The 19th SEFI Mathematics Working Group Seminar on Mathematics in Engineering Education

26th – 29th, June 2018
Coimbra, Portugal

Proceedings

Politécnico de Coimbra - Instituto Superior de Engenharia de Coimbra

Publisher: European Society for Engineering Education (SEFI), Brussels
ISBN: 978-2-87352-017-5

Principal Editor: The Department of Physics and Mathematics, Coimbra Polytechnic - ISEC, Coimbra, Portugal.

Local Organization Committee
Cristina Caridade, Deolinda M. L. Dias Rasteiro, Maria do Céu Faulhaber, Maria do Céu Marques

Introduction

Steering committee of the SEFI Mathematics Working Group proudly presents proceedings of the 19th SEFI MWG Seminar on Mathematics in Engineering Education organized by Coimbra Institute of Engineering in Coimbra, Portugal, on June 26 – 29, 2018.

The aims of SEFI MWG, stated in 1982 when the group was established, remain long 36 years up-to-date and relevant also for the present time:

- to provide a forum for the exchange of views and ideas among those interested in engineering mathematics
- to promote a fuller understanding of the role of mathematics in the engineering curriculum and its relevance to industrial needs
- to foster cooperation in the development of courses and support material
- to recognise and promote the role of mathematics in continuing education of engineers in collaboration with industry.

18 seminars on mathematics in engineering education were held by the SEFI MWG since 1984, to fulfil these aims and maintain international participation. The current 19th seminar taking place in beautiful city of Coimbra is the next event in this long series of successful meetings of enthusiastic maths teachers. Seminar is aimed to provide a forum for the exchange of views and ideas amongst participants interested in engineering mathematics, in order to promote a fuller understanding of the role of mathematics in engineering curriculum, and its relevance to industrial needs and continuing education of engineers in the economic, social and cultural framework of Europe.

Various identified important topics by the SEFI MWG Steering committee and all other relevant issues in the mathematical education of engineers will be presented and discussed. The overarching theme of the seminar is the concept of mathematical competencies reflected in the following themes:

- Putting the concept of mathematical competencies into practise
- Rules for assessing mathematical competencies

Programme of the seminar includes three plenary keynote lectures presented by excellent invited speakers, professors teaching mathematics at universities in different European countries. Professor Edwige Godlewski from the Pierre and Marie Curie – Sorbonne University, Paris, France will speak about “Mathematics for engineers and engineering mathematics, evolution in the French education system“. Professor Jaime Carvalho e Silva from the University of Coimbra, Portugal, will present talk on “Teaching and assessing mathematical competencies and understanding“, and finally Professor Morten Brekke from the Faculty of Engineering and Science, Agder University in Norway will discuss topic “Teaching mathematics for engineers as the NORWEGIAN national framework says – is it possible?“. Special guest is Professor Carlota Simões from the Department of Mathematics of the Faculty of Sciences and Technology of the University of Coimbra, with the talk entitled “Teaching tiles“.

SEFI MWG seminars are traditionally focused on guided discussions among participants during special discussion sessions. Proposed topics include:

• Putting the concept of mathematical competencies into practise

• Rules for assessing Mathematical competencies

Good response to the seminar call for papers, represented by 37 accepted high quality papers with direct relevance to the seminar themes, resulted in very promising programme including poster session with 11 presentations and 26 paper presentations related to important topics in mathematical education of engineering students. The paper presentations are divided into several topics, most of them in parallel sessions, such as *putting the concept of mathematical competencies into practice, assessment of mathematical competencies, motivation and activation of students, technology and software for teaching mathematics, new trends in education.*

All accepted contributions are included as full papers in the proceedings that are freely available at the SEFI MWG webpage, to provide a summary of the topics dealt with at the seminar and free access to presented papers to all interested party. The group´s main objectives are to sustain the accumulative process of gathering published materials and reports related to all identified important topics in mathematical education of engineers for building up a sound body of knowledge in this field.

Finally, the author would like to thank all members of the SEFI Mathematics Working Group Steering committee, the language editors, and the local organizers for doing the language check and editing of the proceedings for the benefit of all potential readers.

In Bratislava, June 2018

Daniela Velichová
SEFI MWG chair

Committees

Program Committee

Burkhard Alpers, HTW Aalen, Germany
Marie Demlova, Czech Technical University, Czech Republic
Tommy Gustafsson, Chalmers University of Technology, Sweden
Duncan Lawson, Newman University, UK
Brita Olsson-Lehtonen, Finland
Paul Robinson, IT Tallaght, Ireland
Daniela Velichova, Slovak University of Technology in Bratislava, Slovakia
Cristina M. R. Caridade, IPC/ISEC, Portugal
Deolinda M. L. D. Rasteiro, IPC/ISEC, Portugal

Local Organization Committee

Cristina M. R. Caridade, IPC/ISEC, Portugal
Deolinda M. L. D. Rasteiro, IPC/ISEC, Portugal
Maria do Céu Amorim Faulhaber, IPC/ISEC, Portugal
Maria do Céu Marques, IPC/ISEC, Portugal

Conference contact: sefimwg@isec.pt

Conference website: https://www.isec.pt/eventos/SEFIMWG2017/

SEFI website: http://www.sefi.be

SEFI MWG website: http://sefi.htw-aalen.de

History of SEFI-MWG 1984 – 2018

Mathematics Working Group (SEFI-MWG) was founded in 1982 under the co-chairmanship of Professor D.J.G. James (Coventry University, England) and Professor K. Spies (University of Kassel, Germany) and succeeded in 1990 by Professor L. Råde (Chalmers University Gothenburg, Sweden), and in 1996 by Dr. F.H. Simons (University of Eindhoven, Netherlands). In 1997the working group chairmanship was awarded to Dr. Leslie Mustoe(Loughborough University, UK), and followed by prof. RNDr. Marie Demlová, CSc. (Czech Technical University in Prague, Czech Republic) in 2002. Prof.Dr.BurkhardAlpers(Aalen University in Germany) was elected as the SEFI MWG chair in 2008, and replaced by the current chair, doc. RNDr. Daniela Velichová, CSc. (Slovak University of Technology in Bratislava, Slovakia) in 2014.

SEFI MWG	Year	University	Place	Country
19th	2018	Coimbra Institute of Engineering	Coimbra	Portugal
18th	2016	Chalmers University of Technology	Gothenburg	Sweden
17th	2014	Dublin Institute of Technology (DIT), Institute of Technology Tallaght (ITT Dublin) and IT Blanchardstown	Dublin	Ireland
16th	2012	University of Salamanca	Salamanca	Spain
15th	2010	Hochschule Wismar - University of Technology, Business and Design	Wismar	Germany
14th	2008	Institute of Mathematics and its Applications	Loughborough	England
13th	2006	Buskerud University College	Kongsberg	Norway
12nd	2004	Vienna University of Technology	Vienna	Austria
11st	2002	Chalmers University of Technology	Gothenburg	Sweden
10th	2000	University of Miskolc	Miskolc	Hungary
9th	1998	Arcada University	Espoo	Finland
8th	1995	Czech Technical University in Prague	Prague	Czech Republic
7th	1993	University of Technology Eindhoven	Eindhoven	Netherlands
6th	1990	Budapest University of Technology and Economics	Balatonfűred	Hungary
5th	1988	Plymouth Polytechnic and Royal Naval engineering College	Manadon-Plymouth	England
4th	1987	Chalmers University of Technology	Gothenburg	Sweden
3th	1986	Polytechnic of Turin	Turin	Italy
2nd	1985	Engineering Academy of Denmark	Lyngby	Denmark
1st	1984	University of Kassel	Kassel	Germany

CONTENTS

One Competency Approach in Mathematics for Engineers in Freshmen Courses

Peter Habala and Marie Demlova

Department of Mathematics, Faculty of Electrical Engineering, Czech Technical University in Prague, Czech Republic

Abstract

In this paper we discuss challenges facing a teacher who wants to address three core competencies related to comprehension of mathematical language. The right place for this is in freshmen courses. We share experiences gained when exploring this path.

The language of mathematics

Among the eight core competencies there are three that are closely related: mathematical reasoning, communication, and handling symbols and formalism. In effect, mastering these competencies means that one can understand text written in mathematical language and also express one's ideas in it properly.

The language of mathematics (mathematish?) is very different from natural languages. Probably the most important difference is its emphasis on logic. People tend to proces human languages as fuzzy bundles of words whose meaning is more guessed (and felt) than derived. When they encounter a mathematical text, they are litttle prepared to extract its contents using logical reasoning.

Consequently, when our freshmen see definitions and theorems, they have trouble forming intuitive ideas about their contents. They may recognize the notions, but they are not ready to build mental structures with them. Thus it is very important that students learn this new language as early and as well as possible.

The benefits of learning the mathematical language are manyfold. On the obvious level, it makes it easier for them to acquire and understand new mathematical concepts, which will help them in mathematical courses to come. Perhaps equally important is the practical experience with logical thinking. Learning to relate facts, to distinguish between the given, the assumptions and the conclusion, these are skills they should serve students well not just in their studies but also in life.

Addressing the competencies

The competencies of reasoning, formalism and communication can be addressed in any mathematical course, starting from the first year. However, how far and how deep we go depends on many factors. Two most important factors are time that we can dedicate to this and the contents of the course that we teach.

The basic level is to develop comprehension and appreciation of logic. Every mathematical statement shown in class can be an opportunity. How do I make sense of what it says? Can I form some mental picture of the contents? Can I actually express it in a sketch? How important are various parts of the sentences? What would happen if we skipped some parts, or changed their order? Thus it makes a lot of sense to think of introductory math courses as of language courses.

Calculus, often the first math course a freshman meets, seems to work really well as the first encounter with the world of mathematics: Most notions are natural, many statements can be easily visualized and situations can be sketched nicely. There are also many nice theorems that encourage discussion about meaning of assumptions.

The next level is the ability to express oneself. A common practice is to ask students to state some statement or definition on a test, but that does not really help. Students would memorize statements and then reproduce them with errors that show their lack of understanding of the language they are using. If we want to truly address this level of competency, we have to give students opportunity to practice the language and give them good feedback. This requires a significant investment of course time.

Every conversation needs some topics, so we naturally end up with proofs. Writing a good proof requires logical thinking, for instance learning the distinction between assumptions and conclusions, and some familiarity with formal language. It also requires skills that are useful not just to students of mathematics: Ability to judge relevance and ability to organize one‘s thoughts.

Besides time, we also need a good supply of simple statements with simple proofs. We want students to focus more on expressing themselves properly rather than inventing clever tricks, which calls for short proofs where we combine some known properties in an obvious way. This is usually enough to show whether a student can organize thoughts well and express them in a concise way.

Where do we find such suitable statements? Calculus seems to be better for more experienced students (some universities offer “Calculus with proofs”), as there is a limited number of conceptually simple “two-liners”. Our experience suggests that probably the best source of proofs for beginners is discrete mathematics. Some fruitful topics are divisibility, calculations modulo n, binary relations and their four basic properties (reflexivity, symmetry, antisymmetry, transitivity), and mappings with their two basic properties (injectivity and surjectivity).

Our practice need not be limited to just proofs of statements. Another good exercise is to take a binary relation or a mapping and investigate its properties. We expect not just a decision (true/untrue), but also a proof (justification) that this answer is correct. We can also practice imagination by asking students to create objects (binary relations, mappings) that would have (or fail) certain properties.

Some examples of proofs

Here we will now show and comment on some typical errors students make when learning to speak the mathematical language. But before we get to these examples, there are some general observations to be made.

Students often fail to put formal bits in their proofs. Typically they would not indicate quantifiers or specify from which set their variables are drawn. Another popular mistake is to indicate that a certain statement is an assumption (typically in induction). A clear distinction must be made between statements that are claimed to be facts and statements that are assumed to be true for the sake of argument (for instance when proving an implication). Many (weaker) students leave such things out because they are working on the cutting edge of their ability. They have to dedicate so much of their brain power to developing the steps of the proof that they have no capacity left to take care of other things.

Sometimes students (especially good ones) would leave out things that they consider obvious. This is a difficult topic, because we almost always leave some bits unsaid in our proofs. There is no clear-cut guideline for that, students generally learn by trial-and-error throughout the semestr as they practice writing proofs and get feedback on it, here it is especially valuable. In general, the level of detail depends on our intended audience and also on the purpose of the exercise. Since students want to show the teacher that they understand things, and the teacher wants to see that they do, it is better to put more than less when in doubt, in effect it pays to assume that the teacher is not very bright and needs a lot of explanations. The teacher should also try to make clear which bits are considered crucial and should not be left out. We show one such example (and some arguments that can be used) in one divisibility problem below.

Now we will give some examples pulled from actual exams. We present them in the way they are typicaly written, the "exam shorthand". It is important to make it clear to students that there usually is some leniency regarding formal precision and style (after all, when we explain our proofs to colleagues, we also take certain liberties), but the logical contents of their sentences should be clear and correct.

Binary relations

Students are given a binary relation and asked to investigate which of the four basic properties (reflexivity, symmetry, antisymmetry, transivity) are valid. They have to prove that their answers are true.

Typical errors:

a) Proof of reflexivity for the relation R on integers Z given by the condition "xy is even":

Take $x = 4$. Then $x^2 = 16$ is even, so xRx.

Comment: This is a fairly typical beginner mistake. Almost all students quickly learn that proof by example does not work.

b) Proof of reflexivity for the relation R on real numbers given by the condition "$xy \geq 0$":

For all real number x: xRx so $x^2 \geq 0$, which is true.

Comment: This is a typical "backward" proof, the conclusion we are supposed to reach is taken as an assumption.

Correct wersion: For all real numbers x: $x^2 \geq 0$, hence xRx.

The reversal of logical reasoning is very common, many students are taught this in high-school and have a hard time shaking it off.

c) Proof that symmetry is not true for the relation R on real numbers given by the condition "$x + y = 13$":

Counterexample: Take $x = 1$, $y = 2$. Then yRx is not true, so symmetry does not hold true.

Comment: This student did not really understand the essence of implication. For the chosen numbers also the assumption xRy is not true, hence the whole implication "xRx implies yRx" is valid for this choice.

Divisibility

We will show three attemps to prove the following statement:
For all integers a,b the following is true: If a divides b, then a^2 divides $13b^3$.

a) Take integers a,b. Assume a divides b, so $b = ka$, k integer.
$13b^3 = a^2l$, substitute, $13k^3a^3 = a^2l$ implies $l = 13k^3a$.

Comment: Another typical example of a backward proof. The conclusion is used as an assumption. Some students are so drilled in this approach that they automatically fall back on it when they are not sure what to do.
Correct version:
Take integers a,b. Assume a divides b, so $b = ka$, k an integer.
Then $13b^3 = 13k^3a^3 = a^2(13k^3a)$, where $(13k^3a)$ is an integer. Thus a^2 divides $13b^3$.

b) Take integers a,b. Assume a divides b, so $b = ka$, k an integers. Then
$13b^3 = 13k^3a^3 = a^2(13k^3a)$, so a^2 divides $13b^3$.

Comment: The algebraic equality $13b^3 = a^2(13k^3a)$, is not enough to reach the conclusion. The condition in the definition of divisibility consists of two components, a suitable algebraic identity and the fact that the multiplicative term in it is an integer. A typical argument one hears is that, well, it is obvious that the number $13k^3a$ above is an integer. Yes, it is obvious, but we should mention it, because:

- formally, divisibility follows from two statements, so they should be shown in the proof to make it complete,
- we want to remind the reader that there are two things that are needed for divisibility,
- we want to remind ourselves that these things should be checked, even if it takes just a fraction of a second, because one day it will not be true and if we get used to not checking, we get in trouble, and
- this is an exam after all, and we want to show the examiner that we know how divisibility works.

c) Take integers a,b. Assume a divides b, so a/b is an integer. Then also $(a/b)\cdot(a/b)\cdot(13b)$ is an integer, that is, $(13b^3)/(a^2)$ is an integer as well. Thus a^2 divides $13b^3$.

Comment: The logical structure of this proof is correct, but it has a fatal weakness. While the original statement is also true for $a = b = 0$, the proof does not cover this case. Students should learn the distinction between divisibility (a property) and division (an operation). While these two are obviously related, they are different notions with different properties (for instance 0 divides 0, but division by zero is not defined).

Induction

a) Proof that $1+2+4+\ldots+2^n = 2^{n+1}\ 1$ for all natural numbers n:

(0) $n = 1$: $1 + 2^1 = 3 = 2^2 - 1$
(1) Let n be a natural number, assume $1+2+4+\ldots+2^n = 2^{n+1} - 1$. Then
$1+2+4+\ldots+2^n+2^{n+1} = 2^{n+2} - 1$

$(1+2+4+\ldots+2^n)+2^{n+1} = 2^{n+2} - 1$

$2^{n+1} - 1 + 2^{n+1} = 2^{n+2} - 1$

$0 = 0.$

Comment: This is a very typical example of a backward proof. Many high-school students were taught to do induction like this. Admittedly, going backwards from a desired equality is a convenient way to find what agebraic steps have to be made, but it does not constitute a proof. It can be salvaged if the steps are equivalent by rewriting it in the correct order, but students often fail to do that. Moreover, this approach fails when induction is applied to inequalities, so students should be discouraged from this practice. A much safer approach (also applicable to inequalities) is to simply use a string of equalities:

$$1+2+4+\ldots+2^n+2^{n+1} = (1+2+4+\ldots+2^n)+2^{n+1} = (2^{n+1} - 1) + 2^{n+1} = 2^{n+2} - 1.$$

b) Proof that a function defined inductively as

$f(1)= 2, f(2)=4,\ \ f(n+1) = 2f(n) - f(n-1)$ for natural numbers $n \geq 2$,

satisfies $f(n) = 2n$ for all natural numbers n.

(0) $n = 1$: $f(1) = 2 = 2\cdot 1$.

(1) Let n be a natural number, assume $f(n) = 2n$. Then

$f(n+1) = 2f(n) - f(n-1) = 2\cdot 2n - 2(n-1) = 2(n+1)$.

Comment: Here we have a failure to appreciate the substance of induction. In the chain of equalities we find one that uses equality $f(n-1) = 2(n-1)$, but we do not know whether this is true at the time. We could ammend this proof by adding this as a second assumption in the induction step (1), then we would also have to fix the base step (0) by adding a statement about $f(2)$.

c) Proof that $1+2+4+\ldots+2^n = 2^{n+1} - 1$ for all natural numbers n:

(0) $n = 1$: $1 + 2^1 = 3 = 2^2 - 1$.

(1) $n = n$: $1+2+4+\ldots+2^n = 2^{n+1} - 1$

(2) $n = n + 1$: $1+2+4+\ldots+2^n+2^{n+1} = (1+2+4+\ldots+2^n)+2^{n+1} = \text{etc} = 2^{n+2} - 1$.

We have seen this structuring of an induction proof repeatedly over the years, so it seems that students are taught this at some high-schools. We fail to see what logical process this is actually supposed to represent. As experience shows, it often happens that when people do not really understand what they are doing, they find solace in formalization and strict adherence to some arbitrary rules. Someone somewhere probably came up with an "induction scheme" and it found its disciples, unfortunately. Our personal favourite is the label "$n = n + 1$" which is a false statement, a succint commentary on this "proof".

As these examples show, it takes quite a bit of work before students learn to appreciate logical structure of arguments, and even more work before they are able to create such arguments on their own. When a teacher decides to include proofs in a course's curriculum, it requires long-term commitment. Students must be repeatedly asked to prove statements on their own, starting from very simple ones, and they need good feedback on what they wrote, so that they can identify misconceptions and address them. It is a bit of a paradox that it is often easier to teach students who never encountered logic before, compared to those who did proofs in high-school but not very well.

Conclusions

The progress of students is greatly facilitated when they become acquinted with mathematicial language, and the ability to recognize valid arguments and formulate one's thought in an organized way is a skill that will benefit them throughout their lives. Mathematical courses offer different opportunities for gaining these competencies. In some courses we can only try to practice math language comprehension, others offer opportunities to reach deeper. Given how important these competencies are, even small steps are worth doing.

References

Alpers, B., et al: (2013) A Framework for Mathematics Curricula in Engineering Education, SEFI, available at http://sefi.htw-aalen.de

Niss, M. (2003) Quantitative Literacy and Mathematical Competencies. In B.L. Madison, L.A. Steen (Eds) *Quantitave Literacy: why Numeracy Matters for Schools and Colleges.* Princeton: National Council on Education and the Disciplines, 215-220.

Niss, M., Høgaard, T. (eds) (2011) Competencies and Mathematical Learning – Ideas and inspiration fot the development of mathematics teachind and learning in Denmark, English Edition. Roskilde University.

Acknowledgement. *Financial support of the Erasmus+ project 2017-1-ES01-KA203-038491 "New Rules for Assessing Mathematical Competencies" is gratefully acknowledged.*

RULES_MATH: New Rules for assessing Mathematical Competencies

Araceli Queiruga Dios[1], Deolinda M. L. D. Rasteiro[2]

[2]*Department of Applied Mathematics, University of Salamanca, Spain*

[2]*Department of Physics and Mathematics, Coimbra Engineering Institute, Portugal*

Abstract

With the starting point of the Framework for Mathematics Curricula in Engineering Education, developed by the SEFI-Mathematical Working Group, a consortium made of higher education teachers from 8 different European countries work together in new rules for assessing mathematical competencies. We present here the first outcome of the project: the learning and teaching platform that will be a central part of the project development.

Introduction

In recent years, one of the university teachers' goals is to change the educational paradigm and make the teaching and learning processes in accordance with students' needs of meaningful learnings. The way of teaching mathematics becomes different depending on the degree. Mathematics for a mathematician has a different approach to that of the mathematics for an engineer.

Realistic mathematics education (Heuvel-Panhuizen and Drijvers, 2014), problem-based learning (Mills and Treagust, 2003), computer-aided learning, and some other methodologies are used to try to incorporate competencies-based methodologies into engineering curriculum.

In mathematics contexts, students could not solve problems if we make small changes, such as the name of variables, the environment of the problem, or the problem statement. Students seem to be "mechanical" actuators repeating some known procedures to solve problems or patching various parts of previous solutions together to match the new problem situations (Woods et al., 1997). The main goal of the majority of students seems to be to end its degree as soon as possible not dedicating the sufficient amount of time to understand how the process evolves starting with the problem, the concepts that are important to obtain and how they can be applied to obtain the solution.

With this in mind, 9 institutions from 8 countries have joined to address the objective of working towards a common way of teaching and assessing mathematical competencies: the Institute of Mathematics and Physics from Slovak University of Technology in Bratislava, Gazi University in Ankara, the Czech Technical University in Prague, the Faculty of Mathematics and Informatics from University of Plovdiv Paisii Hilendarski, the Spanish National Research Council (CSIC) in Madrid, the Coimbra Polytechnic - ISEC, the Dublin Institute of Technology, the Technical University of Civil Engineering Bucharest, and the University of Salamanca.

The main objective of the Rules_Math project is to develop assessment standards for a competencies-based teaching and learning system specifically designed for mathematics in engineering education. From September, 2017, to September, 2020, we plan to work in 3 specific objectives: (1) To develop a collaborative, comprehensive and accessible competencies-based assessment model for mathematics in engineering context, (2) to elaborate and collect the resources and materials needed to devise competencies-based assessment courses, and (3) to disseminate the model to European higher education institutions through the partner networks and also promote the dissemination all over Europe.

To address these aims and objectives we will use a competencies-oriented methodology. Generally speaking, preparation for a mathematics teaching profession is completely insufficient if it is just about acquiring mathematical mastery, no matter at what level this occurs. Usually teachers of high school mathematics universities and institutes pursue different major from the ones that intend to be school teachers. Thus pedagogic and didactic disciplines do not make part of their formation curriculum resulting, sometimes, if there is not the necessary self-learning ambition, in a relation teacher/student that is not complete.

The institutions involved in Rules_Math project have long experience in innovation and they have adapted their degrees to the Bologna Accord. Despite the differences in teacher training and the organizational frameworks for teaching mathematics, there is a great similarity in the problems, perspectives and discussions regarding mathematics teaching in the different partner institutions. Not only one can learn from the good ideas of other colleagues, but also improve the teaching-learning systems and processes. In this way, from an international perspective, teaching engineering mathematics is a global laboratory which can be of advantage to our objectives.

Rules_Math project pretends to renew existing forms of teaching mathematics as a way to strength education and training paths of educators, and the achievement of high quality skills and competencies. This will be done using digital resources and online platforms.

Methodology

Engineering students are not going to become mathematicians or chemist, so the way of learning maths and acquiring maths competencies is different.

With the starting point of the Framework document from Alpers et al. (2013), the proposed methodology includes on one side the identification of the competencies-based teaching and learning methodology. As the final goal is the establishment of rules to assess mathematics, the first step is the use of competencies-based techniques and activities during lectures and for the whole courses. Furthermore, the identification of components and description of competencies-oriented learning activities will allow to define the working context.

On the other hand, we propose to share examples about contents, competencies and maths applications, from partners' experiences and that work has already been done. This will establish the start point to work together in the definition of assessment rules and standards.

The methodology that we follow for the development of Rules_Math project could be summarized in 3 steps. The first one is a discussion about our current situation: What we are doing when we teach mathematics in engineering degrees. After the Bologna changes, we started to move from a contents-based to a competencies-based system. We are not teaching maths as we have learned maths. As an example, we found a linear algebra book specifically design for engineering students where the definition of a symmetric group over a set with n elements as the group of bijections of a set with n elements, $X_n = \{x_1, x_2, ..., x_n\}$, i.e. S_n = Biject (X_n) is the start point for the definition of a determinant. After some other definitions of permutations, theorems with the corresponding demonstrations, etc., we got the definition of the determinant. It is important to be rigorous when presenting contents that is a fact, but in the very beginning is the way referred in the previous example the most correct one? How many students will stay inside the classroom after the first class?

Once we know what we are doing, we planned to analyze what other educators are doing. The PISA/OECD competencies (2009), based on the Danish KOM project (Niss, 2009), are widely used to assess young students in a “non-classical” way. Some EU countries use the competencies-based system for secondary and high school. In fact, some recent laws force to competencies-based assessment. More recently (Niss et al., 2017) formulated the questions “What does it mean to possess knowledge of mathematics? To know mathematics? To have insight in mathematics? To be able to do mathematics? To possess competence (or proficiency)? To be well versed in mathematical practices?” and gave a big insight to this discussion. They attempted to present significant, yet necessarily selected, aspects of and challenges to what some call “the competency turn” in mathematics education, research and practice.

The third phase of the Rules_Math project will be the design of competencies-based courses that will be implemented during the 2018-2019 academic year. This will allow us to test our proposals and redraw conclusions about the procedures proposed, as long as to choose among all the ones that best fit our objectives.

Conclusions

The mathematical education will be integrated in the surrounding engineering courses to really achieve the ability to use mathematics in engineering (and real) contexts. Synergies between teaching, research and innovation will be established. The new approach for mathematics education will allow linking higher education institutions and local communities and regions. Innovative approaches to improve the relevance and high quality of curricula, including using information and communication technologies and open educational resources, are of great importance for this project.

The competencies-based scheme that is proposed in this project will support teachers, trainers, and educational staff in improving the use of ICT-based resources for teaching and learning. This project will develop a common learning competencies-based environment for mathematics for engineering students in Europe, promoting the use of learning outcomes in the design, delivery and assessment of the curriculum in favour of students and trainers. Furthermore, several basic and transversal skills and competencies will be developed to contribute to the development of a European area of common education.

As the assessment rules defined in this project will be valid for the whole EU, the validity of learning is guaranteed. The quality of higher education will be improved. The Rules_Math project (https://rules-math.com/) will serve to integrate good practices and innovative methods, elaborated by the partnership of the project, from local to European level and enlarge the results into activities in the all project partners' universities.

Acknowledgment

This work has been co-funded by Erasmus+ programme of the European Union under grant 2017-1-ES01-KA203-038491 (RULES_MATH).

References

Alpers, B. et al. (2013) “A Framework for Mathematics Curricula in Engineering Education.” *SEFI.* Available at: http://sefi.htw-aalen.de/

Heuvel-Panhuizen, M.V. and Drijvers, P. (2014) *Realistic mathematics education. Encyclopedia of Mathematics Education*, 521–525. Springer.

Mills, J.E., and Treagust, D.F. (2003). “Engineering education—Is problem-based or project-based learning the answer.” *Australasian journal of engineering education*, 3(2): 2-16.

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagatsis, S. Papastravidis (Eds.), *3rd Mediterranean Conference on Mathematics Education*, Athens, Greece: Hellenic Mathematical Society and Cyprus Mathematical Society, 115-124.

Niss M., Bruder R., Planas N., Turner R., Villa-Ochoa J.A. (2017) Conceptualisation of the Role of Competencies, Knowing and Knowledge in Mathematics Education Research. In: Kaiser G. (eds) Proceedings of the 13th International Congress on Mathematical Education. ICME-13 Monographs. Springer, Cham

OECD (2009). PISA 2009 Assessment Framework – Key competencies in reading, mathematics and science (www.oecd.org/dataoecd/11/40/44455820.pdf, accessed on 17 May 2011).

Woods, D.R., Hrymak, A.N., Marshall, R.R., Wood, P.E., Crowe, C. M., Hoffman, T.W., and Bouchard, C. G. (1997). “Developing problem solving skills: The McMaster problem solving program.” *Journal of Engineering Education*, 86(2): 75-91.

The use of Mathematics Skills in Economic Disciplines

Bobalová Martina and Novotná Veronika

Faculty of Business and Management, Brno University of Technology, Czech Republic

Abstract

In courses at economic or technical universities, mathematics plays a significant role, which is mainly apparent in the first years of study as they cover most of the mathematics courses. Mathematics courses are presumed to be the most difficult ones in terms of the results achieved by the students and they may be considered one of the main reasons why the number of students in the first or sometimes second year of study decreases. Usually, the significance of mathematics courses in the schooling of the future graduate is not questioned, yet in the long-term perspective we can see that the number of classes dedicated to mathematics is going down and this results in a reduction of more problematic parts. This process is often associated with the idea of making these courses easier to pass, but the experience shows us that this idea is mostly wrong.

When preparing the curriculum for math courses at any higher education institution it is important to assure the right flow of subjects. In math teaching, the motivation of students is vital. Students should know why they learn. One of the techniques strengthening the motivation to study mathematics is to teach it within the common framework with subjects that make use of it and to use math as a tool wherever the specialized subjects need to use it.

Introduction

The teaching of mathematics at schools of all grades has been facing many problems for years. The subject perceived as one of the unquestionable bases of education for decades got to the verge of public interest. At schools, math is often seen as unpopular, boring, difficult, detached from life, and even useless.

These troubles are naturally reflected in the higher education teaching. Slightly more favorable is the situation of fields of study directly linked with mathematics and usually attended by students who like it. But we now see problems in technical studies where students struggle with math although many faculties have mitigated their requirements compared to the past. The least favorable situation is in humanities, such as economics, sociology, law, etc. Here, the most frequent opinion is that math (statistics, logic, …) is a waste of time and we can hear it from students and sometimes even the teachers.

Hence, the question is how to prepare the teaching of math mainly in fields of study for „non-mathematicians“, how to make it more attractive for the students, make it easier to understand for them, and primarily how to show them that math can play an important role in life, as a discipline that offers various tools usable in practice and as a field of study that refines systematic and logical thinking and develops the ability of objective perception of reality and facts, and last but not least facilitates a creative approach to solve problems of everyday life.

One of the possibilities how to present mathematics to students in an easier way and how to stimulate their creativity is to use the tools offered by ICT.

The use of ICT in math teaching has got to the center of interest of math teachers. Some of the ICT tools are now a natural part of math teaching: graphic calculators and computerized graphing, specialized software, programmable toys or floor robots, spreadsheets and databases.

The integration of computers and all ICT in the process of teaching may have different quality and forms. We can see the computer as a tool by which the teacher prepares the teaching aids, as well as an environment the whole class works in to solve a specific mathematics problem.

Basic Mathematics Tools Used in Economics Teaching

The objective of basic economics courses is not to prove by using sophisticated mathematics methods how variable X affects the development of variable Y, but to familiarize students with the basic economic concepts, to make them capable of explaining what the subject of its research is and why it is good to study economics. On the other hand, in many cases it is not possible to avoid using mathematics apparatus. For basic economics courses it is vital for the student to have skills for the solution of a system of equations in two unknowns and above all to be able to „read" diagrams. These two capabilities are the unconditional basis without which the students of basic economics courses will not do.

Basic courses are usually taught at the bachelor grades of universities, advanced courses in the master's degree studies. The students´ ability to use mathematics as a tool for the solution of economic models is not sufficient although students consider math and statistics courses important. It is shown by the fact that students believe that the practical usage of math and statistics will not be negligible. Problems come up mainly in the usage of basic mathematics knowledge and its application to economic problems. Students often have pointless prejudices and fear to interlink mathematics knowledge with the economics and give up early and easily. Students of the basic microeconomics courses most frequently face the problems of reading a diagram showing a function. Demand can be a good example. If we define demand as the consumer´s willingness to buy a specific quantity of goods at various prices it does not need to be understandable for the students. However, if we give a specific example, such as the number of theatre visits based on the ticket price (Figure 1), the demand will be easier to understand.

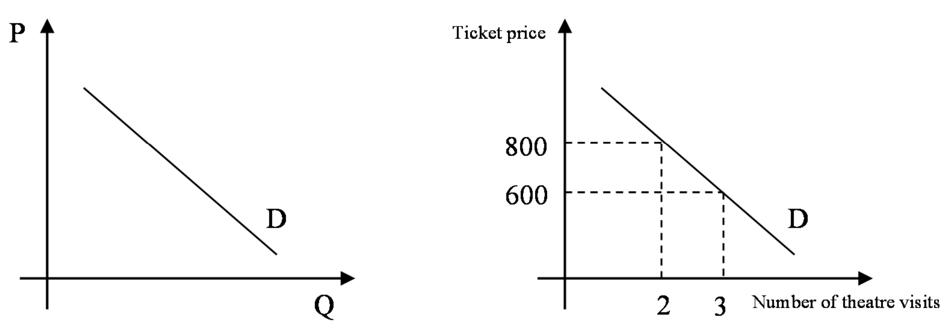


Figure 1. Demand function

Another problem is the confusion of the slope and elasticity of a function. Students generally tend to see these two quantities as one. It is known that the slope of a function speaks about the ratio of absolute changes, while the elasticity relates to the ratio of variable changes. The explanation of the difference between slope and elasticity may again be shown on the example of the demand function, see Figure 2.

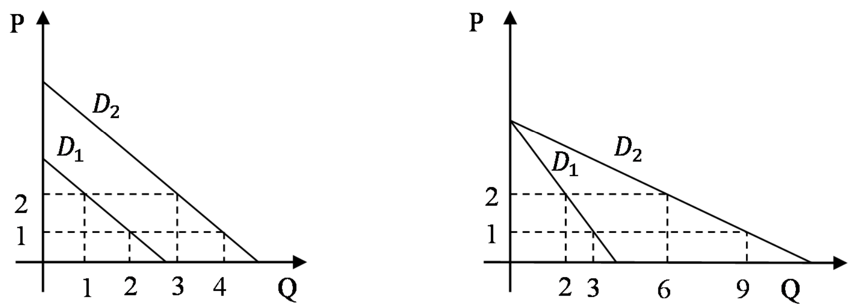


Figure 2. Demand function - elasticity

Figure 2 depicts two situations. On the left, there are two demand functions which are parallel and thus have the same slope all along their length. But their price elasticity is not the same. Demand D_1 is more elastic, i.e. more flexible in price. On the right, however, it is an example of divergent demand functions, which have different slopes but the same price elasticity.

In the case of medium-advanced microeconomics courses students should have mathematics skills mastered by high school graduates as standard. These courses are taught mainly in the subsequent master's degree studies. Particularly for optimization problems it is necessary to use derivations to find the extremes of utility, production, profit or cost functions, but also to solve problems where we measure demand elasticities, from the values of which we subsequently draw conclusions about the examined goods and services.

In optimization problems it is important for the students to understand and apply the Lagrange multipliers method. As a model example we can use the finding of a maxima of the utility function $U(Q_1, Q_2)$ constrained by the condition in the form of a budget constraint $g(Q_1, Q_2) = Y - P_1Q_1 - P_2Q_2$. Here, to get a better idea of the solution, it is convenient to show the constrained maximum in graphic form, see Figure 3.

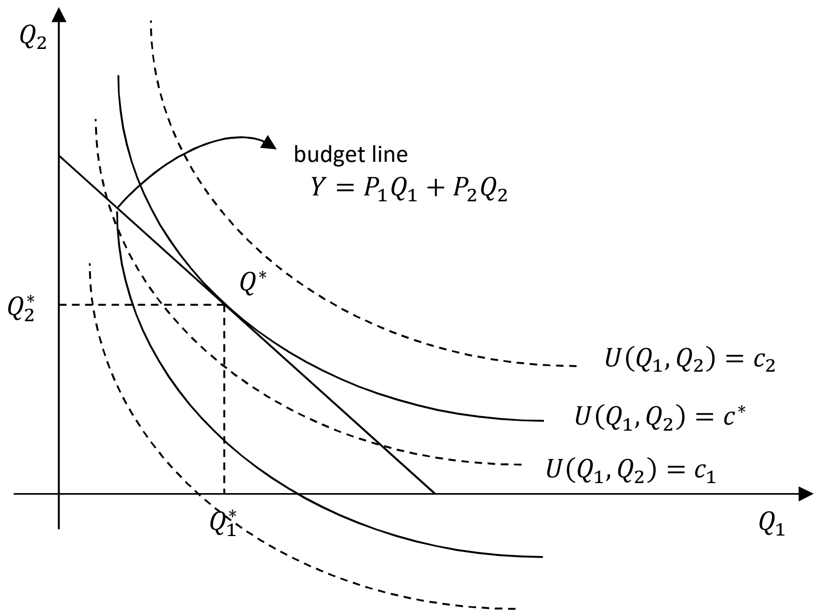


Figure 3. Constrained maximum

In the geometrical interpretation (Figure 3), the solution is point $Q^*[Q_1^*, Q_2^*]$. It is the point of contact of the budget line $Y = P_1Q_1 + P_2Q_2$ and the utility curve $U(Q_1, Q_2) = c^*$.

In advanced courses, students meet functional differential equations and their applications. Mathematical models describing regular problems of everyday life include applications of functional differential equations.
Mathematical models using functional differential equations with delay may be now applied in areas where it would not be expected several years ago. For example, in medicine they are used to describe the process of effect of substances on the organism or spreading of contagious diseases. In ecology, we know models describing the population of a single species, as well as models of multi-species population dynamics. In physics, there are models describing the pantograph motion on the track, or nuclear reactor dynamics. In microeconomics, there are models describing the fluctuation of prices, and in macroeconomics there are the economic cycle theories.

The primary task of the teaching of economic subjects at the university should be to provide knowledge but along with it also to stimulate the interest in applying the acquired knowledge in practice. And on the contrary, the practice should not underestimate the scientific approach – it should look for educated thinking people capable of handling their knowledge pragmatically, used to be responsible, independent and creative, working efficiently. Economic profile disciplines give students many possibilities for the selection of topics and writing of theses. The using of mathematics disciplines in connection with the knowledge from practice-oriented subjects when writing theses at the university gives students of higher grades enough room for their self-realization and along with the numerous ICT tools forms a background to deepen and strengthen the knowledge acquired.

ICT Usage

There are three broad categories of the applications of computers in the field of mathematics education: computer assisted instruction (CAI), student (educational) programming and general purpose educational tools such as spreadsheets, databases and computer algebra systems (CAS). (Aydin,2005)

At present, it appears as very beneficial for university teachers to put information technologies and mathematics together to show the students the possibilities how to use information technologies in mathematics.

ICT technologies may be perceived as

a) a tool for teachers

The easiest form of using computer technology in mathematics classes is to use the computer as a tool for the teacher to prepare for the class. In a period when the university teacher becomes an editor and often also the publisher of its textbooks and reading materials, both in printed or more often in electronic form, the foregoing skills are considered the minimum standard.

b) a tool for demonstrations

The computer may be used as a visual demonstration tool in mathematics classes, by which the teacher presents new pieces of knowledge. For computer supported classes you need an equipped room (computer + interactive board or projector), which is now a standard at higher education institutions.

c) practical aid for exercise

The usage of computers as practical aids used by the students in mathematics classes is the most difficult form of instruction in terms of its requirements. This kind of instruction must take place in a fully equipped computerized classroom, where each student, or a pair of students in the worst case, has a computer of its own. However, the capacity of computerized classrooms at schools often does not cover the teachers´ requirements, and then preference is given to specialized subjects.

Conclusions for Education

The crucial point is that when studying mathematics, students often do not see any practical application of what they are supposed to learn. And this may generate the prejudices against mathematics as mentioned above and lead to the fact that economics is frequently seen as another mathematics-oriented subject. But economics is a science about human acting and mathematics is only a tool for a better understanding of the relations between real economic phenomena.

The solution could be to "humanize" mathematics and to explain and justify consistently why students are required to know how to solve equations, examine the behavior of a function, to derive, or else integrate. Students should know that it is not just learning for the sake of learning something they will not use in further studies or in the practice. Likewise, the same appeal goes to the teachers of economics to be consistent in explaining, demonstrating on examples and pointing out to relevant relations any time they try to enrich the students´ knowledge by such a fascinating science the economics certainly is. Mathematics will certainly never create a universal model of human behavior, but it surely is a very useful science that enriches the economics and helps us understand real economic phenomena, explain them and predict how this or that measure or this or that change will develop, or how the economic reality will be affected when external conditions change.

References

De Witte, K. et al. (2015) *The effectiveness of a computer-assisted math learning program.* Journal of Computer Assisted Learning, 31(4), pp. 314-329. Accessed via http://doi.wiley.com/10.1111/jcal.12090.

Dockendorff, M. & Solar, H. (2017). *ICT integration in mathematics initial teacher training and its impact on visualization: the case of GeoGebra.* International Journal of Mathematical Education in Science and Technology, 49(1), pp. 66-84. Accessed via https://www.tandfonline.com/doi/full/10.1080/0020739X.2017.1341060.

Dostal, P. et al. (2016) *Multicriteria Evaluation of Gifted Students.* In Innovation Management and Education Excellence Vision 2020: From Regional Development Sustainability to Global Economic Growth. Milan, Italy: IBIMA, pp. 262-268.

Drábeková, J. & Švecová, S. (2015) *Some Notes on the Financial Literacy of the Slovak Students – Case Study.* Mathematics in Education, Research and Applications, 01(02), pp. 68-72. Accessed via http://meraa.uniag.sk/docs/2015-02/drabekova-svecova.html.

Jehlička, V. & Rejsek, O. (2018) *A Multidisciplinary Approach to Teaching Mathematics and Information and Communication Technology.* Eurasia Journal of Mathematics, Science and Technology Education, 14(5), pp. 1705-1718. Accessed via http://www.journalssystem.com/ejmste/A-Multidisciplinary-Approach-to-Teaching-Mathematics-and-Information-and-Communication,85109,0,2.html.

Misut, M. & Pokorny, M. (2015) *Does ICT Improve the Efficiency of Learning?* In Procedia - Social and Behavioral Sciences. Las Vegas, NV: GLOBE-EDU, pp. 306-311. Accessed via http://linkinghub.elsevier.com/retrieve/pii/S1877042815017000.

Mathematics competencies in higher education: a case study

Cristina M.R. Caridade & Deolinda M. L. D. Rasteiro

dml@isec.pt;caridade@isec.pt

Coimbra Polytechnic - ISEC, Coimbra, Portugal

Abstract

The mathematics competencies in higher education are so important like the learning of mathematical concepts, aptitude and skills for engineer students. The mathematics competencies were identified by Niss who refers to them has being: the ability to understand, judge, do and use mathematical concepts in relevant contexts and situations, which is the predominant goal of mathematical education for engineers. This study pretends to demonstrated and recognize what are the competencies that engineering students should have or, acquire, when new mathematic contents are taught to them. A questionnaire was performed (with the same questions) before and after mathematics contents were presented to the students. Then, and according with the competencies defined by Niss, we analysed the students' acquisition perception of competencies regarding mathematics that were taught to them. The results from the questionnaires indicate that all the students have acquired the intended competencies, although some more than others. Some of Niss's eight mathematics competencies were more developed than others. The most relevant elements in mathematical competencies are the interaction with the problem, its comprehension, how to describe the problem in mathematical form and its resolution.

