The 21st SEFI Special Interest Group in Mathematics – SIG in Mathematics

June 11th to June 14th 2023

Tampere University of Applied Sciences (TAMK)

Tampere, Finland

PROCEEDINGS

Tampere University of Applied Sciences

SEFI European Society for Engineering Education

STEM CENTER TAMK
Physics Chemistry Mathematics Technology Pedagogy

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INTRODUCTION

The Steering Committee of the SEFI Mathematics Special Interest Group is pleased to present the proceedings of the 21st SEFI MSIG Seminar on Mathematics in Engineering Education. This seminar was locally organized by the Tampere University of Applied Sciences, and has taken place from June 11st to 14th, 2023. Since its establishment in 1982, the SEFI Mathematics Working Group (MWG), now known as Mathematics Special Interest Group (MSIG), has consistently upheld its goals, which remain pertinent and applicable even after 41 years. These objectives include:

- Providing a platform for the exchange of perspectives and ideas among individuals interested in engineering Mathematics.
- Promoting a comprehensive understanding of the role of Mathematics in the Engineering curriculum and its significance in meeting industrial needs.
- Cultivating collaboration in the development of courses and supporting materials.
- Recognizing and advocating for the role of Mathematics in the ongoing education of engineers, in collaboration with the industry.

This seminar edition was based on mathematical competencies in didactical research with emphasis on practise roles of researchers and practitioners and their cooperation. How to assess competencies and the major aim of teaching Mathematics as a tool to future Engineers was also among the discussed topics.

Since 1984, the SEFI MSIG, former known as MWG, has organized 20 seminars on Mathematics in Engineering Education, dedicated to fulfilling its objectives and encouraging international participation. The 21st Seminar took place at the beautiful and welcoming city of Tampere, and marked the continuation of this successful series, bringing together passionate Mathematics educators. These seminars are designed to serve as a platform for the exchange of perspectives and ideas among participants interested in innovative methodologies in teaching Mathematics to Engineering students, i.e., participants that are interested in exploring and learning about new and creative approaches or techniques for teaching Mathematics specifically to Engineering students.

Its overarching goal is to advance a comprehensive understanding of the role of Mathematics in the Engineering curriculum, emphasizing its relevance to industrial needs and the ongoing education of Engineers within the European context. This seminar featured presentations and discussions on various crucial topics identified by the SEFI MSIG Steering Committee, as well as other pertinent issues in the Mathematical Education of Engineers. The central theme of the seminar revolves around the concept of mathematical competencies, encompassing the following themes:

- Mathematical competencies in practice and didactical research
- How to assess competencies?
- The goal of teaching
- Active learning strategies
- Mathematical Engineering and Informatics

Programme of the seminar included three plenary keynote lectures presented by excellent invited speakers, professors teaching mathematics at universities in different European countries.

Professor Markku Saarelainen from Tampere University, Finland spoke about “Flipped Classroom as a strategic approach in teaching – a brief history from theory to practice in Finnish university level “.

Professor Per Henrik Hogstad from the University of Agder, Norway, presented the talk on “Visualizations and simulations in mathematics and physics“, and finally

Professor Ion Mierlus-Mazilu from Bucharest Technical University of Civil Engineering, in Romania, discussed topic “The Use of Mathematical Software in Teaching Mathematics “.

Despite the limitations imposed by the global health crisis that we faced, your commitment to advancing knowledge, working with your students, and sharing insights in the field of engineering education has been perseverant. As a result of this perseverance, a positive response to the seminar's call for papers led to the acceptance of 23 high-quality submission directly aligned with the seminar themes, all addressing crucial topics in the mathematical education of engineering students.

It is important to acknowledge the unique circumstances that have shaped our experiences over the past couple of years. The pandemic has presented unprecedented challenges, disrupted traditional modes of education, and forced us to adapt to online platforms for learning and collaboration. It is a testament of dedication that all teachers have embraced this new normal, making the most of the virtual environment to engage in fruitful discussions and expand yours and your students' horizons. One of the key aspects we must explore is the significance of Mathematics in Engineering Education. Mathematics forms the foundation upon which Engineering principles and concepts are built. It provides the necessary tools for problem-solving, critical thinking, and analytical reasoning, all of which are essential for engineering students to excel in their chosen fields. Without a solid understanding of Mathematics, it would be challenging to comprehend and apply complex Engineering theories. However, it is equally important to strike a balance between the rigor of mathematical concepts and their practical applications. While Mathematics provides a theoretical framework, it is through practical classes and real-life problem-solving that students truly grasp the essence of Engineering. The application of Mathematical principles in practical scenarios helps students bridge the gap between theory and practice, fostering a deeper understanding of engineering concepts. Integrating ICT into Mathematics classes not only enhances students' engagement and understanding but also prepares them for a technology-driven world where mathematical skills are in high demand. It is important to strike a balance between mathematical rigor and the utilization of ICT tools, ensuring that students develop a deep understanding of mathematical concepts while leveraging technology to enhance their learning experience.

All accepted contributions are incorporated as full papers in the proceedings, which are freely accessible on the SEFI MSIG webpage. This initiative aims to offer a comprehensive overview of the seminar's topics and provide unrestricted access to the presented papers for all interested colleagues. The primary goals of the group are to uphold the ongoing process of collecting published materials and reports on all crucial topics identified in the mathematical education of engineers. This aims to construct a robust body of knowledge within this field. Lastly, the author expresses gratitude to all members of the SEFI Mathematics Special Interest Group Steering Committee, language editors, and local organizers for their efforts in conducting language checks and editing the proceedings. These contributions are intended to enhance the quality of the proceedings for the benefit of all potential readers.

In Coimbra, June 2023

Deolinda Dias Rasteiro
SEFI MSIG chair

Examining Data on Student Utilization of Recorded Lectures

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Abstract

As a result of the covid era distance teaching, many institutions are in possession of recorded live lectures. Given the benefits of such recordings for students, it is natural to make these recordings available also to on-campus students. This changes the way students study, and may impact their learning. We report on feedback from students and statistical data regarding use of recorded lectures within the framework of a traditional course.

0. Introduction

Distance education during the covid era typically included streamed lectures that were often also recorded. When lockdowns ended and we returned to on-site education, our institution was – without prior intent – in possession of recorded lectures for many of its courses. Students repeatedly expressed their desire for continued access to these recordings. While there were some concerns about this, a number of teachers made recordings available to on-site students. Typically, they would just let students know about their existence and teaching their courses as usual. This, in our experience, is a very common approach also at other institutions, and it is important for the educational community to know more about the impacts of such use of recordings.

There are grounds for being concerned. Focusing on mathematics and “casual” use of videos as outlined above, there is a variety of reported results, see e.g. the recent survey Lindsey & Evans (2022), and enough of them are negative to show that there is a good reason to worry. We need to understand the situation better.

The most visible impact, reported in the literature and experienced by us when we offered recordings of lectures in our courses, is a visible drop in live lecture attendance. Unfortunately, we cannot quantify it because at our institution lecture attendance is not recorded. Our perception was that the attendance dropped perhaps by a quarter or more.

However, the real question is whether this decreased live lecture attendance lowers learning. In the three courses with available recordings that we consider here the failure rate and grades were within the usual pre-covid variation. However, it should be noted that these courses are not entirely typical, as they include a strong interactive feedback component that can mitigate to some extent the impact of recordings. It is also very likely that the observed changes in student approaches to learning were not in their final form yet and we do not know how the learning outcomes in our courses change when students fully adapt to the new circumstances.

We will see student attitudes reflected in our exploration of data on student use of recordings.

1. Student surveys on recorded lectures

We will look at two surveys, both taken in the spring of 2022, the first post-lockup semester. At the time, the incidence of the virus was still relatively high, and students were explicitly encouraged by instructors to stay home and watch recorded lectures if they did not feel healthy. Consequently, the student response to recordings may not accurately reflect their natural attitudes.

The first survey was taken in course TAL (theory of algorithms) given to first year MSc students. Students were surveyed at the start of the exams and of the 131 enrolled students, 75 completed the questionnaire. The second survey was taken by students of course DRN (differential equations and numerical analysis) taken by freshmen. They were asked to fill it out after exams, at the beginning of the summer break, which may explain the lower response rate: 75 out of 325 students completed the questionnaire.

In both surveys students were asked how many live lectures they attended and how many recorded lectures they viewed. In Table 1a we see in percentages how students in the two surveys answered.

Table 1a. Interest in live and recorded lectures (TAL, DRN)

	None	Few	About half	Almost all
TAL: Live	14.7	20.7	37.3	28.0
TAL: Rec	13.3	16.0	40.0	30.7
DRN: Live	1.3	16.0	22.7	60.0
DRN: Rec	8.0	42.7	34.7	14.7

Both surveys confirm high use of recordings. The relatively high self-reported lecture attendance in the second survey does not quite fit the instructor's perception of attendance and may be in part a selection bias. While the true difference is questionable, it is likely that the real DRN attendance was higher compared to TAL as indicated here, because this would fit with general pre-covid patterns that MSc students have generally lower attendance than BSc students. As expected, students whose live lecture attendance was low tended to view recordings more and vice versa.

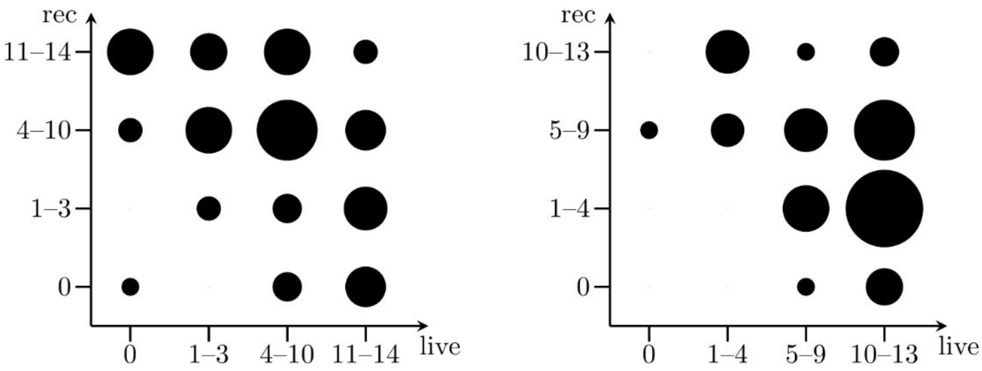

Figure 1b. Live versus recorded lectures (TAL left, DRN right)

We asked students whether they think that without recordings, they would attend lectures more. The two surveys differed in the way this question was asked, in Table 1c we again show percentages.

Table 1c. Perceived impact of recordings on attendance (TAL, DRN)

	Yes	Perhaps yes	Perhaps no	No
TAL:	28.6	26.5	14.9	30.6
DRN:		21.9		78.1

The significantly higher number of negative responses for DMA is related to the high self-reported attendance, leaving little room for improvement.

In both surveys we also addressed use of recordings during exams. Since the first survey was taken before exams, we could only ask about the intent and 90.3 percent of students answered that they will definitely use recordings when preparing for exams. The second survey took place after the exams and 74.7 percent of students reported using recordings during exam preparation.

Finally, we asked students to estimate the benefit of recordings, assuming that they watched them.

Table 1d. Benefit of recordings (TAL, DRN)

	Significant	Moderate	Negligible
TAL:	84.3	15.7	0.0
DRN:	67.6	32.4	0.0

2. Utilization of recorded lectures

We complement the results of our surveys with some statistical results about use of the recorded lectures. We use analytics supplied by Youtube, namely number of accesses and viewing time. Both measures are difficult to interpret without knowing context. Therefore, a proper analysis would have to be done individually for each recording, taking into account all relevant information. While such analysis is useful for an instructor, it is too specialized and extensive for this report.

Note that Youtube logs views based on physical time rather than on what portion of the recording was viewed. For instance, an hour long lecture viewed at double speed only counts as 30 minutes of viewing time, although the viewer saw it all. To assess how serious this distortion of data is we surveyed students in one current course while preparing this report, with over 120 students replying. All of them view videos for the course, and all except three tend to view it at increased speed. Roughly half of them use double speed, while the other half use factor 1.5.

We start with the TAL course given in spring 2022, for which we already saw a survey. First we look at total number of accesses (red) and viewing time (in hours, blue) for the

14 recordings of lectures, where the totals are taken only over the semester The graphs were scaled to offer comparable curves.

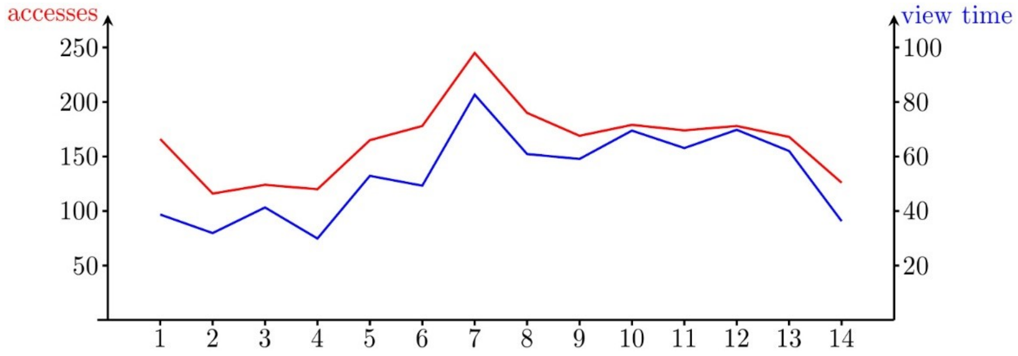

Figure 2a. Accesses and viewing time of lectures (TAL)

Both measures provide roughly comparable information, but sometimes they differ. In particular, the first video in a playlist typically shows a disproportionate number of short accesses ("curiosity effect"). The number of accesses ranges (roughly) between the enrolment and its double. An average student saw 11.2 minutes out of every hour of recording. Obviously, this includes students who saw very little, and others who saw a lot. Now we look how students viewed the 14 recorded lectures in individual weeks.

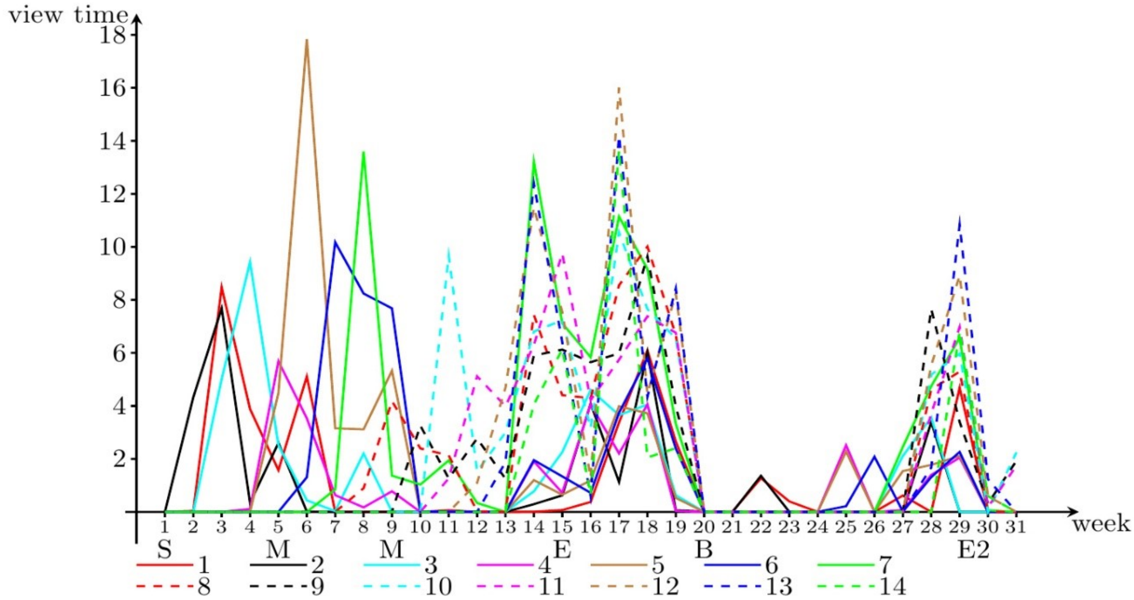

Figure 2b. Timeline for viewing time (TAL)

Here S marks the week when semester starts, M shows midterms, E the beginning of exam weeks, B is when summer break begins and E2 is extra week of exams in early September. We see expected patterns, with student using recording first in weeks when lectures took place, then around midterms and heavily during exams. The five main exam weeks before summer comprise 46% of the total viewing time, which is more than the use during the weeks of instruction (36%).

Next we look at daily patterns in student use of 13 recorded lectures for the DMA course (discrete mathematics) taken by freshmen in fall 2022. In the following graph we shifted daily viewing time data for individual recordings so that they align at the day of lecture, marked L_n in the graph.

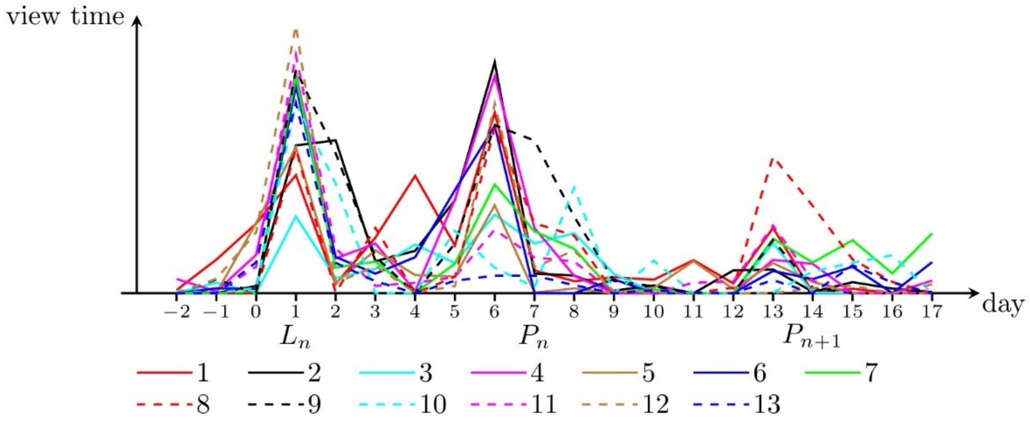

Figure 2c. Daily viewing time, scaled (DMA)

Recordings have their main peaks on the day of lecture, which was Tuesday. The topics introduced there were then explored in the practice class marked P_n the next Monday, and many students reviewed for it on Sunday, showing another peak on day L+5. A week later, in practical class on Monday P_{n+1}, the homework on topics from week n was due, which shows another spike of interest in video n on day L+12, because majority of students seem to work on their homeworks on Sunday again, with some spilling over to Monday and some asking that they be allowed to turn it in later.

The data show that students use recordings in a fairly predictable way.

The 2022 run of DMA offers an interesting comparison, because some lectures were cancelled and students were officially directed to recorded lectures, while they viewed the others at their own discretion. A similar situation happened this spring with the DRN course. In Table 2d we compare data from these two courses and data for the 2022 TAL course discussed above. Namely, for each recorded lecture we look at how many minutes out of every hour of this recording an average student saw within the four-week window that starts a week before the corresponding live lecture and ends two weeks later. In the first four columns of the table we see recordings that were accessed at student discretion, while the next three columns show data for officially designated video replacements for cancelled live lectures (there were no such lectures for TAL).

Table 2d. View time per student per hour of lectures (DMA, DRN, TAL)

DRN	4.3	3.2	6.0	2.3	30.4	25.7	24.8
DMA	7.7	9.7	8.9	6.2	29.6	20.8	18.4
TAL	6.1	5.4	4.7	5.6			

As expected, utilization of officially recommended recordings is significantly higher compared to non-promoted recordings. For the same type of recording, the numbers are fairly similar for all the courses, but we do see differences. There are several factors that can cause the differences and we are hoping to get a clearer view with new data.

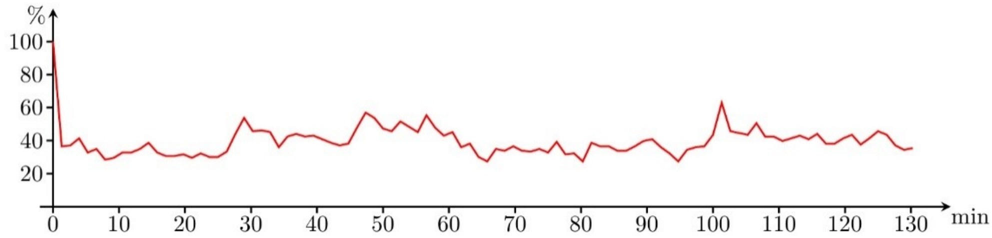

Figure 2e. Video audience retention (TAL)

We conclude our overview of available data with information on audience retention. This shows how many of those who accessed the recordings saw its particular segments. Typically, we see local oscillations, and often also longer segments of lower or higher interest. These are typically easy to explain based on contents, and this information is very useful for the instructor. The local peaks are less tractable. Many of them occur when the slide in the lecture changes, suggesting that students skim videos and start watching when they see a prominent change. However, there are peaks that do not conform to this hypothesis, and for some surges in interest we did not find an explanation yet.

3. Conclusions

The information that we processed above did not really bring any surprises. It confirmed student appreciation of recordings and their impact on live lecture attendance, and data on use generally follow our intuitive expectations. It seems that student reaction to recordings is rational and to a large extent predictable. This is a useful information for an instructor who plans on making recordings available to students.

It would be interesting to compare some of the data with students from other majors and other countries. We also introduced weekly online quizzes to be taken after every lecture (but before practical class) and by the next year we should see whether they in any way influence patterns in use of recordings. The authors hope that the next run of their courses will not be influenced by cancelled lectures and that we obtain a clearer picture of the natural student use of recordings.

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Lindsay E., Evans T. (2022) "The use of lecture capture in university mathematics education: a systematic review of the research literature." *Math. Ed. Res. J* 34, pp 911–931.

Youtube analytics, downloaded throughout March 2023 for the 2022 courses, on May 13, 2023 for the courses in spring of 2023.

The effect of using Padlet on mathematics collaborative learning in an engineering course

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Abstract

Collaborative learning is a process of continuous interaction and joint and mutual construction among members of a given group with the aim of acquiring knowledge and skills. The Padlet application is a collaborative digital tool that allows the construction of knowledge, through the creation of a dynamic wall, in an interactive digital environment for debate and sharing ideas. The use of collaborative methodologies in higher education has been incipient. It is pertinent that greater reflection on didactic knowledge and the application of collaborative learning be initiated in higher education institutions. Thus, this paper intends to describe, analyse, discuss, and evaluate the interest in using a collaborative learning environment for the development of knowledge and for student motivation in the teaching-learning of mathematics for engineers. The students and teachers' visions were also observed regarding their motivation and interest, allowing enrichment of the necessary discussions for the future development of activities.

Introduction

Teachers are increasingly concerned about the motivation and success of their students. Collaborative learning (CL) is a process of continuous interaction and joint and mutual construction among members of a given group with the aim of acquiring knowledge and skills (Figueiredo et al. (2014)). In this contribution, we consider the questions “Can CL be a strategy to improve the teaching-learning process in higher education?” and “Is it possible for students to succeed through the application of CL in the classroom?” A review of empirical results on CL suggests that motivation is socially constructed through interactions with others in the classroom context (Järvelä et al. (2010)). The use of collaborative methodologies in higher education has been incipient. There are few studies in the literature that refer to the application of these methodologies in the classroom. It is pertinent that a greater reflection on didactic knowledge and the application of CL be initiated in higher education institutions (Freeman et al. (2014), Yoshiyuki et al. (2022)).

The Padlet application (Padlet (2023)) is a digital tool that allows the construction of knowledge, through the creation of a dynamic wall, in an interactive digital environment for debate and sharing of ideas (Ramachandiran et al. (2018)). It allows user to easily share information and/or lead conversations using posts, reactions, and comments. According to Udosen (2020), the Web 2.0 Padlet tool and its use with senior secondary students in teaching Agricultural science concepts showed significant evidence of greater retention and academic performance among students. Thus, this paper presents the reflections of two higher education teachers and their students on participation in CL using the Padlet tool by sharing thoughts, creations, discoveries, and experiences related to the syllabus to help students build new learning.

Research method

During the second semester of the academic year 2022/2023, the curricular unit (CU) of Mathematical Analysis of the Degree in Electrical and Computer Engineering is offered to all students who did not pass this CU in the first semester, and do not want to postpone it into the next year. The contents and assessments are identical to those of the first semester, however, as the students already attend the referred CU, they already have many of the theoretical and practical bases, so in this semester the classes are essentially practical, supported by collaborative methodologies. In laboratory classes, students may use computers and investigate the contents of the CU referring to numerical methods. The application of CL was planned and organized according to the four phases suggested by Ponte et al. (2020): Problem, Preparatory work, Observation and Reflection.

- Problem: This year, classes were planned and organized collaboratively, in groups of 2 students, through autonomous learning using the Padlet tool. Each 1h30m class was divided into 3 distinct phases: autonomous learning, verification of learning and assessment of learning.

- Preparatory work: The first phase of the class, was for individual assessment of learning, where students solve individually an exercise on paper about the content taught in the previous class. Lasting 15 minutes and using all available technologies, such as GeoGebra, WolframAlpha and a calculator. In the second phase, autonomous learning takes place for 1 hour. The groups solved one or two exercises to which they have access through a QR-code (the exercises were different for each two students' group) and at the end they upload the solutions to the Padlet area provided for the group inside the Padlet CU area. To carry out the exercises, it is sometimes necessary to remember the techniques and methods involved, consulting the documents available on the CU's Moodle about the contents. In the third phase, learning is verified, lasting 15 minutes. The exercises are shuffled, and each group must correct another group's exercise. At the end, they must comment on the solution in Padlet and, in cases where they find an error, they must upload the correct solution. With Padlet use, students find a set of solved exercises, as if it were a notebook, which they can access any time, and which is extremely important for studying the assessment in the following week. On the other hand, it is a sharing space built by everyone and for everyone, where autonomous learning is the main objective.

- Observation: For laboratory classes, seven topics were covered in seven weeks (Absolute and Relative Error: Definition, Formula; Approximation by Differentials; Graphical method for finding roots of nonlinear equations; Bisection Method: Definition, algorithm, and error; Newton-Raphson Method: Definition, algorithm, and error; Numerical integration: Trapezoidal Rule; Numerical integration: Simpson Rule). The first four of these took place during the period from March to April and these are the ones that are pictured here. In Figure 1, an example of Padlet environment is presented. Throughout this learning process, the teacher accompanies the students, guides them, and gives their final feedback on the students' solutions.

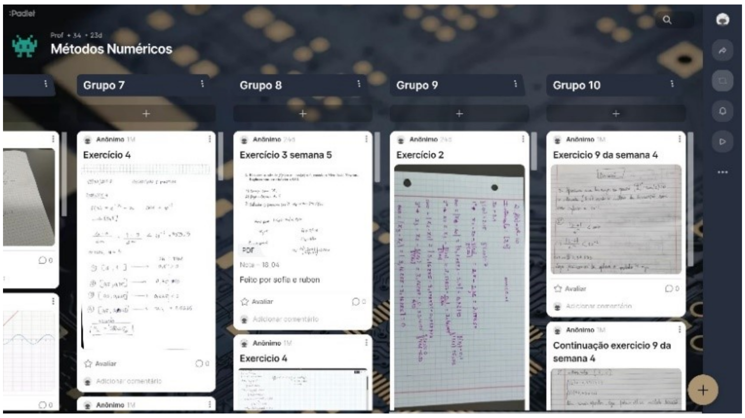

Figure 1. Padlet environment.

- Reflection: The reflection was carried out by the teachers, at the end of each week highlighting the positive factors and identifying the points that needed to be revised.

Research Results and Discussion

From the analysis of the results presented, conclusions were drawn from three different perspectives: the students' view, the teachers' view, and the assessment.

The students' view

Through a questionnaire distributed at the end of the 4 weeks, completed by only 14 students out of the 20 enrolled, it was verified that all students, except one, affirm that their contribution was important for the group to suggest paths and boost the development of the work (8 students), to apply mathematical procedures (7 students), to structure the work (7 students), to organize ideas and reasoning to build new knowledge (6 students), to mobilize prerequisites (knowledge already acquired in previous learning) (5 students), to lead the group (5 students), to justify the reasoning developed (5 students) and to involve the different elements of the group (3 students).

All students answered that the group work dynamics contributed to the development of the proposed exercise. In Figure 2 (left), it is possible to see the words, chosen by students to best describe their performance in the activities of the laboratory classes, such as: cooperation, commitment, perfection, and motivation, through a word cloud. Furthermore, 78.6% of students are confident in what they learned in the classroom and 92.9% enjoyed using the Padlet and say it promotes greater participation and collaboration among group members. Figure 2 (on the right) represents the expressions, chosen by students, that best express how the use of the Padlet was related to the acquisition of new knowledge. Enabling knowledge exchange was the most chosen expression.

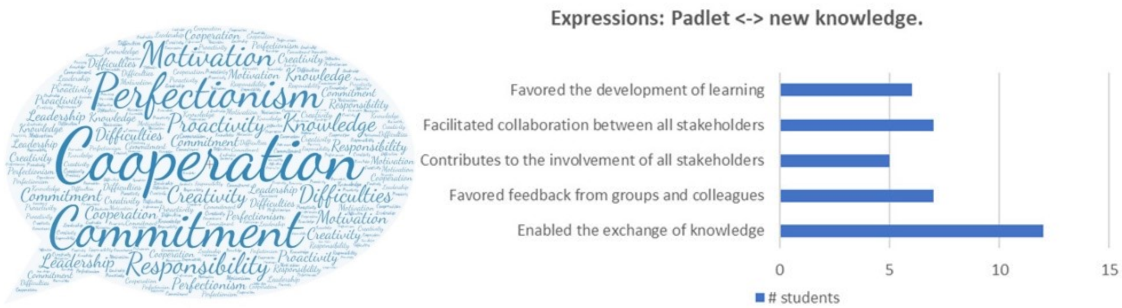

Figure 2. Expressions which best describe their performance in the activities of the laboratory classes - Word Art (left) and expressions that best express how the use of Padlet was related to knowledge acquisition (right).

All students like that innovative learning activities are applied in the classroom and consider that this learning methodology can contribute to the development of skills inherent to academic (engineering) and professional choices. 100% of the students consider that the teacher/student mediation and 92.9% that the mediation between the different students of the group was essential for overcoming the different stages of work design. Regarding the three open questions asked to students: *What did the students like most about these classes? What was the students' biggest challenge during laboratory classes? and What are the students' opinions about the laboratory classes?* the answers obtained are represented in Figure 3. Students like group work and the ease of learning with the CL methodology (Fig. 3 (left)). The biggest challenge is overcoming basic difficulties in mathematics due to lack of secondary knowledge (Fig. 3 (middle)). The laboratory classes were referred to by the students as being good classes, interesting, motivating, creative and fun to learn, which becomes essential for a greater involvement in the CU (Fig. 3 (right)).

The teachers' view

In the first week of the activity, most students showed no experience with the Padlet, although they quickly acquired the necessary knowledge to use it. Cooperation between them existed from day one, as this group of students was already familiar with CL. The greatest difficulty was the autonomous study and to carry out the activities for which they still did not feel confident, because they did not have sufficient skills to allow them an autonomous resolution. For this reason, the teachers were obliged to give much greater support to most groups. In this sense, this routine is very important for achieving inclusive education at school, not only because it allows for difficulties to be overcome and generates advances in learning of the students that would not be able to succeed independently, but, above all, because of the differentiation of work groups and the difficulties of each one. As the weeks went by and the students had already understood the methodology of learning in classroom, the teacher's help in autonomous study was progressively reduced and the progress of the students was clearly improved. The classroom environment became more relaxed and inviting to the promotion and acquisition of knowledge and skills.

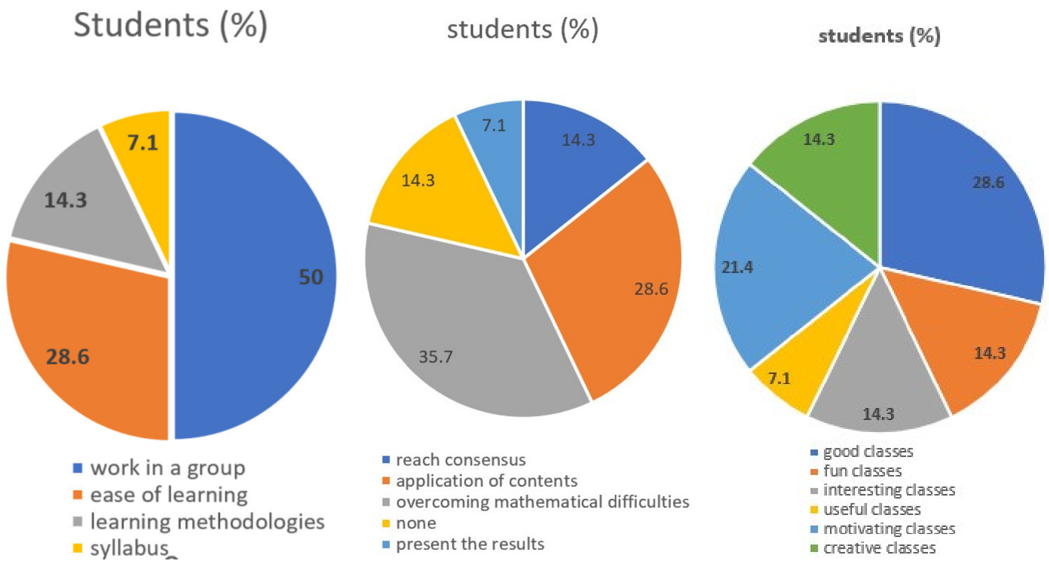

Figure 3: *What did the students like most about these classes?* (left), *What was the students' biggest challenge during laboratory classes?* (middle) and *What are the students' opinions about the laboratory classes?* (right).

The assessments

The assessment at the beginning of each class was carried out individually through a question related to the content addressed in the previous class, for 10/15 minutes. This question is part of the continuous assessment process and was used as a formative assessment since the corrections were presented to students a week later. Figure 4 shows the grades obtained in the assessments from week 2 to week 5. In week 2 to 4, 18 students took the assessment and in week 5 only 13.

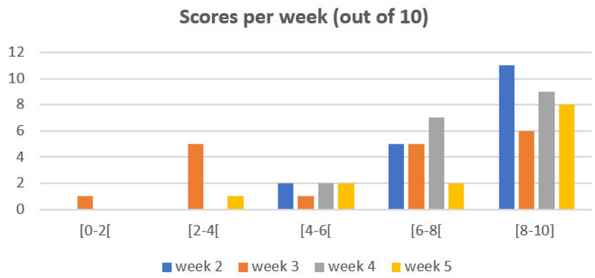

Figure 4: Student scores each week – maximum score is 10.

We may observe that most students have high scores, in the range of 8 to 10 out of 10 (61.1% in week 2, 33.3% in week 3, 50% in week 4 and 61.5% in week 5). This demonstrates the effective and meaningful learning that students achieved with the teachers' support. Some students fail to achieve a high score, especially in the third week, which corresponds to the content of "Approximation by differentials", where students have more difficulties. It should be noted that the low scores are not always from the same students and that they were obtained not for lack of knowledge of the content of the CU, but for lack of mathematical foundations that students should have gained during secondary education.

