a valuable perspective on the potential of SL to support diverse learners. Moreover, authentic engagement and the possibility to contribute with analytical skills to a tangible community-oriented outcome promoted a sense of competence and inclusion for this student, suggesting that SL can be a powerful pedagogical strategy to support neurodivergent learners within Engineering Education.
The projects developed were as follows:
Group 1 investigated ethical consumption and sustainability in the fashion industry, analysing student behaviours regarding fast fashion platforms such as Shein and Temu. Using a structured questionnaire, they examined the influence of gender, price, and social media on purchase decisions and environmental awareness.

Group 2 partnered with the Angolan company Dcanjamba to conduct a customer satisfaction and service effectiveness study. Their survey assessed perceived service quality, clarity of communication, responsiveness, and customer loyalty indicators. The project led to actionable recommendations to improve operations.

Group 3 explored financial literacy and investment behaviours among students at ISEC. Their data collection focused on investment habits, risk tolerance, financial planning, and educational needs. Statistical correlations were used to map relationships between behaviour and financial preparedness.

All groups used Microsoft Excel and/or IBM SPSS (version 27) for data analysis, supported by continuous authors' feedback. Deliverables included a written group report and an oral presentation of findings to peers. These were complemented by structured reflection exercises throughout the semester, allowing students to critically evaluate their experience and identify areas for personal and professional growth.

Assessment of student learning was conducted through a combination of formative and summative instruments. Formative assessment occurred during regular guidance meetings and feedback on interim results. Summatively, students completed a pre- and post-questionnaire designed to evaluate perceived competence development across areas such as problem-solving, teamwork, communication, civic engagement, and critical thinking. The results of these questionnaires are analysed in the following section.

From an ethical perspective, all student groups were instructed on good practices for data collection, including ensuring voluntary participation, anonymisation of responses, and transparency about the educational nature of the activity. Participants in all surveys were informed of these terms, and all data were processed following institutional guidelines for student-led research. It is important to state that the SL activities were aligned with the intended learning outcomes of the respective curricular units. In Statistical Methods, these included competencies in sampling, descriptive statistics, inferential techniques, and data interpretation. In Algebra, logical reasoning and the handling of data structures were reinforced through applied tasks. This constructive alignment ensured students developed content mastery and transversal skills within a coherent pedagogical framework.

Finally, students who did not opt into the SL track followed an alternative learning path based on more traditional problem-solving activities. This parallel structure ensured flexibility while maintaining equal opportunities for curriculum achievement.

Results and Discussion

The three Service-Learning (SL) projects developed by the student groups provided rich empirical data and insight into the transformative learning processes that occur when academic knowledge is meaningfully connected to societal concerns. Each project addressed a distinct theme, yet all shared common features: autonomous questionnaire design, purposeful data collection, and application of statistical analysis methods aligned with the learning outcomes of the Statistical Methods curricular unit. This section presents a synthesis of the main findings and discusses their implications for engineering education and the development of mathematical competencies.

3.1 Ethical Consumption and Sustainability Awareness

The first group focused on the topic of fast fashion, particularly the purchasing behaviours associated with online platforms such as Shein and Temu. The students explored correlations between socio-demographic variables (e.g., age, gender, income) and attitudes towards sustainability, price sensitivity, and social media influence. Their findings revealed a gender effect: female respondents were more likely to express concern about environmental impacts while simultaneously reporting higher purchase frequency from low-cost platforms. This duality led to a discussion of the attitude-behaviour gap, which students interpreted using statistical correlations and thematic clustering.

Pedagogically, the project enabled the development of competencies in survey design, data cleaning, cross-tabulation, and chi-square testing. More importantly, it prompted students to reflect critically on the ethical dimensions of consumption. This reflection, rarely triggered in traditional mathematical tasks, was later referenced by students in their post-questionnaires as one of the most "eye-opening" aspects of the learning experience.

3.2 Community-Oriented Data Analytics for Service Quality

The second group partnered with Dcanjamba, an Angolan company, which assisted students' online presentation, offering services in the tourism and business consultancy sector. Students designed a comprehensive questionnaire to evaluate customer satisfaction, perceived service quality, and competitive positioning. The final data set included over 100 valid responses and was analysed using descriptive statistics, cross-tabulations, and measures of central tendency. In their report, the group highlighted that perceived clarity of service conditions and response time were the most significant predictors of overall satisfaction. Results also indicated a strong likelihood of customer recommendation, although communication issues were noted in open-ended responses.

This project was particularly effective in demonstrating the value of data-driven decision-making in a real organisational context. Students engaged deeply with practical challenges such as designing unbiased response options, dealing with ambiguous qualitative data, and interpreting results for non-technical audiences. Their presentation to the company included actionable suggestions, demonstrating that SL can function not only as a learning method but also as a form of community consultancy grounded in rigorous analysis.

3.3 Financial Literacy and Investment Behaviour among University Students

The third project addressed an internal community challenge: the lack of financial planning and investment awareness among university students. The group conducted a detailed survey involving over 100 participants, focusing on areas such as risk perception, investment channels, saving habits, and planning behaviours. Through correlation analysis (e.g., Spearman's rho), they observed a statistically significant relationship between financial planning and the presence of an emergency savings fund, as well as between risk tolerance and investment value.

The group concluded that although many students expressed interest in investing, a lack of structured knowledge and confidence hindered them from doing so. These findings support ongoing calls for the integration of financial education into engineering curricula, as advocated in recent European Commission recommendations. The SL process helped students articulate real statistical questions, draw valid conclusions from real data, and present them with clarity to an academic audience, thereby reinforcing mathematical competencies while addressing a problem of social relevance.

3.4 Cross-Project Reflections and Competency Development

Across all three projects, students consistently reported growth in transversal competencies such as teamwork, communication, initiative, and social responsibility. The pre- and post-questionnaire included 21 items, adapted from Coverdell's World Wise Schools rubric (Peace Corps, 1998), assessing dimensions such as teamwork, critical thinking, problem-solving, empathy, and the ability to connect theory and practice. A complete list of the items is provided in Table 1 (Annexe 1). From a mathematical point of view, students exercised statistical reasoning, modelling, and representation using tools like Excel and SPSS. An analysis of the pre- and post-questionnaire scores revealed perceived gains in a substantial number of competencies as may be observed in Figure 2. Specifically, improvements were noted in areas such as collaboration and teamwork (Q2), leadership skills (Q3), and civic responsibility (Q6), which are central to the social dimension of Service-Learning (SL). Additionally, students reported higher levels of engagement with the community (Q7), an increased sense of agency in effecting change (Q8), and a stronger ability to apply theoretical knowledge to real-world contexts (Q9, Q18). Gains were also observed in problem analysis and critical thinking (Q10), as well as in self-confidence (Q12) and conflict resolution (Q13), indicating growth in interpersonal and reflective skills. Further areas of perceived development included the assumption of personal responsibility (Q14), acquisition of practical workplace skills (Q15), experiential learning capacity (Q16), and empathy and sensitivity to others (Q20). While these changes were not statistically significant due to the small sample size, their breadth and alignment with the intended learning outcomes of the SL experience suggest a positive impact on both cognitive and socio-emotional dimensions of student development.

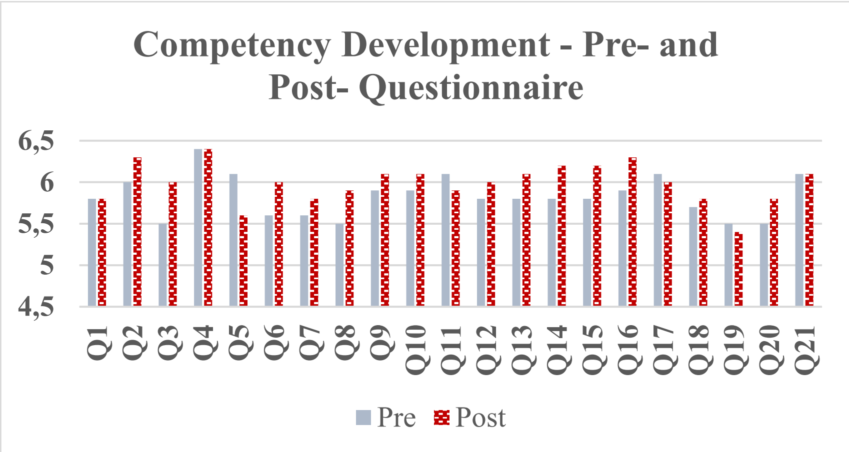

Figure 2. Average Likert scores (1-7) for selected competencies in the pre- and post-questionnaires.

While visual differences are observable, these results reflect the students' self-assessment of competencies before and after the Service-Learning experience. Further analysis is required to determine the statistical significance of perceived changes.

To complement the visual analysis of the pre- and post-questionnaire responses, paired statistical tests were conducted for the selected competencies (Q1, Q2, Q3, Q6, Q16, and Q18). Both paired t-tests and Wilcoxon signed-rank tests were applied to assess whether the observed score variations were statistically significant. Results indicate that, although there were visible increases in mean scores for several items, particularly those related to teamwork, leadership, and learn from experience, none of the changes reached statistical significance ($p > 0.05$). This outcome is consistent with the relatively small sample size ($n = 10$ out of 13 students who participated in the study) and the moderate scale of change reported by students. Nonetheless, the directionality of the results, aligned with student reflections and group outputs, supports the pedagogical relevance of SL for fostering transversal competencies, even if these gains were not statistically confirmed within this sample. These gains tend to validate the role of SL in bridging the technical and human dimensions of engineering education. Particularly, the inclusive design of the activities also supported differentiated learning pathways: the student with Asperger syndrome reported benefiting from the structured phases, the clear goals, and the predictability of SL work, suggesting that this methodology may offer a more accessible environment for neurodivergent learners.

3.5 Interdisciplinary Collaboration with Linear Algebra Students

Aside from the primary SL projects, an interdisciplinary framework was established among students of the Linear Algebra (Electromechanical Engineering) and the Statistical Methods (Mechanical Engineering). Students in Linear Algebra were engaged in a voluntary and extracurricular activity centred on enabling data collection and organisation of mathematics for SL projects. Even though the Linear Algebra syllabus is not directly related to statistical data analysis, such as in the Statistical Methods course, these students participated enthusiastically in the interdisciplinary practice, which proved useful in the transversal skills reinforcement, like communication, initiative, and interpretation of data. All students received a grade from 0 to 4 depending on the quality of their contribution, the algebraic content of their statistical results, and the quality of their analysis. The median score was 3.04, and the first and third quartiles were 2.58 and 3.5. Figure 3 displays the grade distribution.

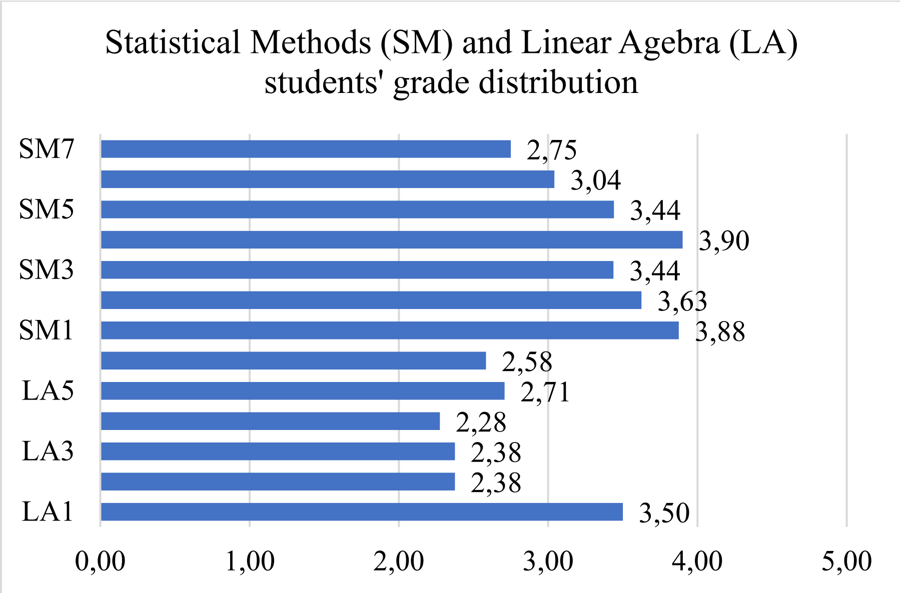

Figure 3. Grade distribution (0-4 scale) of students from Statistical Methods and Linear Algebra involved in the interdisciplinary Service-Learning activity

Conclusions:

This study explored the integration of Service-Learning (SL) into the teaching of Statistical Methods and Linear Algebra within engineering education, as part of a broader effort to link mathematical content with socially relevant challenges. Through three student-led projects, the initiative demonstrated that SL is not only effective in consolidating curricular content, such as survey design, data analysis, and statistical inference, but also in fostering critical transversal competencies, including teamwork, leadership, social awareness, and the ability to connect theory with practice.

An analysis of the pre- and post-questionnaire scores revealed perceived gains in a substantial number of competencies. While these changes were not statistically significant due to the small sample size, their breadth and alignment with the intended learning outcomes of the SL experience suggest a positive impact on both cognitive and socio-emotional dimensions of student development. These findings align with previous literature supporting the use of SL to enhance motivation and learning in STEM contexts (Jacoby, 2015; Queiruga-Dios et al., 2023) and reinforce the value of engaging students with real-world issues as a means of deepening both conceptual understanding and civic consciousness.

Beyond cognitive gains, this experience highlighted the inclusive potential of SL in diverse classroom settings. The successful engagement of a student with Asperger syndrome revealed that the structured, socially relevant, and purpose-driven nature of SL can provide an enabling environment for neurodiverse learners. This points to the need for further investigation into how active methodologies, particularly those involving community engagement, can support differentiated learning and foster a sense of belonging in engineering education.

The integration of Linear Algebra students from the Electromechanical Engineering course into the service-learning project, albeit voluntary and extracurricular, was an enriching complement. Collaboration with Statistical Methods students from the Mechanical Engineering course provided all students with an interdisciplinary environment in which mathematical reasoning, data analysis and teamwork were developed together. This experience validates the great potential in developing different programmatic contents through teamwork in problem-solving activities. The positive results suggest this complementary involvement can be pedagogically rewarding and academically enriching.
Finally, the projects conducted this year, focused on ethical consumption, service quality, and financial literacy, reflected students' ability to identify, formulate, and analyse problems of societal relevance using mathematical tools. They served not only as pedagogical experiments but also as authentic contributions to the community. Future work will focus on refining assessment tools, exploring long-term impacts on student learning trajectories, and expanding interdisciplinary SL opportunities across engineering curricula.

Acknowledgements

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Annexe 1:

Table 1. Competency Development Items Used in Pre- and Post-Questionnaire

Adapted from Coverdell's World Wise Schools rubric (Peace Corps, 1998)

Code	Item
Q1	Experience personal growth
Q2	Ability to work well with others
Q3	Improve my leadership skills
Q4	Improve my communication skills
Q5	Gain a greater understanding of cultural and racial differences
Q6	Improve my social responsibility and citizenship skills
Q7	Increase community involvement

Q8	Ability to influence the community
Q9	Apply information learned in the classroom to real-life scenarios
Q10	Enhance problem analysis and critical thinking
Q11	Apply problem-solving techniques
Q12	Build my self-confidence
Q13	Improve my conflict resolution skills
Q14	Take personal responsibility
Q15	Learn practical skills relevant to the workplace
Q16	Learn from experience
Q17	Develop organisational skills
Q18	Connect theory with practice
Q19	Establish meaningful interpersonal relationships
Q20	Develop empathy and sensitivity to others' situations
Q21	Demonstrate that I am someone who can be trusted

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Rethinking Mathematical Bridging Courses

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Abstract

Mathematical bridging courses offered prior to the beginning of the first semester are often designed in Germany with the goal being to get students all at the same level of knowledge, as a way for students to fill the gaps in their school mathematics. This lofty goal has been shown to be highly unrealistic. In general, it seems that the students who need these courses the least are more likely to attend in comparison to those who need these courses. The gap between students' abilities seems to become larger, which shows that the needs of diverse groups are not suitably addressed with this format.
In this contribution, we look at some of the reasons why bridging courses fail to alleviate the problems that they were designed to create. Some different approaches to redesigning bridging courses are given along with some observations concerning their effectiveness.

Keywords: Bridging courses, activating students, individualisation

Introduction

Offering mathematical bridging courses is a way to prepare students for their mathematics courses at the college level. In the German system, this type of course, aimed at beginning engineering students, is often a compressed review of mathematics taught at school. One of the main goals is to fill in the gaps in order to attain some sort of homogeneity in terms of the abilities of the students so that instructors of college courses can rely on students having the prerequisite knowledge and skills. [see e.g. 1, 2] This was also the case, at least part of the time, in the different universities where the author has been working over the past 20 years. Unfortunately, the classroom situation in first semester mathematics courses shows that this goal is often unmet. It seems as if the students who were good in mathematics in school gain the most from the mathematical bridging course, which in turn only widens the gap between student abilities.

In this contribution, some observations made at the Technische Universität Berlin (TUB), the Technische Hochschule Ostwestfalen-Lippe (TH OWL), and the Technische Hochschule Ingolstadt (THI) will be examined to make some suggestions for revamping these courses.

Mathematical Bridging Courses

Clearly not all bridging courses are the same. Most of the focus in this article will be on the in-person courses that review school mathematics with some insights concerning online refresher courses. The main evaluation of student success in relation to bridging courses was conducted at the TUB in the year 2009 [3, pp. 15-20, 152-157]. During that time, there was an in-person presence bridging course, an online bridging course, and a so-called Early Bird course. The latter course invited talented students to complete the first semester analysis and linear algebra courses before the first semester began. Thus, any student included in the survey who attended Early Bird had either not passed the examination in linear algebra prior to beginning the first semester or they were unsatisfied with their grade. The comparison of this course to the others may be unfair because the goals are different. It is included for reflection purposes.

The scope of each course was as follows. The introductory course was a 2-week in-person course. Students went to a daily lecture and a daily tutorial with the content matching a subset of the online bridging course. The online bridging course was quite lengthy. There was no recommended timeline for this course. Early Bird was a two-month course with a daily lecture and exercise session.

Students were asked to indicate which type of course(s) they attended: in-person, online, Early Bird, none. Of the 1361 students responding, the percentage of students attending a particular form were nearly the same for men (1087 participants) and women (274 participants). Taken together, 42% of the students attended the bridging course in-person, 46% online, 6% Early Bird, 35% none. We see that some students used more than one offering simultaneously. With these different formats, we were able to reach nearly 2/3 of our students.

The same students were asked to estimate how much of the course they had attended: up to ¼, more than ¼ up to ½, more than ½ up to ¾, and more than ¾. Again, the responses of men and of women were very similar. Whereas 47% of the students in the introductory course stayed past the ¾ mark, 62% of the Early Bird participants remained as long. Since the online course is rather lengthy, only 12% claimed to have finished at least ¾ of the course. Whether the students were demotivated by the length of the online course is not clear from the data. All in all, we can say that the average student attending the bridging course completed about 63% of the course. For the online course this figure was about 40% and for Early Bird 65%. Thus, both in-person courses were similar in terms of the amount attended by the participants. The amount of the bridging course attended was not dependent on whether the student had taken an advanced course in mathematics during high school or not: bridging course (63% vs. 65%), online course (42% vs. 41%). Not surprisingly this was not the case for Early Bird (71% vs. 58%).

In an attempt to understand the effect bridging courses play, we will compare the results of students attending the first-semester course "Linear Algebra for Engineers" at the TUB in the winter semester of 2009/2010. Although the results are from 15 years ago, the message is still valid.

A key to success in this course was, as expected, previous experience at school. The following table shows the percentage of students successfully completing the course based on participation in an advanced course in mathematics at the school level and the amount of the bridging course attended.

School math	51-100%	1-50%	0% of bridging course
Advanced course	59%	53%	42%
General math only	41%	33%	27%

Another factor that influenced performance was whether a student liked mathematics. The next table illustrates the percentage of students successfully completing the course taking into account whether the student likes mathematics and if a bridging course was attended or not.

Attitude Towards Mathematics	Bridging Course	No Bridging Course
Like mathematics	57%	42%
Indifferent to mathematics	33%	31%
Do not like mathematics	21%	20%

Students who said they really enjoy mathematics and attended a preparatory course successfully completed the course at a 15% higher rate than those not attending. Students who said they were indifferent to or did not enjoy mathematics successfully completed the course at about the same rate regardless of whether they took part in a bridging course.

Relating performance and self-estimated abilities in mathematics based on participation in a bridging course shows a bigger differentiation than liking mathematics. The following table shows the success rates in the course.

Estimated Abilities in Math	Bridging Course	No Bridging Course
Above average	63%	52%
Average	41%	29%
Below average	20%	21%

These findings raise the question of whether bridging courses are really the appropriate way to prepare students who estimate their abilities as below average and/or do not like mathematics for their college-level mathematics courses.

At the TH OWL's Höxter campus we observed the bridging course in 2015. It was a two-week course offered in the morning in-person. Notes were taken as to the interactions between the instructor, who was a retired mathematics teacher from a local high school, the tutor, and the participants. Due to the small sample size of 24, the results are not representative. The observations are thus summarized qualitatively. The students who demonstrated more knowledge tended to continue until the end. The instructor interacted with these students considerably more frequently than with the students who demonstrated that they did not understand. Those who did not understand for more than one day tended to not return. This experience reinforced some of the findings at the TUB. It was clear that a bridging course needs to be more individualized to meet the needs of the actual target audience.

The THI also offers a review of school mathematics in its two-week bridging course before the first semester of studies. The experiences of the instructor parallels those recorded above. There is low attendance in terms of the number of incoming students in technical disciplines needing mathematics. Traditionally, about 70%-80% of the students enrolled in the in-person course remain at the end of the two weeks. Some of the main reasons for students not attending the bridging course were that students were unaware of the course, students did not feel the contents were important or relevant for their studies, and/or students were physically unable to attend.

The project THI-Success^{AI} (funded by the foundation "Stiftung Innovation in der Hochschullehre", project identification FBM2020-EA-1690-07540) aims to combine AI with the learning process. Two of the main features of this Moodle-based platform, which was conceived for supporting students in the self-learning phase, will be a recommendation system, which puts together individual learning paths for students based on their previous history, and a learning companion, which allows students to network with each other or a peer mentor to discuss a specific learning activity. These features are expected to be helpful for students to learn and discuss mathematics (online) at their own level. Course contents have been created for two semesters in the areas of mathematics, statistics, programming and electrical engineering.

This platform is currently being extended to offer a different kind of bridging course concurrent with the mathematics taught in the first semester (here, calculus). The idea is to distribute the topics of school mathematics over the course of the semester with the featured skills of the week being used in the next lecture. This shows the use of the skill in context, adding to the feeling of usefulness of the skill. The reminder of the skill directly before its use theoretically enhances the assimilation process.

The method used for the integration of the bridging course is called JITR (Just In Time Review) due to certain similarities with JITT (Just In Time Teaching). Before the next class, the student can take a Moodle quiz having to do with a skill from school mathematics, such as how to form the equation of a line in preparation for the tangent to the graph of a function at a point. The student receives a grade for the quiz and can review the school topic if necessary. In class, a method for finding the tangent line is presented. The students have already refreshed a necessary component and see their efforts being rewarded during the lecture. Furthermore, the fact that there are fewer questions regarding school mathematics during the lecture, there is more time to spend on the topic at hand.

Two groups of students have been considered: Winter Semester 2023/24 (59 participants) and Winter Semester 24/25 (23 participants), which we will abbreviate as WS 23 and WS 24. Of these students, 35 and 21, respectively, completed identical entry tests dealing with topics from school mathematics. The average student in the earlier year achieved 56% of the points compared to 66% in the latter year.

Below is a list of school topics refreshed needed for the college level topic(s) in the same line. Of the 21 total participants in WS 24, the number of users and the average number of views per student is given for each school topic. Unfortunately, the data from WS 23 is difficult to interpret because the quizzes were contained in two different places, and it was not possible to follow overlap in usage. It can be said that the usage drastically went down at the end of the semester in both years.

School Topic [Participants/Avg. Views]	Calculus Topic
distributive laws [14/6.7]	mathematical induction
factoring polynomials [12/4.5]	mathematical induction
fractions [10/3.7]	number sets, complex numbers
sine and cosine [15/7.1]	complex numbers, polar representation
functional notation [14/4.8]	functions, difference quotient
laws of exponents [12/4.7]	Sequences
roots [8/2.6]	Series
logarithms [6/4]	Continuity
equation of a line [6/5]	Tangent
area [3/3.2]	Integration

Whereas the students felt comfortable with topics such as roots and area, where the average number of views per person was 2.6 and. 3.2 respectively, students demonstrated less confidence in topics such as sine and cosine and distributive laws (views per person 7.1 and 6.7 respectively).

In WS 23, 45% of the students did some portion of the quizzes pertaining to school mathematics. This percentage rose to 78% in the next year (WS 24), which can be explained by the fact that time was given at the end of the exercise session for students to begin taking the quiz(zes) during the earlier part of the semester. Unfortunately, the portion of students taking the quizzes drastically decreased for the last several chapters in both years.

Note that the number of students still in attendance at the end of WS 23 was 31 of the original 59 participants (53%). In WS 24, 17 out of 23 (79%) of the students attended class until the end. Integrating the quizzes into the exercise session at the beginning of the semester seems to have been beneficial for class participation throughout the semester.

Clearly, we do not expect the individual examination results to correlate heavily with the school quizzes. For the sake of completion, we note some of the overall results. In WS 23 approximately 51% (WS 24, 78%) of the students completed the examination at the end of the semester, which was similar to class participation at the end of the semester. The average grades of those taking the examination were substantially different (C- in WS 23, B- in WS 24), which was expected since the average entrance test scores were somewhat different (56% vs. 66%). The percentage of students successfully passing the course was 41% in WS 23 compared to 65% in WS 24. The success rates for the exam with respect to those taking the Moodle quizzes for school mathematics were similar: 80% in WS 23 and 83% in WS 24. Thus, students who took the Moodle quizzes were much more likely to continue participating and succeed in the examination. How much of this was due to abilities and motivation is not clear.

Conclusion

The method of Just In Time Review (JITR) was presented in this contribution. Just like its counterpart, Just In Time Teaching, there is a saving in time that can be used in other ways during the lecture. We do not expect our best students to profit as much as the average or below average student. Nothing can replace talent and a solid background in school mathematics. Additionally, there is more to the learning process than just resources and methods. The students who dislike mathematics will generally not become fans by taking a few quizzes. As we saw from the study at the TUB, enjoying mathematics and feeling confident are paramount for success at the tertiary level. Thus, JITR cannot be thought of as a one size fits all type of solution for succeeding in mathematics. It does appear to serve its function as a reminder for the best students and as a wake-up call for others. What students do with that information is up to them. Perhaps requiring additional work if a quiz is not passed would be helpful. The main benefit appears to be increased participation in class, which leads to an increased success rate in the examination.

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Cryptography and information security: Sharing knowledge and digital competences with the community

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Abstract

The transfer of knowledge to society has become increasingly relevant within academia, extending beyond traditional spaces such as conferences, scientific publications or patent development. In this context, the delivery of lectures on cryptography and information security or learning methodologies, as a service-learning activity, represents an excellent opportunity for master's degree or PhD students and other researchers to apply their highly specialized knowledge in a more accessible and socially engaged environment and for the students to acquire competencies and skills useful in their future professional or academic work. This study presents an activity aimed at various actors in society, including people who, although they have basic knowledge in the use of digital tools, are not familiar with key aspects of cryptography and computer security or teachers that are not familiar with active educational methodologies, in the case of trainers from different educational levels. This activity is the initial part of a service-learning proposal as the community needs are highlighted after some outreach activities.

Introduction

The Erasmus+ GIRLS project (Generation for Innovation, Resilience, Leadership and Sustainability. The game is on!) is a proposal from the partners to promote several important aspects in Europe such as inclusion and diversity, equality, digital transformation and sustainable development goals. The authors of this project are project's partners. Nowadays, many teachers use innovative active methodologies in their classes to motivate and involve students in their own learning, but sometimes this does not happen in higher education, where it is not so common to change the classical lecture system. The GIRLS project promotes the use of active methodologies in higher education and gets more teachers to use them. Among these methodologies, service-learning (SL) is particularly important. This study details the first stage of this service-based learning. Several professors who are experts in two different fields (cryptography and cybersecurity) and in active methodologies give training sessions to other professors and professionals. The aim is to identify gaps in these fields in order to bring this proposal to master's and doctoral students and get them involved in the training of the community as these topics are highly relevant and important for any group.

On the one hand, the dissemination of knowledge and the effort to foster digital competencies within society have become a critical factor in strengthening the understanding of non-experts in the face of the constantly evolving landscape of cyber threats. In an era where digital interactions underpin a wide range of social functions, the level of comprehension and proficiency in protecting digital assets directly influences the security and well-being of individuals, organizations and the social structure in general. Active methodologies are used in today's university teaching. In cybersecurity and cryptography courses, innovative methodologies are widely accepted by students and teachers, as it encourages attention and competence development (Queiruga-Dios et al., 2022). By equipping individuals (and organizations) with the necessary knowledge and skills, we can collectively achieve a stronger defence against the pervasive and increasingly sophisticated threats that characterize the modern digital age. This study addresses higher education and how service-learning will motivate students in their future careers (Queiruga-Dios et al., 2021; Rasteiro et al., 2024).

Service-learning constitutes a pedagogical approach that strategically interweaves theoretical constructs with practical application. This approach provides students with two opportunities: active participation in service initiatives that directly address community needs and subsequent in-class reflection upon these experiences. The intended outcomes of this reflective process are a more profound assimilation of curricular content and a heightened sense of civic responsibility. Contemporary scholarly research has increasingly substantiated the merits of service-learning within the domain of inclusive education, particularly through the exposure of prospective educators to heterogeneous demographic cohorts within school or community settings (Resch & Schrittesser, 2023).

More specifically, SL projects can originate from diverse organizational entities, encompassing educational institutions at different levels, civic associations, non-governmental organizations, vocational training centers, residential facilities for the elderly, social welfare agencies and groups experiencing vulnerability. The activities undertaken within these projects can be situated across a spectrum of domains, including but not limited to education, healthcare provision, social accompaniment, environmental stewardship, support for the elderly, violence prevention and other areas of social concern. The genesis of SL projects is rooted in a rigorous analysis of the needs prevalent within proximate realities. It is crucial to distinguish SL from both voluntary action and professional internships; rather, it constitutes the execution of a targeted service predicated upon a meticulously identified community need (Rasteiro et al., 2025).

The pedagogical approach of SL has garnered widespread global recognition for its efficacy in cultivating students' civic participation and upholding democratic and pluralistic principles. Furthermore, its significant influence extends to educators and the broader community. Although scholarly inquiry has extensively examined the determinants of initial teacher motivation, a notable lacuna persists in comprehending the factors that perpetuate and sustain teachers' motivation and engagement throughout their professional trajectories. Ultimately, the experiential dimension of community engagement within service-based learning cultivates constructive interpersonal relationships with colleagues and fosters positive connections with communities at local, national, and international levels (Compare et al., 2024; Diez-Ojeda et al., 2025; Rattigan-Rohr & Murphy, 2014).

To determine potential needs within the immediate environment, a process of observation and investigation can be undertaken to discern prevailing conditions within our community. This involves identifying existing civic associations and social entities, understanding their respective scopes of operation and stated purposes and actively seeking to identify challenges or issues they address. Compilations of citizen organizations and entities can serve as valuable resources in this exploratory phase. Furthermore, the examination of previously executed projects can offer insightful ideas for the development of analogous initiatives or the proposition of enhancements to existing endeavors (Angrosino & Rosenberg, 2011; Stukas et al., 1999).

Method of Investigation

As this activity is part of the Erasmus+ GIRLS project, the group of participants in this research came from a group of professors and administrative staff of the Vasco de Quiroga University (Mexico), facilitated by the university's internationalization coordination. Of the 40 participants, approximately half participated in sessions related to cryptography and cybersecurity and the rest attended teacher training sessions on active methodologies and their implementation at different educational levels.

Based on the Kolb’s learning styles model, we proposed the process detailed in Figure 1 (Diez-Ojeda et al., 2025; Gordo-Herrera et al., 2024; Kolb, 1984).

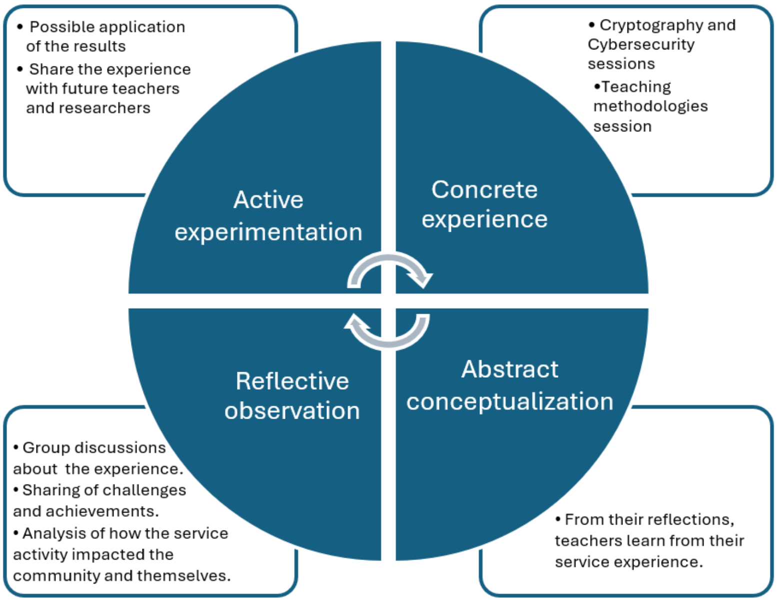

Figure 1. Application of Kolb's cycle to service-learning activities.

The first stage in the Kolb's cycle is the concrete experience, i.e., the observation of community needs in the context of training sessions: Teachers directly observe the needs of the community attending the sessions. This includes:

- Identifying the prior knowledge of the participants in cryptography, cybersecurity or teaching methodologies.
- Detecting doubts, questions and concerns that arise during the sessions.
- Evaluating the level of understanding of the topics covered.
- Observing the participation and interest of the attendees.
- Collecting informal feedback on the expectations and usefulness of the training.
- Analyzing the specific context of the community (e.g., small businesses, associations, public, etc.) to understand their challenges in these areas.

The second stage is the reflective observation, i.e., after the first training sessions, teachers reflect on their observations, thinking about the following questions:

- What topics generated the most interest or difficulty?
- What specific needs were evident in the community?
- How could the content or methodology be adapted to be more effective?
- From the SL perspective: What opportunities exist for master's and doctoral students to contribute meaningfully to meeting these identified needs? What kinds of service projects could be designed?

The third stage is abstract conceptualization, i.e., formulation of ideas and possible SL projects that master's and doctoral students could undertake. This involves:

- Identifying specific areas where students' advanced knowledge could be valuable (e.g., developing cybersecurity tools for small businesses, designing awareness workshops, creating educational resources on active methodologies, etc.).
- Defining possible learning objectives for students within these projects (applying theories, working in teams, understanding the social impact of their discipline, etc.).
- Considering the feasibility and sustainability of service projects.
- Connecting potential projects to the curriculum of master's and doctoral programs.

Finally, the fourth stage is active experimentation, i.e., planning and implementation of SL projects. If faculty decides that the environment is suitable for SL, this stage will involve master's and doctoral students planning and implementing conceptualized service projects. This could include:

- Designing workshops or educational materials specific to the identified needs.
- Developing tools or solutions in the field of cryptography or cybersecurity or teaching methods for the community.
- Providing consulting or advice based on their specialized knowledge.
- Evaluating the impact of their actions on the community.

At this stage, the Kolb's cycle would restart for the students within their SL projects: they would have a new concrete experience (the implementation of the service), they would reflect on it, they would conceptualize learnings and they would carry out new actions.

Discussion and Findings

Reflection at the reflective observation and abstract conceptualization stages will be crucial for faculty to make an informed decision about whether community training sessions are an appropriate setting for SL with graduate students. They will need to consider the clarity and relevance of the needs identified in the community, the real opportunities for students to apply their knowledge in meaningful ways, the possibility of designing SL projects that are both beneficial to the community and valuable to student learning and finally the resources and support needed to carry out successful SL projects.

After the faculty sessions, they discuss about that and decided that this is a suitable environment to implement SL project with students. Thus, for each Kolb stage, these are the findings:

1. Concrete experience (SL implementation) with students could include developing specific and advanced cryptography and cybersecurity training workshops or modules for particular groups in the community (e.g., small businesses, non-governmental organizations, public interested at a deeper level); creating educational materials and teaching resources (guides, tutorials, infographics) on innovative teaching methodologies for local educators; providing cybersecurity consulting sessions or technical advice to community organizations in need; designing and implementing more detailed training needs assessments in specific

locations; or organizing outreach and awareness events on the importance of cybersecurity or on new pedagogical methodologies.

2. Reflective observation (reflecting on the SL experience): After participating in the implementation of their service projects, students will individually and collectively reflect on their experiences: What worked well during implementation, what challenges did they encounter, how did the community react to their efforts, what feedback did they receive, what did they learn about the real needs of the community in these areas, etc.

3. Abstract conceptualization: From their reflections, students will seek to understand the general principles, theories, and academic concepts underlying their service experience. E.g., in cryptography and cybersecurity they could analyse how theoretical concepts of computer security manifest themselves in practical community challenges. They could research on the adoption of security measures in different social and economic contexts; in teaching methodologies they could connect pedagogical theories to the effectiveness of the strategies they implemented in their workshops or materials.

4. Active experimentation: The application of students' knowledge and competencies will affect their future actions and approaches in the following ways: Based on their experience and reflection, they could redesign or refine their interventions to make the SL projects more effective and relevant to the community; apply knowledge gained from their research to the activities with the community; incorporate the service perspective into their professional development; and explore new ways to collaborate with the community.

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Peer learning: Professor and Students unite in a flexible bridging mathematics course

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Abstract

This contribution details the transformation of our six-week intensive mathematics course from a traditional on-campus format, featuring half-day lectures and training sessions with a final written exam, to a more flexible structure incorporating pre-recorded videos and a portfolio of digital tests, with a minimum passing requirement of 60% on all tests. The course is offered both on campus and online, emphasizing independent learning through self-paced study, digital resources, and interactive problem-solving sessions. The teaching was organized and conducted by three learning assistants who provided support and conducted lectures on demand. The course is designed to ensure a smooth transition to engineering mathematics, specifically Calculus 1. We aimed to compare the performance of students in Calculus 1 who completed this redesigned course with those who followed the traditional summer course. This paper presents the results of this comparison, along with insights from semi-structured interviews with both campus and online students to understand their experiences.

Introduction

As part of our efforts to recruit more students to the engineering program at our university, we offer a six-week intensive mathematics course, MA-006, every summer. This course is designed for students with university entrance qualifications who lack mathematics and for students with vocational certificates and it attracts between 130 – 150 students each year. Passing this course is a prerequisite for entering a bachelor's program in engineering. Previously, the course was taught in a traditional manner, with three hours of lectures and three hours of training with student assistants every day. The requirements to pass the course included three written midterm tests (above 40%) and a final written three-hour exam (above 40%). In 2024, I took over the course and implemented substantial changes. I had previously offered the same course online, demonstrating that students from these courses performed as well in engineering mathematics Calculus 1 as regular students with full specialization in mathematics (Brekke, 2014). Experience with flipped classrooms and portfolios with digital tests showed improvement for the bachelor's program in electronics (Brekke, 2016). The setup of the MA-006 course builds upon well-documented results. Ideas from educational research have shaped the design of the course, with a particular focus on how individual tasks are designed using STACK (Kinnear, 2018). *Retrieval practice*, such as the *testing effect*, shows that *"retrieval of information from memory produces better retention than restudying the same information for an* *equivalent amount of time"* (Roediger & Butler, 2011). Therefore, we use *"test-enhanced learning"*, where tests are used as a tool to help students learn during the course (Brame & Biel, 2015).

We have also changed the Calculus 1 course from a traditional final written exam to portfolio assessment using STACK, with very good results. This change came after investigating how the teaching was done and how it was perceived by students (Zakariya et al., 2022). Our experience from these changes includes increased motivation, better self-regulation, and less test anxiety among students. We avoid procrastination, see more cooperation between students, and observe them spending more time on mathematics. We have extended the use of teaching assistants in our teaching. These are experienced students trained in guiding students and in basic pedagogy and didactics.

MA-006 was set up with pre-recorded short videos and a portfolio of three tests in the online assessment system STACK, with a requirement of a minimum of 60% on all tests to pass. The course is designed to provide a smooth transition to Calculus 1. MA-006 was offered both as a campus course and as a fully online course. The campus teaching was organized and conducted by three teaching assistants. The course emphasizes independent learning through self-paced study, digital resources, interactive problem-solving sessions on campus. It has an intensive format that requires students to have strong skills in self-regulation and stress management. Students must be able to organize their own learning, seek help when needed, and find a balance between rapid progression and understanding. Initially, students receive a suggested progress plan with tasks covering the curriculum and learning objectives. For each topic in the book, video lectures are provided and can be used as needed. Daily task reviews are conducted, where teachers go through tasks that students have found challenging. Students are assessed through three partial tests during the course, with each test allowing four attempts. The tests can be taken from any location within the specified time window, with all aids available. To pass the course, students must achieve at least 60% correct on each partial test.

Here, we Morten (professor and course responsible), Emma, and Luis (teaching assistants and fellow students) present how students experienced the course. We will show how this group of students performed in Calculus 1 compared to regular students and previous cohorts. Semi-structured interviews were conducted with both campus and online students to understand their experiences.

Method: Semi-structured interviews

To gain a deeper understanding of how students experienced the redesigned version of MA-006, we conducted a series of semi-structured interviews with students who completed the course during the summer of 2024. Our goal was to explore how the course structure, particularly its flexible format, peer-led support system, and digital assessment model was perceived by students, and how these elements influenced their motivation, engagement, and sense of mastery. We chose a qualitative approach, as we were not only interested in measuring outcomes but also in understanding why and how the course worked or didn't for individual students. The interviews followed a semi-structured format, meaning we prepared a set of guiding questions but allowed the conversation to flow naturally. This ensured consistency across interviews while leaving room for individual perspectives, allowing for in-depth responses, follow-up questions, and reflection. The questions were designed to cover four main aspects:

1. **Academic experience:** This included the use of the textbook and video lectures, experiences with problem-solving sessions and peer instruction, and whether students felt ready to begin Calculus 1.
2. **Motivation**: We focused on how students managed their own learning (self-regulation) within a flexible course structure.
3. **Practical aspects**: This covered students' experiences with the STACK testing system, the use of video lectures as a learning tool, and the role of peer learning.
4. **Overall impressions of the course**: This included expectations, satisfaction, and perceived value of the learning experience.

We interviewed both students who attended campus daily and those who completed the course remotely. This distinction allowed us to identify differences in how the learning environment shaped their experiences. The students' backgrounds also varied in terms of previous mathematical education, which offered further insight into how accessible and effective the course was for a diverse group of students. All interviews were transcribed and analysed thematically, focusing on recurring patterns as well as outliers.

Students' perceptions

Several strong themes emerged from the data. Many students emphasized how helpful the teaching assistants were, especially because they were fellow students themselves. As one participant noted, *"It's much less intimidating to ask for help when you're talking to someone who's been in your shoes recently."* Another added, *"The assistants explained things in a way that made it click, and I didn't feel judged for not understanding right away."*

Students also reflected on their use of learning materials and tools. Most relied primarily on the course-provided resources, particularly the textbook and Canvas. Some made occasional use of external AI tools like ChatGPT and Symbolab to clarify difficult concepts. "*I used ChatGPT to explain concepts in simple terms when I got stuck,"* said one student, "*but I didn't trust it to actually do the math for me."*

Regarding the flexible nature of the course, responses varied. Several students thrived in the self-paced format, especially those with strong self-discipline. One student commented, *"It was motivating to follow the progress plan and check off tasks—I liked having control over my own learning."* However, others found it more challenging to stay on track without the structure of daily lectures, especially those studying from home. One online student noted, *"The flexibility made the course possible for me, but I did miss having people around me—it's easy to fall behind when you're alone."* Another student reflected, *"The flexibility was great, but it was also easy to say, 'I'll do it later' and then suddenly you're behind."*

From the mandatory course evaluation, we see in Table 1, that the overall feedback from students was positive. However, many found the pace and the workload overwhelming, as reflected in comments such as: *"A bit too much material for the available time*" and *"Very well-organized program. Although the workload was large and challenging, it was manageable thanks to a good plan and structured content on the Canvas page."*

	Average score (from 1 – 5, were 5 is best)
The work and teaching methods in the course contribute to a good understanding of the subject matter.	4.18
I perceive the course as relevant to my studies.	4.23
Teaching contributes to my academic development.	4.25
The work and teaching methods are varied and appropriate.	3.95
The use of digital tools in teaching contributes to effective learning.	4.29
Academic collaboration among students (in groups/study circles) contributes to a positive learning environment.	4.24
Academic collaboration between students and teaching assistants contributes to a positive learning environment.	4.27

Table 1 Student's feedback

Students' performance:

To determine if the changes made to the course had any effect on student performance, anonymous grade data was extracted from the university's database for the fall semesters of 2023 and 2024 for the Calculus 1 course. Students retaking the subject were not included in the analysis, as the focus was on first-time, first-year students.
The students were then categorized into "TRES" (summer course students) and "Ordinary" (regular students) based on their type of enrollment, and the frequency of each grade for the two groups was counted. As shown in Figure 1, this also includes a small number of students registered as "N/A," indicating registration but no participation from these students. While counted as a fail, these students are separated to provide a more realistic picture of the distribution of grades. It is important to note that some students enrolled as "TRES" are exempt from MA-006 due to previous education, creating some discrepancies in the data as they did not participate in the summer course. Although no definite data exists for the number of students this applies to, the difference in the number of students is between 5-6%, which is unlikely to significantly affect the data; average course size indicates this applies to between 5-10 students.