Introduction

In higher education, mathematics has an important role in engineering courses (OECD (1996)). From the curriculum of the first year there are Curricular Units (CU) in the area of Mathematics that are fundamental for students to acquire the necessary basic knowledge to most specific CU of each course. Without this well-established mathematical foundation, success in applied CUs is seriously compromised. During an Engineering course, students learn and consolidate the basic principles of mathematics to solve practical problems, reinforcing their conceptual mathematical knowledge. However, although mathematics is a basic discipline regarding the admission to any Engineering degree, difficulties related to mathematics' basic core are identified by almost all engineering students at each CU. In this context, it seems relevant to identify the mathematics competencies attained by engineering students so that they can use these skills in their professional activities.

Mathematics competencies is the ability to apply mathematical concepts and procedures in relevant contexts which is the essential goal of mathematics in engineering education. Thus, the fundamental aim is to help students to work with engineering models and solve engineering problems (SEFI (2011)). According to Niss (2003) eight clear and distinct mathematics competencies are: thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modelling mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating in, with, and about mathematics and making use of aids and tools.

Gaps were detected between engineers' required mathematics competencies and acquired mathematics competencies of engineering students under the current engineering mathematics curriculum (Firouzian (2016)). There is a need to revise the mathematics curriculum of engineering education making the achievement of the mathematics competencies more explicit in order to bridge this gap and prepare students to acquire enough mathematical competencies (Rules_Math Project, (2017-2020)). Hence an important aspect in mathematics education for engineers is to identify mathematical competencies explicitly and to recognize them as an essential aspect in teaching and learning in higher education. It is the fundamental that all mathematics teaching must aim at promoting the development of pupils' and students' mathematical competencies and (different forms of) overview and judgement (Niss (2011), Alpers (2013), Rasteiro, D. D. (2018)).
This research pretends to evaluate and recognize what are the competencies that engineering students can have or, acquire, when new mathematic contents are taught to them. A questionnaire approach was performed before and after mathematics contents were presented, with the same questions. Then, and according with the competencies defined by Niss, we analysed the perception of students, in the acquisition of the taught competencies regarding mathematics.

Description of the study

During the first semester of the 2017/2018 academic year, in the Calculus I course (first year) of Electrical Engineering in Coimbra Institute of Engineering, we proposed to present the improper integrals contents in a different way. Using a graphical visualization problem the theoretical content was presented and explored. Then and for the first time students were introduced to the concept of improper integral using a real and very informative example.

The surface of revolution obtained by revolving the hyperbola $y = 1/x$ around the x-axis cut off at $x = 1$ is known as Torricelli's Trumpet and also as Gabriel's Horn (Côté (2013)) represented in figure 1.

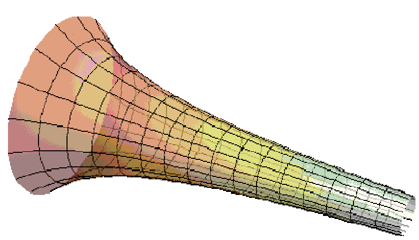


Figure 1. Torricelli's Trumpet (image from https://www.cut-the-knot.org/Outline/Calculus/TorricellisTrumpet.shtml.)

Playing this instrument poses several challenges: 1) It has no end for you to put in your mouth; 2) Even if it did, it would take you till the end of time to reach the end; 3) Even if you could reach the end and put it in your mouth, you couldn't force any air through it because the hole is infinitely small; 4) Even if you could blow the trumpet, it would be kind of pointless because it would take an infinite amount of time for the sound to come out. Other additional difficulties occur (infinite weight, does not fit in universe …), that you can imagine (Bogomolny (2018)).

The volume of Torricelli's Trumpet is given by the integral:

$$V = \pi \int_{1}^{+\infty} \left[\left(\frac{1}{x} \right)^2 - 0^2 \right] dx = \pi \int_{1}^{+\infty} \left(\frac{1}{x} \right)^2 dx$$

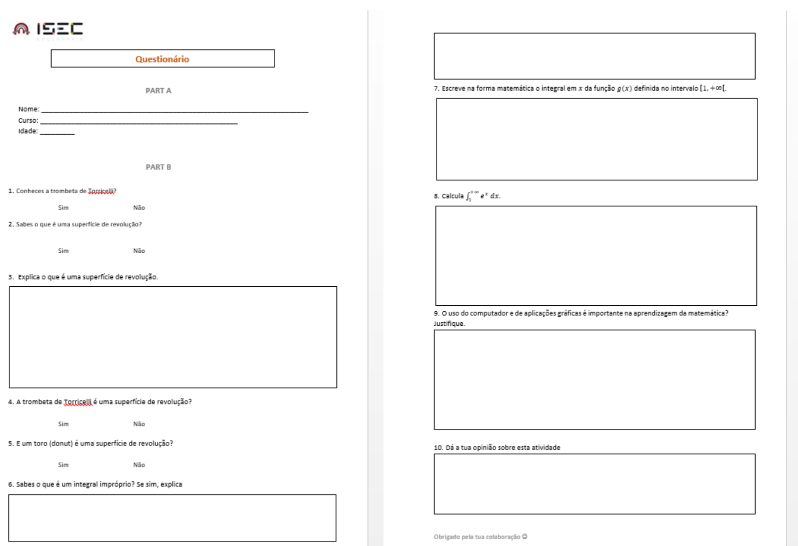
and its area by

$$A = 2 \int_{1}^{+\infty} \frac{1}{x} dx \,.$$

Both integrals are improper once their integration interval is infinite. Despite the fact that Torricelli's Trumpet has a finite volume, it has an infinite surface area and that is something that always leaves students astonished! The volume of Torricelli's Trumpet and its area can be presented to the students as an applied example of the improper integral concept. Thus, students can explore the inherent mathematical concept and mathematical competencies that were developed by them.

Findings and Discussion

In order to evaluate the effectiveness of the proposed activity in the development of the mathematics competencies in the concepts of improper integrals and the students' interest in its realization, data was collected based on the development of a questionnaire (Figure 2).


ISEC

Questionário

PART A

Nome: ____________________
Curso: ____________________
Idade: ________

PART B

1. Conheces a trombeta de Torricelli?

Sim Não

2. Sabes o que é uma superfície de revolução?

Sim Não

3. Explica o que é uma superfície de revolução.

4. A trombeta de Torricelli é uma superfície de revolução?

Sim Não

5. E um toro (donut) é uma superfície de revolução?

Sim Não

6. Sabes o que é um integral impróprio? Se sim, explica

7. Escreve na forma matemática o integral em x da função $g(x)$ definida no intervalo $[1, +\infty[$.

8. Calcula $\int_1^{+\infty} e^x \, dx$.

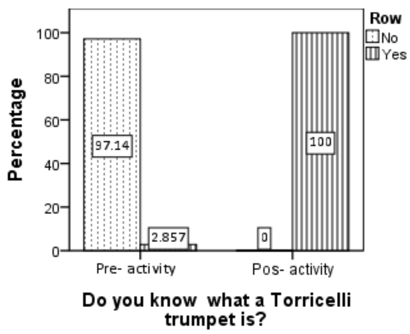
9. O uso do computador e de aplicações gráficas é importante na aprendizagem da matemática? Justifique.

10. Dá a tua opinião sobre esta atividade

Obrigado pela tua colaboração ☺

Figure 2. Questionnaire.

The mathematics competencies questionnaire is composed by ten questions about what a revolution surface means, improper integrals and Torricelli's trumpet; how we can describe and represent it mathematically, how we can model it, and how can we resolve it using or not graphical applications and computers.



The first two questions aim to recognize if the student knows what the Torricelli's trumpet and a surface of revolution is. In the third question the aim is to realise if the student is able to define what a revolution surface is. The fourth and fifth questions are intended to find out whether students associate Torricelli's trumpet and torus with a revolution surface. The sixth question refers to the definition of improper integral and the student is asked to explain what an improper integral is. The seventh and eighth question refer to how an improper integral is represented and how it is calculated. Question 9 asks about the importance of graphics and computer applications in mathematics learning. Finally in question 10 the students can give their opinion about the activity developed.
The mathematical competencies that authors recognise to be present at the above referred questionnaire are listed in table 1.

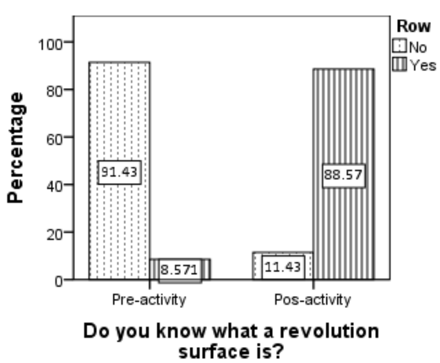
Table 1: Distribution of mathematical competencies by questionnaire questions

Mathematical competencies	Questions
Thinking mathematically	Q3, Q6
Posing and solving mathematical problems	Q8
Representing mathematical entities	Q7
Handling mathematical symbols and formalism	Q7
Communicating in, with, and about mathematics	Q6

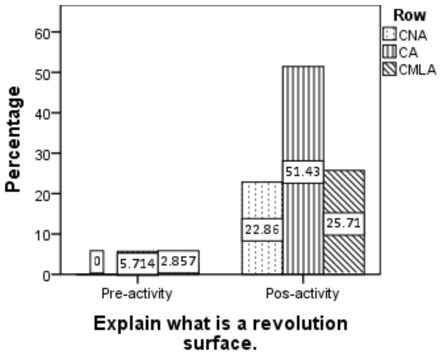
The questionnaire was applied to 35 students of the Calculus I. The same questionnaire was handed twice, one before the activity (pre-questionnaire) and the other at the end of the activity (post-questionnaire).

The data obtained from students was analysed and is presented below. The results obtained before and after the activity will be presented together.

On the first question students were asked if they knew what a Torricelli trumpet was and only approximately 3% of them did. After the activity all of them knew.

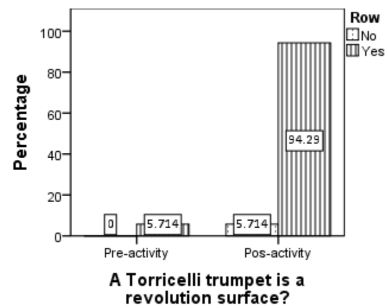


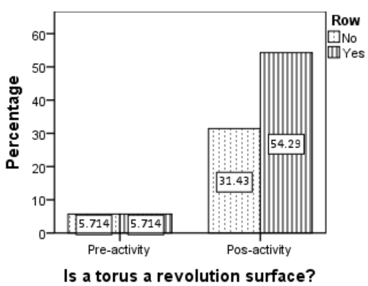
The second question was not successful after the activity as the first one. It was intended to transmit the concept of a revolution surface. Initially 91.43% of the students did not know what a revolution surface. After the activity 88.57% of the students acquire the knowledge but 11.43% did not.

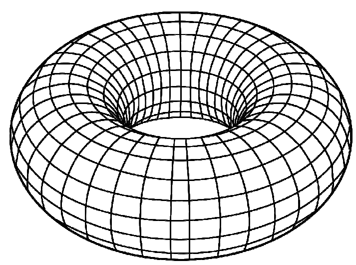


On the third question students were asked to explain, using their own words, what a revolution surface is. Before the activity 5.714% of the students knew (CA), yet after the activity only 51.43% of students were able to define it (CA) and 25.71% of them were able to give an idea, although not completely correct, of the concept (CMLA). Work has to be done in class with the students in order to completely achieve the initial purpose and understand what kind of misunderstanding happen.

On the fourth question the students had to be able to relate Torricelli trumpet with revolution surface. Is a Torricelli trumpet a revolution surface? Before the activity 5.714% students answered "No", yet after the activity 94.29% of them changed their opinion.

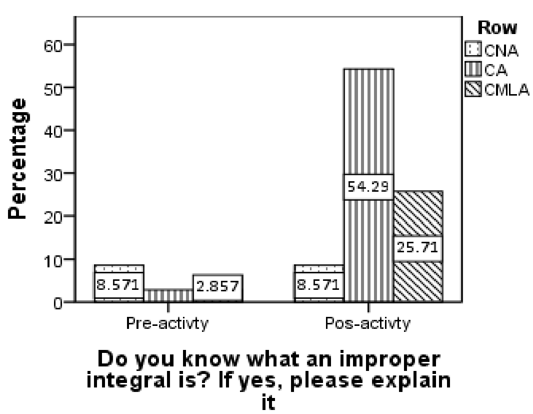




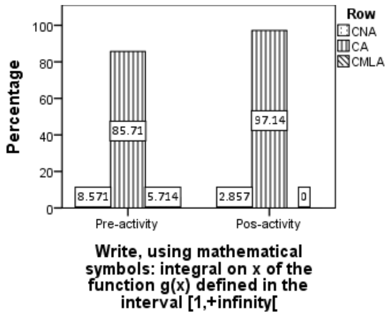


Torus

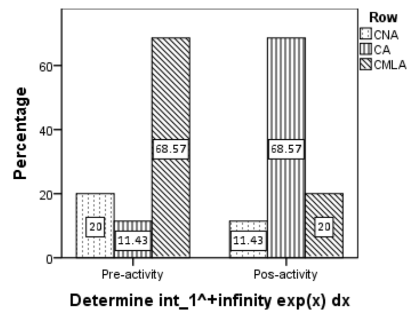
Afterwards, fifth question, the concept of revolution surface was related with other mathematical entity, a torus (the suggestion of a donut was given to students' *imaginarium*). Surprisingly after the activity only 54.29% of the students answered positively to the question –"Is a torus a revolution surface?"



On the sixth question, competencies of thinking mathematically and communicating in, with and about mathematics were tested. The students were asked about what an improper integral is, and were defied to explain it on their own words. The results obtained after the activity were better than before (54.29% knew and 25.71% were almost correct when explaining it) still 8.571% were not able to answer “yes” and explain it. We may say that the concept was learnt (CNA).



The seven question pretended to evaluate the mathematical competencies of representing mathematical entities, handling mathematical symbols and formalism. Almost all students knew, *a priori*, how to answer correctly to this question, only 8.571% of the students failed. After the activity 97.14% answered correctly.



The eight question evaluated the thinking mathematically competence. At the beginning only 11.43% of the students answered correctly but after the activity 68.57% acquired the pretended competence.

Ninth question was an open one. Students were invited to justify their accordance or discordance with the importance of using computers and graphical applications in the learning mathematical process.
Their answer is clear, before and after the activity, they recognize the importance of technology to comprehend mathematical concepts (“Sim" in Portuguese means “YES” in English).

Pre-activity

Pos-activity

Students’ opinion was collected with question tenth. The words that have the highest frequency are “atividade interessante” which means “interesting activity”.

Conclusions

This paper provides an overview of a case study in mathematics competencies in higher education.

The results from the questionnaires indicate that all the students have acquired the intended competencies although some more than others. Some of Niss's eight mathematics competencies are more developed than others. The most relevant elements in mathematical competencies are the interaction with the problem, its comprehension, how to describe in a mathematical approach the problem and how to solve it. The example presented helped to improve the understanding of the theoretical definition of improper integrals and placed the student in a practical and engaging situation of using mathematics. As one of the students said "An enriching situation because I acquired new knowledge that I did not have".

Acknowledgment

This work has been co-funded by Erasmus+ programme of the European Union under grant 2017-1-ES01-KA203-038491 (RULES_MATH).

References

Alpers, B. (2013) "A Mathematics Curriculum for a Practice-oriented Study Course in Mechanical Engineering.", *Aalen University*, Aalen.

Cardella, M. () "Which Mathematics Should We Teach Engineering Students? An Empirically-Grounded Case for Mathematical Thinking.", *Teaching Mathematics and its Applications.* Vol. 27, No. 3, 150-159.

Côté, G. B. (2013) "Mathematical Platonism and the Nature of Infinity.", *Open Journal of Philosophy.* Vol.3, No.3, 372-375.

Firouzian, S., Kashefi, H., Yusof, Y.M., Ismail, Z., Rahman, R.A. (2016) "Mathematical competencies as perceived by engineering students, lectures, and practicing engineers.", *International Journal of Engineering Education*. Vol. 32, No. 6, 2434-2445.

Niss, M. (2003) "Mathematical Competencies and the Learning of Mathematics: The Danish KOM Project.", *In Proceedings of the 3rd Mediterranean Conference on Mathematical education.* Athens: Hellenic Mathematical Society.

Niss, M. (2011) "Competencies and Mathematical Learning Ideas and inspiration for the development of mathematics teaching and learning in Denmark.", *IMFUFA, Roskilde University*.

OECD (1996), "The mathematical education of engineers.", *Report of the Seminar, Paris*.

Rasteiro D. D., Martinez, V. G., Caridade, C., Martin-Vaquero, J., Queiruga-Dios, A. (2018), "Changing teaching: competencies versus contents.", EDUCON 2018 - "Emerging Trends and Challenges of Engineering Education". Tenerife, Spain.

New Rules for assessing Mathematical Competencies ‖ RULES_MATH Project identification: 2017-ES01-KA203-038491; KA2 - Cooperation for Innovation and the Exchange of Good Practices; KA203 - Strategic Partnerships for higher education- https://rules-math.com/

SEFI, (2011) "A framework for Mathematics Curricula in Engineering Education.", *Europe Society for Engineering Education*.

Bogomolny, A. "Cut The Knot Software". https://www.cut-the-knot.org/WhatIs/Infinity/Paradoxes.shtml (visited in April 2018)

Implementing Computer-assisted Exams in a Course on Numerical Analysis for Engineering Students

Karsten Kruse and Christian Seifert

Institut für Mathematik, Technische Universität Hamburg, Germany

Abstract

This contribution reports on implementing computer-assisted exams within a course on numerical analysis for engineering students. As in many courses on applied mathematics, in order to give a glimpse on realistic problems one is faced with large computations which are typically done by computers. However, when it comes to exams on such topics students are often asked to apply the learned methods, which are suited for large systems, to very small problems by pen-and-paper. We will explain how we did overcome this gap by adjusting the examination procedure appropriately. The framework for the exam may also serve as an example which can be easily transferred and adjusted to other courses, institutions and needs.

Introduction and Pros and Cons for e-Exams

Introducing new technologies within teaching is a hot topic nowadays. Especially taking advantage of computers and appropriate software packages in class enables the lecturers to visualize contents, talk about more realistic, i.e. real-world, problems and also to increase the practical relevance. This, however, generates the need to also provide computers during examination. Such exams are then called computer-assisted exams or e-exams, and there are various different functions the computers can take.

The usage of computers during exams has some advantages, see Doukas and Andreatos (2007), Sindre and Vegendla (2015), Conole and Warburton (2015), Küppers and Schroeder (2017), and references therein. First of all, the lecturer can use the computer as a tool and thus the exam can be constructed to fit much better to learning goals, applications and practical relevance. Moreover, the effort on grading can be decreased, either when the computer can grade automatically (see below), but also just because of the fact that typed solutions are much easier to read and thus there is no need to decipher handwriting. If the problem set for an e-exam is generated in an intelligent way, say there is automatic randomization involved, then this set of problems can be reused very easily. Thus, constructing good problems can be very sustainable in the long run. Also, in this way exams can be more comparable during time, and hence long-term evaluations of student developments can be measured more reliable.

Of course, e-exams also have disadvantages. On the one hand, e-exams may tend to standardize the exam in a way that constructiveness is not rewarded any more (or even penalized), for example in case the computer expects a particular form of the solution. In order to overcome this difficulty – which is typically more present in case of automatic grading – the effort to conceive a good e-exam is typically much higher compared to a classical pen-and-paper exam. Also, not all question which can be asked for when using pen and paper can be copied to an e-exam without adjustment. Since software packages may change in time due to updates, one is also in need to check compatibility and adjust the problems and setup if needed. Hence, the lecturer is also in charge to keep up to date with corresponding developments regarding the used setup.

Different Types of e-Exams and Topics to consider

Providing computers during exams enables two basic features, which can also be used to classify e-exams:

A) Use computers and appropriate software packages as a tool to create and solve more advanced problems during exams. Grading is done exclusively by lecturers.

B) Use computers and appropriate software packages as a tool to create and solve more advanced problems during exams. Furthermore, let the computer also automatically grade the solutions. Of course, lecturers can always adjust the automatic grading afterwards.

Hence, the variant B) can be seen to include A), or put differently, A) is a preliminary step to reach B). Note, however, that different tools may be needed for A) and B), respectively. Indeed, for B) one needs a tool to grade automatically the solutions, which in practice may result in different software requirements and thus B) may not just be an upgrade of A). Of course, either variant (or both together) can be combined with classical pen-and-paper exams to obtain hybrid types of exams.

In order to plan computer-assisted exams one has to take into account four major topics:

1. legal topics
2. organizational aspects
3. technical aspects
4. educational topics

Note that these major topics influence each other; for example think about a particular educational concept which requires certain technical features.

Concerning legal topics, first of all the usage of computers during exams should be allowed by the examination regulations. Moreover, the bijective mapping between students and their electronic solutions should be ensured; this can for example be done by means of official student accounts or special examination accounts generated for the particular exam. Moreover, secure storage of the students solution has to be considered. Last but not least, (personal) data protection has to be ensured as well.

The most important organizational aspect is to provide every student with an appropriate computer during exam. This can be achieved by means of special e-examination centers within the university, by using the existing computer labs on campus, by delegating it to a service provider, or by allowing students to bring their own device. In order to ensure equal conditions for all students the “bring your own device” option may be discarded for security and cheating reasons, see also Dawson (2016) and Sindre and Vegendla (2015). For large numbers of students one may need to subdivide the whole group into smaller subgroups in order to have sufficiently many computers available for every examinee. In any case, one may consider to reserve a few spare computers in case of technical troubles at some devices. Also, especially when introducing e-exams for the first time, one may think about an alternative option in case of total breakdown of the computer architecture. Also, in order to test the scenario one may perform a test run in order to train the procedure of how the exam is done.

There are a few technical aspects to consider when planning e-exams. First of all, all the computers used during the exam should be ready-to-use. Also, the possible communication channels of the computers should be blocked such that only the needed features are available. Special environments and browsers can be used as tools to achieve this. Moreover, logins should be provided (this is only applicable in case one uses separate examination accounts). Then, if needed, data sets required for the exam should be provided such that the students can directly start working on the problems. When finishing the exam, the students' solutions should be stored safely and made available to the lecturer. Also, it may be required to log the whole behavior of the examinee during the exam. At least, an auto-safe mode should be enabled to safe the students solutions regularly. There are tools available to provide more advanced logging.

Educational topics include the conception of the whole course and its aligned exam, the choice of software packages, posed question types and a definition of the format of the solutions to hand in. There are various ways ranging from little programs or scripts, calculations, texts, images or graphics generated by help of the software, designs or drawings, generated data sets, or portfolios or collections of it. This heavily depends on the subject of the course as well as the possible software tools and the learning goals to achieve. In case special software packages are used, one may include a tutorial within class in order to train the students on how to use them. For more refined advice one may consult the local centre of teaching and learning.

Introducing Computer-assisted Exams within a particular Course

We introduced computer-assisted exams within a course on numerical analysis for engineering students in winter term 2017/18. The planning process began two month ahead of the first lecture and we settled legal topics and organizational aspects as a start. Our goal was to provide the software MATLAB during the examination to solve practical exercises by implementing short algorithms. Our exam was of hybrid type, i.e. pen-and-paper exercises were combined with electronic ones. For this purpose we gave a MATLAB tutorial in the beginning of the course in order to acquaint the students to MATLAB. In the associated exercise classes the students were assigned tasks and little projects which needed MATLAB to be handled. About four weeks before the examination we did a test run in the designated room for the examination to train the technical procedure and make the students familiar with the circumstances.

At the end of the term we had seventy registered examinees which we divided into three groups a twenty-five persons with thirty laptops being available in our designated room for the exam. Due to the number of available laptops we had to examine the three groups successively. Thus, we created three hybrid exams and one back-up pen-and-paper exam in case of technical issues. These exams were sufficiently different since the first and the third group of examinees had enough time to communicate with each other (during the examination of the second group). Our university data centre generated examination accounts for each examinee and the associated identifier was printed on the exam permitting a one-to-one correspondence between the examinees and their electronic solutions. The laptops were put into a so-called kiosk mode via the Safe Exam Browser of the ETH Zürich which only allowed to access a given website (for downloading data sets and uploading solutions) and to use MATLAB. The data sets which we generated beforehand were located on a webserver created by the university date centre and were provided as a download on this website. The same website was used to transfer the examinees' solutions to the webserver which were equipped with the proper identifier and a timestamp in the process. The examination supervisors had access to the upload directory to ensure that every examinee had uploaded something. Since MATLAB can be used to get access to the whole system of the device where it is installed, the examination supervisors had to guarantee that the examinees do not break out of the kiosk mode (e.g. via the web command in MATLAB). But this was no problem and no additional effort in comparison to a classical exam where one has to prevent attempts to deceive as well.

We needed a lead time of fifteen minutes for each group preliminary to the examination for the usual announcements, the login on the laptops, starting MATLAB and the download of the given data sets. Also, we needed a follow-up time of five minutes after the examination to collect the exams, for the students to upload their electronic solutions (and to check if this was done right) and to log out the users on the laptops. Since we planned with a time of thirty minutes between consecutive groups, this was no problem.

Our costs of introducing e-exams were mainly induced by hiring a half-time research assistant for a period of eight month to carry out the project. We had virtually no other additional costs since we already had the licenses for MATLAB and the hardware needed and the Safe Exam Browser is an open source software.

Student Evaluations and Feedback

An evaluation conducted by our centre of teaching and learning showed that the majority of the students (63.8%) felt well prepared for using computers and MATLAB during the exam. Only a few students expressed the need of more help in MATLAB and more exercises to train for. In contrast, for 53.7% of the students the required MATLAB skills were transparent, for 39.0% at least partially transparent. The technical course of action meant no problem for the participants (only for 2.4%). Almost all students (97.5%) were convinced that the practical implementation of programming matched the course. More than 50.0% of the students wished for more computer-assisted examinations in their course of studies. Finally, 92.5% thought that our project was carried out well and that the resources used were adequate.

How to transfer the Framework to other Courses, Institutions, Countries

In order to transfer the framework developped in our course, we provide a checklist to efficiently work through all the relevant tasks needed for introducing e-exams. This may be seen as a guideline. Of course, it may need to be adjusted corresponding to the general conditions present at the university. We did not include a timeline into this checklist. Clearly, most of the items should be settled before the actual course starts, but can still be adjusted in time.

General tasks before the course starts

☐ Prepare your desired e-examination scenario

Legal topics

☐ Examination regulations checked

☐ Secure storage of students solutions planned

☐ Personal data protection considered

Organizational aspects

☐ University data centre contacted to discuss wishes and needs

☐ Test run planned

☐ Students divided into subgroups

☐ Alternative option conceived

Technical aspects

☐ Computer ready-to-use

☐ Communication channels configured

☐ Examination accounts generated

☐ Initial data sets provided, submission of solutions resolved

Educational topics

☐ Centre of teaching and learning contacted to discuss wishes and needs

☐ Tutorial for software packages provided

☐ Variants of the exam including data sets generated

Discussion and Conclusion

To summarize, e-exams may not always be more efficient compared to classical exams, but the possibility to use computers during exams will enable the lecturer to conceive exams fitting much better to the learning goals of a course, especially in courses on applied topics, since practical relevance can be increased using computers.

Acknowledgement

This project would not have been possible without the help and support of the institute of mathematics, the centre of teaching and learning and the university data centre of the

Hamburg University of Technology. We gratefully thank Patricio Farrell from the institute of mathematics, Katrin Billerbeck, Nicole Podleschny and Timo Kamph from the centre of teaching and learning as well as Frank Lamers, Iman Sayadinejad, Sebastian Eicke and Alexander Lehning from the university data centre.

References

Conole, Gráinne and Warburton, Bill (2005) *A review of computer-assisted assessment.* ALT-J, 13:1, 17-31.

Doukas, Nikolaos and Andreatos, Antonios (2007) *Advancing Electronic Assessment.* International Journal of Computers, Communications and Control, Vol II, No 1, 56-65.

Dawson, Phillip (2016) *Five ways to hack and cheat with bring-your-own-device electronic examinations.* British Journal of Educational Technology, Vol 47, No 4, 592-600.

Küppers, Bastian and Schroeder, Ulrik (2017) *Vergleich von Papierklausuren und elektronischen Prüfungen.* Informatik 2017, Lecture Notes in Informatics (LNI), Gesellschaft für Informatik, Bonn 2017, 307-318.

Safe Exam Browser, Accessed via https://safeexambrowser.org (27 April 2018)

Sindre, Guttorm and Vegendla, Aparna (2015) *E-exams versus paper exams: A comparative analysis of cheating-related security threats and countermeasures.* NISK 2015.

Sindre, Guttorm and Vegendla, Aparna (2015) *E-exams and exam process*

Potatoes drying process – a vehicle to put into practice mathematical competencies in engineering students

Madaleno, R.O. (MSc Student in Chemical and Biological Engineering)
Instituto Politécnico de Coimbra, ISEC, DEQB, Portugal

da Silva, P.M.
Instituto Politécnico de Coimbra, ISEC, DFM and CMUC, Centro de Matemática da Universidade de Coimbra, Portugal

Castro L.M.M.N.
Instituto Politécnico de Coimbra, ISEC, DEQB and CIEPQPF, Chemical Engineering Processes and Forest Products Research Centre, Faculdade de Ciências e Tecnologia da Universidade de Coimbra, Portugal

Coelho Pinheiro, M.N.
Instituto Politécnico de Coimbra, ISEC, DEQB and CEFT, Centro de Estudos de Fenómenos de Transporte, Faculdade de Engenharia da Universidade do Porto, Portugal

Abstract

Identifying a problem, develop and implement a mathematical model to describe the physical phenomena associated, analyse the predictions obtained and gather it with data for model performance evaluation, are fundamental steps for students to build up modelling skills and understand the importance of mathematics as a tool in science and technology education. The work proposed to the MSc student involved in this paper included all of those steps. To improve the student' learning motivation a practical application was selected, the potato drying process.

Assuming that water diffusion within the potatoes' layer is the dominant mechanism in the mass transfer process during the falling drying rate period, a simple model was developed using the classical diffusion equation. A numerical method was used to obtain simulations and the predictions were compared with data obtained from experiments performed in a convective air drier cabinet with controlled temperature and air velocity at 50°C and 1.6 m s^{-1}.

The successful case presented, while integrating mathematical modelling and problem-solving analyses, is very useful for students to realize the importance of integrate subjects/knowledge putting into practice the difference between interdisciplinary and multidisciplinary approaches in applied problems solutions.

Introduction

This work arose from the growing importance of improving students' modelling skills. Bridging "reality and pure mathematics" and learn how to "apply math as an engineer" as reported by Wedelin *et al*. (2015) is certainly an efficient way to improve learning outcomes in engineering students and, in particular, in the Master of Chemical and Biological Engineering students'.

The Master in Chemical and Biological Engineering, from Coimbra Institute of Engineering (ISEC), Polytechnic of Coimbra, initiated in the academic year of 2009/10 with the mission and goal to graduate masters with a high quality preparation and a profile markedly professional. This 2nd cycle aims to deepen the students' knowledge acquired in the 1st cycle in Chemical/Biological Engineering and to develop skills appropriate to the grade of master, in areas oriented for the national and european development. Graduates should be able to exercise their professional activity with a high level of technical and scientific competences and with sense of technical, ethical and social responsibility. Practice-oriented courses with laboratory activities integrate the 1st year master syllabus. The 2nd year of the master is mainly devoted to Thesis/Project/Internship, corresponding to 54 ECTS. Whenever possible the students have an internship in industrial environment or stay in the academic institution to initiate an applied research study. This practice has proven to be a great instrument to improve students' skills and practical competencies. The field of intervention of graduates is wide, going from the production of cosmetics, food and beverages, pharmaceutical products to the production of pulp and paper, petrochemicals and polymers, reflecting the broadband academic formation in chemical and biological processes.

This paper describes the work carried out by one MSc student in the sequence of her dissertation developed in the applied research and development department on a multinational industry concerning the effect of pre-treatments and post-treatments on drying food products. The proposal of this work arose from the increasing effort to show the importance of interdisciplinary to the students using emergent topics, as is the case of food dehydration as a technique of product preservation.

The Problem

The air drying process applied to food industry is a procedure commonly used to preserve food, extending the shelf life of a product. If the moisture content in foodstuffs is reduced to a level under the minimum necessary for microbial growth or spore germination the microbial activity is inhibited. For that reason, dehydration of food products using a drying process is a frequent operation in the industry to preserve the product quality. In the majority of industrial dryers (more than 85% according to Mujumdar, 2006) the heat is supplied to the humid solid by convection from a hot stream contacting directly the product to be dried. This convective type of driers uses normally a hot air stream as drying medium, as is the case of the tray driers where the air flows tangentially to the humid solid surface.

Drying processes of humid products as foodstuffs, in general, encompasses different periods. After an initial period where the solid thermal conditions are adjusted, the moisture in the surface starts to be removed resulting in a period of constant drying rate (evaporation rate, per unit of drying area), under constant drying conditions. In this period, the unbound moisture is removed until the critical moisture content in the solid is reached. Hence, starts the falling drying rate period where physically/chemically bound moisture to the product will be take away from the solid. While bound moisture is removed, the evaporation rate decrease continuously and a falling drying rate period will be observed. The drying process stops when the moisture vapour pressure in the solid equals the vapour partial pressure in the gas, i.e. the equilibrium conditions between the moisture content in the solid and air humidity under the prevailing conditions is reached.

Understanding the transport phenomena associated to the drying operation, modelling it using simplified situations, obtaining predictions by numerical methods and comparing simulations with data from experiments, it will be a good way for the student to recognize the importance of subjects' integration and counteract the tendency to compartmentalize topics. Moreover, this applied problem results in an extra motivation for the student and contributes to enlarge versatility and critical thinking skills.

This work is devoted to the drying process of fresh potatoes (*Red Lady*) with the aim to study the water diffusion within it cellular structure to the surface exposed to the hot air stream, followed by evaporation. Simultaneously it served as a vehicle to put into practice and develop student mathematical competencies.

The Physical Situation

Drying is a complex unit operation comprising heat and mass transfer. The water transfer through the solid may occur by different mechanisms: liquid or vapour diffusion, capillary or hydrodynamic flow due to pressure gradients developed in the solid during drying. Also, associated to drying it could be observed changes in the physical structure of the solid, as shrinking, glass transition and puffing, which greatly influence transport parameters like diffusivity.

During constant drying rate period, the controlling steps are external heat and mass transfer rate and the process is determined by the air stream conditions. For the falling drying rate period, water/vapour movement by diffusion and capillarity towards the solid surface are the rate-controlling steps, whereby internal heat and mass transfer rates are the determinant mechanisms in moisture removal.

Assuming that water diffusion within the potatoes' layer is the dominant mechanism in the mass transfer process during the falling drying rate period, a simple model was developed using the classical unsteady state one-dimensional diffusion equation.

The Mathematical Approach and Numerical Resolution

Ignoring potatoes' shrinkage during drying and the diffusivity (D) dependence with solid moisture content (X), the Fick's second law for direction z throughout the layer thickness of potatoes is given by:

$$D\frac{\partial^2 X}{\partial z^2}=\frac{\partial X}{\partial t} \quad . \qquad (1)$$

The initial and boundary conditions required to completely describe the mass transfer problem are:

$$X(z,\, t=0)=X_i\,, \qquad -D\frac{\partial X}{\partial z}(z=0,\, t)=0 \qquad \text{and} \qquad X(z=\ell,\, t)=X_e \qquad (2)$$

indicating, respectively, that: the initial moisture in the potatoes (X_i) is uniform throughout the layer of product to dry; the moisture flux across the plane at $z = 0$ is null during drying and the potatoes surface in contact with the hot air stream (at $z = \ell$) reaches rapidly the moisture content in equilibrium (X_e) with air conditions used for drying. If the humid solid is dried with only one surface exposed to the hot air stream, as in a tray drier, the plane $z = 0$ should be located at the surface in contact with the tray bottom indicating that is an impermeable boundary. In the case where the humid solid has two open areas to mass transfer (upper and lower surface), the plane $z = 0$ refers to the middle of the solid thickness indicating the existence of symmetric conditions.

A numerical method was used to solve the diffusion equation for both cases, considering one or two open areas to mass and heat transfer. An explicit method with finite difference was applied to discretize the mass transfer problem and the simulations were obtained with a MATLAB code. The average moisture content inside the solid at instant *t* was found by spatial integration of $X(z)$ using the trapezoidal rule.

The Drying Experiments

The fresh potatoes (*Red Lady* variety) were cut into thin slices with almost the same thickness, with an electric food slicer with adjustable cutting blade, and then in squares as depicted in Figure 1. The layer of potatoes formed were immediately dried to avoid browning in a convective air drier cabinet with controlled temperature and air velocity at 50 °C and 1.6 m s^{-1}. Once the steady state conditions were reached, the tray/net was placed on the dryer and the weight of (humid) potatoes was monitored and acquired each 40 s. More details about drying experiments can be found in previous studies of the authors (Castro and Coelho Pinheiro, 2016; Madaleno *et al*., 2017).

For the same air conditions, two sets of experiments were carried out using sliced squares of potatoes in a metallic tray and in a metallic net in the dryer compartment. Care was taken in order to completely fill the metallic support, as can be seen in Figure 1. The evolution with time of potatoes' moisture content for these two cases allowed to show the differences between drying a single layer of potatoes exposed to the air stream from only one side and from both sides.

a) b)

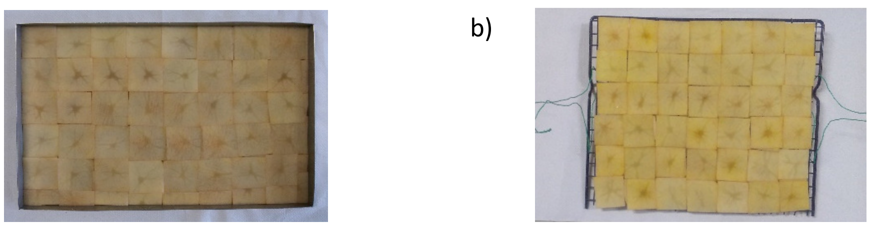


Figure 1. Potatoes samples arrangement (a) in the metallic tray and (b) in the metallic net, used in the drying compartment.

Predictions *versus* Data

From the weight of the wet potatoes registered during the drying process and knowing the weight of the dry solid used in the experiment, the moisture content in a dry basis (X) can be calculated (i.e. mass of water in the potato/mass of dry potato). In order to obtain the mass of dry potato, the tray/net with the sample, after the drying process, was transferred to an oven at 104 ºC, until constant weight. A simple mathematical approach proposed previously by the authors (Castro and Coelho Pinheiro, 2016) uses a linear and a third degree polynomial functions to fit data acquired in the two drying periods (constant drying rate and falling drying rate). From this approach outcomes the value of critical moisture content (X_c), the moisture content in the potatoes from which starts the falling rate period. To compare data obtained in the performed experiments with predictions from the simple mathematical model proposed before only the results corresponding to the falling drying rate period were considered. In fact, the assumption of water diffusion within the potatoes' layer being the dominant mechanism in the mass transfer process only makes sense in that period, where moisture removal depends on the solids characteristics.

For the sake of clarity when comparing data and predictions, a moisture ratio in the potatoes was used. By definition, the moisture ratio is calculated dividing the free moisture content at any instant by the free moisture of the solid at the critical time $(X - X_e)/(X_c - X_e)$. Therefore, this dimensionless parameter varies between 1 and 0 in the falling drying rate period considered. Figure 2 presents the comparison between data and predictions for both cases considered in potatoes drying, with one or two open areas to mass and heat transfer. The effective moisture diffusion coefficient used in the numerical method was obtained from the results acquired during the falling drying rate period in both experiments. The methodology applied was described in detail at Silva and Coelho Pinheiro (2013). For the drying experiment performed with the potatoes in the tray (average thickness 2.44 mm) a value of 7.954×10^{-10} m^2/s for effective moisture diffusion coefficient (D) was calculated and when the potatoes were dried in the net (average thickness 2.39 mm) the value obtained was 3.003×10^{-10} m^2/s.

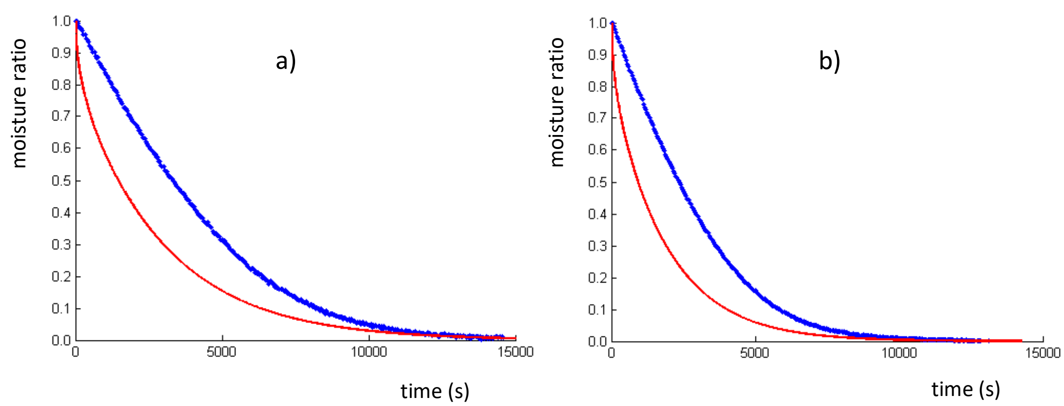


Figure 2. Comparison between moisture ratio evolution with time obtained from simulations (red curve) and data (blue points), for the falling drying rate period, when potatoes were drying (a) in the tray and (b) in the net.

As expected, the obtained results showed that fresh potatoes dried with two open areas to mass and heat transfer dried faster compared to those dried in a tray with only one surface exposed to the hot air stream. To decrease water content in potatoes from 4.42 $kg_{water}/kg_{dry\ solid}$ to 0.17 $kg_{water}/kg_{dry\ solid}$ takes about 16620 s (4.6 h) when potatoes were dried in a tray compared to 11820 s (3.3 h) for the case where drying takes place with potatoes in the metallic net. It should be noted that 4.42 $kg_{water}/kg_{dry\ solid}$ is greater than X_c and, thus, the drying time indicated include a period of constant drying rate.

Concerning the predictions obtained from the simple model considered, it seems that it includes the principal phenomena occurring during the falling drying rate period because qualitatively simulations and experiments are in accordance. Although, the predicted removal of moisture from the solid at the beginning of that period is overestimated being the main cause of the quantitative discrepancy shown in Figure 2. A future improvement of the boundary condition imposed at the potatoes surface exposed to the air drying stream must be considered to increase the model performance. Obtain different effective moisture diffusion coefficients for the early stage of the falling drying period and for the later one when X_e is approached should also be taken into consideration to improve predictions.

Student Learning Outcomes' Testimony

Along all my academic progress as a MSc student in Chemical and Biological Engineering I acquired vast and intensive competences that allowed me to be the professional I became. In my opinion, the master is sufficiently organized in detail to provide understanding of different engineering subjects always combining theory with practice. This fact provided me a cohesive learning once the experimental part is crucial to understand the theory and all the mathematics behind it. Furthermore, and taking into consideration that social skills are one of the keys to succeed professionally, the students are taught how to work in a team while teachers established a close relation with them. During the 2nd year of the master I had my first professional contact with a multinational company where I was confident to solve the presented dissertation topic autonomously. In addition, this experience was very educational once I had contact with a different reality and I was in a multicultural environment. I am strongly pleased to made part of Coimbra Institute of Engineering which gave me all the professional and personal tools to proceed in life as a responsible professional.