Conclusions for Education

The lack of a consistent mathematical base has been one of the main causes for students' failure in engineering courses, generating demotivation and even abandonment of studies by students. The need for higher education teachers to analyse and reflect on their teaching practices emerges, to capture, motivate and improve student learning. The use of a CL environment for the development of knowledge and for student motivation in the teaching-learning of mathematics for engineers was presented here and positive feedback was demonstrated in the engagement and group work on the students' side as well as the success obtained in learning the content. Therefore, it is recommended that mathematics courses integrate, whenever possible, collaborative technologies into student training to facilitate understanding and knowledge acquisition. Through the direct observation of teachers during the classes, the experiences of the authors and the evaluation of the students in these contents, it was possible to collect information that allowed demonstrating that CL with the use of the Padlet was efficient. The students' views and the teachers' views were also observed regarding their motivation and interest, allowing the enrichment of the necessary discussions for the future activities' development.

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Lectures with Worksheets

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Abstract

We report on a teaching intervention in which mathematics lectures were modified to include the use of worksheets with tasks that students were asked to work on with peers periodically during the lectures. Do the students find this format beneficial, and does it help them with the challenges during lectures? The data consists of students´ perceptions about the intervention, collected as free-text answers to questionnaires, and was collected in two calculus courses, one in the USA (n=246) and one in Europe (n=106). The data is coded and categorised, and some basic descriptive statistics are compiled. As a framework for the analysis, we use three didactical challenges for lectures: (1) activating prior knowledge for all students, (2) retaining student focus, and (3) a lack of opportunities for students to get back on track during mathematics lectures. We find that the respondents mostly find the worksheets beneficial but time-consuming. There are indications that the respondents perceive that using worksheets diminishes problems related to the didactical challenges above. However, the challenge regarding activating prior knowledge for all students is not satisfactorily met. Practical implications of these results are discussed, including challenges with the time use and the need to understand which properties of the worksheets influence the students' experiences the most.

Introduction

Lectures are a standard mode of teaching at universities worldwide despite evidence in the educational literature that traditional lectures are less effective than active learning (Freeman et al. 2014). Lectures are often seen as opposed to student-active forms of teaching (ibid.), such as peer instruction or teaching that emphasises feedback (Hattie 2008). That seems to imply that the student learning environment will improve if lectures are removed and replaced by student activities, preferably with feedback.

However, Hattie and Timperley (2007) write, "If students lack necessary information, further instruction is more powerful than feedback". Research also exposes problems with methods activating students through minimal guidance and constructivist methods. The argument is that these methods are incompatible with our understanding of how the human brain works and causes high cognitive load (Kirschner, Sweller, Clark 2006). There is also evidence for the effectiveness of worked examples in learning and teaching, i.e., the learner is shown through direct instruction how something is done, which traditionally is an essential part of lectures (Schwont et al. 2009).

Merrill (2002) identifies the first principles of instructional design and states that demonstration/instruction and learners' application/activity are two. From that perspective, direct instruction and student activity are two essential and complementary parts of effective teaching. Instead of abandoning lectures, one should improve the format and study how lectures can contribute effectively to the learning environment (Harrington, Zakrajsek 2017).

Three challenges for lectures

In search of ways to improve lectures, we focus on three didactic challenges relevant to lectures. First, lectures deliver the same content to a large group of students, even though students individually have different prior knowledge and understanding. It is well known that learning is supported by connecting new knowledge with prior knowledge (Christen & Murphy 1991, Merrill 2002). Hence, the lack of differentiation between students in the lecture can impede the learning for students whose relevant prior knowledge is not activated, maybe due to lack of it.

Second, the students' focus decreases significantly after about 10 minutes of lecturing, and the student's thoughts may wander off to unrelated things (Risko et al. 2012). It is common for universities to have lecture sessions that are 40-50 minutes. In the age of smartphones, the focus span may be even shorter as students tend to multitask during lessons (Joshi, Woodward, Woltering 2022). This loss of focus may hinder students' cognitive participation and activity in the lecture. If a student is unfortunate to lose focus at a critical point of the lecture, it can be difficult to learn from the remaining part.

Third, students have limited opportunities to overcome individual knowledge gaps and untangle misunderstandings during a lecture. Few students ask the lecturer, and if so, other students may see it as disturbing the lecture flow. Resolving cognitive conflicts through peer discussion is inhibited by not wanting to disturb other students listening to the lecturer. The lecture keeps moving forward, leaving the student behind. In cumulative subjects, such as mathematics, this can make the rest of the lecture incomprehensible. A lecture is like a train – if you are not aboard, you get left behind. Hence, during mathematics lectures, students may lack opportunities to get back on track (Harris & Pampaka 2016).

The teaching intervention

We construct a one-page worksheet for each lecture with a handful of mathematical tasks and room for students to write their solutions (see appendix for an example). The worksheet is printed on paper, handed out at the beginning of the lecture, and is not supposed to be handed in. At certain times during the lecture, about every 10 minutes, the students are asked to work on a task with peers sitting close.

The lecture started with the students doing the first task, the refresher, in which prior knowledge essential for the lecture is recaptured. About five minutes is used for this. Then the lecturer quickly illustrates the correct answer to the task before continuing with regular lecturing for about 10 minutes. The students are then given about three minutes to do the second task on the worksheet. The task is designed to fit into the content discussed at that specific moment in the lecture, asking the students to do something related to what the lecturer just has done, work through an example or a part of a faded example (Retnowati 2017). Meanwhile, the lecturer walks around, listening to the student to see what is problematic and what is understood. All students are not expected to complete the task in the approximately three minutes allotted for this activity. The lecturer then briefly lined out a solution to the task on the board before continuing with the next part of the lecture. This toggling between worksheet tasks and regular lecturing continued cyclically till the end of the lecture.

Research question

1. How are lectures with worksheets experienced by students, as beneficial or disadvantageous for their mathematics studies?

2. Do the respondents find that lectures with worksheets strengthen the lecture format regarding the following three challenges: (1) activating prior knowledge for all students, (2) retaining student focus, and (3) small opportunities for students to sort out misunderstandings that they get into?

Method

The respondents are calculus students. We have collected data during two courses, one of which was a continuation course in calculus with approximately five hundred students at a major university in California, USA. There was a mix of students majoring in many different fields. The number of respondents was 246, giving a response rate of approximately 50 %. The other course was an introductory calculus course with engineering students at a technical university in Sweden. These students studied a three-year engineering program. This was their first course. Of the 275 students enrolled in the course, 106 answered the questionnaire, giving a response of 39 %.

We collected student responses on paper during a lecture in the middle of the course, with the benefit that the students are more likely to complete the questionnaire. However, it also means that the students that are not present are not given the opportunity, leading to a sample that may be positively biased towards the teaching. We chose not to pose questions pertaining directly to the intervention. Instead, the anonymous questionnaire contained three broad, open-ended free-text questions: What do you like about the course? What do you dislike about the course? How can we help you learn better? Broad, open-ended questions are helpful in eliciting people's first-order, intrinsic concerns (Ferrario & Stantcheva 2022). A weakness of this approach is that not all respondents may choose to comment on the questions in the focus of this study, the worksheets. However, we used open-ended questions to minimise our influence on the students' opinions, letting them focus on what they found most essential to bring up.

The data is analysed using thematic analysis (Clarke & Braun 2013). A typical answer given was either one short sentence or a list of points, each point consisting of a few words. We started by dividing each response into units of analysis pertaining to one identifiable aspect of the course. This typically resulted in 2-5 units per respondent. When the same thing came up several times from a respondent, that was considered as one unit. The units were gathered into labelled code groups. These code groups were then collected into themes or categories of related codes. The categories were then collated and checked for alignment with code groups and units. In this process, we chose to disregard categories with fewer than five units, categories that referred to aspects relevant to one of the courses only, and categories regarding things outside of the courses. We used responses to all three questions in this process, using the question to identify whether the respondent was positive or negative towards the phenomenon mentioned. The analysis resulted in the following eight categories: Worksheets, Pace, Theory, Textbook, Use of IT, Examination, Enthusiasm, and Pedagogical Ability. For examples of units and their categorisation, see the Results chapter below.

Result and discussion

We will only use two categories to answer the research questions: Worksheets and Pace. The remaining six categories only contain units that do not pertain to the use of worksheets and will therefore be excluded from this report.

Quantitatively, the category Worksheet was the biggest of all eight, with more than one-third of the respondents commenting on them. A great majority of them consider worksheets beneficial (RQ1); see Figure 1. Examples of units are as follows (in parenthesis: if it was liked or disliked and the respondent ID).

> *I like the worksheets given out in lectures. It helps me understand basic concepts and learn more in-depth when the lecture goes on* (like, S1234a). *The process: 1. Review related topics 2. Teach 3. Try problems ourselves. 4. Go over the problem. Being able to try the problem on my own helps me learn* (like, S1209b). *Students get to do practice problems and apply concepts immediately* (like, S1206b). *The tasks we get during lectures. Only through them, one understands* (like, S2089).

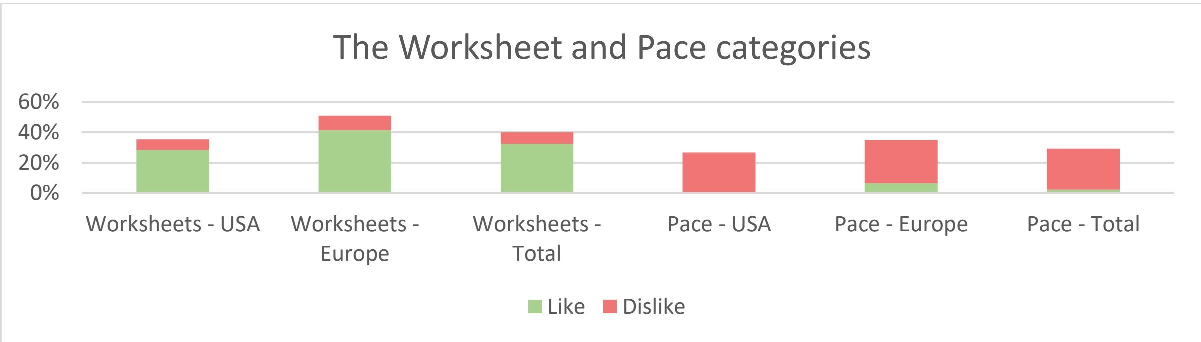

Figure 1. The percentage of students who commented on worksheets or pace, positively and negatively, at each of the two universities.

As can be seen in Figure 1, there are significant similarities between the responses obtained in the USA and Europe. Moving forward to research question 2, we consider the three challenges separately, not separating the two cohorts.

It is of value for learning that prior knowledge is activated (Christen & Murphy 1991, Merrill 2002). The respondents address prior knowledge, pointing to the first task on each worksheet, the refresher.

> *I like that we have warmups to see what we know and don't know. It shows us what we have to study* (like, S1122a). *Telling us what we should know for the lecture in part 1 of the worksheets* (like, S1216b).

The purpose of the refresher, to highlight knowledge that the students need, and to activate necessary prior knowledge to understand the lecture, seems to be fulfilled for some students. The idea is to bridge any gaps in prior knowledge and equalise the difference in prior knowledge between the students. However, some students find the remaining set of questions hard.

I feel like I don't know enough before attempting the questions (dislike, S1217c). *Worksheets, but have almost never been able to answer the questions. Think they are too difficult* (like, S2063).

Hence, we conclude that the refresher is helpful for some students to activate their prior knowledge and may help other students to identify what they need to study. However, for the students lacking knowledge, the refresher is not enough to help them cope with the lecture's content, and they are still left behind.

During lectures, declining student focus is a problem (Risko et al. 2012). The respondents addressed focus.

- *The exercises during the lecture make it easier to stay focused* (like, S2033). *Worksheets, if they are quick, make me not lose focus* (S2085). *Good with worksheets variation in the teaching* (like, S2079).

Hence, alternating between lecturing and worksheets seems to help students with their focus. While not mentioned by the respondents, a lack of self-regulation can be a problem (Joshi et al., 2022). It is worth noticing that there is more significant social pressure to stop using the smartphone when discussing with peers than when listening to the lecturer. Hence, we believe that the habit of multi-tasking during lectures can be partly remediated by peer collaboration on worksheet problems.

In mathematics lectures, students have limited opportunities to get back on track when lost (Harris & Pampaka 2016). The respondents highlighted the aspect of staying on track.

The worksheet provided every lecture. They really help in following the path of the lecture (like, S1117b). *That you have the possibility to think by yourself already during the lecture* (like, S2072). *Going over an example and then working on a very similar problem on our own* (like, S1243c).

It appears that the students appreciate having the opportunity to apply something that has been introduced, do an example, or investigate one of the concepts.

The respondents also point to the benefits of talking to a peer.

Working on problems with peers (like, S1237b). *It is good to be able to discuss during lectures* (like, S2076). *Interaction with peers* (like, S2095).

The worksheets allow the students to ask each other questions, fill in gaps, and sort out misunderstandings with a peer. There is also evidence that communication with fellow students has a strong positive impact on learning. Studies show that peer communication helps students to understand concepts even without any influence from authorities about what is right or wrong (Smith et al. 2009).

As time is a limiting resource when lecturing, the benefits of worksheets must be weighed against the time used. Some students think other things should be prioritised, for example, doing more worked examples on the board.

The system with worksheets. Better to do more examples on the board (dislike, S2011).

The time used for the worksheet means less time for lecturing. In the courses reported here, this was dealt with by increasing the pace of the lectures, limiting the notes on the board, and leaving more material for the students' self-study.

I hate how sometimes lectures can go too quick and some problems are not clearly explained (dislike, S1105). *I hate how we sometimes run out of time* (S1116). *Sometimes lecture is too fast and I could not fully catch up* (dislike, S1210).

Most comments about pace are negative, but not all.

The pace is good (like, S1213b). *I like the flow of the lectures* (like, S1249b). *I like the pace of the class and the worksheet problems complemented by the teaching* (like, S1219c).

There is little time given for each worksheet problem. This is also affecting the students.

I wish we could have more time to work on the problems in lecture (dislike, S1132b). *Not enough time to go over all problems on worksheet* (S1214). *How we don't go over all the questions in the first part of the worksheet* (dislike, S1221a).

Conclusions

Some respondents expressed that the worksheets affected them negatively, preferred traditional lectures, or pointed to weaknesses in the content of the worksheets.

That the worksheets and homework don't prepare me for midterm, since the problems are very different (dislike, S1124). *How the worksheets are generally a lot simpler than the homework problems* (dislike, S1207). *Wording of worksheets sometimes confusing* (dislike, S1132a). *Not covering everything in worksheets* (dislike, S1113).

However, the respondents generally are positive towards the worksheets and indicate that they help them focus and stay on track. The data shows that the respondents appreciated the refresher in the worksheet, aimed at activating their prior knowledge. It is, however, less clear that the worksheets helped students with weak prior knowledge. It helped them identify gaps in prior knowledge but was not enough to help them during the lecture.

An effect of using worksheets was the increased pace of lecturing in this intervention. Did this increase in pace outweigh the benefits of the worksheets? This is still an open question, but the strong support for the worksheets from the students from both universities suggests that worksheets are worth their cost. Further research is needed. The overall positive response about the worksheets call for further experiments, including calibrations of the implementation regarding the use of time, the difficulty of worksheet tasks, and the design and use of worksheet refresher.

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Engagement and Solidarity while Learning

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Abstract

The process of learning Mathematics in Engineering courses is the subject of varied and in-depth studies and investigations. The years 2020 and 2021 were difficult years for students who were already attending higher education but also for those who were in the final years of secondary education preparing themselves to enter higher education. As a result of the pandemic, teaching processes were adapted, and the methods of assessments used were ones that were possible in the context that we all lived and experienced. Thus, students arriving at higher education, and even those already there, needed innovative and stimulating teaching and learning practices that quickly motivated and involved them in the teaching/learning processes. At the same time, Information and Communication Technologies (ICT) tools and digital platforms have seen indiscriminate use in the last two years. Sometimes teachers and students have not questioned whether these platforms were used in the best conceivable way and whether they were being taken full advantage of. Simultaneously, face-to-face group work and involvement with the needs of colleagues lost ground, being harder to achieve and, consequently, lost effectiveness. The preference, assumed or implicit, for individual work and the noticeable reduction in solidarity with colleagues with greater difficulties, except in niches of friendship that come from previous school groups, was an issue/question that teachers raised at the beginning of this study. In this paper we will present a semester-long work scheme for students of Statistical Methods of the Graduation Degree in Informatics Engineering. This curricular unit enrols 533 students, eighty-five on an after-work basis, many of which are student workers. The objective of this work scheme was to create a collaborative learning platform where students could interact with each other within the scope of the curricular unit, even if virtually. Furthermore, it was also the objective of the scheme for students to deepen their knowledge of the topics taught in class, investigating them in more detail (including reading the bibliographical references provided and reviewing exercises conducted by colleagues), looking for more useful and creative resources that would involve them in a logical and organized way. Students who signed up to participate in this study path simultaneously shared their resolutions of proposed exercises and corrected or constructively commented on their colleagues' resolutions. At the end of each week, the curricular unit professors corrected the resolutions proposed by the students. All students who participated had access to all the work developed by their colleagues. The Padlet platform (https://padlet.com/) was used as a platform for engagement and collaboration. The evaluation of students' involvement in their learning process and their collaboration and solidarity with colleagues in the same course, in addition to the results (including pass rates), will be discussed and presented.

Keywords: Collaborative Learning, Active Methodologies, Padlet, Mathematics, Solidarity

1 Introduction

1.1 Activity Contextualization

Learning mathematics in engineering courses has been the subject of extensive research and study by educators. However, the outbreak of the pandemic led to significant disruptions in educational systems worldwide, forcing schools and universities to quickly transition to remote learning to comply with social distancing guidelines and mitigate the spread of the virus. This sudden shift to online education presented substantial challenges for both students and educators, who had to adapt to innovative technologies and teaching methods while also coping with the social and emotional impacts of the pandemic. For students preparing to enter higher education, the pandemic introduced additional obstacles. College and university campuses were either closed or operating at limited capacity, reducing opportunities for campus visits, extracurricular activities, and group study sessions that facilitate engagement and socialization among students. Moreover, the economic repercussions of the pandemic, including job losses and financial strain, added further difficulties for families, potentially impacting their ability to afford higher education. Overall, the pandemic created a difficult and uncertain learning environment, particularly for students aspiring to pursue higher education, necessitating resilience and adaptability to significant changes in their educational plans. As a result, there emerged a need for innovative and motivating teaching and learning practices that can effectively engage students and foster their active participation in the learning process. The indiscriminate use of information and communication technologies (ICT) and digital platforms became prevalent during the pandemic. However, concerns arose among teachers and students regarding whether these tools were optimally utilized by all involved and if their full potential was truly being taken advantage of. The shift away from face-to-face group work and reduced interaction with peers resulted in a visible decline in solidarity and collaboration among colleagues, raising questions about the preference for individual work. Considering these educational challenges, the authors of this study sought a solution that could engage students and facilitate peer support. Considering the advancements made in digital platforms during the pandemic, the authors aimed to find a platform that would enable easy use, interaction, and visualization among students. Based on the recommendations of various authors (e.g., Fisher, C. D., (2017), Mehta K. J., Miletich I., & Detyna M., (2021), and QiaoZhi, MuSu, (2015)), who highlighted Padlet as an excellent online collaboration tool for collaborative knowledge building in and outside the classroom, Padlet was chosen to facilitate a collaborative activity and engage students.

2 Methodology

2.1 Padlet Activity Proposal

A semester-long activity was introduced to students enrolled in the Statistical Methods course within the Informatics Engineering program. This course had a total of 533 students, with eighty-five of them attending on an after-work basis. The primary objective of this activity was to establish a collaborative learning platform, enabling students to interact and engage with one another in the context of the course. The activity aimed to deepen students' understanding of the topics covered in class, including the provided references, as well as encourage them to review exercises completed by their peers. To facilitate this process, a Padlet platform was utilized, where registered students were able to publish their solutions to questions from previous exams. The teachers required students to provide their identification (name and student number) while participating in the activity. Teachers also encouraged students to publish their resolutions and provide constructive comments on the solutions shared by their colleagues. Student scores were calculated based on the proportion of their participation across different content areas. The student with the highest number of correct participations in the greatest variety of content areas received the highest rating. For instance, if there were X distinct content areas, a student who correctly solved 1 exercise from content A and 1 exercise from content B would receive a bonus value of (2/X) * 2 in the final classification. Similarly, a student who correctly solved 2 exercises from content A and 0 exercises from content B would receive a bonus value of (1/X) * 2 in the final classification. Participating in exercises of the same content, although not contributing to the final bonus values, still benefited the student by reinforcing their personal study and contributing to the collaborative learning within the group.

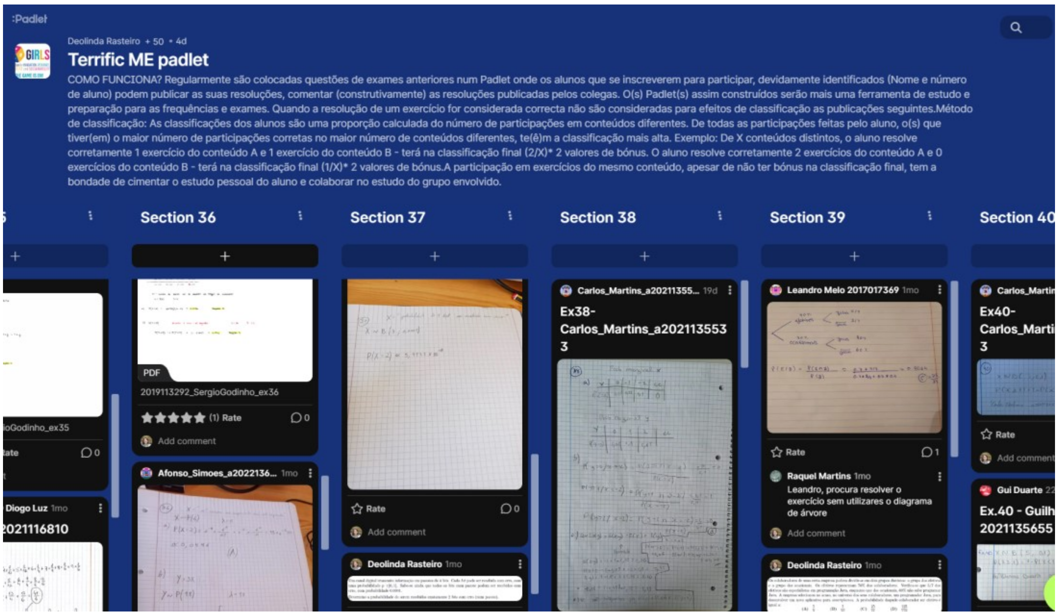

Figure 1. Padlet activity.

The materials published by the students were reviewed and corrected by the professors, who provided feedback on whether the resolutions needed revision or if they were correct. All participating students had access to the entire body of work developed throughout the activity. At the conclusion of the activity, students were eligible to earn a maximum of 1 (out of 10) additional point that would be added to their final grade.

The evaluation of students' involvement, collaboration, and solidarity, along with the results obtained, will be discussed, and presented in the results section.

2.2 Working Sample

A total of 533 students were registered for the Statistical Methods course, with eighty-five of them attending after work hours. All students received an invitation via email and had the opportunity to sign up for the Padlet activity, which was made available on the Moodle platform for a two-week period at the beginning of the semester. Out of the 533 students, 137 expressed their intention to participate, including 28 students from the after-work course. Thus, approximately 24.3% of the regular students and 33% of the after-work students engaged in this activity (approximately 25.7% of the total students registered for the course). However, despite the expressed interest from 137 students, only 50 of them actively posted and interacted with their fellow colleagues.

3 Results

3.1 Padlet posts, satisfaction questionnaire and written assessment

The initial result of the activity revealed a less-than-encouraging engagement rate, with only 25.7% of the students responding to the activity invitation, despite the opportunity to earn 2 bonus points. Over the observation period (from 18/03/2023 to 28/04/2023), a total of 448 posts were made, and their daily distribution can be observed in Figure 2.

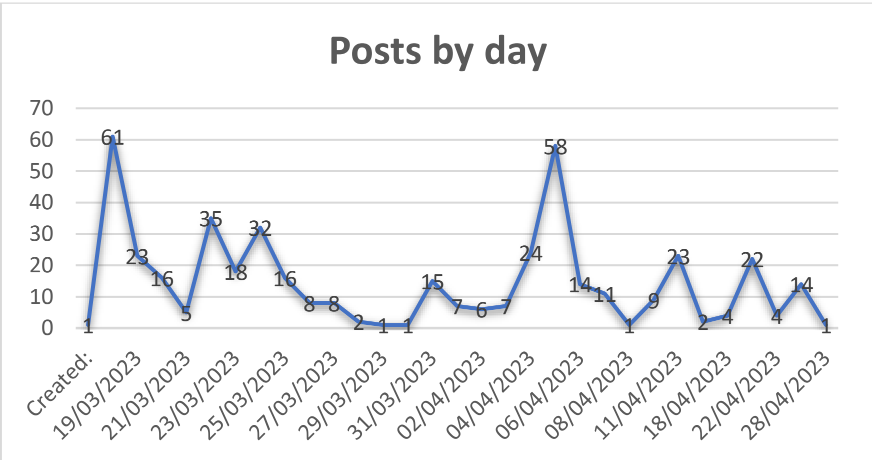

Fig. 2. Distribution of posts by day

The distribution of posts by Exercise is depicted on Figure 3, below. As we may observe, in the beginning there was a higher response rate that may be attributed to two distinct reasons: one is natural curiosity, the other is because initial exercises were simpler that the following ones.

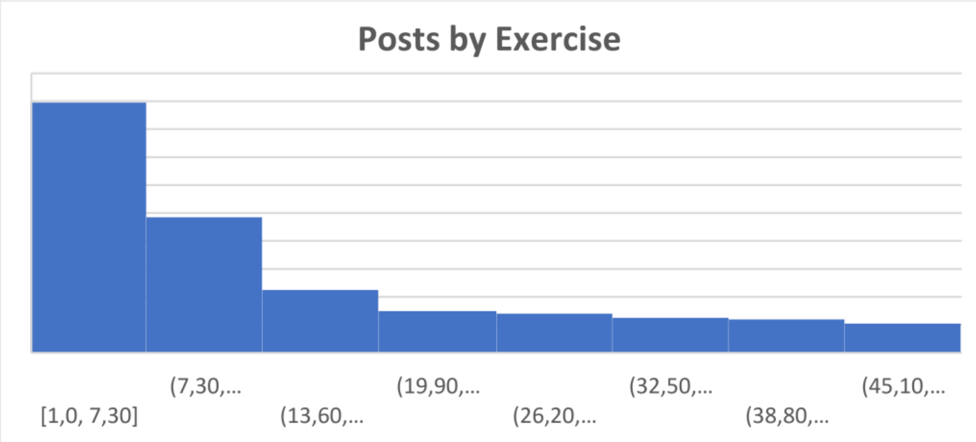

Fig. 3. Posts by Exercise

Since the response rate stabilized during the period of observation and the difficulty of the exercises was regularly increasing the authors tend to explain the initial high number of responses as curiosity.

To gather feedback from the participating students, a satisfaction questionnaire was delivered by the teachers using Google Forms, (Figure 4). The questionnaire aimed to assess students' satisfaction and perceived utility of the Padlet activity.

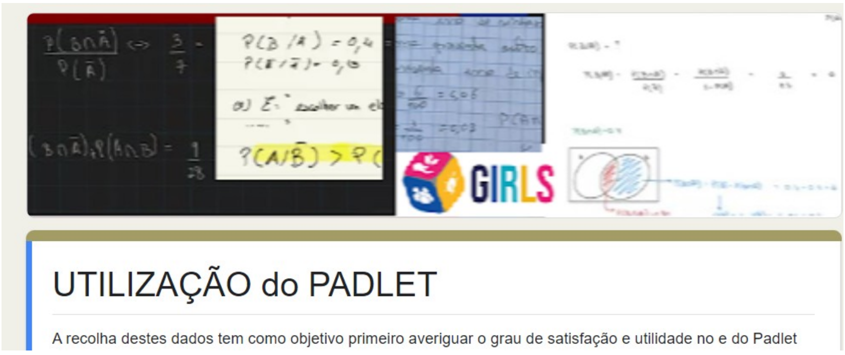

Fig. 4. Satisfaction Questionnaire

From the data collected and analysed we conclude that 61.9% of the responding students had never used Padlet. Since the activity engagement in the beginning was only approximately 25.7%, the authors questioned if the students were only involved because of the 2-points bonus proposed. Besides the 2-points bonus proposed, 90.5% of the students agreed that this activity helped them to study. When asked «Do you consider that, in addition to the bonus, which you may have in your mark, this activity helped you to obtain a better classification in the written assessment? », the obtained answers indicate, as shown in Figure 5., that 71.4% of the students consider the activity has helped them to prepare to the written assessment.

To corroborate these received answers, we compared the number of students passing the written assessment to the number of those students that participated in the activity.

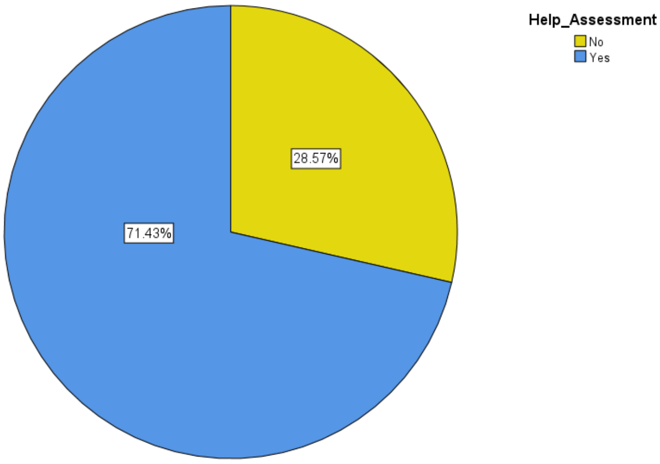

Fig. 5. Positive activity influence in the assessment

In fact, 44.7% of after work students were assessed and, of these, 60.5% passed, and from those 60.9% participated in the activity. Regarding regular students, 43.9% were assessed, and, of these, 70.1% passed, and from those 37.8% participated in the activity.

Table 1. Statistics from the written assessment

Student Type	Total Students	Assessed	Passed	In Activity	Pass Rate
After Work Students	85	38	23	14	60.53%
Regular Students	533	234	164	62	70.09%

Regarding the feedback given by professors, 90.5% of the students considered that it was enough, and 85.7% of them wished that colleagues gave more feedback. As for the materials posted by their fellow colleagues, 95.2% considered that they were of help to complement their own study. When asked «Would you like there to be more interaction with and from your colleagues? (Comments and questions to your resolutions)? », 66.7% of the students wished for more interaction from their colleagues. Finally, to be able to conclude whether students' solidarity was only towards the colleagues participating in the activity or in general, it was asked to students «Do you agree with the sharing of the published resolutions with all your colleagues enrolled in the course? ». It was found that 95.2% agreed with this.

4 Conclusions

From this activity two main conclusions may be drawn. The first conclusion is that the percentage of students engaged in this activity was below the professors' expectation and the percentage of students that undertook the written assessment was also surprising (≅51%). Students prefer to clarify their questions about content and exercise resolutions at office hours or by e-mail where the only participants are the professor and themselves. Therefore, individual study is preferred by the majority of students. The other conclusion is that, even though we have experienced times where ICT was widely used, Padlet, which is a widely known collaborative platform was an unknown tool for 60% of the students. Almost all students agreed that this activity helped them to study, and the feedback provided by professors was enough. A small percentage, 14.3%, of the students wished that colleagues gave more feedback. It is important to recognize that student preferences and attitudes towards online activities may vary. As a teacher, being responsive to student feedback, monitoring participation levels, and adapting activities based on student engagement can help create a positive and inclusive online learning environment. The willingness of students to participate in online activities can vary depending on several factors such as their personal preferences, the nature of the activity, the level of engagement fostered by teachers, and the overall course structure. While some students may readily embrace online activities and actively participate, others may be less inclined to engage. To share their collaborative work with all the other students is, for the students engaged in the Padlet activity, not a problem. Therefore, we may conclude that although only a small percentage of students wish to work in collaboration, those who do want solidarity with all the others.

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How engineering students read the definition of double integral: An eye-tracking study

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Abstract

This paper investigates engineering students' engagement with a mathematical text in relation to double integral. The paper reports a study using eye-tracking technology following three engineering students who were selected using convenience and purposive sampling from one university in Norway. The students were asked to learn the concept of double integral while their eye movements were tracked using screen-based eye-tracking. The double integral text was designed by the researcher based on different calculus textbooks. The text includes the definition of double integral, which is the focus of this report. The results reported here indicate that students tried to make sense of the definition of double integral by engaging with different representations, mostly figures and text. Based on the data from eye-tracking, students appeared to focus more on the relation between volume and double integral. This study suggests that it would be beneficial if lecturers use different forms of representations during their teaching. Moreover, considering the students' struggle, lecturers might focus more on the relation between the representations of double integral and volume when teaching double integral.

Introduction

The concept of double integrals is an important part of the multivariable calculus. The learning and teaching of double integration usually start from the second year of university study for engineering students and is an important part of their curriculum (Tisdell, 2017). Double integrals have a wide range of applications in mathematics, such as finding plane areas, mass, and volumes (Milenković et al., 2022). Several studies reported that most engineering students have difficulties in learning and understanding the double integral (e.g., Hamidreza et al., 2010). Tall (1993) reported that students have several learning difficulties when they try to connect algebraic and graphical representations of multiple integrals and use the appropriate ones.

The importance of using different forms of representations, such as mathematical formulas, figures, and texts in mathematical learning and teaching, cannot be underestimated when it comes to engaging with mathematical concepts and fostering a deep comprehension of mathematical concepts (Ott et al., 2018). However, the effectiveness of these multiple representations is not consistent and can vary based on factors such as the learners' prior knowledge (e.g., Kalyuga et al., 2000), and the characteristics of the combined representations (e.g., Kalyuga et al., 2004).

The eye-tracking as a method can provide useful insights into the way students read mathematical representations (Andrá et al., 2015; Ott et al., 2018). Students' eye movements when they read a text can provide insights on how they engage with the material and which part of the material was the focus of their learning (Beitlich et al., 2015). Considering the importance of the concept of double integration and its representations for engineering students, this paper aims to explore how engineering students read and engage with the definition of the double integral.

Literature review

Double integral

Double integration is an important concept in calculus that has several applications in different fields such as engineering (Tisdell, 2017). Double integration is used to compute the areas, volumes and masses. It involves the integration of a function of two variables over a two-dimensional region in space (Stewart, 2015). The introduction of the double integral concept in calculus textbooks starts with the way we can study the volume of a solid enclosed by two variables function in a closed region. Many students have difficulties in understanding the double integral (Tisdell, 2017). For example, Mahir (2009) highlights that the difficult part of the multiple integral for students is to visualize figures in 3D and the relation between graphical and algebraic representations of space.