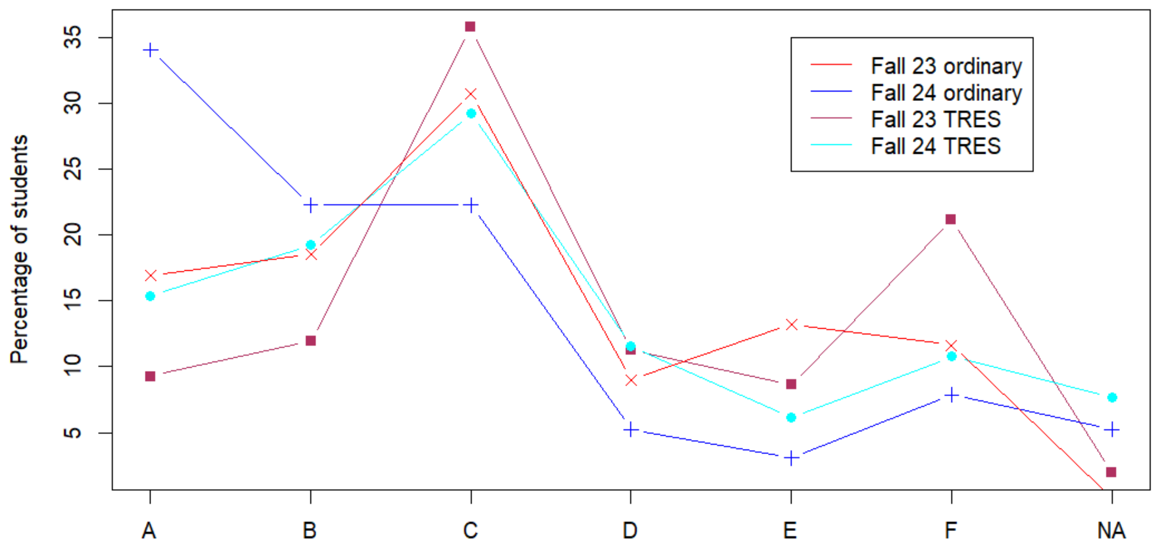

Figure 1. Students' performance data.

For both groups, a general increase in high grades and a reduction in students failing the course can be observed. Most significant is the reduction of fail grades among "TRES" students, falling from 21% to 11%. Overall, these numbers indicate a positive impact of the changes in course structure.

Conclusion and findings

Conclusions so far shows that teaching assistants, were valued for their approachability and understanding. Students relied heavily on course-provided materials and AI tools like ChatGPT for clarifying concepts. Motivation and self-discipline were crucial for success, with campus attendees finding the collaborative environment motivating. Students reported a smooth transition to Calculus 1, with no significant knowledge gaps. The course's intensity and segment-based testing system were particularly appreciated.

The opportunity for multiple attempts on the partial tests has encouraged students to take the tests without aids to see how well they understand the subject. The partial tests provide feedback on what they need to work on and the chance to improve their results immediately. The possibility of multiple attempts has also helped reduce stress levels in an otherwise hectic school environment. However, we see that the great freedom in the course also makes it possible to use AI. ChatGPT answered all the questions on a test and achieved 81% correct answers. In other words, it is possible to pass the course without working and without the necessary competence. However, this will not be possible when students start with university-level mathematics, where tests must be taken under supervision and without aids, something we have been clear about.

To evaluate the extent to which the course has been successful, we must first clarify our objectives. Are we aiming to educate the best students with deep understanding and high competence, or to make our engineering studies accessible to the widest possible range of students? This is a crucial discussion. If our goal is to educate the best students, we may need better control and more secure assessment methods. With the current structure, where students have significant freedom to use various aids such as ChatGPT, it is challenging to determine if the tests truly assess the students' individual knowledge and skills. Students who pass but lack a fundamental understanding of the subject may contribute to lower average grades and higher failure rates in subsequent mathematics courses. To ensure we are educating the best, we must develop assessment methods that minimize the risk of external aids distorting the results. On the other hand, if our overarching goal is to make engineering studies more accessible to a broader range of students, we are on the right track. The flexibility of the course allows more students to participate regardless of geographical location or other commitments. This accessibility enables students to adapt their learning to their own life situations, making education more inclusive and available to all who wish to participate. These two goals are not necessarily mutually exclusive, but they require different strategies and approaches. For future courses, we should consider how to balance the need to ensure high-quality education with the desire to include and support a diverse range of students. Regarding this course, our department's current strategy is to give students in the summer course the freedom to use all aids (to attract more students), rather than being strict. Students who choose to "cheat" their way through the summer course will be exposed in engineering mathematics, where tests must be taken under supervision in our newly built examination facility.

Finally, students have given positive feedback on the support from us teacher assistants. They have trusted that we master the course content, and they have appreciated the low threshold for asking questions.

The findings from these interviews not only inform our understanding of the student experience in MA-006 but also provide a valuable basis for further development of the course. As we prepare to offer MA-006 again next summer, insights from this research will help us refine the course structure, improve student support, and strengthen the overall learning experience.

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Purposeful utilization of digital learning and teaching

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Abstract

In teaching technical subjects, it is important to get students to actively engage and build conceptually accurate basic images related to technical subjects. Pedagogically well-implemented teaching can also reduce dropouts and increase the attractiveness of learning. Various digital methods also increase accessibility, benefitting students with learning difficulties and, helping teachers meet the needs of diverse groups. This article presents the DigiSTEM Erasmus+ project that aimed to increase digital and pedagogical competence of HEI educators and the availability of digital resources in STEM subjects. By increasing such competences of educators, it should give them tools and knowledge to produce and implement digital resources and activities for different personalised learning scenarios in pedagogically meaningful ways. The project developed and disseminated digital learning resources and methodologies, promoting active and self-regulated learning using educational technology and learning analytics.

Introduction

The field of digital education is changing rapidly, which is why it must be continuously studied, and the competences of the educators require continuous development and updating. Also, the Digital Education Action Plan set by the Commission has a priority to make better use of digital technology for teaching and learning. Different educational research studies (Deák & Kumar, 2024; Rahman, 2024) have highlighted that educators have an inadequate competence regarding effective use of digital tools and resources in education from both pedagogical and technological perspectives. Barriers such as lack of confidence, competence, and access to resources continue to hinder the integration of digital methods in education (Santoveña-Casal & Rodríguez López, 2024). Research has shown that students would like to use digital learning possibilities and methods more widely in STEM courses (i.e. Rinneheimo et al., 2018; Kinnari-Korpela & Suhonen, 2017, Santoveña-Casal & Rodríguez López, 2024). However, utilizing educational technology in STEM subjects is very limited and it lags behind the expectations, even though using instructional design that utilizes educational technologies has a great and recognized potential to increase students' motivation, the attractiveness of the subject, and to promote active learning and improve learning outcomes (e.g. Kinnari-Korpela, 2019; Kinnari-Korpela & Suhonen, 2017).

The Erasmus+ funded three-year DigiSTEM project (2021-2024), "Promoting Digital Learning in STEM Subjects," aimed to increase the digital and pedagogical competence of HEI educators and the availability of digital resources in STEM subjects (Science, Technology, Engineering, and Mathematics). By increasing such competences of educators, it should give them tools and knowledge to produce and implement digital resources and activities for different personalised learning scenarios in pedagogically meaningful ways.

This article presents the main outcomes of the DigiSTEM project. The objective of the project was to increase the digital and pedagogical competence of HEI educators and the availability of digital resources in STEM subjects on a large scale to achieve long-lasting effects in everyday activity in European HEIs. The objective was to build HEI educators' competence of such instructional design that improves students' active learning, self-regulated learning and learning engagement with the help of educational technology and learning analytics to provide more effective and personalised support of learners. By increasing such competences of educators, it should give them tools and knowledge to produce and implement digital resources and activities (e.g. learning analytics, digital languages, screencasts, visualisations, and intelligent assessment) for different personalised learning scenarios in pedagogically meaningful ways. This is expected to increase digital competence and skills among the students, to motivate and engage students. The focus was on the combination of pedagogics and technology. In the next section we briefly recap the implementation of the project. We then describe the principal results, advertise the Moodle DigiSTEM platform that is available free on-line and draw some conclusions for the STEM education of engineers.

Implementation

To address the growing need for digital transformation in higher education, the DigiSTEM project equipped educators with the tools and knowledge to design and implement digital learning resources and activities—tailored to diverse and personalized learning scenarios in pedagogically meaningful ways.

The DigiSTEM project consortium consisted of four university partners: Tampere University of Applied Sciences (Finland), Slovak University of Technology in Bratislava (Slovakia), Technical University of Civil Engineering Bucharest (Romania) and Technical University of Madrid (Spain).The project brought together the skills and knowledge, as well as best practices, of several HEI educators, both of STEM subjects but also educators of online pedagogy and combined their expertise into creating the learning resources. On one hand, the STEM educators had the latest knowledge of both the technology that could be introduced in the learning resources and also the needs of their peer educators, and on the other hand, the pedagogical experts were able to offer new, perhaps not previously utilised, viewpoints into online education. While STEM educators contributed subject-specific knowledge and insight into the practical needs of their peers, pedagogical experts introduced innovative approaches to digital teaching and learning, particularly in fostering student engagement and self-regulated learning.

The project implemented the DigiSTEM methodology, which encapsulates innovative pedagogies, best practices, and concrete examples for implementing digital learning and teaching of STEM and similar subjects. Following this methodology, the project planned and produced learning materials for STEM educators and has published two Massive Open Online Courses (MOOCs) on the Moodle platform:

- *Teaching STEM Subjects Digitally*
- *Pedagogy for Teaching STEM Subjects Digitally*

The materials were created by STEM educators from each of the partners, and further modified by experts in accessibility in order to make them more useful for different types of learners. The courses were designed by STEM educators from each partner, which ensured the quality of the created learning resources and their suitability specifically for HEI STEM educators. The material creators also included experts in online pedagogy, who implemented pedagogical learning resources regarding online education and student involvement, with an especial focus on teaching students to take responsibility for their own learning. The learning resources consist of a rich variety of formats - text, videos, images and interactive materials, as well as examples and several practical demonstrations and instructions on how the users may utilise mathematical software in their educational activities, digital assessment tools, and active learning strategies. All the learning resources have been published on-line using a Creative Commons licence, making them freely available via the DigiSTEM platform and DigiCampus Moodle for any STEM teacher interested in the topics. This open-access approach ensures that any interested STEM educator can benefit from the materials, supporting the broader goal of enhancing digital competence across European higher education institutions.

Project outputs

The main concrete outputs were the creation of two online learning courses (MOOCs) on the Moodle platform for STEM educators: Teaching STEM Subjects Digitally and Pedagogy for Teaching STEM Subjects Digitally. The courses have been published on-line on the DigiSTEM Platform and DigiCampus Moodle, using the CC-licence, making them freely available for any STEM teacher interested in the topics. Both courses can be found as follows:

- via the project's website (https://webpages.tuni.fi/stem or digistem.eu)
- on DigiCampus (https://digicampus.fi)

These MOOCs were designed to support the professional development of higher education STEM educators by providing high-quality, research-informed digital learning resources. Below is a brief overview of each course:

- **Teaching STEM Subjects Digitally**

This course is specifically designed to enhance the skills and knowledge of STEM teachers, lecturers, and professors in using digital tools and resources.

The course consists of micro modules about the following topics:

- Using Technology in Teaching and Learning
- Learning Environments for STEM Teaching
- Top Tools for Learning and Teaching
- Videos on Teaching and Learning
- Learning Analytics
- AI Tools
- Find Innovative Tools for Chemistry and Physics: Digital Modeling
- STACK
- Mathematics Software, GeoGebra, LaTeX, Wolfram Mathematica

This course is specifically designed to enhance the skills and knowledge of STEM teachers, lecturers, and professors in using digital tools and resources. Available micro modules bring a comprehensive overview of the best digital and online educational tools for enhanced learning experiences, basic information on how to introduce specific tools in STEM subjects with several examples of the best practice. Instructions on how to design and create their own digital materials as interactive applets, educational videos or simulations are readily available for use. Learning analytics and how they can be used to prevent students from dropping out of courses is presented in separate module. A detailed description of STACK as a system for computer-aided assessment (CAA) in mathematics and how to use it to create efficient and even time-saving exercises is also available. A separate module on AI and its utilisation in university education is also included with links to sites providing current information on this “hot” topic. Teachers of STEM subjects Physics or Chemistry might find here many inspirational educational videos, simulations and other materials. A general overview of some popular free mathematical software packages GeoGebra, Wolfram Mathematica and Latex, pointing to their key features and capabilities is presented with numerous examples.

- **Pedagogy for Teaching STEM Subjects Digitally**

This course is specifically designed to develop the pedagogical competences of teachers/lecturers/professors of STEM subjects in higher education institutions.

The course consists of micro modules about the following topics:

- Best Pedagogical Practices of STEM Teachers
- Active Learning and EduScrum Active Learning Method
- Self-Regulation
- Assessment
- New Trends in Teaching Engineering Mathematics
- The Use of Mathematical Software in Teaching

The course is composed of independent modules, so it is free to choose to complete the topics that are interesting and useful for the individual users´ needs. Various active learning methods are presented with information on how to introduce them within the specific educational scenarios in order to activate students and enhance their motivation and understanding. The EduScrum method promoting cooperation of small groups of 4-5 students is described in detail, as one of the most appreciated and best suited methods from the perspective of the students themselves. Methods to promote students´ self-regulated study approach and learning are described, while several ideas on how to proceed in applying these methods are suggested. Quite a lot of ideas might be found in the module dealing with assessment. Traditional and innovative methods of testing and verifying acquired knowledge are analysed and annotated with valuable advice and notes based on practical experience and basics about new forms of assessments that are flexible, continuous, versatile, and authentic. The module 'New trends in teaching engineering mathematics' brings information on how to fully utilize visualization, illustrations and animations in teaching physics or mathematics, in order to unlock their full potential to improve conceptual understanding of complex topics. Presented examples of materials that might be developed using available digital tools serve as inspiration for active teachers eager to improve their teaching methods. What software should be used, and how to introduce it to their own teaching practice for achieving expected results or to develop interesting engaging learning materials might be found in the section 'Use of mathematical software in teaching'.

Conclusions and Implications for STEM Education

The rapid evolution of digital education necessitates continuous research and the ongoing development of educators' digital and pedagogical competences. The DigiSTEM project addressed this need by bringing together the expertise of STEM educators and specialists in online pedagogy to co-create high-quality, openly accessible learning resources. This interdisciplinary collaboration ensured that the developed materials were both technologically current and pedagogically sound. STEM educators contributed practical insights and subject-specific knowledge, while pedagogical experts introduced innovative approaches to digital teaching and learning, particularly in fostering student engagement, self-regulation, and active learning (Deák & Kumar, 2024; Rahman, 2024; Santoveña-Casal & Rodríguez López, 2024). By enhancing digital competences of educators, the DigiSTEM project gives them tools and knowledge to produce and implement digital resources and activities for different personalized learning scenarios in pedagogically meaningful ways.

Acknowledgments

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Project-Based Learning as an Assessment Tool for Statistical Literacy in Non-Mathematics Undergraduates

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Abstract

This study investigates the value of project-based learning as the main assessment strategy in an introductory statistics course taken by undergraduates whose majors lie outside mathematics. Student feedback gathered across multiple consecutive semesters is analysed to understand perceptions of course design, teaching quality, and the authentic project that asks learners to begin with a prescribed statistical method and work backwards to suitable data and research questions. Results show strong approval of assessment transparency and fairness, instructor expertise, constructive feedback, and the relevance of learning resources. Comments indicate that the reversed design of the project deepens understanding of statistical reasoning and underscores the practical relevance of quantitative inquiry. The analysis also uncovers moderate concern about the dominant theory-and-practice-based approach and the organisational demands of group work. The study concludes that a well-planned, real-life project can effectively improve students' understanding of statistics, especially when students have the chance to practice with simple tasks early on and are supported in working together as a team.

Introduction

Statistical literacy encompasses the ability to comprehend, interpret, and critically assess statistical data and information, as well as to effectively communicate and apply statistical reasoning across diverse contexts (Wallmann (1993)). According to Cal's model (2002), statistical literacy consists of two primary components: knowledge elements and dispositional elements. The knowledge elements encompass literacy skills, statistical and mathematical knowledge, contextual understanding, and critical questioning abilities, whereas the dispositional elements include a critical stance alongside beliefs and attitudes. This conceptual framework informs the design and learning objectives of the undergraduate course "Statistics", which is examined in this study (hereafter referred to as "the course"). The course is structured to provide students with a comprehensive understanding of statistical methods and tools, equipping them with the necessary skills to collect, analyze, and interpret data. The intended learning outcomes of the course include using descriptive and inferential statistics terms and methods correctly; formulating research questions in a justified manner and selecting appropriate data analysis methods accordingly; analyzing statistical data using software, applying descriptive and inferential statistical methods; interpreting and critically evaluating datasets and the results of statistical data analysis; and synthesizing and articulating conclusions based on data analysis results.

The course employs a range of assessment methods, among which Project-Based Learning (PjBL) is serving as one of the primary strategies. PjBL engages students in an extended inquiry process, requiring them to investigate and respond to complex questions, challenges, or problems, culminating in the production of a final artefact (in this case, the project report). Thomas (2000) highlights that PjBL combines academic content with practical applications, promoting both knowledge acquisition and the development of essential soft skills, including collaboration and presentation. Investigating real-world statistical problems reflects the inquiry-driven essence of PjBL, reinforcing its alignment with authentic learning experiences. The defence of the final report further reinforces accountability, critical thinking, and effective communication. Moreover, the PjBL approach aligns with the principles of authentic assessment, a method in which students demonstrate their learning by performing tasks or solving problems that replicate real-world situations. Authentic assessment prioritizes the application of knowledge and skills in meaningful, practical contexts (Herrington and Herrington (2006); Wiggins (1998)) and evaluates competencies through projects, presentations, portfolios, or other artefacts, rather than standardized testing (Lombardi (2008)).

This paper presents the findings of a student feedback survey conducted at the end of each semester, with particular emphasis on the PjBL assessment component. The study outlines the contextual background and data collection procedures, followed by a detailed analysis of the survey responses. The final section provides a discussion of the results and proposes recommendations for future improvements.

Methodology

Context of the study

The course under examination is a first-year undergraduate course classified as a core STEM subject and recognized as a foundational course that supports the learning of discipline-specific subjects. It is a compulsory course for Business students; however, each year it also attracts a diverse group of students from other programmes, such as Business Information Technology, Informatics and Artificial Intelligence, Food and Biotechnology, Public Administration and Governance, etc., who enrol in it as an elective. As such, the student cohort consists predominantly of non-mathematics majors.

The course employs a combination of traditional assessment methods, such as individual open-book Moodle tests and semi-open-book written and Excel-based in-class tests, alongside a PjBL approach. This PjBL component is referred to as the “final exam” of the course, and it contributes approximately 40% to the overall course grade. The final exam consists of a group project (3 - 4 students per group) that includes the following components: formulating research question(s) based on the group’s area of interest; collecting original data or identifying suitable datasets from public sources; analysing the data using prescribed analytical methods; compiling a research report based on the findings; peer-reviewing another group’s research report; evaluating one’s own and others’ contributions, as well as group dynamics, in the form of a reflection; and participating in an open oral defence of their group’s work through a presentation followed by questions from lecturers and peers. The distinctiveness of this particular task lies primarily in the fact that what is prescribed is the statistical method, not the research question or dataset, as is typically the case. Consequently, students are required to approach the task in reverse: they must begin by considering which types of research problems and questions could be addressed using the given method. From there, they need to determine what kind of data would be appropriate for such an analysis and how to either collect or source it. This reverse technique requires students to reconceptualize the entire research process from an unconventional perspective - not starting with a theoretical framework and then proceeding to questions and data collection - and in doing so, deepens their understanding of statistical reasoning.

Data and methods of analysis

The data in this study is derived from student feedback collected at the end of each semester through the university's official Study Information System. Although the feedback survey addresses the course as a whole, including its overall content and assessment methods, and does not specifically target the exam project, the responses nevertheless provide partial insights into the PjBL component. Therefore, the general feedback is analysed and interpreted through the lens of the exam project for this study.

The survey in question is structured using Likert-scale statements ranging from "1 – completely disagree" to "5 – completely agree," and comprises three sections: students' self-assessment (3 items), feedback on the course (4 items), and feedback on the lecturer (4 items). In addition to the quantitative items, students may also provide open-ended comments in each section, offering reflections on both positive aspects and perceived shortcomings of the course and the lecturer. For relevance to the present analysis, only the items concerning the course and the lecturer are included in this paper.

The analysis relies on secondary data, as the dataset is pre-existing and was provided to the author in an aggregated form. Accordingly, the quantitative component consists of average values based on aggregated scores across five consecutive academic years. To enrich the analysis and provide additional context, selected excerpts from students' textual comments[i] (presented in quotation marks) are incorporated alongside the quantitative findings.

Findings and discussion

This section presents the average values of the survey statements' averages, based on the responses from 410 students collected over five consecutive years (see Figure 1). To further illustrate and contextualize these results, selected direct quotes from students are included to highlight underlying patterns and tendencies.

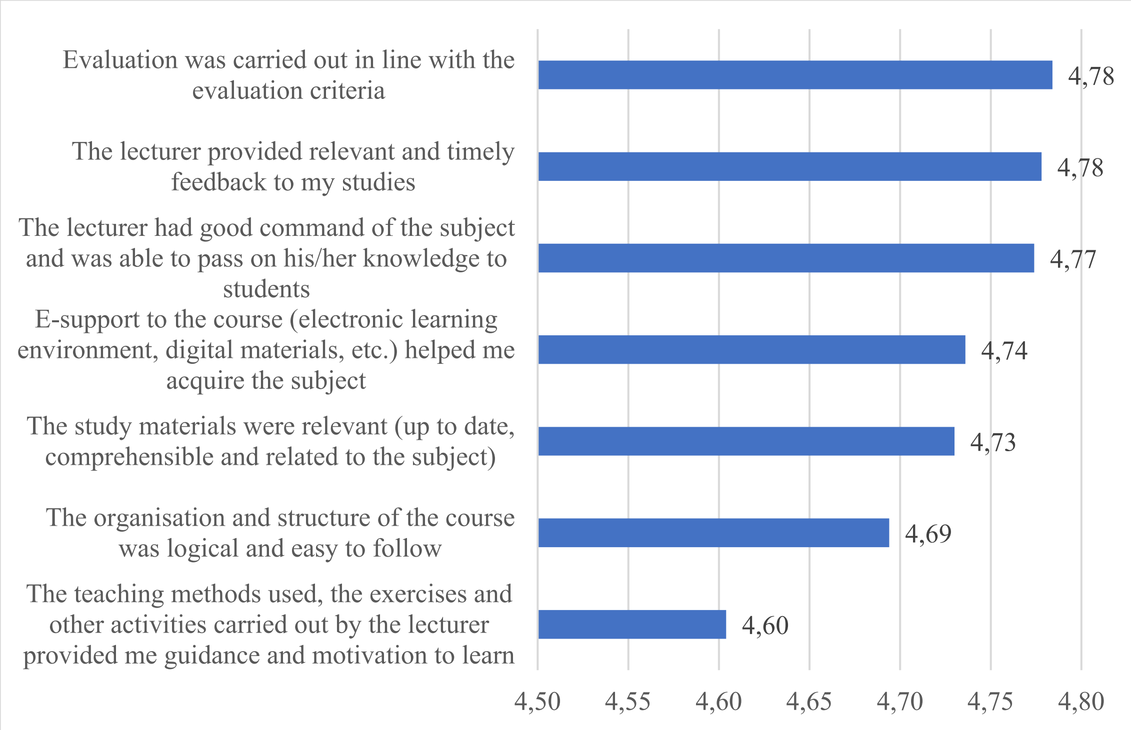

Figure 1. Five-Year Averages of Student Survey Responses

The responses indicate that the evaluation process is clear and understandable for students (over five years, the maximum average score is 4.93, and the minimum 4.61). This may also suggest that the PjBL component of the assessment is consistently well communicated and effectively implemented throughout the years. This interpretation is supported by one student's words: “The assessment system was simple and understandable for everyone”.

Also, the lecturer is seen as an expert in the field and has done a good job teaching the subject (*max* average 4.91, and *min* average 4.41), and giving feedback to the students (*max* average 4.91, and *min* average 4.54). One respondent notes: “The lecturer was a subject matter expert with extensive knowledge in the field, provided relevant examples, and also offered useful recommendations regarding software tools”. This is further supported by another student’s comment: “*[The lecturer]* was very good at explaining different concepts clearly and brought in great real-life examples. It’s wonderful to see a lecturer who can make a difficult subject easier through their teaching.” About giving feedback, one respondent argues: “The feedback was clear, specific, and constructive, with a forward-looking focus on future work and thesis writing”.

The learning materials are consistently rated positively, with students highlighting their relevance (*max* average 4.9, and *min* average 4.57). Similarly, the digital learning materials are considered helpful in supporting subject comprehension (*max* average 4.9, and *min* average 4.54). Student comments reinforce these quantitative findings. As one student notes: “The learning materials were relevant, up-to-date, comprehensible, and aligned with the course content." Another student reflects specifically on the value of the digital resources: "I also borrowed the textbook, but since the digital materials were very comprehensive, it was not necessary to use the textbook to better understand the course content."

A slightly lower average score is given to the organisation and structure of the course (*max* average 4.86, and *min* average 4.54). In the open-ended comments, students express criticism regarding two main aspects: the lack of a final report template and the format of group work. Some students suggest that a ready-made, "fill-in-the-blanks" style template for the final report would be helpful. However, this suggestion is not implemented by the lecturers for several reasons. Firstly, throughout the course, lecturers encourage students to explore relevant academic sources, such as journal articles and final theses, particularly focusing on the methodology and methods sections. This approach aims to deepen students' understanding of discipline-specific literature alongside statistical techniques. Secondly, a predefined template could constrain students within a fixed framework, potentially limiting their creativity in problem-solving and reporting.

The second area of concern relates to the group work format, which is a core component of the assessment, aligned with the PjBL framework. The primary concern relates to the organizational complexity of group work, including challenges in communication, coordination of schedules and pacing, and differing expectations regarding the depth and quality of contributions. Despite these challenges, many students also recognized the value of group work. Over the years, students' comments have consistently emphasized its benefits. For instance, the respondent notes: "The group project was very engaging and taught us best how to actually do statistics!" Another student points out: "The group project and final assignment were very interesting, and the collaboration with course peers was a pleasant change compared to other subjects. The opportunity to continue the group project as the final assignment was a good and motivating aspect." A third student summarizes the experience: "I really liked the structure of the exam [*project*] in this course. Although completing it as a group project is somewhat risky, since much depends on the contribution of other group members, we worked very well together, and we all agreed that doing the exam alone using all the statistical formulas would have been too difficult. However, thanks to the way the exam was designed, we were able to practically revisit everything and revise the most important things we had learned."

The lowest average scores are assigned to the teaching methods employed in the course (*max* average 4.82, and *min* average 4.26). Several factors may explain this pattern. Although part of the assessment integrates PjBL, the overall course structure and the majority of the assessment remain rooted in a traditional lecture-practice format. The course is delivered in a conventional structure where theoretical content is introduced during lectures and reinforced through practice exercises. Moreover, most of the tasks addressed during lectures and practice sessions follow a classical instructional logic: objective → research question → statistical method → analysis → interpretation. In contrast, the final exam, designed as a project, adopts a reversed structure (see discussion above), which may be difficult for students to grasp without prior exposure or practical experience.

Given that this course is classified as a core subject and is intended to provide students with a fundamental theoretical understanding of statistical methods and their underlying mathematical principles, a full transformation into a project-based course is highly complex, if not unfeasible. Therefore, it is unlikely that the teaching format will undergo a major shift shortly, despite student critique. That said, the reversed logic of the exam project, though initially challenging for students, has also received positive feedback. As one student noted: “The exam [*project*] forced us to apply the knowledge we had learned in practice.” Another student added: “The exam [*project*] reflected our understanding and outcomes the most, although the ongoing tests and assessments also contributed significantly.”

Conclusion and educational implications

Five consecutive years of student survey data show consistently high ratings for assessment clarity and fairness, lecturer expertise, feedback quality, and the relevance of learning materials in the course that uses project-based learning as an assessment tool for enhancing statistical literacy. Students’ open-ended comments further suggest that the project’s reversed design logic - beginning with a prescribed statistical method and working backwards to research questions and data - deepens their appreciation of statistical reasoning and its real-world application. Nevertheless, the survey also reveals moderate reservations about the overall teaching methods and the organisational demands of group work. While a full conversion of this core STEM course into a wholly project-based format is impractical, the evidence indicates that targeted adjustments could improve coherence and student experience without sacrificing the rigour required of a foundational statistics module.

Educational recommendations emerge in several areas. Firstly, to scaffold the unfamiliar “method-first” workflow, lecturers could introduce low-stakes micro-projects early in the semester and provide annotated exemplars that illustrate how a chosen analytical technique can guide question formulation and data selection. Secondly, the effectiveness of group work can be improved by offering a brief workshop on team roles and managing conflicts, along with mid-project peer evaluations that help identify unequal participation before the final submission. Additionally, segmenting the final report writing process into distinct stages and submitting sections to lecturers and peers for feedback and presenting preliminary findings already during the course would facilitate a more seamless integration of project-based learning into the course. Implementing these measures could strengthen the alignment between authentic assessment and supportive instruction, foster deeper statistical literacy, and reduce the logistical challenges of collaborative inquiry, all while maintaining the theoretical foundations expected of a core statistics course.

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Project-based learning: Design and development of a service-learning project in Statistics

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Abstract

Service-learning (SL) is a methodology widely used in some universities, whose purpose is for students to acquire knowledge and skills while doing a service to the community.

A SL project was proposed to students of Industrial Engineering to collaborate with students of the degrees in Statistics and Labor Relations of the University of Salamanca. The goal of the activity was to develop teamwork in such a way that each team included students from the different degrees.

The University of Salamanca has institutionalized the SL projects and each year there is a call to all teachers. A group of these teachers who teach statistics, together with the Spanish Network of Entities Against Leukemia and Blood Diseases (AELCLÉS), proposed the project "Learning Statistics in Collaboration with the Community", with the aim of connecting university teaching of statistics with practical applications in the community, highlighting its value in improving the quality of life of hematological patients.

From an interdisciplinary approach, this project addresses challenges such as data cleaning and the generation of statistical reports through descriptive and inferential analysis, using statistical tools such as R, SPSS and Excel.

The SL project highlights the benefits of integrating the Sustainable Development Goals into the teaching of STEM disciplines, demonstrating how mathematics can be transformed into an essential tool for turning data into meaningful knowledge. This approach is particularly relevant in collaboration with the AELCLÉS association, whose work focuses on improving the quality of life of haematology patients and their caregivers. Through statistical analysis, data-driven solutions are provided that enable organizations with a strong social commitment to optimize their support and care programs.

In addition, some of the learning outcomes achieved by students are improved critical thinking and technical competence, along with the social impact of collaboration, creating a bridge between technical knowledge and its application in real life.

Introduction

In recent years, universities have incorporated active learning methodologies that allow students to develop academic and professional competences through direct experience.

Among these methodologies, Service-Learning (SL) has established itself as a pedagogical strategy that integrates academic learning with the provision of community service, fostering civic engagement and social responsibility among students (Jacoby, 2015; McNatt, 2020). This methodology is usually developed following well-defined stages, ranging from the identification of community needs to the evaluation and recognition of results, as Queiruga-Dios et al. (2023) point out.

Several studies have highlighted the benefits of this methodology in the university environment, both at an academic and personal level, favouring the acquisition of technical and transversal competences, the improvement of academic performance and the development of social values (Muñoz-Medina et al., 2021). In the specific case of technical and scientific degrees, ApS allows students to apply the knowledge acquired in real situations, while contributing to the improvement of the quality of life of certain social groups, as occurs in projects linked to the health or social fields. This application in technical and quantitative contexts is still rare, as pointed out by Queiruga-Dios et al. (2021), who estimate that less than 1% of the studies on P2P in Scopus are related to mathematics.

Interdisciplinary work has become an essential element in contemporary university education, especially in social projects, where the complexity of the problems to be addressed requires the collaboration of professionals from different areas of knowledge (Bowland et al., 2015). This approach makes it possible to integrate different methodological and conceptual perspectives, favouring a more complete and enriching analysis of social reality.

This is the context of the project carried out in collaboration with ASCOL (Association against Leukaemia and Blood Diseases) and the Spanish Network of Organisations Against Leukaemia and Blood Diseases (AELCLÉS). The initiative consisted of the cleaning and analysis of a database from surveys of haematological patients and their carers, with the aim of improving the care and support offered to this group. The work has been developed through the participation of students from different university degrees (Industrial Engineering, Statistics and Labour Relations and Human Resources), integrating technical and social skills through an interdisciplinary approach.

The main objective of this paper is to analyse and present a Service-Learning (SL) experience developed with the active participation of students from different degrees, both from engineering and other disciplines. This interdisciplinary approach has enriched the learning process by integrating diverse perspectives and knowledge, fostering meaningful and collaborative learning. In addition, the project has promoted the social commitment of the students by involving them in solving real problems with an impact on local communities or organisations. Through this experience, the aim is not only to strengthen specific competencies in each discipline, but also transversal skills such as teamwork, communication and ethical responsibility, which are essential for training professionals committed to society.

The SL experience involved statistical analysis of real data from haematological patients and carers, promoting technical skills through applied learning. The project also generated useful knowledge to optimise social and healthcare resources for this group, highlighting specific needs and proposing concrete improvements. In this way, it contributed to the advancement of several Sustainable Development Goals (SDGs), in particular SDG 3 (Health and Wellbeing), SDG 4 (Quality Education) and SDG 10 (Reducing Inequalities), aligning academic work with current global challenges.

Method of Investigation

Thirty students from the University of Salamanca, from different degrees: Degree in Industrial Engineering (specialising in Mechanics, Electronics and Automation, and Electricity), Degree in Statistics and Degree in Labour Relations and Human Resources, participated voluntarily. They were divided into 6 groups, with 5 members in each, all of them interdisciplinary.

The activity was carried out during the first four-month period of the academic year 2024-2025 (September 2024-January 2025), in collaboration with ASCOL and AELCLÉS. The entire process was carried out entirely in the facilities of the University of Salamanca, using both conventional classrooms and computer rooms equipped with the necessary resources and statistical software (Excel, SPSS and R).

The project was divided into four phases, described below:

Phase 0: Introduction (September-October 2024)

An initial session was held with all the students and teaching staff involved and the president of ASCOL to explain what the APS project consisted of. During this session, the questionnaire used, the study population, the sample, the variables selected and the database, which was divided into five sub-bases for subsequent analysis, were explained.

Phase 1: Constitution of interdisciplinary working groups (October 2024)

Working teams were organised with students from the different degrees involved, ensuring an interdisciplinary approach. Each group assumed specific responsibilities and was coordinated by a lecturer. Each team was assigned one of the five sub-bases, with the following themes:

- **Group 1:** Diagnosed disease type, previous exposure to radiation or chemicals, family history and involvement in novel therapies.
- **Group 2:** Current treatment status, type of treatment, compliance with treatment received and side effects.
- **Group 3:** Emotional reaction to diagnosis, medical information seeking and use of information resources.
- **Group 4:** Immunological status, history of serious infections, hygiene precautions and emotional health after the illness.

- **Group 5:** Current emotional state and perception of care received during the disease process.
- **Group 6:** Multivariate analysis of all the previous bases, integrating all the variables to identify significant relationships and characteristic profiles within the group of patients.

Phase 2: Debugging, data analysis and reporting (November-December 2024)

The teams conducted an initial debugging process, identifying and correcting errors or inconsistencies in the data. They then organised the sub-bases into formats suitable for statistical analysis and selected the most appropriate techniques for each specific objective. Excel, SPSS and R software were used to develop descriptive, inferential and even multivariate analyses. Each group produced a preliminary report with the main results obtained.

Phase 3: Presentation of results (December 2024-January 2025)

The final reports and conclusions of each group were presented in person to the members of ASCOL, a member of AELCLÉS, in a session held at the University of Salamanca. During this session, the most relevant findings were presented and proposals for improvement aimed at optimising care and support for haematological patients and their carers were formulated.

Phase 4: Evaluation of both teachers, students and ASCOL members involved in the project.

The last phase of the project focused on the evaluation of the experience by the different agents involved: teachers, students and ASCOL, also representing AELCLÉS. As for the teaching staff, a joint assessment was carried out based on the participation and monitoring of pupils throughout the project. The teaching staff carried out a joint assessment based on students' participation and project follow-up.

The students participated voluntarily in a questionnaire designed to gather their perception of the project’s development, the learning acquired and the usefulness of the experience. A service-learning rubric adapted from international models was applied before and after the project to evaluate its formative impact. To this end, two questionnaires were applied: one before the start of the experience and the other at the end of the ApS project, in order to evaluate the formative impact of the intervention. This rubric makes it possible to identify whether the activity carried out corresponds to quality service-learning, based on seven key criteria that integrate the academic context with a commitment to social practice. The results obtained were favourable, highlighting aspects such as practical learning, teamwork and the social value of the project. The responses collected show a high degree of satisfaction with the experience, as well as a perceived improvement in technical skills, communication, critical thinking and sensitivity to health-related social issues.

Finally, the members of ASCOL and AELCLÉS also carried out a qualitative evaluation of the results, focusing on the usefulness of the information generated to support decision-making and to propose improvements in the care and accompaniment of haematological patients and their carers. The association expressly stated the relevance and applicability of the analyses carried out, positively valuing both the rigour of the data and the social sensitivity shown by the participating students.

Results

The Service-Learning project developed made it possible to gather a comprehensive assessment of the experience by the different agents involved: students from the different degree courses taking part, the teaching staff responsible and the president of the ASCOL association. Through satisfaction surveys, self-evaluations and direct observation during the final presentations, it was possible to identify the educational, personal and social scope of this activity. The following are the main evaluations according to the degrees and roles involved.

In the case of the Degree in Industrial Engineering, a total of 13 students took part, some of whom had personal links with the subject addressed, such as the close experience of a friend affected by leukaemia. However, face-to-face participation in the joint activities was lower than expected, a circumstance mainly attributed to the location of the Engineering campus, 72 km away from the faculties of Statistics and Social Sciences, which made face-to-face collaboration difficult despite the students' familiarity with online work. The final presentation was attended by only two students from this degree programme. In spite of the low level of participation, the work was positively evaluated, highlighting the practical application of the statistics subject, which usually does not attract the interest of engineering students. It was pointed out that this type of initiative improves motivation, broadens university perspectives beyond academics and facilitates the acquisition of relevant competences for their professional future.

The nine first-year students of the Degree in Statistics showed notable involvement and a proactive attitude. They took responsibility for data debugging and organisation, demonstrating technical rigour and teamwork skills unusual for their academic level. The experience was highly valued by the students themselves, who were not only confronted with real and complex databases but also became aware of the harshness of the social reality they reflected. They pointed out that, thanks to this project, they were able to carry out real field work, consolidate their knowledge and apply new tools for data cleaning and processing, which enabled them to see at an early stage the practical dimension and social value of their discipline. From the teaching point of view, their maturity, technical rigour and sensitivity to the context were underlined, especially valuing their role as a link between the raw information and its statistical analysis, favouring interdisciplinary cohesion with the students of the other participating degrees.

Twelve students from the Degree in Labour Relations and Human Resources participated. Their involvement was remarkable from the very first moment, showing a very receptive attitude and interest in the project from its initial presentation. Most of them attended the informative meetings and the formal presentation of the project, participating in an active and committed way. Throughout the implementation, they expressed their doubts and concerns on several occasions, always oriented towards rigorously complying with the proposed objectives, which indicates a reflective and responsible attitude towards the proposed challenges. They revealed that they had some initial problems with the distribution and assignment of tasks, which they finally resolved. Their assessment of the experience was very positive, highlighting both the learning acquired and the opportunity to contribute to an initiative with a clear social impact. In this sense, they expressed their wish that their work could, to some extent, have a favourable impact on the quality of life of haematological patients and their carers. The teaching team particularly valued their ability to connect the disciplinary knowledge of the degree with the human and organisational needs of the healthcare context, as well as their sensitivity to the welfare of the people involved.

The 4th-year students of the Statistics degree, the only group from the same program, applied multivariate techniques learned during their studies. Although the data limited the use of some methods, they chose the most suitable ones and complemented previous analyses, bringing a more advanced statistical perspective.

Finally, the president of ASCOL, who also spoke on behalf of AELCLÉS, gave a very positive assessment of the development and results of the project, highlighting the responsible and committed attitude of the students, as well as the usefulness of the reports produced. He stressed that the students were able to treat the data with sensitivity and rigour, aware that behind each figure there were personal stories. Moreover, although some of the results did not match the initial expectations of ASCOL and AELCLÉS, this made it possible to identify aspects that merited further research. Both the association and the network highlighted the importance of this type of collaboration in bringing the university closer to the real needs of the community.

Overall, the ApS experience was valued as highly enriching, allowing the integration of academic learning, social engagement and interdisciplinary work in a real context, aligned with the Sustainable Development Goals and with the values of higher education committed to social transformation.

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Calculations by Hand And Comprehension: A Case Study

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Abstract

The debate over impacts of introducing computers and calculators to classrooms lasts half a century now. While there is a strong case for introducing computing technology to education, the utility of preserving a meaningful component of work done by hand is less studied. In this contribution we report on changes applied to an introductory course on numerical methods, namely reintroduction of calculations done by hand, and their impact on student comprehension and their attitudes.

Introduction

The advent of calculators and computers has brought fundamental changes to education at all levels and raised some crucial questions. Computing skills became less important, which is in particular true about engineering. In the past decades, engineering practice shifted from calculations to the use of software packages. Engineers of the future will be informed users of mathematics; they will need to be able to formulate problems in the mathematical language, and interpret and validate the output of software solvers. Correspondingly, the focus of engineering education in mathematics shifted from procedural knowledge (derivation, integration) to conceptual knowledge.

Much work has been done on what we should teach future engineers, see SEFI (2013) and e.g. Pepin et al (2021) for overview. Students should gain certain mathematical competencies, see Niss (2003), but teaching students to "think mathematically" remains a significant challenge. This has traditionally been difficult, but there are two new factors connected with the current ubiquity of computing and information technology, namely, the reduction of memorization components and work done by hand in schools.

While these trends represent a natural reaction to changes in our civilization, excessive removal of memorization and hand work is detrimental to learning. Results from cognitive science, see e.g. Gilhooly (2004), show that knowledge stored in working memory is crucial for reasoning. In particular, Price et al (2013) address the effect of long-term memory learning on mathematical ability. This knowledge should be well structured. The internalization of knowledge can be helped by work done by hand. For instance, there is mounting evidence that making class notes and preparing study text by hand-writing, in particular using the connected cursive writing, is more beneficial compared to typing the same text, see e.g. Ose Askvik et al (2020). The underlying cognitive mechanism potentially applies also to mathematics.

There is numerous anecdotal evidence pointing to a relationship between significant practice in hand calculations and understanding of mathematics. For instance, there is a common impression (but it is hard to find supporting evidence) that insufficient experience with calculations by hand (in particular working with fractions) can be detrimental to the better appreciation of rules and of the importance of going by them, which later causes difficulties when manipulating abstract objects. It is conceivable that mental and hand calculations may stimulate the brain to develop certain capabilities that later support understanding of advanced mathematical concepts.

Some aspects of this problem have been well studied, for instance the impact of calculator use in classrooms in elementary and middle education. There seems to be a consensus that elementary operations should be mastered by hand before moving to calculators, but the utility of preserving hand calculations in later years and high school is less investigated. For an overview see e.g. Ellington (2003). Another aspect that deserves more study is the impact of pre-college work done by hand on mathematics performance at university. The studies that the author found (Wilson & Naiman (2004), Mogary & Faleye (2012)) suggest that there is a reason for concern, but typically they are small and specific. A more recent meta-analysis Mao et al (2017) does include delayed use of handheld devices until operations were mastered as one of the factors. It did not confirm a negative influence of insufficient work by hand, but this question was not the focus of the study.

If there is little data about the relationship between college performance and pre-college work by hand, there seems even less addressing the impact of work done by hand at university level. This contribution does not bring a definite answer either. It reports on a specific situation that shows that the questions we raised here may be important and deserving a further study.

The Beginning - A Computer-rich Course

In 2014, the author had the opportunity to design a new introductory course on differential equations and numerical methods. Initially it was to be a course offered to an honors program with a small enrolment (up to ten) of highly skilled and motivated students, and with a generous class time donation. A mixed-model was adopted, where students would practice solving ODEs analytically by hand and explore numerical methods using a user-friendly software in computer labs. The exam included a question asking students to explain the workings of a specific numerical method.

The course ran for about three years in this form with considerable success, and it was decided that it would be given to students of the majority of programs offered by the faculty, bringing the enrolment to about 300. Also, the number of weekly classes was lowered by one third by the administration. This brought significant challenges. In particular, surprisingly many students were not comfortable enough with classical computation technology (desktops, operating systems), and helping them took away a significant part of the already decrease computer time.