References

Wedelin, D., Adawi, T., Jahan, T. and Andersson, S. (2015) Investigating and developing engineering students' mathematical modelling and problem-solving skills. *European Journal of Engineering Education*, Vol. 40 (Nº 2): 1–16

Mujumdar, A.S. (2006). *Handbook of Industrial Drying*, 3rd ed., Dekker, M., New York

Castro, L.M.M.N. and Coelho Pinheiro, M.N. (2016) A simple data processing approach for drying kinetics experiments. *Chemical Engineering Communications*, Vol. 203: 258–269

Madaleno, R.O., Castro, L.M. and Coelho Pinheiro, M.N. (2017) Drying kinetics of granulated cork: Effect of air drying stream conditions and granule size. *Biomass and Bioenergy*, Vol. 107: 8–19

Silva, A.S.G. and Coelho Pinheiro, M.N. (2013) Diffusion coefficients of timolol maleate in polymeric membranes based on methacrylate hydrogels. *Journal of Chemical and Engineering Data,* Vol. 58(8): 2280–2289

Using Quizzes on a Regular Basis to Motivate and Encourage Student Learning

Marjeta Škapin Rugelj[1] and Jože Rugelj[2]

[1]*Faculty of Civil Engineering and Geodetics, University of Ljubljana, Slovenia*
[2]*Faculty of Education, University of Ljubljana, Slovenia*

Abstract

A lot of students have problems in the first semester to adapt to studying mathematics at a university. Some of them do not have enough basic calculating skills and most of them have problems because they are not used to absorb so much new knowledge in a short time. We tried to solve this problem by motivating students for independent blended learning in Moodle environment. We prepared a short quiz every week and correctly answered questions in quizzes were a prerequisite for taking the exam. Students became familiar with different question formats. Some types of questions checked calculating skills and other types of questions check understanding. Some questions have deferred feedback with fully worked explanations. The results of the exams at the end of the semester showed statistically significant improvement of students' grades. The reasons for the improvement have been examined with a survey.

Introduction

We have noticed that many students in the first year at the university have difficulties studying mathematics. Since studies at the Faculty of Civil Engineering are not currently popular due to problems with employment in our country, many students with poor mathematics skills enrol our study programs. We try to solve this problem with the preparatory course in the last week before the start of the lectures, where they practice the basic calculating skills. The aim of the preparatory course is to help them repeat topics from high school mathematics, to find out where they have learning gaps, and to help students to fill them.

In addition to problems due to poor previous knowledge and skills of mathematics, students also had a problem that they are not accustomed to having to study a lot of learning content in a short time. Since students need a lot of mathematical knowledge skills in professional courses, they have 10 hours of mathematics lessons per week in the first semester, which means that without studying on a regular basis they have problems to follow the lectures.

Quizzes for independent and blended learning

Independent learning is a method of education in which a student acquires knowledge by her own efforts and develops the ability for inquiry and critical evaluation. It places increased educational responsibility on the student for the achieving of learning goals. It is usually supported by information-communication technology that provides students with access to learning resources and with virtual learning environment with interactive responses and other functionalities. Blended learning combines traditional face-to-face classroom methods with computer-mediated activities regarding content and learning activities in online digital virtual classrooms. It can require the physical presence of both teacher and student, with some element of student control over time, place, path, or pace. A motivated student is most prepared to face a task, focused on handling it, and persistent in addressing the difficulties faced, as well as one that invests more time and effort in learning than the unmotivated student. One way to produce such motivation is to stimulate it through suitable formative assessment methods (Reyes, Enfedaque and Gálvez, 2017).

Quiz is a tool for independent learning. In our case it is implemented in learning management system Moodle and is used to support blended learning for our students. Moodle's quiz module represents an alternative to traditional assessment tools, such as paper-and-pencil tests (Blanco and Ginovart, 2012; Lowe, 2015; Berrais, 2014). Effective online formative assessment, using quizzes, can foster a learner and assessment centred focus through formative feedback and enhanced learner engagement with valuable learning experiences (Gikandi, Morrow and Davis, 2011). The online quizzes focus on some learning goals that might be overlooked by students. Using quizzes can improve student learning and grades. An additional challenge is to reduce student anxiety. They can answer to question in a quiz without feeling badly about having a wrong answer as can happen in a class. Nobody else can see his or her failures, which can in this way become a good opportunity for learning. Quizzes are nonthreatening and all students get credit. If students frequently take quizzes, they learn more, self-efficacy increases, and test anxiety is reduced. The formal or informal class discussions of quizzes often reveal student misunderstandings. Quiz test questions must be academically sound, authentic, and important, similar in format and style to those used on examinations (Snooks, 2004).

The Quiz activity module in Moodle allows the teacher to design and build quizzes consisting of a large variety of question types. Questions are kept in the question bank and can be re-used in different quizzes. Teachers can quickly analyse in what topics students are successful and in what areas they have demonstrated learning gaps. They can select an appropriate learning strategy for each student for class. The teachers can use the online quiz's graphing analysis to see if any learning gaps are class wide. Such real time data improves the formative assessment process. Both students and teachers can see the students' progress over time as they see the online quiz scores.

Quizzes in blended learning environment

In order to motivate our students and encourage them to study on a regular basis, we already prepared quizzes in Moodle last year. We prepared a quiz for each topic that we elaborate in lectures. This means that we added a new quiz every week. Students were able to make any number of attempts. Most of the questions in quizzes were simple multiple-choice questions or the computational result of the tasks had to be entered. Some questions were also more theoretical. Four computational simple quiz questions were also integrated into midterm and final exams. If students did not answer at least three of the above-mentioned four questions correctly, they did not pass the exam.

Their opinions on the quiz were not reluctant. However, more students found it inappropriate to have questions from a quiz condition for successfully passing the exam. They thought that this was too stressful and that, therefore, they sometimes wrongly answered the questions they would otherwise have answered correctly. It actually turned out for some students that they successfully solved other tasks, but they did not answer correctly at least three of the four questions from the quiz.

We were not satisfied with last year's results, because we wanted to motivate students for real-time learning using quizzes. It turned out that the students took all the quizzes only at the end of the course. Many students were satisfied with a single attempt, although they did not achieve good results. We expected students to use them several times, since each question was randomly selected from a pool of each category. So they would get a similar question in the attempt again and could check that they already know how to solve the task properly. Therefore, we decided in this academic year to change the rules. Correctly solved quizzes are a prerequisite for taking the midterm exams. Students are allowed to take final exam even if they have not solved all the quizzes, but in this case they have to answer correctly to three of the four simple quiz questions that are positioned at the beginning of the exam. If they are not successful, they do not pass the exam, regardless of how they have solved the remaining tasks. It turned out that there was hardly any student in the winter term that decided for this option. This confirms our assumption that students prefer to opt for a more secure, though more time-consuming path.

Since we want students to solve quizzes so long as they can correctly answer all questions, the quiz has to be short. We prepare a short quiz with five questions every week. Each question is randomly chosen from a pool of each category. Some categories contain the same question with different numerical values, while others contain different questions on the same topic. Students can retake a quiz as many times as they want to improve their score. The program was set to keep all scores, but only the highest one is taken into account. The student is considered to have passed the quiz when he correctly answers all questions in a single attempt. Students become familiar with different question formats: multiple choice, embedded answers, numerical, and true/false. Some types of questions check calculating skills and the others check understanding. Some questions have deferred feedback with fully worked answers. Feedback is very important, since students can get the strategy for improvement. When they retake a quiz they can use new strategy when they solve a task that is a version of a question that they did not answer correctly in the previous attempt. If they respond correctly, this is a confirmation that they have correctly applied the strategy. A further incorrect response is a sign that the problem needs to be more thoroughly studied. They can always find additional explanation in their notes from lectures and in textbooks, or seek help from a teacher or from colleagues. During breaks between the lectures we noticed that students discuss quiz questions with each other. This suggests that quizzes have an additional positive impact, since they motivate students to solve problems through co-operation and peer learning.

Students were also asked to tell us if they found any errors in quiz and offered them a bonus for each discovered error. In this way we corrected some tiny mistakes that we did not notice during testing. In addition, we encouraged students to be critical and more active. It even happened that the student reported an error that did not exist at all. In this case we explained where he made a mistake. Since we have many more questions collected in the database than we needed for weekly quizzes, we have also published quizzes for repetition in Moodle. These quizzes have questions arranged by chapters.

Method of Investigation

Participants in the study are 125 students of the first year of three study programs of the Faculty of Civil Engineering and Geodetics: Civil Engineering, Geodesy and Geoinformation, and Water Science and Environmental Engineering. We collected data for our empirical study from three different sources. Moodle provided overall attempt summaries form the quiz module, individual responses for each student, and overall quiz results. The second source of data was the scores from exams for courses of Mathematics I and of Physics for the current and for the previous academic year. The third source was a survey that we conducted among the students participating in the course Mathematics I. An anonymous online survey based on the Likert and open questions was used to obtain quantitative data and feedback about students' satisfaction and usefulness of online Moodle quizzes to support their learning of mathematics. 72 out of 125 students have validly participated in the survey.

Findings and Discussion

As teacher can closely examine in Moodle how the students answered questions in the quiz, we were able to find out which questions are the most difficult for students. If students did not answer the question correctly after taking quiz several times, we improved the explanation of that particular topic in the quiz and in the lecture.

Analysis of the results from exams

Since we did not have a control group to compare the effects of the use of the quizzes with the results achieved, the results of the midterm exam in Mathematics I were compared with the results of the exam in Physics for the current (2017) and for the previous academic year (2016). Last year, students received 26.4% of the points in mathematics on the midterm exam, while in physics they got 35.4% points. This year, the average in mathematics was 50.6% and in physics 49.0%. Obviously this year the results in both courses are better. However, the difference of scores could originate from other sources as well, such as the generally higher average of one generation towards another. Quick look at the samples' statistics shows us, that both physics and math exams average have increased drastically from 2016 and 2017, but the average on math exams has improved more. To test, whether this additional improvement by math exams is random or not, we try to predict students' scores, if relations from 2016 were still valid in 2017. Linear regression of math results on physics results on 2016 population shows, that physics results clearly have a predictive power on math results with P-value below 0.0001 although the R-squared of our simple model is relatively low, just above 20%. With the use of regression parameters, we calculate central predictions of 2017 population math scores from physics scores. This sample cannot be directly compared to real results due to the fact, that central predictions are way less volatile and also perfectly correlated with independent variable (physics score). To produce more realistic sample, additional noise must be added as a normal random variable with mean 0 and standard error from the regression. Now we have two comparable sets of math scores for 2017 students - the true ones and the ones based on their physics score, predicted under the assumption that 2016 relations between physics and math scores hold for 2017 generation as well. We perform t-test of two samples assuming equal variances, to see, if the math average in 2017 is statistically significantly higher than the one from our linear model. P-value with just over 0.0001 shows that we can reject the null hypothesis that true average is lower than it would have been, if the relation from 2016 was still valid. There clearly exist additional reasons that the math scores were higher in 2017 than in 2016, others than just a fact, that 2017 students have higher scores on average. We believe that one of those reasons is the introduction of short regular quizzes.

Analysis of the survey

An anonymous online survey based on the Likert and open questions was used to obtain quantitative data and feedback about students' satisfaction and usefulness of online Moodle quizzes to support their learning of mathematics. The quantitative results of the survey are presented in a graph where agreement Likert scale was used with *1 - strongly disagree*, and *5 - strongly agree*.

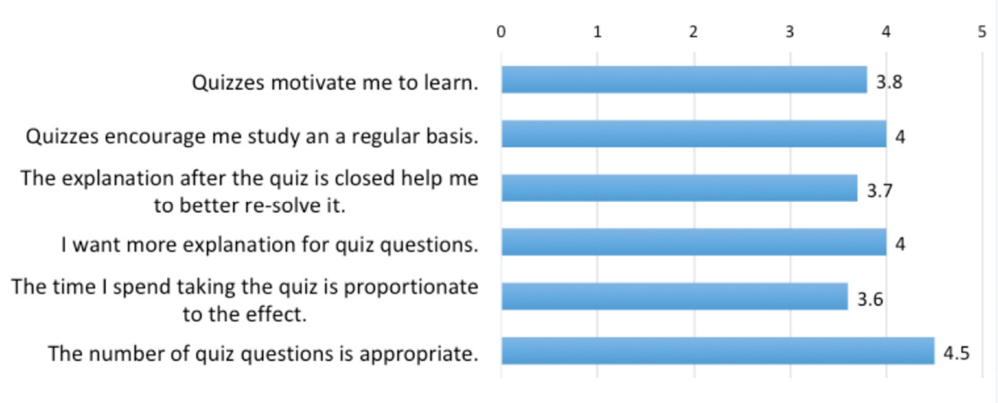


Figure 1. Average results of the survey

We also asked students to make suggestions for improving quizzes. Below we will summarize the most common and most interesting student proposals to improve the efficiency of the use of quizzes. Several students proposed that more feedback at the end of the quiz and better explanations on some more complex issues would be beneficial. Some of them also proposed additional, graphic presentation of learning materials for more effective learning. Another student came up with the idea of guidelines for procedures for solving specific tasks. One of the students argued that he would like to retake that task that he missed in the quiz, but he can only take the whole quiz. Some students expressed a wish that required threshold of 100% for all quizzes is too high for some students, who proposed that three attempts above 90% should be enough.

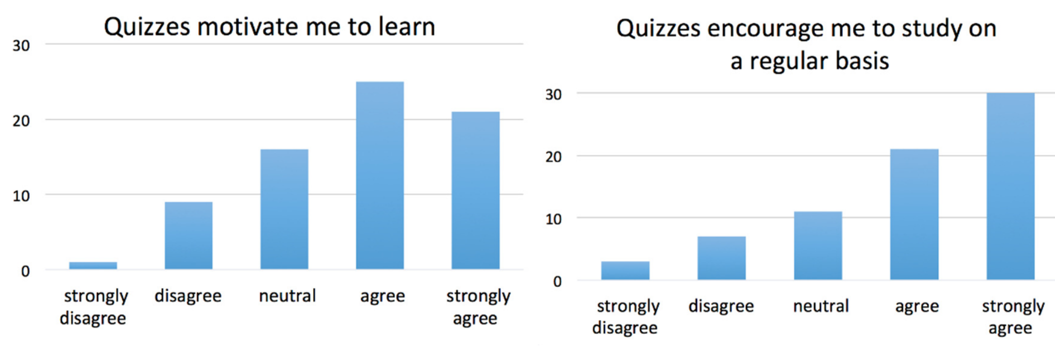


Figure 2. Detailed results on motivation and encouragement for studying regularly

Another respondent is not satisfied that the pool of "theoretical" questions is too limited and they get the same theoretical questions when they have to retake the quiz due to mistakes in computational tasks. He also missed additional, more difficult tasks, comparable to tasks in tutorials and the ability to select difficulty level in the tasks.

Conclusions

Results achieved by students in the exams this year are much better than in the previous academic year. This convinced us to continue using the quizzes. We are pleased that students also feel that quizzes motivate them and encourages them to study on a regularly bases. As students want more explanations for individual more difficult quiz questions, we will upgrade and supplement them.

References

Berrais A. (2015). Using online Moodle quizzes to support the teaching of mathematics to foundation engineering students, QScience Proc. (Engineering Leaders Conference)

Blanco, M. & Ginovart, M. (2012). On How Moodle Quizzes Can Contribute to the Formative e-Assessment of First-Year Engineering Students in Mathematics Courses, *Mathematical e-learning, Universities and Knowledge Society Journal 9*(1), 354-370.

Gikandi, J. W., Morrow, D. & Davis, N. E. (2011). Online Formative Assessment in Higher Education: A Review of Literature. *Computers & Education*, *57*(2), 2333-2351.

Lowe, W.T. (2015). Online quizzes for distance learning of mathematics. *Teaching Mathematics and its Applications: An International Journal of the IMA*, *34*(3), 138-148.

Reyes, E., Enfedaque, A. & Gálvez, J. (2017). Initiatives to Foster Engineering Student Motivation: A case study. *Journal of Technology and Science Education*, *7*(3), 291-312.

Snooks, M.K. (2004). Using Practice Tests on a Regular Basis to Improve Student Learning. In Achacoso, M.V. & Svinicki, M.D. (Eds.). *Alternative Strategies for Evaluating Student Learning* (pp. 109-113). San Francisco: Jossey-Bass Publishers.

Mathematical Economics - Marginal analysis in the consumer behavior theory

Jorge Marques and Rui Pascoal

CeBER and FEUC, University of Coimbra, Portugal

Abstract

In the neoclassical theory, the economic value of a good is determined by the benefit that an individual consumer attributes to the last ("marginal") unit consumed. Marginal analysis was introduced to the theory of value by William Jevons, Carl Menger and Léon Walras, the founders of marginalism. Since the so-called "marginalist revolution" of the 1870s, differential (or infinitesimal) calculus has been applied to the mathematical modelling of economic theories. Our goal is to present some consumer behavior models, their advantages and limitations, using the methodology of economic science. It should be emphasized that each (re)formulation is based on different economic principles: diminishing marginal utility, diminishing marginal rate of substitution and weak axiom of revealed preference.

1. Introduction

Economics is the social science that has incorporated the most mathematics into its theories and models. The formulation and application of mathematical methods to represent economic principles gave rise to a new area of study called Mathematical Economics. The theory of value was one of the first theories to be analyzed using a mathematical framework. The so-called "marginalist revolution" in Economics, at the beginning of the 1870s, is intimately related to the use of differential (or infinitesimal) calculus. For W. Jevons, C. Menger and L. Walras, the founders of marginalism[1], the economic value of a good is determined by the benefit (satisfaction or pleasure or utility) that an individual consumer attributes to the last ("marginal") unit consumed. Jevons and Walras assumed that the marginal utility of a good could be measured by the rate of change of utility as the quantity consumed changes in infinitely smalls units. In its mathematical formulation, the marginal utility of a good is represented by a first order partial derivative of a utility function with respect to the quantity consumed. Unlike Jevons and Walras, C. Menger presented a table of consumer needs-satisfaction to describe the subjective nature of value in a more qualitative analysis. [2] The classic diamond-water paradox is then explained by the existence of two meanings of value: value-in-use and value-in-exchange.

[1] New ideas about the economic value of goods were expressed in independent works by Jevons in England (Jevons, 1871), Menger in Austria (Menger, 1950 [1871]) and Walras in Switzerland (Walras, 1874).

[2] The marginalist analysis refuted the classic "labor theory" of value.

2. Carl Menger – An economist who kept the focus on the meaning of value

Menger kept the focus on the main point underlying the determination of value, arguing that human nature determines decisions leading to action in the economy.[3] In the beginning of his book (Menger, 1950 [1871]) wrote: "All things are subject to the law of cause and effect". The cause-effect relation is inherent to every decision an individual makes in the particular circumstances he faces at each moment. The definition of the economic value of a good as the benefit of the marginal unit consumed remained in Menger's mind. This is evaluated in opposition to alternative uses of other goods. In fact, economic goods are scarce and human effort must be made to provide for their availability. Not all needs associated with these goods can be satisfied by all individuals. Their value is defined by the importance that an individual attributes to the satisfaction of needs that result from the consumption of the last unit of the good that he can dispose of. Each individual establishes a scale of importance for additional units consumed of diverse economic goods.[4] The value attributed to a good is inherently subjective, depending on the needs and preferences of each individual which are also determined by the particular context he faces at each moment and subject to rapid changes. Hence, population heterogeneity cannot be avoided and the existence of a stable function across time representing aggregated demand for the good is precluded. The optimization models created later on (see next section) are based on oversimplified assumptions such as homogeneity which is implicit in the kind of economic agent idealized in economics and for whom optimization is meant to be performed. Menger remarked that value is not an intrinsic property of goods but results from the importance attributed by individuals to concrete units of goods. He strongly stressed the distinction between value-in-use and utility. All goods, including noneconomic ones, have utility to the extent that their consumption satisfies needs, but it is only when a good is scarce for all needs in the population that it becomes economic and its units acquire value. Thus the term "value-in-use" signifies what other economists call "utility" and "total utility" is a nonsensical concept.

3. Mathematical Models and Methodologies

Like in Physics, neoclassical economic theories focus in the equilibrium concept (Mirowski, 1991). The main goal of consumer behavior models is to explain the relationship between the prices and quantities of goods demanded by markets.[5] Mainstream economics preferred the partial equilibrium analysis developed by Marshall[6] (Marshall, 1920) over the general equilibrium analysis of Walras. The highlight on mathematical representation is the Marshallian cross diagram illustrated in the introductory economics textbooks. It is used to show the equilibrium price of a good that results from the

[3] The method leading to the study of human action is called Praxeology, used as a research method by the Austrian School of Economics, founded by Menger. This school defends that the use of differential calculus is deemed excessively simplistic for analyzing the complexity of economic decisions.

[4] For instance, the first unit of the most essential good, e.g., food, has the highest importance, its second unit has less importance but may have as much importance as the first unit of the second most relevant good, and so on.

[5] On the supply side, producers sell goods in markets by minimizing their costs.

[6] Marshall introduced a fundamental assumption in economic analysis, known as *ceteris paribus*, to study a relationship between two variables while holding others constant in a short period of time.

intersection of the demand and supply curves. We will present three models[7] of consumer behaviour using distinct methodologies. The first two are neoclassical consumer models based on cardinal and ordinal utility theories, respectively. The demand curves can be derived from utility maximization in both models. The third model is formulated from restrictions on observable data (choices) and is called Samuelson's revealed preferences model.

3.1 The neoclassical consumer model based on the cardinal utility theory

Neoclassical microeconomics adopted the following definition (Robbins, 1984): "Economics is the science which studies human behavior as a relationship between ends and scarce means which have alternative uses". The neoclassic consumer model is a theoretical model in which an individual consumer is an economic agent whose behavior is influenced by three assumptions. First, there is an allocation of scarce means, that is, the consumer spends his (or her) income I buying a vector of quantities of n goods, $q = (q_1, q_2, \dots, q_n)$, at given unit prices vector $p = (p_1, p_2, \dots, p_n)$ in a market. The consumer's behavior then depends on a subjective utility that he (or she) attributes to goods, which is represented by a unique function, $u(q_1, q_2, \dots, q_n)$, in the cardinal utility theory. Finally, the consumer will maximize utility by the rationality principle. In order to use marginal analysis, the utility function is assumed to be twice continuously differentiable that satisfies the following three axioms: First, goods are continuously divisible which implies continuity[8] of the utility function; Second, the marginal utility of each good is positive, which means the consumer prefers more rather than fewer goods, so that utility increases as the consumption of one good increases, holding the consumption of the other goods constant; Third, the diminishing marginal utility principle states that if the consumption of one good increases, then its marginal utility decreases, holding the consumption of the other goods constant. Thus, it is supposed that

$$\frac{\partial u}{\partial q_i}(q) > 0 \qquad \wedge \qquad \frac{\partial^2 u}{\partial {q_i}^2}(q) < 0, \text{ for all } \ q = (q_1, q_2, \dots, q_n),\ i = 1, \dots, n.$$

The consumer problem is to choose a vector of goods that maximizes the utility function $u(q)$ subject to the budget constraint $p_1 q_1 + p_2 q_2 + \cdots + p_n q_n = I$. The mathematical model is then represented by a constrained optimization problem on the set $\mathbb{R}_n^+ = \{(q_1, q_2, \dots, q_n): q_i > 0\}$, which can be solved by applying the Lagrange multipliers method. The first order necessary conditions are given by

$$\frac{\frac{\partial u}{\partial q_1}(q)}{p_1} = \frac{\frac{\partial u}{\partial q_2}(q)}{p_2} = \cdots = \frac{\frac{\partial u}{\partial q_n}(q)}{p_n} = \lambda \quad \wedge \ p_1 q_1 + p_2 q_2 + \cdots + p_n q_n = I.$$

It is well known that if the utility function is strictly quasiconcave, then the bordered Hessian matrix is negative definite, hence the problem has a unique solution $P_e = ({q_1}^e, {q_2}^e, \dots, {q_n}^e)$. It is said that P_e is the optimal bundle in the market for the consumer. For arbitrary prices and income, n equilibrium demand functions on the set $\mathbb{R}_{n+1}^+ =$

---

[7] These models are normative models that only describe what rational consumers should do (Thaler, 1980).

[8] Marshall adopted the expression *Natura non facit saltum* in his book (Marshall, 1920).

$\{(p_1, p_2, \dots, p_n, I): p_i > 0 \wedge I > 0\}$ are deduced, defined by $q_i{}^e = f_i(p_1, p_2, \dots, p_n, I)$, by solving the first order necessary conditions explicitly in order to determine q_i.

3.2 The neoclassical consumer model based on the ordinal utility theory

Following Pareto's idea of ordinal utility (Pareto, 1909), Hicks asserted: "The quantitative concept of utility is not necessary in order to explain market phenomena".[9] Rejecting the marginal utility notion and consequently the diminishing marginal utility principle, the concept of the marginal rate of substitution (MRS) between goods was introduced by Hicks and Allen (Hicks, 1934) to develop indifference curves analysis. Given any two goods X e Y, the MRS of Y for X measures an amount of good Y that the consumer is willing to give up in order to gain an incremental increase of consumption of X. The neoclassical consumer model is a theoretical model in which a rational consumer seeks to maximize her (or his) utility subject to the budget constraint $p_1 q_1 + p_2 q_2 + \dots + p_n q_n = I$. In the framework of the ordinal utility theory,[10] an individual consumer has a scale of preferences, which could be represented by a utility function, $u(q_1, q_2, \dots, q_n)$, (Debreu, 1959). Consumer behavior is limited by three assumptions: First, if consumer's preferences are defined on the set $\mathbb{R}_n^+ = \{(q_1, q_2, \dots, q_n): q_i > 0\}$, then given any two bundles, she (or he) will prefer one of those or will be indifferent (an indifference hypersurface is defined as the set of all bundles of goods which have the same preference rank or utility level); Second, the consumer will prefer more to fewer goods, meaning that she (or he) will choose the vector of goods that belongs to the indifference hypersurface with the highest rank among those she (or he) can afford; Third, the diminishing MRS principle states that the rate will decrease as Y good is substituted for X along an indifference hypersurface. Given n goods, X_i, $\tau_i(q)$ denotes the MRS of X_n for X_i, $i \neq n$. If we suppose that the marginal rate of substitution, $\tau(q) = (\tau_1(q), \tau_2(q), \dots, \tau_{n-1}(q))$, is a continuously differentiable mapping satisfying the properties of positivity and convexity, then there is an indifference map that consists of a one-parameter family of indifference hypersurfaces. From the economic point of view, the best bundle P_e satisfies $\tau_i(q) = \frac{p_i}{p_n}$, and from the geometric point of view, the optimal bundle P_e, solution of the constrained maximization problem, belongs to the indifference hypersurface with highest parameter (utility level). We note that, assuming that the expression of a utility function is unknown, there is an alternative approach in which the consumer's preferences can be characterized by the marginal rate of substitution between goods, $\tau(q)$, using ordinary differential equations (we can observe the particular case of $n = 2$ goods in (Marques, 2014)).

---

[9] "The equilibrium conditions [first order conditions] and the stability conditions [second order conditions] for an individual consumer have been written out assuming the existence of a particular utility function u. This is, indeed, the most convenient way of writing them; but it is important to observe that they do not depend upon the existence of any unique utility function" (Hicks, 1939).

[10] In the framework of ordinal utility, a utility function u is not unique because $U = g \circ u$ is also a utility function whenever g to be is a strictly increasing function.

3.3 Samuelson's revealed preferences model

Samuelson[11] provided a step forward in getting rid of the unnecessary and explicit reference to the utility concept. He proposed a new methodology based on observable market data. In his approach, called "revealed preferences", it is assumed that an individual's choices (rather than preferences) are empirically determinable from the prices of goods and the income available for consumption. Samuelson's revealed preferences model is designed to deduce the conditions to be imposed on demand by formulating three axioms: First, the existence of n continuously differentiable demand functions $q_i = f_i(p_1, p_2, \dots, p_n, I)$, subject to the budget constraint $p_1 q_1 + p_2 q_2 + \cdots + p_n q_n = I$, is assumed; Second, it is assumed that demand functions are homogeneous of degree zero, meaning that these functions are independents of monetary units[12]; The third axiom is known as the "weak axiom of revealed preference" (WARP), which states that, for any pair of bundles q^1 and q^2, if q^1 is preferred to q^2, $q^2 < q^1$, then $q^1 \nless q^2$. From these axiom, expressing the consistency of consumer behavior, Samuelson deduced that the Slutsky substitution matrix must be negative semidefinite (see (Mas-Colell, 1995)). This is deduced as follows: given prices vector $p = (p_1, p_2, \dots, p_n)$, it is supposed that the bundles $q^1 = (q_{11}, q_{21}, \dots, q_{n1})$ and $q^2 = (q_{12}, q_{22}, \dots, q_{n2})$ have the same total cost, that is, $\sum_{i=1}^{n}(q_{i1} - q_{i2})p_i = 0$. If $q^2 < q^1$ (so that, at price p, q^1 was chosen instead q^2) then WARP implies that when prices change (from p to p'), consumer preferences are unchanged so that $\sum_{i=1}^{n}(q_{i1} - q_{i2})\, p'_i > 0$. Let $q_{i2} = q_{i1} + \Delta\, q_i$ and $p'_i = p_i + \Delta\, p_i$, after some algebraic calculus, we have $\sum_{i=1}^{n} p_i \Delta q_i = 0$ and $\sum_{i=1}^{n} \Delta p_i \Delta q_i < 0$. Taking these expressions to the limit and using $dI = \sum_{j=1}^{n} q_{j1} \mathrm{d}p_j$ we obtain

$$\sum_{i=1}^{n} p_i \mathrm{d}q_{i1} = 0 \;\wedge\; \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial q_{i1}}{\partial I} q_{j1} + \frac{\partial q_{i1}}{\partial p_j}\right) dp_j\, dp_i < 0.$$

A decade later, Samuelson (Samuelson, 1948), recognized that the revealed preferences logic is complementary to the preferences theory based on ordinal utility. Indeed, Houthakker (Houthakker, 1950) has shown that if consumer preferences are transitive[13], then the revealed preferences approach should be able to empirically reconstruct the indifference map on which the ordinal utility theory relied.

Samuelson's model is an economic choice model that draws conclusions exclusively based on observed behavior, making no psychological or philosophical considerations which may be misleading if based on more or less arbitrary assumptions. Samuelson claimed that what matters are the exchanges that a consumer really makes, not the exchanges he claims he would make. The WARP formulation eloquently exhibits the flaw pointed out by the Austrian School, that is, it assumes time stability in individuals' choices, which may be hard to justify except in special circumstances. We further suggest reading Wong's book (Wong, 2006) for a critical analysis of Samuelson's model using Popper's method of rational reconstruction.

[11] "I propose, therefore, that we start anew in direct attack upon the problem, dropping off the last vestiges of the utility analysis. This does not preclude the introduction of utility by any who may care to do so, nor will it contradict the results attained by use of related constructs. It is merely that the analysis can be carried on more directly, and from a different set of postulates." (Samuelson, 1938).

[12] It allows the expressing of demand functions in terms of relative prices of each good with respect to a numeraire good having a price equal to one.

[13] It means that the "strong axiom of revealed preference" holds.

4. Karl Menger – A mathematician with a heavy heritage

Karl Menger, Carl Menger's son, was a mathematician with some works in mathematical economics, but his father, the founder of the Austrian School of economics, gave priority to other methods of economic research rather than mathematics. He was led to try to connect these two antagonistic perspectives (Menger, 2003). In his opinion these were, above all, two different forms of expressing ideas on subjects they could agree. While mathematical economics used mostly mathematics, descriptive language was the privileged means for the Austrian School. On the issue of goods valuation, he considered that, unlike what mathematical economists might think, formal mathematical presentation did not add anything in generality and precision to Austrian reasoning. On the contrary, he argued that mathematical analysis often imposed unnecessary assumptions, for instance, continuity and differentiability properties, which are not based on observed facts in the economy. In the case of the marginal utility of a good, this concept is interpreted as the limit of the rate of change of utility when the quantity increment of that good tends toward zero. Karl Menger assayed the mathematical formalization of the Austrian reasoning by defining a non-decreasing and convex utility function, f, to express the idea of a decreasing rhythm of utility change. For simplicity, f is assumed to be a function of quantity consumed of only one good such that

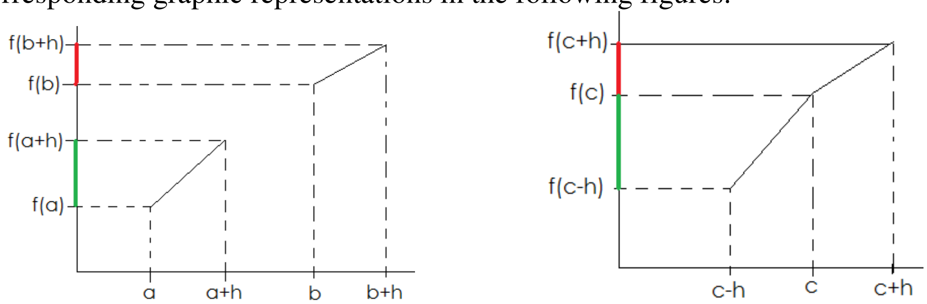
$$\frac{f(c)-f(b)}{c-b} \leq \frac{f(b)-f(a)}{b-a}, \text{ for } a < b < c.$$

He also considered generalizations given by

(i) $For\ a < b, h > 0, \quad |f(b+h) - f(b)| \leq |f(a+h) - f(a)|;$

(ii) $For\ h > 0, \quad f(c+h) - f(c) \leq f(c) - f(c-h);$

with corresponding graphic representations in the following figures.



It is highly doubtful that Karl Menger succeeded in making the two perspectives compatible. In fact, this formalization does not account for specific dimensions of Austrian analysis such as subjectivity in valuation or the importance of time in decision-making, namely the implications of the absence of a time-stable utility function.

5. Conclusion

In a famous quote, Hayek said: "The curious task of economics is to demonstrate to men how little they really know about what they imagine they can design" (Hayek, 1988). At first sight, it would seem that it is just the definition (Robbins, 1984) of scarce means to unlimited ends, but this is not so. Taking a more humanistic approach to economics, Hayek's logic goes far beyond Robbin's since an affectation of scarce means to multiple ends does not imply only a "mechanic" model of constrained optimization. In this approach, it would be necessary to impose more effective and realistic assumptions. For instance, exploring the motivations of each individual in his complexity, taking into account his specificity and subjectivity. The subjectivity concerns not only individual's idiosyncratic preferences but also the unique environmental circumstances he faces at every moment. In the richness of everyone's freely lived life, there necessarily exists a highly heterogeneous population, most of the time not represented in a representative agent model reflecting everybody's choices. Another important point is the recognition that economic individuals are limited in their resources as well as economic researchers and political decision makers. In a highly complex framework with constant novel information, economic analysis is more efficient using a network of individual decision makers where each one manages little information, rather than using central planning where effective decisions are usually not available even to the most potent supercomputer. However, the mathematical models presented here have made important contributions to understanding consumer behavior theory. Nowadays, an interdisciplinary approach involving concepts from all social sciences concerned with human nature is taken to study this complex subject.

References

Debreu, G., 1959. *Theory of value: An axiomatic analysis of economic equilibrium.* Monograph nº 17 ed. New York: John Wiley & Sons: Cowles Foundation Yale University.

Hayek, F. A. v., 1988. *The fatal conceit: the errors of socialism.* Bartley, William Warren ed. London: Routledge.

Hicks, J. &. A. R., 1934. A reconsideration of the theory of value. *Economica,* New series, 1(2), pp. 196-219.

Hicks, J., 1939. *Value and capital.* 1st ed. Oxford: Oxford University Press.

Houthakker, H. S., 1950. Revealed preference and the utility function. *Economica,* Volume 17, pp. 159-174.

Jevons, W. S., 1871. *The theory of political economy.* 1st ed. London: Macmillan.

Marques, J., 2014. An application of ordinary differential equations in economics: modeling consumer's preferences using marginal rates of substitution. Em: F. M. a. M. M. N. Mastorakis, ed. *Mathematical Methods in Science and Mechanics: Mathematics and Computers in Science and Engineering.* s.l.:Wseas Press, pp. 46-53.

Marshall, A., 1920. *Principles of Economics, An Introductory Volume.* 8th ed. London: Macmillan.

Mas-Colell, W. M. D. G. J., 1995. *Microeconomic theory.* Oxford: Oxford University Press.

Menger, C., 1950 [1871]. *Principles of Economics.* English translation of the book published in 1871 ed. Glencoe: Free Press.

Menger, K., 2003. Austrian marginalism and mathematical economics. Em: S. B. e. al, ed. *Selecta Mathematica.* Vienna: Springer, pp. 531-553.

Mirowski, P., 1991. The when, the how and the why of mathematical expression in the history of economic analysis. *Journal of Economic Perspectives,* 5(1), pp. 145-157.

Pareto, V., 1909. *Manuel D'Economie politique.* 1st ed. Paris: V. Giard et E. Brière.

Robbins, L., 1984. *An essay on the nature and significance of economic science.* 3rd ed. London: Macmillan.

Samuelson, P. A., 1938. A note on the pure theory of consumer's behaviour. *Economica,* Volume 5, pp. 61-71.

Samuelson, P. A., 1948. Consumption theory in terms of revealed preference. *Economica,* Volume 15, pp. 243-253.

Thaler, R. H., 1980. Toward a positive theory of consumer choice. *J. Econ. Behavior and Org.,* Volume 1, pp. 39-60.

Walras, L., 1874. *Eléments d'économie politique pure.* 1st ed. Paris: Guillaumin.

Wong, S., 2006. *The Foundations of Paul samuelson's revealed preferences theory: a study by the method of rational reconstruction.* London: Psychology Press.

On Grounds for Competence Oriented Teaching and Assessment

Daniela Richtáriková

Slovak University of Technology in Bratislava,
Institute of Mathematics and Physics, Faculty of Mechanical Engineering, Slovakia

Abstract

The paper discusses the very current topic of recent years on acquiring mathematical competencies in education at technical universities. It presents a glance into history on concept of competency for everyday life, formulation of learning outcomes and used methods. It takes into consideration psychological and social aspects of teaching at universities today and arguments the importance of pedagogical education of university teachers. With respect to goals of new international Erasmus plus project Math_Rules, coordinated by Salamanca University, the paper deals also with mathematical competencies assessment.

Mathematical competence

Competence oriented teaching has become widely discussed topic especially in the context of Recommendation of the European Parliament and of the Council of 18 December 2006 on key competences for lifelong learning (2006/962/EC) (hereinafter as EC document). The document defined further direction of education in countries of European Union, turning the main aim of education to key competencies for everyday life. The document outlines also the definition of Mathematical competence:

"Definition:
Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, charts)",

together with "specification of essential knowledge, skills and attitudes related to this competence:
Necessary knowledge in mathematics includes a sound knowledge of numbers, measures and structures, basic operations and basic mathematical presentations, an understanding of mathematical terms and concepts, and an awareness of the questions to which mathematics can offer answers. An individual should have the skills to apply basic mathematical principles and processes in everyday contexts at home and work, and to follow and assess chains of arguments. An individual should be able to reason mathematically, understand mathematical proof and communicate in mathematical language, and to use appropriate aids. A positive attitude in mathematics is based on the respect of truth and willingness to look for reasons and to assess their validity."

The EC document release was preceded by huge activity of Danish KOM project that "strongly influenced the description of educational goals in the famous OECD-PISA study (OECD 2009)" (Alpers et al., 2013). SEFI MWG Group in "A Framework for Mathematics

Curricula in Engineering Education" (Alpers et al., 2013) adopted for engineering education KOM definition of mathematical competence as

"*the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role*" (Niss, 2003)

with eight competencies:
Thinking mathematically, Reasoning mathematically, Posing and solving mathematical problems, Modelling mathematically, Representing mathematical entities, Handling mathematical symbols and formalism, Communicating in, with, and about mathematics, Making use of aids and tools, which together constitute the overall competence,

and three dimensions for *specifying and measuring progress*:
1. Degree of coverage (the 'reproduction' level, the 'connections' level, the 'reflection' level (corresponding to taxonomies of education: Bloom, Nemierko)*, 2. Radius of action, 3. Technical level.*

The contents and content-related learning outcomes in previous edition organised in four core levels were elaborated into competencies learning outcomes using active verbs formulated in the form: "As a result of learning this material you should be able to" carry out, express, calculate, evaluate, represent, manipulate, obtain, distinguish, recognise, interpret, plot, understand, etc.

The fact that "*competence needs are not static; they change throughout life. The competences acquired at school need to be developed to be adequate throughout life; keeping competences up to date, and acquiring new ones in response to changing needs is a lifelong process. With regards to time, content scope and overall importance awareness, learning.*" has been in the focus of proposal for new Recommendation on Key Competences adopted by the European Commission in January 2018 (Proposal, 2018).

Historical grounds

Concept of competence for everyday life

The concept of competencies for everyday life in education is not a new one, especially in the territory of Central Europe. If we look into the period of 15th-17th century, when elementary schools for children started to be widely established in Central Europe, contrary to church schools where learning was based on memorisation, we can find e.g. in the reformation pedagogical work of Johannes Sturm (16th century) that in addition to memory, also judgment and expression of ideas has to be practiced. In Didactica Magna, Jan Amos Komensky (J. A. Comenius, Moravian philosopher and educator, "The teacher of nations", 17th century) writes that young people have to learn what they need for life, mainly to read, write, count, and measure, but also to sing, narrate, act well, and know something about crafts. He considered counting to be as important as reading and writing, so all of them should be taught at each school. Under the reign of Maria Theresia (18th century) compulsory schooling was introduced comprising compulsory teaching of mathematics too. Universities prepared officers for state administration, guardians of manors, surveyors, rangers, economists, etc. In the 1930s, in Czechoslovakia, the aim in arithmetic curricula

was “to cultivate in pupils routine of calculating thinking, and skills to measure and solve practical, numerical tasks of their everyday environment independently, with certainty and swiftly. To educate pupils to accuracy, rationality and careful enterprise, driven by the idea of general good.” (Divisek, 1989).
After the World War II, the conception of mathematical education in Czechoslovakia was strongly influenced by experience of soviet psychology and pedagogy. With respect to “Law, 1948” the school of that time "leads learners to independent thinking, purposeful acting, active work and cooperative work. The school does not provide only knowledge, it also develops the senses of pupils; it teaches them to think logically and encourages them to goal-directed behaviour”. Ten years later, further tasks for education and training were set. The school had to be in closer connection with life. Youth in schools were to be acquainted with real needs of society; they were to find real stimuli for study and work activities in praxis. “**The school has to prepare for life and work**” (Resolution, **1959**). In sixties, “sufficient attention is devoted to natural sciences and mathematics, social and linguistic subjects, including the field of work, the basics of production, politics and ethics, physical education and subjects of aesthetics. This makes this basic education more versatile and more closely related to the life of society”. (Concept, 1960). In 1976, new curriculum and textbooks were introduced in all schools. The aim of mathematics was to give pupils such knowledge, skills and habits that could use in their future vocation by solving various practical problems and real situations, as well as use them in further studies. (Development, 1976).

After November 1989, major changes in Czechoslovak society have been established. Modified curricula brought "de-ideologisation", releasing strict bindings of teaching plans, thus creating space for diversification of schools, providing more autonomy for teachers, and also creating space for new methods and new forms of work. On the other hand, natural sciences orientation controlled by state dissipated, mathematics stopped to be obligatory at leaving exams, which resulted into strict reduction of mathematics teaching hours at schools of all levels, weakening consideration of its importance and moreover, the readiness of applicants to study technical and science branches. In addition, the state stopped regulating preparation for future occupation in all respects and market itself has not been able to regulate it nor to meet the needs of its own.