Eye-tracking

Eye-tracking technology enables the collection of useful data in empirical studies of human perception, cognition, and behavior. Eye-tracking data is used to determine various aspects of people' sensory perception in the visual modality where they are looking (Was et al., 2016). Eye-tracking technologies are increasingly used in educational research, especially in mathematics education (e.g., Andrá et al., 2015). Eye-tracking has the potential to provide deep insights into individual's mathematical thinking and learning and helps researchers to understand what students focus on while they are working on mathematical problems (Strohmaier et al., 2020). Moreover, eye-tracking can be considered as a useful tool to explore students' learning from multiple representations (Ott et al., 2018). For example, Beitlich et al. (2015) investigated how students make use of different types of representations in mathematics when they were working with examples using eye-tracking. They reported that students tried to make a connection between different types of representations, and they focused more on pictures than on symbolic or textual representations.

Method of Investigation

The data of this paper comes from a larger project and only some parts of the data will be presented in this paper. The present study takes an exploratory approach. Three engineering students from one university in Norway were chosen using convenience and purposive sampling. A mathematical content in relation to double integration[1] was designed based on different calculus textbooks (e.g., Anton et al., 2012; Stewart, 2015). In the content, the definition of double integral was explained in detail. To have more accurate data, it was ensured that the participants had not engaged with the content prior to this study. The designed content consists of different form of mathematics representations such as formula, text and figure. In this paper, text representation is considered when the logical explanation is expressed with a written verbal statement. The formula represents the information in symbolic forms.

The mathematics content was presented on a computer screen to track students' eye movements. Students' eye movements were recorded with Tobii Pro Nano eye-tracker, set to sample at 60Hz. The Tobii Pro Nano is a remote eye-tracker with one binocular

[1] This topic was chosen based on students' previous knowledge and their lecturers' recommendations.

camera; it typically achieves eye-position tracking accuracy of 0.3°. Stimuli were displayed on a screen that participants viewed (without head restriction) from a distance of approximately 65cm. Each participant took part in this study individually in a quiet room. After reading the content, the participants were given a post-test based on the content of double integration. The test was also designed by the researcher. The validity of the test and mathematical content was examined by two senior lecturers in mathematics and two senior lecturers in mathematics education and they were piloted with two students as well.

Data analysis

To analyse data, first, we distinguished between texts, formulas, and figures by drawing area of interests[2] (AOIs) for each representation. Drawing AOIs helps us to analyse students' eye movement for each specific part in detail and it gives us the opportunity to see how students move between different parts of content. Two different parameters from students' eye movement were considered and calculated for this study: dwell time and revisits. Dwell time refers to the amount of time participants spent to look within an AOI. Dwell time helps to understand the level of interest in a certain AOI. Revisits refers to the number of times that one participant look within an AOI, leaves it, and returns back to it. Moreover, the pattern of students' eye movement was examined to understand how students read the text in detail and which parts they focused on more.

Findings

The mathematical content which was given to students is communicated through several representations such as texts, formulas, and figures. To analyze students' eye movements, AOIs were defined for each representation type and their eye movements were analyzed based on two parameters namely revisits and dwell time. In general, the results showed that students tried to make sense of the definition of double integral by engaging with all the three representations (text, formula and figure). However, they focused mostly on text, followed by figure, and then formula. Moreover, they focused mostly on the specific part of the definition which was the explanation of double integral using volume. They tried to make a connection between this part and other parts of the definition.
In detail and regarding to the dwell time, students spent time to engage with all forms of representations in the content, but in general they spent the most amount of time engaging with texts, and figures (Figure 1).

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[2] Area of interests (AOIs) are predefined areas of the stimulus that can be used to analyze eye movements on specific elements of the stimulus.

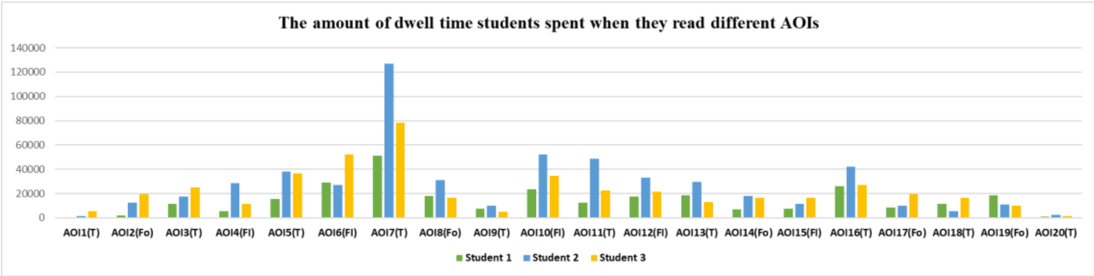

Figure 2. The amount of dwell time for different representations in the definition of double integral

As you can see from the figure 1[3], students engaged with all forms of representations, however, the amount of time spent was different for each student. Although they engaged with the content for different amount of time, they spent more time reading specific texts and figures. For example, all three students spend most of the time engaging with AOI 7 which was a text and related to the explanation of double integral using volume (Figure 2).

AOI 7 (Text)

To find the volume, the first step is to divide the rectangle R into sub rectangles. We accomplish this by dividing the interval $[a, b]$ into m subintervals $[x_{i-1}, x_i]$ of equal length $\Delta x = \frac{(b-a)}{m}$ and dividing $[c, d]$ into n subintervals $[y_{j-1}, y_j]$ of equal width $\Delta y = \frac{(d-c)}{n}$. By drawing lines parallel to the coordinate axes through the endpoints of these subintervals, as in Figure 4, we form the sub rectangles

Figure 3. AOI 7 which is one of the representations in the double integral definition.

Regarding the revisits, students had revisits between the texts, formulas and figures to make connections between these representations, but they had more revisits when they engaged between texts and figures. Moreover, students had more revisits when they tried to make a connection between AOI 7 and other representations. For example, one of the students tried to make a connection between AOI 6 (figure) and AOI 7. When he reached some parts of this text, he went back to the figure and tried to make a connection between this figure and text (Figure 3).

[3] In figure 1, FO refers to formula, FI refers to figure and T refers to text.

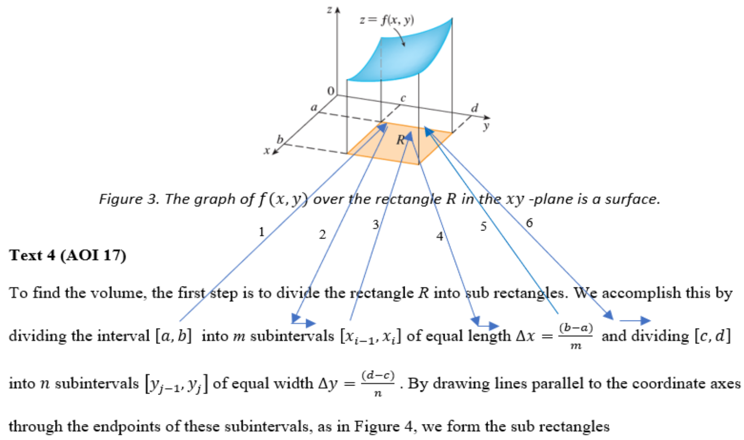

Figure 4. one student's eye movement pattern when he read the definition of double integral.

As you can see from figure 3, the student came back to the figure several times and tried to make sense of different parts of the text using the figure.

Discussion and Conclusion

The present study explored how engineering students read and engage with the definition of double integral using eye-tracking technology. The findings showed that students tried to make sense of the definition of double integration by engaging with different representations (text, formula and figure). Overall, they engaged with all the representations and tried to make a connection between them. This result is consistent with previous studies (e.g., Beitlich et al., 2015) which highlights that students tried to make a connection between information using different types of representations. Moreover, students focused mostly on text, followed by figure, and then formula. This result is consistent with Otte et al. (2018) which reported that text was the one representation that was attended to most and can be considered as a reference representation in useful combinations. Although there are some studies which reported contrasting findings (e.g., Beitlich et al., 2015). Furthermore, the students focused mostly on the specific part of the definition which was the explanation of the double integral using volume and tried to make a connection between this part and other parts of content. The reason for this result could be that understanding the double integral in terms of volumes is difficult and students tries to understand it using visualization.

To conclude, this paper suggests that considering engineering students tried to make sense of content using text and figure representations, it would be beneficial that lecturers use different forms of representations when they teach this topic to students. Moreover, as all three students focused on the relationship between a double integral and volume in the content, it would be helpful that lecturers can focus on this part during their lectures.

There are some limitations of this study that need to be addressed. Considering only three participants were the focus of this paper, it would be beneficial if the number of participants could be increased. Moreover, considering the results are only based on eye-tracking data, it would be helpful if other methods such as interview would be applied after tracking students' eye movement to be able to provide insights into how students read and engage with the definition of the double integral.

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Mathematical education of future architects – playing and learning with and in Iterated Function Systems

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Abstract

The principal methods of architectural design and construction have been inseparable from mathematics since time immemorial. Even nowadays, mathematical ability is an integral part of the professional competence of any architect, and therefore, mathematical education is naturally a very significant segment of architectural studies. At the University of Belgrade, the Faculty of Architecture, the leading higher education institution for architecture and urban studies in the Western Balkans region, undergraduate students primarily meet with mathematics on the core course Mathematics in architecture. As the name suggests, this course covers the basics of analytic and fractal geometry, focusing on application possibilities. Namely, after understanding and adopting the theoretical foundations using standard teaching methodology, it often happens that students hardly recognize the connection between the acquired mathematical knowledge and their future professional activities. Therefore, after each thematic unit during the course, some experimental work or functional research is carried out. This paper briefly presents an active learning method implemented on Iterated Function Systems (IFS), as one of the units. The accompanying experimental student task involves creating various analog models of IFS which could be interpreted architecturally.

Introduction

The use of mathematics in architecture has a long and rich history, dating back to ancient times when architects and builders utilized mathematical principles to create various structures that were both beautiful and functional. Even today, mathematical proficiency is essential for the professional competence of any architect, and therefore, mathematical education is a significant segment of architectural studies at the University of Belgrade's Faculty of Architecture, the leading higher education institution for architecture and urban studies in the Western Balkans region.

The first section of this paper provides a brief overview of the architectural studies at the Faculty, with a focus on classical mathematical courses as well as those that primarily rely on mathematical reasoning. While students acquire a solid theoretical foundation through standard teaching methodologies, they may not always grasp the connection between the mathematical knowledge they receive and their future professional activities. Therefore, it is not sufficient to only learn the theoretical foundations of mathematics. To be successful in their future careers, architecture students must also be able to apply their knowledge in practical ways. As such, our mathematical courses include experimental work or functional research after each thematic unit.

The subsequent sections of the paper concentrate on one such active learning method that has been implemented on Iterated Function Systems (IFS). This method has demonstrated significant effectiveness in helping students comprehend the relationship between their mathematical knowledge and their professional careers. The experimental work conducted through this method has resulted in the creation of numerous analog models of fractals that can be interpreted in an architectural context, some of which are presented in the paper.

Lastly, the paper concludes with insights on the described learning method and the approach to teaching mathematics for future architects, in general.

Mathematical education of future Serbian architects

The University of Belgrade’s Faculty of Architecture is the premier institution for architectural and urban planning education in Serbia, with a history of over 170 years. The results achieved in the domains of education, professional and artistic creation and scientific research have ensured the Faculty’s high-ranking position in the region. The Faculty provides a comprehensive education for future architects, offering a diverse range of studies that facilitate the sharing of knowledge and development of skills required for practising architecture within an interdisciplinary environment.

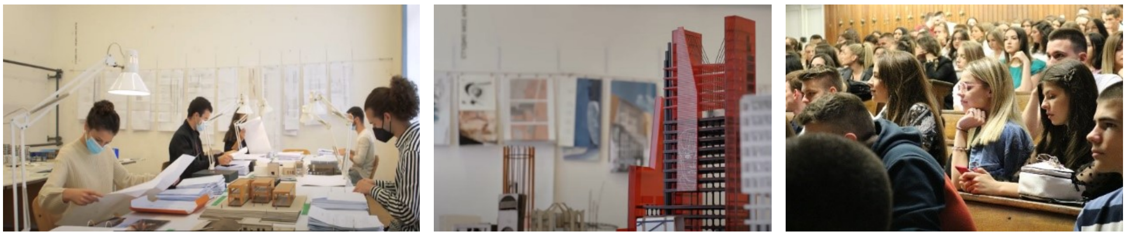

Figure 1. Faculty of Architecture University of Belgrade

The learning process is structured around the activities of the Studio Project, which comprises more than half of the active class capacity and serves as the basis of architectural education. This project-based curriculum emphasizes practical work on specific projects and is complemented by a range of theoretical and research courses. A significant part of these courses focuses on mathematical concepts and techniques, both classical and those based on the latest advancements in technology. By integrating mathematical courses into architectural studies, the Faculty is helping to ensure that future architects have the necessary skills to design buildings that are both aesthetically pleasing and functional, meeting the needs of society. The inclusion of mathematics in the curriculum is essential for producing well-rounded architects capable of meeting the challenges of the ever-evolving architectural profession. In addition, a quality mathematical education can increase the competitiveness of graduate architects in the job market, where companies and clients seek experts with a broad range of knowledge and skills, including mathematical proficiency, in today's rapidly evolving world.

We feature courses taught by the authors of this paper:

- Mathematics in architecture (a core course in the first year of undergraduate studies);
- Architectural geometry I/II (two core courses in the first year of undergraduate studies);
- Applied mathematics in the field of constructive systems (a core course in the first year of master studies in the programme Construction Engineering);

- Parametric modelling of architectural forms (an elective course in the first year of master studies available in all programmes);
- Integrated modelling of architectural form (an elective course in the first year of master studies available in all programmes);
- Building information modelling (BIM) I/II (two elective courses in the first year of master studies available in all programmes).

The Mathematics in architecture course introduces students to the foundational concepts of analytic and fractal geometry, providing them with the technical tools they need to understand the geometrical principles that underpin architectural design.

Architectural geometry courses focus on descriptive geometry and aim to develop students' logical thinking, spatial perception, and ability to imagine three-dimensional space. Students learn about the geometry of architectural forms and use this knowledge to define constructive elements for both exterior and interior design. They also learn various geometric-constructive methods for processing and representing 3D forms applicable in architecture, such as "orthogonal views" obtained for mutually parallel rays of perception and modern digital technology requirements.

The Applied mathematics in the field of constructive systems provides the necessary foundation for master's students to understand the complex mathematical principles of differential and integral calculus that are essential in solving extremal geometric problems, as well as determining the curvature of curves and surfaces and calculating the areas and volumes of irregular figures and solids.

Other courses listed above, related to digital modelling, programming, and other information technologies, enable students to create complex design more efficiently and accurately. These technologies also allow students to simulate and visualize their designs in a virtual environment, which can help to identify and solve problems before construction begins. In order to be able to use digital modelling and programming tools effectively, students need to have a solid understanding of mathematical principles. Hence, the integration of mathematics and digital technologies is crucial for the education of future architects, as it enables them to develop the skills and knowledge required to meet the demands of a rapidly changing profession. It is not uncommon to deviate from the planned curriculum on these courses in order to address specific, current issues or ongoing problematic situations that students are facing during their Studio project. In such cases, mathematics, supported by computer technology, can offer solutions, enabling students to apply a flexible approach and think creatively in problem-solving.

Playing and learning with and in IFS

After students acquire a solid foundation of theoretical mathematics using standard teaching methods, they often struggle to see the practical application of this knowledge in their future professional activities. Many students fail to recognize the potential benefits of mathematics and struggle to understand how to integrate it into their work. In order to bridge this gap between theory and practice, it is crucial to incorporate experimental work or functional research into the curriculum after each thematic unit. By doing so, students are better able to understand the practical implications of mathematical concepts and can more effectively apply their knowledge in real-world situations. To this end, the active learning method implemented on IFS during the Mathematics in architecture course, serves as a prime example of how to incorporate hands-on, practical experiences into the curriculum.

Fractals are complex geometric patterns that are self-similar at different scales. They are often used in architecture, design, and art because of their ability to create visually interesting structures. IFS are one of the techniques used to create fractals. An IFS is a set of mathematical functions that are iteratively applied to a set of points, creating a self-similar pattern. The process can be repeated many times, with different parameters and initial conditions, resulting in a wide variety of fractal shapes. Fractal geometry is useful in architecture for creating buildings with irregular shapes and patterns, as well as for optimizing structural efficiency. For more theoretical details about IFS, see [1] and [2].

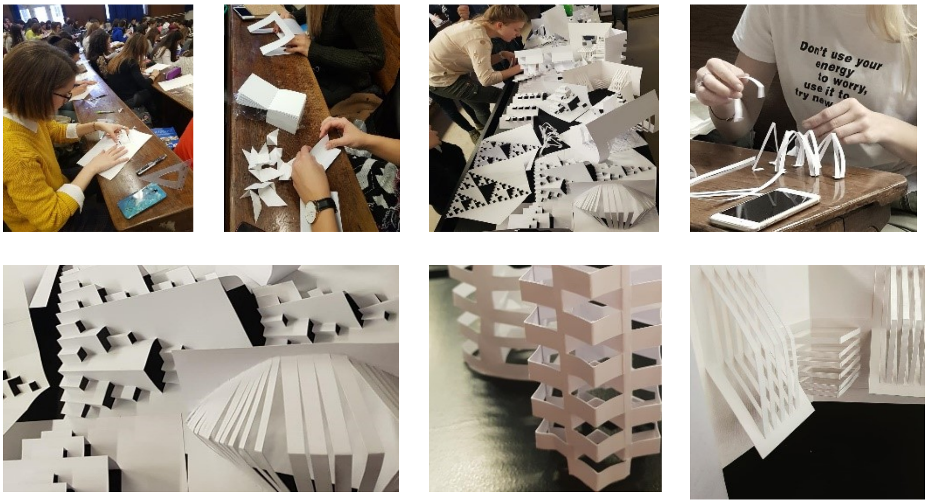

Figure 2. Initial exploring during class time

After learning the theory related to IFS, students are assigned to create an IFS fractal of their own design. This project aims to develop the students' skills in constructing mathematical models and exploring the creative possibilities of fractals in architecture and design. To be precise, we conduct a workshop in which students create several analog fractal models using paper or other inexpensive and easily available materials. The initial goal is for each student to explore the creation of a sequence of different models during class time. This is achieved through a process of rough selection followed by careful refinement, using different initiators and various affine transformations (translation, rotation, scaling) that are iteratively repeated the desired number of times (see Figure 2.).

At the end of the class, we group the works based on their quality in terms of creativity, originality, and architectural potential. We discuss with the students and give them the

opportunity to act as evaluators and express which models they would choose as the most high-quality and provide a justification for their selections. This collaborative approach not only fosters a sense of community among the students, but also allows them to reflect on their own design choices and learn from the successes and challenges of others. Through comparative analysis, they simultaneously consider ways to improve their own models. This phase of the process is especially enjoyable for the students.

Afterward, the students have one week to complete the task, i.e. to generate their final model. They must first decide on the initial form, select the materials to be used, choose the transformations to be applied, and determine the number of iterations. In this way, they are encouraged to experiment with various forms and materials, and to consider the potential for architectural or design applications of their fractal creations. At the end, once completed, students are required to photograph their IFS fractal and submit the photograph that highlights the architectural or design potential to an online forum created specifically for the course. Through this forum, each student has the opportunity to showcase their work to their colleagues, to positively react to the results they consider most successful and receive valuable feedback on their own creations. As evaluators, teachers also benefit from the forum by gaining a more comprehensive understanding of the outcomes considering that there are a large number of enrolled students (over 300). By observing the students' work, we can objectively evaluate their projects and assess their understanding of the underlying concepts of IFS fractals. Additionally, the forum enables us to identify any areas of the course that require further clarification or improvement, and to provide timely feedback to the students. Overall, the online forum serves as a valuable tool for both the students and instructors, facilitating a collaborative and dynamic learning environment. Figure 3 shows the most successful results of this workshop and serves as a prime example of a picture speaking a thousand words.

Conclusions

Expanding on the recommendations for the mathematical education of architects, our observations and findings highlight the crucial role of practical application in reinforcing theoretical concepts. It is important to create an environment where mathematical concepts are not just taught as abstract theories, but are also reinforced through concrete and visually tangible applications that have clear architectural potential. By providing opportunities for hands-on experimentation and active learning, students can not only deepen their understanding of mathematical principles but also appreciate how those principles can be practically applied in their future careers as architects. The ability to visualize and see the potential applications of mathematical concepts in architecture can also spark creativity and inspire innovative design solutions. Therefore, a curriculum that emphasizes both theoretical and practical aspects of mathematics can provide students with a well-rounded education that prepares them for the challenges and demands of the architectural profession. Regular refinement and updating of the curriculum to keep up with the changing technological landscape is also essential to ensure that future architects are equipped with the necessary skills and knowledge to succeed in their careers.

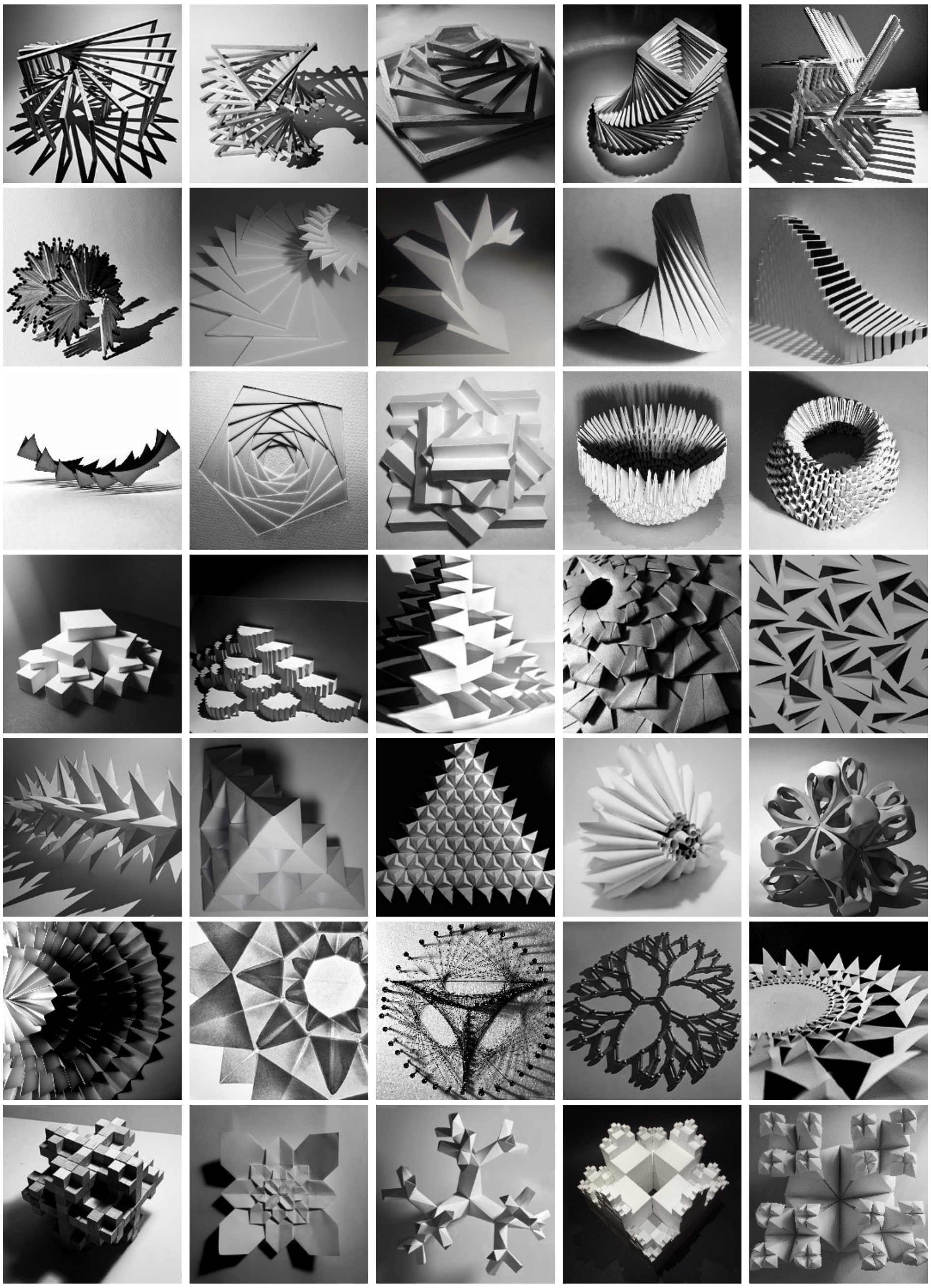

Figure 3. Workshop results – IFS models

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Sparking curiosity and opening doors: using active learning in Mathematics as a tool for promoting access to college.

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Abstract.

Under-performance in, and fear of, mathematics can be perceived as a barrier to progression to higher education by secondary school students. This is particularly true for students who come from areas of social disadvantage. In this paper we describe a programme which seeks to help address this perceived barrier by providing active learning opportunities in mathematics at Technological University Dublin, for second level students who come from schools in socially disadvantaged areas. There are two aims of the programme. The first aim is to spark the students' curiosity with well-chosen active learning workshops, which show them that engaging with mathematics is needed in real world situations. The second aim is to contribute to the broader access to college agenda, by engaging the students in a positive way with the university and nudge open the door of possibility that attending higher education may provide for them. In the paper we will describe the key aspects of the programme and feedback on the programme from participants. We will also identify the features central to it being repeatable in other educational settings.

Keywords: active learning; motivation; access to college.

1. Introduction.

It is widely recognised nationally and internationally that students from socio-economically disadvantaged backgrounds have lower performance across various educational outcomes (Weir & al (2017), Crenna-Jennings (2018)). Since 2005 the Delivering Equality of Opportunity in Schools (DEIS) programme has been the Irish government's main policy initiative to tackling educational disadvantage (DES (2005), DES (2017a), DES (2017b)). The DEIS programme focuses on targeting additional resources to those schools included in the programme to ensure every child has an equal opportunity to achieve their potential. Schools that are part of the programme are known as DEIS schools. Mathematics is a key focus of the DEIS programme as results from PISA 2018 showed that the average mathematics performance of students attending DEIS schools was significantly lower than that of students in non-DEIS schools, and whilst the achievement gap in mathematics between students in DEIS and non-DEIS schools narrowed in 2018 compared to 2012, this effect was not statistically significant (Gilleece & al (2020)).

Technological University Dublin (TU Dublin) was established in 2019 as Ireland's first technological university and has a student population of over 30,000. It was formed by the amalgamation of three existing institutes of technology in the Dublin area: Dublin Institute of Technology (DIT), Institute of Technology Blanchardstown (ITB), and Institute of Technology, Tallaght (ITT). Engagement with communities and widening access to education is viewed as a core part of the educational mission of the new university. The programme *TU Dublin (Tallaght) Access Office Mathematics Initiative*

(TAOMI) began in 2004 as an initiative of the ITT Access Office. It is a collaboration between TU Dublin and local DEIS designated schools in the Tallaght campus catchment area. The main goal of TAOMI is to highlight to pariticapating students the opportunities for access to Higher Eduation in their local area with a particular emphasis on the importance that Mathematics plays in achieving their potential. In the current academic year 10 schools were given the opportunity to participate in the complete range of TAOMI activities and three new schools were invited to participate in one aspect of the programme.

2. Implementation of TAOMI in the academic year 2022-2023

There are **several strands** of the TAOMI programme which vary slightly from year to year based on annual feedback from the schools. Before describing the programme as it is currently offered this year, it is important to give some background as to how it has evolved, to highlight the importance of ongoing dialogue between TU Dublin and the participating schools to ensure that it responds appropriately to the identified needs. TAOMI arose in April 2004 when the ITT Access Officer set up a meeting between Mathematics teachers from Killinarden CS (KCS) and Mathematics lecturers from ITT, at the request of the school. The purpose of the meeting was to consider what, if any supports the college could provide to help motivate the students to explore their potential for pursuing higher education, and in particular to see how mathematics might play a key role in that pursuit. Arising from this dialogue it was decided to find ways to show the secondary students the level of maths they might expect in their first year at college and some of the supports available at ITT so that they would see the level required was well within their reach. As a pilot, fifth year (penultimate year of secondary education in Ireland) students from KCS visited the Tallaght campus in May 2004 to do two active learning workshops, one in Mathematics and one in another discipline. The feedback from students and teachers was very positive, with the school career guidance teacher saying that he saw an unprecendented increase in queries from students about attending college afterwards. Based on this positive impact, the 5th year scheme was expanded quickly to five other schools. To ensure the ongoing relevance of the programme each year the ITT Access Office arranged an annual meeting between ITT and teachers to plan and agree the activities for that year. Arising from the meeting in 2006 teachers requested an intervention for Junior Cycle students and in 2009 they highlighted a gap requiring extra supports in Mathematics for their higher education aspiring 5th year students. In both cases an appropriate intervention was designed and successfully piloted with three schools and then expanded to include all schools. In the academic year 2015/2016 the Access remit of the college expanded to the Ballyfermot suburb and gradually over the next two academic years, four schools from the area were added to TAOMI, as building the trust of the teachers and students was viewed as key to success.Next we describe in detail the three stands of TAOMI in the academic year 2022/2023.

2.1 Senior cycle student visits to the Tallaght campus for workshops.

The first purpose is to show the school students the level of maths they might expect in their first year at college, with an emphasis on the applied nature of the mathematics being taught. The second purpose is to give students an insight into the breadth of disciplines to study in TU Dublin. The fifth year students (16-17 years old) are bussed from the schools to the Tallaght campus of TU Dublin for two workshops: a one hour Mathematics activity followed by a one hour active workshop in another discipline. Students are provided with refreshments in between the workshops in the canteen so that they experience that aspect of student life. The schools choose between two types of mathematics workshop: "Mathematics in the real world using Geogebra" or "Revision resources". The teachers decide in advance which workshop would suit their students. These workshops take place in computer labs. In the Geogebra workshop the students are guided through a set of examples that use Geogebra to explore how linear, quadratic and cubic functions can be found in the real world in photographs. The session culminates with the students fitting cubic functions to x-rays of adult and paediatric patients with abnormal spinal curvatures such as scoliosis thus giving them the opportunity to link their mathematics to its worthwhile real life application such as the resesearch of the Spine Biomechanics Group (local hospital surgeons and TU Dublin Bioengineering researchers) (Purcell et al (2021)). In the "Revision resources" workshop students explore a carefully curated playlist of free online resources to help them in their mathematical studies with the resources matching what they have recently studied. For the other discipline workshop, the students get to sample what it is like to study either engineering, business, computing, science, culinary arts or digital media. These one hour small group workshops are facilitated by staff from those disciplines and the teachers choose in advance which workshops best suit particular students.

The on-campus workshops were run for 440 students from the ten schools across three days in January 2023 whilst TU Dublin students are engaged in examinations so as to maximise space and staff availability. A wide range of staff from the college contributed to the successful implementation of this programme and their commitment to widening participation is critical to the success of these on-site visits. As with all aspects of TAOMI, a detailed feedback survey was conducted afterwards and responses from this will be discussed in section 3.1 below, but even before this was sent out one teacher emailed to say, *"Thanks again for the brilliant workshops yesterday, the students thoroughly enjoyed themselves and had a lot of questions and good discussions after we left.",* highlighting that the workshops seemed to have the desired effect of prompting interest among the students.

2.2 In-school workshop for junior cycle students

This aspect of the programme is for second year junior cycle students (13-14 years old). It is an active learning workshop which takes place in their schools. The facilitators are a lecturer and technician from TU Dublin but the workshop design encourages teachers from the school to take an active part with their students. The workshop requires a "double" school period (1 hour to 1 hour 20 minutes typically) to complete. It can be done with all students in the year, regardless of level of mathematical ability. It consists of two activities:

(a) A **Mathematics exploration** in which the pupils engage for 35/40 minutes in a guided discovery activity. The students try to optimise the number of pieces of a circular pizza that can be achieved for a given number of cuts. The students use geometry to experiment and proceed to building a table of values from which a pattern can be found. They are then guided to the realisation that this underlying pattern can be described by a quadratic equation. It is adapted from a problem suggested by Banks (2002).

(b) An **Engineering exploration** in which the students use “Snap” electronic kits to build a variety of electronic circuits highlighting the key role of co-ordinate geometry in placing the components in the correct positions.

The in-school workshops ran for 420 students from 10 existing TAOMI schools, as well as 186 students from three schools new to TAOMI who participated in this aspect alone.

2.3 Mathematics Volunteer Programme.

The Mathematics Volunteer Programme (MVP) is a tutoring scheme, where TU Dublin students assist 5th year Leaving Certificate pupils from the secondary schools with the development of their mathematical skills. The emphasis is on support to school students rather than the provision of extra classes. It is an excellent vehicle to enable involvement of TU Dublin students in civic engagement and to develop transferable skills associated with that activity. This is done by training suitably mathematically competent undergraduate and postgraduate TU Dublin students to tutor the second level students. The MVP sessions ran from 5 pm to 6.30 pm on a series of four Tuesday evenings in semester 2, with the second level students being bussed to and from the college and refreshments provided for both students and tutors. During the MVP sessions students were encouraged to work on homework or revision problems and the tutors responded to student queries as they arose. A Mathematics lecturer was present in the room to answer any queries from tutors when they felt unsure of how to respond to a student question. Typically in the first week there were several queries of this type from tutors but in subsequent sessions the number of such queries was very low. At the end of the MVP a presentation ceremony was held in which a TU Dublin School of Mathematics and Statistics certificate of appreciation was presented to the TU Dublin tutors, and a certificate of participation was given for the school students to acknowledge their role in the MVP.

From a practical perspective the Access Office provided a member of administration staff to act as a point of contact between the college, the schools, the tutors and organised child protection vetting of the TU Dublin tutors. They also undertook the practical organisation of transport and refreshments and tracked attendance. The co-ordinating mathematics lecturer trains the tutors, with the main emphasis being placed on the key listening and questioning skills which are critical to maths tutoring scenarios as distinct from maths teaching situations. The students who were given the opportunity to attend the MVP sessions were chosen by their teachers and could be doing any level of Mathematics. The number of students able to avail of the MVP sessions is dependent on the number of suitable TU Dublin tutors who volunteer each year, but based on the ratio of 1 tutor to 3 students typically 6-8 students from each school were selected to attend the sessions. Five schools chose to participate this year, 29 students completed the sessions and there were seven TU Dublin tutors.

3. Indications of impact of 2022-2023 TAOMI activities.

Several approaches are employed to gauge the impact of the TAOMI programme. The student numbers progressing to TU Dublin from the schools in the programme is monitored on an annual basis and shows an increasing trend over time. However, there other factors such as other college access programmes and DEIS interventions that could contribute to that trend, so although the correlation is noted, causality is not being asserted here. Feedback from three aspects of the academic year 2022-2023 TAOMI Programme, (second year workshops, fifth year workshops and the MVP) was also examined for evidence of impact. For the 2nd and 5th year workshops, feedback surveys were completed by teachers from each school. For the MVP, feedback surveys were completed by individual second level students. The survey questions were a combination of Likert scale questions and open response form questions.