While technically the course worked, the compromises and generally weaker background and lower motivation of students compared to the special course resulted in lowered learning outcomes. The worst impact was seen in the numerical part of the exam: The majority of students failed to answer the "explain how it works" question. And not only did they fail it, about half of them did not even attempt to answer it.

The End - A Paper-and-pen Course

In 2019 we decided to prepare a new version of the course where students would familiarize themselves with numerical methods by applying them by hand once, before passing to (significantly reduced) computer experimentation. We started to teach this way in the spring of 2020, which was the right time, because when the first covid lockdown came, we were not hit as hard as we would be with the previous (more computer dependent) version of the course.

Running the course in this way in 2020 and 2021 we observed improved results and gained valuable experience. In particular, we found that many students found participation in discussion about numerical experiments sufficient and gave up on running them themselves while attending online sessions from home, without observable ill effects. Moreover, the overhead related to running the computer part of classes became out of proportion to its utility. We therefore decided to drop the active student computer component altogether.

In 2022 we therefore transitioned to a new format. All numerical work done by students in practical classes is done by hand (and perhaps calculators), to which end we prepared a database of computation-friendly problems that can be solved just using small integers or (at worst) simple fractions. Moreover, at the exam, points for a practical numerical problem (also solved by hand) are awarded mostly for applying the right method in the right way, for interpretation of results etc., so numerical errors do not really influence grades when/if they happen.

There is some computer experimentation preserved in practical classes, but it is ran by the instructor with student input. For motivated students we prepared annotated worksheets that they can run at their convenience.

While losing the practical computer component of the course has not been an easy choice, it made the practical classes more efficient, giving students more time to learn. As the results below show, this computer-less version of the course has comparable results to the mixed model from years 2020 and 2021, and students' comments are also very complimentary. Moreover, it simplified organization of the course.

The Outcome

When we decided to look for ways to improve student performance in the numerical segment of the exam, we decided to collect data. Going back to 2018, we recorded student performance on the question asking students to explain a specific numerical method. We ranked the answers as complete, partial, and fails. Moreover, we recorded the attempt rate as an indication of self-confidence of students.

It should be noted that the results are comparable, as the form and difficulty of this question did not change over time, and even in the covid years the exams were taken on-site, under exactly the same conditions as before or after (the lockdowns affected only weeks of instruction).

In the following table we show the results. The number of exams taken by students is absolute, while the numbers in remaining columns are percentages taken against this number of exams. It should be noted that the actual enrolment in the course is typically 15 to 20 percent higher, but not all students make it to exams.

year	exams	attempt	correct	fail
2018	210	53.8	26.2	58.1
2019	245	55.9	25.3	56.3
2020	270	73.0	28.9	46.7
2021	317	94.0	48.9	17.4
2022	280	95.3	46.1	24.6
2023	272	96.3	49.6	22.4
2024	325	95.4	47.1	21.3

Table 1: Course results

The first percentage shows how many students did attempt the theoretical question. The introduction of hand-solved problems makes a clear impact in 2020. The gradual improvement makes sense, as in 2020 we were learning how to teach the new format properly and also adjusting to online teaching.

The second percentage number shows how many answers were satisfactory. Note that this number is also with respect to exams, not against the number of attempts. Since these two percentage numbers (attempts, success) are computed against the same base, they are directly comparable and show that consistently about half of the students who attempt this question succeed.

The last number shows failures. This number thus includes students who did not attempt this question and those who attempted but their answers were not even partially correct. The complement of the last two numbers to 100% shows answers that managed to capture the substance of the required method, but some important aspects were wrong or missing.

Since 2021 the results seems fairly consistent.

The author also observed a gradual change in student behavior. Many students come to university with the habit of pulling out their calculators at the first sign of trouble, and instead of trying to reason out the ideas behind a problem to be solved, they start punching in data and combine them in various ways, hoping to get enlightened. The new form of the course lead to a visible decrease of this behavior. The majority of students learn fairly soon during the course that they need not worry about calculations and it is better to focus on “how”. In particular, there has been a significantly decreased use of calculators during final exams. However, since we did not quantify it before, we cannot support this impression by hard data.

Conclusion, Discussion

When interpreting statistical results, we have to consider various influences. When we observed a problem in the course, we naturally also tried to improve our instruction, making students more aware of what is expected of them. Thus it is not clear what part of the impressive improvements is due to students doing hand calculations. However, while changes in instruction were incremental, we see discrete changes in learning outcomes corresponding to introduction of problems solved by hand. Thus it is arguable that it contributed significantly to the reported results.

They seem to confirm observations made by other authors: When students use technology to solve problems, they tend to focus on the process of inputting data and reading answers, without worrying too much about what is happening in between. Related experience with project-based learning suggests that when students use some tools when solving an applied problem, they do not sufficiently focus on them to internalize them. Calculations done by hand channel student attention to the underlying ideas. The author feels that giving students experience with both calculations by hand and experimenting with software should provide the best result. Unfortunately, reduction in class time prevented having these two as meaningful components in the course.

Thus it is arguable that even in the computer age, calculations done by humans still have their place at universities. However, they should not be an end to itself and an empty drill. Rather, they should be used judiciously to help in development of mathematical competencies. When we decide to include problems worked on paper in our course, we should try to reduce student anxiety about making mistakes. Computation-friendly problems and a grading policy lenient toward numerical errors help to alleviate these concerns.

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Making Statistical Inference Click for Engineering Students

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Abstract

Statistical inference is a foundational yet conceptually challenging topic in undergraduate education, particularly within technical disciplines where students often approach data analysis with a deterministic mindset. Traditional instruction frequently emphasizes procedural fluency over conceptual reasoning, leaving learners with limited understanding of how sample data can inform population-level conclusions. This study explores an alternative instructional approach grounded in a model that emphasizes active, student-centered learning through data generation and real-time visualization. Implemented in an undergraduate statistics course for engineering students, the intervention was designed to make abstract concepts more tangible by engaging students in constructing sampling distributions using their own data. Observations and interviews indicate that this approach fostered meaningful engagement and helped students develop a clearer understanding of key ideas such as variability, distribution shape, and the logic of generalization. Students expressed increased confidence in their comprehension of core concepts, particularly the Central Limit Theorem, though challenges remained in fully grasping the probabilistic reasoning underpinning inference. Peer collaboration and visual feedback were particularly effective in supporting conceptual development, while technical difficulties with data tools revealed the need for targeted scaffolding. Overall, the findings suggest that interactive, context-rich instruction can connect procedural knowledge and conceptual insight, offering a promising direction for teaching statistical inference in applied settings.

Introduction

Statistical inference; drawing conclusions about a population based on sample data, remains a challenging yet fundamental concept in undergraduate statistics education, particularly within engineering and computer science contexts. Students often struggle to internalize the logic that connects sampling variability to population-level conclusions, especially when instruction relies heavily on procedural or formulaic approaches (Kula & Koçer, 2020). This disconnect between conceptual understanding and computational practice has been widely acknowledged in the literature and continues to prompt calls for more effective, interactive, and contextually rich pedagogical strategies (van Dijke-Droogers, Drijvers, & Bakker, 2022).

In technical disciplines, such as Computer Science (CS) and Business Information Technology (BIT), the challenge is compounded by students' varying familiarity with data analysis tools and their tendency to approach statistics from a deterministic rather than probabilistic standpoint. This study explores an alternative instructional approach rooted in the *Construction Direction Model* (Kula & Koçer, 2020), which emphasizes student-generated data and the iterative building of conceptual understanding. By leveraging real-time data aggregation and visualization during a classroom intervention, the study investigates how engineering students develop an understanding of statistical inference through hands-on, interactive learning experiences.

Theoretical Foundations: The difficulties students face in learning statistical inference are well documented. A central issue is the abstract nature of inference itself; using incomplete sample information to make claims about a larger population. According to Kula and Koçer (2020), this challenge arises partly from the conceptual misalignment between how inference is constructed in practice (based on random sampling and variability) and how it is often presented in instruction (as a fixed application of pre-defined procedures). They argue that instructional approaches must prioritize conceptual construction over mechanical application, proposing the *Construction Direction Model* as a means to bridge this gap. This model encourages students to generate their own data, construct sampling distributions, and reflect on variability in an exploratory and inductive fashion.

Complementing this approach, van Dijke-Droogers et al. (2022) propose that the introduction of statistical inference can be greatly enhanced through repeated sampling activities and the use of digital tools for statistical modelling. They emphasize the educational value of interactive representations, such as simulations of repeated samples drawn from a "black box", in helping students intuitively grasp key inferential ideas. These forms of engagement provide a natural segue into more formal topics like hypothesis testing, supporting a trajectory of increasingly sophisticated reasoning.

Other scholars have similarly underscored the importance of informal reasoning as a foundation for understanding statistical inference in the undergraduate level. Blomberg (2024) stresses that inference should be recognized as a central concept in school-level statistics education, which necessitates careful curriculum design and evaluation. De Vetten, Keijzer, and Schoonenboom (2023) further highlight that pre-service teachers, and by extension, undergraduate students, benefit when instruction focuses on the logic of generalization from sample to population, rather than on symbolic manipulation alone. Their findings reinforce the idea that fostering inferential reasoning requires opportunities for learners to engage directly with data, test assumptions, and reflect on their interpretations.

Together, these studies suggest that conceptually grounded, interactive, and contextually meaningful instruction is critical to advancing students' understanding of statistical inference. The present study aligns with this body of research by implementing the *Construction Direction Model* in an undergraduate statistics course for engineering students, with the goal of investigating how such an approach can make inference "click" in both conceptual and practical terms.

Methodology

This study adopts a Design-Based Research (DBR) methodology to investigate how the *Construction Direction Model* can support engineering students in developing a conceptual understanding of statistical inference. Given the complex, context-dependent nature of learning statistical inference and the need to improve both theory and instructional practice, DBR offers an appropriate and flexible framework. It enables iterative design, implementation, and analysis of educational interventions within authentic classroom settings, allowing researchers to examine not only the effectiveness of the intervention but also the process by which learning improvements are achieved (Anderson & Shattuck, 2012).The research was conducted during a scheduled statistics course at the University of Twente, with second-year Computer Science (CS) and Business Information Technology (BIT) students. A total of 63 students attended the intervention, which was based on voluntary sign-up. The intervention took place during Week 3 of the course module, following the students’ prior instruction in fundamental statistical concepts such as hypothesis testing, error types, and sampling distributions. The lesson was implemented after students had covered the foundations (e.g., random sampling, sample statistics) and just before the more technical sections on population mean testing under unknown variance.

The instructional design was guided by the *Construction Direction Model* (Kula & Koçer, 2020), which emphasizes the active construction of statistical concepts through student-generated data. Prior to class, students were asked to analyze a small given dataset by calculating the sample mean and constructing a histogram. During the intervention/session, students calculated sample means and submitted their results through Microsoft Forms. A histogram of these sample means was generated and displayed in real-time, illustrating the emergence of the sampling distribution and reinforcing core ideas from the Central Limit Theorem.

The intervention followed a detailed lesson plan, which included both pre-class tasks (data preparation and individual sample analysis) and in-class activities (interactive comparisons of population, sample, and sampling distribution histograms; guided discussions; and application of inferential reasoning). Digital tools, including Microsoft Forms, facilitated data submission and visualization. Throughout the session, emphasis was placed on peer discussion and teacher-guided interpretation of emerging statistical patterns.

The data collection process was multifaceted and aligned with the DBR principle of triangulation. First, classroom interaction data were gathered through Microsoft Forms and teacher observations. A colleague with expertise in education observed the session to document student engagement, instructional pacing, and fidelity to the intended design. The lesson plan itself was reviewed in advance by four colleagues (two statisticians and two educational experts) to ensure both content accuracy and pedagogical soundness. In addition, two follow-up semi-structured interviews were conducted with student participants to elicit deeper insights into their understanding, experiences, and perceived challenges around two months after the implementation. These interviews were conducted with informed consent and recorded for later analysis. Ethical approval for the study was obtained through the university's internal review process.

The analysis phase involves a qualitative evaluation of student responses (e.g., explanations, interpretations during discussion), interview transcripts, and observational notes. Preliminary trends suggest increased student engagement and improved conceptual clarity regarding the behavior of sample means, variability, and the logic behind statistical inference. Particular attention is being paid to students' ability to connect sampling variability to inferential conclusions, and to recognize the role of randomness and distribution shape in estimating population parameters.

Findings and Discussion

Student Engagement and Focus: The voluntary nature of the implementation led to the participation of students who were inherently more motivated. Classroom observations and the teacher's diary indicated that student engagement was significantly higher compared to prior lecture-based or traditional Q&A sessions. Students were notably more attentive during the data collection and histogram analysis phases. According to the teacher, students asked more conceptually rich questions than in previous years, showing deeper engagement with the material.

When students submitted their sample means via Microsoft Forms (see Figure 1) and viewed the live sampling distribution histogram (see Figure 2), they displayed sustained focus. Peer chatter decreased notably during this stage, replaced by intense observation and interest in how the distribution evolved. These behaviors align with the observer's notes, which confirmed heightened attentiveness and student participation, particularly when the teacher referred to the realism of incomplete data and the importance of not fabricating responses.

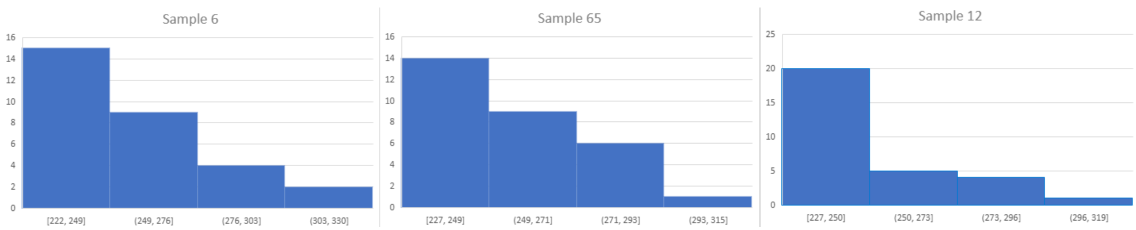

Figure 1. Examples of student-submitted sample histograms.

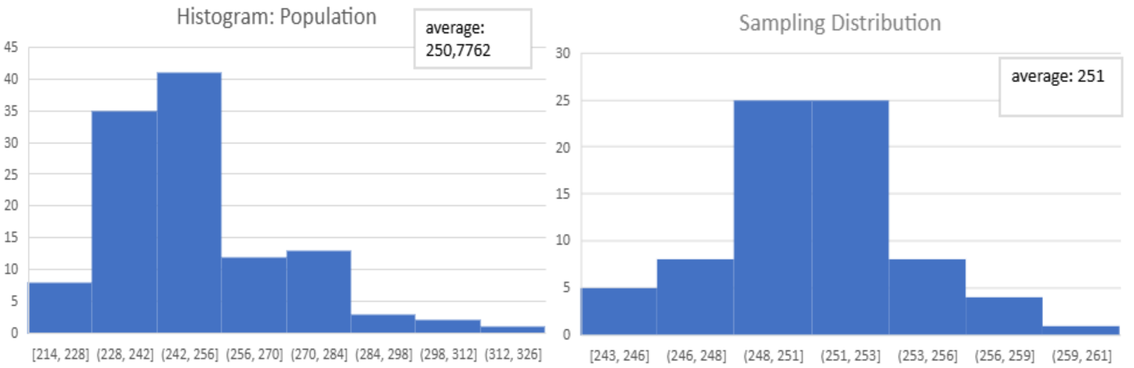

Figure 1. Population histogram and the resulting sampling distribution with displayed sample mean values.

Evidence of Conceptual Understanding: Analysis of students' submissions showed that nearly all students successfully calculated their sample means, indicating procedural fluency. However, interviews revealed some difficulties related to the use of Excel, particularly among BIT students. While all students produced accurate sample means and histograms, BIT students initially struggled more with constructing histograms, a challenge resolved through peer support during the session. Students from both CS and BIT backgrounds worked together, though the CS students demonstrated greater technical ease. The differentiated technical skill levels did not hinder the lesson, due to peer-led scaffolding and the collaborative structure.

Both interviews and classroom interaction suggested that the session supported students in forming conceptual links. One student remarked during the session, "*I really understand the CLT now, I thought I understood it but I think now I really really get it.*" The observer noted spontaneous peer discussion following more complex questions and suggested allowing more time for this interaction to evolve. This collaborative environment also fostered better engagement, with students appreciating the teacher's continuous encouragement and responsiveness.

Interview Insights: In post-session interviews conducted more than two months later, students were still able to articulate the concept of the CLT, despite minor omissions in technical language. They expressed appreciation for using their own data in the lesson, which personalized and contextualized the content.

However, the interviews revealed lingering difficulty in understanding why sample statistics can inform us about population parameters. This appeared tied to a weak grasp of cumulative distribution functions (CDFs), particularly the role of the normal distribution in inference.

Students also struggled to interpret the probabilistic nature of the sampling distribution. They asked insightful questions during the session about the distance 'd' between their sample means and the population mean, reflecting their effort to grasp the probabilistic nature, variability and uncertainty in sampling. These conceptual struggles were productive, highlighting where future instruction might be deepened.

Discussion: The implementation of the *Construction Direction Model* provided a clear pedagogical advantage in bridging the gap between theoretical and applied statistical reasoning. By constructing their own sample statistics and visually experiencing the emergence of the sampling distribution, students moved beyond abstract or hypothetical explanations.

This hands-on approach helped build conceptual understanding of the CLT and addressed key issues identified in the literature (e.g., Tintle et al., 2015), especially regarding students' misconceptions about variability and inference. Rather than memorizing statistical formulas, students engaged directly with the process of sampling, contributing to a more embodied understanding.

The interactive design—using students' own data, real-time histogram visualization, and peer collaboration—was highly effective. One area for improvement lies in scaffolding students' technical skills in Excel, particularly for those less familiar. Additionally, varying sample sizes (e.g., n=50, n=100) could further enrich understanding of distributional changes and reinforce the CLT.

The classroom observer reported no evidence of the Hawthorne effect influencing student behavior. On the contrary, students appeared genuinely engaged, and classroom dynamics reflected authentic participation.

Future lessons would benefit from additional supports for visualizing abstract statistical concepts like the CDF, and perhaps incorporating interactive tools to gather anonymous feedback about student difficulties during class.

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Changes Inspired by Experience from Covid Lockdowns – An Example

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Abstract

During the Covid lockdowns, most universities were forced to switch from on-site to online teaching. When in-person instruction resumed, some of the methods developed for remote learning were integrated into traditional teaching practices. This paper shares insights from such adaptations, as described in Habala (2022), specifically in the course Languages, Automata and Grammars taught in the third year of the Bachelor's program Open Informatics at the Faculty of Electrical Engineering, Czech Technical University in Prague. The paper discusses how these changes impacted students' understanding, and influenced course outcomes and success rates. Also, the enhancement of critical thinking that is taking place.

Keywords: Mathematics education, assessment, homework assignments.

Introduction

Universities and teachers/instructors face significant challenges due to rapid technological and societal changes, particularly in mathematics education for future engineers. Over the past 50 years, technological advancements have transformed both teaching and learning. This evolution raises important questions, one of them being: How can we best prepare students for their future careers? As highlighted in Bates (2015), one of the main educational goals today is fostering critical thinking.
It means the ability to analyse available facts, evidence, observations, and arguments to form sound conclusions or informed choices. It involves recognizing assumptions, providing justifications, evaluating these against diverse perspectives, and assessing their validity and consequences (see e.g. Brookfield (1987)). Also to form conclusions through application of rational, sceptical, and unbiased judgments (see e.g. Glaser (2017)).

During the Covid lockdowns, nearly all universities adopted remote learning. When students and teachers returned to classrooms many strategies from distance learning—such as recorded lectures and encouraging student independence—were retained. This paper presents the experience of integrating such practices into the Languages, Automata and Grammars course, which provides a theoretical foundation for students of informatics. We focus on the effect of structured homework assignments with personalized feedback on final grades, and overall course success. At the same time, this approach helps develop students' critical thinking.

Method

The course "Languages, Automata and Grammars" is mandatory for four specializations in the Bachelor's programme Open Informatics at CTU in Prague. The contact teaching includes two hours of lectures, and two hours of tutorials. The course expects students to spend 1 to 2 hours per week in self-study. The course is typically taken in the fifth semester (third-year winter semester). The course content can be divided into two main parts: (1) regular languages and finite automata and (2) context-free languages and context free grammars.
Lectures are non-compulsory (as all the lectures at CTU); tutorials are mandatory. The course usually enrols between 50–100 students annually.

- *Before 2020*. In academic years 2018/19 and 2019/20 there were classical lectures and tutorials. During semester two midterm tests were written. Homework assignments were optional; usually completed by 3–4 students per tutorial group (25 students); by those who wanted additional feedback.
- *2020/21 and 2021/22 (online teaching):* Lectures were given online. Instead of tutorials, students obtained one homework assignment. Solving it was mandatory and replaced tutorial attendance. Students got feedback before the next tutorial. An end-of-semester student survey to find out what students liked about the online classes showed that students found the homework solutions to be very beneficial.
- *From 2022/23 onwards:* 1. Lectures were streamed as supplementary study material. 2. Students received eight homework assignments over 14 weeks (four on regular languages and automata, four on context-free languages and grammars).

  To qualify for assessment (a necessary condition to be able to take the exam):

  - Students had to submit at least two homework assignments from each topic area. It was left to students which tasks to solve. Homework assignments emphasized explaining solutions and justifying correctness. Students received detailed feedback within a week. Homework assignments were not graded; partial or incomplete solutions/justifications were acceptable.
  - Students had to pass two midterm tests (one per topic) with at least one-third of the maximum score.

Examples of two such homework assignments:

1) *Homework A:*
   a) Design a finite automaton *M* that accepts the language *L* consisting of all binary words *w* that satisfy the following three conditions (1) *w* starts with *0;* (2) *w* contains *011* as a subword; (3) *w* ends with *0.* Carefully justify the fact that *M* accepts *L.*
   b) Show that for the language *L* consisting of all binary words *with* the same number of *0* and *1* there does not exist a finite automaton that accepts it.
2) *Homework B:*

a) Design a context free grammar that generates the language *L* consisting of all binary words of the form 0^i1^j for $0 < i < j$. Carefully justify the fact that the grammar generates *L*.
b) Reduce a given context free grammar. Describe all necessary steps and justify their correctness.

Findings and Discussion

Before the academic year 2022/23 either the homework assignments were not compulsory (2018/19, 2019/20) or the homework assignments substituted the tutorial attendance. For this reason, we only summarise the evidence gathered on homework assignments for the academic years 2022/23 to 2024/25.

Table 1: Homework Submission Statistics

Year	number students	1st	2nd	3rd	4th	5th	6th	7th	8th
22/23	91	90.1%	86.1%	74.3%	77.2%	69.3%	79.2%	78.2%	33.7%
23/24	82	89.9%	77.5%	76.4%	60.7%	75.3%	78.6%	86.5%	31.5%
24/25	74	86.5%	90.5%	77.0%	73.0%	54.1%	89.2%	93.2%	55.4%

Table 1 illustrates the percentage of students submitting each of the eight homework assignments across academic years. The number of students submitting the final assignment was consistently lower, possibly due to the increased workload from other courses or the complexity of the task (constructing pushdown automata). In 2024/25, submissions of the last homework rose when students learned the task would appear on the final exam.
Interestingly, for the first two homework assignments in 80 % of the submissions there was no justification of correctness of the solutions. The situation gradually improved until in the 7th assignment only 5% of students skipped the justification part. This shift could be also due to the midterm test where justification was required.

Table 2: Homework Completion Distribution

Year	8 (in %)	7 (in %)	6 (in %)	5 (in %)	4 (in %)	Average #
22/23	19.8%	37.5%	19.8%	10.4%	12.5%	6.10
23/24	22.6%	22.6%	23.8%	19.7%	12.0%	6.09
24/25	27.0%	24.3%	13.5%	20.3%	14.9%	6.28

Table 2 shows how many students did 8, 7, 6, 5, or 4 homework submissions during the semester. The average number of submissions per student can be found in the last column. During the three academic years the number of students submitting all 8 assignments has increased. In the academic year 2024/25, the number of students doing the bare minimum increased. This may have been due to the fact that in previous years some students who met the number of homework assignments failed the midterm tests and therefore did not receive the assessment.

Table 3: Final Grades Before 2021

Year	#assess	A	B	C	D	E	F	N
18/19	53/54	13.21%	3.77%	18.87%	24.53%	33.96%	5.66%	0%
19/20	61/68	10.48%	19.67%	13.11%	11.48%	36.06%	6.56%	1.64%

Table 4: Final Grades After 2021

Year	#assess	A	B	C	D	E	F	N
22/23	91/101	15.38%	4.38%	13.2%	19.78%	34.06%	11%	2.2%
23/24	82/90	15.85%	10.9%	8.5%	19.51%	36.56%	6.25%	2.43%
24/25	74/74	18.92%	9.5%	13.5%	29.72%	29.73%	1.25%	0%

Tables 3 and 4 compare grade distributions before and after implementing the changes from the Covid period. The second column gives the number of students that qualified for assessment (and were allowed to take the final exam) compared to the total number of students enrolled in the course. It can be seen that the percentage of top grades (A) increased slightly, and failing grades slightly decreased. It should be noted, that the results from the academic year 2022/23 are partly influenced by Covid, because these are the students who, during the first two years of their university studies, had only online teaching.

Table 5: Success Rates Before 2021

Year	1st Resit	2nd Resit	Average	Final Avg.	Success Rate
18/19	26.4%	5.66%	2.72	2.31	92%
19/20	27.8%	3.28%	2.74	2.34	82%

Table 6: Success Rates After 2021

Year	1st Resit	2nd Resit	Average	Final Avg.	Success Rate
22/23	23.07%	6.59%	2.79	2.44	79%
23/24	28.05%	6.09%	2.57	2.08	81%
24/25	24.32%	5.40%	2.58	2.17	98%

There was a success rate decline in 2022/23, likely because students were exposed to online teaching as freshmen. However, by 2024/25, the success rate improved. It is important to note that students may retake a failed exam twice. Additionally, some students repeated the exam because they want to achieve the best possible results in the exam to obtain scholarship.

Conclusions

To develop student competencies and critical thinking, assessment alone is not enough. Students need active opportunities to practice argumentation, receive immediate feedback, and identify flawed reasoning. Structured assignments that require justification and offer feedback are effective in nurturing these essential skills. In the future we will study the influence of solving such structured assignments to enhancing critical thinking in more details.

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Problem-based Teaching and Learning and Authentic Assessment of Mathematics for First-year Engineering Students

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Abstract

Engineering higher education has undergone and continues to undergo enormous change over the past decade and a half. Features of 'emerging leaders' in this trend are noted as having a curricular emphasis on, among other aspects, student-centred, active learning. The New Model Institute for Technology and Engineering (NMITE), itself represents an emerging leader in the UK.

Given this changing landscape, it is important to consider how mathematics embeds within engineering education; this itself is an emerging field of research. This paper is a case study, showing how mathematics was embedded into a first-year thermodynamics module at NMITE, with the aim of promoting discussion in the sector around the pedagogic rationale behind such an approach.

Presented here are the underlying pedagogic principles to mathematics education at NMITE, followed by a description of the practices involved in embedding mathematics into a first-year thermodynamics module. Findings show a broadly positive student experience with mathematics embedment in the module. The attainment data did not provide statistically significant effect sizes by whether students possessed a prior FHEQ Level 3 mathematics qualification.

Introduction

The New Model Institute for Technology and Engineering (NMITE), a self-described 'disruptor' (Hitt, et al., 2020), offers a distinctive approach to teaching and learning in the UK higher education sector. Among its differentiating qualities include:

Problem-based, challenge-led learning – Learning in each NMITE module is driven by an industry-provided challenge, reflecting what students might encounter as professional engineers. This provides a basis for learning facilitation which itself is problem-based.

Block teaching – All modules at NMITE are ran in blocks, i.e. are not taught concurrently with other modules over the course of a semester.

9-5 studio-based learning – Groups of no more than 30 students are taught face-to-face in a studio environment, enabling a collaborative learning space, more representative of an engineering workplace than a traditional lecture hall or seminar room.

Embedded academic skills delivery – 'Academic skills' (namely mathematical, communicational, digital and professional) are embedded in module learning content and assessment, as opposed to being taught via pre-requisite, for example, 'mathematics' or 'employability' modules. This appears as 5 credits in every module and are planned by the Academic Skills and Knowhow (ASK) Centre. Additionally, as is common in the sector (Lawson, Grove, & Croft, 2020), the ASK provides out-of-module elective support.

Inclusive student recruitment – Unlike most undergraduate engineering programmes, NMITE students do not require a Framework for Higher Education Qualifications (FHEQ) level 3 qualification in mathematics or physics to be enrolled (Graham, 2018).

The teaching and learning of mathematics for engineers is itself an emerging field of research, sitting in a niche cross-section of mathematics and engineering pedagogy (Pepin, Biehler, & Gueudet, 2021). Naturally, mathematics education at NMITE is underpinned by the institution's wider pedagogy. In producing 'work ready' engineers, it follows that mathematics be taught and assessed in a way that reflects what a professional engineer's experience may be with it. As highlighted by Goold (2012), 'mathematical thinking' is of superior relevance to professional engineers in their work than 'curriculum mathematics'. Definitions range, but one perspective presented by Schoenfeld (1992) identifies mathematical thinking as 'developing a mathematical point of view' and 'mathematical sense-making'. This contrasts with curriculum mathematics, which can be understood as the topics/domains of mathematics as they appear in the university curriculum. Building on this, mathematics education at NMITE emphasises mathematical thinking. As outlined by Verhage and de Lange (1997), mathematical thinking or 'mathematisation' is considered a higher-order thinking skill, as opposed to the lower-order thinking skill of reproducing algorithms and techniques.

It is well established that a constructivist, student-centred approach to teaching and learning, which emphasises the learner making sense of mathematical problems in their own way, is preferable for developing creative and higher-order thinking skills (Brooks & Brooks, 1999). Accordingly, mathematics at NMITE, as in the rest of its teaching and learning delivery, is predominantly a constructivist, problem-based approach.

A Learning Plan for Mathematics Embedment

The principles detailed above are reflected in the ASK learning plan for mathematics embedment into the FHEQ level 4 'Thermodynamics and Fluids' module. The time-based plan, outlined in Table 2, has two sections: Sessions 1-4 focus on a formative assessment, while Sessions 5-7 prepare students for a summative assessment.

In Session 1-4, the ASK was able to satisfy the learning outcomes of its own 'maths thread' running across the programme. The formative assessment required students to email a descriptive solution to the 'Silo Problem' that involved determining the dimensions of a liquid storage silo with diagrams showing how it may drain. The summative assessment was split into two stages. Stage 1 mirrored the formative assessment, requiring an emailed solution to a modelling and mass balance problem. Stage 2 was an open-book paper test on core module principles, with a generous completion time.

The learning plan contains a range of standard mathematical topics, including quadratics, simultaneous equations, basic calculus, and assumptions and approximation as problem-solving mechanisms. Skills-based learning facilitation was provided: using Microsoft Equation Editor; simple digital diagram-making and using online tools (e.g. Symbolab, ChatGPT, Wolfram Alpha). A key aim in designing the learning plan was to address students' relationships with and preconceptions about mathematics. This includes addressing issues of mindset and anxiety relating to mathematics (Boaler, 2024); critical openness towards use of digital tools and generative artificial intelligence to support problem-solving; and challenging the notion that there is only ever one valid approach and solution to mathematical problems.

***Table 1** - Tabulated learning plan for mathematics embedment in the Thermodynamics and Fluids module*

Teaching Objectives	Mathematical Topic(s) / Student Progress with Problem(s)
<u>Session 1: Introduction to Module and Silo Problem</u> • Develop students' familiarity with how taught sessions will work (explain problem-based learning) • Set expectations by emphasising resilient mindset towards difficult problems and encourage students to take their own unique approaches to problem-solving • Send Silo Problem emails, written such that each student receives unique specifications for their silo • Demonstrate how to translate a written problem statement into mathematical equations/ideas • Demonstrate when systems of equations are not solvable (informal degrees of freedom analysis) • Encourage students to use digital problem-solving tools (Symbolab, ChatGPT, Wolfram Alpha) • Show students how to use the Microsoft Equation Editor in drafting an email response	• Mathematisation • Degrees of Freedom • Simultaneous Equations • Digital Equation Writing • Students determine main dimensions of their liquid storage silo • Students start drafting email responses, particularly written narrative and mathematical workings
<u>Session 2: Diagramming and Differentiation</u> • Task students with using the dimensions from the previous session to create labelled drawings of their liquid storage silos, using a variety of suggested digital tools (Miro, Figma, MS PowerPoint, Canva) • Task students with estimating rates of change on graphs provided via A3 handouts • Suggest a more accurate means of determining instantaneous rates of change, namely, differentiation. Deliver interactive presentation on differentiation, with emphasis on conceptual understanding. Procedural understanding reinforced via 'quickfire round' problems • Demonstrate how students may define a function for their respective graph, then differentiate to provide more accurate solutions to earlier estimates • Introduce students to idea of liquid storage silo drainage flow as a derivative, namely $Q = \frac{dV}{dt}$	• Diagramming • Graphical Interpretation • Graph Fitting / Function Modelling • Differentiation • Students create labelled drawings of liquid storage silos • Students understand drainage flow as a derivative
<u>Session 3: Integration</u> • Repeat of Session 2, but with focus on accumulation of quantities and integration • Derive a differential equation describing change in fluid volume as silo is drained and emphasise use of integration as means of solving it	• Graphical Interpretation • Graph Fitting / Function Modelling • Integration • Students have a differential equation describing silo drainage

Teaching Objectives	**Mathematical Topic(s) / Student Progress with Problem(s)**
<u>Session 4: Box With a Hole in It Practical</u> • Demonstrate how the differential equation from Session 3 may be solved and use as a basis to introduce an equation describing how a given box (physically shown) may theoretically drain • Introduce the idea of a 'discharge coefficient', which may be inserted into the equation to bridge the gap from how the box drains in theory, and how the box drains in practice • Lead students to a new setting, where students are tasked with determining the discharge coefficient in the 'box with a hole in it' scenario, which involves the production and solving of a quadratic equation • Reemphasise formative submission deadline, as at this stage students have everything they need to complete the assessment task	• Solving Differential Equations • Experimental Design • Quadratics
	• Students have a discharge coefficient value, which may be used in their Silo Problem formative assignment submissions • Formative submission deadline reemphasised
<u>Sessions 5 & 6: Mass Balance</u> • Facilitate students' learning of how to approach mass balance problems, Session 5 on single-unit problems, and Session 6 multi-unit problems • Draw on learning from Session 1, and deliver an interactive presentation on degrees of freedom analysis, this time with explicit reference	• Degrees of Freedom • Simultaneous Equations
	• Students learn how to approach mass balance problems, which will be one focus of the summative assignment
<u>Session 7: Assumptions and Approximation for Problem Solving</u> • Allocate students a set of problems, where solving necessarily requires them to define unprovided information, namely, to make assumptions • Deliver interactive presentation on 'assumptions' as a problem-solving tool, also emphasising approximation • Task students with approximation problems, for instance, estimating the rate of cocoa butter production expected from a cocoa bean plantation with pre-defined land area	• Assumption Making • Approximation
	• Students learn how to approach approximative modelling problems, which will be another focus of summative assignment • Students have all they need for summative assessment preparation

Findings and Discussion

A key aim for the ASK at FHEQ level 4 is to minimise the performance gap between students with and without a prior FHEQ level 3 mathematics qualification. Beyond level 4 students' grades contribute to final degree classifications, meaning a significant performance gap may undermine fairness and widening participation. Table 3 summarises attainment gaps between the two groups, alongside one-sided p-values from Mann-Whitney U tests ($\alpha = 0.05$). The results of the Mann-Whitney U tests failed to reject the null hypothesis that enrolment with a FHEQ level 3 mathematics qualification has no influence on attainment.

Table 2 *- Mean grades from summative tutorial questions assessment, grouped by students with and without a prior FHEQ level 3 mathematics qualification*

	Mean Grade Without FHEQL3 Mathematics Qualification	Mean Grade With FHEQL3 Mathematics Qualification	Delta	Mann-Whitney U one-sided p-value
Stage 1	60.2%	62.4%	2.2%	26.1%
Stage 2	47.7%	53.1%	5.4%	14.6%
Overall	51.5%	55.9%	4.4%	23.5%

One month after completion of the module, all 39 participating students were contacted to gather feedback. Of those, 19 students (49%) completed a survey, and 7 (18%) elected to also take part in an in-person conversation about the mathematical elements of the module. A graphical summary of the quantitative data from the feedback survey is given in Figure 3.

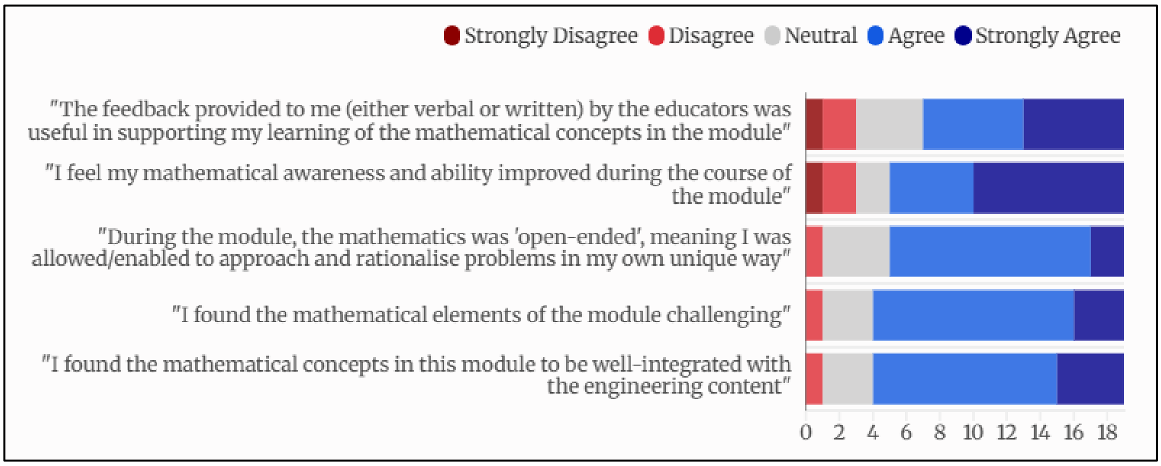

Figure 1 *– Graphical summary of quantitative data from student feedback survey*

The majority of the students who attended the conversation were individuals who have expressed concerns of their mathematical skills. Hence it was particularly pleasing that their reflections were broadly positive. The questions asked were open-ended and included:

What mathematical topics or elements did you find challenging?
What mathematical elements did you find enjoyable? Or not enjoyable?
Can you recall the mathematical topics that were covered in this module?
Did the mathematics in this module 'feel' different to maths in other modules?

All named the Silo Problem and the Box with a Hole in It practical as being particularly enjoyable: "It was just great!", "I liked how it made me think", "oooh, I wonder how this, say that the base of the box was not perfectly flat, affects the flow rate." Students also noted the way these activities were organised: "You didn't tell us - you showed us how to do these problems", "They were very well-planned", "I could see an end goal and why we were doing the maths", "You gave me a reason to care about maths." Some mentioned other topics: "The mass balance problems were really fun! … They were like solving an incomplete puzzle."

In exploring the covered maths topics, it was revealing to hear some students explain 'Assumptions and Approximations' was "not really maths", or that "this session was about something bigger than maths". Others differed: "I liked that it was making your own maths", "There were no given formula which was interesting." They also identified aspects of maths that were not part of the formal learning plan, such as geometry. One student seemed surprised that they were now looking forward to more maths, "I am excited to continue calculus in other modules"; another noted, "I am not elated about maths, but I won't mind."

Conclusion

This paper introduces the pedagogical principles underpinning teaching and learning at NMITE, an emerging leader in UK engineering higher education. NMITE's approach to mathematics education is outlined, and its practical application is exemplified via a detailed learning plan for mathematics embedment in an FHEQ level 4 thermodynamics module.

Evaluation of the learning plan, through attainment and student feedback analysis, yielded largely positive findings, with interesting perceptions of mathematics and on what they felt they had learnt. Attainment data showed insignificant attainment gaps between students with and without a prior FHEQ level 3 mathematics qualification, while feedback suggested broadly positive learning experiences with the mathematical elements of the module.

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School mathematics boost for first semester students having major problems with maths

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Abstract

Successful technical studies rely on a solid mathematical base. Our undergraduate candidates must have solid maths knowledge at least at the intermediate school level to follow their very first university lectures. However, the admission requirements for technical higher education are much lower than actually needed. For this reason, many candidates start their technical studies with insufficient mathematical skills. Long breaks between school and study can also cause mathematical knowledge decay. In order to close the gap between actual and necessary mathematical level, we are offering a short refresher school maths course before the first semester starts and a long boost course during the first semester. In this report we evaluated four post-pandemic semesters and looked at the possible impact of these two interventions on the first semester maths exam. Our findings show, that students who participated in the short refresher course took and passed the first maths exam significantly more often than students who did not take the course. We conclude, the first group is highly motivated to study. Moreover, students of electrical engineering, who just failed their placement test at the beginning of the first semester (reached 40-50% points) and had to take part in the one semester long booster course passed their first maths exam as well as those students, who just passed the placement test, i.e. reached 50-60% points: no significant differences could be found between those groups. This may indicate the positive impact of the long booster course.

Keywords: school mathematics deficits, refreshing course, placement test.

Introduction

Undergraduate candidates often lack the necessary maths skills for engineering study. Intermediate level mathematics, such as adding fractions, operating on symbols, calculating terms, using binomial formulas and power rules, plotting simple function graphs, calculating logarithms and trigonometric functions, solving equations and equations systems, and handling units, are very often unknown to our students. All of these themes are covered in our two remedial courses and placement tests.

Our students also need to understand that maths is not something they can just pay little attention to and forget about, as they did it at school. We try to show them that maths is not a goal in itself, but a necessary step for future studies. For this reason, we include many mathematical problems related to daily life or technology itself in the refresher courses. This makes maths more interesting.

Methods

The first short refresher (SR) course is open to all students at the university and lasts one week, just before semester starts. It is also a great opportunity to get to know each other and the campus. The latter is undoubtedly the main reason why that course is well attended.

At the very begin of the first semester a placement test (PT) in maths takes place for students of electrical engineering. The test starts with simple questions, e.g. adding numerical fractions, goes through square roots, logarithms, 2x2 linear equation systems, Pythagoras' theorem with analysis of units, quadratic equations with parameters, factorization of symbolic fractions with the use of binomial formulas and analysis of quadratic function.

The second, long refresher (LR) course is mandatory for students who failed their placement test at the begin of the semester, i.e. reached less than 50%. Students, who obtained 50-75% of the points and therefore passed the test, are invited to join up the booster course. That course takes place during the first semester, twice a week for 90 minutes, i.e. a total of 15 x 90 minutes. We monitor the progress of maths skills through written homework four times in the semester and two unsupervised online tests. Only students, who have passed every homework assignment, i.e. gained more than 50%, are permitted to take the maths exam in the first semester (MA1 exam). We decided to try unsupervised homework as a way to measure progress because we know that supervised tests always lead to stress. We deliberately sought to avoid the latter, since every student must pass three supervised tests during the regular first semester maths lecture (regardless of the refresher course) to be allowed to take the first maths exam.

Both refresher courses are carried out in a similar way: first a topic, e.g. adding fractions, is presented and solution methods for one or two tasks are demonstrated. Afterwards students are given another task on the topic, which they have to solve independently or in small groups. After a set time, the proposed solutions are discussed. This point is tricky. It is about how to show the students that their solutions are often wrong and where they went wrong in their reasoning without upsetting them or making them feel stupid. The latter may lead to student absence in the following meetings, since the attendance is not compulsory.

The first remedial course is supervised by students, whereas the second course is run by university staff. Student-tutors may significantly lower the inhibition threshold for asking questions, an important skill in gaining understanding. In the second course, we must first gain the students' confidence. We want them to be able to ask questions or share their own solutions.

Findings

We analyse the possible influence of the remedial courses for four semesters, starting with winter semester 2022, up to summer semester 2024, since we admit new students every half a year. Our university began offering the short refresher course in summer

2023, so it has been available for evaluation for three semesters so far. The long refresher course is being offered to students in our department for a much longer time, but most of the results are only available since winter 2022.

We analysed the impact of the short refresher (SR) course on the maths (MA1) exams after the first semester. Students who participated in the SR course wrote the MA1 exam more often (21 out of 27), and passed it more often (20 out of 21) than students who did not participate (45 out of 71 and 35 out of 45 respectively), as can be seen in Figure 1.

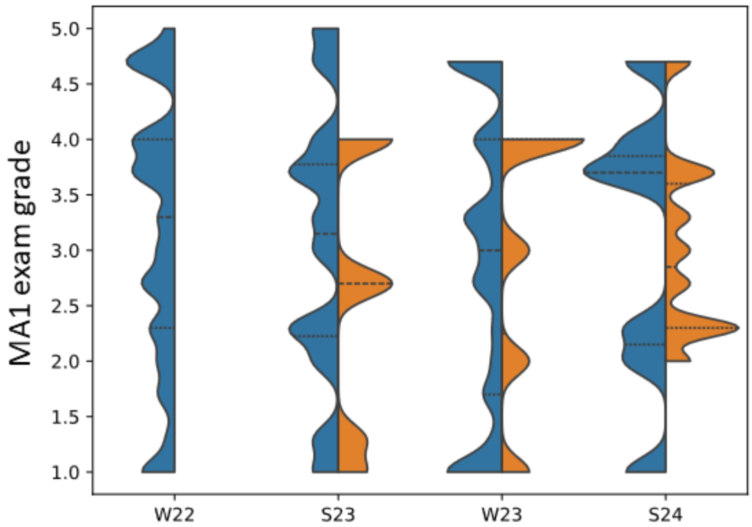

Figure 1. Violinplot of MA1 exam results for four semesters. The results are split between participants of the SR course (orange) and students who did not participate in the SR course (blue). On the left-hand scale, “1” is the best grade, “4” is “sufficient”, and anything above “4” means that the MA1 exam has been failed.