Creating the common European space, to supply the demands in profession structure has become a transnational issue, and **preparation for life and work,** as the main aim of education, is in the focus again.

Mathematical education outcomes formulation

In Didactica Magna, Komensky introduced four six-year-long levels of education: kindergarten school (up to 6 year olds), national school (6-12 year olds), Latin school, and university. Although in different detail, he provides also the formulation of requirements for knowledge, skills and moral qualities of graduates. For instance, in Informatorium of Kindergarten (1632) he writes “The foundation of arithmetic will be *to know*, what is many and what is few, to know calculation up to twenty, *to understand* what is odd and what is even, and *to judge* that three is more than one, *to be able* to add one to three in order to obtain four”, etc. Pupils from 6 to 12 *could* count with numbers as well as with little stones with respect to needs; to measure lengths, widths, distances, by art and in any way. In the proposal of Latin School (12 to 18-year-olds) he wishes students to become arithmeticians and geometricians due to various needs of life on one side, and due to the fact that these sciences in connection to other things provoke wit and polish it. (Mikulcak, 2010)

The school law of Austro-Hungarian Empire from 1805 "Political establishment of schools" returned school supervision under the church. In the spirit of principle "Austria does not need wise people but good serfs", in elementary schools it ordered: not to deal with counting very deeply but to drill skills in counting in mind or by heart with numbers without using digits, to only use digits starting in the third grade, to restrict on four basic operations only with natural numbers and to reach high skills in counting with fractions and in simple proportions.
In 1877, pupils at elementary schools (obecná and mešťanská) should learn:

- In arithmetic – promptly perform elementary calculation operations, cleverly proceed calculations of civic (bourgeois) life, master simple accounting.
- In measuring and drawing – be sure in learning, comparing, calculating and measuring spatial variables.
- In drawing by hand – be clever in sketching plane figures, be clever in sketching space figures with respect to perspective, be clever in sketching according to ornamental samples and models. (Osnovy učebné pro české školy obecné a měšťanské v Čechách podlé usnesení c. kr. zemské školní rady z dne 15. března 1877" (Sedivy, 1988)

As seen above, at first, the outcomes of education were formulated implicitly in school laws and pedagogy works. Usually they were stylized in sentences as the skills and knowledge that were to be acquired (by learners) or that were to be taught (by teachers) - depending on what author (or state educational committee) wanted to emphasise - not explicitly distinguishing between contents and desired skills of learners. In 1948, curricula were radically re-elaborated with respect to the new school law, and contents and skills started to be formulated as two separate issues: Contents and Tasks. (Learning plan and curricula for national schools in Bohemia, Moravia and Slovakia, 1948)

- *Contents:* Reproduction on measures and weights, Solution of math word problem tasks, record of procedure, test of calculation correctness.
- *Tasks:* to acquaint pupils with basic features of decimal numerals, learn them to… Students have to know basic geometric properties of the most important bodies; they can solve word problem tasks from practical life.

Ten years later, in curricula from1959 the following issues were stated (Divisek, 1989):

- *Goals of mathematical education:* pupils have to acquire basic knowledge and skills in arithmetic. Simultaneously, they have to be acquainted with economic phenomena of their surroundings, and they should learn how to obtain the necessary practical data.
- *The requirement of polytechnical education* is realised by acquiring skills in measuring, drawing and solving of practical problems.
- *The development of logical thinking* is carried out by systematic and planned solving of judgment tasks.
- *Pupils have to learn* be accurate, critical, endure, helping to create elements of "scientific world-view".

Although curricula were written in different assortments during following years, they were specifying contents and outcomes of learners in more addressed way. In 1980s, the outcomes were formulated as (Curricula, 1987):
the targeted knowledge on:

- defined mathematical concepts and relationships between them, including the inferences
- mathematical terminology, phraseology and symbolism
- methods of mathematical work
- mathematizing the real situation

graphical-algebraic approaches to the solution
the application of the mathematics curriculum in the given field

the targeted skills:
to know how to find, correctly evaluate, sort and use mathematical information
to work actively with mathematical terminology, phraseology and symbolism
to know how to use acquired knowledge in solving tasks and problems
to master algorithms for solution of mathematical problems
to solve tasks and problems rationally and accurately in hand as well as aided by calculators, tables, and computers
to know how to apply mathematics in natural sciences and in the particular field of study
to interpret correctly obtained mathematical results
to read mathematical text with understanding appropriately to learners' age and level of their mathematical knowledge / e.g. text in the textbook of mathematics

Today the learning outcomes at elementary and secondary schools are formulated as **educational standards in contents and performance** in order to develop the mathematical competence in the way how it was formulated by EC. (details can be seen in Slovak on http://www.statpedu.sk/sk/svp/statny-vzdelavaci-program/).

The first attempt to specify the minimal standards for elementary and secondary schools, introduced in 1957, had to diminish the disproportion between high demands of curricula outcomes set before and real possibilities of education. The requirements on outputs were minimalized to such level, that led to insuficiet preparation for universities. One could hardly not to notice the resemblance to today's situation in Slovakia.

Methods

First schools in our territory were established in 6th century. They were church schools that thought people to read, sing and narrate the bible text. With respect to this, memorisation and then unconscious narrating were the only used learning methods. Later, in towns, the children of merchants and craftsmen were taught practical arithmetics and geometry. Procedures were also trained mechanically, in the form of instructions: where to write digits in operation layout, which numbers to multiply, etc. They were narrated in verses and so better remembered. To make multiplications easier, the multiplication tables of products were used.

Although, memorisation and mechanical learning stand for the most passive and the least effective learning method; the method is somehow natural. "A drowning man will clutch at a straw" – that is the only method that works in case a learner is not able or willing to understand a concept. The method has persisted until now.

In 17th century, Jan Amos Komensky worked out four general principles of didactics which have had their validity until now – principles of consistency, systematicness, activity and proportionality. He did not like non-conscious memorisation. In order to avoid it, he introduced school in theatre (Schola Ludus) where usually didactician, teacher and pupils were discussing on some topic. The didactician instructed the teacher, the teacher was questioning pupils who were to answer him. (Kopecky, 1992). In the part on arithmetics and geometry, mathematician and three pupils Numeratius (calculator), Metritius (measurer) and Tritanius (weigher) acted. (Mikulcak, 2010). Komensky took great pains to find out methods, using those, learning were not exhausting. Not to learn Latin by heart, he created an illustrated book "The World in Pictures", where words from common life were accompanied by pictures, so children could have learnt them in practice together with their meaning. He carried into life two postulates of didactics from his "Didactica Magna" (Kopecky, 1992):

1. To search and find such method, by which teachers teach less, but students learn more, there is less noise, flabbiness, and idleness but more well-being, engaging activity, and lasting learning success at schools.
2. Well worked out order in everything is the basis for reforming schools. It means, such arrangement of things that each thing naturally appertain to: first and following, higher and lower, larger and smaller, similar and dissimilar – with respect to place, time, quantity, measure and weight. Order is the soul of all things.

In 18th century, arithmetics was taught as science, systematically, not taking into account the age of a learner. As a reaction to this praxis, Johann Ignaz von Felbiger, Austrian school reformer, in his book on ways how to teach, influenced teaching process in Austria-Hungary for a long time. He emphasises methodical process from easy to more difficult, from familiar to unknown; he gives the great importance to reading from textbooks by all learners together, and he appreciates the teacher's contribution. (Kopecky, 1992). In four items, he also presents how to teach mathematics (Sedivy, 1987):

- to show the first example on the blackboard, and then let to solve similar problem by a pupil also on the blackboard. Other pupils solve the same on their writing slates. The procedure is corrected with respect to the example
- to let pupils solve the problem of the same type individually, walk around the class and check the progress of pupils, not commenting the correct process, only appointing mistakes with the word "mistake", not showing where the mistake is
- to take care about record arrangement of solving procedure, to check calculations collectively or to execute pupil-to-pupil check
- Not to forget, each pupil has her/his book of computations, where all of solved types are theirself recorded. Every Saturday, the repetition is held

The procedure has been still carried out.

The beginning of 20th century was characterised by boom of psychological research. Pedagogy was substantially influenced by pragmatism, gestalt psychology of the global whole, and behaviouristic psychology. Constructed schemes of calculations were substituted by natural methods inspired by real situations, children's interests and games. The "*game on merchants*" was one of the first didactical plays included in teaching process that diversified the "boring" artificially constructed procedure of computations. It involves more senses (sight, touch, smell, and sound), raised attention; the learners started to work independently and to cooperate. In 1920s, experimental schools, where new trends were applied, started to be developed parallelaly to official schooling. One of new tested trends was *project learning*. While solving a problem from real life, particular knowledge of several subjects were applied. It was the way, how learners learnt these subjects, including also mathematics – with no system, only in accordance with current need. Drawbacks of the project learning were revealed very soon. Due to *combination method* selection of artificial methods were linked with situation method of plays. Each item of computation should have been accompanied by real performance: paying, weighing, etc. Topics and their order were set with regard to mathematical contents and logic, while exposition and learning took into consideration psychological aspect.

Gestalt psychology introduced the idea that the mind forms a global whole with self-organizing tendencies. This was reflected by *global method* of learning which refused analytic-synthetic approach and provides ready solution learned by mechanical repetition of calculation junctions. Junction durableness depended on number of repetitions – *drill*. Although, the method appeared to be insufficient at higher levels of schools, the research on difficultness of calculation junctions invented the procedures that led very quickly to automatisation of operations.

In late 1940s the concept of teaching mathematics in scientific way was promoted again. Mathematics had to be built as scientific system at all levels of schools, new terms had to be defined in exact way, theorems proofed, solution had to be preceded by analysis, and problems practiced in application tasks. And again, it turned out that such strict presentation of mathematics does not correspond with learner's level of psychological development, and that curricula are not able to provide enough time for repetition, skill practice, application tasks, etc. The will to educate in effective mode, and the interest of government to aquire as high results as possible led to instituting didactics as the scientific discipline with national research programs. New methods, contents and textbooks were tested in experimental schools all over the country. The school system garanteed the continuition of education; having spiral curricula structure it was able to link scientific approach with psychological possibilities of learners. Based on own experience, influenced by new trends of education abroad and supported also with soviet pedagogy literature, new ways of solving mathematical tasks were introduced. Programmed learning developed inductive way of thinking, propeudic approach allowed first deductive ideas to arise. Analytic-synthetic procedure in solving complex word problems and in constructional geometric tasks have had their value until now as well as individual work with textbook or scientific text, sometimes followed by reporting to classmates (today we speak about flipped classrooms). Symbolic language had to be used until it simplified the form in record as well as in understanding. The accent was put on logical and critical thinking. It was emphasized that the application of new methods is more important for mathematics understanding and usage than modernisation of contents. Compulsory education was supported by remedial groups for students who had gaps, and talented students who were interested in mathematics attended mathematics clubs and mathematical competitions. The positive atitude to mathematics and conscious of its importance was formed also by TV shows. (Mikulcak, 2007)

Teaching mathematics at technical universities

While the overall aim of education was formulated generally for all levels including the university one, learning outcomes and methods were articulated particularly only for elementary and secondary schools. The reason for this could be prosaic. There was no need for that.

The university education was necessary only for "scientific" professions (educators, physicians, scientists, etc.) and high industry management. Students at universities were very clever students, well prepared at secondary schools, a selection of interested applicants who passed the final secondary exam from mathematics and the university entrance exam, ones who satisfied admission procedure and met limited quota set by the government. They studied autonomously supported with traditional forms of university education (math lectures exposing definitions, theorem and proofs, practicals where sample problems were solved, computer laboratories, seminars, and consultations).

Today, the situation is completely different. First, having minimal requirements, technical universities are open to all interested parties. Practically, each student, finishing secondary school with maturita can attend the university. Universities changed their structure. Five-year-long study, with comprehensive theoretical basis including math, was replaced by bachelor and master degrees with dramatically restricted amount of math and physics teaching hours, requiring no entrance exam. Our students do not undergo selection process, many of them have big gaps in their mathematical preparation from secondary school, having no final secondary exam from mathematics. Generally, they are much less motivated to study; moreover, many of them seem to be less "grown up", less capable of making decisions, less responsible. They are having difficulty to overcome transition period from secondary school, to find their learning style, etc. Many students are not able or willing enough to develop their higher cognitive abilities, especially in their first year at university, demanding a lot of care.

All this bestows much more requirements on pedagogy mastership of university teachers, driving them to develop, test and apply activating methods in order to cultivate students' competencies. It also creates the space for dealing with mathematical competencies in detail, incorporating them more explicitly, if necessary, into study curricula also at universities. From this point of view, sufficient pedagogical education is essential for university teachers and their work in this area should be as valuable as their work in math or technical sciences.

On assessment in relation to competence

Generally, to have a competence means to be able to do something. It refers to knowledge and skills, the person has learned in past, to abilities, the person is able to do in present time, and also to aptitudes, what person is able to learn, and do in future (e.g. ability to develop mathematical thinking in mathematical competence definition). All these items describe inner capabilities of a person. The question is, how inner capabilities could be tested. The answer can be sought in psychometrics, a field concerning with theory and technique of psychological measurement, that has been under huge development since the first half of 20^{th} century. Psychometric tests account for a large part of recruitment process and of assessing employees' qualities, and nowadays they enjoy great business interest. It is natural, that it is not possible to test inner capabilities directly. They can be tested only through **performance**, usually on test items, evaluated by scores. The core lies in quality of test construction and results interpretation. Psychological testing is based on hypothesis that score of standardised test reflects an examined psychological construct (cognitive ability, aptitude, etc.) validated in advance by statistical processing. Maybe here we can find the answer, how to assess knowledge, understanding, and similar inner abilities. Psychometrics tests to large extent involve mathematical competencies tasks (numerical reasoning test, inductive reasoning test, diagrammatic reasoning test, error checking test, etc.) and they may improve competencies assessment at schools. It is important to realise that no competency test result will guarantee the reaction of tested person in supposed way – i.e. having competency in something, the person certainly solve a problem; this is more complex issue, it depends also on personality character, willingness to do it, etc. Going back to school, we have to be also aware that once taken test will not guarantee the consistency of knowledge or skills, and moreover, it could distort results of persons who are not able/ do not like/ are not willing to take that particular kind of test or they have qualities indicating the competence in different way. Anyhow, competence testing should be finally appraised personally, taking into account all impacts.

Tasks usually comprehended in school tests have convergent character. It means they are factographic or they require step by step procedure of prescribed method. Although creativity, critical thinking or error checking make also important part of mathematical competencies, they are assessed rarely or not at all. Our experience shows that students consider such issues to be important and time worthy to study and train, which are examined and evaluated. Considering the competence point of view, it would be very important to evaluate all parts that are valuable for further study, occupation or life. Tasks like: to determine as many ways of solution as possible, to find different ways of utilisation in other disciplines / life (Richtarikova, 2016), to explain, to estimate the value without exact calculation, to find out why the result is not correct – recognise a mistake in calculation / in the way of thinking, give a chain of arguments, what is a point of … (Bender, Thiele, 2014), what is your first idea when..., etc. help to form overall mathematical competency and consciousness of its importance and should not be omitted in examining and specification of courses outcomes.

Questionable is also, to what extent the intercourse tasks directly dealing with utility of mathematics in other disciplines should be included in assessment. Our first year students at bachelor degree, exposed to complex task requiring knowledge from another branch (physics, mechanics), had serious difficulties in composing the suitable formulas and then they were completely lost also in mathematical solution as they often were not able to keep the sense of given task. Providing them a clue in the form of partial formulas with general notations showed much better results. Due to above, we think that with respect to careful formulation of learning outcome it is worth to omit all overloading elements and assess specifically only the desired outcome. Complex, time demanding intercourse tasks are very valuable in higher years of study, in the form of theses or "student scientific works". Testing different forms of project tasks we conclude that to learn the specific outcome of simple mathematics course, especially at bachelor degree, will more profit from simple, student familiar application. This is also the pool where students can learn to identify the mathematical concepts, better understand them, and acquire a broader view on mathematics role among other disciplines or in everyday life.

Conclusion

"*Who wants to know the present with no knowledge of the past can never understand the present.*"

G. W. Leibniz (1646–1716) in Mikulcak 2007

Aiming at everyday life needs of learners is an essential attribute of education itself. It has been concerning each person since compulsory schooling was introduced, and it excessively effects one's life. Today we respect the EC document on competencies defining the goal of education in EU, and seek the effective concept of education also at technical universities. Although our focus is on tertiary education, it is worth to look into rich history of education in our countries, and realise how contemporary concepts, principles, methods came into existence and their implications. Comparing the mathematical outcomes formulated in Curricula 1987 with the competencies stated nowadays for engineering education in the "Framework" one can find them quite similar. Analyses of benefits as well as drawbacks of the work that resulted from extend research in didactics after the World War II can be a very precious source of valuable information as a lot of problems solved at that time mostly for elementary and secondary schools are nowadays still topical and they concern also university studies.

Harmonisation of mathematical standards in secondary and tertiary education could lead to the elimination of the disproportion between the requirements of universities and the level of mathematical preparation at secondary schools. Cultivation of competencies depends at great extend on teacher's qualities, the right selection of methods and positive working atmosphere. Sufficient pedagogical education is essential for university teachers

Rules and the style of tasks in psychometric testing could be helpful for competence oriented testing. It is also important to involve divergent tasks and application tasks of corresponding level. The structure of test should comprise tasks of all relevant areas, it helps to form the student's cognizance of important outcomes.

References

Alpers, B. et al. (2013) "*A Framework for Mathematics Curricula in Engineering Education.*" SEFI, Brussels, 2013. ISBN: 978-2-87352-007-6.

Bender G., Thiele K. (2014) "*Feedback and formative assessments in mathematical lectures*". In Proc. of 17th SEFI MWG Seminar, Brusel, SEFI, 2014. ISBN 978-2-87352-011-3. available on http://sefi.htw-aalen.de/.

Concept (1960). "*Poňatie základnej deväťročnej školy a učebný plán tejto školy*" - por. č. 38/1960 Vestníka MŠK (in Slovak) [The concept of the basic nine-year school and the curriculum of this school]

Curricula, (1987). "*Učebné osnovy pre študijné odbory stredných odborných učilíšť*" (in Slovak) [Curricula for secondary voacational schools], SPN, 1987 (p.199-233).

Development (1976). "*Další rozvoj československé výchovně vzdělávací soustavy. Projekt a důvodová zpráva*". (in Czech) [Further Development of the Czechoslovak Educational System. Project and explanatory report]. Praha, SPN, 1976.

Divisek, J., et al. (1989) "*Didaktika matematiky pro učitelství 1. stupně ZŠ*". (in Czech) [Didactics of mathematics for teaching at 1st level of elementary school] Praha, SPN.

Kopecky, J. (1992) Komenského Škola hrou. In: "*Jaromír Kopecký o Komenského spisu Škola hrou*", (in Czech) [Jaromir Kopecky on Schola Ludus, the act by Komensky], 2015. Available on: https://www.ascestinaru.cz/skola-hrou-nebo-hrani-si-ve-skole-jaromir-kopecky-o-komenskeho-spisu-skola-hrou/)

Law (1948) Zákon č. 95/1948 Zb. "*Zákon o základní úpravě jednotného školství (školský zákon)*". (in Czech) [Law on basic adjustment of unique education. School Law]. Available on http://www.zakonypreludi.sk/zz/rocnik/1948

Mikulcak, J. (2007) Jak se vyvíjela pedagogika matematiky ve druhé polovině 20. století (in Czech) [How the mathematics education has been developing in the second half of the 20th century]. In: Bečvářová, Martina (editor); Bečvář, Jindřich (editor): "*Matematika v proměnách věků*". V. Praha: Matfyzpress, 2007. 978-80-7378-017-3, pp. 249-315. MSC: 00A35

Mikulcak, J. (2010). J. A. Komenský o vyučování matematice. In: Jiří Mikulčák (author): "*Nástin dějin vyučování v matematice (a také školy) v českých zemích do roku 1918*". (in Czech) [An outline of the history of teaching mathematics (and also schools) in the Bohemia until 1918]. Praha: Matfyzpress, 2010. pp. 59–65. Persistent URL: http://dml.cz/dmlcz/400981

Niss, M. (2003). "*Mathematical competencies and the learning of mathematics: The Danish KOM project*". In A. Gagatsis, S. Papastravidis (Eds.), 3rd Mediterranean Conference on Mathematics Education, Athens, Greece: Hellenic Mathematical Society and Cyprus Mathematical Society, 115- 124.

Proposal (2018) "*Proposal for a Council Recommendation on Key Competences for Lifelong Learning* (Text with EEA relevance) {SWD(2018) 14 final}" available on: https://ec.europa.eu/ education/sites/education/files/recommendation-key-competences-lifelong-learning.pdf

Recommendation (2006) "*Recommendation of the European Parliament and of the Council of 18 December 2006 on key competences for lifelong learning* (2006/962/EC)", available on https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri = CELEX:32006H0962&from=EN

Resolution (1959). "*O úzkom spojení školy so životom a o ďalšom rozvoji výchovy a vzdelania v ČSR*". (in Slovak) [The resolution of Central Committee of the Communist Party of Czechoslovakia on close connection of school with life and the further development of education in Czechoslovakia], Bratislava 1959.

Richtarikova, D. (2014) "*To see Mathematics Useful*." In Proc. of 17th SEFI MWG Seminar, Brusel, SEFI, 2014. ISBN 978-2-87352-011-3. available on http://sefi.htw-aalen.de/.

Richtarikova, D. (2016) "*Enhancing Ability to Identify and Use Mathematical Concepts*." In Proc. of 18th SEFI MWG Seminar, Brusel, SEFI, 2016. ISBN 978-2-87352-013-7. available on http://sefi.htw-aalen.de/.

Sedivy, J. et al. (1987). "*Antologie matematických didaktických textů*". Období 1360 –1870. (in Czech) [Anthology of mathematical didactics texts. Period 1360-1870]. Praha. SPN, 1987.

Sedivy, J. et al. (1988). "*Antologie z učebnic matematiky. Období 1860 – 1960*". (in Czech) [Anthology of mathematical textbooks. Period 1860-1960] Praha, SPN, 1988.

New Methodologies for Teaching Math Courses in Engineering Degrees

Carla Pinto[*†], Susana Nicola[*,**], Jorge Mendonça[*], Daniela Velichová[¥], Ariane Heler[‡]

[*]*ISEP/P.Porto,* [**]*INESC-TEC,* [†]*CMUP,* [¥]*STU Bratislava,* [‡]*Chemnitz University of Technology*

Abstract

The traditional way of teaching math classes is based on a 'teaching by telling', or 'chalk and talk', approach, especially in the first years of the university degrees. It is based on single lecture-based delivery to large classes. Recently, there has been a growing interest, by the engineering professionals and the bodies for accrediting engineering degrees, in promoting a change in this paradigm. As Einstein used to say "We must revolutionize our thinking, revolutionize our action". The "change" in the teaching process consists in the implementation of active learning (AL) methodologies.

AL consists in instructional methods that engage students in the learning process, i.e., which require students to do meaningful learning activities and think about what they are doing. Students become an active part of their learning process, by reading, writing, talking, listening, debating, applying principles, and reflecting on the topics they are studying. AL, as opposed to the passive learning, moves the responsibility for learning from a teacher-centered to a student-centered basis. AL fosters skills-development rather than just conveying information to students. All of this aims to promote higher-order thinking, i.e., critical thinking, analysis, and development of soft skills (agility, curiosity, imagination, collaboration, communication). Some AL instructional frameworks include problem-based learning (PBL), hands-on, eduScrum, Jigsaw.

Some practical examples of possible implementation of these innovative learning/teaching strategies in engineering mathematics courses will be presented in the paper.

Keywords: Active learning, Jigsaw, eduScrum, Student-centered learning, Problem solving, Hands-On, Engineering

Introduction

The pursuit of ideal teaching and learning methods has summoned educators for centuries. Challenges are twofold: (i) increased efficiency and competitiveness in a global changing environment project the responsibility of education and training to the post-secondary sector. Students in higher education are defying not only to learn, and articulate subjects, but also to present new solutions to new and interesting problems, on the fly. The second challenge is (ii) the need for an assertive response and adaptability to the increased international competition between institutions for more and better students, to the greater social and geographical students' body diversity, to the constant demand for profit, to the persistent evolving technologies, to name a few. New students require novel teaching techniques. Moreover, there is a change in the nature of the interactions between students and professors in the classroom, due to the appearance of the modern technologies. Every key player in Education, from politicians, to students and families, employers, wish and demand more efficiency on teaching, Kandiko and Mawer (2013).

Educators, worldwide, are trying to make school more interesting, more motivating, more fun to learn, i.e., are trying to transform schooling and education. In this sense, new methodologies, namely active-learning (AL), are gradually being implemented in a variety of contexts and start enduring, Bonwell and Eison (1991), and Eison (2010). AL encourages creativity, promotes cognitive processing, critical thinking, and fosters resilience. Critical thinking consists in identifying misleading advertisements, weighing competing evidence, identifying assumptions or fallacies in arguments. Teachers must encourage the acceptance of divergent perspectives and free discussion in the classroom, Lyman (1987). With AL, learning is seen as ongoing process, where students engage and learn everywhere, anytime, not just a constant or fact to know or memorize. The responsibility of the learning is shifted to the students. Student-centred learning is a major key point in modern teaching. Teachers want students to be owners of their own knowledge, to be self-motivated to seek new knowledge, and to develop new skills, Mendonça *et al* (2018), Nicola *et al* (2018). This is only achievable by a great understanding by the teachers of each and every student. Student-centred learning focus on experience and hands-on. John Dewey preconized that education should be "grounded in real experience" and built around inquiry and exploration, Felder and Brent (2009), Felder and Brent (2010), Fagen *et al* (2002), and Smith *et al* (2009).

Two AL teaching methodologies are eduScrum and Jigsaw. EduScrum is an active collaborative education process that allows students to plan and determine their study activities and their learning process by themselves, supporting responsibility for keeping track of their study progress. While teacher determines why and what to study, the students determine how to organize and manage it. This is resulting in intrinsic motivation, fun, personal growth and better results. Such personalized learning method has a very important role, as it is positively effecting student's creativity, mutual collaboration, professional communication and critical thinking Mendonça *et al* (2018), Nicola *et al* (2018).

Jigsaw is one of the efficient cooperative learning strategies which enable each student in the small working group to be directly engaged with the material, instead of receiving it presented in a passive way, which considerably fosters the depth of understanding Mendonça et al (2018), Nicola et al (2018). Students gain practice not only in self-teaching, but also in peer teaching, which requires them to understand the material at a deeper level than they typically do when they are only asked to perform at exam. This AL strategy is quite opposite to the exam-driven learning strategy that is usually adopted by the majority of weakly motivated students.

Bearing the aforementioned ideas in mind, we outlined the paper as follows. In the next section, we describe in more detail the active-learning methodology and how we adapted it to our Math courses in Bachelor Engineering Degrees. In the Section "Results", we describe the purpose of the study, and analyse and discuss the results. In the last section, we draw some conclusions of our work and shed some light on future research.

Method of Investigation

One of the most important concerns/"fears" teachers usually face is the lack of (deep) understanding from the students of the contents taught in their classes. Students are taught by the same teacher, the same topics, at the same time, and they respond differently. How is this possible? Most of the teachers feel confident about what they explain, since they try very hard to motivate students to learn and do their best to provide them with the most significant scientific material, as well as real examples. In this sense, applications of some topics are put in practice, so students can sense a flavour of the real life meaning of what they are studying. Nevertheless, when the exam grades come out, the results can be frustrating and the optimism goes, sometimes, down the drain. Moreover, all teachers know that the worst thing that can happen is that lower grade students begin to feel less and less confident, and less and less motivated. The later contributes to higher and successive failure rates, and may even increase university studies dropouts.

With this scenario in mind, we, as educators began to think and reflect on how we could change the teaching paradigm of math classes. The learning process requires an active environment and cognitive effort. It is thus our responsibility to create learning strategies that motivate students to accept responsibility for learning, the so-called student-centred learning.

In order to implement this learning approach, we applied in the practical classes of the Linear Algebra and Analytical Geometry course, the eduScrum methodology, as a part of an AL process. In Figure 1, we depict the full teaching framework. The course consists of 12 weeks. Each practical class of 37 students is split in 6 or 7 groups, of 5 or 6 students each. Every two weeks, students have Sprints, where they have to do a set of proposed exercises and are evaluated accordingly. In this scenario students are exposed to some discussion between their peers and with the teacher, which is an essential part of a mathematical classroom. According to Ariana Sampsel (2013) *"discussion as a class or in small groups also allows students to practice critiquing others' reasoning and to practice constructing their own arguments"*

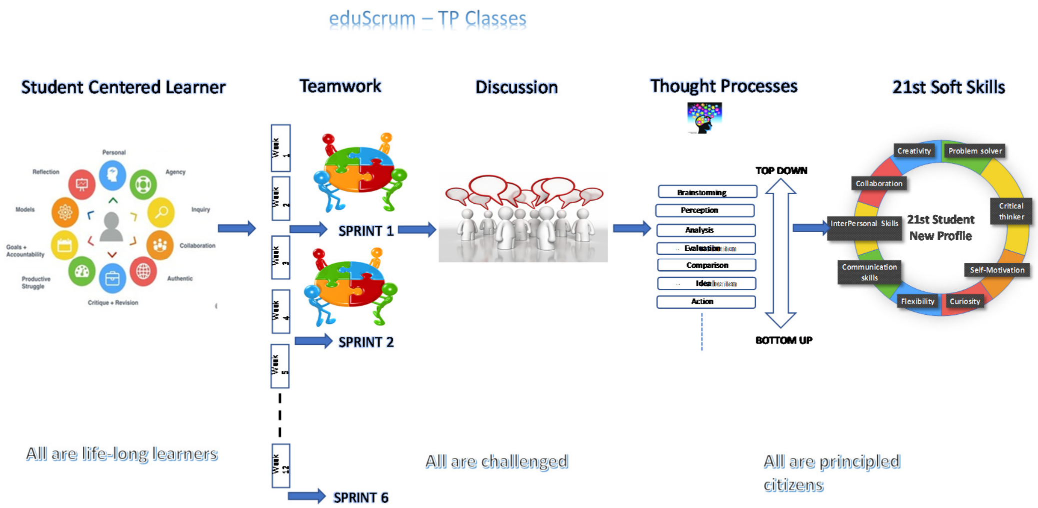


Figure 1 Framework of the practical classes

In addition, working in groups will be contributing for the thinking processes, namely:

- Brainstorming (ideas) – where each student can develop his own creativity, by developing ideas and solutions for given problems, through intensive and free group discussion.
- Perception – where students may visualize the problem in their minds, observe, picture or detect;
- Analysis – students will be able to detail steps, parts, reasons or even sketch a mind map to solve a problem;
- Evaluation – through peer interaction, students can formulate an opinion about the information, share points of view and give meaningful steps towards the solution of what is being learned;
- Action – students do the exercise, and construct the true meaning of the problem.

Thus, AL, as an educational tool, may contribute to the development of the 21^{st} soft skills, namely collaboration and teamwork, creativity and imagination, critical thinking, problem solving, which students need to meet the challenges and opportunities of today's global world.

Findings and Discussion

In this section we outline the essential results from a questionnaire made to students attending the Linear Algebra and Analytical Geometry class, 1st semester of 2017/2018, of the Informatics Bachelor Degree. The students replied to some questions concerning the context of their teaching framework, either eduScrum (EDS) or the traditional method (TM), namely:

- “Brainstorm different possible solutions to a given problem”
- “Assume responsibility for learning material on my own”
- “Discuss concepts with classmates during class”
- “Solve problems in a group during class”

Students replied according to the classification Strongly disagree (1); Disagree (2); Slightly agree (3); Moderately agree (4), and Strongly agree (5).

The summary statistics of the data are given in Table 1 and Table 2. The results from the application of the Mann-Whitney test can be seen in Table 3.

		Mean	S.D.	S.E.
Brainstorm different possible solutions to a given problem	TM N=106	3,35	,936	,091
	EDS N=104	3,56	,774	,076
Assume responsibility for learning material on my own	TM N=106	3,60	,973	,094
	EDS N=104	3,68	,958	,094
Discuss concepts with classmates during class	TM N=106	3,83	1,064	,103
	EDS N=104	4,15	,822	,081
Solve problems in a group during class	TM N=106	3,50	1,181	,115
	EDS N=104	4,21	,809	,079

Table 1. Summary statistics.

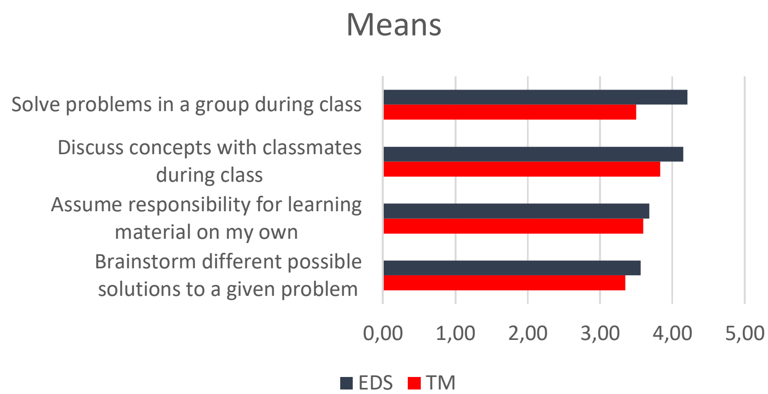


Table 2. Means.

	Brainstorm different possible solutions to a given problem	Assume responsibility for learning material on my own	Discuss concepts with classmates during class	Solve problems in a group during class
U - Mann-Whitney	5018,000	5196,500	4644,500	3550,500
p_value	,224	,451	,037	,000

Table 3. Results from the application of the U-Mann-Whitney test.

We observed significant differences between the two groups (p_value<0.05), TM and EDS in two questions: "Discuss concepts with classmates during class" and "Solve problems in a group during class". For the other two questions, "Brainstorm different possible solutions to a given problem" and "Assume responsibility for learning material on my own", the results do not show significant differences between the EDS and the TM methodologies. In the four questions, it is observed a higher variability in the responses of the TM group, when compared to the EDS group.

The results for the two questions, related to the brainstorm and responsibility for new learning material, are, in our own perspective, due to the students' lack of experience in these novel pedagogical techniques. Upper school students are comfortable with the TM methods, in which they act like vessels, waiting to be filled by the information provided by the teacher. Since this is the first semester they are experiencing different and active teaching approaches, they do not yet know how to take advantage of their full potential. We, as teachers, need to help deconstruct the passive listener scheme of the TM methods. This is a hard and complex task, since students, when arriving to University, face a challenging environment, physically and psychologically, and they have to adjust to all very fast. University constitutes a major achievement but is also an insurmountable *obstacle* to be overcome to reach success. Students must learn how to be independent in all senses, wisely choose how to spend their time, adjust and prepare themselves for the radical and overwhelming 21st century workspace.

Conclusions for Education

In this paper, we described a framework to implement AL methodologies, in particular, eduScrum, in the practical classes of the Linear Algebra and Analytical Geometry course, of the Informatics Bachelor Degree at ISEP.

The results from a questionnaire made to students reveal differences of the students' perceptions with respect to eduScrum and traditional methods. Students feel like discussing and solving problems in group contexts is enhanced in the eduScrum environment. As to brainstorming new solutions and taking responsibility of new teaching materials, there is no significant difference between the students involved in EDS vs students taught with TM. We believe the later is somehow explained by the novelty of the new teaching approaches and the need for an adjustment time from the students. This points to the design of strategies to motivate students to these new pedagogical tools, highlighting their importance to their future knowledge and to their personal and inter-personal skills required in the fourth industrial revolution, which will dramatically alter the way we live, work, and socialize.

Acknowledgments

CMUP's author was partially supported by CMUP (UID/MAT/00144/2013), which is funded by FCT (Portugal) with national (MEC) and European structural funds (FEDER), under the partnership agreement PT2020. All authors wish to thank Eramus+.

Co-funded by the
Erasmus+ Programme
of the European Union

References

Bonwell, C.C., Eison, A.J. (1991) Active Learning: Creating Excitement in the Classroom" in ASHEERIC Higher Education Report No. 1. Washington, DC: George Washington University Press.

Eison, J. (2010) Using active learning instructional strategies to create excitement and enhance learning, 2010. Retrieved from http://www.cte.cornell.edu/documents/presentations/Eisen-Handout.pdf.

Fagen, A.P., Crouch, C.H., Mazur, E. (2002). Peer instruction: Results from a range of classrooms. *The Physics Teacher*, 40(4), 206-209.

Felder, R.M., Brent, R. (2009). Active learning: An introduction. ASQ Higher Education Brief, 2(4).

Felder, R.M., Brent, R. (2010) Navigating the Bumpy Road to Student-Centered Instruction, *College Teaching*, 44 (2): 43-47.

Kandiko, C.B., Mawer, M. (2013) Student Expectations and Perceptions of Higher Education. London: King's Learning Institute.

Lyman, F., 1987, Think-Pair-Share: An expanding teaching technique: MAA-CIE Cooperative News, 1:1-2.

Mendonça, J. Nicola, S., Pinto, C.M.A. (2018) ACTIVE LEARNING: SELF-MOTIVATION IN MATH COURSES, Proceedings of INTED2018 Conference 5th-7th March 2018, Valencia, Spain, 1870-1877.

Nicola, S. Mendonça, J., Pinto, C.M.A. (2018) ACTIVE-LEARNING TECHNIQUES IN MATH COURSES IN ENGINEERING DEGREES: THE PURSUIT FOR A NEW ENGINEER PROFILE, Proceedings of APLIMAT2018, 17th Conference on Applied Mathematics, February 6 – 8 2018, Bratislava, Slovakia, 9 pages.

Petersen, C., Gorman, K. (2014) Strategies to Address Common Challenges When Teaching in an Active Learning Classroom, *New Directions for Teaching and Learning*, 2014 (137):63-70.

Sampsel, A. (2013) Finding the Effects of Think-Pair-Share on Student Confidence and Participation, Honors Projects. 28. https://scholarworks.bgsu.edu/honorsprojects/28

Smith, M.K., Wood, W.B., Adams, W.K., Wieman, C., Knight, J.K., Guild, N., Su, T.T. (2009). Why Peer Discussion Improves Student Performance on In-Class Concept Questions. *Science*, 323(5910), 122-124.

A competence-oriented learning center for first-year students at Mannheim University of Applied Sciences

Susanne Kreim[1], Klaus Beck[1], Steffen Rasenat[2], Wiebke Werft[3]

[1]*Department of Electrical Engineering,* [2]*Institut für naturwissenschaftliche Grundlagen,* [3]*Department of Mechanical Engineering; Mannheim University of Applied Sciences*

Abstract

German universities of applied sciences observe a profound discrepancy between required and applicable prerequisites in mathematics of first-year students of engineering departments, often paired with a lack of self-assessment of one's own level of performance and autonomy in self-study. Mannheim University of Applied Sciences is establishing a learning center with a course on basic topics of mathematics at its heart. The main objective is educating students in the targeted application of mathematics to the specific requirements of the respective engineering discipline. Lasting proficiency in the key skills is achieved by linking mathematical content to the student's daily life as well as to applications in engineering and real-life practice. A three-step methodology is employed to ensure mastery of key skills integrating training of generic competencies (personal and social skills) into the development of professional competencies. The course has been designed to cover up to ten quintessential mathematical topics over the course of roughly one semester. Its modular design allows faculty to choose the topics most relevant to their department. The course has taken place twice and has been evaluated by a good 40 participants. Overall, the course has been perceived well and the workload as adequate. The overwhelming majority stated that they were able to close their knowledge gaps and acquire the respective skills.

Introduction

Germany is a leading location for technology and innovation. Leading-edge technology and the innovative strength rest on the distinct German engineering culture with engineers as a driving force of economic and social development as discussed by Ungeheuer (2015). Regarding continuous economic growth, skilled engineers are indispensable. German universities succeed only partly in satisfying this demand. On the one hand, this is due to a lack of applicants resulting from a lack in open-mindedness towards engineering. On the other hand, this is caused by an average drop-out rate in engineering of about 10%. Performance problems and level of motivation are among the main reasons, the former including missing expertise with respect to prerequisites according to Heublein (2017). The greater aim of the measures presented here is to render possible lasting academic success, thus impacting drop-out rates.

In engineering departments, mathematics plays a key role at the beginning of one's studies. The targeted application of mathematical competencies – for a definition, see Weinert (2001) – to technical problems is imperative for their successful solution and thus essential for engineers. The required knowledge is imparted as part of the core curriculum based on educational standards of mathematics published by the *Kultusministerium* (2015). Yet, a profound discrepancy between required prerequisites according to the working group COSH (2014) and applicable prerequisites in mathematics of first-year students of engineering departments is observed. The COSH working group is composed of representatives of high schools and universities of Baden-Württemberg and has published a catalogue listing minimum requirements of mathematics for studies of engineering disciplines. Speaking with Scholz (2016), the above mentioned discrepancy is classified as vertical heterogeneity due to diverging ability. This strongly heterogeneous level of prior knowledge results from the various ways in which the matriculation standard can be acquired. The vertical heterogeneity is complemented by a horizontal heterogeneity, examples of which being learning strategy, motivation, and cultural diversity. Combined with the inability to self-assess one's own level of performance and the lack of autonomy in self-study this poses an enormous challenge on the curriculum and is of primary concern.

According to basic assumptions of impact research, study success is considered to be the result of a successful fit between student's prerequisites and university requirements. Pursuant to Merger (2015) the success of a measure can be determined by how much the level of fitting has improved for specific groups of students. The aim of the learning center is to devise a teaching-learning process such that students receive individual support according to their respective needs in order to academically succeed. Focus is on mathematical competencies namely "solving problems", "proceeding systematical-ly", "making plausibility considerations" as well as "communicating and reasoning mathematically" as defined by COSH (2014). Furthermore, recent results of Heublein (2017) show that the drop-out rate is highest in the first two semesters. Finally, curricular activities geared to generic competencies are considerably worse perceived than those regarding professional competencies. As a result, the course fundamentals of mathematics (LV MAG - *Lehrveranstaltung mathematische Grundlagen*) was designed as a first-year curricular measure spanning over one semester taking into account the impact pattern shown in Fig. 1. Different activities were implemented with the aim that students not only develop the key professional competencies for a targeted application of mathematics but also promote generic competencies (Fig. 1, outcome for audience).

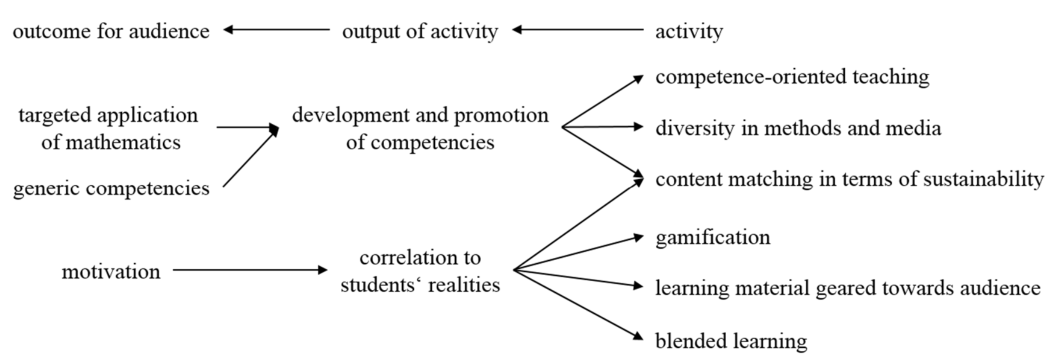


Figure 1. Impact pattern.

In particular, the advancement of professional competencies is closely linked to the development of generic competencies relevant for a future job such as endurance, motivation, communication and feedback skills, assertiveness, leadership or teamwork.