3.1 Feedback on 5th year on-campus workshops

For the on-campus workshops for 5th year students the teachers indicated very strong agreement that the workshop was worthwhile for their students, whilst also indicating agreement that the workshop helped their mathematical confidence and fostered their interest in Mathematics (Table 1). The majority of the teachers (70%) noted very strong agreement that the visit to campus helped to stimulate interest among their students in further education (Table 1). The positive feedback indicated in Table 1 was also echoed in open form responses, with all ten schools highlighting both student and teacher enjoyment of the workshops as illustrated by these sample responses: *"The students loved the visit as did the teachers."* and *"They really enjoyed the visit this year. The on campus visit is excellent"*.

Table 1: Teacher responses re 5th year on-campus 2022-23 TAOMI workshops (*n = 10*).

Survey questions: "The fifth year visit to campus...	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
... was **worthwhile** for the students				2	8
... helped in **their confidence** in maths.			1	3	6
... helped in **their interest in maths**.				4	6
..helped to **stimulate their interest in further education**			3		7

The value of the opportunity to connect with TU Dublin staff in the college setting while engaged in practical active learning was also highlighted by the majority of schools with commentary such as: *"They enjoyed it. Geogebra went well and they could see the connection to real life. The practical activities were liked by all. It gave them a small glimpse into courses available at the college."* and *"From the maths activity to their visit to the canteen to their different workshops where they got hands on experience. It was a joy to see them participate and see that there might actually be a place for them in college."*.

3.2 Feedback on 2nd year in-school workshops

For the second year in-school workshops the teachers indicated strong agreement that the workshop was worthwhile for their students and indicated agreement that the workshop helped their mathematical confidence and fostered their interest in Maths (Table 1).

Table 2: Teacher responses re 2nd year in-school 2022-23 TAOMI workshops ($n = 10$).

Survey questions: “The second year workshop...	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
...was **worthwhile** for the students				3	7
... helped in their **confidence** in maths.				5	5
... helped in their **interest** in maths.			1	4	5

Student positivity, the opportunity to engage in experimentation and the hope that the workshops could be expanded, were the three themes that were present in the responses to the open form survey questions. The positivity of the students regarding the active learning approach in the workshops was the most prevalent theme as illustrated by these sample responses: *“Very positive experience, didn't want it to end.”; “Students found it extremely interesting and were amazed that they were engaged doing maths problems for nearly an hour. Not one asked how long was left throughout workshop.”* and “*Students were delighted with the workshop, "best Maths class ever" they claimed ..””*. The next most prelevant theme highlighted the students being encouraged to engage in exploration, with teachers indicating for example: *“It gave them an idea of what experimental approaches can be employed”; “...enjoyed the hands on nature of the circuits. Encouraged an investigative approach.”* and *“...they got into the problem solving and the most important thing for them to realise is that its ok to make mistakes.”* Finally when responding as to how the workshop programme might be improved both expanding the range of areas explored and workshops for younger students were indicated: “*If other areas of the course could be covered in practical workshops would be great”;* and *“...to have something for 1st years to get them involved from an earlier stage.”*.

3.3 School student Feedback on MVP 2023.

Of the 29 students who took part in the MVP, 14 completed the survey. A large majority of the students rated the MVP highly with 12 students rating it very useful or useful and two rated it as neutral, and nobody selected not useful or not at all useful. When commenting further 12 students had positive comments about the tutors and how the material was explained, for example, “*The tutors offered alternative ways to solve equations”*. A further four commented on the environment being a good aspect of the MVP, highlighting the chill and fun atmosphere, “*The chill non-toxic environment, really good tutor*!”; “*I was able to ask question freely and joke while studying”*. This view was reinforced in responses to other survey questions where the atmosphere of the MVP was rated as “Friendly” by 13 students and where 13 students also strongly agreed or agreed with the statement: “The atmosphere in the MVP sessions makes me feel that it is ok to ask questions”. When commenting on what they perceived as unsatisfactory aspects, five comments mentioned that the MVP could be improved with the addition of more sessions and the provision of extra material to work on. Three stated they were unsatisfied with the bus journey getting to and from TU Dublin "*Not giving us an exact bus timetable to follow*", and a further two had difficulty with understanding their tutor, *"His accent was difficult to understand"*.

In terms of the student perception of mathematical confidence, a majority of students agreed when asked "Has the MVP helped you feel more confident about your ability in Mathematics?", with three strongly agreeing, eight agreeing, one selecting neutral, and two disagreeing. They were asked to comment on this, most of the comments were positive, with some sample comments being: *"I feel more comfortable doing math questions by myself"*; *"I feel a lot more comfortable asking for help when I need it"*; "*MVP made me feel more confident in answering questions without feeling shame for not knowing or understanding something."*. In terms of students perception of Mathematics, 9 students stated that the MVP had changed their view of Mathematics whilst 5 said it hadn't. Sample comments expanding on their choices included: "*Yes, I realised it isn't so difficult unless you believe it is."* and *"Really helped us to enjoy and engage with maths and learn from a different perspective."*

Students were asked about whether attending MVP sessions would influence their decision about attending TU Dublin, with 8 of the 14 students indicating that it had positive effect (Table 3). For the eight students that said Yes to both questions in Table 3, four commented about being able to see the campus and what student life is like, for example, *"it showed us some of the campus and students life and made us feel comfortable with maths"*.

Table 3: Student responses re MVP influence on decsion to consider TUDublin *(n = 14)*.

Survey questions:	**Yes**	**No**
Would you consider going to TU Dublin?	11	3
Has the MVP influenced your decision?	8	6

4. Discussion

The feedback presented in Section 3 is similar in nature to the feedback gathered in previous academic years and indicates that TAOMI is valued by the teachers and the students in the participating schools. The following incident provides some extra evidence to support this. Due to financial constraints a decision was made in September 2013 to cut the TAOMI resource allocation, hence causing the programme to end. Immediately all schools involved in the programme individually contacted the college seeking its reinstatement. This was unprecedented in the area of college access programmes for the institute and led to a unanimous decision in January 2014 by ITT Senior Management

Team to restore the necessary resources. Allied to this process the schools were surveyed at the end of the 2013-14 academic year to rank which of the aspects of TAOMI they would most like to see continue if some had to be removed. Interestingly there was no agreement as to what aspect of the initiative might be removed. As one teacher commented, *"It would be a shame if any of the initiatives were to stop. Each has its own merit. I think in a school like ours it is important to expose students to third level possibilities at a young age and feel that the 2nd year visits are very worthwhile from this point of view. They also get to see what 3rd level staff are like and the content is excellent. Likewise I feel the visit in 5th year to ITT shows our kids what a 3rd level institution is like. It can be especially helpful if they bump into some of our past pupils. I find it can focus them in a way us talking could never do. Even though it is only for a smaller group, I think the MVP is the best value option as it lasted the whole year for the students and I got very positive feedback from my 5th Years last year."* This view has persisted to 2023. One teacher indicated that the 5th year aspect of the programme was *"......invaluable. It shows them possibilities that are available after school."* with another commenting that *"Aside from the benefits of the initiatives in the moment, they generate great conversations afterwards about the possibility of college and even the possibility of STEM subjects at third level."* Other teachers highlighted the early intervention value of the 2nd year workshops commenting *"(2nd year) Students were delighted with the workshop ...... Also it was a great way of introducing third level to those who haven't thought about it yet. Also realising about the university campus in Tallaght and how accessible it is for them"*. It is also interesting to note that no school has ever left the TAOMI programme and this year three new schools applied to join.

Given the perceived value of TAOMI, we would like next to discuss the two features which we have identified as key to this type of programme being repeatable in other educational settings: (1)a truly collaborative approach and (2) an active learning methodology.

4.1. A truly collaborative approach

We employ a Listen/Respond/Facilitate/Feedback(s)' (LRFF) cyclic paradigm, Figure 1, to maximise the impact of TAOMI. In the 'Listen' phase of the LRFF cycle, we ascertain what is needed by the schools. We do this by meeting annually the key mathematics TAOMI teachers who have been nominated by the management of the schools (buy-in from school management is crucial). In the 'Respond' phase, we design a response for that year which will meet the schools needs. It is important to note that this provides flexibility as to what aspects of TAOMI each school would like to participate in, as they know their students best. In the 'Facilitate' phase we implement that response with an ongoing interaction throughout the year regarding timing and content of visits. In the Feedback(s) phase we gather data on the interventions and reflect on how to improve the impact of the next iteration of TAOMI.

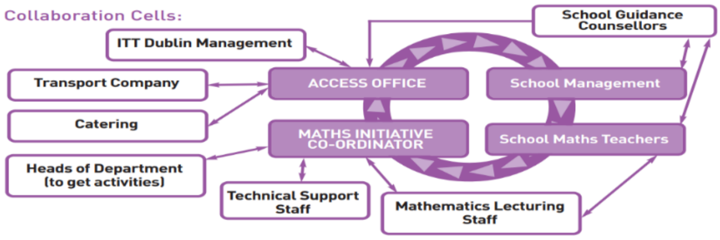

Figure 1: LRFF cycle

Figure 2: TAOMI communication diagram

We also ensure that the workload associated with TAOMI is as distributed as possible *within* TU Dublin. The success of the programme relies heavily on interaction between several functions within the university. TAOMI has always been an Access Office outreach initiative which is coordinated by Mathematics lecturing staff. The Access Office provide the finances and the vital administration support needed to run the programme, whilst the coordinating mathematics lecturer communicates with the mathematics staff, technical staff, the school maths teachers, and the Head of Departments (now Disciplines) to arrange for necessary resource inputs for workshops. Input from each staff member who helps on the programme (aside from the coordinating mathematics lecturer) is well defined and low in time commitment to ensure the sustainability of the programme. It is a testament to TU Dublin staff that since the programme began in 2004 only one member of staff has declined to be involved. Figure 2 above gives an overview of how the various “collaboration cells” are connected.

4.2. Employing active learning methodology.

The value of employing an active learning approach is already evident in the feedback presented in Section 3. However, its importance was highlighted starkly by feedback for the 5th year on campus visits which occurred in May 2022 (7 of the 10 schools availed of the opportunity). The May 2022 visits were different to previous iterations both in timing and content, as at that point the university was only just returning to normal on campus activities post Covid. As there were still social distancing concerns only the Maths workshops ran in the usual active learning mode and discipline workshops were replaced with a talk on accessing college and a short walking tour of the campus. Feedback from the schools was still very positive about the visits but the importance of the active learning element of the visits was highlighted in comments from 6 of the 7 schools, the tone of which is captured in these sample comments: *“I think it is important that the students get to experience some of the practical workshops that we had before. I know this was not possible this year due to covid and the time of the visits. However, I really think our students need the practical element .”* and *“I cannot emphasis enough how highly thought of this college visit was in our school. In previous years we always had students who were full of chat on the numerous workshops they did… culinary , sports , engineering, 3-d printing. As this is always one of the first college visits our students get even getting to the canteen for their snack opened their eyes and minds to the possibility of going to college. Unfortunately they didn’t get any of these experiences this year and for the first time in many years, student had no opinion either way on the visit.*
In contrast for the comments regarding the more normal January 2023 5th year visits

highlighted the appreciation for the return of the more hands-on aspects of the programme, "*We have attended the 5th year workshop many times and it has been thoroughly engaging and enjoyable for our students, the last couple of years have not had the same amount of hands on subject practicals but this year we got to experience some wonderful workshops again."*. These comments highlight that employing active learning methodology is key to student satisfaction in TAOMI and this is why we believe it is necessary toadopt such an approach to maximise impact for students in Mathematics outreach programmes in other settings.

To conclude, we would like to mention how the continuing growth of TAOMI demonstrates the impact TU Dublin believes the programme has had "sparking curiosity and nudging open doors of possibilty" for the students involved. Firstly, it is significant that TU Dublin is seeking to expand the programme beyond the Tallaght campus to include schools linked to its other campuses. Also, a new strand of TAOMI involving 6th year students was piloted this year following a request from schools in September 2022 demonstrating the continuing organic growth of this successful collaborative programme.

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Discovery and Communication in the Mathematics Classroom

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Abstract

University courses in Ireland were taught online during the covid lockdown from March 2020 right up to September 2021. Return to on-campus teaching since then has been gradual and piecemeal, with campus life much changed now with many lecturers continuing to work from home where possible. For some lecturers, including myself, the return to teaching in the classroom was welcome. The experience of teaching online and the innovations adopted during that period gave a new perspective and suggested possible changes to on campus teaching. In this paper we look at recent changes to the teaching of mathematics to a group of engineering students at Atlantic Technological University (ATU) Sligo. In particular how adopting a tutorial style has proved to be a successful model for engaging students and allowing time for exploring different approaches. We look at the use of a large bank of questions and solutions to create randomised assessments.

Introduction

A number of sources have commented on the benefits of encouraging students to become more active in their learning, and to communicate their mathematical thinking. For instance, in her findings from a survey of over 365 engineers in Ireland reflecting on their maths education, Eileen Goold (2014) suggests that an active social learning environment would benefit engineering education. Students would be required to present and defend their mathematical solutions to both their peers and their lecturers. This type of learning environment would develop students' mathematics communications skills and enhance their mathematics thinking and confidence, providing a better match to the mathematics required in engineering practice.

In his book "A Mathematician's Lament", Paul Lockhart (2009) suggests a way to teach students by choosing engaging and natural problems suitable to their tastes, personalities and levels of experience, by giving them time to make discoveries and formulate conjectures, by helping them refine their arguments and creating an atmosphere of healthy and vibrant mathematical criticism.

The classes described in this paper are face to face. The mathematics is taught over 5 semesters to engineering students doing civil, mechanical, robotics and electronics. Topics include linear algebra, calculus, multivariable calculus, vector analysis, Fourier analysis, statistics and probability. The mathematics is common to all programs and class sizes are small with around 40 students in each class.

Video Recordings

During lockdown a typical mathematics class was delivered from home using Microsoft Teams. Demonstrations, previously done on the whiteboard were now done with pen and paper, captured by a visualiser, and shared live. In each class there was a 10 to 20 minute lecturer demonstration of a mathematics problem or a new concept. These parts of the class were recorded using Panopto. The videos consisted of the lecturer's voice and the page showing a demonstration of a mathematics problem. Within a few months, over 230 such videos were recorded. These were initially stored and shared on Moodle (the ATU learning management system) for students to view later. However, it soon became apparent that a better sharing system was desirable to enable multiple student groups to access the same videos and make them available in subsequent years. Thus, a new website was created (Gallagher 2021) to share the videos. The website consists of 11 separate pages for different topics, such as algebra, geometry, vectors and so on.

In an evaluation of distance learning, Habala and Demlova (2021), write that when students at their institution were asked what they would preserve when things got back to normal, the predominant answer (over three quarters) was to preserve availability of recorded lectures. Echoing those results, the videos on the website have proven popular with students at ATU Sligo.

Worked Solutions

For these courses, we don't use a textbook. Instead, each module has a set of lecture notes written in latex. Students work on tutorial sheets in class and in their own time. Tutorial sheets, also written using Latex, contain around 2000 questions. Over the years and particularly during lockdown, many handwritten solutions were written. These solutions were scanned and saved as pdf files. The website provided a great opportunity to share the tutorial sheets and the pdf solutions. Over 600 of these solutions are now displayed on the website, each with a unique reference number. Where a solution is available, this number is referenced alongside the question on the tutorial sheet. Student feedback is positive. Having this bank of videos and solutions is probably the students' favourite aspect of their mathematics courses. For the lecturer, the large question bank has made writing exams and assessments easier and faster. Questions can be randomly chosen from the bank, making them less predictable, and so rewarding students for understanding learning concepts rather than memorising procedures.

Return to the classroom

It became apparent on returning to campus, that online teaching had not suited all students with some not engaging at all and effectively missing a whole semester or two of teaching. This author gained a new appreciation of the benefits of classroom discussion, of being able to observe students working at problems, and the possibility to again have some fun while learning mathematics. With the bank of videos, it became possible to flip the classroom, and have students watch a short video prior to class, and then spend time in class working at problems and discussing them. Because the class sizes are relatively small there is a good opportunity for student engagement and discussion.

Direct instruction and active learning

Direct instruction is the explicit teaching of a skill set using lectures or demonstrations of the material to students. Active learning, meanwhile, is "a method of learning in which students are actively or experientially involved in the learning process" (Bonwell and Eisen 1991). There is much debate as to which is better. In my experience the two styles are not mutually exclusive and in fact are both desirable.

The mathematics classes in this particular case study contain a mix of both styles of teaching. The direct learning takes place both before class when students watch a video and again at the start of each class when a demonstration takes place at the whiteboard. A criticism of direct learning is that it can be dull or passive for the student. To mitigate this, the classroom part of the direct learning here can involve the lecturer asking questions to randomly chosen students, thus keeping them focused and engaged or "on their toes" as one student put it.

After that, students work at their own pace on printed out worksheets, sometimes going beyond what the lecturer has demonstrated. This active learning can lead to some students finding their own innovative solutions, and other students getting stuck. Students are encouraged to communicate with their colleagues to make progress on a tricky problem. Students can also check the handwritten solutions on the website already mentioned using their phones in class. Not all questions have solutions done out, however. When most students have gotten as far as they can on their own, then it is time for another demonstration at the whiteboard. Getting the balance right between instruction and discovery requires the lecturer to be aware of the changing mood and dynamics in the classroom. When students struggle with a problem, there is a possibility for them to learn tenacity and be creative. If they struggle for too long, there is a possibility of them becoming frustrated or abandoning the problem.

Velichova (2021) acknowledges the pedagogical challenge in introducing active learning methods in engineering mathematics. However, she writes how active participation of students in the educational process

> *can contribute to better understanding and more positive approach to learning itself, which becomes more a discovery of dependencies and investigation of activities and processes than memorizing of scattered facts and not connected data*.

One of the arguments against active learning is that it may not be the quickest or most efficient way for students to acquire knowledge. However, while I agree with that in principle, my experience of having students arrive at an alternative way to solve a problem (even if it takes longer) adds richness to the learning experience and can increases motivation and confidence. Ultimately, one of the benefits that a lecturer can bring to a group of students is to share their passion and enthusiasm for mathematics. Students enjoy getting stuck into problems and it is rewarding for this lecturer to see carefully crafted worksheets challenging them.

Attitude

The full-time students at ATU Sligo are typically young (approx. 18-22 years old) and are mostly drawn from the surrounding area. There is a wide range of mathematical ability and attitudes towards mathematics.

Research has shown a positive correlation between students' attitudes towards mathematics and their performance (Lipnevich et al 2016). To help nurture a more positive attitude towards mathematics among engineering students at ATU Sligo, the focus here is not only to make lectures interactive and engaging, but also to provide questions that are relevant and at the desired difficulty level.

Difficulty level of questions

Worksheets begin with relatively easy questions and become gradually more difficult. Bjork and Bjork (2011) emphasise the need for *desired difficulty* in learning. The level of difficulty that is optimal, will, in their words

> *vary with the degree of a learner's prior learning. In general, for example, it is desirable to have learners generate a skill or some knowledge from memory, rather than simply showing them that skill or presenting that knowledge, but a given learner needs to be equipped by virtue of prior learning to succeed at that generation.*

Another key component of learning emphasised by Bjork and Bjork is interleaving. If a topic is taught in a block, it can quickly be forgotten, but spacing out the learning over time can help students to practice memory retrieval. In my experience this happens quite well in mathematics. For example, a skill like integrating by parts is not taught once and then forgotten about. It can be returned to when solving differential equations, or in vector calculus or in calculating Fourier series. So, creating the questions on the worksheets follows a certain trajectory from easy to difficult, but also weaves in ideas from previous topics and the aim is to make the level challenging yet attainable for the student. When students achieve success along the way, while taking incremental steps forward, the idea is that they will gain confidence and be motivated to persevere.

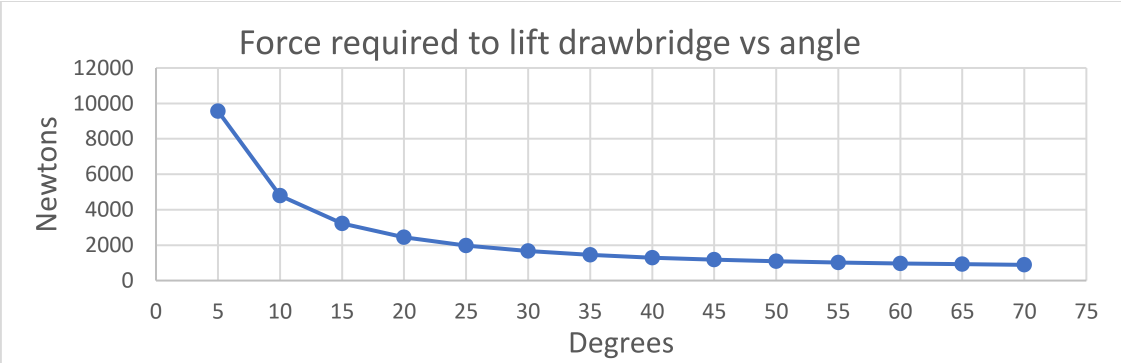
Relevance of questions

A crucial factor in the motivation of engineering students is the *relevance* of the mathematics. Jakobsen (2021) in writing about engineering education highlights a concern that content that is perceived as "unnecessary" can lead to frustration. Schaathun (2021) notes that engineering students

> *are unlikely to solve routine problems by hand in their future careers, that routine solutions have long been computerised. It is the non-routine problems that still require human attention. Thus, there is no point in learning just the basic skills.*

In the same paper, Schaathun references the "coverage issue" (Yoshinobu & Jones 2012). He explains the problems with having to cover excessive material on the syllabus, that it leaves no time for exploration or deep understanding.

It can be challenging to cover all topics required in a syllabus, given the time constraints. Teaching fewer topics may allow for more depth, more understanding, and may enable alternative approaches to problems to be used, for example using software such as Excel, Geogebra or MATLAB to model problems. It may also allow time for students to work on mathematical projects that align with their own particular engineering interests. In recent years, this author has tried to do more modelling using Excel. For example, looking for the force required to lift a drawbridge connected by a rope at a certain angle, using excel we can find the force for various angles. The result can be represented on a graph as shown below.



In the example above, the student not only learns about torque, but can see that changing the angle from say 55 to 65 degrees has little impact on the force.

This author has tried to include more applications of mathematics to help motivate the students. For example, solving differential equations relating to the number of people vaccinated over time, or the change in the amount of CO2 in a room over time, both of which are relevant to our post covid return to the classroom. Indeed, engineering students are forever asking for applications. However, when presented with such questions, they don’t actually like them! Especially when they are asked to explain the results in context.

Creativity

Finally, a quick quote on humour and creativity from Rob Eastaway (Eastaway 2019):

> *About 50 years ago, Arthur Koestler wrote a book called the Act of Creation, and in that book, he said that creativity manifests itself in three ways: Art, Discovery and Humour. Somebody else reduced this to three shorter words: Creativity is Ah, Aha and Haha. I love those three words, because I think they are a quick way to spot that creativity is happening, whether that’s in everyday life or in maths lessons. If a lesson has included no moments of Ah, Aha or Haha then whatever else it has achieved, I would suggest it has not involved any creativity.*

Developing worksheets is a creative exercise. Games provide a rich source of questions, especially in probability. For example, in the boardgame Risk there is a complicated dice system to determine the outcome of a battle (3 red dice versus 2 blue dice – whichever colour has the higher number on any one dice wins, if it is a tie then blue wins). The probability of success for red turns out to be 46.64%, and it is a challenge for students to work this out. Even more fun is for students to then play this out numerous times to see that the distribution approaches the theoretical probability.

Questions can arise from conversation in class. For example, two students comparing their ratings and percentiles on chess.com realise that they can work out the mean and standard deviation of the distribution (assuming it to be normal). Thus, we get this question:

> *Two players compare their rating on a chess website. Anna has a rating of 1325 and a percentile of 74.2%, while Bart has a rating of 1140 and a percentile of 45.65%. Assuming the distribution of ratings is normal, find the mean and the standard deviation of the distribution.*

Occasionally a new question on a worksheet cannot be answered, because the lecturer did not try it out first. This can lead to new discoveries in the class. Consider the following question:

> *In a particular binomial experiment, the probability of getting 3 successes from 5 trials is 0.62. Find the value of p (the probability of success on a single trial).*

This is a lovely question and involves forming a polynomial in p and solving it. Unfortunately, it has no sensible solution. This was only discovered in class after the students started working on it. The value of 0.62 turns out to be too high. But then this leads to the question, "what is the maximum value that could have been used in the question?" and so the following question is born:

> *In a binomial experiment, the probability of getting 3 successes from 5 trials is A. That is P(X=3) =A. If p is the probability of success in a single trial, express A as a function of p, and hence find the maximum value of A. Graph A=f(p) over a suitable domain for p.*

A feature of the question above is that is ties in calculus with binomial distribution and the results can be visualised on a graph. This is a nice example of interleaving topics. It turns out that such polynomials are called Bernstein polynomials.

Conclusion

There are lots of ways to teach mathematics to engineering students. This author has tried many of them over the last 12 years with varying degrees of success. Ultimately, in my opinion, lecturers need to find what works for them. Part of this involves experimentation and feedback. What works for one lecturer might not work for another, due to variations in their personality, interests, technical expertise, equipment, not to mention variations amongst the students. This paper is one example of a mathematics lecturer finding a method of teaching that works given a unique set of constraints. It is by no means a finished method but is continually being amended. Just like the students, the lecturer is always learning in the classroom.

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Learning through discussion

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Abstract

The paper discusses the role of dialogue in the teaching and learning process. It deals with forms of communication in pedagogic use, and especially it focuses on not so traditional handling of discussion in examinations, the gains it makes for students as well as limits, which have to be taken into consideration within implementation. The changes, extorted by covid restrictions transferring education into an on-line space, influenced the communication sphere maybe in the most visible way and its consequences can be seen to this day. The long lasting lack of face to face conversation allowed students to absolutely misunderstand what a presentable learning outcome should be. The irreplaceable on-site position of a teacher is undisputed in this context, as well as that their presence is very important for students' common comprehension along with their motivation and overall activation. We will present good practice forms of communication which result in the improvement of mathematical competencies and which are worth using also in the post covid time recovery process.

Introduction

The forms of communication in class come into consideration from a pedagogical point of view predominantly within the application of the active learning concept, where students are to perform actively, working usually in small collaborative groups using various techniques such as role play, games, problem based learning, etc. The mentioned techniques emerged mainly after the boom of psychological research in the beginning of 20th century; e.g. the "game on merchants" was one of the first didactical plays, or experimental project learning concepts in the 1920s, aiming to teach mathematics only in the form of solving problems from real life (Richtarikova (2018)).

In the latest period, many reviews and researches have been done for primary and sometimes also for secondary schools. They deal predominantly with effort to improve the quality of dialogue between teacher and pupils, and to engage pupils into class or group mathematical discussions. The case studies and researches aim to find and classify questions that help teachers control and move discussion on (Sjöblom, (2022), JMTE, (2022)); to study the effect of interactive dialogue on the students' competence in mathematical reasoning (Smit et.al.(2023); or to examine the organisation and conditions that influence the quality of discussions. The Swedish Institute for Educational Research prepared for Swedish pedagogics a systematic review of works in databases on leading dialogue "where pupils are active, constructively critical participants in collaborative mathematical reasoning." (Fredriksson K., (2022)). P. Liljedahl, (2016), examined in his research nine elements influencing building and maintenance of the "thinking classroom", out of which two, not very often explored, had interesting results for activation of all pupils on site. First, the de-fronted space of group work using vertical non-permanent surfaces (whiteboards, blackboards, or windows), placed in a random configuration around the room, "makes visible all work being done, not just to the teacher, but to the groups doing the work." To facilitate discussion there should be only one felt pen or piece of chalk per group.". The second, in contrast to strategic or randomly formed groups when some students complain about their groupmates, the groups were formed frequently and in a visibly random manner (the learners see how randomization is done) to eliminate this social aspect and "students become agreeable to work in any group they were placed in".

Discussions in tertiary education

Looking at the occurrence of discussions in tertiary education, we can see their place primarily in seminars devoted to theses preparation and in the defence of final theses. Besides this, they naturally appear in many optional activities of students, taking part at university scientific competitions or working on projects as a form of indirect education. On the contrary, they are not very frequent in the direct teaching process, and their implementation depends essentially on a teacher. The inclusion of verbal communication techniques could be an object of dispute, but they are not among the favourite methods for a large number of teachers. With respect to the opinion of many teachers at technical secondary and tertiary schools responding to our survey, the teachers of mathematics can be divided into two main categories:

1. the teachers who prefer to teach through a lot of calculation in pre-defined examples and scenarios. They think that it is mainly the drill of calculation techniques that provides sufficient mathematical knowledge and skills for students at technical universities

2. the teachers who see the main point of teaching and learning mathematics in reaching the acceptable level of conceptual understanding. They prefer more guided dialogue, discussions, chaining arguments and explanation in face to face teaching; and they leave extended calculations for homework supported by consultation.

After the general acceptance of the specification of mathematical competencies (Recommendation (2006), Alpers (2013), McGehee (2021)) there is no doubt that communication on mathematics definitely belongs to the key objectives of mathematical education and moreover, it helps substantially to increase the level of all other mathematical competencies and vice versa. In accordance with the aforementioned it unavoidably leads to combining both approaches.

Discussion as a part of the exam

Traditionally, almost all school summative assessment in mathematics used to include written and oral examination. In written part, tests with closed tasks are very popular due to their quick computer aided evaluation. Besides them there are also tests with open answers or some form of coursework is used. Although the oral examination is taken at secondary schools minimally twice a year, at technical universities the oral examination is, due to a large number of students in a course, often replaced by written tests (usually in the form of a closed multiple choice test), and in some cases theory questions are not even asked at all. Indeed, assessment by written closed tests has many benefits, especially saving time and objectivity. On the other hand, the talk between a teacher and a student in any form brings for the student a freer way of presenting acquired knowledge and skills where articulation in one's own words stimulates further clarification, comprehension, and memory. The most conventional form of oral examination at university is answering randomly picked up questions on a particular topic or answering a kind of supplementary question about a work or problem done within a course or exam.

In order to achieve an overall better understanding of the course sense and of the main principles of the taught methods we decided to include, experimentally, discussion into the assessment of two compulsory-elective mathematical courses. The first one was the subject Applied mathematics at master's degree in the first year and after superior results, we tried to apply the exam structure also in the second year of the bachelor degree in the subject of Numerical mathematics.

The exam consisted of three components:

- compulsory written part: an open problems test which also required giving short reasons for a particular method limitations, a process of calculation, or a result, etc.

- compulsory oral part: discussion in a group.

- project work: compulsory for master's degree, optional for bachelor studies

The written open problems test served as the basis of the final assessment result. The minimal score of 60% entitled the examinee to enter the oral part. The final mark was set by all three parts.

The discussion was conducted in five stages:

1. Creating an ambience. We started with speaking about the topics of the course, which parts students consider to be interesting and why. Such an opening encouraged students to make quick revision of the content and focus their mind on the substance and issues which were not obvious, that surprised them or that caused them difficulties.

2. The deeper discussion on one of the above issues. Basic characteristics, principles, necessary and sufficient conditions of use, pitfalls, what made difficulties, how to solve them, applications in engineering.

3. Posing a non-specific problem which was not addressed within a course but the students can come to solution by a combination of their knowledge and skill experiences.

4. Specification of the most important conclusions of the course, their message to other fields, how to treat with learned methods in the engineering area.

5. Self-assessment or peer-assessment in the discussion and overall assessment.

Each discussion was different. The order of the points 2.- 4. varied with respect to the flow of discussion. The way of implementation and the point 3. depended on the students' level of knowledge and their ability to discuss.

The arrangement, observation, critical moments

The discussions were held in a class with blackboard and chairs arranged into a circle. The blackboard was the common place for notes and visual explanations. The usual number of students was up to 8. They could sit or stand around the blackboard. The maximal time of discussion was one hour.

An examiner served as a chairperson controlling a framework of content and flow giving maximal space to students. It would be ideal if the teacher was only an observer and the whole discussion was realized only by the students. Whatever the reality was, the discussion here did not work as a role play; it required a high level of knowledge on the side of students and a very responsive approach on the side of the examiner. The students had to be attentive, listen to others and react in real time. They did not have the time for preparation, so they had to think aloud, to present openly their style of thinking, and to reveal what they really knew, how they were able to work with their pieces of knowledge and skills. To reach such open discussion, the great mutual trust between the teacher and students had to be built up.

On the one hand, during the semester, the students were trained to articulate their mathematical opinion, to give an agreeing or disagreeing statement and justify it. We regularly inserted short whole class discussions into the lessons. Students had also the opportunity to discuss in groups when developing projects outside the lessons or when they prepared themselves for midterm tests. Group discussions in class did not work well for this purpose, as they were difficult to follow and students could easily deviate to private topics. This corresponds with the research results of P. Liljedahl, (2016), mentioned above. Contrary to this, we have to notice that group discussions held on internet channels during on line teaching were excellent from this point of view, because the teacher's presence kept the students engaged in work (see more in Richtarikova (2021)). Our observations clearly showed that in situations where students do not have their own direct motivation, the teacher’s presence (though passive) on training talks is imperative, no matter what age the learners are.

On the other hand, “creating an ambience” – the first stage of discussion, met the task of developing mutual trust and it was the examiner’s role to create and keep a positive atmosphere, where students felt safe and were able to ask sincere questions without doubt that their grade will be lowered. It was important for the teacher to control his or her immediate reactions. Direct negative evaluation could stack the discussion, so one had to focus on facts and act positively and encouragingly. In a positive atmosphere and when the students were well prepared, the discussions were fluent and required only minimal intervention from the teacher. In addition, the following items were seen to be effective.

- Forming discussion groups from students of approximately the same competence level. It allows examiners to adequately adapt the methodology of conducting the discussion: to challenge individual students to reactions, to pose appropriate supplementary questions, or let students intentionally formulate questions themselves on the most important issues and answer them.

- Observation, how particular students are involved in discussion. This can guarantee each student the same opportunity.

- Watching time spent on discussion. The purpose is not for the discussion to be long and exhaustive.

Conclusions

The feedback from students was excellent. Despite the demands described above, the students considered this way of examination to be less stressed, although they were exposed to direct comparison and in the end they have to evaluate themselves. No one wanted to be assessed by traditional oral form or by written theory test. The most appreciated benefits were:

- The "wow" effect of the third stage, where students realised that they were able to synthetize the learned knowledge and skills and come to new conclusions and opinions.

- The summary of important facts and relations in mutual conversation helped many students to understand relations they were not able to comprehend alone.

- The way they were treated.

Subjectively we saw students leaving proud of themselves for being successful in their presentation of what they knew in front of the group and even what they had invented on the spot, sometimes not realizing that it was the work of all of the group. We have to conclude that we noted an increase mainly in critical thinking and in ability of abstraction. We also saw an increase in the ability to articulate a mathematical opinion, in the way the students give information to others, in the way that students were able to chain arguments and to defend their opinion, as well as in the ability to listen to others. Although to conduct the assessment using the discussion is not easy for examiners and it requires good planning, hard communication work with students during semester, and prompt reactions on the spot, its contribution to developing the mathematical competence is evident.