The placement test (PT) concerned intermediate school mathematics. Questions about factorization, fractional and root equations, quadratic equations with a parameter, power rules and logarithms were found the most difficult, with the least points achieved. The easy questions included adding numerical fractions, simple Pythagoras’ theorem, analysis of simple quadratic equations and a simple 2x2 linear equation systems.
Of the 63 students who passed the placement test, 51 also passed the maths exam, two failed and eight left the faculty. Out of 87 students, who failed the placement test, 67 successfully completed the long refresher course and 40 of them passed the MA1-exam. A short overview is given in Figure 2.

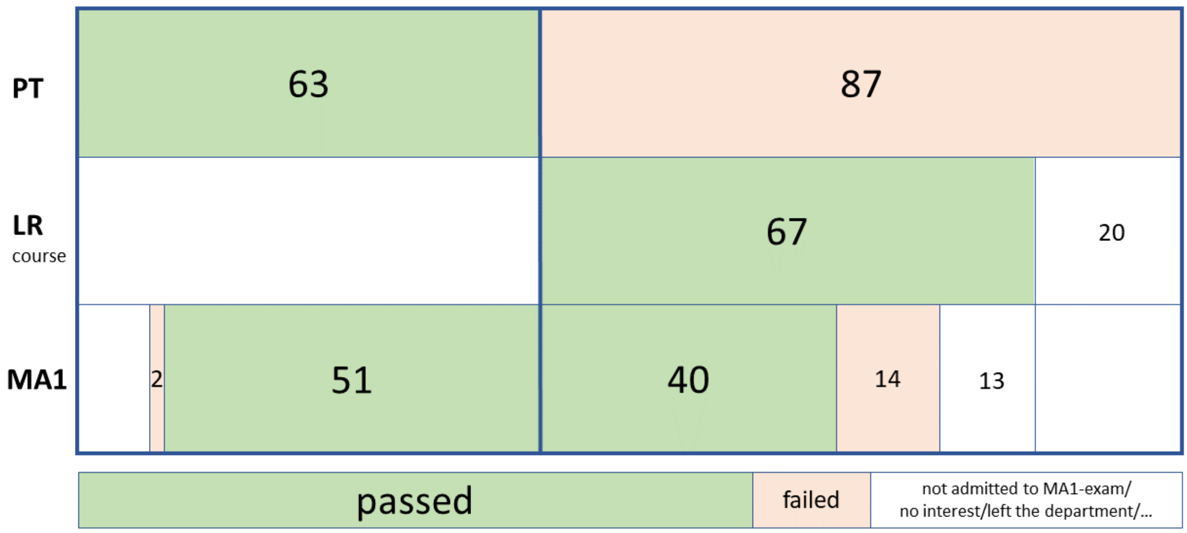

Figure 2. Consolidated results of the placement test (PT), long refreshing (LR) course and MA1 exam for four semesters: winter 2022 - summer 2024.

The relationship between the PT results and the MA1 exam grades can be found in Figure 3.

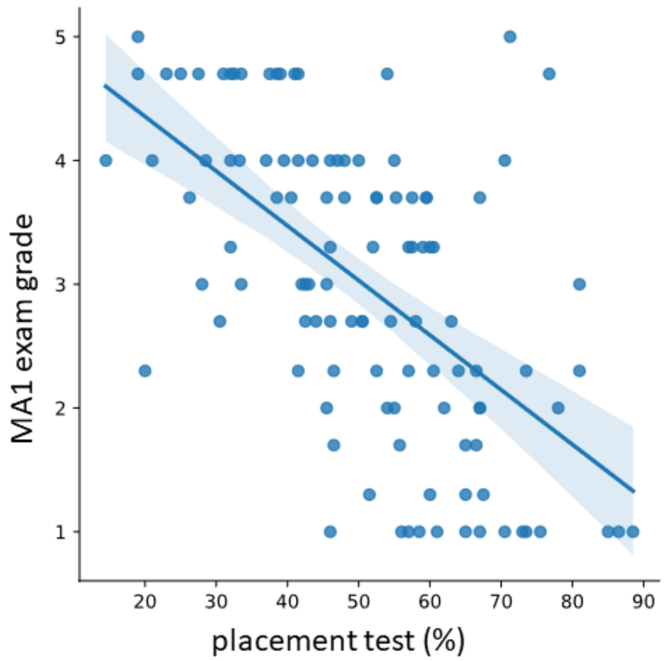

Figure 3. Correlations between placement test results and MA1 exam grades. MA1 exam grade “1” means the best grade, “4” is “sufficient”, and anything above “4” means that the MA1 exam has been failed.

The Pearson’s correlation coefficient between PT results and MA1 exam grades was -0.59 and there was a significant difference in MA1 exam grades between the group of students who participated in the LR course and those who did not. However, based on the PT results alone, one could not predict the MA1 exam results.

We analysed the group of the LR course participants, who passed the MA1 exam. Figure 4 shows the distribution of PT percentage points achieved by the LR course participants and how many of them passed the MA1 exam. The high PT scores did not guarantee MA1 success, nor did lower PT scores exclude it.

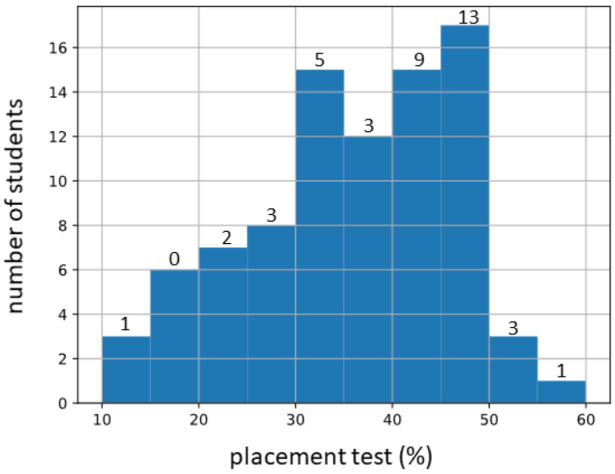

Figure 4: Histogram of the placement test results for the LR course participants only. The numbers above each bin show how many students from that bin passed the MA1 exam later on.

When comparing the MA1 exam results for the strongest group in LR course, who just failed the placement test (obtained 40-50% points) including the LR course volunteers, with the group of students who just passed the test (obtained 50-60% points) and therefore did not take part in the LR course, no differences could be found. It is worth mentioning that the first group achieved significantly lower PT scores in its most difficult questions: factorizations and powers and logarithms, as can be seen in Table 1. The differences could be levelled out within one semester.

	p-2samp-KS	40<PT<50 + volunteers	50<PT<60
PT (in total)	3.12e-14	45.5	55.8
PT: factorisation (max. 12 points)	0.0108	2.67	4.17
PT: powers and logarithms (max. 13 points)	0.000363	4.11	6.66
MA1 exam (German score)	0.59	2.97	2.95

Table1. Results of two-sample Kolmogorov-Smirnov for the group which just passed the PT (50<PT<60) against the group which just failed it (40<PT<50). P-values of that test and the mean values for both groups.

Conclusions and Outlook

We conclude that students participating in the short refresher (SR) course were really interested in successful study, since they could take and pass the first maths (MA1) exam significantly more often than students who did not take this course.
The placement test for students of electrical engineering at the beginning of the semester correlated coarsely with the MA1 exam at the end of the semester, and thus gave a rough insight about math skills. However, it could not be used to predict future maths success. Moreover, students who just failed the placement test (PT) at the beginning of the semester showed the most interest in and benefited the most from the long refresher (LR) course. This was clearly reflected in their MA1 exam grades, which were as good as those of students who just passed the PT.
We are confident that our steps taken to bridge the gap between the existing and required mathematical knowledge and skills of the first semester students have been successful. However, our future approach will change slightly due to the common use of the generative artificial intelligence (GenAI). Over the past two semesters, GenAI has partially dominated the way homework is solved. On the one hand, we encourage students to use GenAI as another cognitive tool or personal tutor, while retaining their own critical reasoning skills. On the other hand, there is a growing danger of mindlessly rewriting everything the AI has written. Unfortunately, the thoughtless use of GenAI last semester leads us to change the format to supervised tests during the booster course.

Using Game-Based Learning to Enhance Student Engagement in Multivariate Calculus for Engineering Students

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Abstract

This study explores the potential of game-based learning (GBL) to improve engagement in Multivariate Calculus (MC) among first-year Aerospace Engineering students. Traditional approaches to calculus education often struggle to sustain student interest, especially within diverse, international classrooms. To address this challenge, a modified board game based on "Snakes and Ladders" was developed and integrated into a core mathematics course. Structured classroom observations and post-intervention surveys provide insight into cognitive, behavioral, emotional and social engagement. Initial findings suggest that GBL can effectively foster metacognitive engagement in STEM education, providing a promising direction for improving abstract mathematical learning.

Introduction

Calculus, often perceived as abstract and challenging, presents significant engagement difficulties for first-year engineering students with diverse educational backgrounds. Traditional teaching methods (teacher-led learning) used during guided self-study activities - where instructors dominate, and students take passive roles - do not positively impact students' behavioural and emotional engagement (see Lim (2023)). To address these issues, this project will implement GBL in the Multivariate Calculus course for first-year Aerospace Engineering students. This course, taught annually in the third quarter, runs from February to March and the students use the book by Adams (2021) which is widely used at Dutch universities for Calculus.

Based on the above considerations and the need in examining the impact of a game-based intervention on student engagement, this study aims to investigate the following research question: *"What is the observed effect of implementing a board game, designed by GBL, in a course of Multivariate Calculus on engagement levels of first-year Aerospace Engineering students?"*

Background

Multivariate Calculus is one of the crucial mathematics service courses like Calculus and Linear Algebra, for the students to have the ability to apply sophisticated mathematical calculations and concepts alongside physical understanding. Despite its importance, the course's abstract nature frequently challenges students, impacting their engagement and retention. Pepper et al. (2012) identify common student difficulties in upper division electricity and magnetism course, which rely on vector calculus pre-knowledge. They highlight a specific challenge: students often fail to select appropriate mathematical tools for problem-solving, a process that crucially involves ‘*decision-making*’ skills. Furthermore, as was emphasized by Craig (2022), observations during the Corona pandemic show that decision-making skills are critical not only in open-book assessments but also in closed-book scenarios, which demand *higher-order cognitive skills* and a deep *conceptual understanding*.

Engaging in game play promotes *metacognitive skills* and an understanding of the interconnections between concepts, offering diverse perspectives. It improves the learning process by helping students to visually structure new information and actively engage in their own learning process (Kustusch (2014)). Research by Anagnostopoulou (2023) supports GBL as a valid and effective contemporary method to enhance students’ *engagement and motivation*.

Through the implementation of the below-mentioned intervention, we aim to investigate how participation in a game designed by GBL principles can enhance student engagement and motivation, as well as improve effective decision-making skills, thereby potentially addressing the noted challenges in Multivariate Calculus.

Intervention Design

The traditional board game "Snakes & Ladders", which can be seen in Figure 1, has been used and adapted into an interactive learning tool to enhance students' engagement with course material. In this version, the game board features numbered squares and players progress by rolling dice, aiming to be the first to finish. The game includes "ladders," which advance players towards the end, and "snakes," which send them back towards the start.

Figure 1. Board Game Layout.

A key modification is the introduction of educational challenges: when landing on a square, players draw a card from one of six categories based on course content and answer the question on the card. Other players evaluate the answers. The question cards that team members hesitate to answer are collected by the teacher. These topics are then addressed during the discussion following the game session. The game concludes when a player reaches the final square or when allotted time expires, with the furthest advanced player declared the winner.

There are four types of cards: *True\False Cards, Decision Tree Cards, Keyword Cards and Power Cards*. Both *True\False Cards* and *Decision Tree Cards* pose a question that tests the player's knowledge/understanding. The optional decision tree diagram for hint is provided during the game. The *Power Card* is a rare bonus and not easily earned. This card allows you to *either* skip a snake (avoiding being sent back) *or* keep your place even if you answer incorrectly on your next turn. You decide when to use it for maximum advantage. The *Power Card* is inspired by the **Core Drive 6: scarcity** from the *Octalysis Gamification Framework* (see Chou (2019)), adding excitement and strategic depth to the game. The *Keyword card* challenges players to connect key concepts from the course material. Each card presents a main keyword related to the course topics. The player's task is to provide 2–3 supporting keywords related to the main keyword and explain their connections.

Before the game, a reflection game activity is introduced during one of the lectures to engage students in active learning and allow them to contribute by designing some of the Keyword Cards. This activity not only gives students additional practice with the course material but also supports **Core Drive 4: Ownership** from the *Octalysis Gamification Framework*.

Methodology

This study was conducted as a pilot using a qualitative exploratory approach to examine the impact of a board game, designed with Game-Based Learning (GBL) principles, on student engagement in Multivariate Calculus. The intervention was implemented during the final session of the course with approximately 30 first-year Aerospace Engineering students, including Dutch, international and HBO bridging students. The redesigned "Snakes & Ladders" game was integrated with core Multivariate Calculus topics, aiming to foster active learning through structured gameplay and peer interaction.

The qualitative data obtained from observations during the game were used to clarify, explicate and enhance the findings from the exploratory quantitative data collected through questionnaires (Hamari (2016); Wang (2016)). The students completed a post-intervention survey consisting of both closed items using Likert scales, to assess cognitive, behavioural, emotional and social engagement, and open-ended questions about the game experience. While this phase focused on qualitative insights, a mixed-method follow-up is planned for a later stage. This will involve a larger dataset, pre- and post-intervention surveys and semi-structured interviews to assess changes in engagement and further explore the educational impact of the intervention over time.

Conclusion

This pilot study demonstrates the promising potential of game-based learning to enhance engagement in abstract mathematics courses such as Multivariate Calculus. By integrating a redesigned "Snakes & Ladders" board game into the curriculum, students were encouraged to participate actively, reflect on key concepts and engage in collaborative decision-making. Initial results suggest improvements across multiple dimensions of engagement, particularly in fostering metacognitive and social learning experiences. While the current findings are based on qualitative data from surveys and observations, the next phase will extend the research through a mixed-method design to include larger cohorts and longitudinal data. During our presentation at the SEFI Special Interest Group in Mathematics Seminar, we will share further insights, including the outcomes of follow-up interviews, to offer a more comprehensive understanding of how game-based activities can support deeper learning and sustained motivation in STEM education.

Acknowledgements

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Peer learning: Professor and Students unite in a flexible bridging mathematics course

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Abstract

This contribution details the transformation of our six-week intensive mathematics course from a traditional on-campus format, featuring half-day lectures and training sessions with a final written exam, to a more flexible structure incorporating pre-recorded videos and a portfolio of digital tests, with a minimum passing requirement of 60% on all tests. The course is offered both on campus and online, emphasizing independent learning through self-paced study, digital resources, and interactive problem-solving sessions. The teaching was organized and conducted by three learning assistants who provided support and conducted lectures on demand. The course is designed to ensure a smooth transition to engineering mathematics, specifically Calculus 1. We aimed to compare the performance of students in Calculus 1 who completed this redesigned course with those who followed the traditional summer course. This paper presents the results of this comparison, along with insights from semi-structured interviews with both campus and online students to understand their experiences.

Introduction

As part of our efforts to recruit more students to the engineering program at our university, we offer a six-week intensive mathematics course, MA-006, every summer. This course is designed for students with university entrance qualifications who lack mathematics and for students with vocational certificates and it attracts between 130 – 150 students each year. Passing this course is a prerequisite for entering a bachelor's program in engineering. Previously, the course was taught in a traditional manner, with three hours of lectures and three hours of training with student assistants every day. The requirements to pass the course included three written midterm tests (above 40%) and a final written three-hour exam (above 40%). In 2024, I took over the course and implemented substantial changes. I had previously offered the same course online, demonstrating that students from these courses performed as well in engineering mathematics Calculus 1 as regular students with full specialization in mathematics (Brekke, 2014). Experience with flipped classrooms and portfolios with digital tests showed improvement for the bachelor's program in electronics (Brekke, 2016). The setup of the MA-006 course builds upon well-documented results. Ideas from educational research have shaped the design of the course, with a particular focus on how individual tasks are designed using STACK (Kinnear, 2018). *Retrieval practice*, such as the *testing effect*, shows that *"retrieval of information from memory produces better retention than restudying the same information for an* *equivalent amount of time"* (Roediger & Butler, 2011). Therefore, we use *"test-enhanced learning"*, where tests are used as a tool to help students learn during the course (Brame & Biel, 2015).

We have also changed the Calculus 1 course from a traditional final written exam to portfolio assessment using STACK, with very good results. This change came after investigating how the teaching was done and how it was perceived by students (Zakariya et al., 2022). Our experience from these changes includes increased motivation, better self-regulation, and less test anxiety among students. We avoid procrastination, see more cooperation between students, and observe them spending more time on mathematics. We have extended the use of teaching assistants in our teaching. These are experienced students trained in guiding students and in basic pedagogy and didactics.

MA-006 was set up with pre-recorded short videos and a portfolio of three tests in the online assessment system STACK, with a requirement of a minimum of 60% on all tests to pass. The course is designed to provide a smooth transition to Calculus 1. MA-006 was offered both as a campus course and as a fully online course. The campus teaching was organized and conducted by three teaching assistants. The course emphasizes independent learning through self-paced study, digital resources, interactive problem-solving sessions on campus. It has an intensive format that requires students to have strong skills in self-regulation and stress management. Students must be able to organize their own learning, seek help when needed, and find a balance between rapid progression and understanding. Initially, students receive a suggested progress plan with tasks covering the curriculum and learning objectives. For each topic in the book, video lectures are provided and can be used as needed. Daily task reviews are conducted, where teachers go through tasks that students have found challenging. Students are assessed through three partial tests during the course, with each test allowing four attempts. The tests can be taken from any location within the specified time window, with all aids available. To pass the course, students must achieve at least 60% correct on each partial test.

Here, we Morten (professor and course responsible), Emma, and Luis (teaching assistants and fellow students) present how students experienced the course. We will show how this group of students performed in Calculus 1 compared to regular students and previous cohorts. Semi-structured interviews were conducted with both campus and online students to understand their experiences.

Method: Semi-structured interviews

To gain a deeper understanding of how students experienced the redesigned version of MA-006, we conducted a series of semi-structured interviews with students who completed the course during the summer of 2024. Our goal was to explore how the course structure, particularly its flexible format, peer-led support system, and digital assessment model was perceived by students, and how these elements influenced their motivation, engagement, and sense of mastery. We chose a qualitative approach, as we were not only interested in measuring outcomes but also in understanding why and how the course worked or didn't for individual students. The interviews followed a semi-structured format, meaning we prepared a set of guiding questions but allowed the conversation to flow naturally. This ensured consistency across interviews while leaving room for individual perspectives, allowing for in-depth responses, follow-up questions, and reflection. The questions were designed to cover four main aspects:

1. **Academic experience:** This included the use of the textbook and video lectures, experiences with problem-solving sessions and peer instruction, and whether students felt ready to begin Calculus 1.
2. **Motivation**: We focused on how students managed their own learning (self-regulation) within a flexible course structure.
3. **Practical aspects**: This covered students' experiences with the STACK testing system, the use of video lectures as a learning tool, and the role of peer learning.
4. **Overall impressions of the course**: This included expectations, satisfaction, and perceived value of the learning experience.

We interviewed both students who attended campus daily and those who completed the course remotely. This distinction allowed us to identify differences in how the learning environment shaped their experiences. The students' backgrounds also varied in terms of previous mathematical education, which offered further insight into how accessible and effective the course was for a diverse group of students. All interviews were transcribed and analysed thematically, focusing on recurring patterns as well as outliers.

Students' perceptions

Several strong themes emerged from the data. Many students emphasized how helpful the teaching assistants were, especially because they were fellow students themselves. As one participant noted, *"It's much less intimidating to ask for help when you're talking to someone who's been in your shoes recently."* Another added, *"The assistants explained things in a way that made it click, and I didn't feel judged for not understanding right away."*
Students also reflected on their use of learning materials and tools. Most relied primarily on the course-provided resources, particularly the textbook and Canvas. Some made occasional use of external AI tools like ChatGPT and Symbolab to clarify difficult concepts. "*I used ChatGPT to explain concepts in simple terms when I got stuck,"* said one student, "*but I didn't trust it to actually do the math for me."*
Regarding the flexible nature of the course, responses varied. Several students thrived in the self-paced format, especially those with strong self-discipline. One student commented, *"It was motivating to follow the progress plan and check off tasks—I liked having control over my own learning."* However, others found it more challenging to stay on track without the structure of daily lectures, especially those studying from home. One online student noted, *"The flexibility made the course possible for me, but I did miss having people around me—it's easy to fall behind when you're alone."* Another student reflected, *"The flexibility was great, but it was also easy to say, 'I'll do it later' and then suddenly you're behind."*

From the mandatory course evaluation, we see in Table 1, that the overall feedback from students was positive. However, many found the pace and the workload overwhelming, as reflected in comments such as: *“A bit too much material for the available time*” and *“Very well-organized program. Although the workload was large and challenging, it was manageable thanks to a good plan and structured content on the Canvas page.”*

	Average score (from 1 – 5, were 5 is best)
The work and teaching methods in the course contribute to a good understanding of the subject matter.	4.18
I perceive the course as relevant to my studies.	4.23
Teaching contributes to my academic development.	4.25
The work and teaching methods are varied and appropriate.	3.95
The use of digital tools in teaching contributes to effective learning.	4.29
Academic collaboration among students (in groups/study circles) contributes to a positive learning environment.	4.24
Academic collaboration between students and teaching assistants contributes to a positive learning environment.	4.27

Table 1. Students’ feedback

Students' performance:
To determine if the changes made to the course had any effect on student performance, anonymous grade data was extracted from the university's database for the fall semesters of 2023 and 2024 for the Calculus 1 course. Students retaking the subject were not included in the analysis, as the focus was on first-time, first-year students. The students were then categorized into "TRES" (summer course students) and "Ordinary" (regular students) based on their type of enrollment, and the frequency of each grade for the two groups was counted. As shown in Figure 1, this also includes a small number of students registered as "N/A," indicating registration but no participation from these students. While counted as a fail, these students are separated to provide a more realistic picture of the distribution of grades. It is important to note that some students enrolled as "TRES" are exempt from MA-006 due to previous education, creating some discrepancies in the data as they did not participate in the summer course. Although no definite data exists for the number of students this applies to, the difference in the number of students is between 5-6%, which is unlikely to significantly affect the data; average course size indicates this applies to between 5-10 students.

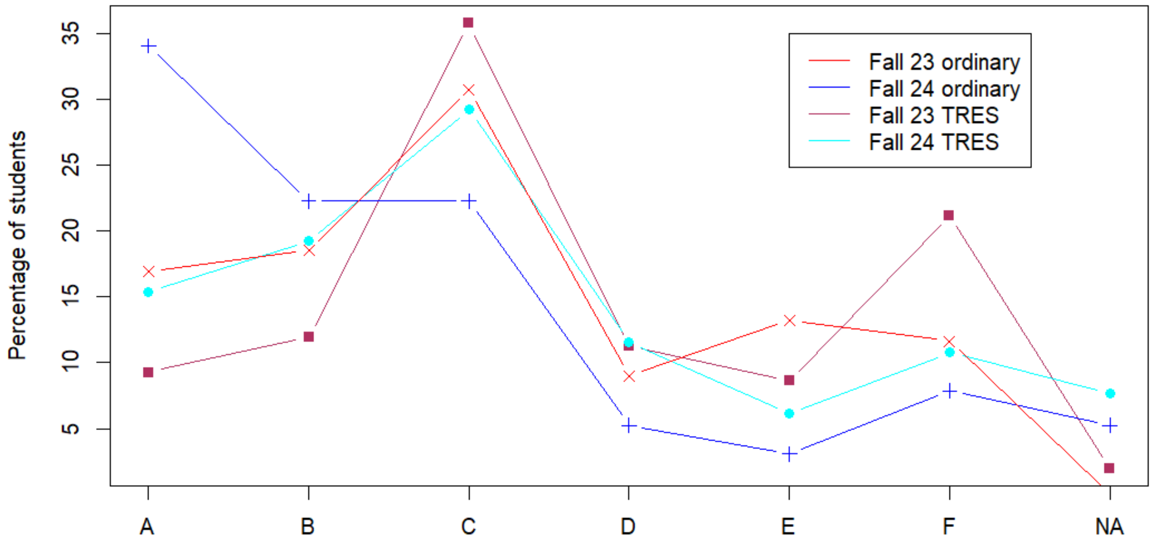

Figure 1. Students' performance data.

For both groups, a general increase in high grades and a reduction in students failing the course can be observed. Most significant is the reduction of fail grades among "TRES" students, falling from 21% to 11%. Overall, these numbers indicate a positive impact of the changes in course structure.

Conclusion and findings

Conclusions so far shows that teaching assistants, were valued for their approachability and understanding. Students relied heavily on course-provided materials and AI tools like ChatGPT for clarifying concepts. Motivation and self-discipline were crucial for success, with campus attendees finding the collaborative environment motivating. Students reported a smooth transition to Calculus 1, with no significant knowledge gaps. The course's intensity and segment-based testing system were particularly appreciated.
The opportunity for multiple attempts on the partial tests has encouraged students to take the tests without aids to see how well they understand the subject. The partial tests provide feedback on what they need to work on and the chance to improve their results immediately. The possibility of multiple attempts has also helped reduce stress levels in an otherwise hectic school environment. However, we see that the great freedom in the course also makes it possible to use AI. ChatGPT answered all the questions on a test and achieved 81% correct answers. In other words, it is possible to pass the course without working and without the necessary competence. However, this will not be possible when students start with university-level mathematics, where tests must be taken under supervision and without aids, something we have been clear about.
To evaluate the extent to which the course has been successful, we must first clarify our objectives. Are we aiming to educate the best students with deep understanding and high competence, or to make our engineering studies accessible to the widest possible range of students? This is a crucial discussion. If our goal is to educate the best students, we may need better control and more secure assessment methods. With the current structure, where students have significant freedom to use various aids such as ChatGPT, it is challenging to determine if the tests truly assess the students' individual knowledge and skills. Students who pass but lack a fundamental understanding of the subject may contribute to lower average grades and higher failure rates in subsequent mathematics courses. To ensure we are educating the best, we must develop assessment methods that minimize the risk of external aids distorting the results. On the other hand, if our overarching goal is to make engineering studies more accessible to a broader range of students, we are on the right track. The flexibility of the course allows more students to participate regardless of geographical location or other commitments. This accessibility enables students to adapt their learning to their own life situations, making education more inclusive and available to all who wish to participate. These two goals are not necessarily mutually exclusive, but they require different strategies and approaches. For future courses, we should consider how to balance the need to ensure high-quality education with the desire to include and support a diverse range of students. Regarding this course, our department's current strategy is to give students in the summer course the freedom to use all aids (to attract more students), rather than being strict. Students who choose to "cheat" their way through the summer course will be exposed in engineering mathematics, where tests must be taken under supervision in our newly built examination facility.
Finally, students have given positive feedback on the support from us teacher assistants. They have trusted that we master the course content, and they have appreciated the low threshold for asking questions.

The findings from these interviews not only inform our understanding of the student experience in MA-006 but also provide a valuable basis for further development of the course. As we prepare to offer MA-006 again next summer, insights from this research will help us refine the course structure, improve student support, and strengthen the overall learning experience.

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Engineer or Math Teacher?
– Career Preferences among Double-Degree Students in Engineering and Education

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Abstract

The shortage of STEM teachers in Europe poses a significant challenge. To address this issue in Sweden, Chalmers University of Technology offers a double-degree program in engineering and education. This study examines why students choose a double degree in engineering and education, and how internal and external factors influence their eventual career choice. Using semi-structured interviews with thirteen students nearing the completion of their studies, we explore their reasons for enrolling in the program and their perspectives on working as either engineers or teachers. Participants were asked to reflect on key individuals or experiences that shaped their views on these career paths. Key themes include intrinsic motivation, e.g., desire to help others, societal contribution, the impact of school practicum experiences, and the tension between perceived prestige of engineering and the meaningfulness of teaching. The teaching practicum in schools had a significant influence on career decisions. Some students found it deeply meaningful, reinforcing their commitment to education. Financial considerations had little impact, while job security and perceived societal respect were more decisive factors. Several students noted that engineering is generally regarded as the more prestigious career, leading them to frequently justify their choice of teaching to friends and family.

Introduction

Europe faces a persistent shortage of qualified STEM teachers, threatening efforts to sustain technological innovation and student engagement in science. (European Commission, 2023). Teachers are pivotal in inspiring future generations to pursue science and engineering careers, yet engineering graduates seldom consider teaching as a primary professional path.

In response to this challenge, Chalmers University of Technology in 2011 introduced a double-degree program in engineering and education, enabling students to qualify both as engineers and as upper secondary school teachers in mathematics and science (Bengmark, Lundh & Gerlee, 2021).

Students enter the double-degree pathway by enrolling in the master's program Learning and Leadership after completing a bachelor’s in engineering at Chalmers. This master's program attracts about 20 students annually, i.e. about 1% of the cohort at Chalmers. About half of the graduates from this double degree program in engineering and education choose to take their first job in schools, while the most of the others pursue careers as engineers in industry.

Their dual qualification raises intriguing questions about career decision-making: what motivates students to enrol in such a hybrid program, and how do they ultimately choose between two distinct professional paths? Zarske, Vadeen and Tsai (2016) found that students studying a similar double degree program in the US identified themselves as both engineers and teachers. Cronhjort (2017) found that student chose a double degree program in engineering and education as they found the combination attractive and because they were unsure of what career to choose.

While studies have explored dual-identity professionals, little is known about how engineering-education students in Sweden navigate their final career decisions. In this study we want to investigate the factors that shape career preferences among students enrolled in the double-degree program at Chalmers in Sweden, who are uniquely positioned at the intersection of two professional identities.

We explore how personal motivations, educational experiences, and social influences interact to guide their career decisions. Both external and intrinsic motivation is considered, especially drawing on Self-Determination Theory where autonomy, mastery, and purpose emerged as key drivers of intrinsic motivation (Deci & Ryan, 2012). By examining the respondent's reflections, we aim to shed light on how career aspirations evolve and what this means for efforts to address the STEM teacher shortage through interdisciplinary education.

We have chosen the following two research questions.

> RQ1: At what time did the respondents decide to enrol in a double-degree program in engineering and education, and why?
>
> RQ3: Which internal and external factors shape students' career decisions when entering the job market with degrees in both engineering and education?

Method

Thirteen students in the final stage of the double degree program were interviewed. All participants had completed undergraduate studies in engineering. They were in the final stage of their masters studies eligible for a double degree in engineering and education, leading to teaching certification in upper secondary mathematics and either physics, chemistry, or technology. All 13 students enrolled in the final year of the Learning and Leadership program at Chalmers during the 2024/2025 academic year were invited and agreed to participate.

Semi- structured interviews were conducted by the author (Ruslin et al., 2022). The semi-structured interview guide was designed to align with the study's two research questions and covered three chronological phases: motivations before university, experiences during the program, and anticipated career decisions.

Each interview lasted between 30 and 60 minutes and was audio-recorded and transcribed verbatim. The interview guide was informed by self-determination theory (Deci & Ryan,

2012) and previous work on double-degree programs (Bengmark et al., 2021, Zarske, 2016).

Thematic analysis followed Braun and Clarke's (2006) six-phase framework: familiarization with the data, generating initial codes, searching for themes, reviewing themes, defining and naming themes, and producing the report. Transcripts were coded inductively, allowing patterns and themes to emerge organically from the data. Coding focused on motivations, perceived barriers and enablers, and social influences. Trustworthiness was enhanced through iterative coding and triangulation with program documentation and prior research on engineering education and teacher recruitment. Informed consent was obtained from all participants.

Results

Some respondents decided to go for the double degree program already when studying at high school. Several of them specifically went to university to be able to get a double degree.

> "I found this program already in the second year of high school [...] it was the plan all along." (S4)

Several of these respondents explained their choice by having double interests and that they could delay their choice of career through a double degree program,

> "I wasn't 100% sure [...] so teaching was my main goal, but to keep the door open, it sounded great to become both an engineer and a teacher". (S6)

Several participants mentioned being influenced by inspirational teachers or family members. For instance, one respondent was inspired by two female teachers who were also engineers.

> "I had two very good high school teachers who were both engineers and teachers. [...] I thought I wanted to become like them." (S4)

In the local region there are other much more broadly known institutions for teacher education. Chalmers is primarily known for engineering education. Still there are early choosers wanting to become teachers that found their way to the double degree program.

> "'When I was about 17, I decided I wanted to become a teacher. [...] then I found out that you could study Learning and Leadership" (S9)
> "I wanted both to study more physics and math and to become a teacher. [...] that was the plan all along." (S4)

There were also respondents who decided to choose the double degree program when they were studying their bachelor at Chalmers. Several of them found an interest in teaching during their bachelor's education.

“I got quite involved in teaching activities during the entire bachelor’s, and I thought that was really, really enjoyable... So that’s kind of the background to why I chose Learning and Leadership later. (S2)

Some of them had been involved in organizations teaching high school students, others just helped their bachelor peers. One respondent describes the culture of peer teaching in their engineering program and the intrinsic satisfaction from those moments as aligning well with a teaching career.

"There is clearly an interest in helping to teach your classmates from the perspective of 'I know this, I can explain it to you' [...] many are really quite good at teaching each other". (S3)

Some refer to the joy in making a difference for others and wanting to make a difference.

“If I can be the teacher I needed [...] then that’s what I want to do.” (S9)
"You can feel greater meaning in school [...] in influencing young people in the right direction". (S1)
“I want to move the world in a positive direction?” (S13)

Some students saw the double degree as a way to keep doors open. They noted the appeal of flexibility—being able to work either in industry or education. It would give economic security and versatility in career options in an uncertain job market.

“It’s also nice to have both paths [...] it makes me feel more secure.” (S3)
“I won’t be unemployed [...] that’s what’s great about my education.” (S8)

Now let us turn our focus to the responses pertaining to the second research question. In the end of their studies, what makes them choose job sector?

Many students highlights that the school-based teaching practicum was formative and sometimes a decisive factor. Some students found that the classroom environment energizes them, confirming their interest in teaching or made them they realize they prefer other forms of education or the structure of industry.

“The practicum was character-defining in a way.” (S4)
“I don’t want this practicum to end.” (S9)
“I’ve done a lot of training [...] but within industry. That appeals to me more.” (S1)

Students sometimes weigh external validation in their final decision. Some feel teaching lacks prestige and must defend the choice to sceptical peers or family, while others feel affirmed in that direction. Students reported feeling the need to justify their career choices, especially when leaning toward teaching.

“A bit like you’re throwing your education away [...] that’s the general attitude.” (S4)

> "They said to me: are you going to be a teacher? [...] I said: you can become both a teacher and an engineer." (S8)

Finally, several students stated that they plan to start their careers in engineering, not because they see it as their destination, but because they believe it will be easier to transition from engineering to teaching than the other way around.

> "It feels easier to go into engineering and fall back on teaching, rather than start in teaching and then try to find my way back into engineering." (S6)

None of the respondents thought that salaries had an influential part in their decision, referring to that the starting salaries for engineers and teachers in Sweden are comparable even if they can differ considerably later in the career.

Discussion

Some students identified their dual interests early, choosing the university specifically for the double-degree opportunity. This early commitment often stemmed from double interests and a desire to keep career options open, aligning with the findings of Zarske et. al. (2016). Others discovered an interest in education during their bachelor's studies, often through teaching-related extra-curricular or peer-support experiences. Both early and late commitment to studying a double degree program aligns with the Chalmers initiative as described by Bengmark, Lundh, and Gerlee (2021),

The students' final career decisions were shaped by both internal motivations and external validations. Practicum experiences were often transformative, providing emotional resonance and practical insights that clarified professional identities. Those leaning toward the teaching profession described a strong sense of agency, purpose, and societal contribution, echoing the concepts of autonomy and meaningfulness from self-determination theory (Deci & Ryan, 2012). Others, however, found greater alignment with the structured, perhaps more predictable environment of engineering work.

Societal perceptions also emerged as critical external influences. Several students noted that teaching, though personally rewarding, is seen as less prestigious than engineering, a view sometimes expressed by peers and family. These perceptions sometimes necessitated justification of their choices, especially when diverging from the expected engineering track, an issue also noted in the Education and Training Monitor (European Commission, 2023) which highlights societal undervaluation of teaching in STEM fields. Interestingly, many students expressed the intention to start in industry and potentially transition to education later. This strategy was seen as both a practical response to labor market dynamics and a way to preserve long-term flexibility.

Notably, salary considerations were not a significant factor for most participants. This contrasts with common assumptions about economic incentives in career choice but aligns with Kahneman and Deaton's (2010) finding that income beyond a certain threshold does not strongly affect life satisfaction. In the Swedish context, respondents viewed both careers as viable in terms of financial stability.

Implications for education

By sharing the experiences, we hope to encourage other universities to address the STEM teacher shortage by offering similar double-degree programs in engineering and education. This study offers insight into aspects that are valuable to consider when designing and running such a program.

For example, it highlights the role of early exposure to teaching, through tutoring, mentoring, or school-based experiences, as a recurring factor in shaping participants' interest in education. Integrating structured opportunities for teaching practice earlier in engineering curricula may help identify and support students with latent interest in teaching. Furthermore, the practicum was a pivotal experience for many students, often confirming or reshaping their career aspirations. This underscores the importance of ensuring high-quality practicum placements.

The double-degree students appreciated the flexibility and job security. Programs should recognize and support this hybridity to make students find a career path that suits them also if it means moving between the sectors over time. By acknowledging and responding to these insights, institutions can better support students at the intersection of engineering and education—and potentially help alleviate the chronic shortage of STEM teachers through more purposeful recruitment and retention strategies.

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The Role of Mathematical Competencies in Engineering Research: Insights from Digital Signal Processing PhD Scholars

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Abstract

In the Digital Signal Processing (DSP) research community, mathematical concepts are deeply embedded in the way signals and systems are represented and used (Broesch 2008). Researchers assume familiarity with both the technological and mathematical concepts that underly signals and systems. This study seeks to explore how engineering researchers perceive and apply mathematical competencies within their research, specifically in DSP. Engineering students often need convincing to see the relevance of mathematics to their future profession (Coupland 2008). Understanding how advanced researchers apply mathematical competencies could offer valuable insights. This study aims to explore the mathematical competencies that PhD researchers in DSP apply in their work, including competencies like reasoning mathematically, using aids and tools, and communicating mathematically (Winkelman 2009). While there is existing literature on professional competencies and the role of mathematics in industry, this paper focuses specifically on the perspectives of DSP engineering researchers in academia. This is a case study used to illustrate what engineers think and perceive the mathematical competencies as and how they connect it to their research. We aim to extend these findings to other engineering disciplines, providing a broader understanding of the role of mathematics in engineering research.

Introduction

Competencies are not unheard of when we think of education and curriculum. There is a variety of competencies ranging from professional, inter-disciplinary and long-term competencies that are currently being researched with the aim of designing a good curriculum for engineering (Alpers 2020). We have always argued that the mathematics that is taught in engineering curricula is relevant, cohesive and important. Mathematical competencies are one of the fundamental groups of competencies that exist and are developed in every engineering curriculum, in most cases implicitly (Wong et al 2022). This paper is based on a case study involving 13 engineering PhD students in the DSP community of KU Leuven to indulge in an exploratory analysis which allowed explicit reflection and discussion on these mathematical competencies.

While professional competencies and industry applications of mathematics have been studied, less is known about how academic researchers apply mathematical competencies in DSP. The motivation of this research is twofold. Firstly, in the context of research on mathematical competencies, the perceptions and discussions of PhD researchers help us consolidate the definitons of these competencies. Secondly, the conclusions can be used to see the applications of these competencies in the activities of DSP research. The questions that we can answer based on this study help us understand the role of these competencies in DSP research and can be broadly summarised below:

- How do DSP researchers perceive the importance of different mathematical competencies in their work?

- How do DSP researchers apply mathematical competencies in their daily research activities?

Methodology

In this study, we employed a workshop-based case study approach to explore how DSP researchers perceive the relevance of different mathematical competencies in their work. The workshop involved 13 PhD researchers specializing in DSP, each receiving a handout (see Appendix) with a description of one of seven mathematical competencies, with a description (Firouzian 2015), based on the framework by Niss & Højgaard (2019). Participants were asked to rate the importance of their assigned competency on a continuous scale from 0 to 5, reflecting its relevance to their research. Additionally, they provided qualitative examples of how they applied the competency in their work. Competencies were assigned randomly to ensure a broad distribution of perspectives. This approach allowed for an exploration of competencies without leading participants toward specific areas of interest. This method allowed us to gather both quantitative data on perceived importance and qualitative insights into real-world applications. The qualitative data were categorised into recurring or highly emphasised themes discussed during the workshop as well as written answers in the handouts, to find insights into the use and perception of these mathematical competencies in DSP research. The methodology not only aimed to collect data but also provided a reflective space for the researchers, allowing them to evaluate their own proficiency in these competencies and encouraging a deeper understanding of their own research practices in relation to mathematical tools and concepts.

Findings and Discussion

The findings revealed that all competencies were rated highly, with participants identifying their importance in DSP research. Researchers were able to explicitly connect these competencies to their work. They provided detailed examples of how these competencies are applied in signal processing algorithms, optimization and acoustic modelling. The observed data also highlights how mathematical competencies are both visible and implicit in their daily research practices. Table 1 shows a concise summary of descriptions of each competency by each researcher and their scoring on a scale from 0 to 5 indicating how they rate themselves in terms of mastering the competency as well as how important they think it is in their field of expertise. The last column indicates the authors' list of extracted themes/ideas that can be concluded from the workshop after analysing the observations. The table is followed by a discussion of each competency where the reflections of the authors as well as the participants have been portrayed.

Competency	Observation Notes	Self	Importance	Main Extracted Idea
Thinking Mathematically	Identifying reusable parts of derivations for generalized algorithms (e.g., acoustic echo cancellation)	3	4	**Generalizing** mathematical structures from specific derivations. **Recognising** relevant parts of a derivation.
Communicating Mathematically	Understanding literature, discussing ideas conceptually, refining derivations, aligning notation, communicating research	3.5, 3.5	4, 5	**Conceptual communication** of mathematics across expert and non-expert audiences.
Using Aids and Tools	Using built-in MATLAB functions, developing new tools, balancing external and self-made tools, optimizing software	4	5	**Strategic use** and critical understanding of mathematical tools.
Symbols and Formalisms	Translating between notations across fields; clear expression needed, notational abuse	2.5, 4	5, 2	**Mastery** and **translation** of symbolic mathematical language, **variability** of symbols across different domains
Mathematical Modelling	Evaluating assumptions; model selection based on context (e.g., GW detectors, sound localization)	3.5	4	Critical evaluation and adaptation of mathematical models, focusing on the **assumptions** made
Representing Mathematically	Choosing appropriate representations (time vs frequency domain); transformation benefits solution properties	3.8, 3	5, 4	Strategic **selection** and **transformation** of representations
Reasoning Mathematically	Building/testing tools; proving convergence; combining models from data	4, 3.8	5, 4	Building, **validating**, and refining mathematical reasoning

Table 1: Overview of the seven mathematical competencies under consideration Niss & Højgaard (2019), with summarized observation notes from handouts given out during the workshop, ratings on a 5-scale indicating self-proficiency in and importance of each

competency, and main extracted ideas from the observed as well as written text. Each competency was analyzed by one or two PhD researchers.

Thinking Mathematically

"How does a species like Homo sapiens learn to think mathematically?" (Tall 2013) It is a question that educators and students have thought about for decades and there is abundance of literature that illustrates the answer to this question in mathematics, philosophy and the sciences. We also sense the vastness of this question and in terms of definitions this is the one that is the most difficult one to put in a closed bracket. It is not surprising that this is one of the toughest competencies to describe as it sits as an umbrella term for all the mathematics that we observe and do.

Mathematical thinking includes the ability to understand and judge what kind of questions can be answered using mathematics and hence where a mathematical approach might be helpful (Alpers et al 2013). In terms of recognising their alignment with the competency, the PhD researcher made an analogy to their work with acoustic echo and noise cancellation. The researcher describes the competency with respect to their work as *identifying* certain parts of a *mathematical derivation* that could lead up to an *initial algorithm*. This in itself is a strong statement as it allows us to see that there exists a potential algorithm that needs to be understood or created and is recognised as well as the mathematical approach that can be taken in order to do so. They further talk about *reusing* or *generalising* what is found to apply to other spaces. Thinking mathematically was evidenced in the reflection about generalizing derivations for example, identifying reusable parts of an integrated algorithm for combined acoustic echo cancellation and noise reduction to extend it for additional interference sources. Rated 3 in self-assessment and 4 in importance, this highlights the skill of abstracting core mathematical structures from specific cases.