Conceptual design of LV MAG

The Ministry of Science, Research, and Art of the State of Baden-Württemberg (2015) views it as a genuine duty of universities to lead as many students as possible to academic success. In particular, the core curriculum should be designed such as to accommodate the different student needs, and its study program should adapt to the variety of different prerequisites, affinities, and competencies. A three-step methodology is employed to reach the aims listed in Fig. 1, outcome for audience. First, requirements analysis is used to identify lack of knowledge and skills. Second, mindset and germane behavior of students as well as lecturers are matched to ensure an effective learning process. Third, an appealing learning environment is generated by meeting state-of-the-art educational standards and integration of modern media.

Requirement analysis is intended to yield a genuine image of preconditions at the beginning of each semester revealing the student prerequisites to the instructor while identifying knowledge and skill gaps to the students. Learning content is therefore matched to the subject-specific requirements in favour of sustainable education (Fig. 1, activity). With an electronic placement test as diagnosis the instructor gets valid feedback on the vertical heterogeneity of the cohort. The placement test is based on the minimum requirements by COSH (2014). The genuine image of preconditions serves as starting point for competence-oriented teaching (Fig. 1, activity). Competence-oriented teaching generates lasting knowledge, which the scholar can actively govern and transfer with a long-term perspective. Result is a promotion specific to each student's needs based on individual prerequisites causing, pursuant to Scholtz (2016), an optimum gain in learning and development progress. Vertical as well as horizontal heterogeneity are thus accommodated while ensuring the high quality of education.

Teaching means interacting. Physically speaking, harmonic and coherent interaction generates constructive interference. To shape this interaction taking on a student perspective constitutes a major challenge for instructors. In particular, the teaching/learning process has to be devised custom-fit to achieve an adequate match between student's prerequisites and university requirements. An effective learning process becomes apparent through a motivating and intriguing atmosphere. Motivation can be aroused following Deci and Ryan (2008) if a link to the reality of student life is established by social integration, topical connection and autonomy. To this end, gaining up-to-date insight into a young person's life at the passage from school to university is substantial. Thereby a correlation can be established between mathematical content and requirements of the respective study program, on the one hand, and the reality of student life, on the other hand. Thus, the learning material can be geared to the audience tying in with individual prerequisites. Vertical heterogeneity can thus be addressed by application-oriented, problem-based assignments with internal differentiation enabling a cumulative and continuous learning process. The worksheets contain problems with different levels of difficulty. This is realized by not only varying the problem type following Bruder (2016) or the kind of action based on the mathematical competencies mentioned above, but also by addressing all the levels of Bloom's taxonomy (1976) from remembering to evaluating. Horizontal heterogeneity can be addressed by initiating the learning process in a diverse and manifold manner including media pluralism and methodological diversity (Fig. 1, activity). A more detailed description of how worksheets and electronic problems are constructed can be found in Kreim (2018). Finally, the course is designed in the format of blended learning (Fig. 1, activity) as defined by Sauter et al. (2004) incorporating on the one hand e-learning scenarios to account for the reality of digital natives as coined by Prensky (2001), and using gamifi-cation elements (Fig. 1, activity) as means to create an intriguing atmosphere following de Sousa Borges (2014). With the successful completion of LV MAG, each student has obtained the professional competence as laid out in the minimum requirements by COSH (2014). The quality of fitting of the course is assessed by the level of proficiency the students have acquired in the targeted application of mathematics to specific requirements. Success of the measure is revealed by lasting, competent student action.

Course format of LV MAG

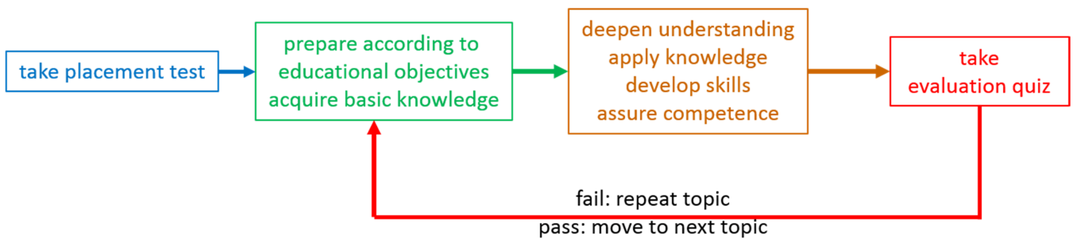


Figure 2. Course loop structure.

The minimum requirements of mathematics as laid out by COSH (2014) have been adjusted to the specific requirements of the engineering disciplines at Mannheim University of Applied Sciences resulting in ten successive topics, which have been integrated in the course curriculum: basic principles, geometry, ratios, power/root/loga-rithm, (linear systems of) equations, fundamental functions, trigonometry, and introduction to vector analysis. Subsequent to the placement test (element for diagnosis, marked blue), students enter a loop structure with one full cycle per topic as schematically illustrated in Fig. 2. First, students prepare themselves according to the educational objectives using the electronic learning platform, Moodle. Here, the teaching form of inverted classroom according to Lage (2000) is used to acquire basic knowledge (asynchronous self-study, online, marked green). Second, students deepen their understanding by applying the knowledge and skills (synchronous study in class, offline, marked brown). The teaching environment is organized with respect to the requirements resulting from orientation to competence. The time in class is used mainly for working on assignments (printed hand-outs) as well as the advancement of generic competencies. Interactive elements are employed strategically depending on the educational objective. To this end, didactical methods adequate for tertiary education according to Macke (2012) are utilized such as brainstorming, concept mapping, pinboard technique, puzzle groups, *Glückstopf*, check list, and feedback. Different social forms behavioral pattern such as question rounds, partner work or group work alternate. As a result, students are enabled to develop the skills required to assure competence. Finally, they take an electronic evaluation quiz (synchronous, online, marked red). After successful completion of the quiz, they move on to the next topic.

Results and Conclusion

The course LV MAG has been conducted twice, each time with a good 20 participants. A subjective student assessment by means of an electronic questionnaire serves as evaluation. The questionnaire is partly based on BEvaKomp developed by Braun (2007) for an operationalized survey on gain in professional and generic competencies. In total, 45 students passed all quizzes, filled the questionnaire, and gave the course overall grade 2 (German system). A good eighty percent of them rated the workload as adequate, about half of them attended the course 80-100% of the time. The quizzes were viewed by about 70% as adequate and their grading by 80% as fair. Other questions were answered using a 6-step scale ranging from "completely agree" to "completely disagree". The table of Fig. 3 quotes how many of the students "completely agree" or "agree" to the respective statement. In the case of the knowledge gaps, the agreement increases to 80% taking into account those who "partly agree". Of the above mentioned students, 25 answered the questions regarding gain in competence. More than half of them stated being able to explain what they had learnt to fellow students as well as to take responsibility for their advance in learning. Taking into account those who “partly agree”, more than 80% of the students stated that their professional conversation skills have improved and that they are able to present complex issues clearly and vividly.

Item	**Agreement**
Teaching philosophy convinced me.	~65%
I could follow the instructor well with my state of knowledge.	~75%
Assignments suited my present state of knowledge.	~65%
Application of math to engineering and real-life practice was pinpointed.	~70%
Individual and competent support by instructor was given.	~90%
I could close my knowledge gaps.	~50%

Figure 3. Results from questionnaire.

In conclusion, a course has been designed to enable students to obtain professional competence in mathematical skills pivotal to an engineering degree. The gain in professional and generic competencies reported by the students strengthens the necessity of this measure. What remains is to survey how competent student action develops over the course of their studies for determining the success of the measure.

References

Bloom, B. ed. (1976) *Taxonomie von Lernzielen im kognitiven Bereich.* Belz.

Braun, E. (2007) *Das Berliner Evaluationsinstrument für selbsteingeschätzte studentische Kompetenzen – BevaKomp.* Vandenhoeck & Ruprecht unipress.

Bruder, R., Leuders, T., Büchter, A. (2016) *Mathematikunterricht entwickeln.* Cornelsen

Deci, E. L. and Ryan, R. M. (2008) Self-Determination Theory: A Macrotheory of Human Motivation, Development, and Health, *Canadian Psychology*, Vol. 49: 182-185.

de Sousa Borges, S., Durelli, V. H S., Reis, H. M, Isotani, S. (2014). A systematic mapping on gamification applied to education. *Proceedings SAC Meeting*

Heublein, U. (2017) *Motive und Ursachen des Studienabbruchs an baden-württembergischen Hochschulen und beruflicher Verbleib der Studienabbrecherinnen und -abbrecher.* DZHW.

Kreim, S., Beck, K., Ellermann, H. (2018) Entwicklung einer didaktisch fundierten digitalen Aufgabenkultur zur individuelle Förderung von Studierenden des Ingenieurwesens in der Mathematik Grundausbildung *(submitted)*

Lage, M. J., Platt, G. J., Treglia, M. (2000) Inverting the classroom: a gateway to creating an inclusive learning environment. *Journal of Economic Education*, Vol. 31:30-43.

Macke, G., Hanke, U. (2012) *Hochschuldidaktik*. Beltz.

Merger, J., Ortenburger, A., Vöttner, A. (2015) *Studienmodelle individueller Geschwindigkeit*. DZHW.

Ministry of Science, Research, and Art of the State of Baden-Württemberg (2015). *Ausschreibung Strukturmodelle in der Studieneingangsphase.*

Prensky, M. (2001) *Digital Natives, Digital Immigration* and *Do they really think differently?* On the Horizon.

Sauter, A. M, Sauter W., Bender H. (2004). *Blended Learning: effiziente Integration von E-Learning und Präsenztraining*. Luchterhand

Scholz, I. (2016) *Das heterogene Klassenzimmer*. Vandenhoeck & Ruprecht

Sekretariat der ständigen Konferenz der Kulstusminister der Länder in der Bundesrepublik Deutschland (2015) *Bildungsstandards im Fach Mathematik für die Allgemeine Hochschulreife.* Wolters Kluwer Deutschland GmbH

Ungeheuer, U. and Appel, R. (2015) *thinking generations – Innovationen für Deutschland.* Stellungnahme VDI Verein Deutscher Ingenieure e. V.

Weinert, F., ed. (2001) *Leistungsmessungen in Schulen*. Belz

COSH – working group cooperation schule:hochschule (2014) *Mindestanforderungskatalog Mathematik.*

Widening Access to Engineering with Mathematics for STEM

P Robinson, Lecturer Institute of Technology Tallaght, Ireland (Paul.Robinson@it-tallaght.ie)

C Breen, Lecturer Dublin Institute of Technology, Ireland (cormac.breen@dit.ie)

C O'Sullivan, Lecturer Institute of Technology Tallaght, Ireland (Ciaran.OSullivan@it-tallaght.ie)

T Brannick, Development Officer Dublin Further Education and Training Board, Ireland (treasa.brannick-ocillin@cdetb.ie)

M Carr, Lecturer Dublin Institute of Technology, Ireland (michael.carr@dit.ie)

ABSTRACT

Many prospective students in Ireland are ineligible for enrolment on STEM programmes because they do not have an acceptable mathematics qualification. For example, a C3 in Higher Level Mathematics (or higher) is typically required for engineering programmes at level 8 (honours). This exam is taken as part of the Leaving Certificate, covering several subjects, and can only be repeated as part of a repeat of the entire Leaving Certificate. Failing, or more often not doing, Honours Mathematics (a lower level may be taken) then effectively cuts that student off from STEM programmes at level 8.

In this paper we discuss a joint project between several third level institutions in Dublin and the further education sector to introduce a new 1 year Mathematics course to address this problem. Currently, most universities in Ireland will only accept a passing grade in higher level honours mathematics as an entry requirement for a STEM discipline. This 1 year module will focus on mathematics for STEM disciplines in particular and forms a viable alternative to the 2 year Leaving Certificate qualification for acceptance onto a STEM course for non-traditional students. The course is now currently running in its second year.

In this paper we will look at the collaboration required to create this course and the outcomes in the first and second year. We will discuss the progression of students into honours STEM disciplines (has it had the right outcomes?) and the student view of the course.

As part of this collaboration an automated testing component of the course has been created between third level and three further education colleges which specifically addresses the basic skills issue. We will discuss the set-up and interaction with teachers and discuss the results of the tests.

Keywords: engineering mathematics, high threshold testing, non-traditional students

INTRODUCTION

Currently in Ireland, students take the Leaving Certificate Examination before progressing to third level. This comprises 6 or more subjects (including English, Mathematics and Irish which are taken by almost every student). The examination can be taken in most subjects at Higher or Ordinary Level. The score for the best 6 results, together with any specific subject requirement, are then used to compete for places at third level institutions through a central applications process. For engineering disciplines at level 8 (honours) a grade of C3 or better in Higher Level Mathematics is usually a specific subject requirement.

For a school leaver not getting a C3 there are very limited options to progress in an engineering level 8 discipline. Leaving Certificate Mathematics cannot be repeated as a single subject, but only as part of repeating the Leaving Certificate Examination as a whole. For this reason, few students repeat it. Worse still, as mathematics is, to all intents and purposes, a compulsory component of the Leaving Certificate, failing it gives the impression of a poor Leaving Certificate transcript and many students opt to take the lower level Ordinary Mathematics, which again excludes them from Engineering at level 8. In response some institutions have developed their own entrance examinations. For example, the College of Engineering and Informatics at NUI Galway offers a Special Entrance Examination in Mathematics [12]. NUIG offers "an intensive five day preparatory course" for this. Several other colleges offer a similar option. Such provision begs the question of why a student should go through a demanding two year Higher Level course! The intensive course with exam is typically a response to "do something" about widening access. Data on the progression beyond first year of students who enter Engineering course using these special entrance Mathematics examinations are not publicly available. There are many other Engineering courses which can be taken without Higher Level Mathematics, but these will usually take longer to complete as students will have to begin on level 7 degree and then progress to level 8.

Outside of the Leaving Certificate route, there are some other courses incorporating a mathematics component which are validated by Quality and Qualifications Ireland (QQI) and meet progression criterion for some third level courses, but none are deemed adequate for entry to level 8 programmes in Engineering by third level colleges. For example, the Dublin Further Education College Coláiste Dhúlaigh offers a FETAC level 5 Engineering award which contains sufficient mathematics to access Dublin Institute of Technology level 7 programmes, but not their level 8 programmes. In response to this, Mary Hickie, the Principal at the Further Education college Coláiste Dhúlaigh and Ms. Patricia Carraher their Maths Teacher proposed the idea of a one year course to meet this need.

For this one year course to be adequate for the purpose of preparing students for honours Engineering programmes at third level, and that third level colleges would have confidence in the course, it was decided that a new form of course development which involved a deep collaboration between mathematics educators from second level, further education and third level would be critical. In this paper, we discuss the design of the one year (300 hours of student effort) mathematics course for students who wish to progress to level 8 programme in Engineering which resulted from the collaborative effort between several third level institutions (Dublin Institute of Technology (DIT), Institute of Technology Tallaght (ITT), Institute of Technology Blanchardstown (ITB)) and the Further Education Sector. This collaboration has been key to the design, implementation and assessment process. Several constraints have had to be met to make this process successful:

- To allow take up by the Further Education Colleges across Ireland, learning outcomes and assessment methods must be provided in detail. Centralised resources for assessment should be available as teachers have heavy workloads (typically 22 hours per week)

- There must be ‘buy in’ on the project from some third level institutions from the start. These institutions must be confident that the course really does produce students who are at a level comparable to C3 on honours Leaving Certificate mathematics.
- Learning outcomes and assessment approaches must be compatible with the current implementation of the Higher Mathematics course at Leaving Certificate, providing a source of textbooks, familiarity for teachers and students and easy comparison with that state exam.
- There must be ‘buy in’ from teachers in the further education colleges. They must see that it is important to third level colleges and collaboration is key to that.

Before describing the design, implementation and assessment elements associated with the one year mathematics course we need to position the development in context. We will provide a brief overview of the Further Education sector and also a description of relevant recent second level Mathematics curriculum reform in Ireland.

1 THE FURTHER EDUCATION SECTOR AND MATHS FOR STEM

In Ireland any education that occurs after second level but is not part of the third level system is known as Further Education. This sector is diverse and includes 32,000 studying on Post Leaving Certificate courses (PLC’s), our target group [13]. A fully comprehensive overview of the Irish Education system can be found at [14] . As with the sector itself, the possible pathways of progression from Further Education are diverse. There are several issues with gaining accurate progression statistics, for instance people can be classed as both employed and in third level education, progression can be attributed back to their Second Level school or there is often no progression data available [15 p. 94]. However, it does appear that main progression routes for students on Further Education programmes are (not given in order) progressing to employment, progressing to third level, progressing to another Further Education programme and leaving to become unemployed. About 20% of PLC students go onto third level education[13]. Many third level colleges, particularly the Institutes of Technology, have small quotas available for students from this sector and also have linked programmes which guarantee access from named Further Education feeder courses [16].

Historically many of these further education students would struggle to cope with the demands of STEM courses at level 8. In an effort to widen access Quality and Qualification Ireland (www.QQI.ie) formed a group of experts to develop a new standard entitled ‘Mathematics for STEM’ as discussed above. It was important that this group comprised of Directors of further Education Colleges (FEC’s) and senior academics, mathematics lecturers and administrators from several third level institutions. This gave the third level colleges confidence that the one year module would be fit for purpose and gave the further education colleges confidence that it would be seen as such.

1.1. Design of the level 8 degree compliant mathematics programme.

Internationally there has been a trend towards more problem-centred mathematics instruction [4]. Significant changes have been made to the second level mathematics curriculum with the introduction of ‘Project Maths’. This new curriculum places greater emphasis on student understanding of mathematical concepts, enabling students to relate mathematics to everyday scenarios with increased use of contexts and applications. The goals of project maths are “strikingly similar to the goals of the reform movement led by the National Council of Teachers (NCTM) in the US” [10]. The ‘Mathematics for STEM’ module was created with this in mind.

To prepare students who enter STEM courses at third level from the further education sector, and to realign the teaching of maths in further education with the changes at second level, an expert group was formed for the design of a one year special purpose award. The view was taken by the expert group that the philosophy, learning outcomes and materials of the new second level curriculum should be absorbed into the new one year programme. With some small changes (most notably the inclusion of logic into the programme) the one year programme is a subset of Higher Level Leaving Certificate Mathematics. The module consists of 6 sections

1. Number (Including percentages, indices, scientific and engineering notation in context, length, area, volume, numerical integration, binary and complex numbers)
2. Set Theory and Logic
3. Algebra (Including linear inequalities)
4. Functions and Calculus
5. Geometry, Synthetic Geometry (as an introduction to proof) & Trigonometry
6. Probability and Summary Statistics
A detailed breakdown and the courses’ validation procedures can be found at [17]. As for Project Maths, the material is set in the context of applied problems and problem solving. The assessment consists of several components:

1. A high threshold short answer/multiple-choice “core skills” section worth 20% with a pass threshold of 80%. Students are allowed three supervised attempts to reach a mark of 80%. Pass in the section is *mandatory* for overall pass of this award. This test consists of 20 questions across a range of learning outcomes involving numeracy and algebra.
2. Two assignments worth 15% each to cover at least two of sections 2, 4, 5 and 6 above.
3. Two exam papers worth 25% each covering all material. Students must achieve at least 50% *on both papers*.

The learning outcomes of the first “core skills” part of the module form a well-known set of materials that students on STEM courses struggle with. We have seen this in the results of diagnostic tests carried out in many Higher Educational establishments in Ireland ([6], [1], [3], [8]) the United Kingdom [9] and in Portugal [2] to name a few countries. The course has this numeracy and algebra problem specifically in mind and the course designers wanted to make it clear to students how important these skills are in STEM disciplines.

Creating an assessment tool for this core material and making it available to all participating Further Education Colleges has been a key focus of the collaborative effort between the teachers in the FEC’s and the lecturers in the third level colleges.

For the other assessment components and as part of this project, assignment and exam materials are to be archived and shared as a resource for all participating FEC’s so as to provide consistency of approach, ease the work burden for teachers and provide ongoing confidence in the qualification for third level colleges. All assignments and exams are also to be independently assessed by an external examiner from third level to further provide confidence in the qualification. Overall, students need 50% for a Pass (including passing three mandatory components), 65% for a Merit (which is classed as equivalent to Honours Leaving Certificate C3) and 80% for a Distinction.

1.2 Maths for STEM take up and student feedback

In the first academic year, beginning September 2016, five Further Education colleges were to take part in the initial running of the course. While all five took part in course design and assessment creation, student numbers meant that only two colleges could run the course with 31 students enrolled in total. The same two colleges ran it in September 2017 (23 students). For 2018 Limerick University will recognize the qualification for access to level 8 and the course is expected to run in local Further Education colleges.

A detailed review will be available through the Education and Training Board [18]. As main points

- 2016 intake: 31 started Maths for STEM , 3 dropped out, 5 dropped to a lower maths level after completing the first online assessments and 23 completed the module. Only 6 students needed the Maths for STEM to progess to a level 8 degree with the rest doing a level 7 degree or other.
- 2017 intake: 23 started Maths for STEM, 8 dropped to a lower level and 15 completed the module (but only one of those in College 2!). Of the 14 who completed Maths for STEM in College 1, 9 have applied for level 8 degrees. The one student in College 2 applied for a level 8 degree.
- Students could do Maths for STEM or a lower level maths award as part of a one year course. Many students in both colleges chose the lower level, especially in 2017 (14 of 22 completed Maths for STEM in college 1 and only one student in college 2). These students were often not interested in applying for a level 8 degree. They also noted that the award had a significant risk of failure and carried no more credit for progression to a level 7 degree than the lower award. It is proposed for 2018 to reduce the failure risk by awarding a Pass for an average 50% (no mandatory pass in 2 exams) and to increase its credit for progression purposes.
- In a 2018 survey, all students who replied from the 2016 cohort (and are in college) said that they were glad they did the Maths for STEM and not the lower level maths. They all found first year maths straightforward, even on level 8 programmes and none failed in first year.

2. CREATION AND OPERATION OF ASSESSMENT TOOL

As part of the project an assessment tool covering material from Number and Algebra was required. Five maths lecturers from DIT, ITT and ITB together with five maths teachers from five FEC’s in Dublin populated the question bank. Documentation was produced by the third level mathematics lecturers on authoring questions, creating and running tests and accessing scores. Training sessions were also organised jointly by the Dublin Education Training Board together with DIT. Further training was provided in September 2017 and will happen on an ongoing basis as issues arise and more FEC's join the project.

The assessment platform is Moodle. The short questions are e a combination of MCQs and short answer questions. Question types are in categories (58) which are grouped into Learning Outcomes. These categories currently comprise over 800 questions, most of which have randomized parameters. Teachers create a quiz by randomly choosing questions from each learning outcome, each choice giving a variety of possible questions, both in content and type. Each student can then see a quite different quiz compared to both other students and to subsequent attempts. Grades and feedback are generated automatically. Feedback is in the form of links to Khan Academy material for that topic. The pass mark for the test is 80%, and repeat tests are easy to schedule.

2.1 Using the assessment tool

The idea in the test design was that students would try similar questions in class and perhaps do an online test as practice. Since they were getting three attempts at the real test they would then link to the learning material in the tests done (and review their notes!) to prepare for further tests. This was far from how testing occurred in practice. **Table 1** below shows that some students had unlimited access to the actual test to practice on in 16 - 17. This was discussed in the 2017 May exam board review and curtailed a little for 2017-18, but not as much as hoped.

practice test attempts	**0**	**2**	**3**	**4**	**5**	**6**	**7**	**9**	**11**	**12**	**15**	**17**	**19**	**21**	**23**	**39**
Coll. 1 (16-17)	8	2			2			2				1	1		1	
Coll. 1 (17-18)		5	3	6	3	3	1	1								
Coll, 2 (16-17)				1	1		2		1	1	1			2		1
Coll. 2 (17-18)																

Table 1: *Uptake of practice tests 2016-17 and 2017-18*

The first row shows the number of practice tests and other rows the number of students taking that many tests. Clearly, several students have attempted to learn the test rather than learn the material with one taking 39 attempts! The teachers running the test were unfamiliar with Moodle and may not have been aware of this possibility. Notably, 8 of the 31 students in 2016 did not attempt any practice tests (only two of those got less than 80% in the real test), and 8 of the 31 students did 12 or more practice tests (of whom 5 got >80%, two did not sit the test and one got 76%)! Only one student chose Maths for STEM in College 2 (17-18) and did no practice tests.

Over the first nine practice test attempts the average score shows a general upward trend as Figure 1 shows. After that (2016 only), scores oscillate for the students taking large numbers of attempts, indicating that they are trying to learn the test rather than the material and techniques.

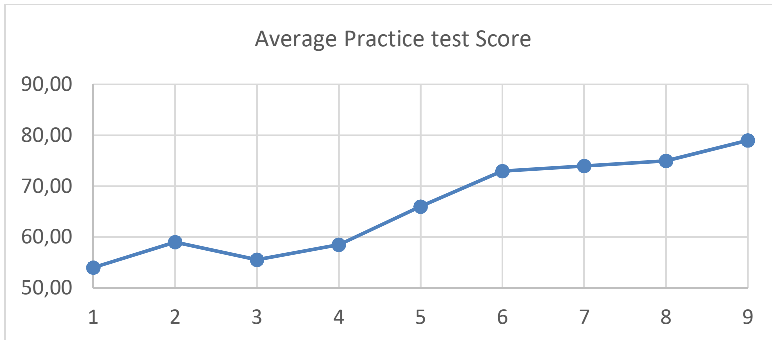


Figure 1: Average score in practice tests in each attempt (Combined 2016/17 intakes)

Further work is needed with participating colleges to understand how the testing system works, how tests are generated and the pedagogical aspects of not allowing too many practice attempts to discourage 'learning the test'.

Conclusion and Further Work

Although college and student numbers are small the module does seem to be well understood and highly regarded by students. The module does seem to meet its purpose in preparing students for third level and allowing access to level 8 courses which were previously denied to Further Education students. We anticipate more colleges in Dublin and also in Limerick to take up the module in 2018. We will also have further training on our online assessment tool and continue the process of archiving and making available test and possibly notes material for all present and future Further education Colleges who participate.

We hope that this unique collaboration will set a template for further collaboration between Higher Education Institutes, further education and second level teachers in Ireland.

References

[1] Carr. M., Ni Fhloinn, E., Murphy, E. & Bowe, B. (2013a) Addressing continuing mathematical deficiencies with advanced mathematical diagnostic testing. *Teaching Mathematics and Its Applications* 32 (2): 66-75.

[2] Carr.M., Bigotte,E., Fidalgo,C., Santos,V,Branco., J.R.,Murphy,E., &Ni Fhloinn., E Mathematics diagnostic test in engineering: An international comparison between Ireland and Portugal, , *European Journal of Engineering Education*,2015,
[3] Cleary, J. 2007. "Diagnostic Testing – An Evaluation 1998–2007." In *Proceedings of Second National Conference on Research in Mathematics Education (MEI2)*, edited by S. Close, D. Corcoran, and T. Dooley, 215–227. Dublin: St.Patrick's College.
[4] Conway,P and Sloane, F (2005) *International Trends in Post Primary Maths Education:Perspectives on Learning, Teaching and Assessment.* NCCA. Dublin 2005
[5] Marjoram. M , Robinson. P, O'Sullivan. C, & Carr, M Improving the key skills of engineering students in Mathematics 41st SEFI Conference, 16-20 September 2013, Leuven, Belgium
[6] Marjoram. M, O'Sullivan. C, Robinson. P and Taylor. P.C "Seven Years of Algebra at IT Tallaght" Proceedings of the LTSN, Wolverhampton 2004
[7] Maths for STEM
http://www.qqi.ie/Publications/Level%205%20Specific%20Purpose%20Cert%20in%20Math%20for%20STEM.pdf
[8] Faulkner, F., Hannigan, A., and Gill, O. 2010. "Trends in the Mathematical Competency of University Entrants in Ireland by Leaving Certificate Mathematics Grade." *Teaching Mathematics and its Applications* 29 (2): 76–93.
[9] Lawson, D. A. 2003. "Changes in Student Entry Competencies 1991–2001." *Teaching Mathematics and its Applications*
[10] Lubienski,S. (2011) "Mathematics Education and Reform in Ireland: An Outsider's Analysis of Project Maths" *Irish Math. Soc. Bulletin* 67(2011), 27-55
[11] National Council for Curriculum and Assessment (2015) 'Project Maths Frequently Asked Questions', online; available at
http://www.ncca.ie/en/Curriculum_and_Assessment/PostPrimary_Education/Project_Maths/
[12] NUIG Website 2014: http://www.nuigalway.ie/engineering-informatics/computer-science-and-information-technology/prospectivestudents/specialentrancemathsexamination/
[13] Department of Education and Skills
https://www.education.ie/en/Publications/Statistics/Statistical-Reports/
[14] Irish education system: https://www.education.ie/en/Publications/Education-Reports/A-Brief-Description-of-the-Irish-Education-System.pdf
[15] ESRI: https://www.esri.ie/pubs/RS35.pdf
[16] The higher Education Links Scheme
https://careersportal.ie/courses/cao_fetac_links.php#.WxAOeO4vzcs
[17] Quality and Qualifications Ireland (QQI)

https://qsearch.qqi.ie/WebPart/AwardDetails?awardCode=5S2246

[18] Brannick, T (2018) Maths for STEM National Programme Review Report 2018, *Publications of the Dublin Further Education and Training Board,* in preparation

Assessing Statistical Methods Competencies and Knowledge in Engineering

Deolinda M. L. D. Rasteiro & Cristina M.R. Caridade

dml@isec.pt;caridade@isec.pt

Coimbra Polytechnic - ISEC, Coimbra, Portugal

Abstract

The concepts taught during a Statistical Methods course make use of different mathematical skills and competencies. The idea of presenting a real problem to students and expect them to solve it from beginning to end is, for them, a harder task then just obtain the value of a probability given a known distribution. Much has been said about teaching mathematics related to day life problems. In fact, we all seem to agree that this is the way for students to get acquainted of the importance of the contents that are taught and how they may be applied in the real world.

The definition of mathematical competence as was given by Niss (Niss, 2003) means the ability to understand, judge, do, and use mathematics in a variety of intra– and extra – mathematical contexts and situations in which mathematics plays or could play a role. Necessarily, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills, in the same way as vocabulary, orthography, and grammar are necessary but not sufficient prerequisites for literacy. In the OEDC PISA document (OECD, 2009), it can be found other possibility of understanding competency which is: reproduction, *i.e*, the ability to reproduce activities that were trained before; connections, *i.e*, to combine known knowledge from different contexts and apply them do different situations; and reflection, *i.e*, to be able to look at a problem in all sorts of fields and relate it to known theories that will help to solve it. The competencies that were identified in the KOM project (Niss, 2003, Niss & H∅jgaard, 2011) together with the three "clusters" described in the OECD document referred above were considered and adopted will slightly modifications by the SEFI MWG (European Society for Engineering Education), in the Report of the Mathematics Working Group (Alpers, 2013).

At Statistical Methods courses often students say that assessment questions or exercises performed during classes have a major difficulty that is to understand what is asked, *i.e*, the ability to read and comprehend the problem and to translate it into mathematical language and to model it.

The study presented in this paper reflects an experience performed with second year students of Mechanical Engineering graduation of Coimbra Institute of Engineering, where the authors assessed statistical methods contents taught during the first semester of 2017/2018 academic year. The questions in the assessment tests were separated into two types: ones that referred only to problem comprehension and its translation into what needed to be modelled and calculated and others where students needed only to apply

mathematical techniques or deductions in order to obtain the required results. The research questions that authors want to answer are:

- What are the competencies that students found, in a Statistical Methods course, more difficult to obtain?
- Having the idea that learning concepts applying them to reality is much more fun and worthy for students, is it really what we should assessed them for? If not, how can knowledge be transmitted to students and be transformed into significant learnings?

Keywords: Mathematics' Competencies; Higher education; Statistical Methods.
Subject: Assessing mathematical competencies and understanding.

Introduction

In higher education, mathematics has an important role in engineering courses (OECD (1996)). From the curriculum of the first and second years there are Curricular Units (CU) in the area of Mathematics that are fundamental for students to acquire the necessary basic knowledge. One of those CU is Statistical Methods. The concepts taught during a Statistical Methods course make use of different mathematical skills and competencies. The idea of presenting a real problem to students and expect them to solve it from beginning to end is, for them, a harder task then just obtain the value of a probability given a known distribution. At least is what students believe.

Often the concept of mastering a subject does not have the same definition for students and math teachers. Regarding students we, as teachers, also should make a difference to which students we are teaching. Mathematics is of course the same but the usage that will be given to their math knowledge is different if they are going to be mathematicians or engineers or else. The authors are math teachers at Coimbra Engineering Institute and for them to teach math is much more than to transmit concepts and resolution methods. It also involves the ability of looking at a real life problem and to be able of selecting, among all the variety of mathematical tools and concepts, the ones that may be applied to solve the problem in hand. (Niss *et al.*, 2017) formulated the questions "What does it mean to possess knowledge of mathematics? To know mathematics? To have insight in mathematics? To be able to do mathematics? To possess competence (or proficiency)? To be well versed in mathematical practices?" and gave a big insight to this discussion. They attempted to present significant, yet necessarily selected, aspects of and challenges to what some call "the competency turn" in mathematics education, research and practice.

During an Engineering course, students learn and consolidate the basic principles of mathematics to solve practical problems, reinforcing their conceptual mathematical knowledge. However, although mathematics is a basic discipline regarding the admission to any Engineering degree, difficulties related to mathematics' basic core are identified by almost all engineering students at each CU. In this context, it seems relevant to identify the mathematics competencies attained by engineering students so that they can use these skills in their professional activities.

Mathematics competencies is the ability to apply mathematical concepts and procedures in relevant contexts which is the essential goal of mathematics in engineering education. Thus, the fundamental aim is to help students to work with engineering models and solve engineering problems (SEFI (2011)). According to Niss (2003) eight clear and distinct mathematics competencies are: thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modelling mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating in, with, and about mathematics and making use of aids and tools.

Gaps were detected between engineers' required mathematics competencies and acquired mathematics competencies of engineering students under the current engineering mathematics curriculum (Firouzian (2016)). There is a need to revise the mathematics curriculum of engineering education making the achievement of the mathematics competencies more explicit in order to bridge this gap and prepare students to acquire enough mathematical competencies (Rules_Math Project, (2017-2020)). Hence an important aspect in mathematics education for engineers is to identify mathematical competencies explicitly and to recognize them as an essential aspect in teaching and learning in higher education. It is the fundamental that all mathematics teaching must aim at promoting the development of pupils' and students' mathematical competencies and (different forms of) overview and judgement (Niss (2011), Alpers (2013), Rasteiro, D. D. (2018)).

This research pretends to evaluate and recognize what are the competencies that engineering students can have or, acquire, when statistical methods contents are taught to them.

In the past few years the CU teachers used to assess students with questions where the recognition of the probability distribution models were necessary to perform further calculus. Students usually complained that they were not able to solve the problem if they failed the first part of the resolution (identification process) and therefore their success was conditioned by the ability to full understand the problem. Even though we consider that, yes the complete success must involve the full understanding of the problem maybe we can accept that not all engineers need to be modellers and some of them will not work directly with the theoretical part of the questions. Once this year we have performed and experience regarding assessment. The questions were, as much as possible, separated into calculus items and models identification and deduction items.

Then, and according with the competencies defined by Niss, we analysed the perception of students, in the acquisition of the taught competencies regarding mathematics. Two tests were performed during the first year semester of 2017/2018 and final exams for students of preferred a summative evaluation. Students were from Mechanical Engineering second year degree.

Description of the study

At the second year of Mechanical Engineering degree of 2017/2018 academic year, 108 students engaged in the Statistical Methods curricular unit (CU). This CU belongs to the second year of their course curricula and has the duration of one semester (1st semester of 2017/2018). At the beginning of the course it was discussed with the students the evaluation possible methods. They chose between regular final exam and distributed tests along the semester. It was also discussed with students that were engaged for the second time, those were the ones that were properly empowered to have an opinion about the subject, which were the main difficulties that they found in the previous year. From the discussion it was agreed that students could choose between distributed assessment tests (two tests along the semester) and final exam. It was also agreed that question of pure calculus were to be separated, as much as possible, from question where students should recognise which was the probability model and also to recognise what should be determined.

To the first test appeared 79 students. The first test had 7.25 out of 20 points dedicated to models identification and 12.75 out of 20 points to calculus. One example of the test questions is given in Figures 1 and 2.

10. One company produces for the national and international markets. The production for the national market is half of the the production designated to the international market for export. Based on the quality control of the company's production, it is admitted that 10% of the products launched in the national market are deficient, being this percentage 3.3% if the production is designated for international market.

    (a) Define the events referred on the text exercise e extract all the available data.

    (b) Indicate, without calculating, what probability value allows you to:

        i. determine the percentage of defective products in the total production of the company?

        ii. in the presence of a defective product, to calculate the probability that it was produced for the international market?

        iii. in the presence of a non-defective product, to calculate the probability that it was produced for the national market?

    (c) Considering a sample of four products of this company. Explain whole the process that conducts to the definition of the variable which allows you to determine the probability that, in these four products, there is at most one defective? What probability value would you calculate? What is the distribution law of such variable?

Figure 1: Probability model and also to recognise what should be determined type of questions

6. Knowing that $P(D) = 0.03$, $P(A/D) = 0.01$ and $P(\overline{A}/\overline{D}) = 0.03$ determine $P(\overline{A})$ and $P(D/\overline{A})$.

7. Let X and Y be 2 real random independent variables such that $X \sim \mathcal{B}(2, 0.2)$ and $Y \sim \mathcal{B}(2, 0.4)$. Determine $P(X + Y = 3)$.

8. Let $X \sim \mathcal{P}(3)$.

    (a) What is the value of $P(X < 3/X > 0)$.

    (b) Determine $E(2X + 3)$ and $V(2X + 3)$.

    (c) Let X_i, $i = 1, \ldots, 8$, be 8 real random independent identically distributed variables with X .

        i. Calculate $P\left(\sum_{i=1}^{8} X_i \geq 12\right)$.

        ii. Determine the expected value and the variance of $\sum_{i=1}^{8} X_i$.

Figure 2: Calculus type of questions

Findings and Discussion

In this section we outline the essential findings concerning the type of assessment chosen by Statistical Methods of Mechanical Engineering degree students and enumerate some of the possible reasons for our findings.

We start by mentioning that more that 73% of the engaged students submit themselves to the distributed assessment. The results obtained were transformed into relative frequencies in order to be able to compare them.

TEST 1 -RESULTS		Statistic	Bootstrap[a] Std. Error	Bootstrap[a] 95% Confidence Interval Lower	Bootstrap[a] 95% Confidence Interval Upper
Modelling_1	N	79	0	79	79
	Mean	,4541	,0197	,4122	,4894
	Std. Deviation	,18013	,01301	,15160	,20375
Calculus_1	N	79	0	79	79
	Mean	,5353	,0204	,4952	,5753
	Std. Deviation	,18488	,01368	,15715	,21082
Test1_n	N	79	0	79	79
	Mean	,5060	,0185	,4695	,5416
	Std. Deviation	,16823	,01317	,14022	,19240

a. Unless otherwise noted, bootstrap results are based on 1000 bootstrap samples

Table 1: Test 1 results.

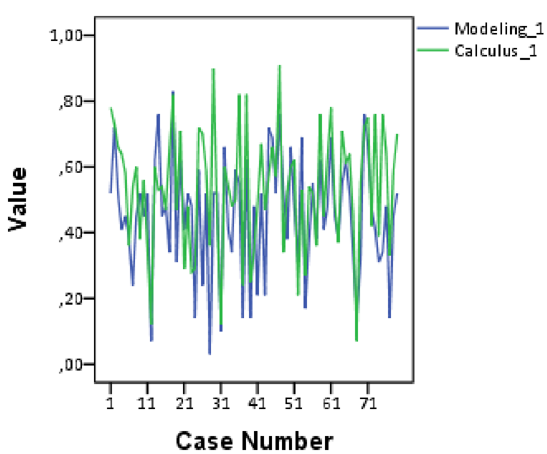


Figure 3: Line graph of test 1 results

From Table 1 and Figure 3 we may see that, although the mean values of both types of questions are very similar, in fact calculus questions have higher classification. We did a t-student test and at significance level of 5% we reject the equality means hypothesis. The mean belongs, with 95% confidence, to the interval]0,4122, 0,4894[in case of modelling / comprehension questions and to the interval]0,4695, 0,5753[in the calculus questions.

TEST 2 - RESULTS		Statistic	Bootstrap[a] Std. Error	95% Confidence Interval Lower	Upper
Modelling_2	N	48	0	48	48
	Mean	,3454	,0391	,2706	,4242
	Std. Deviation	,26942	,01493	,23664	,29412
Calculus_2	N	48	0	48	48
	Mean	,3446	,0278	,2898	,3969
	Std. Deviation	,19292	,01492	,15935	,21888
Test2_n	N	48	0	48	48
	Mean	,3448	,0300	,2867	,4045
	Std. Deviation	,20663	,01361	,17723	,23098

a. Unless otherwise noted, bootstrap results are based on 1000 bootstrap samples

Table 2: Test 2 results.

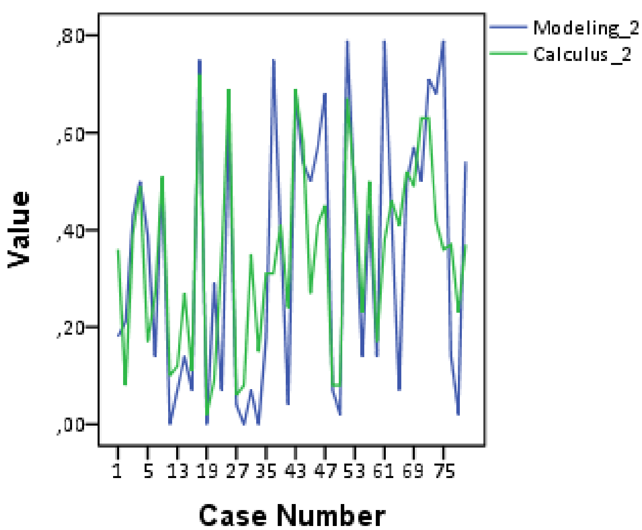


Figure 4: Line graph of test 2 results

Only 44,4% of students engaged in the CU submitted themselves to the second test.

From Table 2 and Figure 4 we may see that the mean values of both types of questions are very similar. Performing the t-student test at 5% level we do not reject, in this case, the equality of means. The mean belongs, with 95% confidence, to the interval]0,2706, 0,4242[in case of modelling / comprehension questions and to the interval]0,2898, 0,3969[in the calculus questions.

Conclusions for Education

Analysing the results obtained namely Table 1 and Figure 3 we may conclude that although the mean values of both types of questions are very similar in fact, calculus questions have higher classification. The same does not happened on the second test. The contents of the second part of syllabus are basically statistical inference. From results obtained on previous years and also this year (Figure 5), we notice that the grades are not that high as the ones obtained on the first test and the difficulties felt on modelling and handling mathematical symbols are smaller.

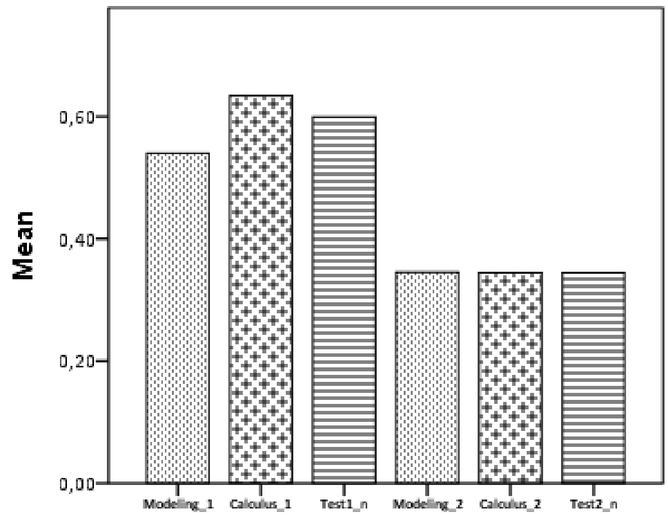


Figure 5: Test 1 and Test 2 results

Still, as teachers and with these results we are convinced that our students did acquire the competencies of modelling mathematically, representing mathematical entities, handling mathematical symbols and formalism yet they sure need to work more on them and dedicate time exploring real life problems in order to become real "doers" and apply their knowledge.