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A Mathematics Less Ordinary: Serious Games Engaging Engineering Students

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Abstract

We discuss the didactical elements underpinning the Horn of ODin (HOD), our recently published mathematics learning object, a serious game in the form of a viking age adventure. The motivation driving development of HOD was the criticism often appearing in student evaluations that traditional learning activities in engineering and pure mathematics are perceived as boring or not helpful for learning. Purpose-made serious games on the other hand are widely accepted as good learning tools for engagement and deeper learning. The challenge is to construct such games to cover systematically all aspects of often complex learning outcomes while avoiding resemblance with traditional lessons. We assembled a multidisciplinary team of games specialists and educators to design HOD's learning activities, focusing on the topic of mathematical relations. HOD is a self-contained prototype learning object, including all theory, exercises and formative assessment, covering this topic, which is at Level 2 of the current SEFI mathematics curriculum framework. The methodology can be applied readily to subjects at Core Levels as well as Level 3 of the framework, and can be extended to create rich cross-disciplinary learning objects exploiting game elements to link mathematics with customised practicals. HOD integrates formative assessment with player progression and gives continuous, personalised feedback; its immersiveness promotes student engagement on a cognitive level as well as in a social setting.

Keywords: Active Learning Strategies, Playing and Learning in and with Mathematics, Serious Games, Future and Emerging Technologies, Student Engagement, Next Generation Classroom, Campus, Virtual and Blended Learning Communities.

1. Introduction

In course evaluations students often criticise traditional learning activities in engineering and pure mathematics (e.g. lectures, practicals and textbooks) as boring or not helpful in their learning. Class attendance is not always at ideal levels, and it is unfortunately not uncommon for a large part of a cohort to fail one or more learning outcomes on a course due to lack of student engagement.

What drove the development of the Horn of Odin (HOD) learning object we present here, was a desire to reduce notably the number of students failing to meet one particular learning outcome, on relations on sets, in a second year undergraduate course in discrete mathematics at Mid Sweden University, as this was the learning outcome on the course with the poorest student performance. Before our intervention over half the cohort regularly failed to meet this learning outcome in their first examination attempt, and it was thus contributing significantly to the poor student attainment in the subject, cf. Taralunga, Heidtmann and Krokos (2021).

The topic of mathematical relations is common in discrete mathematics courses around the world. It is at Level 2 on the current, third mathematics curriculum framework by SEFI (2013), and has many applications in computing subjects as diverse as databases, scheduling and neural networks. Traditionally it is introduced to students through one or two lectures (c. 4 hours) followed by a workshop, where the students work with well-chosen problems illustrating each concept involved. At Mid Sweden University the lectures and workshop are also followed by a computer-graded formative assessment.

Through student interviews and questionnaires, the challenges with the topic at Mid Sweden University could be attributed mainly to poor student engagement: low attendance at lectures and practicals, disinterest in the subject (as it was perceived to be more trivial than other topics studied in parallel courses competing for student attention), and not reading the textbook (which was characterised as boring). Finally many students also were disinclined to working with others, preferring to work individually rather than in a collaborative way.

The objective was thus to create a learning object on relations that would (i) engage the students, (ii) make them reflect on the subtleties of the concepts involved in the topic, and (iii) retain their engagement for long enough for them to realise that there are deeper discoveries to be made. We also wanted to encourage them to (iv) engage in discussions with each other in order to (v) help each other getting to the deeper levels of learning.

Purpose-made serious games are widely accepted as good learning tools for engagement and deeper learning, cf. Garris, Ahlers, and Driskell (2002) for example. The challenge was to design our serious game so as to systematically and methodically cover all aspects of the complex learning outcome in the topic without the resulting learning object resembling traditional lessons. A multidisciplinary team of games specialists and educators have worked together to design the learning activities within HOD to address this challenge.

In the next section we explain how the learning outcome was broken down into subtopics and analyse it in terms of cognitive skills required from the students to master it. In section 3 there follows a description of the resulting learning object.

2. Analysis of the learning outcome

The learning outcome on relations that is the learning objective for HOD is from a second year course for students on a civil engineering degree in computer science at Mid Sweden University and reads as follows: *In simple cases the student must be able to decide whether a relation is an equivalence relation or not.*

The first step in the game design process was to break down this learning outcome systematically into bite-sized subtopics. Each subtopic represents a well-defined bite-sized chunk of knowledge students must grasp, but is large enough to reveal their errors and misunderstandings in subsequent testing. For each of the subtopics, errors made by students were carefully and methodically identified by experienced teachers from answers to quizzes and exams spanning several years. The subtopics are listed below in the order they are traditionally taught in the course, and the game largely follows this order in order to complement the traditional lessons, not merely replacing them. Students are advised that they can play the game, follow the normal lessons or do both.

1. What is a relation?
2. How to depict a relation by using a digraph;
3. The reflexivity property;
4. The symmetry property;
5. The transitivity property;
6. An equivalence relation is one that is reflexive, symmetric and transitive;
7. An equivalence relation induces a partition on the underlying set into equivalence classes;
8. The anti-symmetry property;
9. A partial order is reflexive, anti-symmetric and transitive;
10. A (total) order is a partial order in which every pair of elements in the underlying set can be compared.

At first glance the list is merely some facts and definitions that have to be learnt, and could consequently be classified as being at base level (factual knowledge) in Bloom's Taxonomy (Bloom & Krathwohl, 1956), but for many students they are a real challenge, as this is often one of their first encounters with abstract concepts. One issue the students face is that mathematical language is exact, and moving as much as a comma or changing a seemingly insignificant symbol can completely change the meaning of a sentence. So much care must be taken when learning these definitions. As an example, consider item 4 in the list, the symmetry property:

In full the definition that must be learnt here is: *A relation R on a set S is symmetric if xRy => yRx for all x,y in S.* A common misunderstanding by many students here is that symmetry is a property of pairs of simple objects *x* and *y* rather than a property of the more complex object of a relation. Students who have not fully grasped the meaning of the logical symbol '=>' often also confuse it with one of the three symbols comma, '<=>' or 'and', but if the symbol '=>' in the definition of symmetry is replaced with a comma or with the word 'and', this definition is no longer the same and no longer describes what a symmetric relation is. Further, if they read or think 'and' instead of 'implies' when they see the symbol '=>', they will never be able to reach the higher levels of comprehension, application, analysis and synthesis in Bloom's Taxonomy with the symmetry concept. Similar logical errors or difficulties impede understanding of the other subtopics. Further, as symmetry is part of the definition of an equivalence relation (Subtopic 6 in the list) along with reflexivity and transitivity, students must reach at least the learning level of synthesis in Bloom's Taxonomy for all these three concepts before they are finally ready to start at the base level again with the factual knowledge that an equivalence relation is one that is reflexive, symmetric and transitive.

Now, because the concepts of reflexivity, symmetry and transitivity are closely connected also with the anti-symmetry property, and an equivalence relation must not be confused with a (partial) order on the set, the students must also learn about anti-symmetry and then follow this up with learning about partial orders and total orders all the way from base level to at least the level of synthesis in Bloom's Taxonomy.

Finally subtopic 7 combines knowledge about partitions of sets, which the students have acquired in an earlier part of the course with a property which they must now be able to deduce about equivalence relations. Therefore most of the learning involved in this subtopic lies at synthesis level in Bloom's Taxonomy. However, getting a deeper understanding of why three abstract properties like reflexivity, symmetry and transitivity of a relation on a set induces a classification of elements on the underlying set, which can then be recognised as a partition, is at the very highest learning level of evaluation.

Once the ten subtopics in the list above are learnt, the student can start considering the learning outcome as a whole, as they will now know all the definitions of its ingredients. The learning outcome on relations is demanding a highest level (evaluation) level of learning about equivalence relations in 'simple cases'. That is, in simple cases students must be able to judge/assess/prove whether a given, previously unseen relation, is an equivalence relation or not. Finally, 'simple cases' in the context of this learning outcome is generally taken to mean relations defined on small, finite sets, or infinite sets of which the students have substantial knowledge, e.g. the set of integers.

3. The resulting learning object

The Horn of Odin is an adventure game set in the Viking Age. The setting was chosen as adventure games like the Valheim game set in fantasy worlds like this are popular these days, and the nine realms of Norse mythology also gave a suitable number of scenes in which to set a suitable number of mini-games covering all the subtopics identified when breaking down the learning outcome.

The players are presented with a narrative taking them through a series of connected tasks, where they learn the subtopics needed to master the learning outcome on relations on sets. The goal at the end is to reassemble a broken drinking horn in Valhalla for which they will be collecting runes throughout the story and throughout the realms of Norse mythology, meeting many of the colourful characters and creatures from Valhalla and beyond. The reward for completing most mini-games is a rune needed to reassemble the horn.

The game is a self-contained learning object teaching theory, while allowing exploration through practical exercises and integrated formative assessment. The tasks to be completed are in the form of individual mini-games teaching the mathematics both through the game structure and the game contents. We found some subtopics too complex to be fully illustrated by a single mini-game, so we ended up with 14 mini-games, designed to help the student appreciate all aspects of the 10 subtopics listed in the analysis of the learning outcome above. The player is engaging with the abstract mathematical concepts and problems through a trail of enjoyable puzzles and games that lead to the rewarding conclusion of reassembling the broken horn. The interested reader can find a short video presenting the HOD game here: https://youtu.be/BtmTclVhNzw.

Player communication is facilitated by access to The Crafty Fox tavern, where they can discuss game play, mathematics, hints and tips and also communicate with the game developers and the teacher. We chose to implement this using a dedicated Discord channel, but other communication tools are possible, and this is an area for future, further development.

In the implementation phase, attention was paid to using simple and intuitive game mechanics, so the player can focus on solving the mathematical problems. Further, artwork and story create an immersive experience encouraging the players to persevere with the tasks and creates an urge to see the end of the story, cf. Taralunga, Heidtmann and Krokos (2021).

Each scene in the story is made up of between one and three mini-games, a theory page and an assessment. The theory pages contain the mathematical theory needed to meet the learning outcome, but it is couched in examples fitting the environment in which the story is set. After reading the theory, the student can play the mini-game(s) and at the end of the mini-game there is either formative assessment inside the mini-game or on the website. Initially we had the assessments being served via the website, but we quickly realised that a more immersive experience was supported by having quizzes inside the mini-games, and rather than scoring points, players could gain lives or strength to slay nasty creatures or potions to get ahead quicker in the game.

4. Conclusions and future developments

The introduction of the prototype HOD game to the discrete mathematics course at Mid Sweden University has succeeded in improving significantly the attainment of students on the learning outcome on relations. In the latest version of HOD, new customised artwork has been incorporated, and we are working on incorporating competitive elements to motivate students further. More on statistics, testing and validation results, and future developments can be found in Taralunga, Heidtmann & Krokos (2021).

It is worth mentioning here that mathematical relations is a topic at Level 2 on the current SEFI mathematics curriculum framework of SEFI (2013). However, our methodology can be applied readily to subjects at Core Levels as well as Level 3 of the framework, and by mathematicians working together with e.g. engineers or physicists can be extended to create rich cross-disciplinary learning objects, where game elements are used to connect mathematics with practical engineering exercises both as part of game play and in the formative assessments.

Finally, while the desired collaborative aspect of learning is supported only by The Crafty Fox tavern in HOD, our latest venture fully embraces collaboration. The Loot of Loki game by Taralunga, Heidtmann & Krokos (2023) is a multiplayer serious game for two players together delving further into the transitivity concept. One player is playing the role of a mage and the other playing the role of a warrior. The two players must collaborate to compute a key and cannot succeed unless both master the transitivity concept. Players choose whether to play the role of mage or warrior and are then paired randomly by the game. It would be interesting in a future development to have a supporting AI component to do this pairing based on the profiles of players, so weaker players are paired with stronger players, for the weaker player to learn from the stronger and the stronger to improve their insight by coaching the weaker.

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The evolution of the Mathematics requirements for entry onto a level 8 Engineering degree in Ireland and how we can increase access to level 8 engineering.

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Abstract

Entry onto most level 8 engineering degrees in Ireland is restricted to students with a H4(60-70) or higher in leaving certificate mathematics. The current mathematics requirement for submission to a level 8 engineering degree in most universities in Ireland has evolved from a decision taken in 1969, based on the old honours and pass grading system. The evolution of this position is traced from 1969 until 2021. At each stage of the process, the emphasis has been on how best to approximate the 1969 position, with little or no reflection on whether this is the correct position or not. In this paper we look at the data of students who have entered level 8 engineering in TU Dublin, and argue that TU Dublin(and other Universities) should use an evidence based approach to open up entry to all students with a H5(50-60) or higher in leaving certificate mathematics. Such a decision would greatly increase the pool of available students for level 8 engineering, and would allow entry to level 8 to a large pool of students who are capable of completing a level 8 degree in engineering but are currently denied direct entry.

Introduction

Currently TU Dublin and many other Universities, provide a second chance exam for entry to level 8 engineering. The grade obtained in this exam if it is 60% or higher can be used instead of a H4 in Leaving certificate maths. In reality almost everyone with a H5 passes this exam and as do many people with a lower grade. This exam is restrictive as many people can not come due to geographical restrictions, or the fact that these exams are run in many Universities in the same week. This negates the power of a common entry system in the first place. In addition the data shows that students with a H5 are as equally likely to succeed as those who have a H4. Such an exam still has a place for a small number of students who may have had a bad day, have come from another educational system etc, but it doesn't make sense to retest students when our data suggests that students with a H5 are as equally likely to pass first year level 8 engineering as those with a H4.

In this paper we will review

1. The evolution of the different Leaving certificate grading systems from 1969 to the present day and how this has influenced the direct entry requirements for a level 8 engineering degree
2. The development and implementation of project maths at second level
3. The introduction of the bonus points initiative for higher level mathematics
4. An overview of the different pathways to a level 8 engineering degree in Ireland
5. The introduction of special mathematics exams by HEI's as an alternative to LC honours mathematics

6. We compare the success rates of students with a H5 grade with those with a H4 grade in a level 8 engineering degree and make a recommendation for the entry level requirements for a Level 8 degree in Engineering

Section 1: Leaving certificate grading system

History of LC Maths grade requirement for L8 Engineering

Table 1: Summary of leaving certificate grading systems at higher level from 1969 to present

<table>
<tr><td rowspan="2">1969 – 1991</td><td colspan="3">D</td><td colspan="3">C</td><td colspan="3">B</td><td colspan="2">A</td></tr>
<tr><td colspan="3">40-54</td><td colspan="3">55-69</td><td colspan="3">70-84</td><td colspan="2">85-100</td></tr>
<tr><td rowspan="2">1991 – 2016</td><td>D3</td><td>D2</td><td>D1</td><td>C3</td><td>C2</td><td>C1</td><td>B3</td><td>B2</td><td>B1</td><td>A2</td><td>A1</td></tr>
<tr><td>40-44</td><td>45-49</td><td>50-54</td><td>55-59</td><td>60-64</td><td>65-69</td><td>70-74</td><td>75-79</td><td>80-84</td><td>85-89</td><td>90-100</td></tr>
<tr><td rowspan="2">2017 – to date</td><td colspan="2">H6</td><td colspan="2">H5</td><td colspan="2">H4</td><td colspan="2">H3</td><td colspan="2">H2</td><td>H1</td></tr>
<tr><td colspan="2">40-49</td><td colspan="2">50-59</td><td colspan="2">60-69</td><td colspan="2">70-79</td><td colspan="2">80-89</td><td>90-100</td></tr>
</table>

Until 1969 students were just given an honour(60 or higher) or a pass(40-60). In 1969, the practice of awarding grades from A to F to students was introduced to replace the previous honours and pass distinction (Kellaghan 1984, Kellaghan and Hegarty 1984) (Coolahan 1981). At this point a C in higher level was designated as the necessary standard for an Honours degree in Engineering This C(55-70) was the closest approximation to what was the old Honour(60 or higher).

In 1992, a decision was taken to increase the number of grading bands used in the LC examination (see Table 1, above). (Mc Coy, Byrne et al.). This increase in bands took place ‘at the request of the higher education institutions’ (Mc Coy, Byrne et al.). This change to a 14-point grading scale for reporting Leaving Certificate results was made to improve the means by which the results of the examination could be used for selection for third-level education i.e. to have a more precise points system. There was no request for this change from the school system

According to (Mc Coy, Byrne et al.) little research regarding the reform was carried out prior to implementation and there were reports of an increase in the number of requests from LC candidates for remarking scripts. Concerns also emerged at this time regarding variation in grading practices across subjects (Kellaghan and Millar 2003).

In 2017, partly as a response to the number of regrading requests see table 1 above, there was a return to broader bands, with eight bands at both higher and ordinary level. The points levels were moved away from multiples of 5 to give points totals differentiated between students(O'Meara, Prendergast et al. 2020).

Table 2: Points awarded for grades achieved in Leaving Certificate examinations(2017 -)

Examination score	Higher Level		Ordinary Level	
	Grade	Points awarded	Grade	Points awarded
100% - 90%	H1	100	O1	56
89% - 80%	H2	88	O2	46
79% - 70%	H3	77	O3	37
69% - 60%	H4	66	O4	28
59% - 50%	H5	56	O5	20
49% - 40%	H6	46	O6	12
39% - 30%	H7	37	O7	0
29% - 0%	H8	0	O8	0

At this point in time a discontinuity appeared between the old grading system and new grading system, and it wasn't possible to map a C3(55-59) directly to a H5(50-59) or H4(60-69) grade.

Section 2 Introduction of project maths

It is well documented both in Ireland(Faulkner, Hannigan et al. 2010, Carr, Bowe et al. 2013, Marjoram, Robinson et al. 2013) and internationally(Edwards 1995, Lawson 2003, Carr, Fidalgo et al. 2015) that incoming STEM (Science, Technology, Engineering, Mathematics) students to courses at third level are mathematically underprepared Undergraduates struggle with basic mathematical skills, have poor understanding, and struggle to solve mathematical problems (Prendergast, Breen et al. 2018, Farrell and Carr 2019, Faulkner, Breen et al. 2020)

In 2005 the National Council for Curriculum and Assessment (NCCA) described mathematics teaching in Ireland as procedural, with a teacher centred approach dominating (Morgan & Morris, 2009). This resulted in superficial understanding (Prendergast & O'Donoghue, 2014) subsequently leading to poor performance at third level (Gill et al., 2010). With the introduction of 'Project Maths" significant changes have been made to the second level mathematics curriculum. These changes aimed to place greater emphasis on understanding of concepts, and to relate mathematics to real world scenarios with more use of applications (Prendergast et al., 2014). Project Maths also aimed to develop an increased focus on problem-solving skills and the alignment of assessment with the updated classroom practices. The goals of the reform are similar to the goals of the change movement led by the National Council of Teachers (NCTM) in the U.S (NCTM, 1991, 2000). Both call for more real-world problems and the use of instructional technology, with an increased emphasis on statistics and probability, algebra, and geometric reasoning.

The syllabi were reworked and separated into five main strands (Statistics and Probability, Geometry and Trigonometry, Number, Algebra, and Functions and Calculus). Project Maths began in 24 'initial' schools in 2008. Changes began to be introduced nationally on a phased basis in September 2010 with the first students sitting the Leaving certificate in 2012.

Section 3: The bonus points initiative

The benefits of studying advanced mathematics at secondary school are well known(O'Meara, Prendergast et al. 2020). Higher level mathematics is essential to develop a variety of skills necessary for a scientifically literate workforce(Chinnappan, Dinham et al. 2008). In Ireland, there is a particular higher Mathematics prerequisite, for entry onto many Level 8 STEM subjects and in particular for almost all Level 8 Engineering degrees. From 1999 to 2011 the number of students studying higher level mathematics in Ireland had seen a steady decline from 10,696(1999) to 8227(2011) (*https://www.education.ie/en/Publications/Statistics/Statistical-Reports/*). This is not a uniquely Irish problem. In Australia for example the main issue seems to have been the perceived effort to succeed and the total number of points available for higher level mathematics (Hine 2019). Many students who are well able for higher level mathematics strategically avoid it in order to maximise their score across several subjects (Kirkham, Chapman et al. 2019, O'Meara and Prendergast 2019). In 2012 the Irish government introduced a bonus points initiative, that increased the number of points available for doing higher level mathematics; in essence reweighting the higher mathematics subject to be more in line with the effort required for the subject.

As we can see from table 3 and figure 1 below there has been a steady and significant increase in the number of students studying higher level mathematics from 2011 through to 2019.

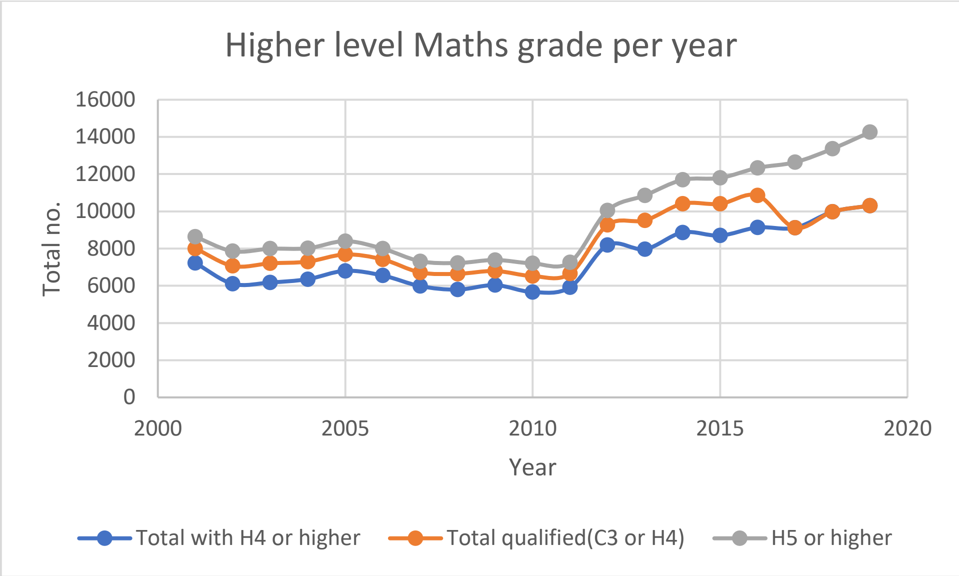

Figure 1: Number of Leaving certificate students with H4(), Qualified to enter Level 8 Engineering(Green), and H5(Red) maths grade. Bonus points were introduced in 2012 and the new grading system was introduced in 2017.

Table 3: Total number of students qualified for Level 8 Engineering

Year	H4 or higher	Total qualified	H5 or higher
2019	10309	10309	14256
2018	9978	9978	13364
2017	9115	9115	12639
2016	9134	10851	12337
2015	8698	10402	11793
2014	8865	10410	11692
2013	7970	9517	10860
2012	8186	9272	10044
2011	5923	6656	7267
2010	5662	6522	7203
2009	6036	6794	7387
2008	5793	6643	7223
2007	5982	6718	7320
2006	6552	7416	7993
2005	6804	7683	8400
2004	6356	7297	8014
2003	6178	7205	7991
2002	6106	7065	7857
2001	7223	8006	8632

From 2011 the number of students with a C3(55) in Maths (6656) has increased to between H4 (60) with 10309 students and with a H5(50) 14256. When we extrapolate using previous data this equates to about 12,100 students with a C3

Section 4: Current alternative entry pathways to a Level 8 Engineering programme in Ireland

In Ireland a variety of pathways exist to graduate with a level 8 Engineering degree without the necessary honours maths requirement from direct entry onto a level 8 degree. These consist of four main approaches:

Entry via an Ordinary degree (level 7) :Students complete an Ordinary Engineering degree (normally three years) and then, provided their grades are sufficiently high, they can enter directly into third year of an Honours programme and complete the final two years to attain their degree(3+ 2).

Foundation year: Students may enter via "foundation" year and progression being dependent again on sufficiently high grades(4+1) . Those who are not successful and may transfer to year 2 of an ordinary degree

Mixed approach In addition to this an alternative approach is used in TU Dublin(Tallaght campus, formerly IT Tallaght) where a mixed approach is used and all students enter the same degree with some exiting after 3 years with a level 7 degree and others (depending on grades) having the option to stay for 4 years and graduate with a level 8 degree.

Post Leaving certificate course :Students may do a post leaving certificate course in a further education course and on successful completion of this and if they have completed the Maths for STEM module(with a grade > 60) (Carr, Robinson et al. , Robinson, Breen et al.) they can enter directly into year 1 of the 4 year degree

Comparing the Irish situation with other countries

Internationally there exist a wide variety of routes into a level 8 degree in Engineering, with huge variety within and between systems. For a full review see Ní Fhloinn & Carr, 2010 (Ní Fhloinn and Carr 2010)

Section 5: The introduction of special maths exams

Recently another route has emerged with many third-level institutions offering their own mathematics exam(almost a mini matriculation) to students who have failed to reach the minimum mathematics requirement of a H4 (previously a C3) or higher for Honours Engineering programmes. Some of these HEI's impose a lower threshold that students must have achieved before they can take the supplementary examination; others do not. (Ní Fhloinn and Carr 2010)

Section 6: Results

An analysis was carried out on the 2018/19 class in the first year common engineering programme. The success rate for each grading band is shown below in table 3. Those with a H3 or higher were significantly more likely to pass first year. Those with a H4(82%) were not significantly different from those with a H5(85%) and even those with a H6/7 grade had a 75% pass rate. Those shows that we need to re-examine the need for a H4 in Higher level mathematics, for direct entry onto a level 8 degree. This could be reduced to a H5.

Table 3: Data from 2018/19, the number of students within each Leaving Cert band and their success rate in first year.

Grade	Number	Pass		Fail	
H1	1	1	100%	0	0%
H2	11	10	91%	1	9%
H3	23	22	95%	1	5%
H4	41	34	82%	7	17%
H5	26	22	85%	4	15%
H6	7	5	71%	2	29%
H7	1	1	100%	0	0%

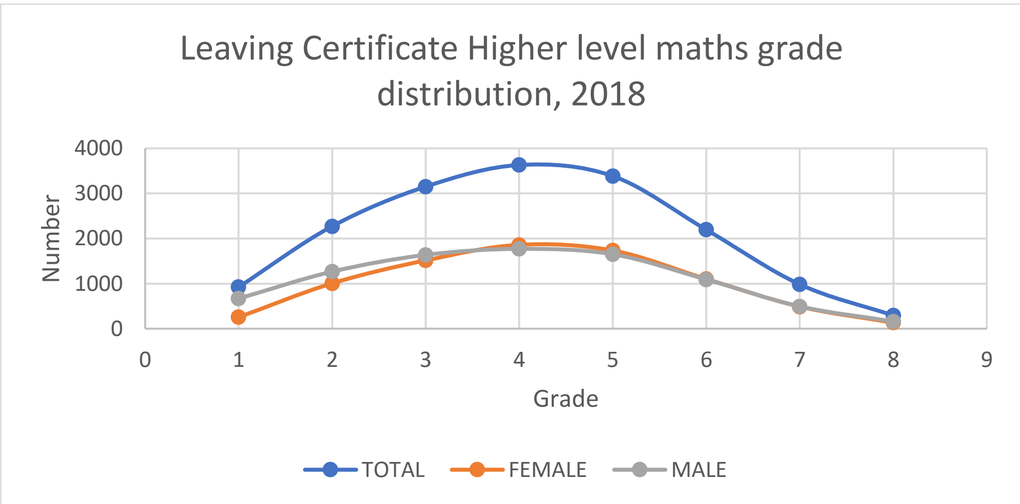

Figure 2: Leaving certificate grade distribution for 2018

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Competence-related learning opportunities afforded by a student-built measurement device – A case study

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Abstract

The concept of mathematical competence as used in the SEFI MSIG curriculum document (Alpers et al. 2013) stresses the ability to understand and use mathematics in relevant situations and contexts. In order to foster the acquisition of this ability, respective learning opportunities have to be designed. This contribution describes how the design, realization and usage of a measurement device in student projects afford such learning opportunities. It outlines the educational settings and identifies potentially acquired mathematical competencies. Moreover, experience with student groups working with the device is reported and discussed.

Introduction

In the curriculum document of SEFI's Maths Special Interest Group (Alpers et al. 2013) the concept of mathematical competence and competencies was adopted for specifying goals of the mathematical education of engineers (Niss and Hojgaard 2019). The concept emphasizes the ability to do, use and understand mathematical concepts in relevant situations and contexts. For making more detailed specifications eight mathematical competencies were identified like mathematical reasoning, problem solving and modelling. If one wants students to acquire the respective goals one has to offer suitable learning opportunities. Besides the classical offerings like lectures and assignments, projects seem to be particular effective for this purpose (Alpers 2020). In this contribution, I report on such project work which was concerned with designing, building and using a measurement device. I state the learning opportunities offered within this environment by identifying subtasks and corresponding mathematical competencies which can be acquired by students. First, I outline the educational setting within which the projects are placed. Then, the learning opportunities coming up when designing, realizing and using the measurement device are identified. Finally, experiences with students are reported and discussed in light of competence acquisition.

Educational settings

In the mathematical education of mechanical engineering students at Aalen University, students get familiarity with the basic mathematical concepts in lectures and assignment sessions during the courses Mathematics I and II. During this rather classical setting, many references are made to the meaning and usage of mathematical concepts in application subjects, particularly in the courses on engineering mechanics running in parallel. Moreover, there are obligatory small group projects related to mechanics (designing and computing the forces in trusses, designing motion functions for special purposes) which put more emphasis on the links between mathematical and mechanical properties of objects like systems of equations and functions. Students create Matlab© Live-Scripts where the computations are performed using Matlab commands. The mathematical education is completed by a course Mathematics III on numerical algorithms for standard tasks (in theory and with Matlab© commands) where students afterwards have to work more intensely on a mathematical application project (2 Credit Points (CP)) (for more information on the concept of these projects, see Alpers 2002). Moreover, in semester 6 the curriculum requires students to work on a larger individual or group project (5 CP) from an area of mechanical engineering which can also have a larger mathematical component. The measurement devices I report on in this contribution were designed and built by student groups as project work in semester 6 because the design, construction, production and ordering of parts is a major effort worth 5 CP. The devices were then used in mathematical application projects in Mathematics III for performing real measurements and solving mathematical tasks related to the generated data.

Measurement devices – learning opportunities in design and realization

In a project for mechanical engineering students during their 6^{th} semester the task consisted of designing and realizing a simple measurement device by which one can measure points within a cuboid with given side length. The main challenge is to set up a construction which enables the user to reach all points from (potentially) arbitrary sides (e.g. from above or from below) and to make it easily usable. It was clear from looking at professional measurement devices that the construction needs translational and/or rotational joints for achieving this. Figure 1 (left part) depicts the result of the first project group working on this task. They used one translational joint for moving the index arm horizontally and four rotational joints including digital angle measurement devices. This construction provides a good reachability of the upper surface and the sides of an object and for smaller objects even of the lower surface. The angles can be easily read off from the digital displays. The drawback was its tedious handling: The rotational joints had a certain slackness and when you want to move the measuring tip to the point to be measured you have to loosen the rotational joints, rotate them and then fix them again.

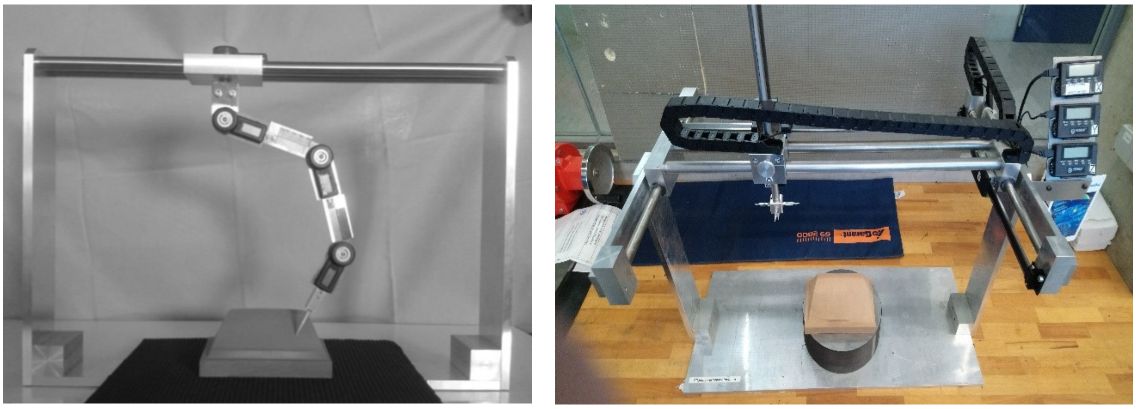

Figure 1: Measurement devices

The following sub-tasks offer competence-oriented learning opportunities, i.e. they require the students to make use of mathematical concepts introduced before in order to solve some practical problems:

- Dimension the links in order to reach points given an object of certain size: This requires geometrical thinking which can be performed via drawing a sketch or with technological support by using a CAD programme where a certain configuration can be constructed in order to test reachability.

- Compute the Cartesian coordinates from the measured angles and horizontal movement (using trigonometry or rotational matrices). This requires the students to go through the configuration and set up equations by which the position can be computed step by step. The reverse computation might also be interesting in order to get a rough idea of how to set up the angles in the rotational joints in order to reach a certain point. Here, the same system of equations might be used but solved for different unknowns and questions of uniqueness come up.
- Investigate how the resulting coordinates depend on the exactness of angle measurement (error propagation; which joints are the most sensitive ones?).

Another project group one year later had the task to modify the measurement device for improving the handling. Since they did not see a chance to improve the setting of angles, they went for a different approach using just translational joints (portal measurement device) as shown in Figure 1 (right side). The drawback of this approach comes from the reduced reachability. This was partially remedied by using a cube with five tips.

The following sub-tasks offer competence-oriented learning opportunities:

- Enable the measurement device to reach points at the sides of objects.
- Make the measurement result independent of the tip used (the project group came up with an approach where all values were recalculated to a reference point) and write a Matlab© script for this such that the user does not have to deal with it.

Measurement devices – learning opportunities in use

There are two types of application-oriented tasks where students in the 3rd semester make use of the measurement device (for some years only the second version is in use):

- Reverse engineering: Here, a machine part is given like a blade of a rotator and the task consist of setting up a digital wire model in Matlab® and a volume model in a CAD programme and to produce a replica. Mathematically, the task consists of reproducing so-called "freeform geometries" using the spline concept and different kinds of splines.
- Investigation of the measurement device itself regarding exactness: How exact can simple geometric objects like lines, planes, circles, balls and cylinders be measured using the machine? Mathematically, this requires to work with elementary geometrical objects formerly defined and investigated in vector algebra as well as to perform model fitting using least-squares methods in non-trivial ways where orthogonal distances are used (as will be discussed below).

In both types of tasks students have to think first about suitable data capture. Since the measurement of a point takes some time students have to argue and decide where to take measurements. The usual line of argumentation used by students is that where curvature is high more points should be measured in order to get a good approximation. Moreover, when students are to create wire models in Matlab©, they have to choose the points in a proper way. With both measurement devices students have to decide how to approach the points. Here, a practical argument that comes from really performing the measurement is that if the tip arm is nearly orthogonal to the surface the measurement has the least error.