Communicating Mathematically

Mathematics is a language and there are formal rules to use it unambigously. In general, engineering students make a concious effort to use appropriate terminologies only if the environment of learning has facilitated both thinking and communication skills (Rahman et al 2012). Most often a researcher needs to present their work in the community that understands all its definitions (conferences and publications) as well as communicate their work to a group of people who are not experts in that field (outreach programs and seminars). This idea led to some substancial reflections that established a description of this competency, based on the response by two researchers. They broke down the competency in 3 major chunks: Understanding literature and own research, collaborating with peers, and aligning definitions and notation. One researcher stated that the competency is used to connect different sources and then *linking* it to their own research in order to establish *connections*.

One key quote was, "Discussing ideas with others requires understanding the maths on a conceptual level," indicating that presenting work, even preliminary findings, demands clarity of thought and expression. This researcher rated their communication competency at 3.5 but rated its importance at a strong 5. This is not the case only with researchers in DSP but for all of mathematics related research. It allows to present one's work in seminars and conferences and hence it is essential to know one's own work well. However researchers are simultaneously answering research questions as well as defining and finding new results, especially in engineering. This can put the researcher in a tight spot as they are still trying to convey to an audience about something that is not yet found or defined conceptually well.

Using Aids and Tools

This is a competency that was discussed quite easily. All researchers agreed that MATLAB optimization routines were the ones that were mentioned explicitly. They reiterated the fact that there are existing (previously designed) solvers that are used in optimization problems. “However many times we are making these tools/aids ourselves from scratch. We work a lot on developing tools using equations from maths. But you cannot do everything, so rely on the tools other people have developed,” explained an engineers way of thinking perfectly!

Mathematics is increasingly mediated by computational tools, from built-in functions in software like MATLAB to specialized solvers for optimization problems. The competency of using aids and tools emphasizes not just the ability to use these tools but also understanding their underlying principles and limitations. As one researcher stated, while these tools can simplify calculations, a deep understanding of how they work is necessary to use them effectively.

In the reflection, a strong emphasis was placed on using tools like built-in MATLAB functions and *optimization solvers,* while also developing custom tools when necessary. The researcher scored themselves highly (Self: 4; Importance: 5), acknowledging that reliance on external aids must be balanced with understanding the underlying mathematics. As noted, "to tighten a screw you better need a screwdriver," illustrating the pragmatism behind tool use. Self-ratings suggest that researchers feel confident in using these aids, especially when *applying existing tools* to their research. However, research in mathematical tool usage indicates that an over-reliance on software without understanding its underlying mechanics can lead to a superficial understanding of the problem at hand.

Symbols and Formalisms

The use of symbols and formalism is an essential part of mathematical communication. However, as one researcher noted, different fields and papers often use different notations, which can be a source of confusion. Understanding how to translate and work with various symbols is a vital skill for researchers and can further be further broken down to the idea of Symbol Sense (Arcavi, A. 2005). Symbol sense broadly describes a student’s tendencies when using symbols and notation.This competency is frequently used when writing scientific articles, where precision in notation is necessary to convey complex ideas clearly.

The researcher's reflection revealed the constant negotiation involved in interpreting differing notational systems across fields and translating complex ideas into mathematical expressions for publication. While self-rated at 2.5 but recognizing its importance at 5, the reflection noted the challenge of "*notation abuse*" being common but needing attention, particularly when translating rather abstract concepts like Riemannian manifolds into practical applications. Understanding formal symbols requires unpacking the *logical structures* behind them, a necessary skill when bridging theory and application (Selden & Selden 1995). Self-ratings suggest students find this competency somewhat challenging, likely due to the complexity and variability of symbols across different domains. Mastery of this competency is necessary not only for academic writing but also for understanding the works of others.

Mathematical Modelling

Even though this competency is most mentioned in engineering mathematics, a systematic literature review indicated that there is a necessity for further theoretical work on conceptualizing mathematical modelling competencies as well as its implementation at various stages in education (Cevikbas et al. 2022). Mathematical modelling involves translating real-world problems into mathematical structures, often underpinned by assumptions that must be critically evaluated. As one researcher pointed out, decisions about which model to use and how to represent signals and systems depend heavily on the problem at hand. This competency is particularly relevant in fields like sound source localization and signal processing, where models are used to interpret data and make predictions. Mathematical modelling appeared prominently in the reflections, where the researcher described *evaluating assumptions* in physical models (e.g., likelihood assumptions in gravitational wave detection, model choices in supernova waveforms). This reflective critical stance was rated 3.5 in self-assessment and 4 for research.

Mathematical modelling constitutes a major challenge for students of different skill levels (Blum & Niss, 1991). Researchers self-assessed themselves moderately in this area, indicating that while they recognize the importance of modelling, they may not always feel confident in applying models to new situations. Effective modelling requires moving fluidly between real-world phenomena and abstract representations, explicitly considering and adjusting the assumptions embedded in models, precisely the skill the researcher reported using when debating the relevance and generality of models.

Mathematical Representation

Mathematical representation is essential in conveying complex concepts, particularly in fields like signal processing where problems are often expressed in the *time or frequency domain.* To solve increasingly complex problems, the biological brain needs to organize concepts in ways that enable them to be manipulated easily and often these entities, known as Procepts, can represent a concept as well as a process, simultaneously. An elementary procept is the amalgam of three components: a process that produces a mathematical object, and a symbol that is used to represent either process or object (Tall 2013). This aligns very well with the researchers' reflections, as different representations of the same problem can lead to insights about the problem's various properties that may not be visible in one representation. The researcher demonstrated a nuanced understanding of multiple representations, noting the choice between time and frequency domain formulations depending on problem properties (e.g. common-zeros problems). They reflected on the shift from variational models in the time domain to iterative solutions in the frequency domain, a practice that earned a 3 in self-assessment and a 4 in importance. Researchers rated themselves and expressed that they understand the value of using multiple representations to clarify and solve problems. Mastery of this competency enables the researcher to communicate their ideas more effectively and tackle a wider range of problems.

Reasoning Mathematically

Reasoning mathematically is a fundamental competency that underpins the entire process of mathematical inquiry. It involves logical thinking, deductive reasoning, and the ability to prove or disprove hypotheses based on sound mathematical principles. Researchers engage in formal mathematical reasoning (e.g., algebraic manipulation, proofs) and informal reasoning (e.g., intuition, estimation) when solving real engineering problems. As one researcher highlighted, mathematical reasoning is particularly critical in algorithmic development, where *iterative processes* must be justified and optimized.

Strong mathematical reasoning skills are developed over time, with Researchers progressing from more intuitive approaches to more formal, rigorous methods. This competency is essential not just for solving existing problems but also for creating new mathematical tools and algorithms. Mathematical reasoning was deeply embedded in the researcher's daily practice, particularly in building and proving iterative algorithms, comparing models, and *connecting theoretical results to empirical data*. Rated highly (Self: 4; Importance: 5), the reflection described "developing new tools based on intuition first, then testing optimality," and "pairing two models based on real data visualization."

Conclusions

This reflective study highlights how essential mathematical competencies are woven into the daily practices of researchers in Digital Signal Processing. Researchers establish that they do not simply apply mathematics but instead they actively shape, adapt, and refine it within their evolving research contexts. The observations show that while competencies are recognized as highly important, they also present significant challenges. Furthermore, the balance between using established tools and creating new ones reflects a deeper understanding of when to trust existing methods and when to innovate. Translating mathematical models across domains (e.g., time to frequency), handling formalisms with care, and critically evaluating model assumptions all emerge as core competencies needed for high-quality DSP research. The reflections also underscore the real-world tensions:

how to communicate incomplete or evolving ideas clearly, how to align notation across fields, and how to reason rigorously even when facing intuitive or uncertain grounds.

Ultimately, these findings suggest that fostering these mathematical competencies should be seen not as peripheral but central to training for engineering research. The results suggest that mathematical competencies are integral to DSP engineering research. Strengthening them can significantly enhance the ability of researchers to innovate, collaborate across disciplines, and contribute impactful results to the broader signal processing community. Additionally, by examining how these competencies are used in advanced research, we can better align teaching methods to meet the needs of future engineers. Further research could extend these findings to other engineering disciplines, deepening our understanding of the role of mathematics in engineering research and education.

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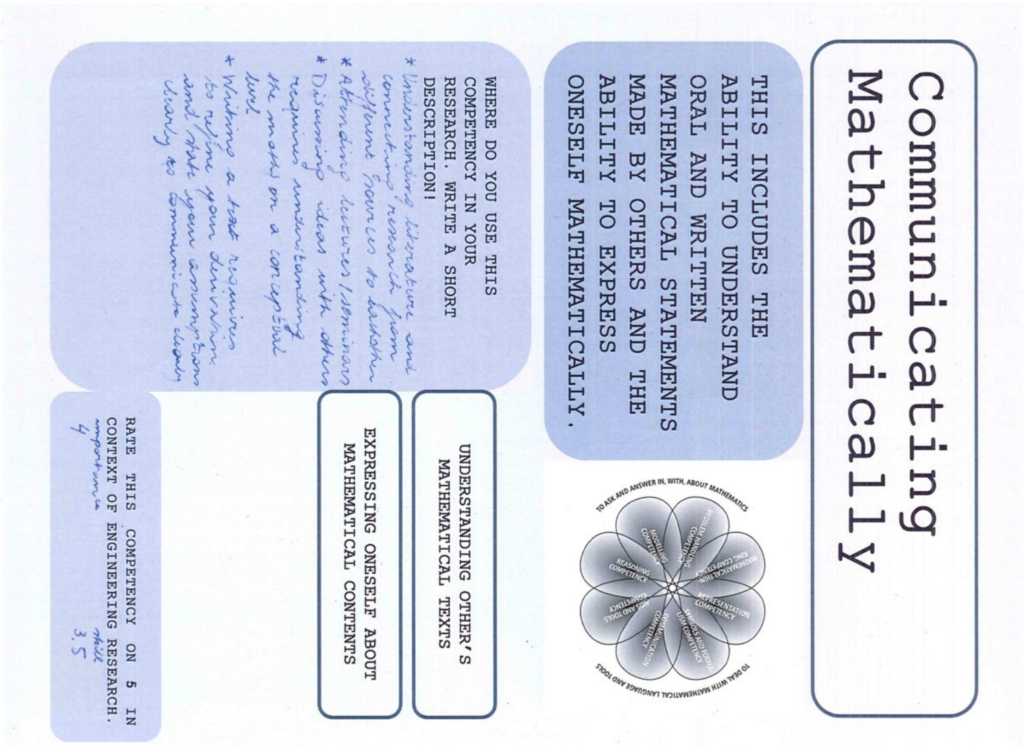
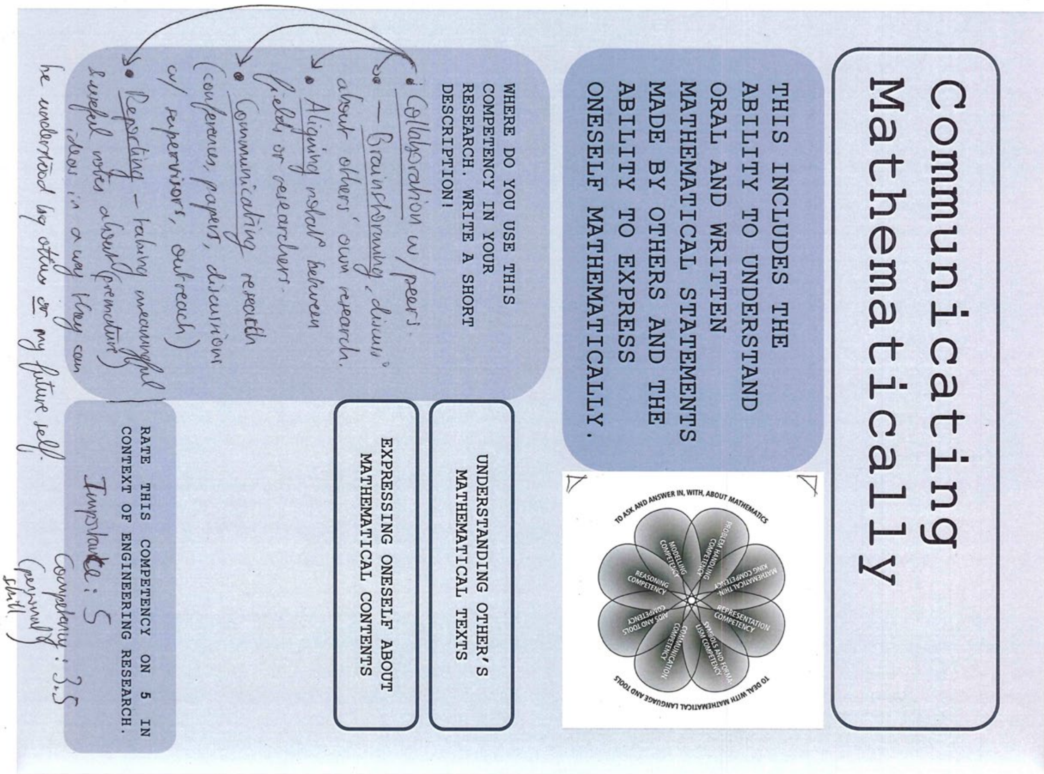
Appendix A


Communicating Mathematically
THIS INCLUDES THE ABILITY TO UNDERSTAND ORAL AND WRITTEN MATHEMATICAL STATEMENTS MADE BY OTHERS AND THE ABILITY TO EXPRESS ONESELF MATHEMATICALLY.
WHERE DO YOU USE THIS COMPETENCY IN YOUR RESEARCH. WRITE A SHORT DESCRIPTION!
UNDERSTANDING OTHER'S MATHEMATICAL TEXTS
EXPRESSING ONESELF ABOUT MATHEMATICAL CONTENTS
RATE THIS COMPETENCY ON 5 IN CONTEXT OF ENGINEERING RESEARCH.
TO ASK AND ANSWER IN, WITH, ABOUT MATHEMATICS
TO DEAL WITH MATHEMATICAL LANGUAGE AND TOOLS
4
3.5


Communicating Mathematically
THIS INCLUDES THE ABILITY TO UNDERSTAND ORAL AND WRITTEN MATHEMATICAL STATEMENTS MADE BY OTHERS AND THE ABILITY TO EXPRESS ONESELF MATHEMATICALLY.
WHERE DO YOU USE THIS COMPETENCY IN YOUR RESEARCH. WRITE A SHORT DESCRIPTION!
UNDERSTANDING OTHER'S MATHEMATICAL TEXTS
EXPRESSING ONESELF ABOUT MATHEMATICAL CONTENTS
RATE THIS COMPETENCY ON 5 IN CONTEXT OF ENGINEERING RESEARCH.
TO ASK AND ANSWER IN, WITH, ABOUT MATHEMATICS
TO DEAL WITH MATHEMATICAL LANGUAGE AND TOOLS
Importance: 5
Competency: 3.5

Should we go back to master classes when teaching mathematics

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Abstract

Mathematics is a fundamental pillar in engineering education, as it acts as a cornerstone for developing the analytical and problem-solving skills essential for future engineers. However, despite its undeniable importance, many students struggle with this subject. The abstract nature of mathematical concepts, difficulty integrating theory with practical applications, and an initial lack of interest in mathematics often hinder success in the subject. Many students enter engineering programs with a preference for subjects directly related to engineering practice, leading them to reject mathematics early in their studies. In addition, large classrooms can make active participation difficult, further complicating the learning process. Continuous assessment and the master class are key elements in higher education, each with advantages and limitations. This paper analyses these two methodologies, highlighting how their combination can optimize academic results, promote student participation and overcome the individual limitations of each approach. Moreover, this paper shows the analysis of students results from the 2021-2022 to the 2024-2025 academic year. Students often prefer continuous assessment but tend not to spend much time on continuous activities during the course. The combination of lectures and continuous assessment offers a promising approach to overcoming the challenges of teaching engineering mathematics. By addressing both the theoretical and practical dimensions of learning, this integrated methodology can help students develop the analytical and problem-solving skills they need to succeed in their engineering careers.

Introduction

Both conventional, teacher-centred teaching and contemporary active methodologies have significant advantages. By integrating the most valuable aspects of both approaches, mathematics educators can design a balanced pedagogical strategy that responds to the diverse needs of students. Thus, whether through classical or innovative methods, the fundamental goal of education is to facilitate the full development of our capabilities in a constantly changing environment (Pandya et al, 2024).

The effectiveness of teaching methodologies has been a recurring topic of research, prompting educators to explore how to refine the efficiency and impact of learning. In mathematics teaching, a common method is to introduce topics through initial examples, then gradually move towards more abstract ideas, allowing students to build their understanding through continuous practice (Norton, 2024).

Several key features distinguish the traditional educational approach, such as (Lee & Paul, 2023; Lessani et al., 2017):

- Oral exposition (lecture-based instruction) wherein educators disseminate content orally to the students. This format is typically characterized by unidirectional communication, with the instructor serving as the conduct of knowledge transmission to the learners.
- Teacher-center teaching in which the educator assumes the role of knowledge holder and director of the pedagogical process, while students are primarily expected to engage in receptive listening, note-taking, and adherence to directives.
- Emphasis on memorization. Students are often evaluated based on their capacity to memorize and reproduce information in assessments, potentially at the expense of fostering deep comprehension and critical reasoning faculties.
- Well-defined curriculum, typically articulated through standardized and structured syllabi and textbooks. The academic content is organized into discrete subjects and topical modules.
- Standardized summative evaluation that predominantly relies on standardized examinations and tests. These evaluation instruments are designed to measure students' ability to recall information and demonstrate competence in specific academic domains according to predefined evaluative criteria.
- Passive knowledge reception: Learners often assume the role of passive recipients of information, expected to assimilate knowledge through auditory and textual input. Aspects related to active participation, development of critical thinking skills or experiential learning activities may be limited.
- Restricted integration of technological resources, i.e., fundamental educational technologies may be incorporated, however, the substantive integration of technology to facilitate interactive and dynamic learning experiences is typically less prevalent when contrasted with contemporary pedagogical approaches.

The traditional paradigm of mathematics teaching is primarily passive, teacher-centered learning. One of the consequences of this approach is lack of engagement, high drop-out rates and lack of understanding among students, a frequent phenomenon in many classrooms around the world. Indeed, aspects such as the absence of critical thinking in mathematics are major concerns. Innovative pedagogical methodologies, such as game-based learning, project-based learning, collaborative learning, flipped classroom model, etc., have emerged as possible solutions to this problem and have shown significant potential to improve student outcomes (Babo et al., 2023; Caridade et al., 2018; Queiruga-Dios et al., 2020; Rasteiro et al., 2018).

Method of Investigation

A cross-sectional quasi-experimental design will be employed, taking advantage of the implementation of a continuous evaluation scheme that coexists with a final exam of considerable weight. This will allow comparison of student performance under a system incorporating continuous evaluation (CE) elements with the performance that would historically have been expected under a purely traditional model (although an explicit control group without CE is not available).

The study population will consist of students enrolled in the second-year engineering mathematics course during the 2021-2022, 2022-2023, 2023-2024, and 2024-2025 academic years. The sample size will vary slightly between courses, ranging from 52 to 61 students per academic year.

The official grade records of the course will be used to obtain the final grade of each student in the courses analyzed, as well as the continuous evaluation records, i.e., the data of the grades obtained by the students in the different activities that make up the continuous evaluation: grades of the problems handed in, computer practices, questionnaires proposed in the course platform and works of application of mathematics to engineering. The grades obtained in the final exam will also be extracted for a more detailed analysis.

The data collected will be analyzed using descriptive and inferential statistical techniques. In a first descriptive phase, measures of central tendency (mean, median) and dispersion (standard deviation, minimum and maximum) will be calculated for each of the quantitative variables, differentiated by academic year. Graphical representations such as box plots will also be used to explore patterns and detect possible outliers. Since the distributions are not strictly normal, descriptive results should be interpreted with caution.

In the inferential phase, three analyses of variance (ANOVA) will be applied, one for each quantitative variable, considering academic year as a factor. Although the data do not fully comply with the assumption of normality, nor with the homogeneity of variances in all cases, the ANOVA analysis will be maintained due to its acknowledged robustness to moderate deviations from these assumptions, especially when sample sizes are similar and sufficiently large, as in this study (all over 50 participants and with balanced sizes between courses). If statistically significant differences are found, a post-hoc analysis will be performed using the Bonferroni test.

The statistical analysis will be carried out with the statistical software: R and SPSS.

Findings and Discussion

The study was carried out on the basis of the grades of a total of 219 engineering students, distributed over four consecutive academic years with relatively balanced sample sizes. In the academic year 2021-2022, 52 students participated (23.7% of the total), as did in the academic year 2022-2023 (52 students, 23.7%). In 2023-2024 54 students (24.7 %) and in 2024-2025 61 students (27.9 %) were assessed.

Table 1 presents a basic descriptive analysis of the three quantitative variables considered: continuous assessment, exam mark and final subject mark. Measures of central tendency and dispersion were calculated for each of these variables.

Table 1. Descriptive analysis of student academic performance (Continuous Assessment, Exam Score, and Final Course Grade) (n = 219)

Variable	n	Mean	Median	SD	Min	Max
Continuous assessment	219	6.69	8.00	3.31	0	10
Exam score	219	4.06	4.50	2.92	0	9.73
Final score	219	4.65	5.16	2.78	0	9.78

Overall, the descriptive results show a higher concentration of high marks in the continuous assessment compared to the final exam, which has a direct impact on the final marks. This first approximation already allows us to intuit differential patterns that will be contrasted in the subsequent inferential analysis.

Analyzing these scores segmented by year, it can be seen that in continuous assessment there seems to be a notable difference between the 2021-2022 academic year and subsequent years (Figure 1). In particular, the median in 2021-2022 is significantly lower, and the dispersion of scores is much greater, with a range that covers practically the entire possible spectrum (0 to 10). From 2022-2023 onwards, the distributions narrow and shift towards higher values, especially in 2024-2025, where both the median and the interquartile range are clearly higher and concentrated at the high end of the scale. These patterns suggest an evolution in performance or in the application of the continuous assessment system over the years analyzed.

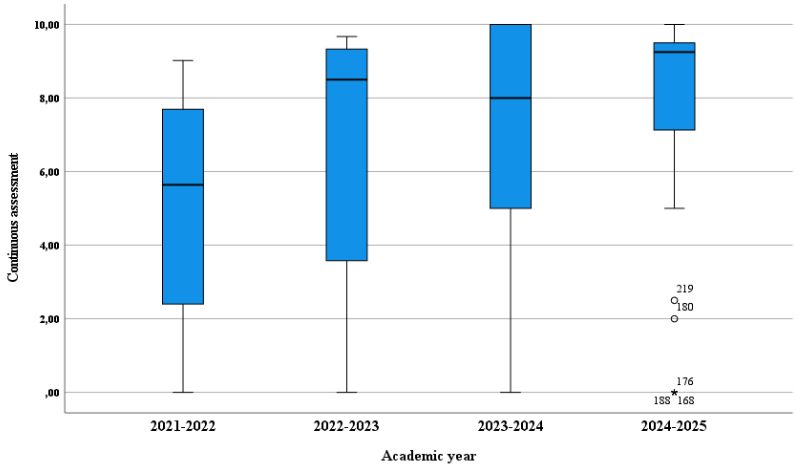

Figure 1. Distribution of continuous assessment scores by academic year

As for the exam grade, there are no relevant differences between the courses analyzed; the medians remain around intermediate values and the dispersion is wide in all cases, suggesting a sustained variability in performance in final tests (Figure 2 left). The final grade shows a similar pattern, although with a slight upward trend in the medians as of the 2022-2023 academic year, which could reflect a greater contribution of continuous assessment to overall performance or a better integration of the assessment system over time (Figure 2 right).

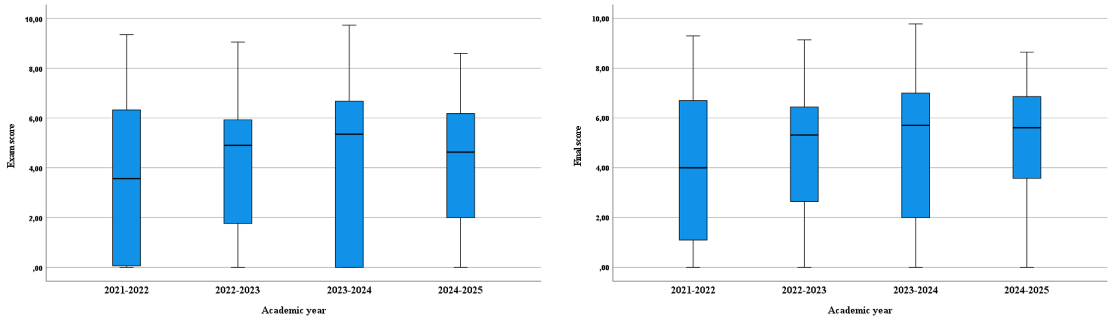

Figure 2. Distribution of final exam scores (left) and final course grades (right) by academic year

In the inferential analysis, a one-factor ANOVA was applied to test whether there were significant differences in the grades according to academic year. The results are shown in Table 2.

Table 2. Results of one-way ANOVA comparing academic performance variables across academic years

Dependent Variable	F-statistic	p-value
Continuous Assessment	8.669	<0.001
Final Exam Score	0.724	0.539
Final Course Grade	2.120	0.099

Statistically significant differences were obtained only in the Continuous evaluation variable (p-value < 0.001), indicating that at least one of the academic years presents a different distribution. The Bonferroni test revealed differences between the 2021-2022 academic year and the three subsequent academic years, suggesting that this academic year presents an atypical pattern with respect to continuous assessment. This divergence could be related to the post-pandemic context, which may have affected both the implementation of the assessment system and student performance. In contrast, the ANOVA results for the final exam grade (p-value = 0.539) and the final course grade (p-value = 0.099) showed no statistically significant differences between the different academic years. These results suggest that, despite possible methodological or contextual changes, the overall performance on the final exam and the final grade remained stable throughout the period analyzed.

Conclusions for Education

Significant differences were found in continuous assessment scores across academic years, with 2021–2022 showing lower and more dispersed results, likely reflecting post-pandemic effects. From 2022–2023 onwards, scores became more consistent and higher. In contrast, no significant differences were observed in final exam scores or overall course grades, indicating stable performance in summative assessments throughout the period.

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Just-In-Time Learning: using Skill Trees to connect Challenge-Based Learning to fundamental skills

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Abstract

The amount of Challenge-Based Learning (CBL) applied in university curricula is growing. A common pitfall when using CBL is that the projects are disconnected from the fundamental theory and skills (e.g., mathematics) that students learn. This results in missed learning opportunities as well as a reduced motivation for students.

This paper presents a solution to this problem using Skill Trees. Skill Trees are an educational tool that allow teachers to plan which skills students need to master, when they need to master it and how they can work towards said mastery. The application to Project-Based Learning, in which all students have a similar project, is still relatively straightforward with little effort required from the teacher. An extension concerns the application of Skill Trees to CBL, where project topics can widely differ. Here the teacher needs to play a more active role coaching student groups. In both cases, the result is a Just-In-Time Learning set-up that more strongly connects fundamental skills and theory to applied project work.

Interviews with students have indicated both an increased motivation as well as a stronger mastery of fundamental skills, as compared to using CBL without just-in-time teaching.

Keywords: Challenge-Based Learning, Knowledge Graphs, Skill Trees, Just-In-Time Learning, Course Structuring

Introduction

In recent decades, STEM education has seen a shift from traditional classroom learning towards learning in projects. Instead of only learning basic skills, students are tasked with solving more practical problems. The idea is that this grows the students' confidence of applying their skills in practice. (Allan, 2007; Taconis and Bekker, 2023)

There are many variations to exactly how to set up such a project. Is the project part of a course, or a separate course? Is theory taught to students separately, or do they need to figure it out on the go? Are the problems to be solved simulated, or from an actual company? And do all student groups get the same/a similar task, or is there a wide variety of topics? Though the literature is not unanimous on the naming (Sukackė et al., 2022) we will consider *Project-Based Learning* (PBL) as having a mostly simulated project, with fixed scope, often coupled to theory, while *Challenge-Based Learning* (CBL) is more broadly scoped, often with assignments from actual companies, where students have to figure out for themselves what skills are needed to tackle the challenges at hand.

A key element of PBL/CBL is that students apply the skills they learned. Ideally this is coupled: students learn a skill and quickly afterwards they get to apply it in practice. This

Just-In-Time Learning (JITL) set-up increases both the motivation of students to learn, as well as the retention of the skill. (Abdulwahed and Balid, 2009; Killi and Morrison, 2015) In practice, this direct connection is often much less present. In many curricula the theory courses and project courses are developed separately. Students complain that they don't know why they're learning the theory they are learning. By the time they get to the project, a large part of the obtained knowledge is forgotten. (Gallagher and Savage, 2022)

To reduce this disconnect, a new competency-based didactic method called Skill Trees has been used to couple fundamental skills to their application in PBL/CBL courses. In this paper, we first briefly present this method. We consider how we can apply it in PBL/CBL courses in general and then look at actual applications in courses. In the end we evaluate these applications to derive lessons learned.

Structuring educational content through Skill Trees

To couple educational contents to projects, it helps to structure the respective contents. In particular, we want a method that deals well with competency-based content: modelling what students can and cannot do. Ideally suited for this are Skill Trees. These are defined and explained in more detail in (Bijl, 2025), but the main definitions and ideas will be summarized here.

The fundamental component behind the Skill Tree is the *skill*: a range of actions that are deemed similar enough to make them comparable. A skill can have a variety of *exercises* associated with it: a challenge of doing a certain action, with requirements to verify if it was done in a satisfactory way. Exercises can typically be split up into steps. Each of these steps are then again exercises, which could each be coupled to other skills. This gives us the *subskills* of the respective skill: a skill *A* is a subskill of a skill *B* if executing *A* is commonly a significant step in executing *B*. By visualizing the links between all skills, we find the so-called *Skill Tree*: a Directed Acyclic Graph (DAG) whose nodes are skills and whose edges are prerequisite relations between the skills. An example is shown in Figure 1.

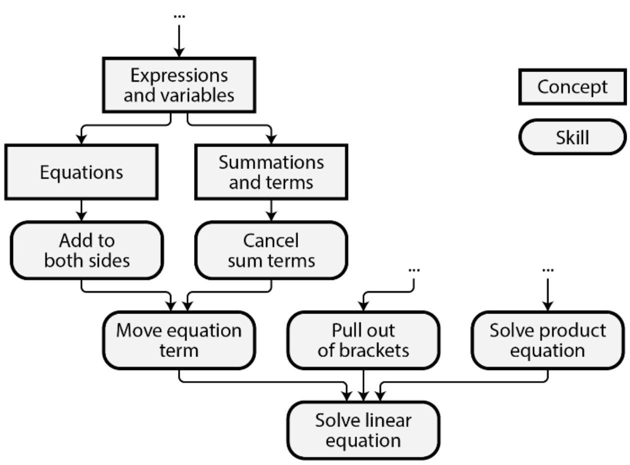

Figure 1: An example Skill Tree (simplified) around the skill *Solve linear equation*.

Coupled to Skill Trees are Concept Trees. A *concept* is an idea or definition (or combination thereof) that one can visualize or have an intuitive grasp of. Commonly a concept is a notional machine (Du Boulay, 1986; Sorva, 2013). We say that a concept *A* is a *subconcept* of a concept *B* if grasping this concept *A* is directly necessary to grasp concept *B*. The overview of all concepts is the *Concept Tree*: a DAG whose nodes are concepts and whose edges are prerequisite relations between the concepts. Concept Trees and Skill Trees are connected, since skills often also require concepts. The converse is not true: concepts never require skills.

Using Skill Trees as tool to plan courses

Suppose that we have developed a Skill Tree for a set of theory. How can we use it? There are three distinct cases that each have their own challenges and possibilities.

In *classical education* there are generally no projects. Education focuses on the fundamental contents. In this case, having a Skill Tree is mostly beneficial for lecture planning: the Skill Tree helps teachers plan in which order to teach concepts/skills and how to divide these subjects over class sessions. On top of this, Skill Trees help teachers and students track progress. By noting each student's mastery level for each skill and visualizing this in the Skill Tree, teachers and students can immediately see how far along each student is. This progress tracking can be done in a variety of ways, from a simple 'You have done three exercises correctly, so you have mastered this skill' up to a more elaborate probability-based tracking algorithm. (Bijl, 2024) Either way, having a clear overview of progress helps motivate students in their studies.

Skill Trees become more useful in the case of *Project-Based Learning* (PBL). In PBL there is generally a mostly predefined project, with pre-determined schedules and steps. In many curricula, a PBL project is often coupled with a theory course; either as two separate courses, or as one joint course together with the project. In this case the Skill Tree can help the theory course in teaching the right contents at the right time. If the project requires students to have mastered (for example) the skill *Solve linear equation* in week 4, then the theory course can immediately see which prior concepts and skills need to be taught, ensuring sufficient mastery at the right moment. Doing so may seem like common sense, but in practice this rarely occurs to a sufficient degree and having a tool like a Skill Tree makes this matter more concrete and visual, facilitating the *Just-In-Time Learning* (JITL) process.

The application of Skill Trees in the case of *Challenge-Based Learning* (CBL) is even more powerful, but also more challenging. In CBL courses, the project is usually not clearly defined in advance and with an (often) external stakeholder involved, the skills needed to successfully tackle the project are unknown in advance and will differ per project group. In this case, having a Skill Tree of all contents that may be relevant for the project is even more beneficial. If students, in consultation with their supervisors/coaches, for instance decide 'We must apply a regression analysis' then with the help of a Skill Tree they can see all the subskills they need to learn to be able to apply such a regression analysis. In this case teachers take more of a coaching role, advising students what they need to study, while the students take charge of their own learning, following the learning path they plotted for themselves. The Skill Tree serves as the map where they plot the learning path. Students decide on their own timeline – what they need to know when – in true JITL fashion.

A simplified example of a study plan based on a Skill Tree is shown in Figure 2. Here a student has decided to master a skill J in four weeks' time. The Skill Tree shows ten skills (or identically concepts) that need to be mastered to get there. For the first week, the goal is to master skill C, which also requires skills A and B. For the second week the focus is on mastering E. The third week is about getting to H. The fourth week then allows the student to master J, just in time for its application in the project.

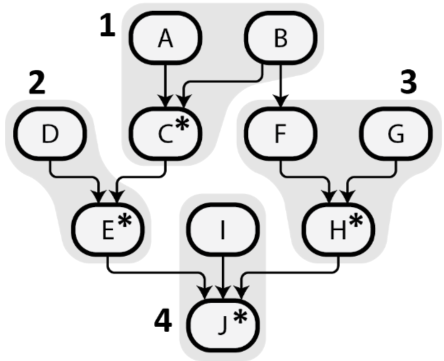

Figure 2: An example Skill Tree used to plan what needs to be learned in JITL fashion. Skills are grouped into blocks (e.g., weeks) based on intermediate targets (asterisks).

Application in practice

The above ideas have been applied at the Institute for Design and Engineering of the Utrecht University of Applied Sciences. Specifically, there have been three places in the curriculum where the Skill Tree methodology has been applied, each in one of the ways described above. We will study their application to show the experiences learned.

The first application concerns the first-year courses of the *Mechanical Engineering* bachelor. The first year is split up into four quarters (15EC each) and each quarter has three courses (5EC each). Two of each quarter's courses are theoretical courses, of which one focuses on STEM (mathematics, physics, etcetera) and the other on related theory (design theory, material sciences, etcetera). The third course is a project course, formally called *Professionalization*. In the past, all courses operated independently of each other and a common complaint from students was that they didn't see how the mathematics they learnt was relevant to what they did. To fix this, the following steps were taken.

1. The theoretical courses formalised/visualised their structure using Skill Trees.
2. The project courses identified the skills they needed in their projects and which weeks these skills were required/applied.
3. The theoretical courses adjusted their schedules to teach these skills directly prior to the respective application.

The restructured schedule worked well at showing students why they learned certain fundamental skills. Agreeing on these connections between courses helped teachers refer to each other's courses, 'You will need this for your project course next week.' It also facilitated students at using the language they learned in their theoretical courses within their project, 'We just have to calculate the *cycle efficiency*, like we learned in class.'

The second application concerns a master course *Data Science*, part of the master *Next Level Engineering*. This 5EC course was set up based on PBL. The course consisted of two parts: theory and project work. The theoretical part was taught in two-hour lectures on Mondays, after which students had to do a few basic exercises to master the presented skills. (First collecting/cleaning data, then setting up a classification/regression/clustering algorithm and finally analysing the developed algorithm to answer the business question.) Each of the lectures used Skill Trees to outline the skills that were taught and how they related to each other. For the second part of the course – the project – student groups (of up to four students) had to define their own Data Science project, subject to various requirements. These requirements ensured that the project was of sufficient depth and that it fit well with the schedule in which the theory was presented. In practice some students chose projects more focused on classification (week 4) while others chose projects requiring more regression (week 5), so not all topics were equally used by each group. Wednesday's classes were used to coach the project groups on their progress. The final grade was partly (50%) based on the project work (written group report) and partly based on each student's theoretical understanding (individual oral exam).

The third application concerns a third-year bachelor course with the (translated) title *Create: Creation of new products and processes*. In this course, students were tasked to design and develop a new product, including a detailed analysis of how it would work in practice. (Think of mechanical stress tests, vibration analyses and more.) There were companies providing potential projects, but students could also define their own projects or use examples provided by teachers. Topics and scales could vary significantly, ranging from a tiny part of a robot arm to a city-wide energy distribution network. Also the timeline varied, with some groups mostly doing design work, while others developing various prototypes. To ensure that students would have access to the right theory, the course offered a large set of potential modules that the students could learn through self-study. Students were not required to do all modules (there would be too many) but decided themselves, in consultation with teachers, on what would be useful to learn for their project. Part of the project reflection revolved around how the theory learned was used within the project. Though there was a minimum amount of modules that needed to be done, many students surpassed this minimum. The final grade was based on the full portfolio of students, which included both small module assignments (individual, often auto-graded) as well as project work (in groups, manually graded).

Results and conclusions

To check the impact of the implemented changes, the course evaluations and course pass rates have been examined and focused interviews have been held with both teachers and students. We will discuss the results of each of these investigations below.

For cases 2 and 3 (Data Science and Create, both new courses) the respective courses consistently had the highest student evaluation ratings in their respective programs. This shows that students generally appreciate the set-up. For case 1 (Professionalization, an existing course) the evaluation ratings were on average somewhat higher than before Skill Trees were used to connect projects to theory, but the differences were small and there were also various outliers, especially during the corona pandemic.

The pass rates for all courses have in general been good. Especially worth noting is that the Create course (case 3) replaced a variety of theoretical courses with traditional exams. These exams always had a relatively low pass rate, resulting in some students being ‘stuck’ in their curriculum because they lacked one course still needed to graduate. The self-study modules and their exercises, on the other hand, worked a lot better for students. They could take their time to do this well, not being subject to the time pressure of exams. This effectively solved the problem of students being stuck in their studies.

In focused interviews, teachers have mainly noted the differences in type of work. For case 1 the differences weren’t very pronounced, but for case 2 and especially case 3 the role of teacher has shifted more from ‘provider of knowledge’ into ‘project coach’. Most teachers indicated that, while this new role comes with more uncertainty/improvisation and is hence more challenging, it is also far more rewarding since there is a lot more in-depth contact between teacher and student. Workload-wise there was not much of a difference. In case 3, a large amount of work went into setting up all self-study modules, but afterwards this effort paid off and resulted in a workload reduction.

Students have indicated in interviews to be positive about the changes. Most students appreciated not having the stress of exams anymore, as well as doing work more connected with practical applications. In case 3 students mentioned they needed some time to get used to the idea of setting own learning goals, but most students found it a far more satisfying experience. A minority of students preferred the classical set-up, ‘Just tell me what to study and I’ll study it.’

The opinions of students and teachers on the use of Skill Trees have generally been positive. In cases 1 and 2 most teachers found them a helpful tool in planning their course, while most students were ambivalent, ‘It’s nice to have a Skill Tree as an overview, but we need to study everything anyway.’ For case 3 students were a lot more enthusiastic about the Skill Trees. Instead of needing a teacher to help them set up steps to reach certain learning goals, they could plan their own path, increasing their autonomy. ‘Checking out all the modules and how they were connected was like planning a vacation. What would be fun and useful? And how do we get there?’

Overall we can conclude that methodically connecting theory to projects improves the learning experiences of students and Skill Trees are a helpful tool, both for teachers to facilitate this connection and for students to plan learning paths towards individual learning goals.

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A Project-Based Learning Case Study in Operational Research for the Construction Industry

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Abstract

In recent years, the academic and industrial sectors have increasingly recognized the significance of Project-Based Learning (PBL) as an effective educational approach that aligns closely with the needs of the modern workforce, particularly in engineering disciplines. This paper explores the application of PBL in operational research within the construction industry, focusing on how it enhances the assessment and development of competencies among engineering students at the Technical University of Civil Engineering Bucharest (UTCB). The study is rooted in a collaboration between UTCB and a local construction company facing a real-world problem: optimizing machine utilization to maximize profit. The project not only provides a tangible context for students to apply theoretical knowledge of linear programming but also challenges them to develop solutions that have direct implications on the company's operational efficiency. This setup allows for an authentic assessment of student competencies in problem-solving, critical thinking, and collaborative skills. Utilizing a comparative study design, this research contrasts the outcomes from traditional lecture-based methods with those achieved through PBL. The methodology includes both qualitative and quantitative measures, such as student engagement surveys, performance assessments, and a detailed analysis of the solutions proposed by the students. Initial findings suggest that students engaged in PBL exhibit a higher level of understanding and practical application skills. They also report greater satisfaction and a deeper sense of accomplishment, indicating enhanced intrinsic motivation. Moreover, the study highlights the role of PBL in fostering a proactive learning environment where students are encouraged to take ownership of their learning process. This shift not only improves educational outcomes but also equips students with the professional competencies necessary for successful careers in engineering. Additionally, the collaboration with industry partners enriches the educational experience, providing students with invaluable insights into real-world engineering challenges and expectations. This paper argues that PBL is not merely an educational tool but a critical pedagogical approach that bridges the gap between theoretical knowledge and practical application, thus preparing students more effectively for the complexities of the modern construction industry. It concludes with recommendations for integrating PBL into engineering curricula to enhance competency assessment and meet the evolving demands of industry and society.

Introduction

The construction industry, a critical driver of economic growth, faces significant challenges related to logistics and transportation management. Efficient movement of materials, equipment, and personnel between sites is essential for project success, yet these operations are often plagued by inefficiencies, delays, and excessive costs. Addressing these logistical challenges is not only vital for the smooth operation of individual projects but also has broader implications for productivity and sustainability in the sector. To this end, operational research (OR) has emerged as a powerful tool for optimizing transport-related decision-making. By applying mathematical models and optimization techniques, companies can significantly improve the efficiency of their transportation logistics, leading to reduced costs, shorter delivery times, and enhanced project coordination.

This paper explores a student-led initiative from the Technical University of Civil Engineering Bucharest that leverages operational research to solve transportation problems in the construction industry. The learning service developed by these students is designed to provide companies with innovative tools and solutions for improving transport logistics. The service emphasizes the use of OR techniques such as linear programming, network analysis, and route optimization to address specific logistical challenges faced by construction companies. The initiative is not only an educational tool but also an example of how academia can directly contribute to solving real-world industry problems.

The problem of transportation optimization in construction is multifaceted. Companies must deal with the physical constraints of moving large and heavy materials across varied and often complex terrains. Furthermore, they face scheduling and cost-related constraints that can result in resource wastage and delays. Traditional methods of managing transport logistics, which often rely on intuition or manual planning, are increasingly insufficient in today's fast-paced construction environments. This is where operational research provides a structured and scientific approach to solving transport problems. By creating models that factor in multiple variables and constraints, OR can help construction companies make more informed decisions about how to allocate resources, plan routes, and schedule deliveries.

The objectives of this paper are twofold: first, to present the concept of the learning service and the role of students in its development; and second, to demonstrate how the application of operational research can provide practical solutions to transport issues in the construction industry. The learning service is designed not only to enhance the understanding of OR techniques but also to offer hands-on tools that companies can implement immediately. Through interactive learning modules and case studies, the service bridges the gap between theoretical knowledge and practical application, making it a valuable resource for construction professionals.

The structure of this paper is as follows: The second chapter provides an overview of operational research techniques with a focus on their relevance to solving transportation problems in the construction sector. The third chapter delves into the learning service model, explaining its design, functionality, and student-driven innovation. Chapter four presents case studies where the learning service has been applied to real-world construction logistics scenarios, highlighting its impact on optimizing transportation routes and reducing costs. Finally, the fifth chapter concludes by summarizing key findings and discussing the future potential of this initiative, particularly in scaling the service for broader use in the construction industry and beyond.

The introduction of operational research-based learning services offers a promising avenue for improving transportation logistics in the construction industry. The student-led initiative from the Technical University of Civil Engineering Bucharest serves as a prime example of how academic knowledge can be translated into practical solutions for real-world challenges. By providing a structured, mathematically grounded approach to transport optimization, this service has the potential to revolutionize how construction companies manage logistics, paving the way for more efficient, cost-effective, and sustainable practices.

Operational Research in Transportation

Operational research (OR) has established itself as a critical tool for addressing complex decision-making problems, particularly in industries such as construction, where efficient resource allocation and logistical planning are paramount. In the context of transportation, OR provides a structured, mathematical approach to optimizing the movement of goods, materials, and personnel. Its application in the construction industry is of particular significance, as transportation accounts for a substantial portion of project costs and can significantly influence project timelines. By leveraging operational research techniques, construction companies can streamline their transportation logistics, reducing costs and improving overall efficiency.