We also defend, as Alpers B. *et al* (Alpers B. *et al*, 2013), that mathematical education aims to provide mathematical expertise needed in the students future but also has to provide the mathematical concepts and procedures needed in application subjects and more theoretically considered contents need to be assessed end the knowledge acquisition measured. Modelling and working with models plays an important role for efficient work. Thus, setting up models and solving problems with models should be an essential part of engineering education without disregarding calculus, solution analysis and communication.

Acknowledgment

This work has been co-funded by Erasmus+ programme of the European Union under grant 2017-1-ES01-KA203-038491 (RULES_MATH).

References

Alpers B. et al: A Framework for Mathematics Curricula in Engineering Education, 2013, SEFI. Available at: http://sefi.htw-aalen.de/

New Rules for assessing Mathematical Competencies || RULES_MATH Project identification: 2017-ES01-KA203-038491; KA2 - Cooperation for Innovation and the Exchange of Good Practices; KA203 - Strategic Partnerships for higher education- https://rules-math.com/

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagatsis, S. Papastravidis (Eds.), 3rd Mediterranean Conference on

Mathematics Education, Athens, Greece: Hellenic Mathematical Society and Cyprus Mathematical Society, 115-124.

Niss, M., & H∅jgaard, T. (Eds.) (2011). Competencies and Mathematical Learning. Ideas and inspiration for the development of mathematics teaching and learning in Denmark, English Edition, Roskilde University.

Niss M., Bruder R., Planas N., Turner R., Villa-Ochoa J.A. (2017) Conceptualisation of the Role of Competencies, Knowing and Knowledge in Mathematics Education Research. In: Kaiser G. (eds) Proceedings of the 13th International Congress on Mathematical Education. ICME-13 Monographs. Springer, Cham

Rasteiro D. D., Martinez, V. G., Caridade, C., Martin-Vaquero, J., Queiruga-Dios, A. (2018), "Changing teaching: competencies versus contents.", EDUCON 2018 - "Emerging Trends and Challenges of Engineering Education". Tenerife, Spain.

Future mathematics project: the experiences and practices for mathematics learning and teaching with technology

Kirsi-Maria Rinneheimo[1], Hanna Kinnari-Korpela[2], Daniela Velichová[3], César Benavente-Peces[4], Ion Mierlus-Mazilu[5]

[1]School of Construction and Environmental Engineering, Tampere University of Applied Sciences, Finland, [2]School of Industrial Engineering, Tampere University of Applied Sciences, Finland, [3]Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, Slovakia, [4]Department of Signal Theory and Communications, Technical University of Madrid, Spain, [5]Department of Mathematics and Computers Science, Technical University of Civil Engineering Bucharest, Romania

Abstract

Future Mathematics (FutureMath) project aims to enhance learning and teaching of engineering mathematics by exploiting educational technology and exploring pedagogical aspects for that. The project's objectives are to explore and develop methods and resources to learn and teach engineering mathematics by utilizing different types of educational technologies and using digital contents. This paper introduces the main outputs of the project and experiences on the use of educational technology in mathematics learning and teaching.

Introduction

Today's students are accustomed to learn with the aid of state-of-the-art technologies. Mathematics educators who wish to implement these technologies in their classroom teaching are often deterred in doing so due to time restraints. FutureMath project strives to meet the current needs of teachers and learners alike.

The FutureMath project aims to respond to the requirements of modern society and to make mathematics' learning and teaching more digitalized, effective and accessible. Additionally, the aim is to explore and develop the most motivational, learner centered methods, techniques and resources for engineering mathematics learning and teaching with the help of digital technologies.

FutureMath project is a three-year project (2015-2018) funded by the EU Erasmus+ Programme. The project consortium consists of four university partners: Tampere University of Applied Sciences (Finland), Slovak University of Technology in Bratislava (Slovakia), Technical University of Civil Engineering Bucharest (Romania) and Technical University of Madrid (Spain). This paper introduces main outputs of the project and experiences on the use of educational technology in mathematics learning and teaching. The main outputs of the project are as follows

a) Mathematics Online Pedagogy (MOP)
b) Mathematics Learning Platform (MLP)
c) Mathematics Learning Resources (MLRs)

Mathematics Online Pedagogy (MOP) combines best practices and pedagogical point of views for meaningful utilization of different types of educational technology in mathematics learning and teaching context. Mathematics Learning Platform (MLP) is a comprehensive framework for mathematics learning and teaching in web which contributes to the digitization of the education. It is a versatile teaching/learning digital platform which implements a repository for technology enhanced digital materials, resources, teaching/learning activities, assessment and other useful educative tools. Due to its nature, the MLP constitutes a platform providing ubiquitous teaching/learning support. Mathematics Learning Resources (MLRs) are different kinds of resources planned, produced and tested in the project. The MLRs encapsulate a vast variety of ICT (information and communication technologies) based on learning and teaching resources such as short video lectures, lecture materials, online learning materials, online assessment components, dynamic interactive applets, authentic learning modules and online resources for learning, for example.

These three outputs are the main outcomes of the project. As an overall outcome of the project, these outputs provide a collection of best practices, useful resources and pedagogical practices for online learning and teaching of mathematics. Outputs such that have a potential to make learning of mathematics more motivational, personalized and interesting but also to increase accessibility and the alternative modern methods for learning.

Project outcomes

To ensure the best starting point for the design and implementation of the work for the project's main outputs, an online survey was conducted in autumn 2015. The main idea was to explore students' expectations about modern teaching methods for mathematics. Survey together with the curriculum document "A Competence-based Framework for Mathematics Curricula in European Engineering Education" by SEFI (2013) layed the bases for the design and implementation work . This section introduces in more detail the produced outcomes of the project.

Mathematics online pedagogy

Pedagogy can be described to be a discipline that deals with the theory and practice of education. Hence, pedagogy concerns issues related to how some substance is best to teach. During the FutureMath project, so called online pedagogy in the context of engineering mathematics has been explored and developed. The project's online pedagogy combines best practices and pedagogical perspectives for meaningful utilization of versatile types of learning technology in engineering mathematics teaching and learning.

To explore engineering students' expectations, the survey for utilisation of learning technologies and learning methods were conducted. Based on literature and the survey results, trends such as flipped classroom, online assessment, learning analytics, short videos and gamification were selected to be more reviewed during the project. The issues related to the project's online pedagogy have been discussed in the blog of the project (FutureMath blog, 2015). As the online pedagogy is a topic that continuously develops, the materials related to the online pedagogy has been delivered through the blog.

During the project, research related to the selected trends has been carried out and published in the conference proceeding. The research discusses more in datail the topics such as utilisation of online assessment (Kinnari-Korpela and Suhonen, 2017; Kinnari-Korpela and Yli-Rämi, 2016; Rinneheimo, 2017; Velichova, 2017), learning analytics (Kinnari-Korpela and Suhonen, 2017; Kinnari-Korpela and Yli-Rämi, 2016), short videos (Rinneheimo, 2017) and flipped classroom (Rinneheimo, 2017).

Mathematics learning platform

The Mathematics Learning Platform (MLP) is the supporting infrastructure for digital based teaching/learning methodologies and resources. Its design is aimed at hosting the digital teaching and learning resources including video-lectures, interactive teaching and learning tools, links to external resources and assessment activities, in an Open Educational Resource setting, thus promoting a supportive community of mathematics educators involved in the digitization process. The MLP is based on an open source kernel where the developed plugins can be shared with the community (Moodle, 2002). The MLP capabilities include the development of teaching activities including interactive tools, assessment activities, and communication facilities to interact with students (forum, mailing, etc.).

During the piloting period, the MLP is including, as exemplary activities for the platform test, courses on Algebra, Geometry and Analysis. Courses include the teaching and learning resources, learning activities using different methodologies and tools, and assessment activities. Among the different kind of digital resources designed for the teaching and learning process, a number of video-lectures have been produced covering the various topics considered for the piloting and test period. Additionally, activities based on the interactive and dynamic tool named Geogebra have been designed and implemented to support the designed education activities. Also, to enrich the education, competencies and skills of the students several links to external resources and tools are provided.

Mathematics learning resources

One key output of the project is Mathematics Learning Resources (MLRs) such as e.g. short video lectures, personalized learning materials, lecture materials, online learning materials, online assessment components, authentic learning modules, online resources for learning etc. Thus the MLRs encapsulate a vast variety of ICT-based learning and teaching resources.

GeoGebra is an intuitive user-friendly free software product suitable for all users without any specific needs and skills in information technology. It is available for download from the webpage (GeoGebra, 2001), and can serve for development of dynamic visualizations and application applets. GeoGebra is available in more than 50 languages accessible on a click from the menu, while complete construction protocol appears directly in the selected language. All constructions can be followed in the step-by-step mode. One of the latest advantages of GeoGebra program are 3D graphics possibilities, which enable presentation of 3D problems. In addition to various projections of 3D scenes, such as orthographic mapping, perspective and axonometry, users can benefit also from available stereoscopic view. Here 2 coloured images, separately for left and right eye in complementary green and red colours are automatically generated. These images can be viewed through colour glasses and provide an excellent real 3D vision, even when projected via data projector to a large screen. Printed form is also working well, when read with suitable glasses; see

Figure 1. Thus, a very powerful didactic tool is available for teaching geometry, enabling not only to explain better stereometric relations in 3D, but also to attract students in an unusual way (visiting 3D cinema) to study and understand space relations better.

Many GeoGebra applets are available in the project MLP - visualizations of various properties of conic sections as planar curves, and their views as intersections on cones of revolution. Development of dynamic models is also an inspiration how to utilize information technologies meaningfully in the role of a novelty didactic tool, which can not only attract learners, but also enable them to realize their own creative work. Both subjects of the educational process, teacher and students, act in this didactic situation more as equal partners, not as it is usual in the classical forms of didactic situations, where the role of teachers is active presentation of new facts and data, while role of learners is usually passive, just receiving presented facts. Dynamics opens easy way to discover connections, and to understand mutual dependencies, which is often more important than a detailed fragmented knowledge itself.

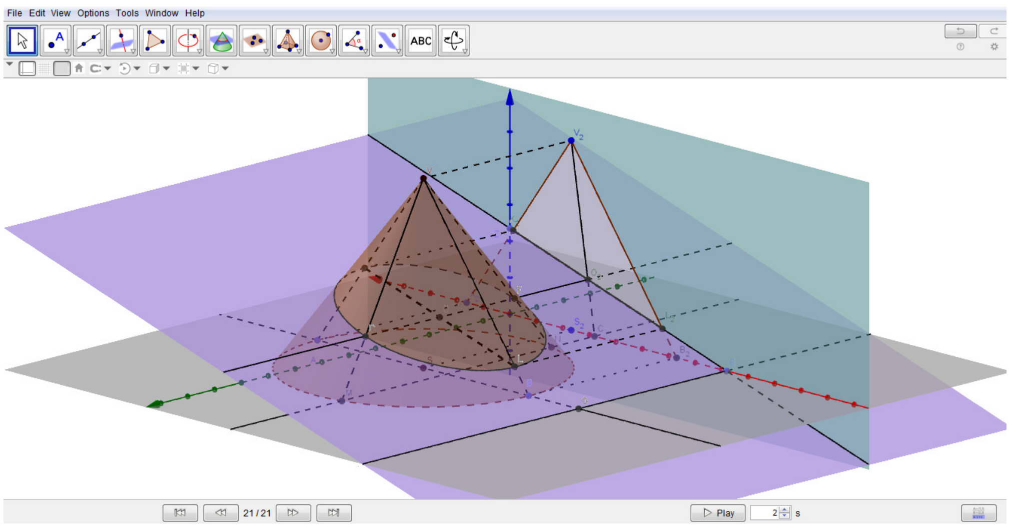
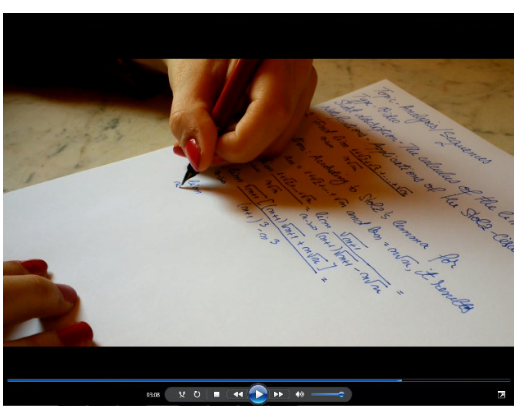


Figure 1. Elliptic intersection on a cone of revolution.



With web-based learning technology, learning is no longer limited to time in the classroom but can be done anywhere at any time. For this reason in our MLP we introduce some video materials for exercices and theory; see Figure 2. Video exercises helps students who prefer visual learning methods. Now, video exercises with the solution are readily available for students.

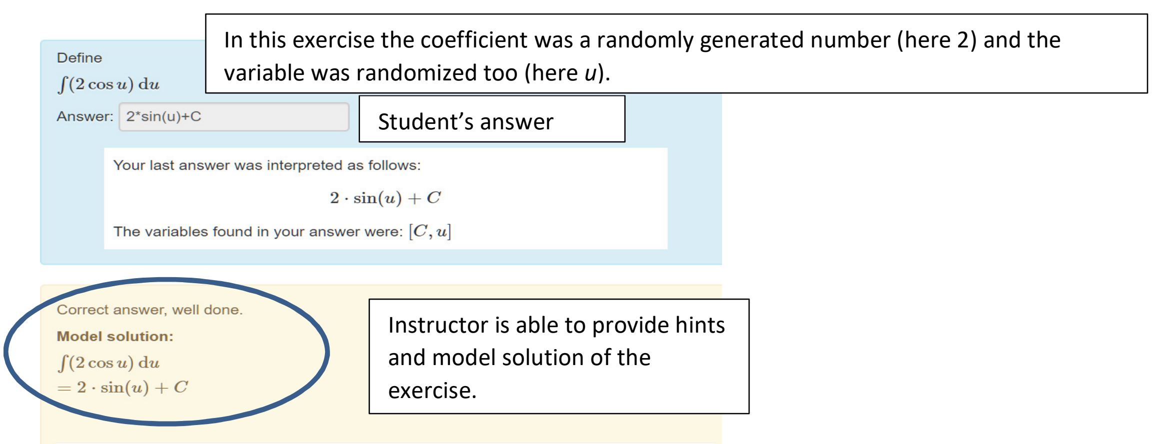
We have produced also in this project STACK (A System for Teaching and Assessment using a Computer algebra Kernel) questions. STACK is an open source e-learning system designed for mathematics but encouraging experiences have been reported also in other mathematical engineering studies (Kinnari-Korpela and Yli-Rämi, 2016; Rinneheimo, 2017).

STACK is a computer aided assessment package for mathematics, which provides a question type for the Moodle quiz. STACK enables an evaluation of student's answers in a number of ways. The system can also provide accurate feedback on the most common errors. STACK system enables giving hints and model solution of the exercises to student.

Based on our experiences, students preferred to have immediate feedback. As the most STACK exercises can be parameterized, meaning that different students get slightly different initial values for their assignments, it enables also instant feedback with the exact same parametres.

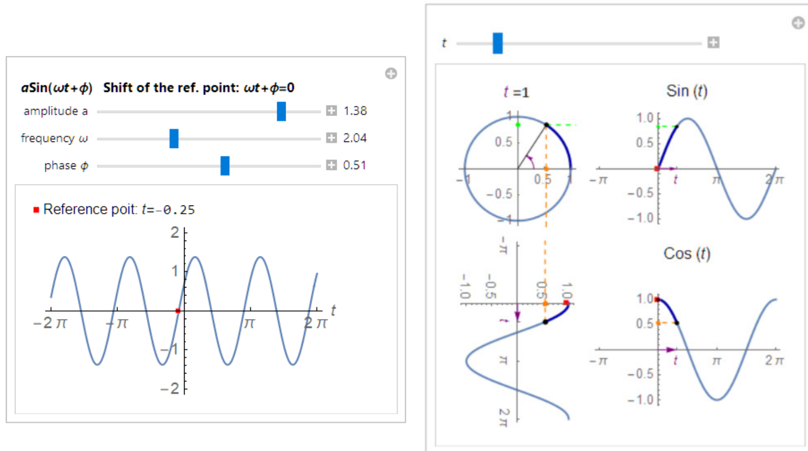
During the FutureMath project, a various amount of STACK exercises has been produced. These exercises are delivered through the MLP which encapsulates a set of different mathematics courses. Many of these STACK questions include randomized versions and fully worked step-by-step solutions for the exercises. To facilitate implementation of STACK exercises, MLP includes ready made quizzes encapsulating a set of exercises. Alternatively, an instructor can use the large question bank of MLP to create own quizzes.

In the Figure 3 is a simple randomised exercise related to a integral function. The Figure 3 demonstrates, how the system works when the student has given a correct answer. The student can see his/her answer and how the system interpretes the answer. The instructor can code feedback that is related to the student's answer and/or the model solution.



•

Learning materials produced in the project includes also visualizations materials. Figure 4 presentes this kind of visualizations.



Conclusions

With this foundational work and the future contributions from mathematics educators throughout Europe, the accessibility of ready-made digitized materials will increase. These flexible alternatives, which can either replace or be combined with traditional teaching methods, are attractive and motivating to students, as the performed surveys indicate. They can help to engage students more intensely in the learning process, thereby increasing their chances of successfully learning mathematics. The individual learning solutions and differentiated feedback are key to students' self-motivation, so that students are also supported in the self-study phase. The use of the MLP to support the teaching/learning process contributes to the digitization and its ubiquitous access to education. Furthermore, it is aligned with the priorities and key actions of the Erasmus+ program [4].

All the learning resources developed in the project will be made available for free under the idea of Open Source or Open Educational Resource (OER).

Acknowledgments

The authors thank the EU for supporting this project funded under the ERASMUS+ Program, with reference 2015-1-FI01-KA203-009044.

References

FutureMath blog. [Online] Available at the address: http://futuremathematics.blogspot.fi/ (Accessed 11 May. 2018)

GeoGebra webpage. [Online] Available at the address: www.geogebra.org (Accessed 11 May 2018)

Erasumus+. [Online] Available at the addrss: https://ec.europa.eu/programmes/erasmus-plus/about_en
(Accessed 11 May 2018)

Kinnari-Korpela, H., Suhonen, S. (2017). HOW MUCH IS TOO MUCH IN E-MATERIALS? CASE STUDY IN ENGINEERING, ***EDULEARN17 Proceedings***, pp. 1512-1518.

Kinnari-Korpela, H., Yli-Rämi, H. (2016). Utilization of computer-aided assessment in mathematical engineering studies. EDULEARN16 Proceedings, pp. 8933-8942.

Moodle. [Online] Available at the address: https://moodle.org/ (Accessed 11 May 2018)

Rinneheimo, K-M. (2017). ENHANCING LEARNING AND TEACHING IN ENGINEERING MATHEMATICS WITH TECHNOLOGY, ***EDULEARN17 Proceedings***, pp. 1136-1142.

SEFI MWG (2013). A Framework for Mathematics Curricula in Engineering Education. Available at:
http://sefi.htw-aalen.de/ (Accessed 11 May. 2018).

Velichova, D. (2017) Computer Assisted Assessment in Mathematics. *Proceedings of SEFI 2017 annual conference.* Quadrado, J. C., Bernardino, J. & Rocha, J. (eds.). Azores, Brussels: European Society for Engineering Education SEFI

INVOLVE ME AND I LEARN – video-lessons to teach math to Engineers

Cristina M.R. Caridade & Deolinda M. L. D. Rasteiro

dml@isec.pt;caridade@isec.pt

Coimbra Polytechnic - ISEC, Coimbra, Portugal

Abstract

Beginning with a Benjamin Franklin's quote "Tell me and I forget, teaching me and I understand, **involve me and I learn**". The students who are engaged in class are more academically successful than disengaged students. Through a variety of strategies, we strive to promote student engagement in order to spark student excitement, dedication, and motivation to excellence. The activity presented in this paper is an example to promote students motivation and their dedication for learning mathematics. Videos have assumed a growing and promising role in teaching, particularly as a different and motivating activity at the service of education. In this sense during this school year, in the Calculus I course of the degree in Electrical Engineering in Coimbra Institute of Engineering, we proposed to students the development of a video-lesson about the contents taught in course. About 80 students (divided into groups of 2 students each) presented their video-lesson and participated in the visualization and evaluation of both their work and the work of their colleagues.

With reflections about the students behind the camera, we believe that the process of producing videos opens space for great exchanges, in which students can express their knowledge through their own language, even under the guidance and mediation of the teacher.

Introduction

One way to enrich mathematical learning experiences is through the use of different types of activities. Although the use of these experiences does not determine learning by itself, it is important to provide several opportunities for contact with different activities to arouse interest and involve the student in mathematical learning situations.

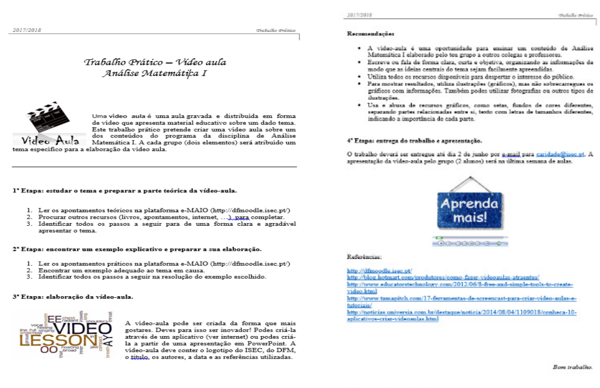
Videos have assumed a growing and promising role in teaching, particularly as a different and motivating activity at the service of education (Stefanova 2014). A number of initiatives and studies have already been carried out in this area, such as Moran (1995), who discusses different possibilities for the use of this material in the classroom, as "simulation" (to simulate an experiment) or as "teaching content" (to show a certain subject) or Willmot (2012) that describes the design and development of an attractive new resource to encourage academics to incorporate video reporting into their student-centred learning activities, among others. Within this context, video as a medium continues to have an on-going impact on higher education, on the role of the student, challenging the (traditional) role of the lecturer and the format of delivering course contents via lectures. Currently the videos available on the internet of small parts of classes are the most viewed by the students, who watch them when they have some conceptual questions. Videos of this nature have also gained space in distance education (Koppelman 2016).

In the scope of mathematics education Clarke et. al. (2011) point out that video can be used in an interactive environment in order to enhance expression and communication, as well as a pedagogical action that motivates learning. Hasan (2017), for example, used models of

e-learning by using power point based screencast o-matic videos to improve teaching mathematics achievement in elementary school students as a mean to support the teaching and learning process and not just to implement the teaching materials, but also to create an atmosphere of interesting and fun learning.
In this sense during this school year, in the Calculus I course of the degree in Electrical Engineering in Coimbra Institute of Engineering, we proposed to the students the development of a video-lesson about the contents taught in course. With this activity, we wanted to motivate the students to dedicate themselves to study contents in greater depth, to plan the information to be presented, to improve the way of expressing themselves especially in mathematical language, to promote creativity and to involve students in learning.

The activity purpose

As described previously, in this research, we analysed the possibilities of use video-lessons in the classroom, in order to disseminate its potentialities. The Calculus I course had 156 hours for classroom lessons 14h of which were tutorials and another 20h attributed to the group work. Within the tutorials, 4h of them were dedicated to knowledge acquisition of some video production materials, to study the mathematical concepts and the way of presenting them and to discuss about the potentialities and limitations of their use in the video-lessons. For some students, this is the first time they have had contact with the production of a video with mathematical contents. The idea is to showcase various types of video to serve as inspiration for their own productions. Video-lessons could be made only with the producer's narration, with animations or with computer screenshot and other videos. In these classes we used several videos available on the YouTube website to exchange ideas and experiences between the groups and the teachers.
The activity purpose (presented in figure 1) intends to create a video-lesson about one of the contents of the Calculus I course.


2017/2018 Trabalho Prático

Trabalho Prático – Vídeo aula
Análise Matemática I

Uma vídeo aula é uma aula gravada e distribuída em forma de vídeo que apresenta material educativo sobre um dado tema. Este trabalho prático pretende criar uma vídeo aula sobre um dos conteúdos do programa da disciplina de Análise Matemática I. A cada grupo (dois elementos) será atribuído um tema específico para a elaboração da vídeo aula.

1ª Etapa: estudar o tema e preparar a parte teórica da vídeo-aula.

1. Ler os apontamentos teóricos na plataforma e-MAIO (http://dfmoodle.isec.pt/)
2. Procurar outros recursos (livros, apontamentos, internet, …) para completar.
3. Identificar todos os passos a seguir para de uma forma clara e agradável apresentar o tema.

2ª Etapa: encontrar um exemplo explicativo e preparar a sua elaboração.

1. Ler os apontamentos práticos na plataforma e-MAIO (http://dfmoodle.isec.pt/)
2. Encontrar um exemplo adequado ao tema em causa.
3. Identificar todos os passos a seguir na resolução do exemplo escolhido.

3ª Etapa: elaboração da vídeo-aula.

A vídeo-aula pode ser criada da forma que mais gostares. Deves para isso ser inovador! Podes criá-la através de um aplicativo (ver internet) ou podes criá-la a partir de uma apresentação em PowerPoint. A vídeo-aula deve conter o logotipo do ISEC, do DFM, o título, os autores, a data e as referências utilizadas.

Análise Matemática I – Engª Eletrotécnica

2017/2018 Trabalho Prático

Recomendações

- A vídeo-aula é uma oportunidade para ensinar um conteúdo de Análise Matemática I elaborado pelo teu grupo a outros colegas e professores.
- Escreve ou fala de forma clara, curta e objetiva, organizando as informações de modo que as ideias centrais do tema sejam facilmente apreendidas.
- Utiliza todos os recursos disponíveis para despertar o interesse do público.
- Para mostrar resultados, utiliza ilustrações (gráficos), mas não sobrecarregues os gráficos com informações. Também podes utilizar fotografias ou outros tipos de ilustrações.
- Usa e abusa de recursos gráficos, como setas, fundos de cores diferentes, separando partes relacionadas entre si, texto com letras de tamanhos diferentes, indicando a importância de cada parte.

4ª Etapa: entrega do trabalho e apresentação.

O trabalho deverá ser entregue até dia 2 de junho por e-mail para caridade@isec.pt. A apresentação da vídeo-aula pelo grupo (2 alunos) será na última semana de aulas.

Aprenda mais!

Referências:

http://dfmoodle.isec.pt/
http://blog.hotmart.com/produtores/como-fazer-videoaulas-atraentes/
http://www.educatorstechnology.com/2012/06/8-free-and-simple-tools-to-create-video.html
http://www.tamapitch.com/17-ferramentas-de-screencast-para-criar-video-aulas-e-tutoriais/
http://noticias.universia.com.br/destaque/noticia/2014/08/04/1109018/conheca-10-aplicativos-criar-videoaulas.html

Bom trabalho.

Análise Matemática I – Engª Eletrotécnica

Figure 1. Activity: development of a video-lesson.

To each group (of two elements) will be assigned a specific subject (chosen by the teacher) for the elaboration of the video-lesson. The activity was divided into 4 steps:

Step 1: study the topic and prepare the theoretical part of the video-lesson.
Read the theoretical notes in course's Moodle platform.
Look for other resources (books, notes, internet ...) to complete the study.

Identify all the steps that have to be followed to clearly and pleasantly introduce the subject.

Step 2: find an explanatory example and prepare it.
Read the practical notes in course's Moodle platform.
Find an appropriate example of the subject matter.
Identify all the steps that have to be followed when solving the chosen example.

Step 3: preparation of the video-lesson.
The video-lesson can be created in the way you, as students, like the most. You should be innovative! You can create it through an application (see internet) or you can create it from a PowerPoint presentation. The video lesson should contain the ISEC and the DFM logotypes, the title, the authors, the date and the references used.

Step 4: delivery of work and presentation.
The student should present the video-lesson to teachers and other colleagues.

Some recommendations and help in the development of the video-lesson were presented, such as:

The video-lesson is an opportunity to teach a content of Calculus I written by your group to other colleagues and teachers.
Write or speak in a clear, short and objective way, organizing the information so that the central ideas of the subject are easily grasped.
Use all available resources to arouse public interest.
To show results, use illustrations (graphics), but do not overload graphics with information. You can also use photography or other type of illustrations.
Use and abuse graphic assets such as arrows, backgrounds of different colours, separating related parts from each other, text with letters of different sizes, indicating the importance of each part.

The activity evaluation

About 80 students (2 students per group) presented their video-lesson and participated in the visualization and evaluation of both their work and the work of their colleagues. The evaluations, presented in table I, focused on the form (argument, sound, aesthetics and editing), language (including mathematical language) and content (clarity, narration, creativity, research and exploration) of video-lessons.

Table 1: the criteria for video-lesson evaluation.

Criteria	1=Poor	2=Fair	3=Good	4=Very good	5=Excellent
Form					
Argument (does the video have an appropriate treatment in relation to the subject? Can the video teach the subject?)					
Sound (sound record used correctly? (Voice, sound level, sound overlap, etc.)					
Aesthetics (can the video align the content and shape appropriately to the subject?)					

Editing (does the video demonstrate that there was an editing work or is it a mere random collage of images, testimonials, etc.?)					
Language					
Language (is the language used appropriate and correct? Are there any spelling mistakes?)					
Content					
Clarity (is the subject transmitted in the video in clearly way?)					
Narrative (the video is presented in a coherent and pleasant way? Does it shows a line of reasoning?).					
Creativity (is the video innovative and creative in the way it approaches the subject? Does it surprise the viewer?)					
Research (does the video show that there was a research for its elaboration?)					
Exploration (does the video present a deepening of the subject?)					

The scores for evaluation are: (80%) by professors, (10%) by the average of ten of his colleagues and another (10%) for self-evaluation.

Student Experiences

The present work aims to analyse part of the results of a research that investigated the possibilities of the use video-lesson in the classroom. In all video-lessons students begin by describing the theoretical content associated with their subject (primitives, integrals and application of integrals) and only after that presents one or two illustrative examples as suggested in the activity purpose. The videos presented were very interesting, enriching and attractive as teaching and learning tools. Several options and choices have been used by students in video-lesson production, so a great diversity of videos are presented as can be seen in the figures 2 to 6.

In the figure 2 are represented images of some of the video-lessons developed. As we can see some of the videos were produced in animated slides with defined periods of time and with sound where the student explains, by his words, the form of resolution that we are observing at the moment. In some cases the auxiliary calculations are presented in a particular zone of the screen, as a way to replicate the resolution process done by the teacher (figure (a) and (b) in left size). Also it is possible to use computer mathematic applications such as Geogebra, Matlab and Mathematica, in order to observe the contents in a more motivating way like we can see in figure 2 (c) and (d).

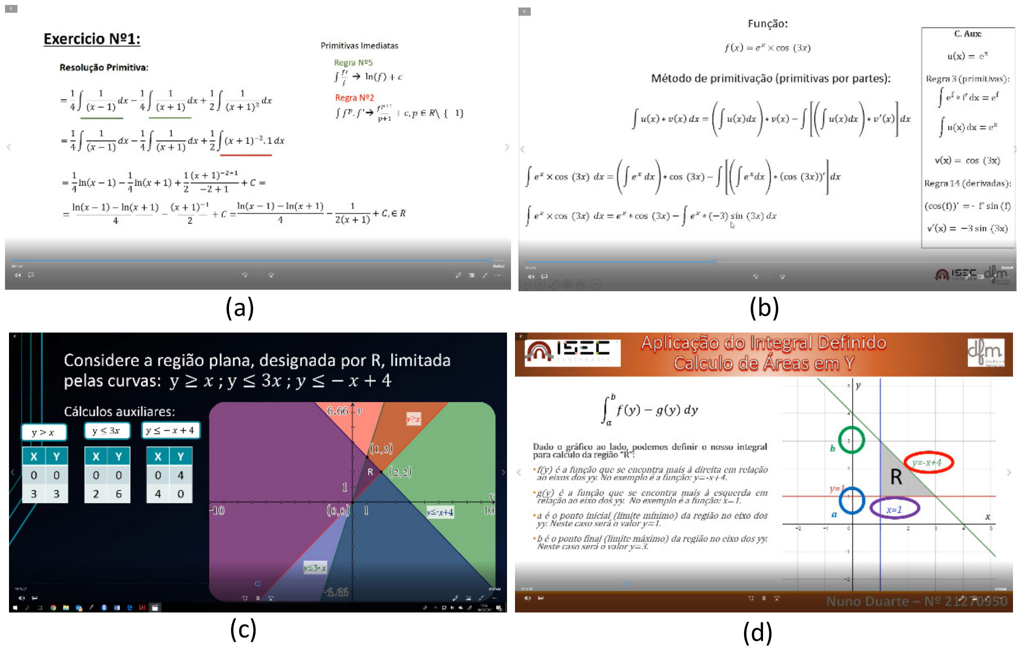


Figure 2: Video-lessons produces by animate slides: (a) and (b) with auxiliary calculations on left size; (c) and (d) using computer mathematic applications.

In case of animated slides it is also possible to hand write on slides. In this case it is intended to highlight some details such as placing arrows, or curves around certain expressions (figure 3 (a)) or explain step by step with handwritten letters in different colours (figure 3 (b)).

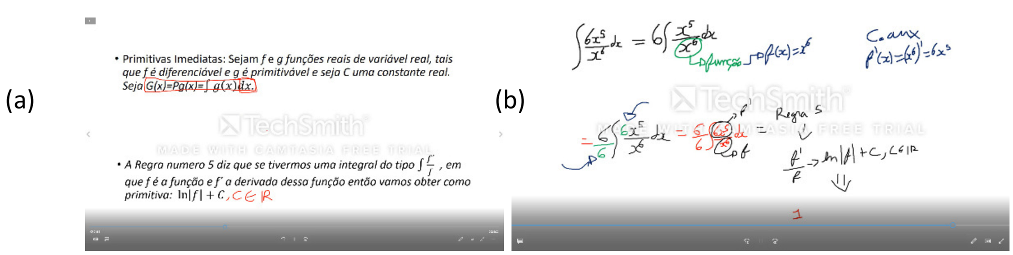
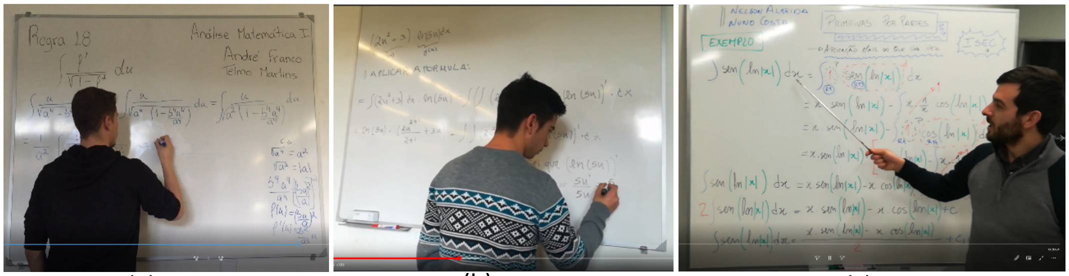


Figure 3: Video-lessons produces by writing slides: (a) with background text; (b) with white paper.

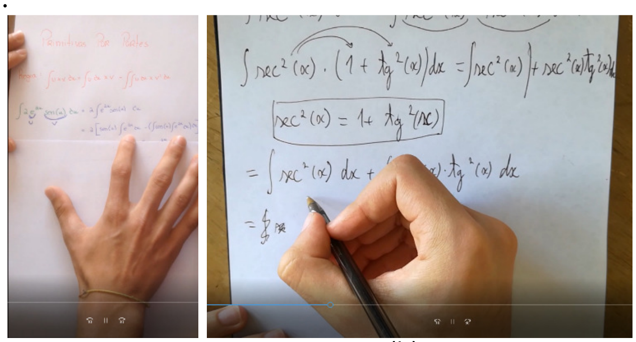
In other videos the student will film the subject on a whiteboard with the use of pens while describing his steps in words (figure 4). Here we noticed, with greater relief, the concern of doing the teacher's replica. In figure 4 (a) and (b) students present the lesson by writing in a whiteboard at the same time they explain it, and in figure 4(c) the student presents the subject using a pointer to explain it.



(a) (b) (c)

Figure 4: Video-lessons produced using recording and filming of the whiteboard: (a) and (b) students write with pens; (d) student uses a pointer.

There are still videos that film a white paper, where the student presents the mathematical contents and the calculations made. In these videos, the student's voice is also important to explaining the steps taken. In figure 5(a) the student moves a white paper with his hand over the writing paper showing only what he intends. In figure 5(b) the student write what he intends to show.



(a) (b)

Figure 5: Video-lessons produces recording and filming a white paper: (a) student moves a white paper on the writing paper; (b) students write on a paper.

Other videos use the applications associated with the calculators to display the contents. In these cases the video-lessons are presented inside a calculator projected on a screen. Figure 6 shows two lesson using this method. The lesson presented is about antiderivatives (a) and applications of integration to determine areas between curves (b).

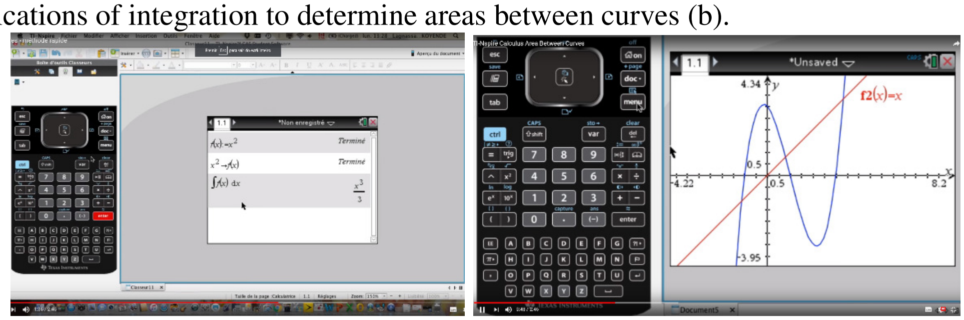


Figure 6: (a) and (b) video-lessons using calculator.

Conclusions for Education

In this paper, we seek to highlight the production of the videos by students and teachers, characterizing what Ferrés (1996) calls the video process, in which the student becomes the protagonist of his learning, deciding how to approach the content and expose it to its colleagues. Surely the performance of teachers can always contribute to student learning.

In the development of this activity the teacher can promote several discussions that integrate mathematics, as well as other subjects that may be of interest to the students. It is important to emphasize that in a video production it is possible to contemplate reading, research, interpretation, creativity, writing, orality, as well as allowing the creation of a communicative link between teacher-student. According to Pires (2002), video production gives students the opportunity to craft their own narrative and gives them a reinvention of the world's writing.

Some benefits can be realized from the practical activity of video lessons:

The lesson created by each student reflects his personalization of the content. In fact, each student presents their lesson according to their difficulties, their points of attention and their strengths.

With these lessons the understanding of the content is more explored, the student becomes more participative, more creative, more communicative and engaged. The student gains confidence in himself and gradually discovers his skills and abilities.

The video-lessons helped students to better prepare classes, learn more about the content they were exploring, get the details and relevant aspects of the content, prepare for exams and therefore to learn better in general.

The relationship between the teacher and the student becomes an experience of knowledge exchange and, consequently, much more enriching. The student, moreover, gains autonomy and freedom to approach and overcome his questions, fears and difficulties.

With reflections about the students behind the camera, we believe that the process of producing videos opens space for great exchanges, in which students can express their knowledge through their own language, even under the guidance and mediation of the teacher.

Acknowledgment

This work has been co-funded by Erasmus+ programme of the European Union under grant 2017-1-ES01-KA203-038491 (RULES_MATH).

References

Clarke, D., Hollingsworth, H., Gorur, R. (2013) "Facilitating Reflection and Action: The Possible Contribution of Video to Mathematics Teacher Education." *Journal of education*, 1(3), 94-121.

FERRÉS, J. (1996) "Vídeo e educação". Trad. Juan Acuña Llorens. 2. ed. Porto Alegre: Artes Médicas.

Hasan, B. "Teaching Elementary Mathematics using Power Point Based Screencast O-Matic Videos." INA-Rxiv.

Koppelman, H. (2016) "Experiences with using videos in distance education. A pilot study: A course on human-computer interaction." *Issues in Informing Science and Information Technology* 13, 269-277. Retrieved from http://www.informingscience.org/Publications/3472.

Leonard, E. (2015) "Great Expectations: Students and Video in Higher Education", SAGE publications.

Moran, J.M. (1995) "O vídeo na sala de aula." *Comunicação e Educação, São Paulo*, 347-362.

Pires, E.G. (2002) "A experiência audiovisual nos espaços educativos." *Comunicação & Educação.* São Paulo, 25, 94 – 100.

Stefanova, T.A. (2014) "Using of Training Video Films in the Engineering Education." *Procedia - Social and Behavioral Sciences*, 116, 1181–1186.

Willmot, P., Bramhall, M., Radley, K., (2012) "Using digital video reporting to inspire and engage students." Retrieved from http://www.raeng.org.uk/ publications/other/using-digital-video-reporting.

Going up and down by the lift to learn Linear Algebra

Araceli Queiruga-Dios[1], María Jesús Santos Sánchez[2], Cristina Prieto Calvo[3], Ascensión Hernández Encinas[1], Juan José Bullón Pérez[4], Jesús Martín-Vaquero[1]

[1]*Department of Applied Mathematics,* [2]*Department of Applied Physics,* [3]*Department of Fundamental Physics,* [4]*Department of Chemical Engineering and Textile University of Salamanca, Spain*

Abstract

We have developed several engineering problems and situations according to the different mathematical levels in engineering degrees. In this paper we show how students could learn the calculation of eigenvalues and eigenvectors solving a mechanical problem of vibrations. From a one degree of freedom system and its physical and mechanical knowledge, students are able to acquire the mathematical competencies and get the values of frequencies and vibrations modes. We present in this study the results from linear algebra course from different years, using different methodologies and also different assessment methods.

Introduction

Calculus, Linear Algebra, Numerical Methods, and in general mathematics, could be considered as tools in the training of engineering students. In recent years, teaching and learning methodologies have changed considerably. We moved away from master classes with separate hours of theory and problems, to different attempts to apply mathematics to engineering courses, which motivates students and make them acquire the mathematical competencies.

The aim of this paper is to present a proposal to make students acquire the 8 competencies proposed in the Framework document from the mathematics working group (Alpers et al., 2013): Thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modelling mathematically, representing mathematical entities, handling mathematical symbols and formalisms, communicating in, with, and about Mathematics, and making use of aids and tools for mathematical activities.

The Bologna process has made university teachers to promote a change in the educational paradigm. As we teach to science and engineering students, we must keep in mind that these students are different from the ones in mathematics degrees. The way of teaching and learning have became a new challenge as mathematics represents a tool and not a goal in itself.

One of the changes that came with the degrees modification is the use of technology for educational purposes. In recent years, apart from using a moodle-based learning platform, we also use different mathematical software, such as Mathematica or Matlab. Moreover, during some courses we propose the use of Socrative or Kahoot as gamification and assessment tools for quick answers from students (Bullón et al., 2018).

In Figure 1 we show our current situation concerning industrial engineering students at the University of Salamanca, from electricity, automatics, and mechanics. There is a continuous decrease of the success rate. In fact, in the 2016-17 academic course, from 74 students (22 from electricity, 27 from electronics, and 25 from mechanics) only 11 students (5 from electricity, 4 from electronics, and 2 from mechanics) were promoted to the next grade. These quantities are not a good quality indicator. This situation leads as to propose a change in the teaching and learning methodology.