The following learning opportunities regarding the mathematical competencies listed in (Alpers et al. 2013) can be identified:

Thinking mathematically: In both tasks mentioned above students experience mathematics as a provider of models to describe an object and to judge on the suitability of an approximate description. The tasks are not posed in a purely mathematical setting but mathematics comes into play in the process of solving the tasks.

Reasoning mathematically: In the reverse engineering task students have to reason about the type of spline they use (e.g. not-a-knot or periodic spline) and the suitable kind of parameterization (equidistant or chord-length). There is more reasoning involved in the model fitting task: students have to argue about the reduction of design parameters for making model fitting easier which will be pointed out in more detail below when discussing representations; students also have to reason about finding suitable initial values for the design parameters such that non-linear least-squares solvers (see the last competency) have a higher probability of finding good approximations.

Problem solving: There is less problem solving involved in reverse engineering since students can apply the concepts and examples treated in earlier lectures; some reasoning on the suitable choice of parameterization and spline type is required as already described above. The model fitting tasks are more challenging since during the lecture only the fitting of functions (polynomials but also functions that are non-linear in the design parameters) is dealt with. Students have to apply the method to fitting geometric models to data points using orthogonal distances. They have to retrieve their knowledge on vector representations of geometric objects like lines, planes and balls as well as on formulae for calculating the distance of a point to such an object. They set up the sum of squared distances and get an expression in the design parameters. For using an available routine in Matlab© ("lsqnonlin") they have to provide a function of the vector of design variables which provides a vector of (signed) distances to the given points.

Mathematical modelling: The tasks require the creation of geometrical models which have been treated before. Students have to judge on the quality of model fitting: In reverse engineering this might simply be a geometric judgment on the "smoothness" of the model whereas in model fitting the sum of squares (or the average) provides a numerical measure.

Representing mathematical entities: In the reverse engineering tasks, students experience how freeform geometries can be represented mathematically extending their repertoire of just a few geometric objects they could represent before. In model fitting students use the vector representations they learnt before, e.g. for a line the parametric representation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \mu \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$ where there are at first sight 6 design parameters $x_0, y_0, z_0, a_x, a_y, a_z$ but the representation is not unique. So, the question comes up how the representation can be made unique by reducing the number of design parameters. For lines that are not parallel to the (x,y)-plane, z_0 can be chosen as 0 and a_z as 1. For lines parallel to the (x,y)-plane, a_z can be chosen as 0 and a_y as $\sqrt{1-a_x^2}$. For other objects similar questions and considerations regarding the uniqueness of representations come up.

Handling mathematical symbols and formalism: In the reverse engineering task students have to compute the parameters in chord-length parameterizations and for doing this in Matlab© they have to work with the respective symbolic expressions but the students have already encountered examples in the previous lectures. Model fitting tasks are more challenging in that students have to use the symbolic representations and formulae for geometric objects and distances from former mathematics lectures and calculate the sum of squares. Moreover, they have to reduce the number of design parameters in the symbolic representation. They also have to provide the input for the least-squares solver in the formal way required by the Matlab© function.

Communicating mathematically: In both tasks, students have to argue and justify their decisions orally during group discussions, discussions with the lecturer and in the project presentation. They also have to do this in written form in the project report.

Using aids and tools: In both tasks, students learn to use Matlab© as a computational tool which is also needed in several application subjects (e.g. control theory); they learn to work with more complicated functions like those for creating freeform geometries which provide a data structure from which a function can be built. They need to look up the documentation of the tool particularly for using the non-linear least-squares method ("lsqnonlin") where it is challenging to understand the input to be given to the function and the output provided. For producing the graphical representations, the students also have to work more deeply with the plotting facilities offered by Matlab©.

Experience and discussion

In the 6th semester projects where the devices were designed and built, the student groups were very engaged which is not surprising since they selected the projects themselves. There was intensive discussion within the groups and with me as their "customer". Geometric thinking was one of the essential constituents of project work, so students experienced mathematics as something deeply embedded in engineering work which has also many other important aspects, though. This might lead to an attitude of "sceptical reverence" as recommended by Gainsburg (2007).

In the 3rd semester projects which were more guided, students had to create their data themselves and thus experienced the inaccuracy of data directly. I gave them a short introduction to taking measurements and then sometimes they did the measurements a few times when they recognized that there were mistakes or the measured points were chosen unsuitably. When working with the data, students made use of concepts from vector algebra, interpolation and approximation. They experienced these concepts as being useful in working on practical tasks in mechanical engineering and thus enriched their knowledge and possibly changed their attitude towards mathematics. Students use "simple" mathematics from vector algebra but have to think about it more deeply (e.g. on representation uniqueness) and apply them in more challenging situations; they experience that concepts learnt earlier can really show up and be of good use which might increase their readiness to look for applicability of things they learnt in mathematics. There were many opportunities for discussion when students got stuck, e.g. when transferring the least-squares method from functions to geometric objects or when using more advanced Matlab© functions.

Compared to the experience gained by the more traditional educational setting in mathematics I and II, one could say that during the latter classes they got acquainted with mathematical coherence with references to application scenarios; whereas in the mathematics III projects, the application problem coherence is emphasised with references to mathematics. The use of devices like the measurement one is by no means restricted to the educational setting outlined above. It might also serve as one component in a mathematical modelling course like the one described by Wedelin et al. (2015).

In the previous section I listed the learning opportunities for acquiring mathematical competencies. One should realize, though, that learning in those projects is exemplary learning and a transfer to other situations and contexts is by no means trivial. Mathematical competencies are stabilized and extended by using mathematics in application subjects and later in professional life as was outlined by Gainsburg (2013) for the modelling competency. It would be an illusion to expect the competencies to be fully developed during mathematics education.

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Individual Digital Learning Recommendations for Pre-Courses in Mathematics

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Abstract

Prior school knowledge in the STEM subjects, especially mathematics, is not sufficient for many degree programmes to ensure a good entry. In order to support the preparation as well as possible, individually and specific to the degree programme, a blended learning concept is used, which is appropriately supported by an online learning environment. The students and teachers benefit from the digital diagnostic knowledge tests which are specific to the degree programme and the topic-specific learning recommendations. On the one hand, the pre-course concept and the digital learning environment with learning videos, many interactive exercises and knowledge tests that was used is presented. On the other hand, the results of knowledge tests and the corresponding learning recommendations before and after the pre-course are shown, as well as a comparison between different study programmes in electrical engineering from several semesters.

Introduction

In many degree programs, prior knowledge of mathematics is required in order to start studying successfully. To refresh the previous knowledge, the universities offer mathematics pre-courses, which are usually held 2 to 4 weeks before the semester begins. Often only a proportion of students takes part in these preliminary courses. In particular, those who need support often do not attend the pre-course for various reasons. It is important to reach these students, motivate them to attend in the pre-course and offer them individual learning opportunities. With the help of the knowledge tests before and after the preliminary course, the effectiveness of the pre-courses of the electrical engineering degree programs at Hamburg University of Applied Sciences is evaluated and assessed.

Concept of Blended Learning in Preparation Courses

The preliminary courses at the Department of Information and Electrical Engineering at the Hamburg University of Applied Sciences are conducted according to the blended learning concept. For each day of the 12-day preliminary course, there is a preparatory online self-learning component, as well as a component for joint learning with other students and a tutor. At the beginning of the pre-course, students take a digital knowledge test, usually in person at the university's PCs. The knowledge test for electrical engineering students includes 10 mathematics topics with 5 questions each and an automatic evaluation. The overall result of the test as well as the results for the individual mathematics topics with the resulting individual learning recommendations give the student direct feedback on their own level of knowledge. Furthermore, these results provide the tutor with information on the performance level of the pre-course group.

The online learning environment viaMINT (Landenfeld et al, 2016) is used for the self-learning part and the online knowledge tests. viaMINT provides video-based learning modules with many interactive exercises and immediate feedback appropriate to the mathematics topics (Göbbels et al, 2016). Furthermore, the learning environment offers the possibility to select a study programme, so that the student receives information about which mathematics topics are relevant for his or her study programme. In addition, the course-specific knowledge test and the appropriate learning modules are provided.At the end of the preliminary course, the students take the knowledge test again so that they can recognise their individual learning progress.

Overall Test and Individual Recommendations

This section shows the results of the knowledge test at the beginning of the pre-course in mathematics in the winter semester 2022/23. The students participating in the pre-course took it on the first day of the pre-course in a protected environment in the PC pool at the university. Figure 1 shows the overall results of the test. It shows the percentage of correct answers and how many students scored in each area. It can be seen that about 2/3 of the students (40 out of 59 students) answered less than 50% correctly. So there is a great need for a refresher course in mathematics.

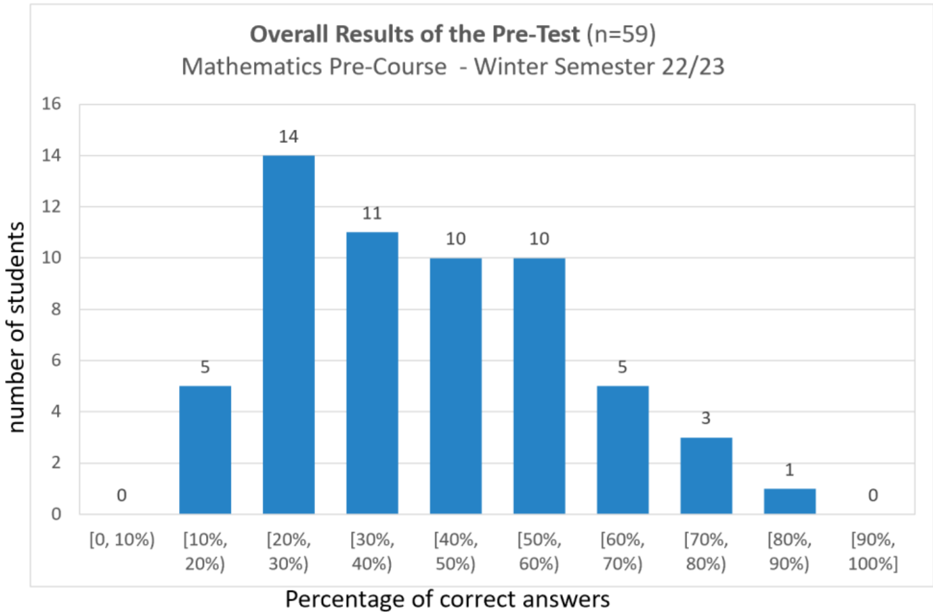

Figure 1. Overall results of the knowledge test at the beginning of the pre-course

Figure 2 shows the learning recommendations to the students at the beginning of the pre-course, summed up on the basis of the knowledge test per mathematics topic. Learning recommendations for a mathematics topic are set according to the following criteria: strongly recommended if less than 40% of the answers are correct, recommended if 40% to 70% of the answers are correct, and sufficient knowledge if at least 70% of the answers are correct. In 5 out of the 10 topics more than half of the students received a strong learning recommendation. The 3 topics that were solved the weakest were: Equations and Inequalities, Vectors and Logarithms. The topic Functions 1 (linear functions, quadratic polynomials) received the fewest learning recommendations, i.e. the students' prior knowledge is best here.

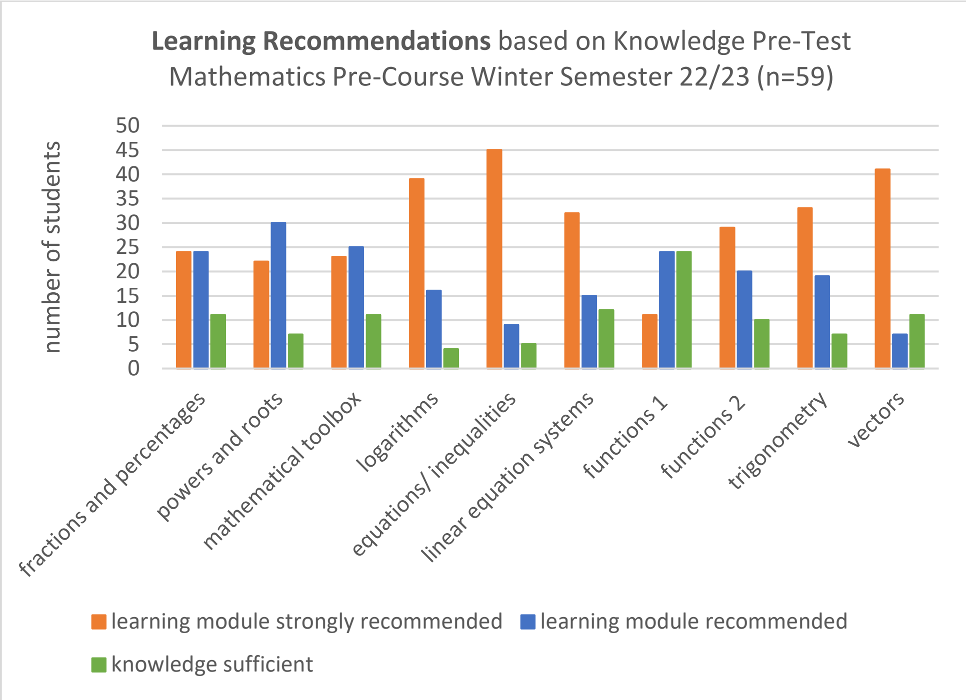

Figure 2. Learning recommendations for the students based on the knowledge test

Figure 3 shows, as an example, three of the five questions in the topic block Powers and Roots. The questions in the knowledge test are implemented with Moodle and the question type STACK (Sangwin, 2013), which supports randomisation so that the questions in the test contain different numerical values when they are called up several times. The questions are based on the tasks recommended in the COSH catalogue (COSH, 2021) for a transition from school to higher education.

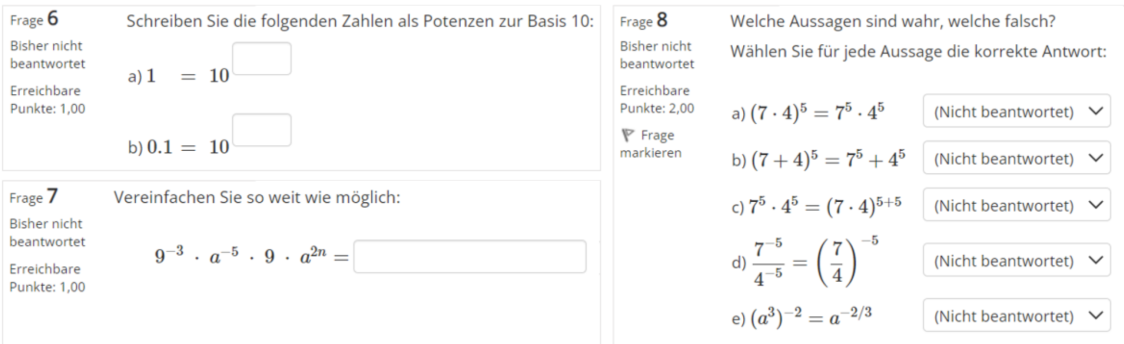

Figure 3. Examples of questions of the topic Powers and Roots.

Knowledge Tests Before and After the Pre-course

Pre-courses in mathematics are necessary to refresh previous knowledge of mathematics and thus facilitate the start of studies. The length of a preliminary course varies between 2 and 6 weeks depending on the university and the degree programme. With a preliminary course, a certain amount of missing or forgotten prior knowledge can be learned or reactivated, but a complete acquisition of the necessary knowledge is not possible due to the short time. Figure 4 shows the results of a knowledge test at the beginning (Pre-Test) and end (Post-Test) of the 12-days preliminary course in mathematics in the Department of Information and Electrical Engineering at the Hamburg University of Applied Sciences in the winter semester 2022/23. The increase in learning is clearly visible in the figures. It must be mentioned here that at the end of the preliminary course only 43 students took the knowledge test, which means that about 25% of the students did not complete the preliminary course.

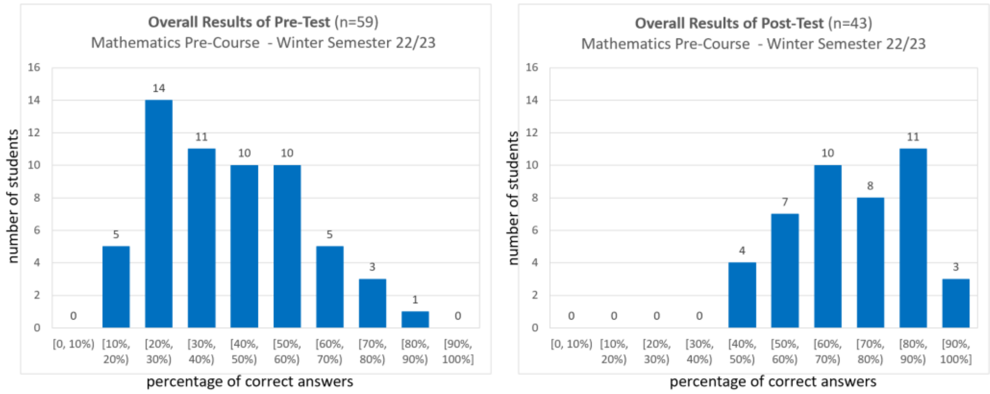

Figure 4. Comparison of overall results of Pre- and Post-Test in winter semester 22/23.

Figure 5 shows the improved knowledge on the results of the questions of the learning module Trigonometry. Based on these detailed results, it is also possible for the teacher to see what has not yet been understood so well, e.g., the answer to question 41 is only 44% correct. On this basis, the teacher can adapt his/her teaching concept or alternatively check the question for fit.

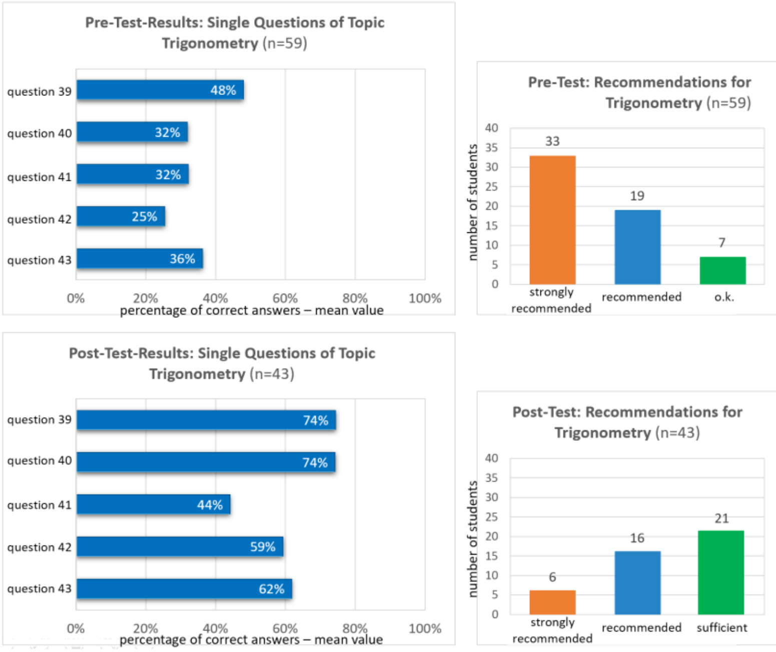

Figure 5. Comparison of the results of Pre- and Post-Test for the topic Trigonometry in the pre-test and in the post-test of the mathematics pre-course in winter semester 2022/23.

Comparison of Knowledge Tests for Different Degree Programmes

The mathematics pre-course in the Department of Information and Electrical Engineering is cross-curricular and is also attended by students from other degree programmes. An evaluation of the knowledge tests before and after the preliminary course, taking into account the degree programme, yields the results shown in Figure 6. The mean values show clear differences in the mathematical entry level of students from different degree programmes. However, it can be seen that all student groups achieve a similar increased knowledge. The mean values of the post-test are between 27.3% and 32.4 % higher than in the pre-test. However, there still remains a large knowledge gap, so that a 12-day pre-course can be considered insufficient to acquire all missing prior knowledge.

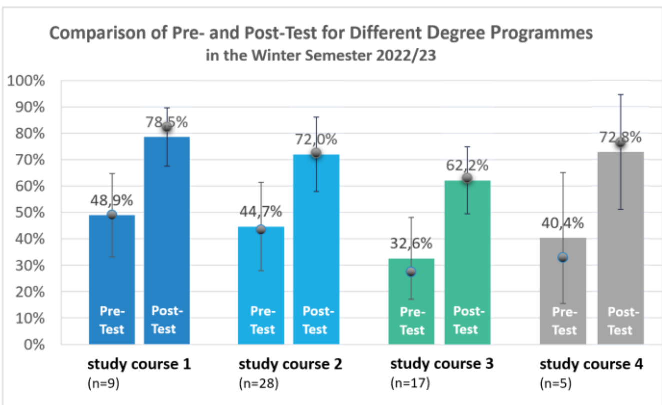

Figure 6. Comparison of knowledge tests for different degree programmes.

Comparison of Knowledge Tests for Different Semesters

The comparison of the knowledge tests over several semesters (see Figure 7) shows a similar behaviour to the comparison for several degree programmes in Figure 6. The percentage mean of the correct answers increases by %-values between 23.1 - 29.6. The values in the summer semester are all slightly higher than the values in the winter semester. This can be explained by the fact that in the summer semester, a disproportionately large number of students start their studies in cooperation with companies and are well prepared.

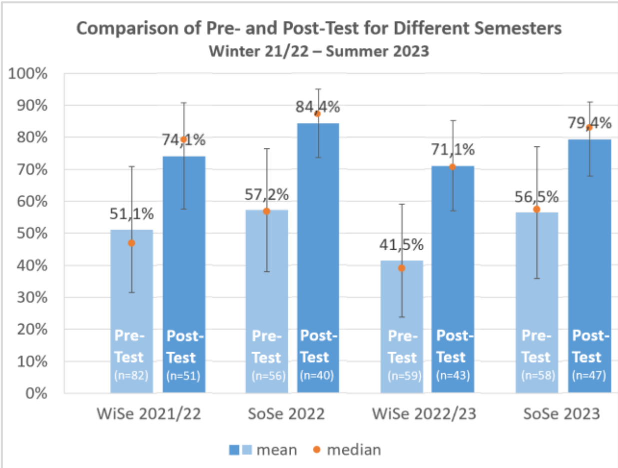

Figure 7. Comparison of knowledge tests for four semesters.

Conclusions

The results of this study show that pre-courses in mathematics are very important, as they can improve prior knowledge and thus facilitate the start of studies. However, a 12-days preliminary course is not sufficient to fully understand and consolidate the mathematical learning content considered necessary for a successful study entry. A knowledge test with differentiated learning recommendations helps students to identify their knowledge in the individual mathematical subject areas. The support of the mathematics pre-courses by an accompanying online learning environment is very helpful as it provides individual learning opportunities. In particular, the possibility to select the course of study in the learning platform, which provides information about the required prior knowledge, promotes acceptance and the need to recognise the required refreshment of prior mathematical knowledge.

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Enhancing Multivariate Calculus Learning using MATLAB

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Abstract

This work illustrates the preliminary results obtained using MATLAB in the Multivariate Calculus module of the first-year Analysis course of the Industrial and Management Engineering Bachelor's degree at LIUC. In the pilot study, MATLAB was made available to a subset of the class during extra-class time, and a questionnaire was administered to this sample of students to gather feedback on their experience. The results were analysed to compare the performance of students who used MATLAB with that of the rest of the class, identifying the strengths and weaknesses of this approach, and suggesting potential future improvements.

Introduction

Mathematical software is considered a powerful tool for supporting the teaching and learning of mathematics, Höltta and Hyötyniemi (2003). The use of toolboxes and plots allows students to visualize mathematical concepts. Indeed, "visualization is a technique to render abstract mathematical concept concrete and through this concrete representation, students can better understand abstract concept", Milenković (2022). Multivariate calculus, which deals with elements living in the plane and space, such as level curves and gradient flows, is one of the mathematical topics that benefit greatly from visualization. While students may rely on illustrations in textbooks, this approach can lead to a superficial understanding of the material, Lipsman and Rosenberg (2017). To promote a more active learning experience, students can draw, manipulate, and analyse the geometric shapes of multivariate calculus using software tools, Freeman (2014).

This work aims at illustrating the actual state of the art to investigate which kind of approaches are used when introducing mathematical software in traditional math courses to support learning and to understand what could be the best way to design this kind of course. Despite the benefits of using software, teachers face challenges in designing and implementing these approaches. In first-year engineering courses, indeed, classes can consist of hundreds of students, making it difficult to guide all of them in learning and using a software. Challenges also arise in terms of coding skills prerequisites and available time for coding activities. To address these challenges, this work illustrates some preliminary results using MATLAB, Mathworks (2022), in the multivariate calculus module of the first-year Analysis course of the Industrial and Management Engineering Bachelor's degree at LIUC. In this case, the opportunity to use MATLAB was offered to only a subset of the class during extra-class time. A questionnaire was administered to this sample of students, and the results were analysed. A final discussion is presented comparing the performance of students who used MATLAB with that of the rest of the class, to identify the positive and negative aspects of this approach. Indeed, a potential risk is that this strategy may create a division in the class group, as those participating in the extra-curricular activity may be more motivated to study, while the rest of the class may not be participating despite potentially needing the learning approach more and expected to benefit more from it.

State of the Art

A literature research has been conducting with the idea of investigating which could be a proper way to design a math course where a software is added to support traditional learning. To properly organize the literature research, a three- step methodology is adopted. Scopus is the selected database, while keywords used in the searching phase are related to the mathematical subject *("Multivariate Calculus Learning" OR "Multivariate Calculus" OR "multivariate mathematical analysis" OR "mathematical learning")* AND to the software used *("MATLAB" OR "mathematical software" OR "software" OR "visualization")* AND to the context addressed *("engineering" OR "engineering courses")*. The resulting 13 documents have been screened on the basis of some eligibility criterion: only English-written articles (no restriction on publication year) and only single paper (no collection of papers). Abstracts have been read for checking the papers consistency with respect to the research subject. The resulting 5 documents have been categorized according to four dimensions: subject of learning, used software, educational level and the learning strategy. The results are summarized in Table 1.

Table 1. Literature research findings.

Reference	Subject of learning	Used software	Educational level	Learning strategy
Milenković et al. (2022)	Multidimensional calculus with specific focus on double integrals	Wolfram Mathematica	University students.	Class were divided in two groups: the experimental and the control one.
Gutiérrez et al. (2019)	Differential equations and linear algebra	-	University students.	Development of physical experiments to be offered in extra-class activities.
MD-Ali et al. (2018)	Shape and Space geometry	GeoGebra	Grade 8 students (14 years old)	Students were divided into three groups (Experimental Group 1, Experimental Group 2 and Control Group).
Kurokawa et al. (2017)	Symbolic and graphical representations	Visual Basic 2010	University students	Ad-hoc experiments to confirm the effectiveness of system functions.
Su et al. (2007)	Differentiation and limits	Mathematica	Undergraduates students	Class were divided in two groups: the experimental and the control one.

According to the literature, geometry and multidimensional calculus are the fields that require special attention. As Milenković (2022) noted, "learning and understanding multivariate calculus causes a lot of difficulties". To address this issue, software tools can be used to facilitate geometric and visualization learning by creating images that help students gain a better understanding of these complex concepts, MD-Ali (2018).

The availability of software that could be used for enhancing mathematical learning is really high. None of the analysed paper is referring to use MATLAB. The choice of the software is usually motivated on the basis of some criterion, such as the easy to use and the possibility to be used for free. For example, GeoGebra has been specifically designed for its use in mathematics education in higher educational institutions, Hohenwarter (2004) and it could be used also online, Zakaria (2012).

Some of the papers that have been results from the Scopus search refer projects developed during high school or targeted towards students with specific needs, such as those with dyslexia. We have excluded these documents from our analysis as they are not relevant to our research on engineering education. However, this leads to observe that only a few works are specifically addressed to engineering students.

Many papers cite experiments that have been designed to demonstrate the effectiveness of the proposed learning strategy. Typically, students are divided in two or more groups based on the type of learning they have received. This choice allows to apply statistical analyses like ANOVA or MANOVA. In Gutiérrez et al. (2019) the learning strategy comprises the development of four physical experiments and it is offered to all the students, but as extracurricular activity.

In conclusion, the importance of having software to visualize mathematical concepts, explore knowledge, and solve problems has been well established in the literature. However, while there is a significant body of literature focused on proving the effectiveness of various learning approaches, there are few resources dedicated to the design of the learning process. MD-Ali (2018) described a study where students were divided into three groups, two of which were given treatment using GeoGebra's dynamic software. However, it should be noted that each group only consisted of a small number of students (six per group), and the author did not provide details on how the lectures were organized. Milenković (2022) divided students in two groups, the control group and the experimental one. Around 50 students belong to each group, but all the material given in the experimental group has been prepared by the teacher, so students are not required to use directly as programmers the Wolfram Mathematica software. Therefore, it results difficult to use these cases as benchmark for our learning process design.

Design of the Multivariate Calculus MATLAB Lab

As we did not find any benchmark for the design of a learning process aimed at introducing mathematical software in the Multivariate Calculus courses, we have proceeded considering certain parameters, including the number of students, available time, and necessary prerequisites. With regards to the number of students, our first-year engineering course comprises about 80 students, making it challenging to guide them all in learning and using a new software. The lecture time is already constrained and may not be sufficient for explaining the course curriculum, let alone introducing the use of MATLAB to the entire class. While students can have asynchronous sessions to learn the software using the MATLAB Onramp platform (2022), this approach requires full class involvement, which may be difficult to achieve, particularly with first-year students who may still be developing their autonomy.

Considering these reasons, two possible solutions seem actionable. The first is to restrict the usage of the software to the teacher, meaning that students can only view on the screen what is implemented on the software by the teacher, as done by Milenković (2022). This approach would limit students to a passive role, as they would not have the opportunity to directly interact with the software environment. However, this solution could help overcome constraints related to limited available time and necessary prerequisites. The second solution is to offer the opportunity to a limited number of students who voluntarily wish to participate in an extra-curricular Lab dedicated to learning and using the software within the context of Multivariate Calculus. The Lab would be organized outside regular lesson hours. While this approach could help overcome the constraints previously described, there is an inherent drawback. Namely, this approach may create a division within the class, with students participating in the extra-curricular activity potentially being more motivated to study while the rest of the class may not participate, despite potentially needing the learning approach more and expected to benefit more from it. Aware of this risk, we decided to choose the second solution.

Another important aspect to consider is the choice of the software used in the Lab. Despite being a pay-for-license software, we have opted from the very beginning to use MATLAB for some reasons. Firstly, it has been specifically designed for engineers and scientists who require a programming language that enables them to directly express matrices and arrays, which are the fundamental numerical objects for numerically solving analytical expressions. Furthermore, in recent years, MATLAB has evolved into a comprehensive environment which includes toolboxes and apps that can be easily used even by those who are not proficient in programming. These features make MATLAB an ideal choice for our purposes.

To start with a pilot case, a group of voluntary students has been enrolled for participating at the MATLAB Lab about Multivariate Calculus, during the Analysis course of the 2021/2022 academic year. The learning process has been designed using as reference the text book written by Lipsman (2017) and it has been completely integrated in the Moodle page of the Analysis Course, available for the students on the Liuc Moodle platform, LIUC Ecorsi (2023). Students have been requested to study in asynchronous way an introductory part, available on the Mathworks website, named MATLAB Onramp (2022). This module is dedicated to autonomously learn the basics of MATLAB through a tutorial on commonly used features and workflows. The tutorial is divided in 13 modules. Not all the modules are really necessary to face then the Multivariate Calculus part, but the students are let be free to learn according also to their personal interests.

The Lab is organized in five lectures of 2 hours. Topics discussed are related to: visualization of functions in two variables, computation and representation of level curves, computation and representation of the gradient of a function, computation of critical points, optimization in two variables, constrained optimization and Lagrange multipliers. The initial part of each lecture is dedicated to introduce a specific argument and to discuss how to implement the solution of problems associated with it in MATLAB. An editor file is provided in advance to the students, in order to let them to follow the explanation directly on their laptop, trying to run functions and having the possibility of modifying pieces of code and prove to implement alternatively solutions. A second part of the lecture is dedicated to the group work. Students could assess their knowledge, both in terms of Multivariate Calculus concepts and MATLAB skills, trying to solve some new problems. Students are requested to implement and assess the goodness of their solutions directly in the Moodle page of the Analysis course. Indeed, thanks to the integration between Moodle and MATLAB Grader (2022) it is possible to have a unique environment where students can solve the problems and the teacher can immediately see students' results and the main problems or challenges they have encountered in carrying out the solution. Furthermore, the teacher has sufficient time to move around the classroom and to provide personalized answers to specific questions while guiding discussions about issues or doubts that arise during the problem-solving group work.

Findings

A total of 26 students did the registration to participate at the MATLAB Lab. To provide a quantitative analysis of what has been reached in term of achievement of the expected learning goals, we have monitored the results during the final test about the Multivariate Calculus module, as reported in Table 2.

Table 2. Participants and results of the Multivariate Calculus Test.

	# of students taking the final test	% of success	Average grade
Participating at the LAB	26 (all the lab participants)	80%	25,5/32
Not participating at the LAB	29	52%	23,6/32

A qualitative analysis has been also performed to evaluate the interest in this learning approach, to assess the main difficulties that students have experienced, to collect feedbacks and suggestions to improve the Lab. For this reason, a questionnaire has been prepared and made available on the Moodle page of the course. Students have been requested to freely answer the questions and 16 students over 26 participants have decided to compile the questionnaire. Some questions are then effectively answered just by 14 students. Collected results are reported in the Annex.

Discussion and Conclusions for Education

The results of the MATLAB Lab on Multivariate Calculus indicate that it is an effective learning approach, as demonstrated by the grades achieved by students and the level of interest generated. However, it was observed that the rest of the class did not achieve the same level of knowledge as those who participated in the Lab. As the primary goal of any good teacher is to create a positive and stimulating learning environment for all the students, a decision has been made to combine both approaches for the current academic year 2022/2023. The lab is therefore still offered to a small group of voluntary students during extra hours, but some of the lab materials is also incorporated into standard lectures. This approach aims to strike a balance between traditional theories and the use of technology, as already done by Milenković (2022). This integration is still in its initial phase, as it requires the teacher to have access to the appropriate technology in the classroom. This includes the ability to easily switch between the whiteboard and software screen. Additionally, the teacher must possess the necessary skills to manage these instruments effectively and organize the lecture in a way that minimizes time lost during the transitions, also to avoid losing students' attention.