One of the key advantages of operational research in transportation is its ability to address the inherent complexity of logistical systems. Construction sites, often scattered across multiple locations, require a constant flow of materials and equipment. Managing these flows involves a multitude of variables, including distance, time, cost, and capacity constraints, making it a challenging task to achieve optimal outcomes using traditional planning methods. OR techniques such as linear programming, network optimization, and simulation modeling allow companies to analyze these variables systematically and generate solutions that maximize efficiency while minimizing costs.

Linear programming is one of the most widely used OR techniques in transportation. It involves creating a mathematical model to represent a real-world problem, with the goal of finding the best possible solution within a set of constraints. In the case of construction transportation, linear programming can be used to determine the optimal routes for material deliveries, taking into account factors such as fuel costs, vehicle capacity, and delivery deadlines. By solving these models, construction companies can identify the most cost-effective routes, ensuring that materials arrive at the right place at the right time, while minimizing the distance traveled and the associated costs.

Another powerful OR technique in transportation is network optimization, which focuses on optimizing the flow of goods through a network of nodes and links. In the construction industry, this could involve determining the most efficient way to transport materials from suppliers to construction sites, while minimizing delays and costs. Network optimization allows companies to visualize their supply chains as interconnected systems and identify bottlenecks or inefficiencies that can be addressed through better route planning or more effective use of transportation resources. By modeling transportation networks mathematically, OR provides construction companies with a detailed understanding of how materials move through their supply chain and how these movements can be optimized to enhance efficiency.

Route planning, a critical component of transportation logistics, also benefits from the application of operational research. In construction, materials must often be transported over long distances, across multiple locations, and under tight time constraints. OR techniques such as the traveling salesman problem (TSP) and vehicle routing problem (VRP) provide mathematical frameworks for determining the optimal sequence of deliveries and the best routes for transportation vehicles. By applying these techniques, construction companies can minimize travel time and distance, reduce fuel consumption, and ensure that deliveries are made in the most efficient manner possible.

The relevance of operational research to the construction industry cannot be overstated, particularly when considering the significant impact transportation logistics has on project costs and timelines. Efficient transportation planning ensures that materials are available when needed, reducing delays and minimizing downtime on construction sites. Furthermore, by optimizing transportation routes and schedules, companies can lower their environmental footprint through reduced fuel consumption and emissions, contributing to more sustainable construction practices. In an industry that is often characterized by tight margins and intense competition, the ability to optimize transportation logistics through operational research provides a valuable competitive advantage.

Operational research also offers a flexible framework that can adapt to the unique challenges of the construction industry. Construction projects are often subject to unpredictable variables such as weather conditions, changing project scopes, and fluctuating material prices. OR techniques, particularly those involving stochastic modeling and simulation, allow companies to account for these uncertainties in their transportation planning. By creating models that simulate various scenarios, companies can develop contingency plans and ensure that transportation logistics remain efficient, even in the face of unexpected changes.

Operational research has proven to be an invaluable tool for solving transportation problems in the construction industry. Through techniques such as linear programming, network optimization, and route planning, OR provides construction companies with the means to optimize their transportation logistics, reducing costs and improving project efficiency. The ability to model complex transportation networks and account for multiple variables and constraints allows for more informed decision-making, ensuring that materials and resources are delivered in the most efficient and cost-effective manner. As the construction industry continues to evolve and face new challenges, the application of operational research in transportation will remain a key factor in driving efficiency, reducing costs, and promoting sustainability.

The Learning Service Model

The learning service model developed by students at the Technical University of Civil Engineering Bucharest represents an innovative approach to solving transportation problems within the construction industry. At its core, the service integrates operational research (OR) techniques into an interactive educational platform, aimed at enhancing the logistical decision-making capabilities of construction companies. This model serves a dual purpose: it provides companies with practical, data-driven tools to optimize their transport operations, while simultaneously offering a learning environment where users can deepen their understanding of operational research methodologies and their real-world applications.

The concept of this learning service stems from the recognition that many construction companies, particularly small and medium-sized enterprises (SMEs), often lack access to advanced logistical optimization tools. Traditional methods of managing transportation logistics, such as manual planning or basic software, may be sufficient for small-scale projects but fall short when addressing the complexity of larger construction operations. The learning service model, therefore, aims to bridge this gap by providing a platform where companies can access sophisticated OR techniques without the need for specialized knowledge in mathematical modeling.

The design of the learning service is based on two key pillars: accessibility and practicality. The platform was designed to be user-friendly, ensuring that construction professionals, many of whom may not have a background in operational research, can easily navigate the system and apply its tools to their logistical challenges. This was achieved by breaking down complex OR techniques into digestible modules, each focusing on a specific aspect of transportation logistics, such as route optimization, resource allocation, or cost minimization.

Each module within the learning service is designed to be interactive, allowing users to input data from their own operations, such as distances between sites, vehicle capacities, and material quantities. The platform then uses OR algorithms, such as linear programming and network analysis, to generate optimized solutions tailored to the user's specific needs. This hands-on approach not only helps companies find immediate solutions to their transport problems but also allows users to gain a deeper understanding of how operational research works in practice.

The service is structured around real-world case studies, drawn from the construction industry, to demonstrate the effectiveness of operational research in solving complex logistical issues. By working through these case studies, users are able to see firsthand how OR techniques can be applied to common transportation problems, such as determining the most efficient delivery routes, optimizing vehicle loads, or scheduling material deliveries in a way that minimizes downtime on construction sites.

The learning service model is unique in that it was developed entirely by students at the Technical University of Civil Engineering Bucharest. This student-led initiative highlights the power of academic environments as incubators for innovation, where theoretical knowledge is translated into practical solutions for industry challenges. The students involved in the project were motivated by a desire to apply the operational research skills they had acquired during their studies to real-world problems, and the construction industry, with its complex logistical demands, presented an ideal testing ground for these techniques.

Collaboration with industry professionals was a key component of the development process. The students engaged with construction companies to better understand the specific transportation challenges they faced and tailored the learning service to address these needs. This collaborative approach ensured that the service was grounded in practical realities, making it not only a theoretical exercise but also a tool that could be immediately adopted by companies to improve their operations.

The success of this student-led initiative also underscores the importance of experiential learning in higher education. By working on a project with direct industry applications, the students were able to develop skills in project management, software development, and user experience design, in addition to honing their operational research expertise. The learning service, therefore, serves as a testament to the value of combining academic knowledge with practical, hands-on experience in creating innovative solutions.

A key feature of the learning service model is its emphasis on user interaction and experience. The platform was designed with the end user in mind, ensuring that it could be easily integrated into the day-to-day operations of construction companies. The interface is intuitive, with step-by-step instructions guiding users through the process of inputting data, running OR models, and interpreting the results.

The interactive nature of the platform allows users to experiment with different variables, giving them a deeper understanding of how changes in one aspect of the operation, such as vehicle capacity or delivery schedule, can impact overall transportation efficiency. This experimentation fosters a learning-by-doing approach, where users not only solve their immediate logistical problems but also build a broader understanding of how to apply operational research principles to other aspects of their business.

Additionally, the platform includes feedback mechanisms that allow users to evaluate the performance of the OR models in real-time. For example, after implementing a suggested route optimization, users can track key performance indicators (KPIs) such as fuel consumption, delivery times, and cost savings, providing valuable insights into the effectiveness of the solutions generated by the learning service. This feedback loop helps to refine the models further and ensures that the service remains relevant and effective for its users.

The learning service model developed by students at the Technical University of Civil Engineering Bucharest represents a significant innovation in the application of operational research to the construction industry. By providing a user-friendly, interactive platform that integrates sophisticated OR techniques into transportation logistics, the service empowers construction companies to optimize their operations while also deepening their understanding of logistical challenges. The student-led nature of the initiative highlights the role of academic institutions as engines of innovation, where theoretical knowledge is transformed into practical, real-world solutions. As this learning service continues to evolve, it has the potential to make a lasting impact on the construction industry, improving efficiency, reducing costs, and fostering a culture of continuous learning and innovation.

Case Studies and Application

The value of operational research (OR) in the construction sector becomes most evident when applied to real-world logistical challenges. Among these, transportation stands out as a critical and often problematic component of project execution. Inefficiencies in the movement of materials between batching plants and construction sites frequently lead to delays and increased costs, especially in the absence of data-driven decision-making tools. OR techniques offer a structured and quantitative framework for optimizing such processes, enabling companies to reduce costs and improve operational efficiency.

This chapter builds on the theoretical and methodological foundation laid out earlier by presenting a practical case study involving a medium-sized construction company based in Bucharest. The company operates several concrete batching plants and active construction sites, requiring daily coordination of material deliveries. The core objective was to develop a transport plan that minimizes costs while meeting site demands and respecting production capacities. The case was modeled using linear programming and is based on real data, developed through collaboration between students and industry professionals.

Beyond demonstrating the computational power of OR, this case also highlights its pedagogical relevance. At the Technical University of Civil Engineering Bucharest (UTCB), the transportation optimization problem was introduced into the curriculum using two contrasting approaches: a traditional lecture-based format and a Project-Based Learning (PBL) format, supported by a student-developed interactive digital platform. This dual integration, technical and educational, provided a unique opportunity to evaluate not only the effectiveness of OR as an optimization tool, but also its role in enhancing student learning outcomes.

By exposing students to authentic engineering problems, the case allowed for a deeper engagement with the material, fostering critical thinking, collaboration, and applied problem-solving. The experience revealed OR's dual utility: as a powerful instrument for solving industrial problems, and as a pedagogical vehicle for cultivating professional competencies in engineering education.

A medium-sized construction company operating in Bucharest manages three concrete batching plants and three active construction sites. Every day, concrete must be transported from the plants to the sites in the most cost-effective manner possible, considering transport capacities and distances.

This recurring logistics task involves balancing the limited daily output of each batching plant with the specific concrete needs of each construction site. Transport costs vary depending on the distance between the origin and destination points, and the company must respect both the supply constraints at the plants and the demand requirements at the sites. Relying on intuition or manual planning often leads to suboptimal decisions that increase operational costs and resource waste. To address this challenge, the company explored the use of linear programming as a decision-support tool to develop a transport plan that ensures efficiency, feasibility, and minimal total cost.

	Site A	Site B	Site C	Supply (m^3)
Plant 1	12	15	13	100
Plant 2	14	9	16	120
Plant 3	11	13	10	80
Demand (m^3)	**80**	**110**	**110**	

Figure 1. Problem data.

- The costs are in €/m³ and are based on fuel, driver time, and delivery schedules.
- Objective: Minimize the total transport cost while fulfilling all site demands.

Linear Programming Model: Let x_{ij} represent the quantity of concrete transported from plant i to site j.

Objective function:

$$Min\,\{12x_{11} + 15x_{12} + 13x_{13} + 14x_{21} + 9x_{22} + 16x_{23} + 11x_{31} + 13x_{32} + 10x_{33}\}$$

Subject to:

$$\begin{cases} x_{11} + x_{12} + x_{13} \leq 100 \\ x_{21} + x_{22} + x_{23} \leq 120 \\ x_{31} + x_{32} + x_{33} \leq 80 \\ x_{11} + x_{21} + x_{31} \leq 80 \\ x_{12} + x_{22} + x_{32} \leq 110 \\ x_{13} + x_{23} + x_{33} \leq 110 \\ x_{ij} \geq 0, for\ all\ i,j \end{cases}$$

		Site A	Site B	Site C
1	Plant 1	70.0	0.0	30.0
2	Plant 2	10.0	110.0	0.0
3	Plant 3	0.0	0.0	80.0

Figure 2. Optimized Transport Plan.

The minimum total transport cost is €3160.

The integration of real-world engineering problems into the academic curriculum represents a powerful catalyst for meaningful learning. At the Technical University of Civil Engineering Bucharest (UTCB), the transportation optimization case described in the previous section served not only as a practical application of linear programming but also as a pedagogical experiment. The same problem was taught using two contrasting methods: the traditional lecture-based approach and the Project-Based Learning (PBL) format, supported by the interactive learning service developed by students. This dual implementation enabled a direct comparison of how teaching methodology influences student engagement, understanding, and skill development.

In the traditional setting, students were introduced to the theoretical principles of linear programming, followed by step-by-step demonstrations of classical transportation problems. Exercises were based on textbook scenarios, and students were evaluated through individual written assignments. While this approach ensured procedural fluency and theoretical comprehension, it often lacked the contextual richness necessary to fully appreciate the relevance and complexity of operational research in real-world settings. Feedback from students suggested that while they understood the techniques, their motivation was generally lower, and they perceived the material as abstract and detached from industry practice.

By contrast, the PBL approach introduced the same transportation problem within a simulated professional scenario. Students worked in teams, assuming the role of consultants hired by a construction company to optimize its concrete delivery logistics. Using the interactive learning platform developed at UTCB, they explored the problem iteratively: inputting data, running linear programming models, analyzing results, and adjusting assumptions based on feedback. This hands-on, exploratory process mimicked the uncertainties and dynamics of actual engineering work, offering students not only technical proficiency but also experience in decision-making under constraints.

Following the completion of the transportation optimization learning activity, all participating students (two groups of 25 students each) were invited to complete a structured feedback questionnaire. The objective was to evaluate and compare the perceived effectiveness of the two instructional methods: traditional lecture-based learning and Project-Based Learning (PBL).

Students were asked to rate the following six items on a Likert scale from 1 (very poor) to 10 (excellent):

- Level of Engagement: How actively did you participate during the learning activity?
- Understanding of Operational Research Concepts: How well did you understand the principles of optimization and linear programming?
- Skill Development: To what extent did you develop practical skills relevant to your future profession?
- Depth of Problem-Solving: How would you rate the complexity and depth of the tasks you worked on?
- Overall Satisfaction: How satisfied are you with the overall learning experience provided?

Performance in Evaluation was completed based on actual test results.)

Administration:

- The questionnaire was administered anonymously using a digital format (Google Forms).
- Participation rate was 100% for both groups.
- Average scores were computed and used for comparative analysis, as illustrated in Fig. 3

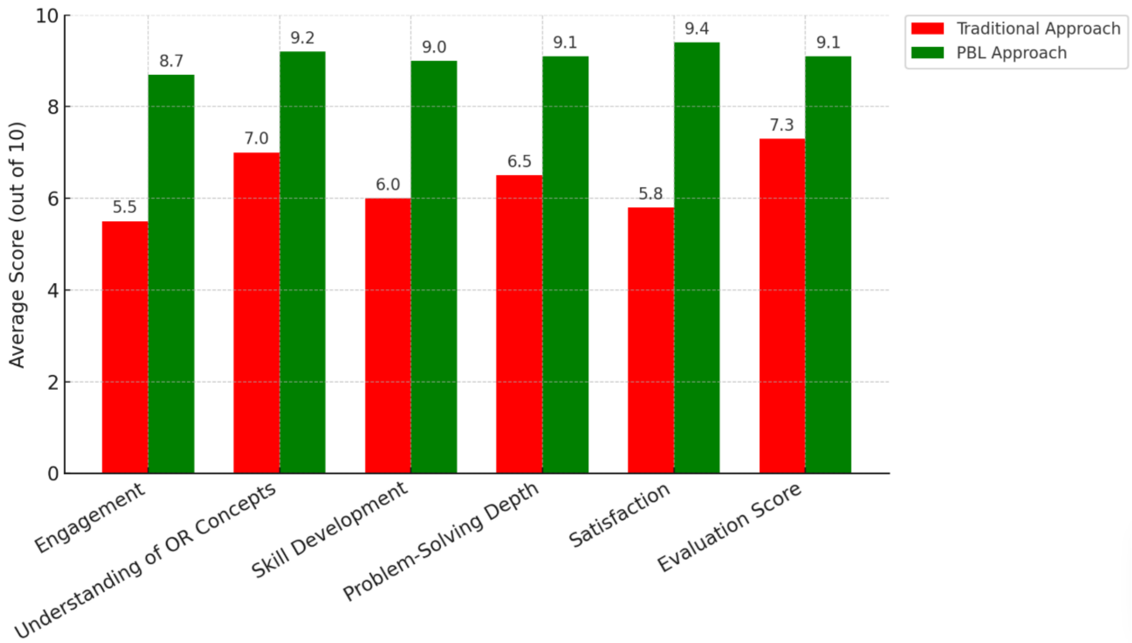

Figure 3. Comparison of Traditional vs. PBL Approach.

The outcomes between the two groups differed significantly. PBL students demonstrated a deeper understanding of the underlying optimization principles, often proposing creative adjustments to constraints or questioning the assumptions of the model. More importantly, they developed a broader range of transferable skills such as communication, teamwork, and critical thinking, that are difficult to cultivate through lecture alone. Their engagement was noticeably higher, and their feedback revealed a greater sense of ownership over the learning process. Rather than passively receiving knowledge, they actively constructed it through inquiry and collaboration.

Assessment results also favored the PBL group. While both cohorts successfully solved the problem, students exposed to project-based methods showed superior performance in tasks requiring interpretation, adaptability, and real-world reasoning. In follow-up interviews, several PBL participants reported feeling better prepared to apply similar models in their future jobs or internships, recognizing the value of operational research not only as an academic discipline but as a professional tool.

The pedagogical implications of this comparison are clear. When operational research is taught as an isolated mathematical topic, it risks becoming another abstract hurdle for engineering students. However, when embedded within authentic, purpose-driven projects, it becomes a gateway to systems thinking, resource optimization, and strategic planning skills central to any engineering discipline. PBL does not merely teach students how to solve problems; it teaches them how to frame them, interpret them, and communicate their solutions with confidence.

The dual use of the transportation problem, both as a technical model and a pedagogical experiment, demonstrated that the choice of instructional method deeply influences the quality and relevance of learning. While traditional teaching offers structure and clarity, PBL offers immersion and authenticity. For future engineers, whose careers will be defined by complexity and interdependence, the latter may prove indispensable. The experience at UTCB strongly supports the adoption of project-based methodologies in operational research education, especially when combined with digital tools and industry collaboration.

Conclusions and Future Directions

The development and implementation of the operational research-based learning service by students from the Technical University of Civil Engineering Bucharest mark a significant advancement in the intersection of academia and industry, particularly within the construction sector. Through the application of sophisticated operational research techniques, this innovative platform has successfully addressed critical logistical challenges faced by construction companies. The case studies presented in this paper highlight not only the practical benefits of using operational research in transportation logistics but also the broader implications for the construction industry as a whole.

The primary conclusion drawn from this research is that operational research serves as a powerful tool for optimizing transportation logistics in the construction industry. The learning service model has demonstrated that by integrating OR techniques, such as the Vehicle Routing Problem (VRP) and linear programming, construction companies can significantly enhance their logistical efficiency, reduce costs, and improve overall project outcomes. The two case studies illustrate the tangible impact of this approach, showcasing how data-driven decision-making leads to more effective resource allocation, reduced fuel consumption, and minimized project delays.

Moreover, the student-led nature of the learning service exemplifies the potential for academic institutions to contribute to real-world problem-solving. This initiative not only provides students with practical experience in applying their knowledge but also fosters a collaborative environment where academia and industry can work together to innovate and improve practices within the construction sector. The engagement of students with industry professionals in developing the learning service underscores the importance of experiential learning and the application of theoretical concepts to address real-world challenges.

While the results of this initiative are promising, it is essential to recognize some challenges and limitations encountered during the development and implementation of the learning service. One significant challenge is ensuring that the platform remains user-friendly and accessible to professionals without a background in operational research. As the complexity of OR techniques can be intimidating, continuous efforts are needed to simplify the interface and provide adequate training for users.

Additionally, the learning service's reliance on accurate and comprehensive input data is crucial for generating effective solutions. In practice, construction companies may struggle with incomplete or inconsistent data, which can impact the model's effectiveness. To address this issue, future iterations of the learning service should incorporate data validation tools and provide guidance on data collection best practices.

Looking ahead, several exciting opportunities exist for further development and expansion of the operational research-based learning service. First, enhancing the platform's capabilities to incorporate more advanced OR techniques and algorithms could provide users with even more robust solutions to complex logistical problems. For instance, integrating machine learning algorithms could enable the system to learn from past delivery data, predicting traffic patterns and material needs more accurately.

Furthermore, expanding the service to include mobile applications could enhance accessibility for users on construction sites, allowing real-time input of data and adjustments to transportation plans as conditions change. This flexibility would make the learning service even more valuable in the dynamic environment of construction logistics.

Collaboration with additional stakeholders in the construction industry could also broaden the scope and impact of the learning service. By partnering with larger construction firms, logistics companies, and academic institutions worldwide, the platform could evolve to include more comprehensive training modules and case studies, providing users with a broader range of scenarios and solutions.

Finally, as sustainability becomes an increasingly pressing concern within the construction sector, future iterations of the learning service should focus on integrating sustainability metrics into the OR models. By optimizing transportation logistics with an emphasis on reducing carbon emissions and waste, the platform could support the industry's shift toward more environmentally friendly practices.

In conclusion, the operational research-based learning service developed by students at the Technical University of Civil Engineering Bucharest represents a significant step forward in addressing transportation challenges within the construction industry. By applying advanced OR techniques, this initiative has proven capable of optimizing logistics, reducing costs, and enhancing overall project efficiency. The collaborative effort between students and industry professionals exemplifies the potential for academic institutions to drive innovation and problem-solving in real-world contexts. As the construction industry continues to evolve, embracing technological advancements and sustainable practices, the future of the learning service holds great promise for further enhancing its impact and relevance in the field.

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Beyond Traditional Learning: Digital Escape Rooms as an Engaging Tool for Mastering Integral Calculus

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Abstract

The integration of digital escape rooms (ER) into mathematics education has garnered increasing attention as a strategy to enhance student engagement and promote active learning (Borrego et al., 2017; Jensen et al., 2022). In today's higher education landscape, students have access to a diverse range of learning methodologies, including blended learning, adaptive learning technologies, and game-based instructional approaches that cater to different learning styles and preferences (Sailer et al., 2017).

This study examines the impact of the MATH-DIGGER digital escape room, ER1, on students' learning experiences in the context of integral calculus. Participants completed pre- and post-questionnaires to assess their expectations regarding escape rooms as a learning tool, their preferred learning styles, their confidence in solving mathematical problems, and their perceived difficulties both in learning mathematics and engaging with the ER 3D-environment. The collected data underwent statistical analysis to identify trends and correlations that provide insights into the effectiveness of this approach. Additionally, the structure and mechanics of the MATH-DIGGER ER1 are briefly described to contextualise its role in the learning experience (Babo et al., 2023; Pinto et al., 2024; Rasteiro et al., 2024).

Preliminary results indicate that students generally held positive expectations about learning through ER and demonstrated higher engagement compared to traditional instructional methods. However, the analysis also revealed variations in students' confidence levels and their ability to navigate mathematical challenges within the ER, highlighting key areas for pedagogical improvement. Furthermore, specific learning difficulties in integral calculus were observed, suggesting potential refinements in the design of future digital ER to better support conceptual understanding and problem-solving skills.

This study contributes to the growing body of research on gamified learning and mathematics education, demonstrating the potential of ER to create a more interactive and immersive learning experience. The findings offer valuable insights into the design and implementation of game-based learning environments in higher education mathematics curricula.

Keywords: Digital Escape Rooms, Mathematics Education, Integral Calculus, Gamified Learning, Student Engagement, Active Learning

Introduction

The design and evaluation of educational escape rooms (ERs) have attracted growing interest as innovative approaches to promoting learning through gamification and active engagement (Piñero Charlo, J. C., 2020; Queiruga-Dios, A. et.al., 2020). A central element in these experiences is the Game User Experience (GUX), which encompasses participants' interactions with the game, their perceptions, and the value they derive from it. Gaining insight into the core dimensions of GUX enables educators and developers to assess the effectiveness of escape rooms not only as engaging learning environments but also as instruments for the development of knowledge and transversal skills (Veldkamp, Alice, 2020).

In educational escape rooms (ERs), thoughtfully developed storylines significantly contribute to the pedagogical relevance of gameplay. By situating tasks within authentic, real-life contexts, these narratives strengthen the alignment between in-game challenges and intended learning outcomes, thereby enhancing both immersion and educational value. In the MATH-DIGGER experience "ER1 – Sustainable Environment on University Facilities" (Babo et al., 2023; Pinto et al., 2024; Rasteiro et al., 2024), the storyline is centred on the reconstruction of a university campus following a severe environmental disaster. Confronted with resource scarcity and strict governmental sustainability mandates, participants engage in missions related to pivotal domains: urban development, energy, environmental stewardship, and demographic change. With conventional energy sources disrupted, players (students) resort to installing solar panels to reactivate university functions, underscoring the necessity of resilience and eco-conscious innovation. The escape room seamlessly combines problem-solving tasks with an inquiry-based pedagogical approach, framed by a narrative of ecological recovery. Evaluating the Game User Experience (GUX) across multiple dimensions offers critical insights into the effectiveness of educational escape rooms. By closely examining student interactions within these environments, educators and designers can both highlight successful elements and detect areas needing refinement. This structured analysis ensures that escape rooms serve not only as captivating experiences but also as meaningful educational tools. A concrete example of this approach is found in MATH-DIGGER Escape Room 1, developed within the framework of the Erasmus+ MATH-DIGGER project. Tailored for first-year engineering students, this escape room weaves together topics of sustainability and energy efficiency with core mathematical content from Calculus I. Grounded in inquiry-based learning (IBL) methodology, the game challenges participants with activities such as interpolation and integral calculus, fostering active learning in a gamified setting.

The paper is organised as follows: the present section provides a brief introduction to the use of escape rooms in educational contexts and highlights the importance of analysing the game user experience to enhance their educational impact. The methodology section describes the participants involved in the study and outlines the data collection procedures. This is followed by the results section, which presents the data analysis and discussion. Finally, the paper concludes with a section summarising the main findings and offering insights for future research.

Methodology

The data collection process was conducted among first-year engineering students of Mathematical Analysis from the graduation in Electrical and Computer Engineering, which is one of the engineering graduation courses of the Coimbra Engineering Institute in the Polytechnic University of Coimbra. Students were invited to participate in a pilot test of the ER1 that was implemented under the MATH-DIGGER Erasmus+ project. Two questionnaires, a pre- and a post-questionnaire, were completed by the students who participated in this pilot study. A total of 66 students responded to the pre-questionnaire, while 33 students completed the post-questionnaire. Students that do not answer the post-questionnaire were students who dropped the pilot test or did not even start it.

To ensure the reliability and validity of the data collected, both the pre- and post-questionnaires were administered through Google Forms and included clear instructions, while emphasising the voluntary and anonymous nature of student participation. Participation in the ER1 activity was incorporated into the course grade for Mathematical Analysis, worth 4 points out of 20 (20% of the final grade). The incorporation was intended to encourage student participation and to acknowledge the pedagogical value of the escape room activity. The pre-questionnaire was completed before the activity to collect baseline information on students' expectations and prior knowledge, whereas the post-questionnaire was administered at the end of the pilot study, allowing participants to immediately reflect on their experience. This dual-survey design enabled the timely and systematic collection of data, offering valuable insights into students' perceptions of ER1 as both a gamified experience and an educational tool. The responses provide a comprehensive basis for evaluating the escape room's effectiveness in conveying key calculus concepts within the context of sustainability.

Results

The data collected was analysed using IBM SPSS version 29 (IBM Corp., 2022).

To evaluate the potential impact of the educational escape room on students' perceptions and experiences, a paired samples t-test was conducted comparing responses from the pre- and post-questionnaires. The results, as may be observed in Table 1,

Table 1. Paired Samples t-test

Paired Samples Test

		Paired Differences							Significance	
					95% Confidence Interval of the Difference					
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	One-Sided p	Two-Sided p
Pair 1	Mathematic Interest - Mathematic Interest	.167	.913	.167	-.174	.508	1.000	29	.163	.326
Pair 2	Mathematic Knowledge - Mathematic Knowledge	-.167	.950	.173	-.521	.188	-.961	29	.172	.344
Pair 3	Trust Level in Solving Math Problems - Trust Level in Solving Math Problems	-.167	.791	.145	-.462	.129	-1.153	29	.129	.258
Pair 4	Games Usage in Learning Invironment - Games Usage in Learning Invironment	-.167	.950	.173	-.521	.188	-.961	29	.172	.344
Pair 5	How Confortable Using TIC in Learning Activities - How Confortable Using TIC in Learning Activities	-.033	.556	.102	-.241	.174	-.328	29	.373	.745
Pair 6	ER Familiarity as a Learning Tool - ER Familiarity as a Learning Tool	-.167	.461	.084	-.339	.006	-1.980	29	.029	.057
Pair 7	ER Efficiency in Math Learning - ER Efficiency in Math Learning	.133	1.502	.274	-.428	.694	.486	29	.315	.631

showed no statistically significant differences across the seven measured dimensions, including mathematical interest, self-perceived knowledge, trust in solving mathematical problems, and perceptions of games and technologies in learning.

Although none of the comparisons reached statistical significance at the 5% level, the item related to ER familiarity as a learning tool (Pair 6) approached significance ($p = 0.057$). This suggests a potential trend toward increased familiarity, although the effect was not strong enough to be conclusive. In contrast, other items such as perceived mathematical knowledge, trust in solving problems, and perceptions of game use in education (Pairs 2 - 4) showed similar mean differences but with wider confidence intervals and higher p-values, indicating weaker and more uncertain effects. Nevertheless, it is worth noting that all confidence intervals lie predominantly within the negative range, which may suggest a tendency toward improvement following the activity, despite the lack of statistical significance.

These findings suggest that while the activity was well-received, it did not produce measurable changes in the surveyed constructs after a single intervention.

Besides the pre- and post- questionnaires, a questionnaire, based on the GUESS-18 (Keebler, J. R., 2020), aimed to capture various dimensions of the escape room experience, including usability/playability, narrative, play engrossment, enjoyment, creative freedom, audio aesthetics, personal gratification, social connectivity, and visual aesthetics. Responses were collected via a Google Forms survey, which facilitated efficient distribution, user-friendly interaction, and secure data collection. A total of 49 valid responses were collected from the same set of students who answered the pre- and post-questionnaires, with no missing data across the variables analysed. The sample was composed primarily of male students (92% male), with a mean age of 19.80 years (SD = 1.92), aligning with the expected demographic profile of first-year students in engineering programmes. All participants were enrolled in the same Higher Education Institution (IPC/ISEC) and academic level, ensuring a controlled educational context for the escape room activity.

Regarding prior experience with Games and Escape Rooms, participants reported moderate familiarity with digital games in general, with a mean score of 4.69 (SD = 1.75) for the item assessing frequency of playing games and 4.37 (SD = 1.83) for perceived usefulness of games in learning environments. These values indicate that students are generally open to game-based learning. Familiarity with escape rooms as educational tools was lower, with a mean of 3.69 (SD = 1.72), suggesting that while the majority had some awareness, direct experience was not widespread. This finding supports the exploratory and potentially novel nature of the activity for many participants.

Student responses to GUX-related items revealed an overall positive perception of the educational escape room design. Usability aspects such as interface (M = 4.24, SD = 1.65) and controls (M = 4.65, SD = 1.81) were rated favourably, suggesting that the technical design supported smooth interaction. Narrative elements were also appreciated, with story coherence (M = 4.16, SD = 1.61) and fantasy elements (M = 4.12, SD = 1.67) both receiving high scores. Emotional engagement was evident, with students reporting the game as fun (M = 4.10, SD = 1.57) and stimulating to creativity (M = 4.18, SD = 1.81) and imagination (M = 3.82, SD = 1.70). The item “feeling bored” yielded a lower mean of 3.88, confirming a generally engaging experience. Perceived immersion, although moderate, was present. Items such as “detachment from the real world” (M = 3.57, SD = 1.77) and “immersion in game events” (M = 3.37, SD = 1.84) reflected varying degrees of absorption in the storyline and gameplay.

Aesthetic and auditory components were well-received. Students positively evaluated the visual appeal (M = 4.08, SD = 1.79) and graphics (M = 3.94, SD = 1.71) of the escape room. The role of sound in the experience was also acknowledged, with sound effects (M = 3.65, SD = 1.99) and audio enhancement (M = 4.00, SD = 2.00) contributing positively to immersion and satisfaction. In terms of social engagement, results indicate moderate interaction, with students reporting social interaction with peers (M = 3.55, SD = 1.75) and enjoyment of playing with others (M = 3.55, SD = 1.87). While the activity was not designed as a multiplayer game per se, these results suggest that peer presence and collaboration were at least partially perceived as part of the learning experience.

Finally, students reported high levels of perceived competence and motivation. The items “focus on own performance” (M = 4.61, SD = 1.54) and “feeling able to do well” (M = 5.35, SD = 1.64) suggest a strong internal drive to succeed and a sense of efficacy during the activity. These findings indicate that the educational escape room supported not only engagement and enjoyment but also students’ confidence in their ability to complete learning tasks effectively. The distribution of responses for each GUX item can be observed in Figure 1.

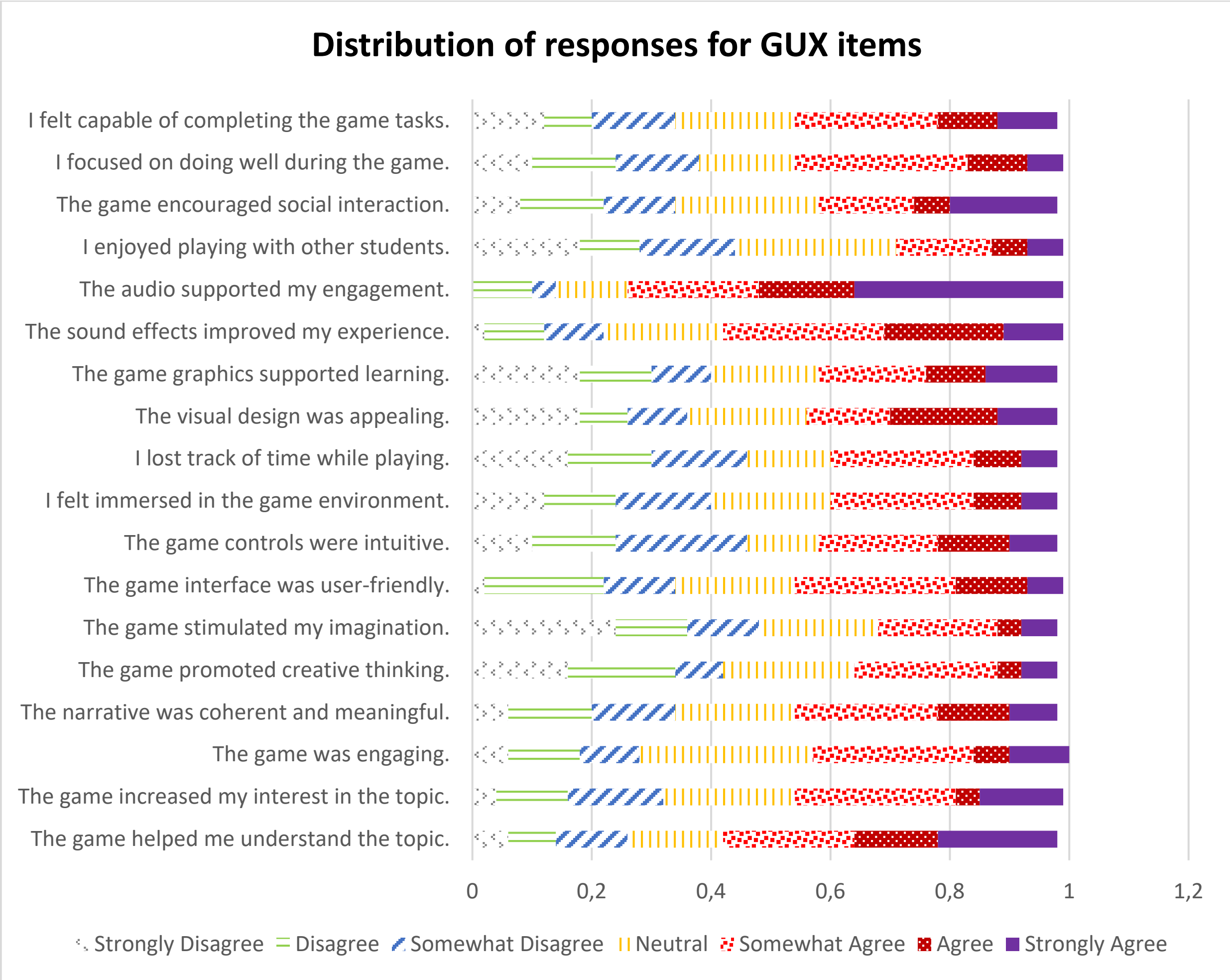

Figure 1. Distribution of responses for GUX items (Excel graphic)

The descriptive and inferential analyses suggest that students perceived the escape room experience positively across multiple dimensions of Game User Experience (GUX), particularly regarding usability, narrative coherence, and perceived learning support. While variability existed across some items, especially those related to emotional immersion and social interaction, the general pattern indicates consistent engagement with the activity. The high self-assessed motivation and confidence reported by participants further reinforce the activity's alignment with learner needs and preferences. In addition to the data collected from the students, an individual assessment of their performance in ER1 was also carried out. This evaluation was based on the criteria presented in Table 2.

Table 2. Evaluation Criteria for ER1

Evaluation Criteria	Description	Weight # (%)
Correct answers.	Proportion of correct answers in mathematical challenges.	1.20 (30%)
Number of attempts made.	Number of times the student attempted to complete challenges, assessing persistence.	0.40 (10%)
Execution time and progression.	Time required to complete levels, considering improvement between attempts.	0.60 (15%)
Collected objects/in-game interaction.	The quantity of educational objects collected reflects attention and exploration of the environment.	0.40 (10%)
Evolution throughout the game.	Comparison between the beginning and end of the game, evaluating progress and learning.	0.60 (15%)
Levels completed successfully.	Number and complexity of levels completed in the game.	0.40 (10%)
Autonomy and efficiency in problem solving.	Ability to solve challenges with few errors and without external help.	0.40 (10%)

The results, on a scale of 0 to 4 points, reveal an average performance of 2.76, with a median of 2.7, demonstrating an overall positive performance. The dispersion of results, with a standard deviation of approximately 0.61, shows moderate heterogeneity in the group. Of the 33 students who responded to the post-questionnaire, 31 completed at least one part of the MATH-DIGGER digital escape room, ER1, and obtained a valid performance score. The remaining two students dropped out after failing to progress beyond the first level, chose not to participate further, and obtained no points. Figure 2 shows the grade distribution obtained by students.

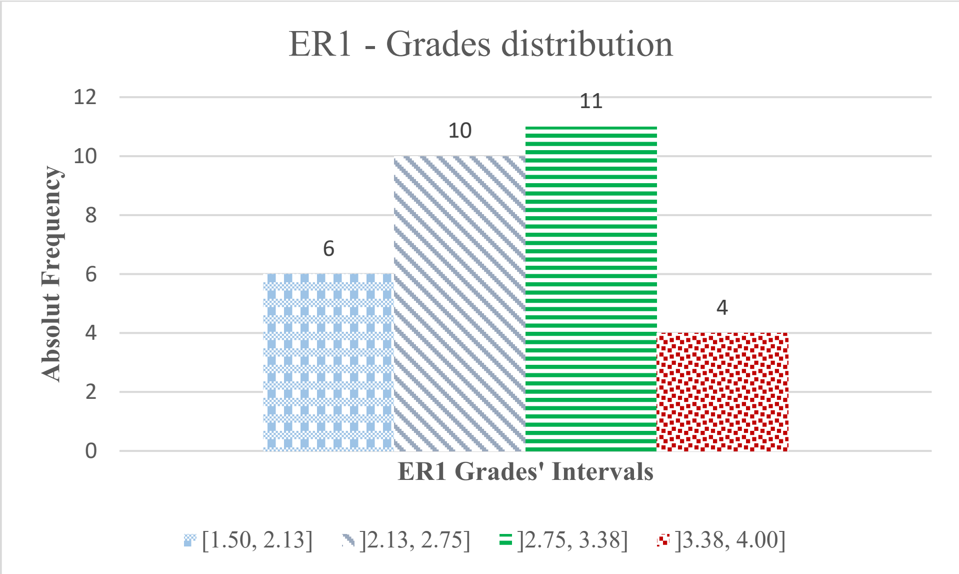

Figure 2. Distribution of student grades (scores) in ER1 across 4 levels. (Excel graphic)

Conclusions

This study aimed to evaluate the educational value of a digital escape room designed to support first-year engineering students in learning key concepts of Mathematical Analysis, within the context of sustainability. Two main data sources were considered: a pre- and post-questionnaire completed by participants, and a detailed analysis of their perceptions of the Game User Experience (GUX) during the activity. The results of the pre- and post-questionnaire analysis revealed no statistically significant differences across the measured dimensions, including students' interest in mathematics, perceived knowledge, confidence in solving mathematical problems, and familiarity with digital learning tools. Nonetheless, one variable – familiarity with escape rooms as a learning tool – approached significance ($p = 0.057$), suggesting a trend toward increased awareness and potential appreciation for this methodology. While this single exposure may not have substantially shifted core attitudes, it introduces a novel pedagogical tool within a traditional academic context. In contrast, the GUX analysis provided a more nuanced view of the learning experience. Students reported high levels of satisfaction with the escape room's usability, narrative quality, and integration of mathematical content. The activity was generally perceived as engaging, enjoyable, and conducive to learning, with strong scores in areas related to interface design and confidence in task performance. While some variability was observed in dimensions such as emotional immersion and social interaction, the overall pattern indicated that students found value in this gamified learning experience.

The assessment of student performance in ER1 revealed that most students demonstrated good adaptation to the game dynamics and reasonable mastery of the program content used in the escape room. The results suggest that ER1 was effective in consolidating mathematical concepts, promoting logical reasoning and problem solving in a playful way. In addition to the overall results obtained, it is worth highlighting the approach used to evaluate student participation in the MATH-DIGGER digital escape room developed.

This assessment was carried out according to a series of criteria, which were defined to measure not only cognitive capacity (skills such as attention, memory, reasoning, critical thinking and decision-making), but also students' involvement, improvement and problem-solving ability. The criteria defined were hits, attempts made, execution time and progression, number of objects picked up during the escape room, evolution throughout the game, levels successfully reached, and autonomy demonstrated in overcoming obstacles. All these parameters became part of the grade given to the student on a scale between 0 and 4 points, previously weighed so that the assessment was balanced and pedagogical. This allowed us to assess not only the mastery of mathematical content but also how students engaged in recreational and educational activities. Statistical analysis indicated that there is a positive correlation between game performance and knowledge consolidation, reinforcing the usefulness of innovative learning environments such as gamification for student engagement and concrete content learning.

Together, these findings suggest that while a single intervention may not be sufficient to produce measurable shifts in student attitudes or self-perceptions, the quality of the experience itself, as captured through GUX dimensions, plays a critical role in shaping student engagement and motivation. Educational escape rooms, when carefully designed and contextually aligned with curricular goals, can be used as powerful tools for supporting active learning, particularly in mathematically intensive courses. Future implementations might explore repeated or scaffolded use of such activities to foster deeper changes in students' perceptions and learning behaviours.

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Mathematical competencies in understanding and using an industrial guideline on motion design

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Abstract

Mathematical competence is used in the SEFI MSIG curriculum document (Alpers et al. 2013) to specify curricular goals. It emphasizes the ability to understand and use mathematics in relevant situations and contexts. In order to identify goals, application subjects and later workplaces should be investigated. This contribution analyses the mathematical competencies needed when using an industrial guideline on motion design issued by the German Association of Engineers (VDI) which is widely used by practising engineers and thus provides insights into real usage of mathematics beyond a single workplace. Potential educational consequences and learning opportunities are also discussed.

Introduction

SEFI's Maths Special Interest Group bases its curriculum document (Alpers et al. 2013) on the concept of mathematical competence and competencies as developed by Niss and Hojgaard (Niss/Hojgaard 2019). The overarching goal is the ability to do, use and understand mathematical concepts relevant in engineering situations and contexts which is further specified by identifying eight competence areas called mathematical competencies like mathematical reasoning, problem solving and modelling. There are two main sources for finding meaningful goals within these areas: application subjects of the study course under consideration where mathematical concepts are used for modelling and problem solving; and usage of mathematics at real engineering workplaces. For investigating the latter, different methods including interviews, observations and analysis of artefacts have been used (for an overview see Alpers 2021). When visiting single workplaces it is always questionable to which extent the results can be generalized. In this contribution I analyse a specific guideline on motion design issued by the German Association of Engineers (VDI). Motion design is a standard task in mechanical engineering, and the VDI Guideline 2143 (VDI 2024) is often used and supported in controller software. Therefore, the guideline offers the opportunity to get information of mathematical competencies needed by a larger group of engineers in a broad spectrum of companies without claiming that every engineer needs to have the competencies. In the next section I give a brief overview of the structure and content of the guideline. The subsequent section analyses the required mathematical competencies. Finally, educational consequences and learning opportunities are discussed.

VDI Guideline 2143 – An overview

The VDI Guideline 2143 supports the definition of suitable one-dimensional motion functions where the dependent variable has the meaning of distance or angle and the independent variable may be distance, angle or time. In a slider-crank mechanism (see Alpers 2015) the driving element is a motor and the driven element is a slider, so the function has the meaning of distance over angle or distance over time. The guideline abstracts from the concrete mechanical situation by considering a function y(x) allowing for all meaningful combinations. In VDI 2143 the overall motion task is subdivided into sections and the boundary points of sections are classified as rest (R) ($y' = y'' = 0$), constant velocity (G) $(y' \neq 0, y'' = 0)$, turn-over (U) $(y' = 0, y'' \neq 0)$ and general motion (B) $(y' \neq 0, y'' \neq 0)$. The user sets up a so-called motion plan where the sections are defined (Figure 1) and the values of the derivatives are provided.