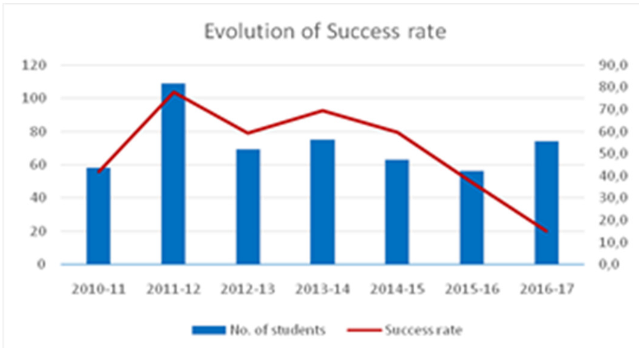


Figure 1. Results from Mathematics (linear algebra) subject from industrial engineering studies at the University of Salamanca.

The linear algebra subject, for engineering students, at the University of Salamanca, includes 5 blocks of contents: (1) Systems of linear equations, (2) vector spaces, (3) linear applications and associated matrix, (4) Euclidean space, and (5) diagonalization. This represents a basic subject in the first semester of the bachelor's degrees.

Methodology

We started with the proposal of using the mobile phone as a pendulum together with a computer algebra system. Once students collect the data from their devices, they were able to calculate the frequency of a pendulum or a spring and this leads them to study the vibration of a system with one, two or more degrees of freedom. Mobile-aided learning and computer-based learning in general help students to be motivated. The use of devices awakens their curiosity and captures their attention. We have tested this with students from the "master degree in teachers from compulsory secondary education and high school, vocational training and language teaching" at the same University. All students (around 10) were very motivated with the use of the mobile phone for mathematics and physics classes. The results were very positive.

The proposal now is to develop a competencies-based course to improve the final marks in the case of students from the first year of bachelor degrees. This will indicate that students acquire the competencies, which improve their engineering education, and make them be prepared for their future careers.

The activity that we propose is the use of the mobile phone to present some algebraic problems to make more understandable the subject's contents and, at the same time, to make students acquire the competencies.

The phyphox App, available for Android and iOS (http://phyphox.org/), includes several physical experiments to work with students: Elastic collision, acceleration (without and without g), acceleration with g, audio amplitude, audio autocorrelation, centrifugal acceleration, Doppler effect, elevator, free fall, etc. All of them include videos and working activities to do it easy to use.

The first activity that we propose for the Linear Algebra curriculum is the use of the "elevator" from phyphox App to make students understand the concept of a vector, as an element with a direction and a value (module). When the mobile is use to measure the acceleration of the elevator students get in their mobile screens the Figure 2.

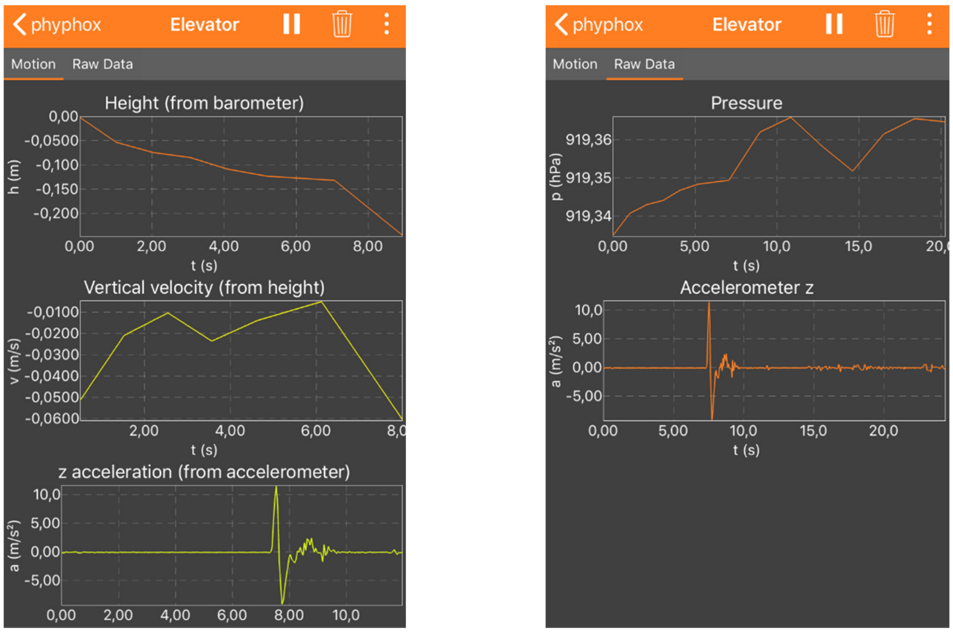


Figure 2. Phyphox working with the elevator experiment.

Once the students understand the concept of vectors and the meaning of different magnitudes, it is possible to work with vector spaces (collection of objects called vectors) and subspaces, as these keep the properties of the vector spaces.

The methodology that has been used in this study is similar to the approaches that are made in the PISA tests, that is, to develop Mathematical competencies from problematic situations, in our case from engineering problems, motivating students to reflect and propose solutions in the context of industrial engineering. In this context we speak about mathematizing, meaning the fact of applying methods and mathematical approach to different experiments.

The experiment called "Spring" allows users to simulate the behaviour of a spring-mass oscillation system. As students are familiar with the pendulum, and also with this spring-mass system from their physical laboratory classes, they should be able to deduce the system of differential equations. The last part of the course is related to the calculation of eigenvectors and eigenvalues, and it is very common that students do not understand the meaning of those ideas. Sometimes, for engineering students the "traditional" definition of eigenvector or proper vector of an endomorphism is more difficult that the concept of vibration modes of a spring, and the same for eigenvalues and frequencies.

With the use of appropriate mathematical language throughout the whole experiments the student acquires the competence of handling symbols and mathematical formalisms, which is closely related to communicating in, with and about mathematics, in which in the process of solving the tasks the student understands mathematical expressions and statements and expresses himself mathematically in different ways.

To solve any problem that arises to an engineering student, he must be able to use the necessary material and tools, which includes knowledge about the resources and tools that are available, as well as their potential and limitations. In addition, it includes the ability to use them carefully and efficiently.

The competencies related to the use of the technology are particularly relevant in engineering careers in general and in mathematics subjects in particular, since one of the causes of the difficulty in learning mathematics is the level of abstraction that these entail. The mobile phone is located precisely between the world of formal systems and the physical world and has the ability to make concrete the most abstract concepts (Turkle and Papert, 1992).

With the use of mathematics the student can experience solving mathematical problems through the computer. It is important to focus on two very important aspects of learning mathematics: the possibility of "experiencing" through the mobile devices and a change in the attitude of the student around the process of teaching and learning mathematics.

Approach of the Students: Problem of a building in vibration

After a computer-aided course we propose the students to solve a problem like this: Consider a two-story building in horizontal vibration by the action of the wind. The mechanical properties of the building are given by m_1 = 8000Kg, m_2 = 8000Kg, k_1 = 4000N/m and k_2 = 3500N/m. Calculate the natural modes and vibration frequencies (initial conditions are also given).

Reading this task, the student, who has knowledge of physics and is studying a engineering degree, can think that he must solve the problem from an algebraic equation and can give a mathematical answer (think mathematically). The relationship between the different forces involved in a system of two degrees of freedom must be translated into mathematical conditions or equations that include the mechanical data of the problem: the masses, m_1 and m_2, and the elastic constants, k_1 and k_2. To do this, he will apply a series of arguments (reasoning mathematically and representing mathematical entities) based on his physics' knowledge and the corresponding mathematical sense:

1. Thinking about the simplest system: a simple pendulum, its displacement is directly proportional to the force that produces it (Law of Hook): $F= k\ x$ (k = elastic constant, x = elongation).

2. The building will be at rest as long as there is no force acting on it that varies its initial state (Newton's first law). The use of drawings and graphs of problems helps to understand them (Domínguez Caicedo, 2014).

3. The forces produce accelerations that are proportional to the mass of a body. When there are several forces acting, they will be added vectorially (Newton's Second Law): $F = m\,x''$ (m = body mass, x''= acceleration).

A building can be modelled assuming that the walls do not have mass and that the mass is concentrated in the floors, so that there is a horizontal rigidity (Rao and Yap, 2011). The problem is equivalent to that of 2 springs and masses, as can be seen in Figure 3.

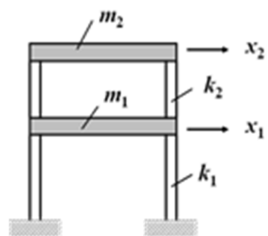


Figure 3. Two-storey building equivalent to 2 springs and masses.

The system of differential equations that models the system can be easily obtained from any textbook or directly searching on the internet (modeling mathematically), and the characteristic equation that will allow to get the eigenvalues (frequencies) and eigenvectors (vibration modes).

Moreover, the use of a computer algebra system, such as Mathematica could help to find the results quickly (Making use of aids and tools). With the use of appropriate mathematical language throughout the whole problem the student acquires the competence of handling mathematical symbols and formalisms, which is closely related to communicating in, with and about mathematics, in which in the process of solving the problem the student understands mathematical expressions and statements and expresses himself mathematically in different ways.

Once the problem is solved, a representation of the two-storey building vibrating could help to understand the meaning of the values of eigenvectors (see Figure 4).

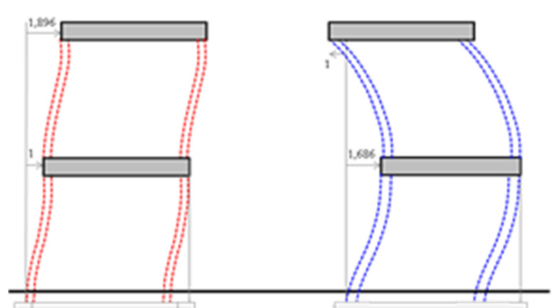


Figure 4. Final solution of the problem: 2 vibration modes.

Conclusions for Education

We started with the proposal of using the mobile phone as a spring together with a computer algebra system. Once students collect the data from their devices, they were able to calculate the frequency of a pendulum or a spring and this leads them to study the vibration of a system with one, two or more degrees of freedom. Mobile-aided learning and computer-based learning in general help students to be motivated. The use of devices awakens their curiosity and captures their attention.

While the traditional methodology starts from a formal and perfectly structured presentation of the contents and later focus on application problems, this methodology proposes to reverse the process: Initially, mathematization is induced through contextualized activities close to real life, sciences and engineering that are familiar to students, in order to later compare results and undertake the formal content fixing process.

Acknowledgment

This work has been co-funded by Erasmus+ programme of the European Union under grant 2017-1-ES01-KA203-038491 (RULES_MATH).

References

Alpers, B. et al. (2013) A Framework for Mathematics Curricula in Engineering Education. *SEFI*. Available at: http://sefi.htw-aalen.de/

Bullón, J.J., Hernández Encinas, A., Santos Sánchez, M.J., Gayoso Martínez, V. (2018) Analysis of student feedback when using gamification tools in Math subjects. In *Proc. IEEE Global Engineering Education Conference (EDUCON)*, 1824-1829.

Domínguez Caicedo, M. (2014) Períodos de vibración de las edificaciones. *Revista de Arquitectura e Ingeniería* 8(2), 1-13.

Rao, S.S. and Yap F.F. (2011) *Mechanical vibrations* vol. 4. Prentice Hall Upper Saddle River.

Turkle, S. y Papert, S. (1992) Epistemological pluralism and the revaluation of the concrete. *Journal of Mathematical Behavior* 11(1), 3-33.

Influence on learning outcomes by human factos

Stefanie Winkler, Franziska Gorgas, Ruth Leskovar, Andreas Körner

TU Wien, Institute for Analysis and Scientific Computing, Vienna, Austria

Abstract

In the last decade, the understanding and use of eLearning and blended learning changed significantly. The growing generation of digital natives and the fast progress in technology increased the importance of online tools in teaching. This development led to adaptions and improvements regarding the availability and usage of eLearning at the TU Wien. The field of application of digital tools at TU Wien covers not only first semester courses but also higher education in various bachelor and master programs. In the last two years, the evaluation of blended learning courses led to modifications in the course structure in order to use online teaching elements more effectively. This paper gives an overview of the course structures, learning methods and tools used for lectures and exercises in the group mathematical modelling and simulation. Additionally, an evaluation of the influence of those responsible, like lecturers and tutors, in such courses is provided. This evaluation shows that even the most elaborate course structure cannot guarantee a perfect course.

Introduction

Since 2006, the research group mathematical modelling and simulation, situated at the faculty of mathematics at TU Wien, developed several blended learning courses. These classes focus mainly on calculus for engineers and on modelling and simulation for several disciplines. For administration of content, all provided courses use an online platform based on Moodle and interactive applications based on Maple as well as MATLAB and OCTAVE. The examples for the latter are hosted on an external web server called MMT – Mathematics, Modelling and Tools. The Maple based platform was purchased by the university and is a commercial computer algebra system called Maple T.A. It was established to enlarge the possibilities of examples, teaching, learning and testing. Maple T.A. stands for Testing and Assessment and is at TU Wien primarily used to support blended learning courses in mathematics for freshmen in general and students in electrical engineering, mathematics and geodesy.

In the following, the different tools will be introduced and their applications will be explained in detail. Additionally, the course structures and online elements will be presented, evaluated and discussed.

Tools used for Blended Courses

The TU Wien only offers courses that require class attendance, but no distance learning courses. Therefore, teachers and professors are not obligated to use any given online tool for their lectures. The employed professors are often more interested in the progress in their research field than in the changes and improvements of teaching techniques. Especially in Austria, depending on the field of study, a lack of development in education on university level is present.

In contrast, one of the main goals of this research group is the implementation and promotion of online learning platforms, such as Moodle, MMT and Maple T.A., in the teaching routine at the university in order to improve students' learning outcomes.

Moodle

Moodle, the open source content management system, is available for all university's students and employees. The university established a department responsible for server maintenance and implementation of additional features to facilitate the adjustment from paper-based learning to eLearning, based on Moodle. Most of the time, the platform is only used for uploading, organising and distributing files. Moodle itself offers a variety of possibilities, yet hardly anybody makes full use of this potential. Moodle was introduced at the university around 2006 but still, only approximately a third of the courses at the university take advantage of this opportunity. One possible explanation might be the basic administration webpage, which manages all the student data and basic course information, such as time and place, but additionally covers simple features, for instance uploading additional materials and offering communication tools, which satisfy most teachers' needs.

In contrast, Moodle is utilised for all courses held by the research group. Presentations and additional materials are uploaded and administered there. Beyond that, a special feature, implemented inside Moodle by the department, allows students to tag examples they have prepared for presentation prior to their attendance classes. The glossary function is used to point out and summarise important mathematical definitions and propositions in the first semester. Additionally, Moodle also helps administrating project groups and the upload of students' exercise marks. Some courses also use the survey and voting tool provided in Moodle to gather feedback from the students. On top of that, the available LTI connection enables direct links to other online learning platforms, avoiding additional account data.

MMT – Mathematics, Modelling and Tools

In the group for mathematical modelling and simulation another frequently used platform is called MMT – Mathematics, Modelling and Tools. After the shutdown of the official MATLAB Webserver, the research group started developing their own interface. On this system, questions for calculus 1 and 2 for engineers are realised. Due to the focus of the research group also modelling and simulation examples are implemented. These examples were designed to help students understand the principles in modelling and simulation. For all courses using this interface, the goal is to enable students to explore provided examples and perform experiments with given models online for a better understanding of the content without installing any additional software.

The fact that the created examples should not only be used for experimenting but also for understanding model behaviour and the motto "testing drives learning" led to the decision to use these examples also in tests. Due to a restriction in Moodle, the tests are composed of multiple-choice questions and questions with numeric answer boxes. Questions for theoretical mathematics are formulated as multiple-choice questions. Questions with numeric answer boxes describe the task and provide the link to the corresponding example on the webserver. There, students have to adapt parameters in the model and run the simulation to determine the numeric value asked for. To increase the diversity, questions with varying parameters were created and organised into groups. From this collection of examples, test questions are chosen randomly. Using this technique, the possibility for cheating decreases since the probability of students sitting side by side with the same exam question minimises.

Maple Testing and Assessment – Maple T.A.

Maple T.A. is an online interface based on the computer algebra system Maple. Therefore, it not only enables the creation of static questions, as in Moodle, but allows the composition of questions with a high variety in variables and functions. Beside the advantage of randomization, the possibility to assess these questions in a mathematically intelligent way is an important feature of the system. Compared to the numeric boxes available in Moodle, this computer algebra system enables mathematical equivalence grading. In certain examples, comparing the correct answers directly to the students' responses is not sufficient. Maple T.A. offers the possibility to create special grading routines to enable partial grading as well as validating certain properties of the students' answers. In order to provide fair and student-friendly grading, the computer algebra grading was adapted to enable partial grades. With this adjustment, we established a customised grading library, containing commands, allowing partial grading of different mathematical structures. This tool has been applied in the research group for teaching since 2008. Every calculus course uses Maple T.A. in one way or another.

In the following sections, we will focus on the different levels of mathematical courses and only mention similarities or contrasts to the modelling and simulation courses if appropriate.

Course Structure of Mathematical Blended Learning Classes

The blended learning course structure of Mathematics for electrical engineers has been adapted over the last ten years to meet different requirements and to include online elements into the concept. The three courses consist of lectures and corresponding exercises. One of the courses' obstacles is the rotation of the lecturers. The team organising and carrying out exercises stays the same but the underlying materials as well as the lecturer change. In order to develop a student-friendly and forward-looking assessment, the lecture content was reviewed and revised. Students and representatives of the faculty of electrical engineering were involved in this process. Due to the content dependency of collaterally offered classes, one critical point is the timing of the content. For detailed planning, the semester was divided into three thematic parts to enhance the learning process of the students.

Before 2008, the attendance exercises were defined by two components: homework examples and their presentation. Now, the concept consists of four main requirements. First, there are two to three formative online tests where students have to obtain a certain amount of points in total. Secondly, they have to prepare at least 60% of all provided online examples as homework. Thirdly, they have to present selected exercises from their homework in front of small groups of students and one tutor. During this presentation, they have to show that they understand the concept and the underlying theory. Fourthly, at the end of each exercise, they have to perform a short, written test. This test consists of one small example and one definition or theorems of the weeks' topic. Tutors grade these small tests and hand them back in the next lesson to give individual feedback.

All these criteria and demands are well defined and well communicated at the beginning of each course to avoid confusion and misunderstanding. This course structure is one of few using online assessment not only for formative but also for summative assignments. As Maple T.A. is used for determining students' grades and not only providing self-assessment, it requires strict but also fair grading. Using state of the art technology often has the disadvantage that machine grading returns either true or false. Often, this neither fits students' nor teachers' needs. Therefore, it is very important to enable adaptive machine grading in order to catch small algebraic errors in the calculations of students without totally losing the systems' time benefit.

Since 2012, the exams of each lecture can be taken in Maple T.A.. For this online exam a special blended grading is applied. Taking into account, that exams of two hours without calculation errors are illusory, students get an online exam but write down all calculations in detail. At the end, they hand in their notes which then serve as prove that the results, entered in Maple T.A., were calculated by the students themselves. Their notes also help understanding the applied arithmetic procedure in order to assign partial points in case of small calculation errors. After grading theoretical questions and calculation errors the students can access the revised points to see the solutions, their mistakes and total points. The blended grading can be realised quite easily gaining maximum satisfaction of students and teachers.

Evaluation of Course Results and Human Factors

At this point, someone might think the human factors of interest are part of the process of blended grading. In fact, this is not the case. However, there are several points, where human factors significantly influence the success of this well-structured and sophisticated concept, described above. On the one hand all tutors, who are lecturing the exercises, are a major factor because they are guiding the students throughout the semester. They have a great impact on the students' motivation and progress in understanding the mathematical background, as mentioned in Krause (2005). On the other hand, the professor giving the lecture dominates the atmosphere of the course and defines the key points and focus of the course. One might think that the tutor, sharing face-to-face time with small student groups, has the greatest impact on students. In contradiction to this assumption stands an evaluation of the last five years, which shows, that the lecturer defines the atmosphere and the level of motivation in the first month. Even if the participation in the voluntary lecture decreases after some weeks, the mood created by the lecturer in the first few weeks remains. Of course, this proposition can hardly be proven directly by any data. However, one fact we received through students' feedback in surveys or directly face-to-face, was that, depending on the lecturer, the response to the online platform changes significantly.

Maple T.A. was first introduced at the TU Wien in 2008 providing an additional learning and testing environment. The first course established was a refresher course in mathematics to help students recall mathematical definitions and methods learned in school they might have forgotten due to a certain time gape before university. The first cycle of Mathematics 1 and 2 for electrical engineers, which are courses in the first and second semester, was established in the same academic year. Both mathematical courses follow the same concept, consisting of the four components explained above. The digitalisation of the homework every week was introduced two years ago. They get two attempts to complete the online homework. All available examples, including the ones of the homework, are available for unlimited practicing in Moodle. The tests done during the semester are executed directly in the system since 2008. In Figure 1 the results of the tests in Mathematics 1 over the last five years are shown. The two diagrams depict the results divided in the two different lecturers. Due to the fact that the students only need to pass two of the three tests or reach a certain point threshold in all the tests combined. In general, the third test has the worst results. Besides that, no significant difference can be found. The average of all three tests of both lecturers are about the same, between 63 % - 75 %, depending on the test evaluated. Still you might get the impression, that the results are slightly better considering the left plot.

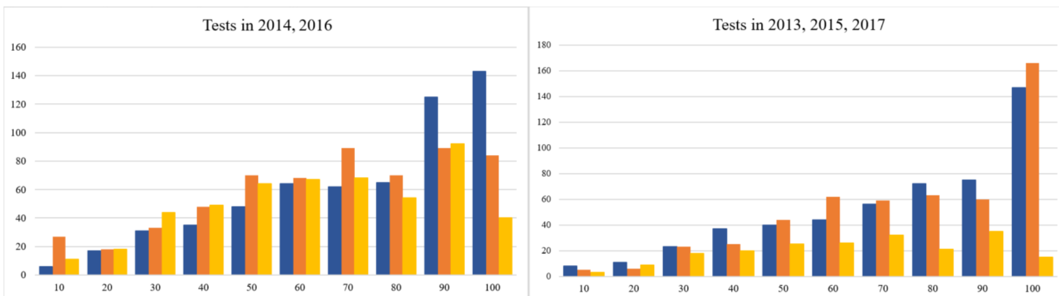


Figure 1. Results of all 3 tests (1st blue, 2nd orange, 3rd yellow) in the exercise course for Mathematics 1 for electrical engineers divided in the two different lecturers. Both plots show the number of students (y-axis) achieving between 0 % and 100 % (x-axis) of the points.

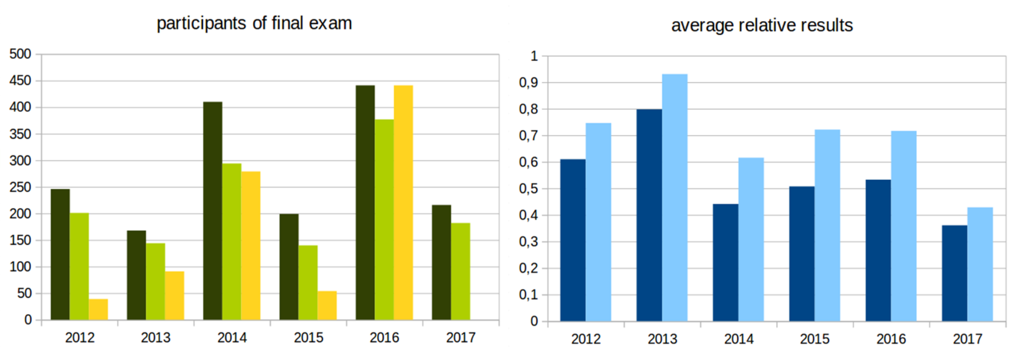


Figure 2. The left plot shows the number of participants for every year (dark green – total number, light green – without repetition, yellow – online exams). The right plot shows the relative results over the year (dark blue – all results, light blue – without repetition).

Since 2012, students have the possibility to attend the written lecture exam on the online platform as well. These exams were first performed optionally in Maple T.A., but are obligatory on the system since 2017 for one of the lecturers. In Figure 2, the number of students taking the exams and the average relative results are shown.

In the left plot of Figure 2 one can see the number of students taking the test every year. The dark green shows the total number of students, whereas light green excludes all repeating students. The yellow data displays the count of exams taken on Maple T.A., annually. One can see that the number of students taking the exam decreases all odd years but increases in the even years. The resisting and conservative opinion concerning Maple T.A. of the second lecturer explains the decrease in online exams. Since last year this lecturer changed back to written exams due to the strong disbelieve that an exam done online equals written exams. For assessing calculation skills, it is satisfactory but not for the two-hour exam. Meanwhile, in the other lecture series the number of online exams taken increases each year. As mentioned above, the online exams are obligatory since 2016. In the right plot in Figure 2 the average results for the exams are listed. The dark blue set shows the relative results of all students while the light blue excludes again all repetitions. In this diagram, one can detect a slight relation between the decreasing online exams and the decreasing results, neglecting repeating students.

Conclusion

Concerning the evaluation results, the lecturer not only teaches content but also expresses an opinion. Although the course structure for both lecturers is the same, the response to the online system has the highest variance. The approach to the online system reduced the motivation of students to invest more time than necessary in practicing online examples. Especially in mathematics, repetition and practicing are fundamental parts of learning. Therefore, including Maple T.A. not only in the small tests during the semester is an important part to enhance independent learning. Due to the randomization in all the online examples, the effect of practicing increases because memorizing examples gets harder and pointless.

Even if there is not enough data to supporting this position, the number of exams taken in one year shows at least, that students are more likely to take the online exams even though there is no significant difference in the students' success rate. One reason might be the possibility of practicing all available examples making the online exam more attractive. The blended grading enables theoretical and practical questions and is therefore equal to a regular written exam. For us the presented structure realizes the meaning of constructive alignment increasing students' motivation, as mentioned in Biggs (2003), and therefore facilitating a successful learning process.

References

Biggs, J. (2003). Teaching for quality learning at university: "What the student does." 2nd edn, Berkshire: SRHE & Open University Press

Krause, K-L. (2005). "Understanding and promoting student engagement in university learning communities". Centre for the Study of Higher Education: University of Melbourne.

Different Views of Mathematicians and Engineers at Mathematics: The Case of Continuity

Burkhard Alpers

Department of Mechanical Engineering, Aalen University

Abstract

In a competence based approach to the mathematical education of engineers, it is important to take into account the engineering "view" at mathematics in order to enable students to understand and use mathematical concepts in relevant engineering contexts and situations. In this contribution, I will first elaborate on the concept of "view" and methods to capture its components. I will use the mathematical concept of "continuity" as an example to demonstrate the different views of mathematicians and engineers. The contribution closes with a discussion of potential consequences for the mathematical education of engineers.

Introduction

Dray & Manogue (2005) state in their paper on the gap between mathematics and the physical sciences: „Mathematics may be the universal language of science, but other scientists speak a different dialect". This way they express that mathematicians and physicists have their particular views on mathematics, its value and potential use. The same holds for mathematicians and engineers. In a competence based approach to the mathematical education of engineers (cf. Alpers et al. 2013), it is important to take into account the engineering view in order to enable students to understand and use mathematical concepts in relevant engineering contexts and situations.

In this contribution, I will first elaborate on the concept of "view" and methods to capture its elements. I will use the mathematical concept of "continuity" as an example to demonstrate the different views of mathematicians and engineers. For doing this, I will investigate the treatment and use of this concept in two widespread German textbooks on analysis (for mathematicians: Heuser 2009) and on engineering statics (Gross et al. 2013), respectively. Moreover, for capturing the engineering view more widely, I will also consider the use of continuity in an industry guideline issued by the German association of engineers and in a practical task from industry I encountered when doing consultancy work. The contribution closes with a discussion of potential consequences for the mathematical education of engineers.

Aspects of a "view" and methods for its investigation

The term "view" is used in order to comprise the following aspects:

- What is considered as a "valid" mathematical statement? Are these well-defined concepts described by axioms based on set theory as well as theorems proved by strict logical argumentation? Or is it "allowed" to use terms that are based on a non-precise imagination (like a vector as object that has length and direction) and assertions where certain properties are implicitly assumed to be true (like the completeness of the reals). In the latter view, mathematics is rather seen as fragmentary (a set of loosely connected pieces) whereas in the former view mathematics is seen as closed theoretical building without logical omissions.

- Which kind of mathematical knowledge is considered to be essential and interesting? Are these assertions which shed more light on terms and relationships between terms, which state properties and provide classifications? Or are these computational procedures that help to solve application problems?

- Which usage scenarios are considered as essential? Are assertions like theorems seen as being interesting for their own sake and as useful instruments for proving further theorems? Or are concepts and procedures used to describe and solve application problems?

Similar to terminology in software engineering, the term “view” comprises the “interface” (mathematical objects/properties/operations) as well as “use cases” (meaningful usage). It is problematic to talk about the view of “the” mathematician or „the“ engineer because there are various different roles. For example, a mathematician can fulfil the role of a mathematical researcher or author of scientific articles written for mathematicians but he/she can also be a mathematics lecturer in application study courses or member of a computational department in industry. Similarly, an engineer might be a lecturer, a researcher on theoretical foundations or a practicing engineer in industry. It is rather likely that the role shapes the view.

In order to identify components of the view of mathematicians (in a certain role) one can apply the following methods

- One can investigate mathematical elaborations in textbooks or journal articles in order to see what counts as valid result.
- If a mathematician acts as lecturer in an engineering study course, he/she has an at least implicit understanding of the view an engineer has on mathematics. For capturing this view one can investigate textbooks on mathematics for engineers.
- One can also analyse texts on the philosophy or epistemology of mathematics since these fields deal with the „essence“ of mathematics as well as valid methods. Moreover, there are also respective books by well-recognised mathematicians like Courant & Robbins (2010), Davis & Hersh (1981), or Gowers (2002).
- Qualitative research methods (like analysis of interviews) can also be applied (see, for example, Holmberg & Bernard (2017) regarding the Laplace transform).

In order to identify components of the view of engineers (in a certain role) one can apply the following methods:

- One can investigate the definition and usage of mathematical concepts, assertions, representations and procedures as well as mathematical argumentations in engineering literature (textbooks, manuscripts, research literature). For example, Hochmuth et al. (2014) and Alpers (2017) analyse mathematical definitions and notations in textbooks on signal processing and statics, respectively.
- One can also interview engineers and analyse the answers using qualitative research methods (cf. Gould & Devitt (2012), Holmberg & Bernard 2017).
- One can investigate the usage of mathematical concepts, representations, assertions and procedures in engineering practice by analysing artefacts with mathematical

meaning like programme input/output, computational procedures (e.g. for dimensioning machine elements) or engineering guidelines (e.g. on motion design, cf. VDI (1980)).

In the sequel I follow the textbook analysis approach and restrict myself to the treatment of the concept of „continuity“ in introductory (German) textbooks on calculus (Heuser 2009) and statics (Gross et al., 2013) which are widely used (12. and 17. edition, resp.). Moreover, I also include the usage of continuity in an engineering guideline and in a problem occurring in my industrial consultancy.

The view of a mathematician at the concept of "continuity"

In the textbook on calculus written by Heuser the concept of continuity is embedded in an axiomatic setting with well-defined terms and complete proofs for all assertions as is well known in mathematics. Continuity is first defined at a certain place in the domain and later for a whole function. After giving an example with a "jump" special examples are constructed like the Dirichlet function (1 for rational x, 0 for irrational x) and the function which maps irrational numbers to 0 and x=p/q (relatively prime) to 1/q for x>0. The latter is continuous at irrational x and discontinuous at rational x. Such examples serve the purpose to get a deeper understanding of the limitations of the concept beyond the simple "jump" image given by piecewise continuous functions. Heuser (p. 35) writes: "… deal with the objects according to certain rules which are given by axioms and find out which corollaries can be shown by using these rules. What the "essence" of the objects is, is of no concern to you. (translation by BA)" (similarly Courant & Robbins, p. XXII).

In order to better understand the definition and to allow for specialisations and generalisations, equivalent definitions are given (with proofs of equivalence). The εδ-definition leads to the notion of uniform continuity by removing the restriction to a certain place, and the definition via open sets allows the generalisation for metric or topological spaces.

In the sequel, Heuser states and proves theorems on the continuity of certain classes of functions and on the construction of new continuous functions from existing ones. Then, important properties of continuous functions are proven (e.g. fixed point theorem, intermediate value theorem) which are later used in proofs of other assertions. Heuser also investigates the relationship with other notions like monotony and (later) differentiability (p. 262).

Continuity is an important concept in the theory of calculus as developed in the textbook and it is used in later parts of the book in order to be able to prove further theorems using the properties proven before.

The view of a mechanical engineer at the concept of "continuity"

In the textbook on statics by Gross et al. (2013) the concept of continuity only shows up in the chapter on "Beams, frames, and arcs" (appr. 40 pages) where load, shear force and moment functions along the object under consideration are set up. Here, the concept plays an important role as can be deduced from the fact that the terms "jump", "discontinuity", and "continuous" are used on 14 different pages. There is a jump in the shear force and the moment function resp., when a single point force or moment acts on the structure. Therefore, such jumps are necessary to model these idealised situations. Other occurrences of discontinuities are of no interest, so all functions are piecewise continuous. Hence, the - from a mathematical point of view - far too restricted understanding that discontinuities only occur as jumps between continuous pieces is completely sufficient. The consideration of “non-standard” functions like the Dirichlet function seems to be rather confusing for engineers since they only need a “partial concept understanding”.

Regarding the formal representation the specification of the domain is of particular interest. If there are jumps caused by single point forces (or moments) the points are excluded from the domain. For example, when a beam of length d carries a single force at a, then the (left and right) bearings 0 and d and the place a are omitted. The authors call these places “discontinuities” (p.179) although mathematically the function is continuous since continuity is only defined for elements of the domain. Heuser, for example, writes with respect to the function $f(x)=1/x$, defined on $R\setminus\{0\}$: “… (this function) is in 0 not discontinuos, not continuous – but simply not defined” (S. 213, translation BA). Gross et al. (2013) also use alternative representations. They use different functions per interval called „fields“, e.g. $Q_I(x)$ and $Q_{II}(x)$ for the shear force where the boundaries of the intervals are omitted. But for formulating conditions on how to connect fields, the interval functions are then evaluated at the boundaries, e.g. $Q_I(a)= Q_{II}(a)-F$ when a single point force acts at a. For achieving mathematical precision, one has to introduce one-sided limits here. Instead of doing this, in subsequent examples the authors use formulations like “making a section (of the beam) immediately before the point where the force acts” (p.103, translation BA) or the value at a place is provided two times, e.g. Q(2a)= “…to the left of the bearing A” and “… to the right of the bearing A”.

If one wants to avoid to have to deal with many „fields“ with many conditions on how to connect fields, authors of engineering mechanics books sometimes use the Macauley symbol (German: Föppl symbol) <x-a>^{n} which is 0 for $x<a$ and $(x-a)^n$ for $x>a$. One gets from load function to shear force function and from there to moment function essentially by integration (additionally point forces and moments have to be taken into account). Since Macaulay symbols can be integrated like power functions one does not need any conditions for connecting pieces. Single point forces can be inserted by using F*<x-a>^{0} (similarly with single moments). Since in the domain of Macaulay’s symbol the point a is omitted, the functions would not be defined there even if they were continuous at this value whereas elsewhere (p.181) such a value was included which is inconsistent.

From the graphical representations of the functions under consideration one cannot conclude an exact mathematical definition since at the „jumps“ there are two “large points” (one at each boundary) and these are connected by a vertical line. If there is a continuous connection of pieces the authors also use a „large point“ (see p. 176). The meaning of these points is not explained.

For an engineer these inconsistencies seem to be irrelevant since points are idealisations anyways and the essential part is whether there is or is not a “jump”. Therefore, the question comes up in which usage scenarios these points have a meaning and play a special role. If one wants to compute the shear and the moment function when there are several “fields” one has to set up conditions for connecting fields. For this one has to know whether one has to equate the expressions or to include an offset caused by a point force or single moment (p. 193). Moreover, when sketching the function one also has to find out whether one needs one or two points at the connection of two “fields”. In the end, the functions are needed to compute the absolute maximum (force or moment). For this, one has to determine the jumps since the absolute maximum might be located at such a point (p. 187).

Continuity is also relevant for mechanical engineers when it comes to motion design, e.g. when setting up the motion function of a slider in a packaging machine. For this task, the German Association of Engineers (VDI) issued a guideline document (VDI 1980) where a piecewise approach using certain types of functions is advocated. In that setting, continuity is required because it has a very important practical meaning: Continuity of the distance over time function is required because you cannot be at two different places at the same time. Similarly, continuity of the first derivative is necessary because you cannot have two different velocities at the same time.

Finally, when doing consultancy work for a company producing milling machines, the problem came up of how to feed into the controller a piecewise-defined function when only basic functions like sine, cosine, polynomials, trunc etc. are available (and no if-then-else construct). For this one needs the Heaviside function in order to switch on and off function pieces. Since the Heaviside function is discontinuous it cannot be constructed by combining continuous functions. Therefore, to solve the problem one has to look for a function with a discontinuity. This is the case with the "round" function which can be used as follows: Heaviside(x)=round((1/π)*arctan(x)+1/2).

Potential consequences for education

The above investigation of two textbooks on calculus and statics, resp., regarding the concept of continuity has shown that there are remarkable differences between the views of a mathematician and that of an engineer (both in the role of a textbook author). In the statics textbook, only a partial understanding of continuity as "no jump" at the connection of two continuous function pieces is required. In order to avoid confusion by including more facets of the concept it is advisable to restrict oneself to this basic meaning in the education of engineers. Moreover, for interconnecting the different subjects of the curriculum it is important that students are aware of the problems of formulations like "immediately before" or "left to" and recognise behind those words the mathematical concept of one-sided limit.

In order to let students actively experience the meaning and usage of (dis-)continuity one should design application-oriented tasks where discontinuous situations have to be modelled or continuous functions have to be designed (as in motion design). Finding the absolute maximum in a discontinuous function might also be included.

References

Alpers, B., Demlova, M., Fant, C.-H., Gustafsson, Th., Lawson, D., Mustoe, L., Olsson-Lehtonen, B., Robinson, C. and Velichova, D. (2013) "A Framework for Mathematics Curricula in Engineering Education", SEFI, Brussels.

Alpers, B. (2017). Differences between the usage of mathematical concepts in engineering statics and engineering mathematics education, R. Göller, R. Biehler, R. Hochmuth, & H.-G. Rück (Eds.). Didactics of Mathematics in Higher Education as a Scientific Discipline – Conference Proceedings. Khdm-Report 17-05 (pp.137-141). Kassel: Universität Kassel

Courant, R. & Robbins, H. (2010). Was ist Mathematik? 5. Auflage, Berlin-Heidelberg: Springer

Davis, Ph.J. & Hersh, R. (1981). The Mathematical Experience, Boston: Birkhäuser

Dray, T. & Manogue, C.A. (2005). Bridging the Gap between Mathematics and the Physical Science. In: Preparing Future Science and Mathematics Teachers, ed. D. Smith & E. Swanson, 39-41, Bozeman: Montana State University

Goold, E., Devitt, F. (2012). Engineers and Mathematics. The Role of Mathematics in Engineering Practice and in the Formation of Engineers, Saarbrücken: Lambert Academic Publishing

Gowers, T. (2002). Mathematics. A very short introduction, New York: Oxford University Press

Gross, D. et al. (2013). Technische Mechanik 1. Statik. 12. Auflage, Berlin-Heidelberg: Springer Vieweg

Heuser, H. (2009). Lehrbuch der Analysis, Teil 1, 17. Auflage, Wiesbaden: Vieweg+Teubner

Hochmuth, R., Biehler, R., & Schreiber, S. (2014). Considering mathematical practices in engineering contexts focusing on signal analysis. In: Proceedings of the 17th annual conference on Research in Undergraduate Mathematics Education

Holmberg, M. & Bernard, J. (2017). University teachers' perspectives on the role of the Laplace transform in engineering education, EJEE 42(2017), 413-428

VDI Richtlinie 2143 (1980). Bewegungsgesetze für Kurvengetriebe – Blatt 1: Theoretische Grundlagen, Berlin: Beuth Verlag

The Effect of using a Project-based-Learning (PBL) approach to improve engineering students' understanding of Statistics.

Fionnuala Farrell[1] and Michael Carr[2]

[1.] *School of Mechanical & Design Engineering, Dublin Institute of Technology, Ireland*

[2.] *School of Multidisciplinary Technologies, Dublin Institute of Technology, Ireland*

Abstract

Over the last number of years we have gradually been introducing a project based learning approach to the teaching of engineering mathematics in Dublin Institute of Technology. Several projects are now in existence for the teaching of both second-order differential equations and first order differential equations. We intend to incrementally extend this approach across more of the engineering mathematics curriculum. As part of this on-going process, practical real-world projects in statistics were incorporated into a second year ordinary degree mathematics module.

This paper provides an overview of these projects and their implementation. As a means to measure the success of this initiative, we used the SALG instrument to gain feedback from the students. The SALG online tool - **S**tudent **A**ssessment of their **L**earning **G**ains - https://salgsite.net/; is a free course-evaluation tool that enables third-level educators to gather feedback specifically focused on what the students gained through the learning exercise they experience. It can be used to measure students' learning gains. Pre-developed surveys are available which can be modified and are stored in a repository for ease of access. Results are anonymous and there is the ability to download comments and basic statistical analysis of responses. Feedback from the survey points to a large increase in understanding of the material coupled with an increase in confidence. In addition we outline some of the limitations of our initial implementation of this approach and what we hope to improve on for the next academic year.

Introduction

A 2013 study of the First Year Experience (FYE) in the eight third level institutions in the Dublin Region, found that one of the key problem areas identified by academics across all eight institutions was the lack of "student engagement" (Roper et al 2013, Cusack et al 2013). This lack of engagement can result in both poor performance and poor retention. Since September 2012, incoming first-year students to higher education in Ireland have studied a revised mathematics curriculum (Project Maths) in second-level (Jeffes et al 2013, Prendergast et al, 2017). This new approach to the teaching and learning of mathematics in Ireland aims to situate mathematics in everyday contexts where possible, so that students will be better able to understand the uses and relevance of mathematics. In particular there has been a huge increase in the amount of statistics taught at second level. Much of the material taught in the early years of mathematics is not explicitly mapped at that point to modules or applications in later years, making it difficult for students to understand the importance of what they are learning at this early stage in their careers. Sometimes this can be difficult for mathematics lecturers to find applications that are easily understood by students at this stage.

Successful service-teaching of mathematics relies heavily on a “sufficient supply of discipline related problems” (Yates 2003). This changing mathematical landscape in Ireland provided the motivation for the development of the project-based-learning approach described in this paper. In particular statistics is ready made for simple applications being introduced early on.

In Dublin Institute of Technology, students are offered two main routes to obtain a Level 8 engineering qualification: via direct entry onto a four-year Honours degree programme (Level 8) or alternatively through a three-year Ordinary degree programme (Level 7) followed by a transfer into third year of the Honours degree (Llorens et al. 2014, Carr et al. 2013). This project is thus an attempt to evolve the teaching of engineering mathematics at Level 7 to both improve the engagement of students in engineering mathematics classes and to provide a deeper understanding of the material, which may ultimately help these students to progress onto a Level 8 degree.

Automation Engineering

The cohort under consideration in this study are second year Automation Engineering students. This involves the design, development and implementation of sensor and robotic systems for applications across a wide range of technological sectors according to the DIT website (www.dit.ie). The cohort under study are in year 2 of a 3 year level 7 degree. There are approximately 30 in the class, this specific year had 23 students. Many of them will go directly into industry upon graduation but some will remain in academia and proceed to do a level 8 course. We need to provide a level of training that will prepare them for industry but will also give the necessary mathematical background to proceed to an honours degree. We feel that using project based learning is an ideal way of covering both eventualities.