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Annex – Questionnaire Results

	Question	Answers				
1	**Matlab Lab has been useful?**	**Yes**	**No**			
		16	0			
		100%	**0%**			
2	**If yes, why?**	It helped me to better understand what we did in class.				
		I managed to have some basis to work from				
		Better understanding of the topic covered also in class				
		It helps to visually understand the topics covered in class				
		Help to better visualize what is being done in class				
		Because this laboratory gives the possibility to "touch" the data and observe complex functions and graphs that otherwise could only have been imagined in the classroom.				
		To better understand the topics covered during the course				
		The ability to visualize functions in multiple variables has been particularly useful for better understanding the theoretical concepts				
		Giving a geometric and visible meaning to what was studied in the classroom. Furthermore, thanks to this tool it is possible to verify the correctness of some exercises carried out independently				
		To better understand and study the topics from another point of view				
		It gave me the opportunity to better understand and visualize the topics covered in class				
		I managed to learn the basics for studying functions on matlab				
		It allowed me to deepen some topics covered in class				
		It allowed me to understand how I can use matlab to solve problems				
		It is a more practical and daily usable method to solve problems concerning the second part of the course, as well as having future usefulness in some working environment				
3	**Did you follow Matlab Onramp before the Lab?**	**Yes**	**No**			
		12	2			
		86%	**14%**			
4	**If yes, how you evaluate it?**	**necessary**	**Useful, but not necessary**	**Partially useful**	**Not useful**	
		2	8	2	0	
		17%	**67%**	**17%**	**0%**	
5	**Was your previous knowledge in the IT field useful to better follow the Matlab lab?**	**Yes**	**No**			
		13	1			
		93%	**7%**			
6	**If your answer is NO, can you explain the reason why?**	The coding language learned in the IT course is Pyton, which is different from Matlab				
7	**Did you struggle to follow the lab lessons?**	**Yes**	**No**			
		0	14			
		0%	**100%**			
8	**If yes, what difficulties did you encounter?**	-				
9	**Do you think the topic addressed during the Matlab laboratory was interesting (Multivariate Calculus)?**	**Yes**	**No**			
		14	0			
		100%	**0%**			
10	**Which topic among those addressed did you find particularly interesting?**	**optimization**	**level curves**	**gradient**	**directional derivatives**	**critical points**
		6	2	3	4	2
		35%	12%	18%	24%	12%
11	**Which topic among those covered did you NOT find interesting?**	**none**	**level curves**			
		5	1			
12	**Which topic would you have liked to address also from a "computational" point of view?**	**Integrals**	**Matrix and Linear systems**	**Differential Equations**	**Eigenvalues and eigenvectors**	**Multivariate Calculus**
		1	2	1	2	2
13	**Any suggestion?**	I think overall it is an interesting laboratory, but there was little effective time to be able to assimilate the basic techniques of using the program and therefore understanding it was sometimes a bit complex				

Investigating Mathematical Thinking and Reflection in First and Second Year Energy Engineering Students.

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Abstract

A new second level mathematics curriculum (Project Maths), was introduced on a phased basis into schools across Ireland between 2010 and 2018. One aim of the curriculum is to improve students' interpretation, formulation and presentation of mathematical ideas. At the end of 3 years students take a Junior Cycle paper followed 2 years later by a Higher or Ordinary Level Paper which counts toward university access.

As part of a wider PhD study of the challenges students may be having when engaging with both procedural and problem-solving skills in mathematics, we have attempted a pilot study to investigate mathematical thinking in a group of first and second year engineering students doing basic procedural problems. These students have completed the full Project Maths cycle and, typically, have entered with Ordinary Level mathematics. The main focus of this study is not so much to find out about the students, but more to find out how well two data collection methods will work with our 'typical' students, so as to inform the investigative aspect of the wider study.

Over a semester students take a series of similar tests. Between tests students complete a Reflection Sheet which describes in words a problem they could not do, where they found material to help them do it, any appropriate formulas they think they need and a fully worked out example solution. Volunteers are also invited to participate in a 40 minute 'think aloud' interview where they articulate their thoughts as they work through some basic algebra problems. Can we capture useful information this way, can we 'see' our students' thoughts?

In this paper we will present insights into how the students engaged with the reflection and 'talk aloud' processes and its implications for the wider study.

Introduction

In 2019 Allied Irish Bank (AIB) made a significant financial contribution to what was then the Institute of Technology Tallaght, to widen access to STEM education in the Tallaght (Dublin) area in general and to support students with their mathematics in particular. The Institute caters for a population of approximately 3,500 full-time students and offers a wide range of programmes. At the same time as the financial contribution, Technological University Dublin (TU Dublin) was established as Ireland's first technological university and has a student population of over 30,000. It was formed by combining three existing institutes of technology in the Dublin area: Dublin Institute of Technology(DIT) , Institute of Technology Blanchardstown (ITB, and ourselves, Institute of Technology Tallaght (ITT).

Using some of this AIB funding, we created a fully funded 4 year PhD position, beginning in September 2022, to look at '*Enablers and Barriers: An investigation into students' thoughts when engaging with procedural and problem-solving tasks in mathematics*'. The scope of the wider study now involves engineering and other students in this much larger TU Dublin cohort. The work builds on *Faulkner et al* (2021), which examined beginning undergraduate students' problem solving and procedural skills, in the context of the phased introduction of the Project Maths curriculum in second level education in Ireland in 2010. This program was introduced over several years, with the 2018 group being the first to have gone through all years since first year. One of the aims of Project Maths was to change the emphasis of second level mathematics education from a focus on rote learning to real understanding and problem solving. Work carried out by *Faulkner et al* (2021) demonstrates that students problem solving skills on entry to third level education have not improved in the lifetime of the Project Maths roll out. This supports an extensive evaluation of the Junior Cycle Program (the three years of Project Maths for 12-15 year old students) for the National Council for Curriculum and Assessment (NCCA) by *Shiel. G & Kelleher. C* (2017). This recent work therefore called for a qualitative investigation into what challenges students may be having when they are engaging with both procedural and problem solving skills in mathematics.

In this pilot study we have focussed down onto two groups taught in Tallaght, first and second year Energy Engineering students. Typically, students on these courses are weak mathematically, with very few, if any, of them having done Higher Level Mathematics at school. Both of these groups have gone through the full Junior and Senior Cycle of Project Maths. Under Project Maths, the lower (Ordinary) Level is a subset of the Higher Level and also requires much less extended multi-faceted problem solving. This pilot study is, for the most part, investigating two data collection methods with these 'typical' TU Dublin student groups, more than it is investigating the students themselves. It is narrowed down further by looking at student's procedural skills rather than their problem-solving abilities, focussing on algebra in particular. Two investigative strands will be described

- Students are given a series of tests on procedural skills over the semester and asked to complete a detailed reflection sheet on a question before their next test.
- Students are invited to verbalise their thoughts while they work through set of procedural algebra problems *Cowan*(2019), *Nielsen* (2012), *Ericsson* (2003).

Key Skills and Reflection Sheets

For many years at then IT Tallaght we have been running Key Skills tests in procedural mathematics for our engineering students *Marjoram et al* (2008), *Robinson et al* (2010). The idea is to continuously test key mathematical skills over a semester until a high mark is achieved (a high threshold competency based test). Such tests are randomized so that adjacent students do not see the same questions. They are repeatable, automatically marked, and provide immediate feedback on learning resources that students might go to, in order to do better next time. The tests are supervised, difficult to cheat on and marks gained go towards the continual assessment for their maths module – a currency all students understand. Such frequent testing has been shown to be effective in motivating students to learn, especially weaker students, see *Tuckman* (2000) for example. For this study we are focussing on two groups, a year 1 group of Energy Engineering students

(39) tested in Semester 2 and a year 2 group (25) tested in Semester 4, both of which groups have gone through the full Project Maths cycle.

Key Skills tests last one hour and have 15 questions each, comprising numerical input and multiple choice. The topics included in the tests have all been studied in earlier semesters. For the Semester 2 Group there are 10 basic algebra questions and for the Semester 4 group 6 such questions. The questions are generally straightforward, such as solving a linear equation (Sem 2 &4), adding two algebraic fractions (Sem 2&4), identifying the equation of a straight line from its graph (Sem 2&4) or performing a differentiation/integration (Sem 4). As the tests are based on material from earlier semesters, the first Key Skills tests are given in Semester 2. Each question comes with feedback linking to either a Khan Academy lesson (https://www.khanacademy.org/) or a book chapter available in the library.

The *best* mark in the semester's Key Skills tests contributes to the module mark. For a 15 question test, a student gets 6 for 10 correct, 7 for 11 correct, 9 for 12 correct, 10 for 13 correct, 15 for 14 or 15 correct and 0 for 9 or less correct. For this year's iteration we have added a Reflection Sheet element as an upload to an unmarked Question 1. The rubric of the test site explains the marking scheme and also provides a link to the example Reflection Sheet so that they can see the level of detail required. During a test a dated blank Reflection Sheet is given to each student. After reviewing their results and before the next test, students are required to complete the sheet for one of the questions, scan it and upload it to Question 1 of the *next* test. The next test may be 1 or 2 weeks later and typically students do up to 6 tests in a semester, potentially uploading five such sheets.
As an incentive to create such sheets, students were told that three 'ok' sheets uploaded would get them an extra 2 marks added to their best mark, to reflect the effort made. Looking at the marking scheme, this could be a considerable bonus. In the Reflection Sheet we were looking for the student to mimic the work of a good student; the student should articulate the problem in words to show they know what it means, the student should give an accurate reference of source material to show they found out what to do, the student should write down a good example and give a detailed solution, explaining what they are doing at each step.

Table 1: *Numbers of students with 0, 1, 2 or 3 uploaded Reflection Sheets*

Sem/Sheets	0	1	2	3	Total
2	24	7	1	7	39
4	10	6	4	5	25

As **Table 1** shows, the results were disappointing. Both groups did 6 tests over the semester but as we can see, nobody did more than 3 sheets. In spite of being told verbally, being sent emails before each test explaining what to do and having it written on the test rubric, the most common response is no sheets. Only 12 students in total managed three sheets and none did more. As **Table 2** below shows, out of 205 tests between the combined 64 students, 147 had no uploaded Reflection Sheet (83 when we discount Test 1). We were generous in our allocation of good, ok and poor to the uploaded reflection sheets themselves, with good often being far from thorough. Fewer than 10 sheets had an accurate reference to learning materials; most common was a vague reference to “Khan Academy”, “last years notes”, the title of a book listed in question feedback and even “talking to friends”. Little evidence of real diligence was there though.

Table 2: *Rank of uploaded Reflection Sheets as good, ok or poor*

Sem/Rank	none	good	ok	poor	Total
2	71	9	15	5	100
4	76	12	11	6	105

Within the OK and Poor sheets, the written description of the problem area was most often vague, particularly with algebra problems. A problem involving the addition of 2 algebraic fractions might simply be described as “fractions” for example. Very often a problem was simply stated and repeated further down the sheet as an example, without any description of what it was about or what area it might lie in. Very often a particularly simple example problem would be chosen and ‘solved’ in one line. Within almost all sheets, while a clear example problem was given, the solution (right or wrong) was minimal in its effort to show what was going on and sometimes only a final answer was presented. **Table 3** below shows the pattern of Reflection Sheets for the seven Semester 2 students who uploaded 3 sheets over 6 Tests. This was the group hoping to have 2 points added to their best score. The shading indicates a sheet about algebra (of which there were 10 of the 15 questions).

Table 3: *Rank of uploaded Reflection Sheets as Good, OK or Poor*

Test Number	1	2	3	4	5	6
Student1		poor	ok	ok		
Student2				poor	good	ok
Student3		ok	good	ok		
Student4		ok	good	ok		
Student5		ok	poor	ok		
Student6		ok	ok	ok		
Student7				ok	good	ok

As we can see, most were not well done, particularly in algebra (a similar pattern is observed in the 5 Semester 4 students who did 3 sheets). Of course, these were not strong students, only 6 students of the 64 in both groups managed a top score within the first 3 tests and all but one of those (an Erasmus student) submitted no sheets. It is also notable that the students in Table 3 did three sheets together, either at the start or end of testing. Weak students, on the whole, tend to be quite dis-organized students and this may be one interpretation of Tables 1 to 3. The Reflection Sheet process was not engaged with by many students in spite of the potential marks boost and, where it was engaged with, the work was not done to the level requested. Very often requirements were mis-understood and verbal reminders during test sessions about how the marking and reflection sheets worked was always required. Some students tried to hand over hard copies or emailed copies after tests, sometimes in batches. Some uploaded the same sheet several times. The students were focussed on the Reflection Sheets as a method of boosting their marks, but very often gave little though to their content. The pattern of test marks below in **Table 4** also indicates that they were using the sheets as a marks boost option rather than as an aid to study. In 2018 (the last pre-Covid supervised second semester tests), 80% of Sem 2 and Sem 4 Energy students who did 3 or more tests did 0 or 1 more attempt after their best attempt. This year the Sem 2 and 4 groups that did 3 Reflection Sheets have 58% doing 0 or 1 attempt after their best. This suggests that they were more interested in handing in sheets than getting a better mark by learning to do problems.

Table 4: *Test marks for students in Sem 2 who did 3 Reflection Sheets (high score shaded)*

Test Number	1	2	3	4	5	6
Student1	8	11	11	9		
Student2	8	12	8	10	8	10
Student3	10	9	10	12		
Student4	10	7	12	12		
Student5	9	7	9	13		
Student6	6	10	4	8		
Student7	5	8	9	6	8	8

Finally, in terms of looking at the relationship between Reflection Sheet engagement and test performance, **Table 5** below picks out the performance in the 10 algebra questions for these 7 Semester 2 students

Table 5: *Algebra question performance for students in Sem 2 who did 3 Reflection Sheets*

Test Number	1	2	3	4	5	6
Student1	50%	60%	70%	60%		
Student2	50%	90%	50%	70%	60%	50%
Student3	80%	50%	70%	80%		
Student4	70%	50%	70%	70%		
Student5	50%	40%	50%	90%		
Student6	40%	60%	30%	40%		
Student7	20%	50%	60%	20%	50%	40%

Comparing with **Table 4** only Student5 shows consistent improvement during or after the Reflection Sheets were submitted. The Semester 4 students follow a similar pattern.
On the whole, for the 64 students doing tests, improvement was seen through the Semester, but less so for the students who did three Reflection Sheets. Reflection sheets were engaged with more, but to a generally weak standard, by weaker students. As an aid to improved work, Reflection Sheets clearly need a little more thought in their implementation. One idea is to add a mark to a score with an uploaded sheet for *that* test only, rather than adding 2 marks to the best score for 3 sheets overall.
What do the Reflection Sheets tell us about how the students think, if anything? Reflection Sheets were banded into Good, OK and Poor. Good sheets had a reasonable problem

description and reference, a well stated example and in particular a detailed solution process (which need not be correct). OK sheets made an attempt at a problem description and learning reference with some attempt at a worked problem solution. Poor sheets had elements missing and/or little attempt at problem solution detail, a simple answer given or the solution of a trivial example.. Focussing on algebra, there were 32 sheets in total (14 Good, 15 OK and 3 Poor) between the two groups. Across all sheets, the type of algebra problem was not articulated well and most often an example of a problem was given. No use was made of technical language like “common denominator” or “exponent”

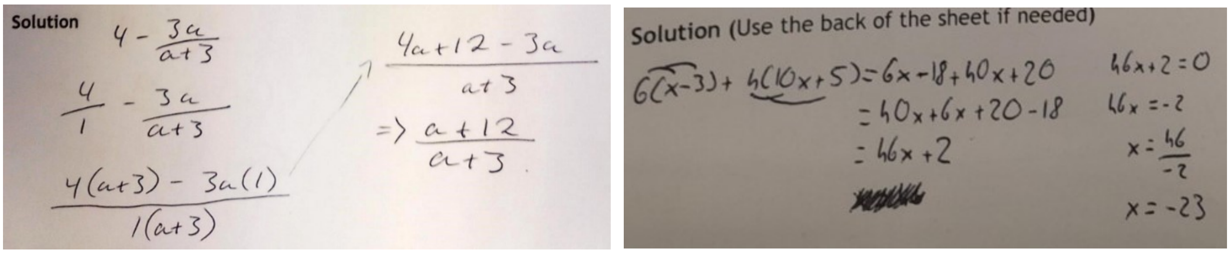

Figure 1: *Two Good algebra sheets, one correct and one incorrect*

Typically, Good sheets had a correct solution process to a problem but not always. Figure 1 shows the combination of 2 algebraic fractions and the solution of a linear equation. In nearly all Good sheets, there was little articulation of the thought process. In the incorrect example in Figure 1 we really want to know how 46x = -2 became x = 46/-2, but there is nothing there. Use of ‘=’ and ‘$\Rightarrow$’ was very often absent as in the first example above, or used in a haphazard way, which led to many ‘chaotic’ solutions

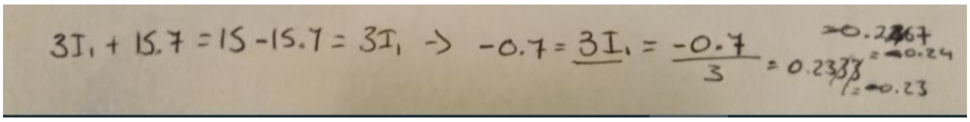

Figure 2: *Part of a chaotic solution process to a set of simultaneous equations*

Consistently, whether correct or not, detailed explanation was missing. This was probably, even mostly due to the way that the Reflection Sheet was written and the instructions given. Beyond the Pilot phase we must be much more explicit with the students about what is expected and provide a better example or set of examples. The effect of this though may be to overburden the students with expectations and reduce the take up of Reflection Sheets even further.
The OK and Poor algebra sheets typically gave an example as the problem type, had no real reference to learning material, presented the same example to solve and did not articulate anything about the usually brief solution process. Again, the thought processes of the student were not visible and, as presented, the Reflection Sheets have failed to capture student’s thoughts and motivations.

The ’Talk Aloud’ Process

Students in both groups were invited to volunteer for a ‘talk aloud’ interview of about 40 minutes where they would articulate what they were thinking and feeling as they worked through 6 procedural algebra problems. A detailed description and analysis of this interview process is made in *Matthews et al* (2023). As with Reflection Sheets, students were again invited frequently during the Semester, both in class and via email, after meeting with the interviewer. Students were offered a lunch voucher as an incentive. As with Reflection Sheets, take up was disappointing with only one student from each Semester taking part. The Semester 2 student, StudentS2, obtained grade 1 in the Ordinary Leaving Certificate, did one Keyskills test (14/15) and no reflection sheet. The Semester 4 student, StudentS4, did not complete the Ordinary Leaving Certificate, did four Keyskills tests (14/15 in Test 4) and two reflection sheets.
The results were much more encouraging than for the Reflection Sheets. Both students found the process difficult, frequently losing their train of thought as they worked. The process was also quite intimate with frustration, doubt and anxiety often expressed as well as thought processes on the material. As an example of the latter, both students struggled to reason clearly when adding algebraic fractions and then separately, solving an equation with algebraic fractions in it. In the first problem they both correctly created a common denominator by ‘multiplying top and bottom by the same thing’ but then both brought those thoughts to solving an equation in a later problem. Instead of multiplying both sides by the same thing to remove fractions, they multiplied top and bottom of both sides by the same thing, clearly confusing equation solving involving fractions with creating a common denominator. Both doubted that what they were doing was right, but could not resolve their confusion in the moment.

It may be that lack of confidence caused a low uptake in doing interviews. In spite of there being no official assessment element, student’s may have been keenly aware that such an interview was very much an assessment and an opportunity to be embarrassed. StudentS2 was, relatively speaking, a confident high achiever and the mature StudentS4 frequently attended learning support and may be passed being embarrassed. Perhaps a greater incentive might be helpful, but with our typical students take up may be low as the study progresses.

Conclusion

In an effort to ‘see’ what students were thinking as they solved procedural problems they were invited to articulate their thoughts in written form on Reflection Sheets and verbally through a ‘talk aloud’ interview. Take up of both was low. Reflection Sheets were found to be mostly done by weaker students as a means to gain marks. On the whole, they did not produce much of a written record of the students’ thoughts. It is suggested that Reflection Sheets need to be more explicit in their instruction and that their reward element be more finely tuned. The ‘think aloud’ interviews produced much more in terms of thoughts, but take up needs to be encouraged with incentives. Participation may remain to be low however.

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Investigating the Level of Mathematical Preparedness of Students who Transfer from the Further Education Sector to Higher Education STEM Courses

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Abstract

A student's level of mathematics as they begin degree courses at Higher Education (HE) in the Science, Technology, Engineering, and Mathematics (STEM) disciplines has been recognised as a key indicator of their success in those courses. Much research has taken place into the teaching of mathematics at second level, and much work has gone into a reshaped Leaving Certificate syllabus designed to better equip students to succeed at third level, with greater emphasis on applicable rather than procedural knowledge. A comparatively under-researched area has been that of Further Education (FE), a sector which supplies a smaller proportion of HE's student intake, typically in the form of one- or two-year Post-Leaving Certificate (PLC) courses.

Such considerations have led the author to investigate the mathematical preparedness of students at FE level, with a particular emphasis on those students hoping to progress to a STEM degree course. The project is being conducted using a mixed-methods approach, incorporating analysis of both quantitative and qualitative data.

The quantitative analysis component will involve analysis of diagnostic testing and examination results from both FE and HE in mathematics, and an application will be made to SOLAS/ETBI to access administrative data linked to education and employment outcomes for students who have progressed from the FE to HE sectors.

The qualitative analysis component will consist of surveys, focus groups and interviews, aiming to explore students' experiences of progressing from FE to HE, difficulties they may have with mathematics, and whether they are at risk of dropping out or failure because of mathematical issues. The principal participants in this component at this point of the research are those studying Engineering-focussed PLC courses for the purposes of progression to courses at HE level, with an intention to broaden this out to other STEM disciplines at a later point.

Keywords: Mathematics Skills, Further Education, Mathematics Curriculum, Problem Solving, Education Transfer, Progression Pathways

Introduction

A student's level of mathematics as they begin degree courses at Higher Education (HE) in the Science, Technology, Engineering, and Mathematics (STEM) disciplines has been recognised as a key indicator of their success in those courses. Much research has taken place on the transition from second level to third level but relatively little research ahs taken place on the transition Further Education (FE) to third level.

In this research we investigate the mathematical preparedness of students at FE level, with a particular emphasis on those students hoping to progress to a STEM degree course. We decided to collect quantitative data in the form of a survey as one part of a broader, mixed-methods approach. The majority of students in FE colleges seeking progression opportunities in HE are doing so on one of three modules ‘Mathematics 5N1833’, ‘Maths for IT 5N18396’, and ‘Maths for STEM 5N0556’.

The survey was to contain both a diagnostic test of key mathematical skills and Likert scale questions exploring commonly held beliefs about mathematical problem-solving.

This design process was to be aided by a pilot survey of students from six PLC classes, which was distributed and collected in April 2023. Of the six classes surveyed, three were from the Computer Science/Computing area, two from Engineering, and one from Science, with three of the classes studying the ‘Maths for IT 5N18396’ module, two studying ‘Mathematics 5N1833’ and one ‘Maths for STEM 5N0556’. In total, 56 responses were collected as part of this pilot process.

Overview of the Further Education Sector in Ireland

The Further Education and Training (FET) sector supports approximately 180,000 unique learners compared to , in 2018 there were 362,899 students in second level education, and 185,474 students in full-time higher education (Education, 2020).

Post Leaving Certificate (PLC) Courses

Post Leaving Certificate (PLC) courses constitute the sector’s biggest single course type, consisting of over 842 courses in 2018, with approximately 28,000 learners primarily at NFQ (National Framework of Qualifications) Levels 5 and 6. PLC colleges tend to have a dual focus; courses which focus on vocational education leading directly to employment, and courses with the primary focus of progression within education, most often to the HE sector. PLC courses with an employment orientation were expected to cater for 36% (12,524) of PLC enrolments that year, while progression-oriented courses were to cater for 45% (15,616) (SOLAS, 2019).

When asked about their primary objective for study between September 2015 to February 2016, 39% of PLC learners surveyed reported that their main objective was to get a job immediately after the PLC course, while 38.6% reported that their main objective was to get a place on a higher education programme. (18.5% cited “personal development”, and 3.9% chose “other”) (McGuinness et al., 2018).

As explained above, most PLC graduates on progression-focused courses complete a Level 5 course before seeking routes into third level. The majority will enrol in HE the same year they graduate from their Level 5 programme, with patterns showing smaller groups enrolling up to 4 years after graduation. 52.64% of PLC graduates across the three cohorts (or 8,031 learners) enrolled in an to STEM degrees. These are areas where a notable lack of literature currently exists.

Using a QQI Level 5 Award to Progress to a HE STEM Degree

The majority of students in PLC colleges are working towards QQI Level 5 awards with approximately 50% of Level 5 and Level 6 PLC graduates who do not intend to remain within the FE sector continuing their education in HE.

We assume that the only QQI Level 5 mathematics modules being studied at QQI Level 5 for the purposes of progression to STEM degrees are ‘Mathematics 5N1833’, ‘Maths for Information Technology 5N18396’, and ‘Maths for STEM 5N0556’.

Mathematics in the Further Education Sector

A Comparison of the Three Mathematics Modules at QQI Level 5

‘Maths for STEM 5N0556’(Robinson et al.) was the last of the three FE mathematics modules to be developed, similar changes to mathematics curriculums both nationally and internationally.

The last 10 to 15 years have seen significant change in how mathematics is taught and assessed in the second level school system, with the introduction of the Project Maths curriculum on a phased basis since 2010. This has sought to change the focus in Irish classrooms more towards problem-solving skills and conceptual understanding rather than a more procedural approach, thereby better equipping school-leavers with the sorts of skills to be successful at HE. A similar problem was also recognised in a Further Education context, where the existing Quality and Qualifications Ireland (QQI) Level 5 Mathematics module was not deemed adequate for entry to level 8 programmes by universities. A collaborative process involving subject experts (from TU Dublin, TCD, and UCD among others), staff from QQI, and the City of Dublin Education and Training Board (CDETB) saw the development of ‘Maths for STEM 5N0556’, a one-year mathematics module at level 5 designed to be accepted by HEIs as an alternative to the HC3/H4 grade in Leaving Certificate mathematics. The module was delivered to its first cohort of PLC students in 2015. The newer module differs significantly from previously existing mathematics modules at QQI Level 5, most strikingly in terms of increased contact hours and the rigorousness of its assessment criteria, as outlined in (Curriculum Development Unit, 2018a, 2018b, 2018c).

Table 1: The differences between QQI Level 5 Maths modules, as per module descriptors published by the City of Dublin Education and Training Board (CDETB) (Curriculum Development Unit, 2018a, 2018b, 2018c)

Module Title	**Maths for STEM**	**Mathematics**	**Maths for IT**
Module Code	5N0556	5N1833	5N18396
Level 5 Major award credit value	30	15	15

Directed learning hours (for a standard term of 26 weeks)	150 (typically 6 per week)	75* (typically 3 per week)	75* (typically 3 per week)
Recommended self-directed (i.e. learner-led) hours	150	75*	75*
Qualification requirements for teachers	Degree with strong maths	No requirements stated	No requirements stated
Number of assessments	5	3	3
Percentage of final grade from proctored assessment	85 – 100%†	40%	40%
Number of specific learning outcomes (SLOs)	97	36	55

**Duration in hours specified by Mathematics and Maths for IT module descriptors as 150 to include both directed & self-directed learning.*
†Option of a 15% Statistics research project which, if not taken, must be replaced by proctored assessment. Also, mastery in the topics of Arithmetic and Algebra must be demonstrated with a mark of >80% in first proctored assessment.

Table 2: Breakdown of the 3 maths modules under consideration

Diagnostic test

This research set out to investigate the mathematical preparedness of students at FE level for progression to a STEM degree course. As part of this research a diagnostic test was developed and piloted. In the diagnostic test the first part of the question is procedural and the second part is more applied/problem solving

Table 2: Topics covered in the test

1,1a	Area, Perimeter
2,2a	Logs
3,3a	Quadratic equations
4,4a	Inequalities
5,5a	Graphing Trigonometric functions
6,6a	Differentiation

7,7a	Equation of a line
8,8a	Trigonometry
9,9a	Mean/median
10,10a	Probability

Example of questions are given below.

Question 1. A square has an area of $196m2$. Calculate the square's perimeter.

Question 1a. Two dairy farmers have plots of land of area $196m2$. One of the farmers has a

perfectly square plot (as above), while the other has a perfectly circular plot. Both

farmers wish to fence the outer perimeter of their land to prevent cattle going

missing. If the cost of fencing is €7.50 per metre, which farmer will end up

spending more money

Table 3: Results from the diagnostic tests.

		Overall			Maths for STEM			Maths L5			Maths for IT		
Qu		Average	Median	SD	Average	Median	SD	Average	Median	SD	Average	Median	SD
1		7.1	10	3.8	9.3	10	1.8	2.5	0	8.9	7.0	10	3.8
1a		5.7	7	3.9	8.4	10	2.9	8.9	1.5	19.6	4.6	3	3.7
2		4.0	0	4.8	6.7	10	4.7	5.0	0	14.5	4.1	0	4.7
2a		3.9	0	4.6	7.6	10	3.9	2.8	0	8.6	3.1	0	4.4
3		3.8	3	4.1	7.2	9	3.4	0.4	0	1.0	3.1	0	3.8
3a		2.3	0	3.1	5.3	3	3.5	7.1	3	14.9	1.2	0	1.8
4		5.4	7	4.2	8.6	10	1.5	6.9	0	19.2	4.0	1.5	4.3
4a		4.1	3	4.1	7.7	10	3.3	11.1	5	26.8	2.8	0	3.7
5		0.7	0	2.2	2.0	0	3.6	2.0	0	5.0	0.1	0	0.6
5a		0.9	0	2.4	2.9	0	3.9	6.3	0	14.3	0.2	0	0.8
6		2.5	0	3.4	3.7	3	3.4	6.9	0	15.1	2.1	0	3.2
6a		0.5	0	1.8	1.4	0	2.8	1.0	0	3.2	0.3	0	1.3
7		2.6	0	3.6	6.6	7	3.1	1.2	0	3.9	0.8	0	2.2

7a		1.4	0	2.9	4.6	3	3.9	6.1	0	14.4	0.3	0	1.3
8		3.8	0	4.4	6.7	7	3.8	5.9	0	15.8	2.5	0	4.1
8a		1.0	0	2.6	3.1	0	4.3	4.3	0	10.4	0.3	0	1.0
9		5.4	7	4.1	6.5	7	3.8	0.8	0	2.3	4.4	3	4.1
9a		2.5	0	3.7	4.5	3	4.2	7.6	0	19.1	1.8	0	3.4
10		5.0	3	4.3	5.3	7	4.7	11.5	7	26.5	4.7	3	4.0
10 a		2.6	0	4.0	4.5	3	4.5	11.6	7	26.5	2.0	0	3.6
Total (/200)		**65.4**			**112.6**			**47.2**			**49.4**		
Total (%)		**32.7**			**56.3**			**23.6**			**24.7**		

Conclusion.

From the initial testing we can see that the Maths for STEM module leaves students much better prepared for transitioning to third level STEM courses. Further work remains to be done on this by following these students over the next few years as the progress to third level. This pilot will also be used to refine the diagnostic test.

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Improving mathematical skills towards undergraduate studies

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Abstract

In the OECD's PISA study, the decline in the level of competence has been visible among youth in Finland during the past decade specifically in literacy, mathematics, and science. The skills of immigrants are also of concern, as the discrepancy in learning outcomes between pupils with an immigrant background and those belonging to the native population is one of the largest in the reference countries. (Ministry of Education and Culture 2019, 122-125.) Based on Pietiläinen's report (2021), ninth graders' proficiency in mathematics has decreased compared to previous studies. Furthermore, Niemi et. al. (2021) reported that the level of competence declines if a student does not enter the upper secondary school.

This article describes an EU funded project called KoKo – Towards Education started in February 2023. Labour shortages, especially in the technology sector, is growing, and the demands of digitalisation do not help the situation. This project will develop an operating model to promote access to education in the Finnish universities of applied sciences and employment for the disadvantaged by improving their mathematical and scientific skills, and thereby improving their chances of graduating on time and/or applying for training.

The aim is also to develop individual learning, self-regulation and motivation, not only in mathematics but also in physics. The students who feel confident about their own abilities and are motivated and succeed better in mathematics (Niemi et al. 2021).

Introduction

The labour shortage, especially in the technology sector, is alarming and the demands of digitalisation do not reduce the problems of employment mismatch. Students applying in engineering are not filling the gap, as enthusiasm to study engineering is decreasing. According to Energiateollisuus (2021), only 60% of engineering students graduate. Decent skills in mathematics predict a higher graduation percentage in engineering studies than the actual educational background. This project focuses on upgrading the mathematical skills of those interested in professional studies but hesitate to apply.

One principal factor of succeeding in professional studies is self-regulation. Students who are excellent in regulating their studies are usually progressing well. One part of this project focuses on studying the self-regulation skills of participating students and aims to find methods for upgrading their regulation level by giving them hints on how to improve and by using some special teaching methods.

Aspects of learning psychology

The target group for the online course produced in this project consists of unemployed people in low-income occupations, students in vocational schools and immigrants. The common factor between all of them is that they might be uncertain of their skills and knowledge, and they may lack the skills of academic studying required in undergraduate studies.

Before the COVID-19 pandemic, an autonomous course design was rare. Since the pandemic, in general, most of the students have some basic knowledge about virtual learning environments (VLE). An international study (Hurme, Brown, 2016) of first year engineering students' preconceived thoughts concerning e-learning and e-assessment revealed that some students think the online assessment might not be a true reflection of their work effort. Students have preconceived ideas about assessment and testing, may be uncomfortable with new forms of computer supported training, and are unsure about how they should interact with the online assessment systems. Considering the target group, this is taken into account by focusing on guidance, motivation and self-regulation.

Motivation and self-regulation are key aspects of academic studies. Students attending this online course are assumed to be motivated and consequently our main interest lays in the aspects of their self-regulation skills. We are utilising a self-directed learning test developed by Fisher et.al (2001) to study what is the main cause for the lack in self-regulation: self-management, self-control, or desire to learn. We also study whether specific areas of self-regulation predict dropping out or poor progression.

It is important that students are informed in which areas of self-regulation they should concentrate to help them achieve their goals. Based on their individual results in Fisher's test, we are giving them advice to promote self-regulation and thereby commit them to completing the online course. We also study whether there are some ideas for teachers to be utilised in course design to "secretly" improve students' self-regulation.

Conceptual learning via learning for mastery

All universities in engineering education include mathematics and physics as core parts of the curricula. Moreover, basic mathematical skills are needed in entrance exams to higher education institutions (HEI).

Brown et. al (2018) conducted an international survey of (HEI) students' (n = 374) perceptions, beliefs, and attitudes across several academic disciplines. Study was based on three issues: prior online testing experience, perceptions of barriers, and confidence and expectancy. This study represented the findings that no major differences exist between groups of students in similar higher education institutions in different countries. The engagement of students, and comprehension of assessment were crucial for their continued motivation and satisfaction. Even though the study was developed among higher education students the results reflect the skills needed in studying at HEI. One of the most important issues that needs to be addressed is the variety of ICT skills and the expectations of students entering third-level education.

Another study of Brown et al. (ECER 2016) considered students (n = 183) not to be from higher academic tracks. A preliminary questionnaire revealed that many learners struggle to engage online with abstract mathematical concepts and consider a loss of reward to be a negative attribute of the assessment process. This reflects negatively to self-efficacy and continuation in studies. Using these findings, we set the framework of the material to conceptual level and underline the fact that without basic understanding of the foundational laws of arithmetic and basic mathematical concepts, the higher-level cognitive skills required in STEM cannot be developed. Therefore, the labour shortage will be catastrophic if the engineering students fail in the core studies of mathematics and physics.