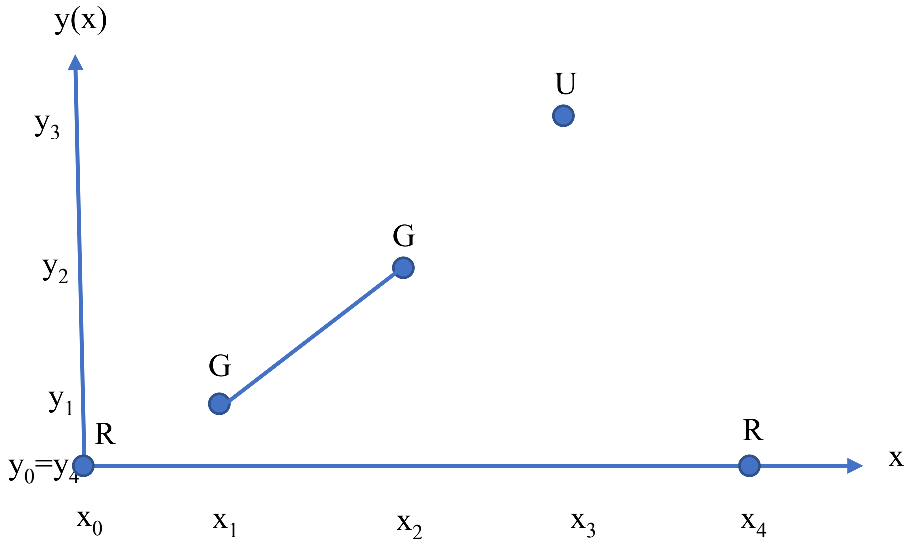

Figure 1: Example of a motion plan

There are sixteen types of sections according to the possible combinations of boundary points and hence sixteen types of so-called motion tasks. For RR and GG it is assumed that the velocity is constant between the two points. Each task is transformed into a normed version f(z) by scaling and shifting, such that f(0)=0, f(1)=1, z=0…1 holds (VDI 2024, Formula (12)). For each of the sixteen tasks the guideline provides one or more normed motion functions f(z) which have continuous derivatives up to second order and which allow to fulfil the requirements regarding the boundary points. In order to choose a suitable function from the offering the guideline defines four quality criteria, the maximum absolute values of $f', f'', f''', f' \cdot f''$. These criteria have concrete meaning regarding application features, for example keeping the maximum absolute value of f'' low results in low mass forces at the driven element such that unwanted vibration can be avoided.

For all motion tasks the polynomial of degree 5 can be used since it has six degrees of freedom such that arbitrary boundary conditions up to the second derivative can be fulfilled. Other functions which have better values regarding certain quality criteria are often constructed by piecewise defining the second derivative and developing from that the first derivative and the motion function itself such that the required boundary conditions are fulfilled. As an example we consider for the RR task ("rest-in-rest") the so-called modified acceleration trapezoid which is shown in figure 2. Here, the second derivative consists of 5 pieces. Because the absolute maximum value can be held constantly over a non-zero subinterval (as opposed to the polynomial of degree 5 for example), one can keep the maximum lower than when using a polynomial.

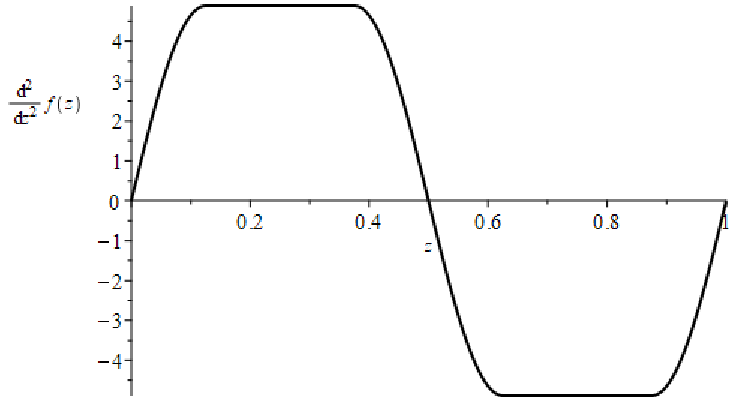

Figure 2: Modified acceleration trapezoid from (VDI 2143)

Mathematical competencies in understanding and using VDI 2143

In the following, we use the framework of mathematical competencies (Alpers et al. 2013; Niss/Hojgaard 2019) in order to analyse the mathematical abilities needed in understanding and using the VDI Guideline 2143:

Thinking mathematically: This competency is concerned with having an idea where and in which ways mathematics can play a role. Readers of the guideline should have the expectation that for motion to be realized by a controller it needs to be described in a formal way using mathematical objects like functions or value tables. Having such an expectation helps to get a quick overview of what the guideline provides similarly as a well-founded expectation of what a CAD programme can do helps in getting familiar with another such programme very quickly. Mathematics should also provide means to make judgements on candidates for motion functions, i.e. identify meaningful quantities as quality measures like the characteristic values stated above. Certainly, for a justification of such measures application aspects beyond mathematics must play a role. Readers don't have to bother about mathematical thinking in form of theorems and proofs since a guideline for practitioners is not meant to provide elaborations like academic textbooks.

Reasoning mathematically: The guideline intends to present the state-of-the-art in motion design for ready use by a practitioner without elaborating on the derivation and reasoning behind all the results stated. Therefore, in general only results are given in form of formulae. For example, motion functions are stated together with the first two derivatives but how one gets from the second derivative depicted in figure 2 (modified acceleration trapezoid) to the function itself is not explained since this is assumed to be part of the standard mathematical repertoire of the reader. This also holds for switching between the original and the normed version of a motion law by shifting and scaling a function. Explanations are given when it comes to the meaning of the characteristic values. E.g., the maximum kinetic energy at the driven element is stated as $E_{max,kin} = \frac{m_{ab}}{2}\left(\frac{Y_{ij}}{T_{ij}} \cdot C_v\right)^2$ (Formula (36) in (VDI 2143) with $C_v = \max |f'(z)|$), hence the characteristic value C_v should be kept low for energy efficiency reasons.

No reasoning is provided as to why a certain function gives a low value for one of the characteristic values. In the modified acceleration trapezoid (Figure 2), for example, the possibility to keep the maximum value over a whole interval (and not just at a single point as is the case with the polynomial of degree 5) allows for a lower maximum value. The reader does not need this reasoning but can just choose the motion law for its low maximum of the second derivative. Only when a reader is not content with what the guideline provides and tries to set up a motion function on his own could such a line of reasoning be very valuable.

The symbolic expressions for the motion laws contain a lot of symbols (e.g. for the boundary values or for parameters) such that the reader must think about which quantities are given by the application task and which ones can be set freely. With several motion functions there is one degree of freedom which can be used to set a parameter occurring in the function or to prescribe a resulting characteristic value within certain limits. Capturing the limits also requires some reasoning because square roots occurring within the expressions must be real.

Problem solving: The guideline is designed to help engineers to solve the problem of constructing a “good” motion function. For this, it provides the section-wise procedure following the problem solving strategy of “divide-and-conquer”. Moreover, for the sections it provides normed function candidates and states quality criteria that can be used for choosing one specific function. The reader still has to make use of the procedure by giving the necessary input in form of points of the motion plan and boundary values of the sections as well as parameter values for some of the functions offered by the guideline. Very often, the application requirements do not prescribe exactly the points in the plan and even for a rest phase small variation might be allowed. Therefore, it is still the task of the reader to modify the motion plan appropriately if the outcome of the current plan is still unsatisfactory, e.g. because the overall cycle time is too large.

Mathematical modelling: As opposed to many other modelling problems, motion design is not mainly concerned with analysis, but with synthesis. The guideline already provides the central initial mathematical model for doing this in form of the motion plan. The reader still has to clarify the situation at hand in order to find out about boundary conditions and their strictness. The guideline contains several motion laws for each subtask to choose from and it states characteristic values as a quality measure to justify the choice. Still, the reader has to investigate the specifics of the application situation in order to judge on the relative importance of the different characteristic values. The characteristic values might be too coarse since they do not give information on where the maximum occurs which might be important for the consequences. Moreover, the reader has to validate the resulting motion function and check whether it is within the limits given by the application (e.g. the required power of the driving motor is not too large making the device too expensive).

Representing mathematical entities: The main mathematical objects dealt with in the guideline are symbols and functions. The symbol table has 89 entries. For motion functions the abstract representation y(x) is used because x can be time, distance or angle and y can be distance or angle. The functions are given in a normed version f(z) as stated in the previous section. The functions are often piecewise defined functions where the pieces are presented giving an expression and a sub-domain. When there are symmetries, the representation is shortened by referencing a former piece, e.g. $f_4(z) = -f_3(1-z)$. For several functions also graphical representations are given showing the function pieces (mainly regarding the second derivative as in Figure 2). The reader has to connect these representations by discovering the sine and polynomial pieces in the figures. The figures also help the reader by giving graphical meaning to parameters and characteristic values and by providing tables of graphical representations of functions offered for a certain sub-task (like RR: rest-in-rest); the reader can easily recognize the differences and make his choice. There are also graphical representations on how characteristic values vary with parameter values appearing in a motion function. This way the reader gets an overview of how all characteristic values depend on the parameter such that he can achieve an adequate combination of characteristic values.

<u>Handling mathematical symbols and formalism</u>: Handling symbols and understanding and using symbolic expressions is very important for understanding and using the guide-line. If the reader does not have algebraic fluency and familiarity with understanding and manipulating symbolic expressions he still is able to take the expressions and type them into a software tool for further processing but even then the correct usage of brackets is still left up to the user. The understanding is much deeper when the role of the symbols within the expressions is recognized. Readers should also be able to switch between the symbolic representation of a function piece for a section and its normalized version (f(z), z=0..1). For some parameterized functions it is helpful when the reader can analyse expression with roots in order to choose parameter values which result in real values. Readers should also understand the sine and polynomial expressions in order to connect them to the corresponding pieces in the graphical representations. Often, the characteristic values are also given as symbolic expressions such that the reader can see how the value depends on the "ingredients". Sometimes, this is not possible because the respective equations cannot be solved symbolically such that the value can only be calculated for a concrete numerical configuration. The reader should be familiar with such a situation and should not always expect symbolic expressions. For the task BB among the suggested functions there are also B-Splines. These are defined recursively and symbolic expressions are given how to choose the control vector in order to fulfil the boundary conditions.

<u>Communicating mathematically</u>: VDI guidelines are not meant to replace academic textbooks and hence they contain very "condensed" knowledge on state-of-the-art methods. They lack the broad explanations and elaborations of the latter. The reader has to capture the procedure of setting up the motion function section by section by transforming the sub-tasks into a normalized form and choosing motion function from a set of suggested ones using characteristic values. The practising engineers does not have to understand everything in the guideline but he has to recognize which information is expected from him and which kind of results he gets. He must capture which parts must be really understood and which can just be used. He might also have a software tool provided with the controller which enables him to set up the motion plan and make choices when the functions of the guideline are implemented there. The reader might also have the duty to document his work using the motion plan with boundary conditions and to justify the choice of functions based on the characteristic values.

Using aids and tools: There are different ways of making use of the information provided by the guideline using technological tools. An engineer might create a value table using the function pieces taken from the guideline using for example a spreadsheet programme. He might also use controller software to build the function there without having to implement anything further. If none of the function pieces offered by the guideline is suitable an engineer might still use the suggested section-wise procedure and construct his own motion pieces. For investigating such pieces a mathematical programme could be very helpful (setting up and plotting functions, computing derivatives, solving equations).

Educational consequences and learning opportunities

The results of the above analysis can be used in several ways. They might corroborate the justification of competencies which are already part of the curriculum. For example, a certain degree of algebraic fluency is quite important for students not to be overwhelmed by the plethora of symbolic mathematics used in the guideline. One can also draw good motivation scenarios from the material, e.g. for shifting and scaling functions or for concepts like continuity and differentiability when one uses piecewise defined functions. Students see that all the basic topics on functions really occur in serious work on application questions which might increase their readiness to look for applicability of things they learnt in mathematics. Moreover, the results might give reason to identify additional aspects like the constructive use of functions which gives deeper insights into their modelling properties. The guideline is also helpful when finding assignments or projects for students which provide opportunities for acquiring the competencies needed for understanding and using the guideline (cf. Alpers 2014; Alpers 2015). For students particularly interested in programming one might even let them implement a development environment for the guideline in Maple® and Matlab®.

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Double integrals in engineering mathematics: Confusing region geometry with integrands

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Abstract

In engineering mathematics, double integrals are encountered in a wide range of fields, such as electrodynamics, aerodynamics and quantum mechanics. Setting up appropriate bounds and integrands is key to successfully solving an integral; computation is often easy, by hand or with software. In a recent study within a multivariable calculus course for Electrical Engineering, Applied Physics and Advanced Technology majors, we investigated students' proficiency at setting up double integrals to represent the area of a two-dimensional region provided in a diagram. In the case of computing an area, the integrand should be unity, yet we found that a significant portion of the students created an integrand out of the bounds of the region. One identical error was incurred by sixty percent of those cases. In this paper we present the different classes of incorrect integrands, how students relate to the geometry of the region and discuss potential causes. In particular, we observe that when using a single variable to calculate area under a curve, the integrand is not unity, which might cause cognitive conflict. We discuss implications for the teaching and learning of double integrals and suggest educational interventions.

Introduction

Integral calculus, both single and multivariable, is a very important subject for engineers and physicists, with many applications (Martínez-Planell & Trigueros, 2020). It is important that engineering students receive a good grounding in how to construct and to solve integrals in their mathematics classes, so that they can then use these mathematical tools confidently and with proficiency in their technical and disciplinary contexts. We have observed, and studies in this area have confirmed, that students struggle with interpreting integrals and their definitions.

Multiple studies investigating students' conceptual understanding of single variable definite integrals reveal that students heavily lean towards describing definite integrals in terms of area (Jones, 2013; Gemechu, Mogiso, Hussein and Adugna, 2021). In fact, there is a strong prototype image employed when describing definite integrals. This image is of a continuous function in the first quadrant displaying certain characteristics such as not varying far from its average value (Jones, 2018). Furthermore, studies show that students struggle to make sense of the (non-geometric) definition of the definite integral, the Riemann sum (Wagner, 2018). Since the definite integral is defined as the limit of a Riemann sum and not as an image of an area, the predominance of an area conception over a Riemann sum conception is troubling.

Certainly there is a link between geometry and integration (Martínez-Planell & Trigueros, 2021; Milenković, Božić & Takači, 2023). Bašić and Milin Šipuš (2022) describe the link as an "intricate connection" (p. 251) and Milenković, Božić and Takači (2023) have found correlations between proficiency at analytical geometry and that of integral calculus. Jones (2018) argues that prototype images can be useful but also have drawbacks such as raising assumptions that a mathematical object will always display certain properties.

Studies into students' conceptions of the double integral find that these conceptions are linked to those the students already have of the single variable definite integral (Jones and Dorko, 2015; Özbey & Dost, 2023). Since area is the predominant single variable integral conception, this link translates to strongly area-correlated conceptions of double integrals. Jones and Dorko (2015) identify ten conceptions of the double integral of a function on a region, of which several are correct and related to area; for example, "double integral as adding up slices". In contrast to these correct conceptions, however, they do find a certain subset of their cohort interpreting $\int_R f(x,y)dA$ as area of a region in the (normatively incorrect) understanding they call "the double integral as boundary and area".

Similar to their single variable counterparts, double integrals are defined in terms of a Riemann sum. Studies have shown that typical textbook treatments of the double integral Riemann sum tend to be less fully presented than that of the single variable Riemann sum, suggesting (against evidence) that students can transfer their understanding from one to the other (McGee and Martínez-Planell, 2014). Martínez-Planell, and Trigueros (2020) find that many students are confused by elements of the Riemann sum construction and struggle to relate the Riemann sum as a whole with the double integral. In the study of Özbey and Dost (2023) they investigated pre-service mathematics teachers' conceptions of double integrals. They find that students struggled to construct double integrals to describe area unless "the function" was given, some students did not recognise "1" as a function, and others who knew that the integrand should be 1 struggled to explain why.

With the exception of some studies such as Özbey and Dost's, in which students were asked to construct integrals, most educational studies of integrals involve presenting students with already constructed integrals and asking for interpretations. Our study differed from that general trend, in that we presented students with a diagram of a region and requested construction of the double integral for area. Given the studies showing the strong link between the symbolic integral and the geometric idea of area, the exercise is apparently well aligned with students' conceptions of integration. Instead of seeing apparent alignment between our exercise and students' conceptions of integral as area, we found that a significant minority of the cohort provided integrands that were either related to the geometry of the region or were at best conditionally correct. Our study contributes to the field by presenting results related to students' conception of integral-as-area in an unconventional form, that is calling for construction of area double integrals. The results we discuss here and elsewhere (Craig & Pehlivan, 2025) respond to the call for more studies into the teaching and learning of multivariable calculus (Martínez-Planell & Trigueros, 2021).

Research Methodology

We carried out a study investigating students' construction of bounds of double integrals representing area of a lamina (Craig & Pehlivan, 2025). The focus of the mentioned study is the bounds of integration. However, one phenomenon apparent in the data was the tendency of some students to express the integrand in terms of one or more of the bounds of the region. It is this phenomenon on which we focus in this paper. Data were collected in a standard calculus exam offered to students in the programmes of electrical engineering, advanced technology and applied physics at a technical university in the Netherlands. 168 students wrote the exam and the exercise on which data was collected can be seen in Figure 1.

Region R is shown alongside. Use double integrals to describe the area of region R:

a) first in the order $dydx$,

b) then in the order $dxdy$.

Do not solve these integrals.

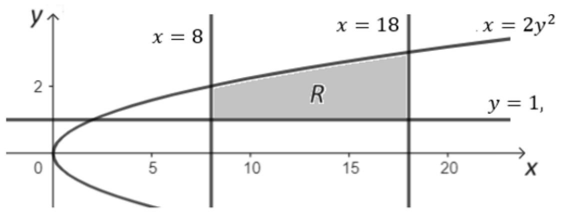

Figure 1. Exam exercise on double integrals

Correct answers to the exercise in Figure 1 are

(a) $\int_8^{18}\int_1^{\sqrt{x/2}} 1dy\,dx$,

(b) $\int_1^2\int_8^{18} 1dx\,dy + \int_2^3\int_{2y^2}^{18} 1dx\,dy$.

Each student's answer was recorded in typed form and the data were anonymised. Each record included bounds of integration and the integrand.

Integrands chosen by the students were divided into three categories, namely correct, conditionally correct and incorrect. Correct integrands included writing the number 1 explicitly, or leaving it out entirely. Conditionally correct integrands were $f(x)$ and $f(x,y)$ since these are correct under the condition that $f(x) = 1$ or $f(x,y) = 1$. Any other integrand was incorrect, with the majority of those being related in some way to the geometry of the region.

Data Analysis

In part (a) (see Table 1), of the total 168 students, 104 provided the correct integrand. A further 32 gave an integrand that was conditionally correct. 17 out of 32 wrote $f(x,y)$ for the integrand while the remaining 15 used $f(x)$ as the integrand. 25 students used the region itself to contrive an integrand, the most common integrand being $2y^2$, provided by 15 students. Two students gave incorrect integrands unrelated to the region.

Integrand	Count (% of 168)	Subcounts [frequency]
Correct	104 (63%)	
Conditionally correct	32 (19%)	$f(x,y)$ [17], $f(x)$ [15]
Related to the region	25 (15%)	$2y^2$ [15], $\sqrt{\frac{x}{2}}$ [3], R [3], $2y^2 - x$ [2], $18 - 2y^2$ [1], $2y^2 - 1$ [1]
Other	2 (1%)	x [1], y [1]
Omitted the exercise	5 (3%)	

Table 1. Integrand categories in student responses to part (a)

In part (b) (see Table 2) 102 students provided the correct integrand. 29 students provided conditionally correct integrands. 16 of them chose $f(x,y)$ as the integrand and the remaining 13 chose $f(x)$. 28 students provided integrands that were either clearly or apparently derived from the region itself. The most common of these was $\sqrt{\frac{x}{2}}$ provided by 12 students.

Integrand	Count (% of 168)	Subcounts [frequency]
Correct	102 (61%)	
Conditionally correct	29 (17%)	$f(x,y)$ [16] , $f(x)$ [13]
Related to the region	28 (17%)	$\sqrt{\frac{x}{2}}$ [12], $2y^2$ [6], $2y^2 - x$ [2], R [2], $2y^2 x$ [1] , $\sqrt{\frac{x}{2}} - y$ [1] , $x - 2y^2$ [1] , 8 [1] , $\frac{x}{4}$ [1] , $\frac{1}{2}\sqrt{x}$ [1]
Omitted the exercise	9 (5%)	

Table 2. Integrand categories in student responses to part (b)

We note that, if we were to express the area of the region as single variable integrals, correct answers for (a) and (b) would be

(a) $\int_8^{18}\left(\sqrt{\frac{x}{2}} - 1\right)dx$ and (b) $\int_1^2 (18-8)dy + \int_2^3 (18 - 2y^2)dy$.

No student provided an integrand of $\left(\sqrt{\frac{x}{2}} - 1\right)$ for either (a) or (b), although three did provide $\sqrt{\frac{x}{2}}$ in part (a). One student provided an integrand of $(18 - 2y^2)$ for part (a), but none did for part (b).

Discussion of findings

In this paper we are primarily interested in students' incorrect integrands. However, it is notable that conditionally correct integrands are slightly more prevalent in the data set. We are reminded of Özbey and Dost's (2023) findings that some of their students' belief that an integral cannot be constructed without "the function" and that 1 is not a function.

Introductory calculus textbooks typically begin defining integration as area under a curve (Adams & Essex, 2021). A graph of a function $f(x) > 0$ is typically depicted, over a closed interval, and the area is approximated by a sum of rectangle areas. By allowing the number of rectangles to tend to infinity and the width of the rectangles to become infinitesimally small, the "true" area under the curve is represented by the limit of a Riemann sum. The height of the rectangles becomes the integrand of the integral. Thus, from the very beginning of the topic of integration, (single variable) integrals are introduced as representing area, with the integrand being the function "under which" is the area in question.

Similarly, when double integrals are first introduced, they are typically introduced very visually, as volume under a surface. Little tiles are defined in the *xy*-plane, with the value of the function $f(x, y) > 0$ providing the height of the rectangular prisms under the surface. The sum of the volumes of the prisms is an approximation of the volume under the surface and, in the limit, becomes the double integral of the function.

Once double integrals are defined in terms of volume under a surface, as teachers we can expand beyond that definition to interpret, for example, $\iint_R \sigma(x, y)dA$ as mass of a lamina, if $\sigma(x, y)$ is area density, and $\iint_R dA$ as area of a lamina, given that this integral represents the limit of a sum of tiles of area ΔA.

We make two observations that we argue are influential factors. First, we teach the various types of Riemann integration in the order single variable, double, triple, line and surface. In all but single variable integration, we teach an interpretation of the integral where the integrand is 1. That is, area of a lamina, volume of a solid, length of a curve, and area of a surface. In single variable integration, however, the nature of $\int_a^b 1dx$ is usually not addressed. Thus, an integrand of 1 is first encountered in double integrals, with no prior similar construction to build upon cognitively. Our second observation is that double integral as volume is often framed as the simplest interpretation of a double integral. "In the simplest instance, the volume of a three-dimensional region is given by a double integral of its height over the two-dimensional plane region that is its base" (Adams and Essex, 2021, p. 833). We suggest that "double integral as volume" is not the simplest interpretation of a double integral, but that area of a two-dimensional region is.

Conclusions

Our study investigated students' conceptual understanding of constructing double integrals for area. We observed that a number of students, albeit a minority, placed a geometric feature of the region in the integrand instead of the correct integrand of 1. We suggest that this error is rooted in the students' understanding of the single variable integral $\int_a^b f(x)dx$ representing area under a curve.

We make three suggestions for teaching.

1. When teaching single variable integration, associate $\int_a^b 1dx$ with the length of $[a, b]$.
2. When introducing double integrals, instead of beginning with an integrand of $f(x, y)$ and an interpretation of the result as volume, begin with an integrand of 1 and an interpretation as area. Thereafter a variety of integrands can be introduced, all with their interpretations (for example volume or mass) equally weighted in importance.
3. When teaching double integral for area, be explicit that the earlier definition of single variable as area is, in effect, a double integral with the first computational step already carried out. That is

$$\int_a^b \int_0^{f(x)} 1dy\, dx = \int_a^b f(x)dx$$

A limitation of the study, as pertains to the results discussed here, is that the study only recorded the double integral constructions for area double integrals. No other types of integrals were addressed and there were no interviews. To further investigate the construction of integrands, a follow up study could include construction of a variety of types of integrals, or area integrals for a variety of different shaped regions. Additionally, interviews would increase clarity.

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Software generation of local extremes tasks of equal difficulty

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Abstract

In this paper, we discuss the possibilities of software generation of mathematical tasks of equal difficulty for the local extrema of polynomial and rational functions of one variable and polynomial functions of two variables. The software we used for this purpose is Moodle with the STACK extension and Maxima.

Introduction

With the development of new and advanced technologies, new possibilities for their use in teaching are emerging. One such possibility is software generation of mathematical problems corresponding to the required area, difficulty, and other input parameters. This allows teachers to create a large number of exercises for practicing the subject matter while maintaining the desired properties of the problems. This has the potential to streamline the teaching process and improve students' understanding of mathematical problems.

Many studies have focused on the integration of ICT into mathematics education. It has been shown that the proper use of ICT in the teaching process improves mathematics education and helps students understand key concepts (Aggarwal, Gupta, 2020). Other research indicates that the use of ICT in teaching contributes to student engagement in the educational process (active learning), supports self-directed learning, and enables students to work remotely (Asare, 2023; Das, 2019).

In this paper, we focus on the software generation of tasks to investigate local extrema of polynomial and rational functions of one real variable and polynomial functions of two variables, with the requirement that stationary points have integer or rational coordinates if calculated from a linear equation, and integer coordinates if calculated from a quadratic equation. We use Moodle with the STACK plugin (utilizing Maxima for STACK) as a tool, but the considerations described are generally applicable.

When creating sets of tasks of the same type, a teacher sometimes wants the results to be from given numerical sets. To generate problems of the same difficulty, the simplest and most natural choice for students is when integer solutions are produced. This is not always necessary, but in structurally more complex problems, it is a simplifying factor. Essentially, it is about being able to separate different types of results when generating them so that they are either specifically included or excluded.

Polynomial functions of one variable

For our purposes, we consider quadratic and cubic polynomials. If a student knows how to find integer roots of polynomials of the third and higher degrees, the procedure described for the cubic polynomial can naturally be extended. The task of the prepared exercises is to find stationary points (we will not deal with investigating the existence and type of extrema here). Recall that stationary points are found as the solution to the equation where the derivative of the function is set to zero.

First, consider the quadratic function $f(x) = ax^2 + bx + c$. Setting the derivative to zero gives the linear equation $2ax + b = 0$, the solution of which, for integer a, b, c is rational and equal to $x = \frac{-b}{2a}$. In this case, any integer coefficients a, b, c can be generated, with the only condition being that a is non-zero.

In the case of a cubic function, the stationary points will be the solutions of a quadratic equation. Depending on the control of the generation, we may want to allow integer solutions, rational solutions, or solutions with square roots in the real domain. Rational and irrational solutions can be achieved by controlling through the discriminant (which could also be elaborated), but we will limit ourselves to integer solutions and show the way to generate them from the back. Thus, we will not generate directly the coefficients of the optimized cubic function, but instead generate the integer stationary points, i.e., directly the solutions of the quadratic equation. The function for the problem statement is then obtained by integrating the intended quadratic function.

We assume $f'(x) = k(x-a)(x-b) = k(x^2 - (a+b)x + ab)$, where a and b are generated integer roots and k is a constant specified below. Thus, $f(x)$ is the primitive function: $f(x) = k(\frac{1}{3}x^3 - \frac{a+b}{2}x^2 + abx) + c$. If we want the problem statement to also have integer coefficients, we choose k as any multiple of six. Another option is to take k as a multiple of three and ensure that $a + b$ is even within some additional condition. Note that the coefficient c can be taken arbitrarily.

Rational function of one variable

Task of finding stationary points, namely points suspected of being extremes. Again, we approach this from the end. The situation is more complex with many more scenarios. We expect the derivative of the function in the form

$$f'(x) = \frac{P(x)}{Q(x)}.$$

The solution of the equation $P(x) = 0$ may require factoring out up to x^2 and solving a linear or quadratic equation. The function $f(x)$ is obtained again by integration, but this time the antiderivative may include not only the intended rational function but also logarithmic and arctangent functions. It is necessary to handle the conditions so that the function specified is exclusively rational, with coefficients of other functions needing to be zero. We will analyse possible variants, specifically describing the solutions of three of them, while for the others we will only state the conditions found.

In further considerations, we assume that the function $P(x)$ is in one of the following forms: $P_{1+k} = x^k(x-a)$, $P_{4+k} = x^k(x-a)(x-b), P_{7+k} = x^k(x^2+c)$ where $k \in \{0,1,2\}$, a and b are integers (non-zero), and c is positive. We expect the function $Q(x)$ to be in one of the four forms: $Q_1 = [x-m]^2$, $Q_2 = [x^2-m^2]^2$, $Q_3 = [x^2+n]^2$, $Q_4 = [x-m]^3$, where m is an integer and n is positive. These combinations yield the most common rational functions used to solve stationary point problems of this type.

Further, let's denote

$$f'_{ij} = \frac{P_i}{Q_j}.$$

For example

$$f'_{11} = \frac{x-a}{[x-m]^2}.$$

This derivative has a single zero point at $x = a$. However, the primitive function of f'_{11}

$$f_{11} = \log(x-m) - \frac{m-a}{x-m}$$

is never rational, therefore it cannot be used for our purposes. Next, the function

$$f'_{21} = \frac{x(x-a)}{[x-m]^2}$$

has two zero points $x = a$ and $x = 0$. The primitive function of f'_{21} is

$$f_{21} = (2m-a)\log(x-m) + \frac{x^2-mx-m^2+am}{x-m}$$

and it is rational only under the condition $a = 2m$. So only integer m can be generated. For the third time,

$$f'_{43} = \frac{(x-a)(x-b)}{[x^2+n]^2}$$

has two zero points at $x = a$ and $x = b$ and the primitive function

$$f_{43} = \frac{(n+ab)\,\mathrm{atan}(\frac{x}{\sqrt{n}})}{2n^{\frac{3}{2}}} - \frac{(n-ab)x + (-b-a)n}{2nx^2+2n^2}.$$

This is rational under the condition $n + ab = 0$. During generation, we generate two parameters (taking into account that $n > 0$) and calculate the third one.

We attach the results of the conditions for different combinations of numerators P_i and denominators Q_j of derivatives f'_{ij}. Note that in STACK or Maxima, the function f_{ij} can be found using the command f_{ij}: integrate(f'_{ij}, x);

f'_{11}: $\emptyset$	f'_{12}: $a = 0$	f'_{13}: $a = 0$	f'_{11}: $without\ conditions$
f'_{21}: $a - 2m = 0$	f'_{22}: $\emptyset$	f'_{23}: $\emptyset$	f'_{24}: $\emptyset$
f'_{31}: $3m^2 - 2am = 0$	f'_{32}: $2m - b - a = 0$	f'_{33}: $\emptyset$	f'_{34}: $3m - a = 0$
f'_{41}: $2m = b + a$	f'_{42}: $m^2 - ab = 0$	f'_{43}: $n + ab = 0$	f'_{44}: $without\ conditions$
f'_{51}: $3m^2 - 2(a+b)m + ab = 0$	f'_{52}: $m = 0$, $a + b = 0$	f'_{53}: $\emptyset$	f'_{54}: $3m - b - a = 0$
f'_{61}: $4m^3 - 3(a+b)m^2 + 2abm = 0$	f'_{62}: $3m^2 + 2(a+b)m + ab = 0$	f'_{63}: $ab - 3n = 0$, $a + b = 0$	f'_{64}: $6m^2 - 3(a+b)m + ab = 0$
f'_{71}: $m = 0$	f'_{72}: $m^2 = c$	f'_{73}: $n + c = 0$	f'_{74}: $\emptyset$
f'_{81}: $\emptyset$	f'_{82}: $\emptyset$	f'_{83}: $\emptyset$	f'_{84}: $m = 0$
f'_{91}: $4m^3 + 2cm = 0$	f'_{92}: $\emptyset$	f'_{93}: $c - 3n = 0$	f'_{94}: $\emptyset$

Table 1: Conditions for generating rational functions

Polynomial function of two variables

In the theory of functions of two real variables, the situation is more complicated already with polynomial functions. Stationary points are sought here as a solution to the system of equations that arise by putting partial derivatives equal to zero. Stationary points now have two coordinates that satisfy a system of two equations for two unknowns. The simplest situation is in the case of a linear system of equations. Considering the difficulty of the calculation, this time we can let the rational coordinates of the stationary points come out. In the case of a non-linear system, however, we will want the coordinates to be integers.

Again, we will approach the preparation of the tasks from behind - we will directly generate the coordinates of the stationary points, which will be the solutions of the specified types of systems. For a system of equations to arise from the partial derivatives of a function of two variables $f(x, y)$, the condition of equality for mixed derivatives of the second order must hold: $f''_{xy} = f''_{yx}$.

We denote the partial derivatives of the first order $f'_x(x, y) = P(x, y)$ and $f'_y(x, y) = Q(x, y)$. Then function $f(x, y)$ is found as the so-called stem function to $P(x, y)$ and $Q(x, y)$ by solving an exact differential equation

$$P(x, y)\, dx + Q(x, y)\, dy = 0.$$

Note that in Maxima or STACK we can use the command ode2('diff(y,x) = - P(x,y)/ Q(x,y), y, x); to solve the exact differential equation.

In the task design, we will focus on the following two groups. First, the stationary point will be one, calculated as a solution to a linear system of equations for two unknowns. Then one of the partial derivatives will have two real roots in one variable. In each such group, multiple options will need to be considered, since the described conditions can be applied to both x and y partial derivatives.

- $f'_x = ax + b,\ f'_y = cy + d$

The mixed partial derivative is zero in both cases. Therefore, it is sufficient to generate numbers a, b, c, d, K integer, a, c non-zero. The stem function then has the form

$$f(x, y) = cy^2 + 2dy + ax^2 + 2bx + K\ .$$

And the stationary point has coordinates $\left[\frac{-b}{a}, \frac{-d}{c}\right]$.

- $f'_x = cy + d,\ f'_y = ax + b$

Mixed partial derivative are $f''_{xy} = c, f''_{yx} = a$. From there we have the condition $a = c$. Therefore, it is sufficient to generate numbers a, b, d, K integer, a non-zero. The stem function then has the form

$$f(x, y) = axy + by + dx + K\ .$$

A stationary point has coordinates $\left[\frac{-d}{a}, \frac{-b}{a}\right]$.

- $f'_x = (x - a)(x - b),\ f'_y = cy + d$

The mixed partial derivative is zero in both cases. It is enough to generate numbers a, b, c, d, K integer, c non-zero. The stem function is then of the form

$$f(x, y) = 3cy^2 + 6dy + 2x^3 + (-(3b) - 3a)x^2 + 6abx + K\ .$$

There are two stationary points: $\left[a, \frac{-d}{c}\right], \left[b, \frac{-d}{c}\right]$.

- $f'_x = (x - a)(x - b),\ f'_y = (y - c)(y - d)$

The mixed partial derivative is zero in both cases. It is enough to generate numbers a, b, c, d, K integer. The stem function is then of the form

$$f(x, y) = 2y^3 + (-(3d) - 3c)y^2 + 6cdy + 2x^3 + (-(3b) - 3a)x^2 + 6abx + K\ .$$

There are four stationary points: $[a, c], [b, c], [a, d], [b, d]$.

- $f'_x = 2xy - (a + b)y + c,\ f'_y = (x - a)(x - b)$

The mixed partial derivatives are equal $2y-(a+b)$. It is enough to generate numbers a,b,c,K integer such that $a\neq b$. The stem function is then of the form

$$f(x,y)=x^2y-(a+b)xy+aby+cx+K\ .$$

There are two stationary points: $[a,\frac{-c}{a-b}],\left[b,\frac{-c}{b-a}\right]$. Note that instead of c in the partial derivative with respect to x, the function $c(x)$ of the variable x could be, but the calculation of the y coordinate would still be from a linear equation.

Other variants of the previous types can be obtained by directly exchanging the variables x and y in the partial derivatives from which the problems are developed. The situations would be analogous, so we will not list them.

Conclusion

The paper presents the possibility of software generation of mathematical tasks of the same difficulty, which is demonstrated in a specific area. We chose the local extremes of one real variable and two real variables, while as a software solution we used Moodle with the STACK extension (including Maxima support).

Educators can use such a pre-prepared environment in mathematics courses and thus make the education process more efficient. By extending it to several mathematical areas and problems, it is possible to create a course that can be used for students' home preparation, test preparation, or directly in the classroom. Within one mathematical area, it is possible to generate tasks of varying difficulty, allowing students to practice their skills on a large number of tasks.

The software we chose is capable of working with mathematical text, randomly generating tasks according to a pre-programmed form, evaluating the correctness of solving individual tasks, evaluating the equivalence of expressions (whether the student's answer is equivalent to the correct answer) and more. All this has the potential to improve and make the educational process more efficient, but a disadvantage is the rather demanding preparation required in this software environment.

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Mathematics with Real-life Applications: Starting with the Big Problem

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Abstract

The author previously discussed the benefits of integrating real-world engineering applications into mathematics instruction for STEM students at the 2022 SEFI Conference (Salim, 2022). The motivation stemmed from research by Burdman (2022), which identified disengagement in traditional calculus courses as a key barrier to completing engineering degrees in the U.S. The published paper listed examples of interdisciplinary content in engineering mathematics and reported improvements in student learning and performance. We also know that traditional mathematics teaching often progresses linearly – starting with foundational concepts before advancing to complex material. This mirrors core textbook structures and may fail to engage students and adequately prepare them for later courses. Therefore, the author decided to do things differently and reorder the delivery of the content in his new work. The modified module began with the teaching of Differential Equations before introducing the foundational topics as supporting tools for solving complex engineering problems. By tackling a larger challenge first, students were encouraged to appreciate the relevance of the mathematical tools and concepts they were learning. Student feedback and performance improved compared to the previous cohorts.

Introduction

Effective learning in engineering requires a strong foundation in primary principles, science, and mathematics. Moreover, developing students' critical thinking and analytical skills is crucial for their success. A solid mathematical base is essential for nurturing competent engineers who can confidently analyse and solve problems, employ various analytical tools and techniques, effectively design and innovate solutions, and communicate their results to diverse audiences.

Recent reports, such as those by Hunter et al. (2019), suggest that disengagement with traditional calculus subjects is a significant reason for student dropout from engineering courses. Personal observations and feedback from colleagues and collaborators indicate a decline in mathematical abilities among senior undergraduate students, who struggle with even basic calculus in their advanced years. Alarmingly, a report titled '*Charting a New Course: Investigating Barriers on the Calculus Pathway to STEM'* (Burdman et al., 2021) further highlights that traditional approaches to calculus contribute to the discouragement of women and students from minoritised backgrounds from pursuing STEM careers. This suggests that this issue is not limited to engineering but also touches upon equity, diversity, and inclusion (EDI) in education.

Engineering educators generally agree that one of the main factors contributing to these issues is the lack of engaging and relevant material that describes, solves, and aids the understanding of the significance of mathematics in an engineering context. The report proposes that STEM academics can address these barriers to mastering mathematics by prioritising collaborations and co-creation across disciplines to transform the teaching and learning of engineering mathematics using interdisciplinary and multidisciplinary approaches (Burdman, 2022). Studies have also shown that active learning environments are more effective than traditional lectures (Eberlein et al., 2008). Furthermore, Deakin (2006) concluded that students value the link between teaching and research, particularly research-led teaching and its impact on their learning experiences.

A paper presented by this author at the SEFI 2022 conference (Salim, 2022) outlines how active-learning and research-led teaching, with a focus on integrating multi-disciplinary content, was adopted in an existing engineering mathematics module to make it more relevant and support effective learning. This approach was well-received by students, as evidenced by positive feedback from module evaluations, surveys, and informal interviews. This encouraged the author to continue considering the importance of framing and describing the intradisciplinary nature within the mathematics syllabus, in addition to its interconnection with other engineering subjects, as recommended in the literature.

In the new paper presented here, the author details the further changes he made to the engineering mathematics module. He broke with convention and reordered the delivery of the content. Instead of teaching the syllabus in the sequence presented in core textbooks, the author started with the big problem and gradually introduced the building blocks as the semester progressed. The new approach was intended to challenge students to think more broadly and develop a better appreciation for the subject matter.

This new strategy (Version 2), together with the previous modifications discussed in the previous paper (Version 1), not only resulted in improvements in student feedback, engagement, and academic performance but also contributed towards equipping students with the skills and tools required to solve engineering challenges.

Summary of Previous Module (Version 1)

Pedagogical literature, student feedback, and discussions with educators at external events has emphasised the benefits of integrating theoretical knowledge with practical application. This informed the enhancements introduced in the first revision of the module as discussed in the 2022 paper (Salim, 2022), drawing from cross-disciplinary research and engineering subject expertise. The mathematics lessons in the module were updated with real-life applications. This aimed to help learners appreciate the relevance of mathematics in the wider engineering context, sparking their curiosity and motivating them to explore further and transfer the skills and knowledge to other subjects.

Outline of Version 1

The ‘Engineering Mathematics 2’ module is delivered to first-year undergraduate engineering students in the second semester. The topics were organised in the following sequence:

- Vectors
- Complex Numbers (CN)

- Ordinary Differential Equations (ODEs)
- Multivariate Functions (MV)
- Series and Sequences (S&S)

The topic order was consistent with the way they were presented in major textbooks, where Vectors and CN were first taught before moving to the more complex contents covering ODEs and MV, and finally S&S.

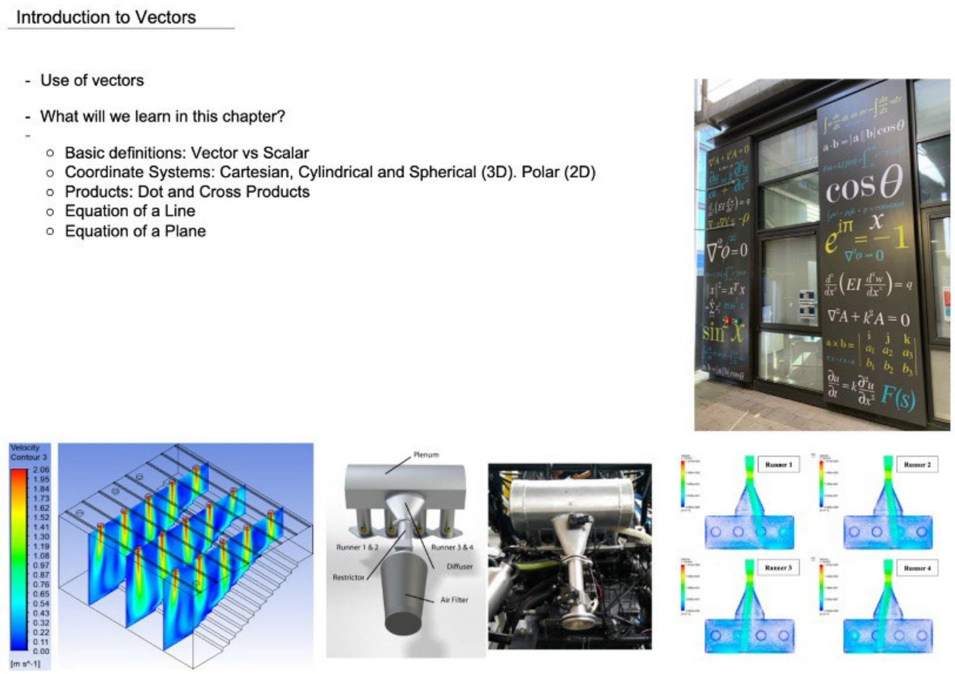

Figure 1. Introductions to Vectors

Delivery of Version 1

Figure 1 shows a screenshot of the previous lecture introducing the first 'Vectors' topic. The lecture included examples of student projects using Computational Fluid Dynamics (CFD) to emphasise the relevance and importance of the topic in an engineering context. The first image shows a student modelling ventilation in a lecture theatre to assess indoor air quality in response to the COVID-19 outbreak. The other two images are simulations of an intake manifold used by the Formula Student race team and their proposed design change to enhance volumetric efficiency.

Reflections and Student Feedback in Version 1

It was evident based on student feedback that this approach engaged students, motivated them, and helped them appreciate the practical applications of vectors. The same strategy of integrating student projects highlighting real-life scenarios was applied throughout the teaching of all topics.

Below are some student comments collected as part of the formal module feedback exercise during the previous delivery. They were in response to the open-ended question:

Q: Please name the one thing in the module that had the most impact on your learning.

'I like the variety of ways he teaches his lectures (example classes, quiz classes, polls etc) as this helps me stay engaged and focused.'
'Lecturer is very likeable and approachable, and this shows in his lectures. Everyone pays more attention because he also makes the lectures more interesting and fun to attend.'
'The lecturer is very engaging. He makes the lectures interesting and his passion for the subject is very clear, making lectures more enjoyable. The lectures are very interactive.'

The author attributed these positive comments to the use of real-life engineering examples to explain and bring mathematical techniques to life. By drawing in content and applications from other engineering subjects and research activities, the module became truly inter- and multi-disciplinary. Interested readers are invited to refer to the published paper (Salim, 2022) for further details, including additional examples of the integration of real-life applications and student projects.