As widely recognized, the gaps of assessment (e.g. Bocanet et. al 2021) and decreased proficiency in mathematical skills Pietiläinen (2021) can form a barrier to continuing study. This means a weak ability to understand and apply mathematical expressions and structures thus preventing deeper understanding of mathematics.

The model we use to widen students' basic mathematical and conceptual skills as a theoretical framework relates to the work of Bloom (1984) and Pelkola (Pelkola et.al 2017). Bloom's LFM requires extensive testing. In large online courses, the required assessments measuring and monitoring learning for mastery by testing is more accessible than in a traditional learning situation of tens of students. In this project, the mastery is achieved with digital tasks generated and initialized using STACK. The learning model emphasizes the power of automated assessment and feedback to provide the seeds to support the growth in self-regulation and learning for mastery in mathematical skills.

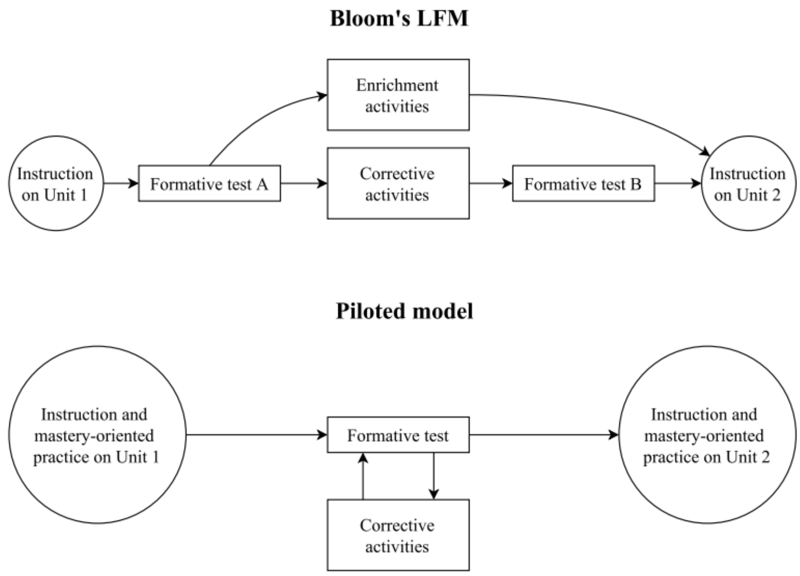

Figure 1. Bloom's model compared to the piloted model of Pelkola et. al (2018).

The results of this model will be reported during the project. The proper preliminary test will be carried out widely to investigate the effectiveness of mastery in the online learning model.

The online course covers the topics of mathematics needed in the general mathematical/logical section of the entrance exam mandatory to all applicants, physics and some basic learning skills required in professional studies (study techniques and how to use a formula book and a calculator). Because not all the participating students apply to engineering studies, the math topics are restricted to the part needed in all undergraduate studies. Physics is an optional part for those interested in physics or who are applying to engineering.

Interactive online course for independent learning

One goal of this project is the creation and development of a shared medium for materials suitable for online training and assessment of mathematics and physics for various types of education, for students who have completed the secondary level and who wish to advance towards higher education or continue in vocational programmes. The material produced in this project includes but is not limited to instructional videos and digital tasks for mathematics and physics, with and without interactive elements. Digital tasks are implemented with STACK and the interactive elements are added with JavaScript library JSXGraph.

According to Guo et. al. (2014) the duration of the instructional videos on an online course should be no more than 6 minutes impregnated with preplanned screenplay, personality, and enthusiastic touch. Embedding the videos within the digital task makes the two of them directly connected and interrelated. This offers a logical progression scheme that connects the theory and the tasks, increasing the students' engagement with the videos.

Combining STACK with HTML, JavaScript, and JSXGraph increases the possibilities of the task design as well as the task variety (Lache, J. et al., 2021). The main importance of the interactive tasks in this project is aiding students to master the basic concepts of mathematics and physics in order to gain a proper conceptual understanding before diving into problem solving. As Rinneheimo et. al. (2022) noticed, languaging might help in understanding the fundamental concepts of mathematics.

In physics, students may get the correct answers for the task but still do not understand the underlying physics concepts (Lawrenz et al., 2009). In mathematics, it is widely known that students use their own "tricks" to solve problems and they might get the correct answers to some particular cases. When interactively exploring the underlying concepts, an insight is gained into the nature of the physical phenomenon or an abstract mathematical concept. Thus, students can actively participate in their own knowledge-building. Once the conceptual understanding is obtained, it is relevant to solve more traditional tasks about mathematics and physics.

Conclusions and future aspects

This project aims to facilitate the entrance to undergraduate studies for people who, for one reason or another, have not had the opportunity to do so before. As an independent online course to study, the threshold for improving one's own knowledge and skills can be lowered because studying is not place nor time bound. For some students it might be also important that the course can be completed without the pressure of the on-site classroom situation, so that the potential lack of competence does not cause further anxiety and distancing from the study of mathematics.

As the project is at the beginning, no results have been obtained yet. However, the expectations are high, as different educational institutions and Employment and Economic Development Centres are cooperating with the project. By developing an innovative learning model and conducting research on its effectiveness, the project has potential to make a significant contribution to the education and employment prospects of disadvantaged individuals. Overall, the KoKo – Towards Education project is an important initiative aimed at addressing the decline in math and science proficiency and increasing interest in STEM fields.

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Levelling up. A post Covid intervention to improve students core mathematical skills

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Abstract

TU Dublin offers students a number of different routes into engineering, allowing many non-standard entrants the opportunity to study the discipline provided they fulfil certain criteria. The final aim of many of these students is to achieve an Honours Degree in Engineering, which takes a minimum of four years. Apart from the first year of the course, the other main entry point is at the start of the third year, at which stage students who have performed well in a three-year Ordinary Degree can begin. However, these students have a wide range of mathematical abilities and prior knowledge, and many are missing the basic skills required for completion of a mathematics module at this level. In an attempt to quantify the problem, it was decided to pilot an Advanced Maths Diagnostic Test which covered many of the key concepts from the early years of Engineering Mathematics. A pass-mark of 90% was set in this assessment. 167 third-year students studying for an Honours Engineering degree were tested during the pilot study, only two of whom achieved the pass mark on the first sitting. To encourage the other students to revise this crucial material, multiple re-sit opportunities were provided, and a weighting of 10% of the continuous assessment mark for the mathematics module was given to the diagnostic test. Online resources and special classes covering the relevant material were also provided, with the result that 131 of the 167 students reached the necessary threshold by the end of the semester.

In our 2013 this process was discussed and we outline the success of this methodology to improve the core skills of students upon articulation. In 2022 we had a cohort of students who were entering 3rd year of our honours degree having spent 2/3 years of their university experience online. These students were diagnostically tested and compared with the results of the students tested 10 year previously in a pre-Covid world. We also compare the final results of the 2023 and 2013 cohorts. We the discuss the success of this methodology and it's potential for use with students who may have been affected by Covid 19.

Introduction

1.1 It is well known internationally that Engineering mathematics students often struggle with a poor grasp of mathematical basics (Carr et al., 2013; Carr et al., 2015; Faulkner et al., 2010; Marjoram et al., 2008; Marjoram et al., 2013). High threshold mathematical tests have been widely used to improve students core basic mathematics skills(Carr et al., 2011; Marjoram et al., 2013). In this scenario students are given multiple attempts at an online test until the reach a high threshold(70%-90 % depending on the test). Such tests have been run for many years in TU Dublin, both for 1st year students and for 2nd and third year students(Marjoram et al., 2013).

1.2 Entry to an Honours Degree in Engineering.

In TU Dublin there are two main distinct routes to achieving a Honours degree (Level 8) in Engineering in TU Dublin(Carr et al., 2013; Carr et al., 2011). Students who have achieved a H4 (60%) or better in higher level Mathematics in the Irish Leaving Certificate (final secondary school exam in Ireland) are eligible to enter directly onto a 4 year Honours degree in Engineering. Students who only have a pass in Ordinary level Maths can enter onto a 3 year Ordinary Degree(Level 7) Engineering .After successful completion of this award students may enter onto to the 3rd year of the Honours degree. These students have a large range of mathematical core skills, and many are missing the basic skills required for completion of a mathematics module at this level. An advanced diagnostic test had been developed in 2010 and used for many years. Due to changes in software and the VLE being used this test had fallen out of use. Post COVID we decided to re write this test for the new VLE and use it i) as a comparison with students basic skills 10 years ago and ii) as a means of increasing the students core skills post COVID.

Advanced diagnostic test

The test has 10 paired questions across each of the subtopics shown below. The test is restricted to 1 hour, as computer rooms in the university are 1 hour bookings. This also allows us to test more students. Lee & Robinson in their 2005 paper found that "to get full advantage of the paired question approach it is essential that the pair test exactly the same skill and have the same number of steps involved. In addition, we would recommend that in order to minimize the potential for making a slip it would be wise to keep the number of steps involved in a question to a minimum"[13]. After several iterations the following 10 subtopics were chosen.

Table 1: Results on each question type.

Topic	Sub-Topic	# Questions	Q1	Q2	Combined %(2023)	Percentage Correct(2013)
Differentiation	Basic	2	93.2	94.1	75.3	88
	Product Rule	2	27.9	12.7	3.1	31
	Quotient Rule	2	12.8	10.5	1.15	21
	Chain Rule	2	39.5	39.5	13	37
Integration	Basic	2	94.1	69.7	56.5	68
	Substitution	2	45.4	44.2	38	37
Differential Equations	1st order ODEs	2	61.6	53.4	28.3	37
	2nd order ODEs	2	41.8	31.4	11.3	1
Matrices	Multiplication	2	8.13	20.9	1.46	5
Complex Numbers	Multiplication	2	24.4	31.4	6.6	3

The test conducted in 2023 was multiple choice, the combined mark is the percentage of students who got both questions correct as opposed to maybe guessing one answer in a multiple choice test. The test given in 2013 was not multiple choice on the first attempt.

As we can see students post-COVID still struggle with the same issues as they did pre COVID and there similar trand as to which questions the students struggles with . The 2023 results are lightly worse but this may be the result of a number of factors, namely the period during COVID of online teaching, the change in the demographic of student entering TU Dublin. One problem with this research was that whilst we used the same test in both years we used MCQs in the initial test in 2023 whereas the initial test in 2013 had a blank space to be filled in. This flaw in the methodology probably masks some of the drop from 2023 to 2103.

When we adopted the high threshold testing we actually see a very positive reaction to online testing.

Table 2: Number of students who passed the Advanced Mathematics Diagnostic Test on their first attempt and the Advanced Core Skills Assessment before the end of semester.

Attempt(2013)	Pass (2013)	Fail(2013)	Pass(2023)	Fail(2023)
First Attempt	2	165	0	151
End of Semester	131	36	151	0

In 2023 everyone who engaged with the online testing did eventually get 90% or higher. This may indicate a higher fluency with online testing in 2023.

Conclusion

We are seeing too effect here. Firstly the original marks from 2023 are worse than in 2013 and secondly by using this method everyone who engaged with the online testing managed to achieve the 90%. In conclusion although there is some evidence of a post COVID slump in students basic mathematical skills, by using high threshold mathematics testing we were able to return the mathematical level of the students to similar levels as pre COVID students.

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Appendix: Diagnostic test

1. Find $\frac{dy}{dx}$ where $y = x^4$

a) $4x^3$ b) $4x^4$ c) $5x^5$ d) x^3

2. Find $\frac{dy}{dx}$ where $y = x^7$

a) $7x^6$ b) x^6 c) $7x^8$ d) $\frac{x^8}{7}$

3. Find the solution of the following second-order Differential Equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$$

a) $Ae^{-3x} + Be^{x}$ b) $Ae^{-3x} + Be^{-x}$ c) $Ae^{3x} + Be^{x}$ d) $Ae^{x} + Be^{3x}$

4. Find the solution of the following second-order Differential Equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 10y = 0$$

a) $e^{-3x}Acosx + Bsin$ b) $e^{1x}Acos3x + Bsin3x$ c) $Acos3x + Bsinx$ d) $e^{1x}Acos3x + Bsin3x$

5. Find $\int x^3 dx$

a) $\frac{x^4}{4}$ b) $4x^4$ c) $3x^3$ d) x^3

6. Find $\int x^{-4} dx$

a) $-4x^{-3}$ b) $-\frac{x^3}{3}$ c) x^{-5} d) $\frac{x^{-5}}{5}$

7. $\begin{pmatrix} -1 & -2 & 1 \\ 2 & -3 & 2 \\ 1 & 2 & -3 \end{pmatrix}\begin{pmatrix} -2 & 7 \\ 0 & 3 \\ 5 & -1 \end{pmatrix} =$

a) $\begin{pmatrix} 7 & -14 & 6 \\ 3 & 3 & 4 \\ 2 & 1 & 7 \end{pmatrix}$ b) $\begin{pmatrix} 7 & -14 \\ 6 & 3 \\ -17 & 16 \end{pmatrix}$ c) $\begin{pmatrix} 3 & -14 \\ 6 & 7 \\ -13 & 16 \end{pmatrix}$ d) $\begin{pmatrix} 3 & -14 \\ 6 & 7 \\ -13 & 10 \end{pmatrix}$

8. $\begin{pmatrix} 6 & 3 & 0 \\ 2 & 5 & 1 \\ 9 & 8 & 6 \end{pmatrix}\begin{pmatrix} 7 & 4 \\ 6 & 7 \\ 5 & 0 \end{pmatrix} =$

a) $\begin{pmatrix} 65 & 49 & 141 \\ 45 & 42 & 92 \end{pmatrix}$ b) $\begin{pmatrix} 60 & 45 \\ 49 & 43 \\ 141 & 92 \end{pmatrix}$ c) $\begin{pmatrix} 65 & 45 \\ 49 & 43 \\ 141 & 98 \end{pmatrix}$ d) $\begin{pmatrix} 65 & 42 \\ 49 & 43 \\ 140 & 98 \end{pmatrix}$

9. Solve $\frac{dy}{dx} = \frac{2x}{y+1}$

a) $\frac{y^2}{2} + y = x^2 + c$ b) $\frac{y^2}{2} + y = 2x^2 + c$ c) $\frac{y^2}{2} + y = \frac{x^2}{2} + c$ d) $y^2 + y = x^2 + c$

10. Solve $\frac{dy}{dx} = (1+x)(1+y)$

a) $1 + y = x + \frac{x^2}{2} + c$ b) $1 + y^2 = x + \frac{x^2}{2} + c$ c) $\ln(1+y) = x + x^2 + c$ d)

$\ln(1+y) = x + \frac{x^2}{2} + c$

11. $\frac{d}{dx}\sin(2x+1) =$

a) $2\cos(2x+1)$ b) $2\sin(2x+1)$ c) $\cos(2x+1)$ d)

$\mathrm{I}\cos(2x+1)$

12. Solve $\frac{d}{dx}\cos(5x-4) =$

a) sin(5x-4) **b**) -5sin(5x-4) c)-5cos(5x-4) d)-sin(5x-4) +5

13. Solve $\frac{d}{dx}x^3\cos x =$

a) $3x^2 cosx - x^3 sinx$ b) $3x^2 + cosx$ **c)** $3x^2 + x^3 cosx$ d) $-x^3 sinx$

14. Find $\frac{dy}{dx}$ where $y = x^2\sin x$

a) $2xsinx + 2cosx$ b) $2xsinx - x^2 cosx$ c) $2sinx + x^2 cosx$ **d**) $2xsinx + x^2 cosx$

15.Find $\frac{dy}{dx}$ where $y = \frac{\cos x}{x^2}$

a) $\frac{(xsinx + 2cosx)}{x^2}$ **b)** $-\frac{(xsinx + 2cosx)}{x^3}$ c) $\frac{\mathrm{I}(sinx + 2cosx)}{x^4}$ d) $\frac{\mathrm{I}(xsinx + cosx)}{x^3}$

16. Find $\frac{dy}{dx}$ where $y = \frac{x^2+5}{2x-4}$

a) $\frac{(20-8x)}{(2x-4)^2}$ **b)** $\frac{(2x^2-8x-10)}{(2x-4)^2}$ c) $\frac{(6x^2-8x-10)}{(2x-4)^2}$ d) $\frac{(2x^2-8x-5)}{(2x-4)^2}$

17. If $z_1 = 9 - 2j$ and $z_2 = 2 - 4j$ find $z_1 z_2$

a) 18-32 j b) 18+40j c)18-40j **d**) 10-40j

18. If $z_1 = 4 + 2j$ and $z_2 = 1 - 8j$ find $z_1 z_2$

a) 4+46j b) 4-46j c)20-34j **d)** 20-30j

19. Evaluate the following integral $\int \cos(x+2)dx$

a) –sin(x+2) b)sin(x+2) c) 2cos(x+2) d)-2cos(x)

20. Evaluate the following $\int x(4x^2-7)^3 dx$

a) $\frac{(4x^2-7)^3}{8}$ b) $\frac{(4x^2-7)^3}{32}$ c) $\frac{(4x^2-7)^4}{8}$ **d**) $\frac{(4x^2-7)^4}{32}$

The Future of Mathematics in the Digital Age

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Abstract

In this paper we will discuss some ideas about possible future engineering mathematics course design in the digital age. Three essential aspects of mathematics engineering education will be addressed: technological issues – ICT supported education, distance learning, on-line courses and assessment, instructional materials and resources; didactical issues – various learning arrangements and scenarios, active learning methods, motivation, content and learning outcomes; and mathematical understanding – conceptual understanding versus calculation skills, epistemological and hermeneutic understanding.

Introduction

"Engineering (service) mathematics is the art of applying mathematical methods to complex real-world problems, combining mathematical theory, practical engineering and scientific computing to address today's technological challenges". This is the introductory sentence of the Call for Papers of the journal Engineering Special Issue on Mathematical Problems in Engineering published in 2013, and a prominent sentence advertising Engineering Mathematics study programmes at the University of Bristol, UK. University education can provide opportunities to experience different aspects of theoretical sciences, work on projects in teams, or to induce meta-cognitive thoughts about the processes involved, but it cannot provide years of experience in real engineering projects. Instead, what university education can do is to contribute significantly to building and developing various competencies. One should not expect university education to deliver 'fully grown' engineers. Engineering students, performing well at examinations in various mathematics and professional courses, do not immediately become a good practising engineer. An ability to do engineering work comes from the experience of working in an engineering environment, watching other engineers estimate and solve real problems, and experience how they view "the bigger picture".

There is strong evidence that students enjoy well designed web-based learning resources. Electronic courses on the Internet can be very useful in providing easy access to various resources, discussion boards, other sources of information (links to various educational sites, informal chats and learning platforms, chatbots, AI). Students can work at their own place and space, and lots of additional explanatory materials are available, like graphs, dynamic illustrations, visualisations, exercises and solved problems, applications, mathematics stories, puzzles, mathematics games, and many others. On the other hand, ill-considered computer aided teaching and assessment may produce frustration and anxiety, as many students are not sure whether they understand the mathematics properly and without misconceptions when they do not have face-to-face explanations and discussions with teachers. Online learning does not have the "human touch", peer cooperation and "careful teacher guidance with prompt direct advice". Nonetheless, developments in Artificial Intelligence and applications like ChatGPT raise the question of how such advances will influence the further development on mathematics education in the digital age.

Students are still concerned about communication with other students and teachers, they find it important to have real meetings and direct cooperation in teams, Velichová et al (2022). On the other hand, they also welcome software solutions that make collaboration easier and more comfortable. Impersonal attitude, teaching just a rigorous course of mathematics without a glimpse on practical applications in the respective engineering field might have quite a destructive influence on the future engineer's overall relationship to mathematics.

Various aspects of engineering mathematics education

Three aspects of mathematics engineering education are discussed here:

- Technological issues – ICT supported education, distance learning, on-line courses and assessment, different forms of instructional materials and resources
- Didactical issues – various learning arrangements and scenarios, active learning methods, content and learning outcomes, motivation
- Mathematical understanding – conceptual understanding versus calculation skills, epistemological and hermeneutic understanding.

Some of the presented ideas were born, discussed, addressed and repeatedly presented at the series of previous MSIG seminars and published in the proceedings of these meetings (which are available at the SEFI Mathematics Special Interest Group webpage https://sefi.htw-aalen.de/).

Technological issues seem to be everlasting topics for discussions, nevertheless, unanswered questions remain:

- How to determine the proper proportion, balancing usage of traditional methods and ICT?

- How to cope with the new role of teachers as facilitator, and no longer the main “bearers of knowledge“!?

- Computer assisted assessment versus traditional paper-and-pencil tests!?

- What are good design principles for web-based learning resources and how to restrict or improve ill-considered computer aided teaching and assessment?

- Should be all calculation skills left to computers?

- May/do computer-based learning environments cause an unexpected transmission of the bases of “traditional mathematical culture”?

- How to recognise an appropriate amount of core mathematical knowledge and ideas, having a strong impact on the overall reasoning, that was always considered by engineers as **natural common sense**?

We can recognise direct and indirect methods of how to utilize ICT in teaching and learning, as one can see described in details at the SEFI MSIG webpage in the section Important Topics, Use of Technology. Both ways brought a considerable added value as they significantly enriched the existing collection of instructional resources and valid didactical tools that might improve mathematical education in general. Assuming, that the enormous amount of work on the development of these materials, which rested solely on the shoulders of educators, would be properly appreciated and used by the student community. Another aspect introduced by digital technologies is the wide range of possible mutual interactions between students themselves and between students and their teachers. New communication technologies enable more contacts in different modes, often much more suitable for the young digital generation, preferring anonymous acquisition of knowledge and not so direct personal contact with teachers during assessments.

Didactical issues are dealing with various problems related to learning scenarios, innovative active teaching and learning methods and ways of assessments. Here an important question is:

- Which learning arrangements are adequate for preparing engineers to make proper use of mathematics at their future workplaces?

Let us briefly describe some of the basic learning arrangements:

- **Lectures for large groups in traditional way versus flipped classroom**

The goal is to give students the basic information to be familiar with the study topic. Further study activities necessary to develop deeper understanding have to be carried out by the learners, individually or in groups. A good lecture should motivate the material, relate it to previous concepts and provide an overall picture. Teachers may use various presentation materials prepared in advance by means of ICT and motivate students with animated illustrations of presented topics and application materials related to their main fields of interest. The activating method of Jigsaw techniques can be used during lectures to activate students (Pinto at al, 2019). The task is to discuss certain assigned topics with neighbours in pairs and compare the findings resulting from these discussions in a group of 4 students sitting next to each other in the lecture hall. Another useful activation tool is to engage students with small tasks related to the topic of the lecture, followed by voting on the correct answers, giving the teacher immediate feedback on the overall understanding and feedback to students themselves on their individual performance. The flipped classroom learning arrangement serve as an alternative option, when students alone prepare their presentations of selected new material and explain the basic concepts to their peers. The role of teachers as the lecture facilitators and bearers of new information is suppressed in this case, but they should follow the lecture carefully, consistently check the correctness of presented facts and guide the lecture progress as moderators. These tasks seem to be much more demanding than to prepare a lecture in advance and present it to the cohort of students regardless of whether they really watch, understand or show any interest, which is often the case of on-line lecturing and ill-designed distance learning courses. The Covid-19 experience with on-line lecturing reflected this issue in many papers presented at SEFI MSIG 2021 seminar in Kristiansand.

- **Tutorials versus eduScrum**

The tutor (teaching assistant or one of the students) works with smaller groups of students in order to improve their understanding related to a lecture in whatever environment that is the most suitable for this purpose – classroom, computer or other laboratory, workshop.

Here, the role of teachers is as an expert consultant and facilitator encouraging self-learning in the classroom or through the use of technology and increased peer-to-peer collaboration. A good facilitator helps the team to communicate more effectively, discuss open problems, find the best solutions and achieve goals – deep understanding and clear knowledge. The eduScrum method is an innovative active learning scenario, striving to acquiring knowledge through cooperation in small teams of students. The whole group of students is distributed to small teams, usually 4-5 students form one team, solving together a sprint consisting of a few tasks. Each team is led by one of the members in a role of Scrum master, and during a semester all students in the team take turns in this role. The Scrum master is responsible for the correct solutions of all tasks, and at the end of the sprint he/she has to present their team solutions to the other teams in the classroom. All students in the team are awarded the same evaluation points for their successful team work. An advantage of this method is a higher engagement of students in small teams as they feel more responsible to contribute equally to getting as many points as possible. Also, it is beneficial for cultivating their soft skills, as team work and cooperation, collective spirit and helping colleagues, respect and responsibility for other members of the team and socialization. Many of these social relations and obligations are nowadays quickly disappearing due to extensive anonymous digital communication tools, consequent social isolation, frustration and fear of society.

- **Formative assessments versus checked home-works or Stack exercises**

Continuous testing via smaller tasks that students have to perform during the maths course helps to build deeper understanding, to acquire better calculation skills and to become more familiar with symbolic notation, mathematical formalism and procedures on solving more complex problems. These formative assessment tests might be performed as short pen-and-paper tests including standard computational tasks that serve to develop calculation skills. Another possibility is to use multiple choice tests with provided answers, or also a more open and investigative assignments, with or without technology. One of the efficient ways of how to cope with the huge amount of tests to be checked is to use ICT in a hybrid mode. This means to use the multiple-choice test online, where students choose answers from the offered alternatives. The test is evaluated automatically by the system immediately after submission, but students submit also written statements necessary to justify their chosen answers with correct calculations. This way, no chosen incorrect answers need to be checked. Another very demanding alternative is to require completing homework assignments frequently, which of course must be carefully corrected so that students receive the proper desired feedback. Stack exercises seem to be a solution here utilizing the power of ICT. Students receive a collection of exercises in their e-platform portfolio, which must be solved within the given deadline. Teachers can follow the progress of individual students and support them if needed with some knowledge problems, or with encouragement in the case of the poor study habits. Stack exercises are developed in the step-by-step calculation mode and students are continuously informed about the successful progress in solving the problem. They are automatically offered short instructions in the respective steps of the solving procedure and there are provided recommendations to go back to the explanations of misunderstood topics in the teaching materials, if necessary. However, to prepare quality stack exercises is a demanding and time consuming burden.

- **Work on projects versus miniPBL (small problem solving)**

Students are asked to work, mostly in groups, on applied problems which are larger, more open and investigative in nature, as related to their engineering specialization. At the end of term they provide a project document and present their work. This type of learning activity is more successful when adopted with advanced students in higher study years, as they are more capable of individual study plans and schedules in order to finish the project work on time. Large projects are time-consuming as it is necessary to explore a quantity of materials related to the main project problem, then develop the proper mathematical model of the problem and find its analytical solution that must be finally correctly interpreted with respect to the original application problem. Currently, miniPBL activities are introduced in basic courses of mathematics in order to support the technical university students, freshman in particular, creating awareness of the general need to use a mathematical approach in solving applied engineering or scientific problems. This demonstrates the versatility of mathematical models, their applicability to various kinds of applied problems and the computational power and precision of the latest ICT enabled mathematics and its methods to penetrate all areas of human activities as an unsurpassed tool. MiniPBL are small applied problems that can be solved using basic mathematical models illustrating thus importance of a core mathematical knowledge in any kind of engineering activities. Tasks in miniPBL are related to current environmental problems of our planet and the further sustainable development of life on it, in accordance with the EU and UN strategy for sustainable development goals (SDG), see Erasmus+ Project Pythagoras webpage https://www.pythagoras-grant.eu/ .

- **Technology enhanced learning**

Many different scenarios are available. Collaborative virtual classrooms allow synchronous or asynchronous communication on available e-learning platforms. These are usually included within the academic information systems of individual universities, to support distance and e-learning courses and on-line testing, or blended learning forms. Often the learning activity is carried out without the presence of a member of academic staff. Much presentation material is available for students to use in such an adopted self-guiding strategy, as they can revisit course content arbitrarily many times in order to gain full understanding. Students might also work individually in university PC laboratories on tasks requiring the use of mathematical software such as numerical programs or various CAS to practise their usage in solving computationally demanding tasks and experimenting with more open tasks of an investigative nature. Several European projects aim to develop learning platforms with materials for students. However, it is necessary to talk also about the need to develop digital didactics and methodology, and to support teachers' efforts to innovate traditional methods and approaches to teaching mathematics at technical universities. The Erasmus + Project DigiSTEM is focused on developing a learning platform for academic staff - practising mathematics educators, bringing materials on how to effectively use ICT and digital technology in STEM education, see Erasmus + Project DigiSTEM webpage https://digistem.eu/index.php/en/.

- **Gamification**

Learning by playing, an old-new strategy that turns the painful, boring and arduous work of acquiring knowledge into fun and interesting activity is experiencing an unprecedented renaissance thanks to ICT. Jan Amos Comenius (1592 - 1670), educational reformer and "teacher of the nations", promoted this didactic method in overall education as early as the 17th Century. His textbook *Orbis Pictus* received much acclaim and greatly influenced the educational system by emphasizing the use of illustrations and real world problems in teaching. One of his prominent works was *Didactica Magna*, which organized the school system in a way similar to the current educational systems, delineating kindergarten, elementary school, secondary school, college, and university levels. Although he did not use the modern words, Comenius addressed such topics as: education for everyone, students' natural tendency to learn, learning by easy stages, financial aid, career preparation, extracurricular activities, lifelong learning, gamification. It is thanks to him that educators today think these issues are important. In honour of his important ideas, which influenced the development of educational systems on a global scale, UNESCO awards the Comenius Medal to a person who has greatly influenced education.

Mathematical understanding and competency remain a topic of endless discussions. Calculation skills as one of the previously highly valued "engineering skills" are becoming less important due to the powerful ICT that is available. Techno-mathematical literacy brought to the fore specific working styles of "trial and error", and new problem solving strategies are recognised, as "guess and verify" and "separate larger problems into smaller ones". The increased power of ICT increases the relevance of the age-old dilemma: what should be the balance between conceptual understanding versus calculation skills. The question is which essential mathematical skills are most useful for future engineers. New skills must be considered: estimation of the forms and values expected as results from computer, predicting the results of calculations, and being aware of the limits and restrictions of mathematical modelling in terms of practical needs. Technical skills of how to use various available CAS, CAD and simulations are not sufficient to enable engineers to solve applied problems represented as mathematical models.

Engineers today need a deep understanding of mathematical principles and basic concepts to apply them reasonably for solutions of complex applied problems. They must be aware of advanced and newly developing mathematical theories and be able to apply these in mathematical modelling to solve challenging engineering problems. Accuracy plays an important role in many advanced technical branches; a "sense of numbers" should be developed as an aspect related to various interpretations of solutions, and a "sense of error" should be fostered, meaning that engineer should be able to check and verify data and detect errors. Technical creativity can be supported by innovative technology and tools, decision making, estimation and confirmation of solution could be developed as a consequence of a competency based teaching approach.

The SEFI Mathematics Special Interest group latest edition of core curriculum document (Alpers et al, 2013) is based on the concept of mathematical competences. This document adapts the competence concept to the mathematical education of engineers and explains and illustrates it with examples. It also provides information for specifying the extent to which a competency should be acquired. The competence-based approach in the mathematical education of engineers requires integration of mathematics within the entire engineering study course to support the real usage of mathematics in engineering contexts. The document presents several forms of how this integration can be realized, as it is essential to the development of competencies and requires close cooperation between mathematicians and engineers. It is also extremely important to adopt the assessment procedures and assessment schemes to truly assess the level of competences acquired during the completion of the course. Ideas on how to introduce these strategies into engineering mathematics are discussed during the SEFI MSIG seminars and also outlined in the document Alpers et al (2013).

Contrary to epistemological understanding suitable for pure mathematicians, where the properties and behaviour of the developed mathematical models are investigated to acquire new knowledge, engineers should acquire hermeneutic understanding, where the mathematical model is designed in order to serve for solution of a real problem. Computing the behaviour of this model is not sufficient, it is the meaning and utility that is important to its users, as mentioned in Schaathun (2022).

Understanding, in general, is the natural state of the mind when it is freely exploring the surrounding world. Understanding cannot be forced nor passed on. Some concepts and techniques have to be learnt, but students need some freedom to find their meaning. Efficient learning depends on pre-understanding and meaning, both of which are highly personal and individual. Each student has their own unique background, and their own visions and motivations for the future. They need freedom to search for meaning within their own personal context and within their existing cognitive schemata. Meaning is personal, and has to be found by the learner, and cannot be given by the teacher, which shifts agency from the teacher to the learner. This attention on meaning and agency might possibly solve some of the known problems in mathematics education of engineers.

Self-motivation is the best learning strategy, so perhaps we should concentrate more on ways of how to foster it in our students, not just to deliver all available information.

Approach of the Students

Students´ opinion, reactions and their attitude to the utilization of new technologies are continuously investigated in many opinion polls, inquiries, surveys and diagnostic questionnaires. Analysis of feedback and summary of findings can produce results which might be quite unexpected. The young generation really appreciates all new communication tools and possibilities, but they are also rather critical with respect to their permanent and sole usage in educational process. Sometimes the excessively large amount of offered instructional materials that need to be studied in order to successfully complete one of the demanding subjects, mathematics including, represents an insurmountable barrier for students. This frustrates them, affects their self-confidence and creates an overall negative attitude to the respective subjects. Centres of Mathematics Support, established at some universities with settled contact hours, represent an excellent support provided to students, where available services are accessible both virtually and in-person. Some extremely radical demands of students, such as the exclusive use of existing CAS in teaching mathematics at technical university without any conceptual backgrounds and theoretical knowledge cannot be considered authoritative. Nevertheless, these ideas sporadically appear at conferences in presentations of students requesting innovation of engineering study programs, often refusing to compulsory mathematics courses. These ideas should not be ignored.

Conclusions

Mathematical skills are essential in STEM careers not just because they are required, or because they are utilized frequently, but because mathematics helps to analyze and solve problems with a methodical approach, while paying attention to detail, and thinking abstractly. These are skills that all of us will greatly value when making key life decisions. Mathematical skills will not be rendered obsolete with time, so we can continue to use them in a meaningful way over the course of our lives, both professional and private. Mathematics is teaching us to seek the truth, to solve problems prudently and rationally, to take objective views and never give up. Often even a negative answer is an acceptable solution in mathematics.

Mathematics gives us a way to understand patterns, to quantify relationships, and to predict the future. Mathematics helps us understand the world — and we use the world to understand mathematics. It can also predict such issues as profits and growth, or loss and bankruptcy, how ideas or the Covid-19 pandemic spread, how previously endangered animals might repopulate, how our Universe is going to behave and evolve.

Mathematics is going to become more important, not less. Computational skill will become and already is far less important. If you only know how to mechanically do some calculations, that does not count for much. Problem-solving ability and knowledge of how to take abstract mathematical ideas and apply them to real world situations is going to become far more important. Currently, the price of the physical labor is relatively high, but automation is making many industrial jobs obsolete. Mathematical competency and understanding is the guarantee of the future success; it will never become irrelevant.

Mathematics is a fun game for those who accept its weird rules. This game teaches us to accept insurmountable boundaries and to wait patiently, while tirelessly seeking to find solutions of, as yet, unsolved tasks, for the right time to be able to solve them.

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