New Strategy: Reordering Delivery of the Content (Version 2)

The author's research expertise and interests are in Computational Fluid Dynamics (CFD), which includes student project supervision and numerous research publications. Since he already incorporates much of this work into his teaching to highlight real-life engineering applications, he believes it is appropriate to begin with the most intriguing and fundamental aspect, namely the Navier-Stokes (NS) equations. These are a series of differential equations that also happens to be a core component of the Engineering Mathematics 2 module.

While this approach deviates from conventional practice - since CFD and NS are typically taught in the senior years of a degree programme - the author argues that starting with the big problem and gradually introducing it at the appropriate level helps students grasp the broader picture and appreciate the relevance and importance of mathematical techniques in solving real-life engineering problems.

In the updated delivery plan, the first lesson opened with an introduction to the NS equations, highlighting their status as one of the Millennium Prize Problems by the Clay Mathematics Institute. This was followed by a discussion on various methods for solving engineering problems, including analytical, experimental, and computational approaches. The class concluded with an overview of the use of the NS equations in the context of CFD, accompanied by real-life examples from senior student projects spanning over the years.

Delivery of Version 2

The intention behind this new strategy of reordering the content is to encourage first-year students to see the wider picture and appreciate the goal of learning mathematical techniques. By gradually equipping them with the appropriate tools and skills, the author aims to adequately prepare the students in the task of analysing and solving engineering problems in later levels of studies.

Table 1 details the previous (Version 1) and reordered lesson plans (Version 2)

Table 1. Order of Topics

Topic	Version 1	Version 2
1	Vectors	Ordinary Different Equations
2	Complex Numbers	Multivariate Functions
3	Ordinary Different Equations	Complex Numbers
4	Multivariate Functions	Vectors
5	Series & Sequences	Series & Sequences

The beauty of the Navier-Stokes (NS) equations lay in their multidisciplinary nature. Students encounter them repeatedly in various forms throughout their engineering studies, particularly in Thermodynamics, Fluid Dynamics, Solid Mechanics, and other core modules. The NS equations are partial differential equations (PDEs), but for the purposes of this module, they are simplified to ordinary differential equations (ODEs). This allows for a smooth transition to other topics as the course progresses through the semester. Different aspects of the NS equation are revisited and related to the mathematical concepts being learned, thereby connecting all the contents.

Students are encouraged to take ownership of their learning by supplementing in-person lessons with virtual asynchronous learning using Pearson MyLab Math™. MyLab Math offers a combination of e-texts, homework, study plans, and online tests. Interactive example classes and discussions about engineering applications are also covered in class. To support active learning, bi-weekly class quizzes are conducted via the Pearson Learning Catalytics™ interactive digital learning resource. This technology, integrated within the Pearson MyLab Math platform, supports student engagement by generating classroom discussions, promoting peer-to-peer learning, and supporting just-in-time teaching.

Observations and Reflections in Version 2

As the semester progressed, the class performance improved as can be seen by the higher proportion of green in the pie charts for the various questions asked during the in-class tests demonstrated in Figure 2, and the class average of the continuous assessments (online tests) shown in Figure 3.

And the most impactful observation was that students performed much better in the 'Ordinary Differential Equation' and 'Multivariate Functions' related questions in the final exam. This could be attributed to two likely factors: a) students had more time to master the more complex topics as it was introduced much earlier in the semester and b) because they started with a lower score in the in-class quiz and online test that motivated them to work harder, and as they encountered easier topics as the semester progressed this built their confidence which translated to much better engagement and outcome in the final exam. This was also reflected by the increased student satisfaction captured in the module evaluation survey.

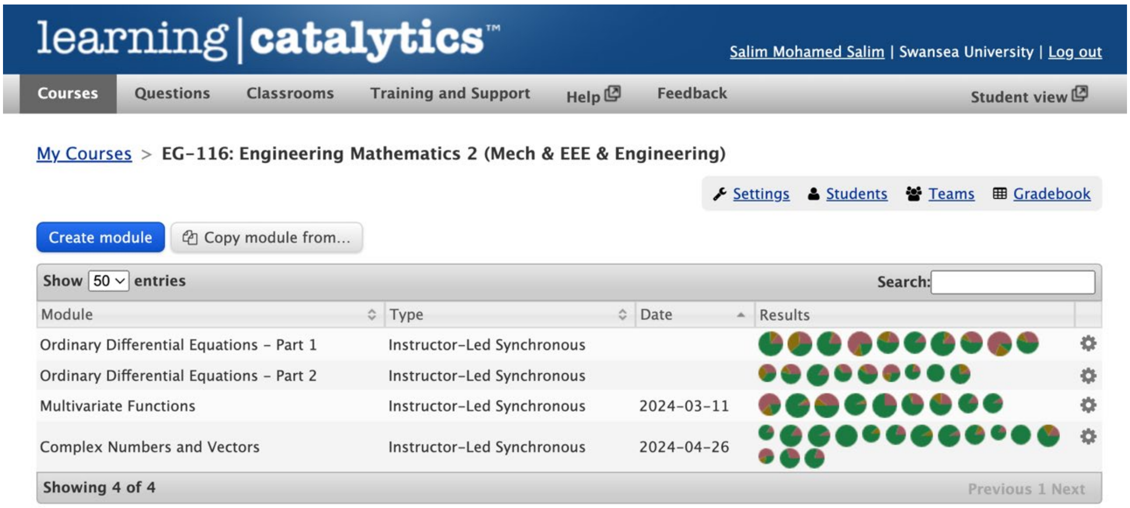

Figure 2. Class performance in in-class quiz (conducted via Learning Catalytics)

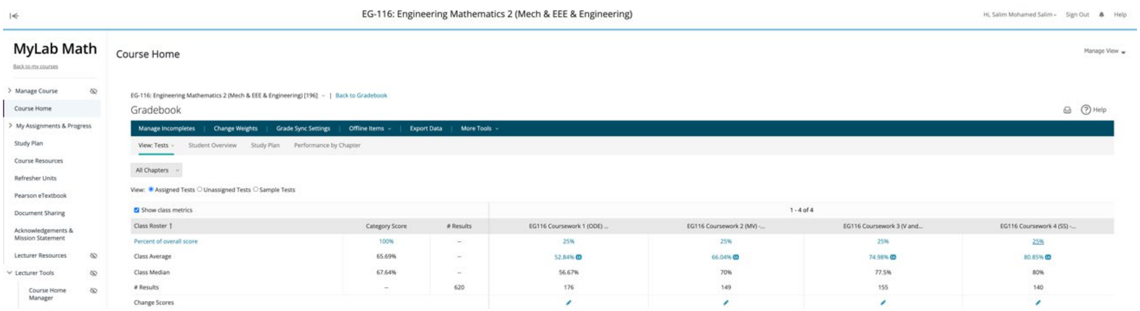

Figure 3. Class performance in online tests (conducted via Pearson MyLab Math)

Conclusion

The gradual enhancements introduced into the module combining real-life relevance and reordering the delivery to start with the main topics clearly lead to improvements in class performance and satisfaction and could be a useful strategy to adopt in other engineering modules.

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Enhancing Metacognition with Competence Lists

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Abstract

While the use of desk calculators has diminished mental arithmetic skills, the availability of powerful artificial intelligence further reduces students' motivation to solve mathematical problems. "Why should I do it?" they ask. However, without a solid mathematical foundation, the ability to critically assess AI-generated results is compromised.

The SEFI competency framework provides a strong foundation for undergraduate mathematics courses. The learning objectives outlined in this framework serve as the basis for our three-volume exercise book "Mathe – kann ich". Together with its competency lists, the book offers students a clear understanding of what is expected of them. Furthermore, by engaging with the material, students focus not on their mistakes but on their progress, as they check off problems they have mastered and competencies they have acquired. This approach enhances metacognition and reinforces long-term retention of the material.

The method of bringing SEFI competencies to life through exercises and worked-out solutions has received excellent feedback from undergraduate students in business IT and industrial engineering. This is further reinforced by the lecturers' feedback on students' solutions to voluntarily selected problems.

Introduction

The use of competence lists is a well-established method to support self-organized learning (Schmidt-Gröttrup 2014). According to Herold & Herold (2017), a competence list is a table of competencies defined by learning objective, activity record, taxonomy and a column to check off. Students use competence lists in order to monitor and check whether they have individually reached the targeted learning objective. In maths teaching the solution of exercises whether formative or summative is central. An example of a competence list is given in figure 1 taken from Schmidt-Gröttrup & Risse (2024) chapter 39 on matrices. Instead of a taxonomy, an expected duration for the exercise is given.

Questions to the lecturer

Sensible working with competence lists requires the lesson to be aligned to the learning objectives. This alignment means the lecturer has to make sure, that

- lessons are based on verifiably formulated learning objectives,
- the importance of the learning objectives is communicated to students,
- lessons, exercises and exams are aligned with the learning objectives,
- exercises and exams are aligned.

Nr	Ich kann	Aufgabe	√
	Matrizen: Grundbegriffe		
359	erklären, was mit einer Matrix gemeint ist,	39.1	
360	die grundlegenden mit Matrizen verbundenen Begriffe wiedergeben wie Diagonale, Spur, quadratische und Dreiecks- und Einheitsmatrix,	39.1	
361	die Transponierte einer Matrix bestimmen,	39.4, 39.8	
	Matrizen: Arithmetik		
362	eine skalare Multiplikation mit einer Matrix durchführen,	39.4, 39.12, 39.22	
363	erkennen, ob zwei Matrizen addiert werden können, und ggfls. ihre Summe berechnen,	39.4, 39.5, 39.6	
364	erkennen, ob zwei Matrizen multipliziert werden können, und ggfls. ihr Produkt berechnen,	39.2, 39.4, 39.6, 39.7	
365 *	Transformationsmatrizen erkennen und verwenden,	39.7, 39.9	
	Determinanten		
366	die Determinanten von 2×2 und 3×3-Matrizen berechnen,	39.11	
367	die geometrische Bedeutung von 2×2 und 3×3 Determinanten erklären,	39.10, 39.13	
368	die elementaren Eigenschaften einer Determinanten bei ihrer Auswertung benutzen	39.12, 39.14, 39.15, 39.16	
	Matrizen: Rang		
369	den Rang einer Matrix berechnen,	39.17, 39.18	
	Matrizen: Inverse		
370	die Bedingung an eine quadratische Matrix angeben, damit sie eine Inverse hat,	39.20, 39.21	
371	die Inverse einer 2×2-Matrix angeben, wenn diese existiert,	39.20, 39.21	
372	die Inverse einer Matrix mit Zeilen-Operationen berechnen, falls sie existiert,	39.22	
373 *	allgemeine Eigenschaften der Invertierung verifizieren.	39.19, 39.21	

Figure 1: Competence List for Chapter 39 in "Mathe - kann ich"

SEFI Curriculum

The curriculum of the SEFI mathematics working group was developed over two decades in a series of seminars and workshops on mathematical education for engineers (Alpers 2014). Its curricular part is divided into four core levels, from Core Zero containing the indispensable basics for any engineering education, to Core 1 containing the basic knowledge for general engineering. Core 2 contains topics that are essential for specific engineering disciplines, and Core 3 contains highly specialized mathematical knowledge for example numerical solution of ordinary differential equations. Within each core, individual topics are organized along five mathematical disciplines, namely algebra, analysis and calculus, discrete mathematics, geometry, statistics and probability. Apart from Core 3, explicit learning objectives are formulated. They are therefore an excellent basis for planning courses using competence lists.

Book Series "Mathe - kann ich"

The learning objectives of Core Level 0 and 1 of the SEFI Curriculum are the basis for the three volumes "Mathe - kann ich" (Schmidt-Gröttrup et al, 2022, 2023, 2024). They contain the basics of algebra and calculus in Volume 1, geometry and analysis of functions in Volume 2 and discrete mathematics and linear algebra in Volume 3. Each volume is organized along the slightly supplemented SEFI curriculum (Alpers 2014).

Limitations of the SEFI curriculum are discussed in Risse (2023). Students find their way around quickly as every one of the 42 chapter has the same structure: Introduction, Competence List, Problems and Solutions.

Students' Feedback

A survey at the end of 2023 resulted in very positive feedback to the first volume. The statement "The exercises are the most important feature of the book for me" is agreed with by the majority, with a mean of 1.47 on the five-point Likert scale: agree (1) - tend to agree (2) - neutral (3) - tend to disagree (4) - disagree (5). There is also clear agreement with the statement "The competence lists are the perfect introduction to the topic for me", mean 2.07. In the open questions at the end, a majority emphasize the clarity and structure of the book. One student wrote: "The author uses appropriate examples and formulations to explain difficult issues." At the same time, more detailed solutions are desired. A student remarked: "In the solution of some exercises more intermediate steps would be helpful for a better understanding." His wish was reaffirmed by many other student in the discussion of the survey.

Metacognition and Motivation

Orientation is crucial for learning security, i.e. the sureness to have learnt the right material and the certainty to be able to apply what one has learned. Methods as learning maps or advance organizers are helpful to show where a student is now, where the student has been and where the student is going (Asubel 1980). Dealing with the meta-level of the learning material is not particularly captivating for most students, however it is significantly increasing the persistence of what has been learned. The competence list brings the meta-level of the learning material to the fore. Working towards the goal of passing their exams, students make intensive use of the competence lists. Students appreciate the transparency of the examination requirements. Checking off completed learning objectives makes their own progress visible and boosts their motivation.

Further features of the learning environment

A series of videos showing solutions to single exercises was created during Corona lock-down. The learning objectives are mentioned in the introduction to the video. Students expressed their high appreciation of the videos and used them in preparation of the exams. Students develop a personal relationship and receive repeated feedback on tasks in exercises in the "Master-or-Die" format according to the approach of Nölte (2021). Here, a self-selected exercise is worked on to perfection with the help of feedback from the lecturer. Only a complete solution is graded ("Master"). Incomplete or incorrect solutions are not accepted ("Die"). The "Master-or-Die" format follows the ideas of "Mastery Learning" by Bloom (2020).

The possibility to choose topics on their own learning path is extremely motivating for students. A wealth of learning material structured by competence lists opens a wider learning landscape that can be offered for self-organized learning.

Conclusion

Competence lists are an extremely suitable tool for enhancing metacognition of one's own studies. The experience of one's own ability to study is particularly important in undergraduate studies. Competence lists strengthen students' motivation and learning success. The SEFI curriculum provides a good basis for developing the basic engineering mathematics skills. Linking a learning objective with exercises helps students to understand the objectives meaning. Additionally, their own learning success is checked. The "Mathe - kann ich" book series supplements Core 0 and Core 1 of the SEFI curriculum with exercises, i.e. problems and solutions. The combination of working with chapters from the book series, supplementary video material and feedback on completed problem solutions worked excellently in practice.

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Improvements of Knowledge Acquisition by Changing Teaching and Learning Methods

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Abstract

During the first year of engineering undergraduate degree programmes, students are expected to develop a profound understanding of mathematics. This requires both time and a willingness to practice on the part of the students. Unfortunately, it has been observed for several years that fewer and fewer students attend lectures and participate in the provided exercises. The prevailing trend is towards short-term preparation of course material shortly before the respective exam, resulting in generally poor exam performances and limited transfer of knowledge to long-term memory.

Based on our experience in the mathematical education for engineering students in recent years, we present our current changes to our teaching and learning methods. By revising the grading scheme and the examination format, we aim to improve examination results and better prepare students for the remainder of their studies.

Introduction

Mathematical training in engineering degree programmes typically occurs in the first year, providing the foundation for many subsequent disciplines such as electrical engineering, engineering mechanics or fluid mechanics. The primary goal of mathematical education is to impart a high level of mathematical knowledge and to foster a deep and long-lasting understanding. Alongside lectures, exercises and tutorials are designed to convey knowledge and promote a deeper understanding of the mathematical concepts and their interconnections. Regular practice throughout the term is essential to reinforce the material and enhance comprehension, cf. Brown, Roediger and McDaniel (2014).

Unfortunately, many first-year engineering students do not recognise the importance of this basic mathematical training and often see it merely as an unnecessary burden. As a result, they engage little with the underlying mathematical concepts, and do not attend or participate actively in lectures and exercises, tending instead to study only for the exam and favouring short-term exam preparation. In recent years, this trend has led towards a decline in the acquisition of mathematical knowledge. Consequently, the knowledge fails to be transferred into long-term memory and is lacking in advanced engineering courses, cf. Roediger and Butler (2011).

In order to counteract this trend, we have revised our teaching and learning methods starting from the winter term 24/25. These changes are inspired by established international systems that incorporate multiple assessments throughout the term. To encourage continuous engagement and to keep students motivated during the whole term, we are introducing several smaller examinations during the semester in addition to the final examination at the end of the term. As a result, we expect more consistent and active participation in the entire course, leading to improved exam results and a deeper understanding of the material.

In the following sections, we will outline our changes and describe our experiences with the help of initial statistical analyses. Particular attention is paid to the correlation between the changes and the performance in the exam. At the end, some impressions of students are discussed and a conclusion is drawn.

Changes in Teaching and Learning Methods

Two major changes were implemented for the winter term 24/25. The first involves continuous assessments throughout the term, and the second concerns the structure of the final exam:

- **Continuous assessments throughout the term:** Until 2024, students could voluntarily submit their exercise sheets (homework) for correction, allowing them to identify their mistakes and receive individual feedback. However, only about 15% of all students took advantage of this opportunity. Starting from the winter semester 24/25, submission and passing (without grade) of weekly homework became mandatory. In addition to the final written exam, students are now required to achieve an average score of at least 40% across all assignments throughout the term to pass the module examination, cf. Sotala and Crede (2021).
- **Restructuring of the final written exam:** As the written exam usually followed a standardised format with always similar types of tasks, many students prepared by simply reworking past exams questions. To reduce this predictability, exams are now designed with an increased variety in task types and also include additional comprehension questions.

Since the changes also affected examination modalities, the examination and study regulations were revised and came into effect for first-year students in the winter term 24/25. This naturally resulted in two test groups of students: one with mandatory assignments (mainly first-year students) and one with voluntary assignments (older students), which are distinguished accordingly in the following discussion and can be used to analyse the effects of the changes.

Assignments

During the winter term 24/25, there were 13 weekly assignments each containing 3 to 5 tasks. As shown in Figure 1, submission rates were generally 90% or higher, with a few exceptions. Assignment 9 coincided with the Christmas break; and the decrease in the submission rates for the last two assignments is attributed to some students having already accumulated enough points to reach the 40% requirement. The overall high submission rate indicates that not only the group of mandatory assignments, but also the group of voluntary assignments submitted their work.

In Figure 1, it can be observed that most students performed quite well in the assignments. In particular, a bonus task in the final assignment allowed as many students as possible to pass. Furthermore, we can see that some subjects were better understood by students than others, likely due to varying levels of difficulty across the mathematical topics and the tasks themselves. This might be taken into account in future terms.

The overall aim of these compulsory exercises was to promote understanding through continuous and intensive engagement with the subject matter throughout the term, in order to improve performance in the final exam, and to better prepare students for advanced courses. Our intention is to motivate students, rather than impose a significant obstacle. Having most students passing these assignments while observing an increased attendance and more active participation in both lectures and exercises indicates the success of our strategy.

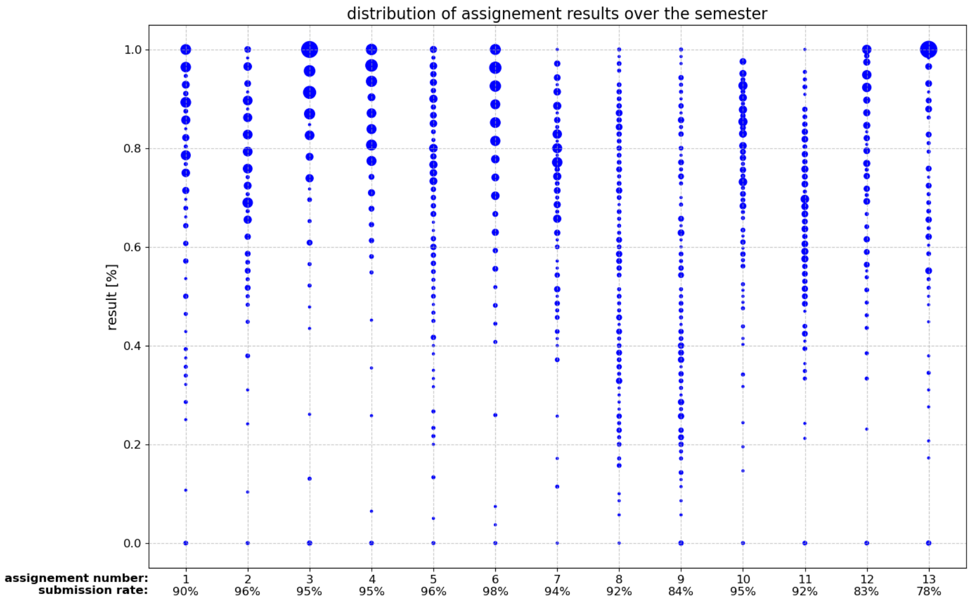

Figure 1. Distribution of assignment results over the winter term 2024/2025. The size of the circles indicates the number of submissions with the respective result.

Restructuring of Final Exams

The written exam was designed to maintain the same level of difficulty as in previous years, but with a greater variety of task types and topics. This included the introduction of new task formats and an increased proportion of comprehension-based questions. Specifically, the exam comprised two standard tasks, similar to those in previous exams (tasks 1 and 2 in Figure 2), four tasks of new type (tasks 3, 6, 7 and 8), and three tasks classified as comprehension questions (tasks 4, 5 and 9). While performance on the standard tasks was relatively consistent, there were greater variations in the comprehension tasks. These fluctuations suggest that students either fully understood and prepared for the respective topics, or not at all - except for task 9, which consisted of multiple independent short questions of different difficulty.

Overall, however, the exam results fell short of expectations. The reasons for this are assumed to be continued low attendance in lectures and too much focus on only practicing previous exams, rather than engaging with the broader range of material.

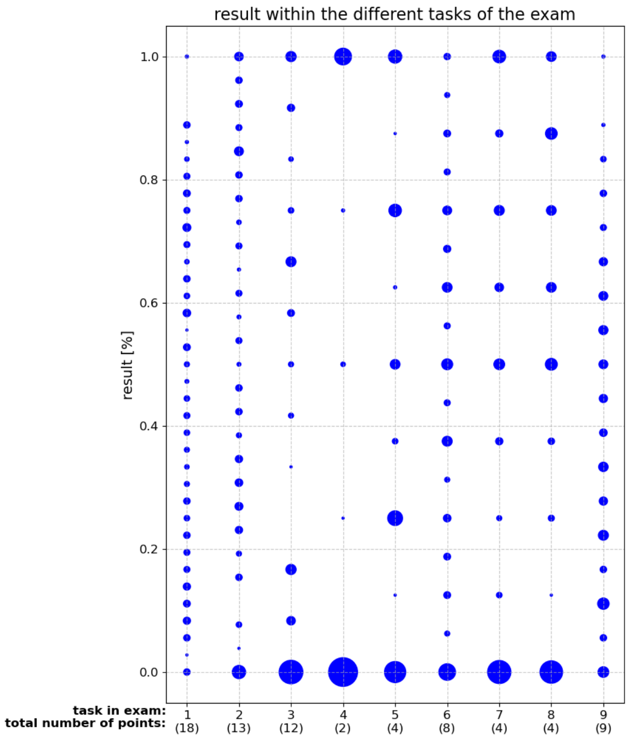

Figure 2. Results within the different tasks of the exam. The size of the circles indicates the number of students with the respective result.

Correlation between Assignments and Exam Results

In this section the correlation between the assignment performance and the respective exam result will be discussed, see Figure 3. In the graph, the different colours indicate the two test groups: blue represents students with mandatory assignments, while red corresponds to those with voluntary assignments. Additionally, dots represent students with active participation (i.e. at least 5 weekly assignments were submitted), whereas crosses indicate inactive participation. This distinction helps to filter out students who did not make a serious attempt to pass the assignments, consisting primarily of those in the voluntary assignments group.

The following positive conclusions can be drawn from Figure 3:

- The better the assignment performance, the better the exam scores, see black regression line.
- Almost no one passed the exam without passing the assignments.
- Almost every student, who actively tried to solve the assignments, also passed them. This aligns with the initial intention to motivate continuous study rather than create a significant obstacle.
- The exam passing rate is substantially higher among students with mandatory assignments (50%), compared to only 25% in the group with voluntary assignments.

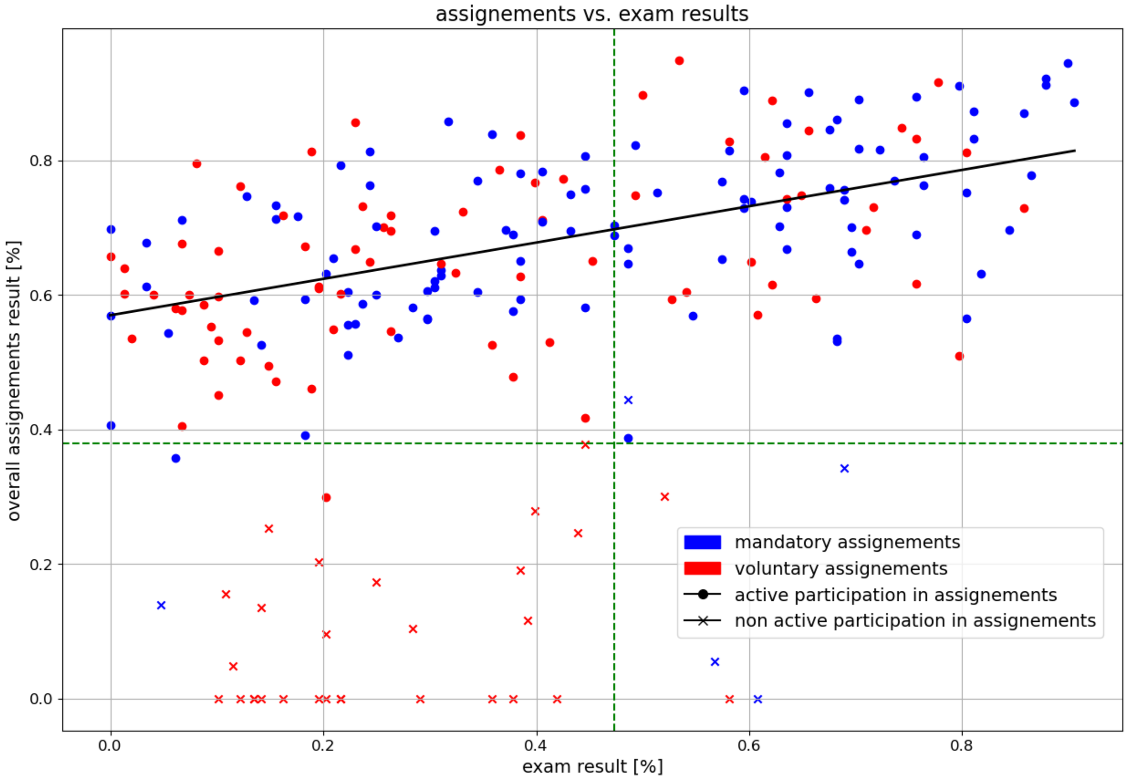

Figure 3. Correlation between assignment and exam results.

An additional survey conducted in one lecture (though not fully representative) indicates that the exam pass rate is even higher (60%) among students who actually attend and participate in the lectures. We assume a correlation whereby mainly first-year-students taking the course for the first time actually attend lectures regularly, while others (such as those who have already attended the course in the past or who only need to pass it as condition) are absent. In summary, active attendance during the current term appears to have a positive impact on exam success.

Despite all the positive conclusions, Figure 3 also reveals one significant concern. Although most of the students performed well in the assignments, many of them were unable to transfer this performance to the exam. This discrepancy suggests that some students may have copied solutions from others or relied on generative artificial intelligence to complete the assignments. Unfortunately, it is still unclear how to effectively address this issue.

While it may be necessary to accept that some students may never engage despite all effort, we can still aim to motivate most students by sharing the results of this study. Emphasizing that the assignments are manageable, even when solving them without assistance or external help, can encourage students to try solving them on their own. Furthermore, highlighting the clear correlation between assignment and exam performance may motivate students to use the assignments as effective preparation for the exam.

Impressions from Students

In addition to the perspective of the teaching staff, the students' views on the changes should also be considered. For this reason, a survey was conducted among the participants in the last lecture of the winter term 24/25. As Figure 4 shows, the feedback from the students was very positive. Furthermore, there were also direct comments requesting to maintain the compulsory assignments. It is encouraging to see that students appreciate being motivated by mandatory exercises and recognise that without this obligation, they tend to be less motivated.

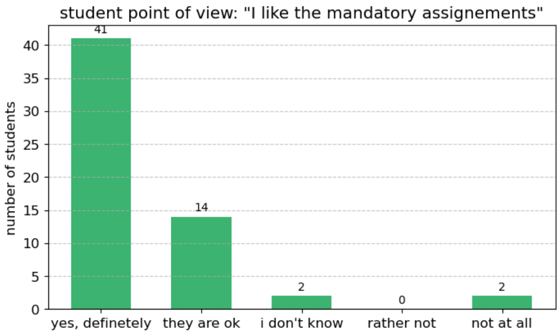

Figure 4. Impressions from students.

Conclusions and Future Prospects

The changes in our teaching methods resulted in some positive effects, such as an increased motivation among students and active participation in class, as well as a high submission rate for the assignments.

Interestingly, students need to be encouraged, or even compelled, to engage continuously with mathematics, as they rarely do so voluntarily. Our study shows that students tend to study primarily for grades and only engage with homework when mandatory, rather than for their own long-term understanding. However, they also acknowledge that consistent engagement with mathematical topics, though demanding and exhausting, is valuable.

However, the examination results still fell short of expectations, which is assumed to be due to the restructuring of the written exam. Improved results are expected in the future terms as soon as students become more familiar with the new system and exam format. Long-term effects, such as a deeper understanding of mathematical concepts and better preparation for advanced courses, cannot yet be estimated after just one term, but are expected to be positive.

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PBL in the Teaching of Differential Equations from Bachelor to Engineer

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Abstract

This paper will present the use of the PBL active learning method in the teaching of Differential equations from bachelor of engineering study programmes at the Faculty of Mechanical Engineering at the Slovak University of Technology in Bratislava.
PBL is an educational approach that focuses on using real-world problems to support student learning. Therefore, we want to emphasize its importance in teaching Mathematics – why we need it, in which mathematical courses we can use it, how to incorporate PBL into mathematics courses, where we can find it, what can students use to solve PBL, and how we want to proceed with PBL further.
The paper is concluded with a demonstration of one PBL used in two mathematics courses – in the basic course Mathematics II at the bachelor´s degree of study and in the Applied Mathematics course at the master´s degree of study.

Introduction

Mathematics represents one of the fundamental pillars for mastering specialized courses at technical universities. Secondary school graduates often struggle to meet the demands of university-level mathematics and to work independently, systematically, and apply their already acquired mathematical knowledge. Active learning methods such as Problem-Based Learning (PBL) and eduScrum help students overcome these challenges. PBL shows students where and how mathematics is continuously used in specialized technical courses. EduScrum develops students' soft skills, such as the ability to work in a team, share responsibility for solving tasks, and more. Based on our previous experiences, students' feedback on these methods has been very positive and their opinion was collected through an evaluation questionnaire (Gabková, Letavaj, Velichová (2022)).

Differential equations form the theoretical foundation of most engineering courses at the bachelor's and master's levels of study at the Faculty of Mechanical Engineering at the Slovak University of Technology (FME STU). They are taught in the basic course Mathematics II in the curriculum Ordinary Differential equations at the bachelor´s degree of study and in the Applied Mathematics course at the master´s degree of study. The Applied Mathematics course has been taught since 2018 and was designed specifically for masters degree students of the study programme Automobiles and Mobile Working Machines.

It is necessary to introduce PBL into teaching mathematics in the first year of study at the university because

1. they demonstrate where and how Mathematics can be used continually in special technical courses,

2. students learn to work with the nomenclature, that is used in technical courses (see Table 1.),
3. students become aware of the parallel between the conceptual apparatus of Mathematics and technical courses (see Table 2.),
4. they give students the opportunity to develop and test acquired knowledge and skills in Mathematics directly through problems they might face in the real-world situations,
5. students better understand the importance of numerical methods in engineering education.

Table 1. The nomenclature in the Maths and Technical courses

	Mathematics course	Technical courses
Different variables	$y = y(x)$	$u = u(\mathrm{t})$
Constants denoted generally	$g = 9.81\ ms^{-2}$	g (only in general)
Different notation of differential equation	$y'' = y''(t)$	$\frac{d^2u}{dt^2} = u''(t) = \ddot{u}(t)$

Table 2. The conceptual apparatus in the Maths and Technical courses

Mathematics course	Technical courses
The stationary point of a function	The equilibrium position of a system
Eigenvalues of the matrix	The stability of a system with forced oscillations
Transformation of **the motion equation** into a system of the 1st order linear differential equations	Transformation of **the motion equation** into a state space

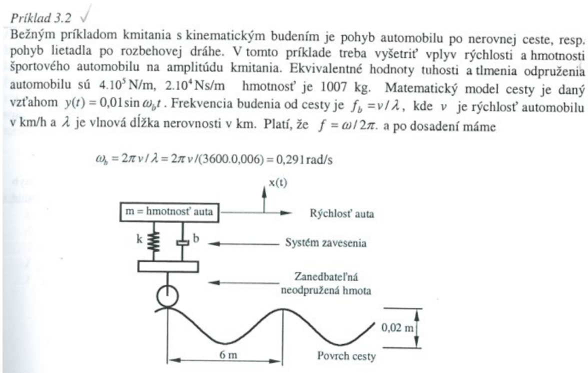
Practical experiences in teaching mathematics show how it is important to select problems for PBL from the lecture notes for specialised subjects taught at the faculty. Why? The advantage of these applications is that once in further study students "will remember them!". Special technical text books (Starek (2009), Žiaran (2003)) exist for study programme Applied Mechanics and Mechanics at the FME STU that are a rich source of PBLs for a teacher of Mathematics. However, the inclusion of these PBLs in mathematics teaching also requires close cooperation between the mathematics teacher and the teacher of technical courses in this study programme.

Some PBL equations can be hand-solved, but most of them require a solution using numerical methods and with the support of some software. FME STU students use Mathematica software, a table of formulas of numerical methods and Wolfram Mathematica type files containing short programmes to solve more demanding PBL equations using numerical methods. Each student "composes" his solution of the project using these short programmes with numerical methods, graphs of numerical methods and evaluation of errors in numerical solution. Why Wolfram Mathematica? Because it is a simple tool that allows to simulate and to solve calculation-intensive tasks of real-world situations through its symbolic and visualization options.

Demonstration of PBL in courses at the bachelor´s and master´s degree of study

Short introduction to a PBL topic. Car passengers are exposed to fatigue and unwanted vibrations that affect their health. Undesirable vibrations (in the vertical, longitudinal and transverse direction) occur when the car drives on an uneven road. For the vertical direction of oscillation, it is necessary to avoid the frequency band in the range of 4-8 Hz (the natural frequency of the human organism in the abdomen), which causes nausea. Several models of the car - quarter, half and full are used to simulate the oscillation of the car, its non-spring-loaded and spring-loaded masses when driving on an uneven road at different speeds. The behaviour of the car when driving on an uneven road allows us to know the solution of the equations of motion for the corresponding model of the car in the physical and state space.

The following PBL was selected from the special technical text book by Starek (2009).



Príklad 3.2 √

Bežným príkladom kmitania s kinematickým budením je pohyb automobilu po nerovnej ceste, resp. pohyb lietadla po rozbehovej dráhe. V tomto príklade treba vyšetriť vplyv rýchlosti a hmotnosti športového automobilu na amplitúdu kmitania. Ekvivalentné hodnoty tuhosti a tlmenia odpruženia automobilu sú 4.10^5 N/m, 2.10^4 Ns/m hmotnosť je 1007 kg. Matematický model cesty je daný vzťahom $y(t) = 0{,}01 \sin \omega_b t$. Frekvencia budenia od cesty je $f_b = v/\lambda$, kde v je rýchlosť automobilu v km/h a λ je vlnová dĺžka nerovnosti v km. Platí, že $f = \omega / 2\pi$. a po dosadení máme

$$\omega_b = 2\pi v/\lambda = 2\pi v/(3600.0{,}006) = 0{,}291 \,\text{rad/s}$$

Figure 1. The special technical text book and the selected PBL

- **PBL in the basic Mathematics II course at the bachelor degree of study**

A sports car weighing m = 1000 kg moves along an uneven road in time T at a constant speed. The mathematical model of the uneven road is given by function $u(t) = \sin(t)$. Example values of the stiffness and damping of the automobile suspension are $k = 2000\,\text{Nm}^{-1}$ and $b = 2000\,\text{Nsm}^{-1}$.

Movement of the car on the uneven road represents a real system with forced oscillation. The behaviour of the car (oscillating system) when driving on an uneven road allows us to know the solution of the equation of motion for a quarter model of a car with 1 degree of freedom in the physical space

$$m\ddot{x}+b(\dot{x}-\dot{u})+k(x-u)=0$$
$$x(0)=0$$
$$\dot{x}(0)=0$$

where
$x = x(t)-$ a deflection of a car moving on an uneven road

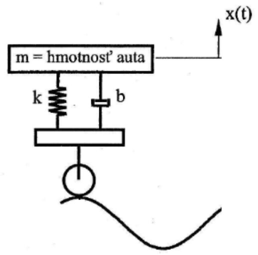


Figure 2.Quarter model with 1 degree of freedom of a car moving on uneven road

Task 1. Adjust the motion equation to the basic form.

Solution: $m\ddot{x}+b(\dot{x}-\dot{u})+k(x-u)=0 \;\rightarrow\; \ddot{x}+2\dot{x}+2x=2\sin t+2\cos t$

Task 2. Find the deflection of a car moving on an uneven road in time T.

Solution: The particular solution $x_p(t)$ of non-homogeneous ODE II is

$$x_p(t)=-\frac{4}{5}e^{-t}\sin(t)+\frac{2}{5}e^{-t}\cos(t)-\frac{2}{5}\cos(t)+\frac{6}{5}\sin(t)\;;\; t\in[0,T]\;;\; T>0$$

Task 3. Interpret the results about a car moving on an uneven road in physical units

$$x(3)=0,54\;;\;\dot{x}(3)=-1,07\;;\;\ddot{x}(3)=-0,64$$

- **PBL in the course Applied Mathematics at the master´s degree of study**

A car is moving along an uneven road in time T at a constant speed. The mathematical model of the uneven road is given by function $u(t)$. Movement of the non-spring-loaded masses and spring-loaded masses of the car on the uneven road represents a real system with forced oscillation.
The behaviour of the car (oscillating system) when driving on an uneven road allows us to know the solution of the motion equations for a quarter model of a car with 2 degrees of freedom in the physical space

$$m_1\ddot{x}_1+k_1(x_1-u)-k_2(x_2-x_1)-b_2(\dot{x}_2-\dot{x}_1)=0 \qquad x_1(0)=0\;,\;\dot{x}_1(0)=0$$
$$m_2\ddot{x}_2+k_2(x_2-x_1)+b_2(\dot{x}_2-\dot{x}_1)=0 \qquad x_2(0)=0\;,\;\dot{x}_2(0)=0$$

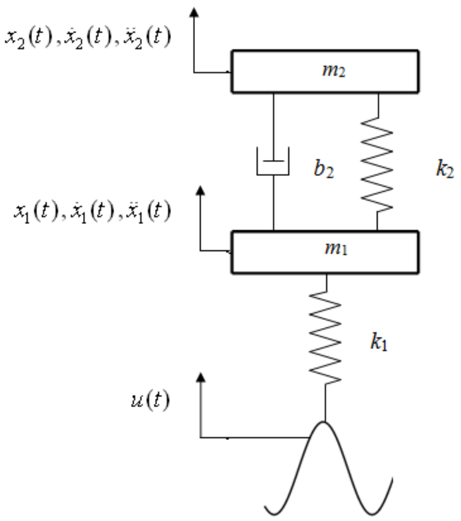


Figure 3. Quarter model with 2 degrees of freedom of a car moving on uneven road

where

– the mathematical model of an uneven road $u(t) = A\sin(t); A = 0.05\,\text{m}$ – the amplitude

– non spring-loaded masses of a car

$x_1 = x_1(t)$ – a deflection

$m_1 = 386.48\,\text{kg}$ – a mass

$k_1 = 1.2528 \cdot 10^6\ \text{Nm}^{-1}$ – a radial stiffness coefficient of the tyre

– spring-loaded masses of a car

$x_2 = x_2(t)$ – a deflection

$m_2 = 2628.7\,\text{kg}$ – a mass

$k_2 = 2.0597 \cdot 10^5\ \text{Nm}^{-1}$ – a coefficient of the spring stiffness

$b_2 = 6.722 \cdot 10^3\ \text{Nsm}^{-1}$ – a damping coefficient of oil shock absorber

Task 1. Transform motion equations (ME) into the state space and solve further tasks in this space. Write motion equations in matrix form for both space.

Solution:

ME in the physical space: $M \cdot \overline{x}''(t) + B \cdot \overline{x}'(t) + K \cdot \overline{x}(t) = \overline{F}(t);\ \overline{x}(0) = \overline{0};\ \overline{x}'(0) = \overline{0}$

ME in the state space: $\overline{z}'(t) = A \cdot \overline{z}(t) + \overline{f}(t)\ ;\ \overline{z}'(0) = \overline{0}$

$$\begin{pmatrix} z_1'(t) \\ z_2'(t) \\ z_3'(t) \\ z_4'(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1 - k_2}{m_1} & \frac{-b_2}{m_1} & \frac{k_2}{m_1} & \frac{b_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{b_2}{m_2} & \frac{-k_2}{m_2} & \frac{-b_2}{m_2} \end{pmatrix} \cdot \begin{pmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{k_1}{m_1} u(t) \\ 0 \\ 0 \end{pmatrix};\ \begin{pmatrix} z_1(0) \\ z_2(0) \\ z_3(0) \\ z_4(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Task 2. Based on presented graphs of numerical solutions, can one state, that the acceleration of the non-spring-loaded and spring-loaded masses of the car are dampened after a certain time?

i.e. The car's movement on the uneven road will be stabilized?

Solution:

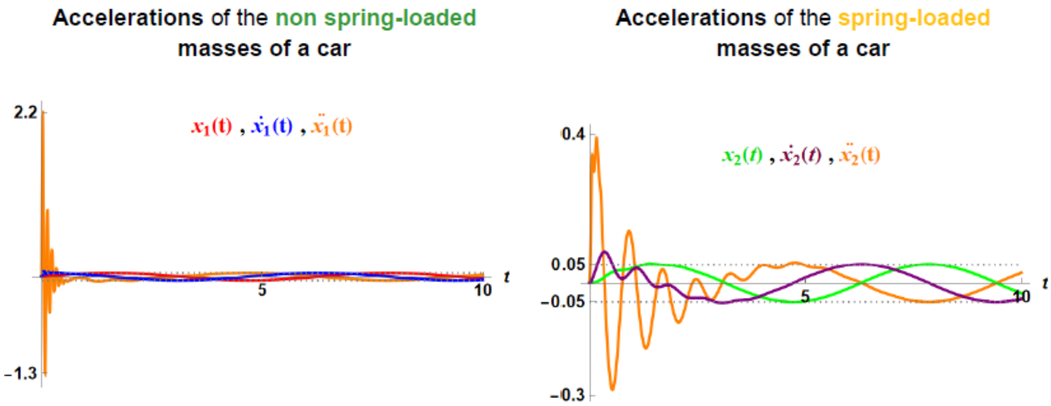


Figure 4. Dampening of accelerations of both masses of a car

Task 3. Display a deflection and a velocity of the non-spring-loaded and spring-loaded masses of a car at time $t \in [0,10]$ in sec using Euler´s method with step $h = 0,01$. If the method is not stable for the given step h, modify step h so that it is stabilized.

Solution:

The unstable Euler´s method ($h = 0,01$) The stable Euler´s method ($h = 0,001$)

the deflection of a car

the velocity of a car

Figure 5. Non-spring-loaded masses of a car

The unstable Euler´s method ($h = 0,01$) The stable Euler´s method ($h = 0,001$)

the deflection of a car

the velocity of a car

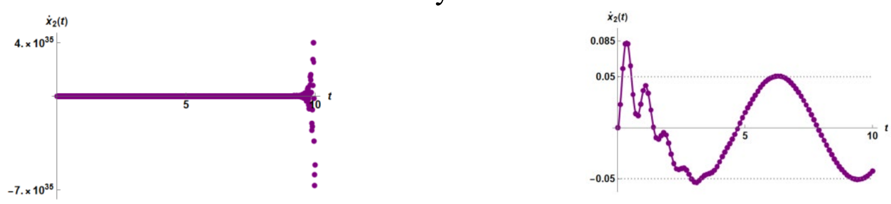


Figure 6. Spring-loaded masses of a car

Comment. Task 2. and Task 3. were solved using numerical methods and the software Wolfram Mathematica.

Conclusions for Education

The teaching process is based on cooperation between the teacher and the students. Therefore, in this process, not only the teacher but also the students must be active. (Maňák, Švec 2003). PBL is a teaching method by which students learn to use mathematics by actively engaging in real and meaningful projects. The PBL method, combined with the eduScrum approach, is what drives students' motivation and learning.

Therefore, we have decided to continue using PBL in the future. We have already selected additional topics involving suitable application problems to show first-year bachelor's students how and where mathematics is used in specialized technical subjects. At the same time, we aim to better prepare master's students in the *Automobiles and Mobile Working Machines* study program for their diploma theses.

Acknowledgements

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