Method

According to Koparan & Güven (2014), in this current ‘information era’, data literacy has become an essential skill and highly relevant in the field of mathematics and science. This is also required in the field of engineering (Ben-Zvi & Garfield, 2008). Koparan & Güven (2014) investigated the use of project-based-learning (PBL) to help develop robust statistical literacy skills of their 8th grade students. Their findings were encouraging, showing that this approach not only helped their understanding on the subject matter but also, via the projects, promoted a cooperative working and learning environment for students. Given these positive results for 8th grade students and the success of previous work by Carr & Ní Fhloinn (2016) with third year ordinary degree students it was deemed appropriate to trial this with the second year ordinary Automation Engineering degree students.

Aligning with the approach investigated by Gratchev & Jeng (2018) and applied by Carr & Ní Fhloinn (2016), a ‘hybrid’ approach of teaching statistics was used in this study. This involved introducing a significant quantity of fundamental statistical material and examples using the traditional teaching in-class approach and then introducing a realistic project to consolidate the theory discussed in class whilst providing the opportunity for students to learn how this is applied in real-world situations.

Improvements to teaching statistics such as providing ‘authentic statistical experiences’ (Bryce, 2005); along with the consensus that it is taught more effectively with real data (Cobb & Moore, 2007) and the increased benefit to students’ learning if they collate their own data (Hogg, 1991) all influenced the design and delivery of the statistical projects taken in this study. Some research also suggested that personal relevance is important for successful learning (Mvududu, 2003).

The objective of the statistical real-world projects was ultimately to give students a better understanding of the material but also to help further develop their problem solving, teamwork and communication skills, linking to their other course module on communications where the emphasizes is strongly placed on the importance of these softer skills when applied to technical problems/situations in reality.

Overview of the Projects

After completing three 2-hour traditional class-room lectures on probability and various statistical distributions (Normal, Binomial and Poisson) the students were divided randomly into groups of three or four. They had to work on a short project together during class-time supported by the lecturer to address any queries, as well as dedicate some additional time outside of class in order to complete it. Each group was given a different project which they randomly selected. The project topics are briefly outlined in Table 1. Students were not told which distribution could be related to their project.

Each group was required to read the project scenario and discuss it as a group to determine what data should be collated. The groups then had to gather this data themselves and analyse it. This was to be completed in one 2-hour session. Students were to use additional time between group sessions to prepare a short presentation on their data and findings. Additional time was provided at the start of the next 2-hour session to finalise their work. Specific questions were asked in each project scenario which the team also had to use their data to help answer and present. At the end of their 10 minute presentation each group of students was asked another unseen question in which they had to use their analysed data on the spot in order to provide an answer. This assessment was worth 5% of the mathematical module overall.

#	Project Topic	Brief description
1	High-volume Production of Mini-bars	High-volume production companies wanting to ensure products are in control based on either weight or length parameters. With a sample of production material, students would be asked to determine the probability of finding a bar of a certain weight/length based on their data.
2	High-volume Production of tear-drop metal components.	
3	College Soccer Club 'Finger Footie' penalty shoot-out competition	Prize for team of students who score the most in 5 attempts. Project involved determining the best shooter strategy and students were asked to determine the probability of scoring a random amount of goals based on their selected strategy.
4	M&Ms Distribution	Identifying the proportion of each colour of M&M's in a random sample of fun size packets and determining the probability of finding any particular colour when randomly selected from a packet.
5	Student Safety Alert: Addressing student Jaywalking main road between college campuses	Traffic Corp have been receiving complaints from drivers regarding the dangerous behaviour of students crossing a main road and not using the pedestrian crossing provided. Students Union would like to see a safe footbridge constructed in a better location and also wanted to gather evidence of the issue raised. They want to be able to determine the probability of a student J-walking v's using the pedestrian crossing in any given 15 minute period.
6	Student Transportation – City bicycle availability	Addressing an issue raised by students due to the lack of availability of city bikes to support their requirement for them to attend lectures before and after lunchtime held on different city campuses. Students were asked to determine the probability that within any 15minute period during lunch at least 4 bikes would be available.

Table 1: Overview of the 'Real-world' Statistical Problems

Gaining Student Feedback

Directly following the presentation session, students were asked individually to complete an online survey using the open source 'SALG' instrument to gain their feedback. The SALG is an online open source survey tool - **S**tudent **A**ssessment of their **L**earning **G**ains - https://salgsite.net/. used to measure students' learning gains. From the experience of previous work in creating a survey for this type of study (Carr *et al*, 2017) and feedback shared in presenting the findings, it was agreed that using this standard tool would beneficial in understanding the impact of the project on student learning. It was also considered optimum to conduct the survey directly following the presentation session under supervision to help support participation and to clarify any questions students might on the survey content.

Overview of the survey questions

This section outlines the survey questions edited specifically for this particular project but which follows the prescribed format designed to help assess learning gains. Results were anonymous and there is the ability to download comments and basic statistical analysis of responses.

The structure of the standard survey remains consistent with 10 headings insofar as the general categories for assessing learning gains. The questions within each of these headings can be edited to suit the particular needs of the learning session i.e. the 'class' or in this case the 'statistics project' Three to four questions were asked under each of the 10 headings captured in Table 2 and were edited slightly to suit this particular student exercise and learning experience. Some questions were open-ended providing the opportunity for students to expand on their feedback with the majority of questions asked via a likert-type 5-point scale, with 1 indicating 'no gains' in their learning and 5 indicating 'great gains' obtained from the particular learning.

1.	Your understanding of class content
2.	Increases in your skills
3.	Class impact on your attitudes
4.	Integration of your learning
5.	The Class Overall
6.	Class Activities
7.	Assignments, graded activities and tests
8.	Class Resources
9.	The information you were given
10.	Support for you as an individual learner

Table 2: SALG Survey: Generic Structured Headings

It took students on average 10 to 15 minutes to complete the survey. In total there were 12 respondents, giving a 57% response rate (two students did not participate at all in the project). Full details of the actual survey used can be obtained on the SALG website searching for Instrument # 79785.

Results

There were five groups in total who participated. This project was run in the last two weeks of the semester which impacted participation. Below is a synopsis of the responses under each theme.

Understanding Class Content: Students indicated they made good gains (4.4) in their understanding of the statistical concepts and good-great gains in how studying this type of material would help people address real-world problems (4.6). When asked how their understanding of statistics has changed as a result of this project students positively responded that it made the importance of statistics and its use in the engineering discipline a lot clearer *'before this class i had no real understanding of statistics, so the resuling change would be 100%'* From the way the class was taught students felt that using real comparisons/data helped them remember the key ideas which were presented 'clearly' and were 'well explained' Using the data to answer additional questions also help in remembering key ideas.

Increases in Skills and attitude: Students felt they had moderate-good gains (4.1 average) in developing their skills in identifying patterns in data, analysing and presenting data and working effectively together. Some commented that their general skills on statistics improved echoing the response from the previous category in gaining better understanding of the subject matter. Overall a positive attitude (3.9) towards statistics resulted through their increase interest and confidence in it with one student commenting that they *'always liked maths but just not the statistics section but after attending a class just specific to the subject, really helped me in changing my view and made me understand it which means like it too'*.

Integration of learning and overall Impact of the Project: Students were asked how they could use this project and apply it to other situations or in problem solving. They all agreed it provided good gains (4.0) with some suggesting they could apply it to 'critical thinking' or for 'designing automation projects' and helped in understanding the value of data collection. In relation to the class overall, specifically focusing on the instructional approach taken the students positively responded (4.6) commenting that things were *'well explained'* and having the lecturer *'open to questions'* and *'repeated questions…ensured all students were on the same page'* The fact that work was *'evenly distributed'* amongst the team as the lecturer encouraged individuals by asking questions also appeared to have a positive impact. That said, however, students were 'indifferent' as to whether the project changed the way in which they studied in general.

Class Activities, assignments and tests: Students all agreed that attending and participating in lectures improved their learning (4.4) as well as doing the hands-on work on the project. Students commented on the fact that the class questions helped them learn with one student commenting that 'questions were asked that I didn't want to ask' enhancing their learning. Students highlighted that this project positively required an extra mental stretch (4.4) which helped also in their learning.

Class resources, information and support: Additional to the class-notes used in the traditional lecture sessions prior to the project, students were also provided with the HELM online notes on statistics and probability. They were encouraged to use additional textbooks to help in their understanding. Whilst there was positive feedback on the use of the classnotes themselves, little or moderate gains were obtained from standard textbooks on statistics according to the students. It is questionable how much time and effort students put into reviewing this additional material however. Interacting with lecturer (4.5) and working with their peers in-class (4.3) proved to be most beneficial for the students in helping their learning of the subject matter overall as it *'gave us a chance to listen and learn off each other'* A student commented that the pace of the project work was good which helped their learning.

Overall the results from the student survey were very positive and students seemed to agree that the practical real-world approach of the project work supported the in-class more traditional approach to the topic. However, very little can be concluded in how this project helped with the terminal exam as only 4 students answered the question on statistics, three of which either didn't participate at all in the project or only attended the first project data-collection session.

Findings and Discussion

The feedback to this approach is very positive; we certainly see strong evidence of engagement and an increase in understanding. Previous work (Carr, Farrell & Ni Fhloinn, 2017) has shown that this approach works at least as well as the traditional approach in terms of answering traditional type questions, but in addition there is an increase in other skills such as problem solving, team work and application of mathematics to the real world. Given that this is just a provisional study we have enough positive findings to justify a more rigorous analysis of this approach next year. In addition it does show that this PBL approach can be introduced in second year and probably into first year of the programme.

The positive feedback of using a combined approach of traditional based lectures supported by real-world type projects is consistent with findings by Gratchev & Jeng (2018) where a similar reluctance to engage with PBL activity was experienced during this study. In this case however, students, once the initial data was collected, found it difficult then to apply/use the data without good support and encouragement from the lecturer. This can be expected however where students are unaccustomed to this type of learning experience and that a hybrid approach could be a good alternative.

Future work

Firstly, the timing of delivery of this class exercise was rather late in the year and may have had an impact on participation and full engagement. Secondly, we need to do a more rigorous quantitative analysis of learning gains. We probably need to include a specific question on the survey to see if they feel this prepares them for the final exam. In particular to this cohort, applying PBL methodology of teaching would be extremely beneficial given that these students take up work-placement in industry during their second year of the automation course. The softer skills gained through PBL activity would help prepare them for this placement and apply what they are taught in supporting modules. Finally these students will do more work on statistics in their third year and we need to extend this approach both into their third year and into the first year of the programme.

References

Ben-Zvi, D. & Garfield, J. (2008). Introducing the emerging discipline of statistics education. School Science and Mathematics, 108, p355–361.

Bryce, G. R. (2005). Developing tomorrow's statistician. Journal of Statistics Education, Vol 13(1). www.amstat.org/publications/jse/v13n1/bryce.html

Carr, M; Farrell, F and Ni Fhloinn, E (2017) 'A project-based-learning approach to teaching first-order and second order differential equations to engineers' SEFI 2017, 18-21 Sept 2017, Azores, Portgual.

Carr. M., Ní Fhloinn, E.,(2016) A project based-learning approach to teaching second-order differential equations to engineers. *SEFI Maths Working Group.* Gothenburg, Sweden.

Carr. M., Ní Fhloinn, E., Murphy, E. and Bowe, B. (2013) Addressing continuing mathematical deficiencies with advanced mathematical diagnostic testing. *Teaching Mathematics and Its Applications* 32 (2): 66-75.

Cobb, G. W. & Moore, D. (1997). Mathematics, statistics, and teaching. The American Mathematical Monthly, Vol 104, p801–823.

Gratchev, I and Jeng, Dong-Sheng (2018): Introducing a project-based assignment in a traditionally taught engineering course, European Journal of Engineering Education, DOI: 10.1080/03043797.2018.1441264

Hogg, R. V. (1991). Statistical education: Improvements are badly needed. The American Statistician, Vol 45, p342–343.

Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheater, R. and Dawson, A. (2013). Research into the impact of Project Maths on student achievement, learning and motivation: final report. Slough: NFER

Koparan, T & Güven,B (2014) 'The Effect of Project Based Learning on the Statistical Literacy Levels of Student 8th Grade' European Journal of Engineering Education, Vol 3 (3) p145-157.

Llorens, M., Nevin, E. and Mageean, E. (2014). "Online Resource Platform for Mathematics Education". In *Proceedings 44th Annual Frontiers in Education (FIE) Conference*, 1865-1872. Madrid, Spain

Mvududu, N. (2003). A cross-cultural study of the connection between students' attitudes toward statis-tics and the use of constructivist strategies in the course. Journal of Statistics Education, 11(3). Retrieved from. www.amstat.org/publications/jse/v11n3/mvududu.html

Prendergast,M., Faulkner,F., Breen,C. and Carr,M., Mind the Gap: An Initial Analysis of the Transition of a Second level Curriculum Reform to Higher Education, *Teaching Mathematics and its Applications*, 2017

Yates, P. (2003) Teaching mathematics to first year undergraduate chemists in the context of their discipline. *Maths for Engineering and Science LTSN MathsTEAM booklet.*

Competence-based Learning in Engineering Mechanics in an Adaptive Online-Learning Environment

Markus Linke, Karin Landenfeld

Faculty of Engineering and Computer Sciences, Hamburg University of Applied Sciences, Germany

Abstract

In engineering sciences like mechanical or automotive engineering pre-knowledge in mathematics is required to deeply follow specific engineering lectures. The mathematical competencies are essential preconditions to complete successfully the engineering lecture. It is necessary that these pre-competencies are identified and tested for each individual student and learning opportunities are provided according to the individual student needs.

Within this paper an approach of competence-based learning in Engineering Mechanics is presented. This approach is assisted by an online-learning environment which is adapted and extended by several features in order to enable a competence-oriented learning strategy. Computer assisted tests are used for measuring mathematical pre-competencies. Moreover, a mastery learning approach based on exercises is utilized in order to secure a certain competence level before the student moves forward to learn subsequent competencies. Test results influence the individual learning path by different suitable learning elements offered to the single student.

Introduction

In many engineering study programmes, the lecture of Engineering Mechanics is a high threshold for a successful university career. More than half of the students at the Hamburg University of Applied Sciences is confronted with delays during their academic studies or even drop out. This is partially caused by challenging lectures like Engineering Mechanics. Simultaneously, we see an increasing heterogeneity within the student body. More and more students lack the necessary preconditions, in particular concerning mathematical competencies, to deeply follow the lectures and to finally pass the exams. In order to improve this situation, we establish an online-learning environment, among others, for the specific needs of Engineering Mechanics courses. One important objective is to enable a better handle of the experienced heterogeneity within the student body. This is achieved by offering different learning paths with a personalised configuration based on the preconditions of the individual student.

In this article, the implementation of the online-learning environment called ELFE™ (ELearning with Feedback Elements in Technical/Engineering Mechanics) is presented (cp. more detailed explanation according to Linke & Landenfeld 2018). ELFE™ is part of the learning platform viaMINT (cp. Landenfeld et al. 2016) which is extended by several features in order to enable a competence-oriented as well as an adaptive learning approach. First, the underlying learning philosophy is described. This comprises the competence-oriented learning approach as well as its transformation into a learning sequence resp. an instructional learning path. Based on that, the personalised learning approach is illustrated enabling adapted learning paths according to student pre-knowledge.

Competence-Orientation in Engineering Mechanics

A competence-oriented learning approach is realised in ELFE^{TM}. The competencies which are intended to be learnt are described using operationalised learning objectives, in particular, regarding the learning taxonomy dimensions according to Anderson & Krathwohl 2001. Operationalised learning objectives are detailed descriptions about a desired and observable change in learning resp. student behaviour. Concerning Engineering Mechanics courses, the students shall achieve the cognitive process dimension of appropriately applying the mechanical relations on the conceptual knowledge dimension.

For the sake of convenience, we discuss this competence-oriented approach based on the exemplarily chosen competence "computation of bearing reactions using the elastic bending line". This learning objective comprises the determination of forces and moments in supports of mechanical structures when the deformation of the structure is known. The definition of the learning objective shall be as detailed as possible as well as necessary. It describes the student behaviour to be observed if the competence is indeed built up. The desired learning outcome consequently equals the definition of the learning objective.

From the mechanical point of view, the intended competence typically covers a wide range of several individual capabilities. In order to distinguish the intended capability from these necessary single competencies, we specify the intended one as the domain capability or domain competence. Furthermore, the individual competencies can be subdivided into pre-competencies (sometimes also called pre-knowledge) and new competencies to be taught for the defined learning objective. The latter one concerns Engineering Mechanics. Pre-knowledge typically deals with the mathematical background which is needed to solve the underlying mechanical concepts. But mechanical preconditions are also partially concerned with pre-knowledge.

Pre-competencies are usually not taught in the context of the defined learning outcome but they are important capabilities in order to systematically achieve the learning outcome, i.e. the domain competence. E.g. the students have to be capable of "defining and using adequately a coordinate system" (Mathematics applied to Mechanics) that is "appropriately chosen for beams in symmetrical bending" (Statics and Strength of Materials), "differentiating polynomials of low degree" (Mathematics) as well as of "knowing the definition of internal resultants and applying it correctly to different beam structures" (Statics). Although these capabilities are pre-competencies, a lot of students lack their correct application having a direct impact on the learning success concerning the domain competence. Therefore, these capabilities are in the learning focus, too.

The domain competence is further separated into individual competencies which we call single mechanical competencies. These have to be taught and exercised in the context of the intended domain competence. Concerning the chosen example, the students have to learn to "differentiate the elastic bending line with respect to the beam axis", "use the differential equation of symmetrical bending in order to obtain the bending moment" and "to apply the differential equilibrium at an infinitesimal beam element in order to obtain the transversal force". As a result, the domain competence is split up into several competencies (pre-competencies and new ones) as schematically indicated in Fig.1.

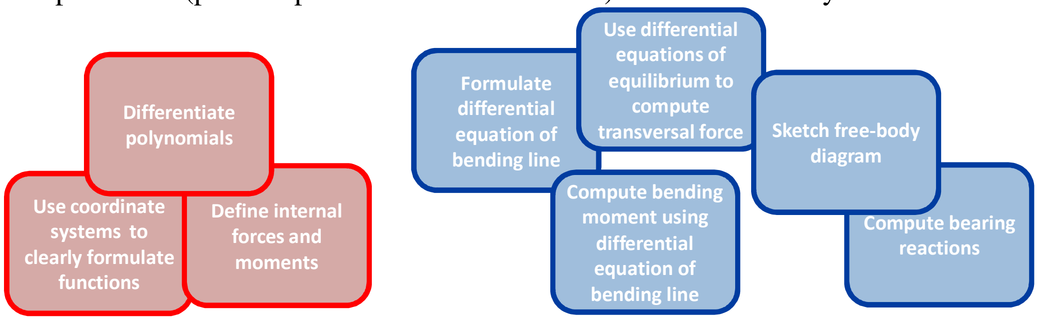


Fig. 1: Single competencies of domain competence "computation of bearing reactions using the elastic bending line" (red: pre-competencies, blue: new competencies)

For each competence we define appropriate online-questions. Based on the answer pattern of the students to these online-questions, we conclude with a certain probability whether the correlated competence is built up or not.

Direction Instruction

Although the competence-oriented learning approach described above can be realized in different learning strategies (like a problem-based learning approach), we use the direct instruction within our online-platform ELFE™. The competencies are sequentially taught and tested in online-questions. The theoretical basics to solve the online-questions are delivered to the students by learning videos as well as learning texts. The videos comprise lecture recordings, practical examples, screencasts of theory derivations and mechanical demonstrator experiments. The instructional sequence is chosen in such a way that the different competencies are logically based on each other leading to the online-questions of the domain competence at the end of the sequence. These online-questions for the domain competence combine finally all the different individual capabilities taught before. In Fig. 2, the competencies of Fig. 1 are exemplarily arranged into a learning sequence.

Usually, so-called question-pools are used where several different tasks are delivered at the same competence level. This allows an independent exercise of mechanical concepts based on a larger selection of questions for one competence. As a result, the students chose by their own the tasks they solve. As the questions are linked directly to competencies, students get consequently well-suited feedbacks to their performance. Furthermore, the competence-oriented approach combined with this online-learning platform leads in the described manner to a very structured student learning based on small step exercises. As this is in principle the basis for Mastery-Learning-Strategies (cp. Bernitzke 1987), we also integrated mastery steps. This means that students are only allowed to proceed in the learning sequence if they passed specific steps before. In the example sequence shown in Fig. 2, mastery steps are integrated at the online-questions where traffic signals are integrated between the competence boxes.

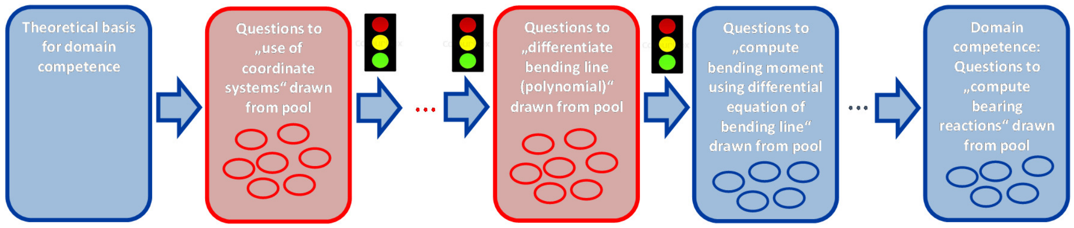


Fig. 2: Direct instruction based on a sequence of different competencies which is used to establish a competence-oriented learning approach

Adaptive Online-Learning Environment

For implementation of ELFE™, the online-learning environment viaMINT has been chosen und adjusted to the specific needs of the ELFE™ approach. viaMINT is an adapted Moodle[14] based online-learning environment which has been developed for refreshment of school knowledge for mathematics, physics, chemistry and programming and is used in different precourse scenarios, e.g. blended learning or individual online-learning. The online-learning modules in viaMINT provide learning sequences with explanatory videos and interactive online-exercises with directly given individual feedback. The online exercises are implemented using the Moodle extension STACK and the Computer Algebra System Maxima[15].

As each student has its own learning background, the students show a wide range of different preconditions when they enter our learning platform resp. sequences. Therefore, an individual set-up of the learning sequences is realised. This comprises the so-called pre-competencies which are not taught within the sequence but which are crucial for a successful pass of the domain questions. In particular, these pre-competencies concern mathematical as well as mechanical knowledge. In the example sequence according to Fig. 2, these pre-competencies are indicated in red. For adapting the learning sequence, a quiz has to be filled out at the beginning of the sequence allowing a personal sequence configuration. The quiz contains several questions enabling the check of the competencies before the learning sequence is really entered by the student. As a consequence, the red competencies can be part or no part of the example sequence with regard to the individual response pattern of a student. Fig. 3 shows an excerpt of the pre-competencies quiz and the resulting learning sequence after the test evaluation.

---

[14] Moodle is an open source online learning platform: https://moodle.org/.
[15] STACK is a system for teaching and assessment using a computer algebra kernel: http://www.stack.bham.ac.uk/
Maxima is a Computer Algebra System: http://maxima.sourceforge.net/

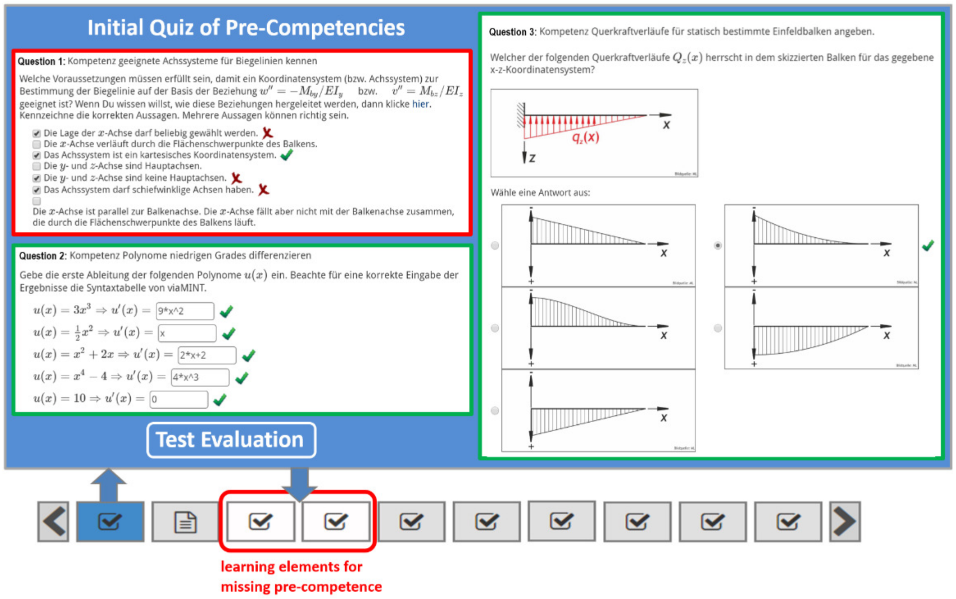


Fig. 3: Interrelation between test of pre-competencies and resulting learning sequence

In Fig. 4 the answer patterns of two students result in individual learning paths. In the case of student 1, there is a lack of mathematical preconditions concerning the correct use of coordinate systems. Therefore, the corresponding online-questions have to be mastered before. Student 2 completely passed the initial quiz so that no questions concerning pre-competencies are offered.

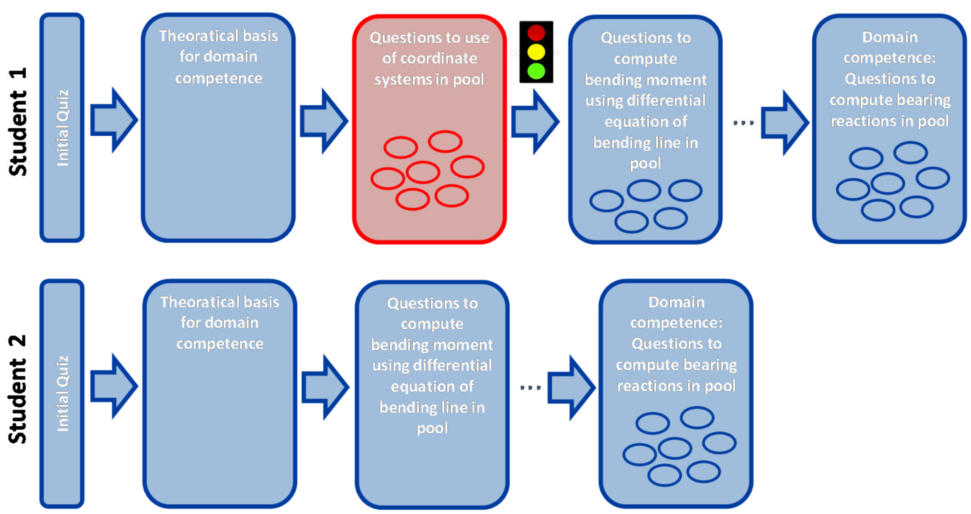


Fig. 4: Individual learning paths for two distinct students adapted according to their response pattern in an initial quiz

Conclusions and Outlook

The online-learning platform ELFE™ based on a competence-oriented instructional learning strategy is described. The competencies to be taught are split up into individual capabilities which cannot be subdivided further from a mechanical point of view. The competencies are linked in a learning sequence where the achievement of specific capabilities is secured by online-exercises combined with mastery testing. In order to consider heterogeneity of learners, in particular concerning their different preconditions, personal learning paths are implemented. The preconditions are tested in an initial quiz at the beginning of the learning sequence allowing to adapt the offered steps in the subsequent learning sequence. In this manner, different measured preconditions lead to distinct learning paths. Preconditions are only tested by initial quizzes so far. However, due to the competence-oriented approach, it is in principle possible to detect whether a competence is acquired or not based on the individual student behaviour within the online-platform as exercises are linked to capabilities. And as several competencies are part of different domain competencies, learning sequences can be modified with regard to the preconditions of students enabling a more sophisticated generation of personal learning paths.

Acknowledgement

The authors gratefully acknowledge the internal funding by Hamburg University of Applied Sciences, Germany, for establishing the online-platform ELFE™ in the framework of HAW-Future Fund.

References

Anderson, L.W. and Krathwohl, D.R. and Bloom, B.S. (2001) *A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives*. New York: Longman, ISBN 9780321084057.

Bernitzke, F. H. (1987) Mastery-Learning-Strategie als Unterrichtsalternative- Empirische Studie zur Effektivität der Mastery-Learning-Strategie und zu Interdependenzen mit Schülermerkmalen, *Europäische Hochschulschriften (Reihe 11): Pädagogik / Education / Pédagogie 305*

Landenfeld, K., Göbbels, M., Hintze, A. (2016) "A Customized Learning Environment and Individual Learning in Mathematical Preparation Courses." 13th International Congress on Mathematical Education(ICME), Hamburg, 24-31 July 2016, ICME-13 Monograph *Distance Learning, E-Learning and Blended Learning in Mathematics Education - International Trends in Research and Development,* Editors: J. Silverman, V. Hoyos, Springer Verlag, ISBN 978-3-319-90789-5

Linke, M., Landenfeld, K. (2018) "Kompetenzorientierte Lehre in der Technischen Mechanik - softwaretechnisch umgesetzt." In: *Begleitband zur 7. Konferenz Inverted Classroom and Beyond 2018*, Josef Buchner, Christian F. Freisleben-Teutscher, J. Haag, E. Rauscher (Hrsg.), FH St. Pölten, 20./21. Februar 2018, ISBN: 978-3-99023-489-1

Mathematical Competencies and Credentials in a Practice-Based Engineering Degree

Emily Cook

Engineering Practice Academy, Swinburne University of Technology, Australia

Abstract

In the context of a new engineering practice-based degree, in which students study as if working and undertake industry-set projects throughout their degrees with no traditional lectures or classes, it was necessary to find a novel way to support their learning of fundamental knowledge such as mathematics. This required a just-in time learning approach with topics taught as required by specific projects. The solution was a series of 'credentials' which are small, online modules. There is a synergistic relationship between credentials and projects, with credentials providing the knowledge required for the project, and the project providing an opportunity for students to gather evidence of their ability to apply the mathematical concepts in practice.

Credentials are not graded but are either marked as achieved or not yet achieved. Assessment is based on the evidence students collate from applying the content in their project work. Students must be assessed as proficient or advanced in all of the credential assessment criteria to achieve the credential. These criteria include aspects of all mathematical competencies such as mathematical communication and using software as well as more traditionally assessed competencies.

Introduction

Practice-based learning is an educational model which bridges the gap between traditional teaching and learning, in which theoretical knowledge is acquired first and applied in practice later, and work-integrated learning in which the work itself is the learning. In practice-based learning the work and the learning occur simultaneously (for details of this educational philosophy see Mann et al. (2018)).

Practice-based learning here is applied in a new Engineering Practice degree which was Industry co-designed (Cook, 2017) in response to the ongoing need to produce graduates with work-ready skills (Beanland & Hadgraft, 2014). This degree has industry-set projects throughout the course, with no lectures or exams. Students, referred to as "Associates" as they are treated like employees, work in teams on four projects per year for the entire four years of their degree. The projects are six weeks in duration and, being set by industry, change each year in response to industry needs. In the first year projects are proscribed and in later years Associates can select projects based on their interests and chosen specialisms. In this context where the course content, in terms list of topics studied, is dependent on the particular projects, it was necessary to find a novel way to ensure that all the fundamental knowledge is covered, in particular the core maths and physics. The solution to this is a series of small, online modules called 'credentials'.

The system of credentials supports the development of all fundamental knowledge across the curriculum, both disciplinary and skills-based, and form an interrelated web of topics. The credentials are divided into domains (self, work, thinking, process and disciplinary) with each domain subdivided into capabilities. The credentials that are prerequisites for other credentials have been mapped and are prescribed across the curriculum to provide a structured pathway for Associates.

Mathematics falls under the 'discipline' domain, but also links to process and work domains (e.g. applying errors and uncertainty links to research processes and using financial maths connects with the budgeting and planning aspects of work). Mathematics credentials are also pre-requisite credentials for many physics and applied engineering credentials that sit within the discipline domain.

Rationale for using Credentials

The framework for credentials in this context is developed and adapted from the concepts of micro-credentials and digital badges. Micro-credentials were initially developed as a means of providing and recognising professional development and have been implemented in higher education to broaden the range of skills that can be acknowledged beyond the information provided in a standard university transcript (Bowen & Thomas, 2014). The digital nature of these badges also allows them to be displayed online and the badges can be linked to the learning outcomes of the micro-credential and potentially also the evidence of the student has provided to demonstrate meeting those outcomes (Casilli & Hickey, 2016). The affordances of digital badges include motivation, recognition and evidence of achievement and provide flexibility about the skills that can be recognised and legitimized (Gibson, Ostashewski, Flintoff, Grant, & Knight, 2015).

Micro-credentials have been suggested for use in tertiary engineering courses to "provide students and employers with better information, support the mixing and matching of courses, give tertiary education organisations (TEOs) more flexibility and encourage innovation" (Mischewski, 2017). Educational badges have also been found to have a positive effect on motivation in some learner groups depending on performance level and the nature of the content of the badge (Abramovich, Schunn, & Higashi, 2013).

The term credential has been adopted in this degree as both the terms 'micro-credential' and 'digital badge' are already used in myriad ways but the key affordances of these items are fundamentally the same.

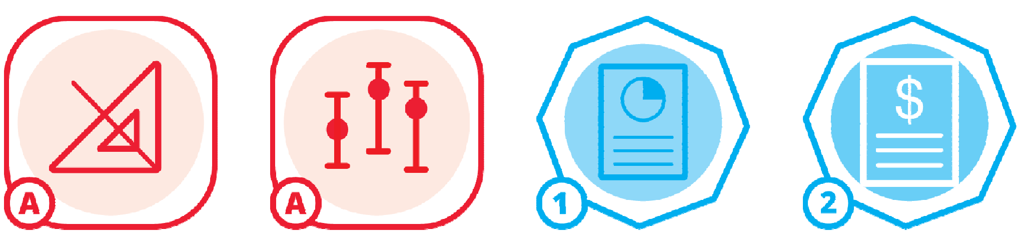


Figure 1. Icons for some digital badges for mathematics credentials in the Bachelor of Engineering Practice (from left to right: Trigonometry, Errors, Technical Communication

and Budgeting & Finance. Red badges indicate the discipline domain, blue the work domain).

Credential Structure

Credentials represent the smallest divisions of the curriculum and span all curriculum domains. Each credential represents approximately six hours work for an Associate. There is a synergistic relationship between credentials and projects, with credentials providing the disciplinary knowledge required for the project, and the project providing an opportunity for Associates to gather evidence of their ability to apply the concepts in practice, which conversely is used to award the credential.

The purpose of credentials is to allow just-in-time learning and flexible pathways through the curriculum. This is needed to support projects and Associate specialisms, interests and abilities. The credentials are online modules with content and assessment tasks housed on the learning management systems (Canvas). Some credentials also require face-to-face workshops (called professional development sessions to reinforce the learning-integrated work concept) and drop-in sessions are timetabled with tutors available to provide in person support with credential content. Each credential consists of three learning tasks, and an application task from which evidence is submitted.

Learning Tasks

The learning tasks contain the content of the credential. These can comprise readings, videos, worked examples, self-test questions, attendance at a face-to-face professional development workshop sessions, workbooks and other relevant activities. Application tasks contain the description of how the Associate can evidence their ability to apply the content in practice. The key point of difference in the assessment of an application task compared to a traditional unit is that the evidence provided must be of the content being applied. This makes a credential not merely a 'small unit', but a fundamentally different way of assessing knowledge. This addresses the mismatch between modes of teaching and assessment described by Niss (2003) as a matter of "designing and adopting assessment instruments that are capable of telling us what we really want to know about students' knowledge, insights, and skills in, with and about mathematics." With this aim, application tasks focus not on what Associates know, but what they can do.

Application Tasks

There are three types of application tasks to demonstrate learning. The first is simply a list of criteria, linked to the learning outcomes of the credential, that are to be demonstrate in a project or related task. The second is a more structured project or investigation. This option can be selected if the Associate does not have the opportunity in a current project to collect the required evidence. However, the task is still applied in a practical context and not merely a series of questions. For example the "Exponentials & Logarithms" investigation option requires Associates to set up a circuit containing a resistor and a capacitor (either physically or using an online simulator). They plot graphs of the voltage when the capacitor is charging and discharging and both measure and calculate the time constant of the circuit for different values of resistance and capacitance.

An application task contains a description of the type and format of the evidence required along with a detailed rubric outlining each criterion to be demonstrated. The rubric has four levels for each criterion: no evidence, developing, proficient and advanced. Credentials are not graded but are marked as either achieved or not yet achieved. Associates must provide evidence which is rated as proficient or advanced in all of the assessment criteria to achieve the credential. Associates can submit their evidence as many times as they need to achieve the credential but must respond to feedback comments in order to resubmit.

Credentials and Mathematical Competencies

Cardella (2008) states that beyond the mathematical content knowledge required for engineering practice, it is necessary to consider all the aspects of mathematical thinking identified by Schoenfeld (1992): problem-solving strategies, resources and use of resources, beliefs and affects. The strong emphasis on team work throughout the course emphasises the use of social resources as suggested by Cardella (2008) as Associates, used to working in project teams from day one of their studies, use the same collegial approach to tackling mathematical problems and concepts. The notion of mathematical competencies as further developed by the Danish KOM project (Niss (2003); Niss and Højgaard (2011)) presents a broad and comprehensive view of mathematical competencies, describing eight competencies - thinking mathematically, posing and solving mathematical problems, modelling mathematically, reasoning mathematically, representing mathematical entities, handling mathematical symbols and formalisms, communicating in, with and about mathematics and making use of aids and tools.

By defining criteria and using rubrics as they are described in this paper, it is possible to move away from assessing recall and mathematical knowledge and towards assessing mathematical competencies. For example there is a strong emphasis on communicating mathematically, with all credentials containing criteria around using correct notation, annotating workings and visually presenting data. The use of aids and tools is also emphasised throughout many credentials with Associates encouraged to use graphical calculators and software packages to perform calculations, analyse and display data. There are also explicit credentials in software aspects such as using spreadsheets and MATLAB. The criteria are not weighted, so a criterion such as ‘correct numerical answers’ must be met just as the criterion ‘uses appropriate software to plot a graph’ is met. The practice-based nature of the course and the requirement that application tasks be connected to project work wherever possible also provides a heavy emphasis on mathematical modelling as Associates need to seek examples in their project work of situations to model.

Within this curriculum model, in which all knowledge and skills are linked to a credential, it is also possible to specify maths competencies in other credentials across the curriculum. An obvious example of this is physics where criteria can include for example a “communicate reasoning clearly” criterion in a credential on electromagnetism, along with “correct numerical answer” and “Correct use of SI units for physical quantities”. Less obvious examples of maths competencies can be found in work related credentials, for example in communication, criteria are specified relating to interpreting and correctly displaying data and in project management to finance and budgeting.

Reflections

The modularity of credentials means there is a complex web of prerequisites, as mathematical knowledge and skills require building from foundations upwards and complex topics cannot be taught before basic ones have been mastered. It is also worth noting that as all criteria have to be rated proficient, it is not possible to achieve the credential if even one aspect is not sufficiently evidenced. Therefore a missing unit or expressing an answer to an inappropriate number of significant figures can mean the credential is not yet achieved and must be resubmitted if those items are listed in the criteria.

Some of the challenges around using this credential model for teaching mathematics in an engineering degree is around the design of application tasks. Students are used to mathematical assessment being exam or question-based. Developing applied, authentic, project-linked assessment tasks is both time consuming for the developer and initially challenging for the student. However, with the increasing recognition of the importance of authentic assessment, applied tasks as a form of assessment will are becoming increasingly common in mathematics education. The design of the rubrics used for assessing the evidence is also crucial both for communicating criteria to students and guiding assessors given a variety of evidence can be submitted from different projects and marking multiple resubmissions can be time-consuming without a suitable guide.

Overall, the benefits of this credential system are the ability to assess all competencies and describe criteria in all areas that must be met, the flexibility to incorporate mathematical competencies in the broader curriculum and the constant connections of mathematics to industry-designed project work, emphasising the relevance of the mathematical content and developing practical modelling and software skills.

References

Abramovich, S., Schunn, C., & Higashi, R. M. (2013). Are badges useful in education?: it depends upon the type of badge and expertise of learner. *Educational Technology Research and Development, 61*(2), 217-232. doi:10.1007/s11423-013-9289-2

Beanland, D., & Hadgraft, R. (2014). Engineering education: Innovation and transformation. *commissioned by UNESCO*.

Bowen, K., & Thomas, A. (2014). Badges: A Common Currency for Learning. *Change: The Magazine of Higher Learning, 46*(1), 21-25. doi:10.1080/00091383.2014.867206

Canvas. (2018). Retrieved from https://www.canvaslms.com/

Cardella, M. E. (2008). Which mathematics should we teach engineering students? An empirically grounded case for a broad notion of mathematical thinking. *Teaching Mathematics and its Applications: An International Journal of the IMA, 27*(3), 150-159. doi:10.1093/teamat/hrn008

Casilli, C., & Hickey, D. (2016). Transcending conventional credentialing and assessment paradigms with information-rich digital badges. *The Information Society, 32*(2), 117-129. doi:10.1080/01972243.2016.1130500

Cook, E. J., Mann, L. M. W. , Daniel, S. A. . (2017). Co-designing a new engineering curriculum with industry *45th SEFI annual conference*.

Gibson, D., Ostashewski, N., Flintoff, K., Grant, S., & Knight, E. (2015). Digital badges in education. *Education and Information Technologies, 20*(2), 403-410. doi:10.1007/s10639-013-9291-7

Mann, L., Chang, R., Chandrasekaran, S., Coddington, A., Daniel, S., Cook, E., . . . Cosson, B. (2018). From Problem-Based Learning to Practice-Based Education: A Framework for Shaping Future Engineers. *European Journal of Engineering Education, In press*.

Mischewski, B. (2017). Micro-credentials: A model for engineering education. New Zealand: Report comissioned by the Tertiary Education Commission (TEC).

Niss, M. (2003). *Mathematical competencies and the learning of mathematics: The Danish KOM project.* Paper presented at the 3rd Mediterranean Conference on Mathematics Education, Athens, Greece.

Niss, M., & Højgaard, T. (2011). *Competencies and mathematical learning: ideas and inspiration for the development of mathematics teaching and learning in Denmark, Ministry of Education*. Retrieved from

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. *Handbook of research on mathematics teaching and learning, 334370*.

Sense and Nonsense of Teaching Math for Engineers

Reinhard Kahle

CMA & DM, FCT, Universidade Nova de Lisboa, Portugal

Abstract

Based on experiences with teaching Calculus II for engineering students at the Faculdade de Ciência e Tecnologia of the Universidade Nova de Lisboa we identify one particularly worrying problem with this teaching: the existence of fundamental errors which, normally, should result in an immediate failure of the student. However, these errors are not about the material lectured in this course, but from Calculus I or even from high school mathematics. The seriousness of the problem results from the fact that these errors compromise the entire knowledge acquired in Calculus II.

Math problems of students

Complains about deficits of students in Mathematics, let it be in Engineering or in other areas, are frequent; and not new - see, for instance, the letter of Eduard Study (1862-1930) to Rudolf Lipschitz (1832-1903) from August 20th, 1898 where he complains about his students in Greifswald, including one believing that $-0=\infty$, Lipschitz (1986).

A particular mistake, I encountered while marking an exam of Calculus II, was that a student was able to solve correctly an exercise concerning the area of a surface in R^3 as long as the numerical calculations were used. The corresponding drawing, however, showed something in R^2. This is bad, of course, as it shows that the student had absolutely no idea what (s)he was calculating. But is was a single case, even if we fear that this type of error is more frequent than one imagines. In fact, there is another example from a statistics course I saw which showed a similar fundamental lack of understanding. The students had to answer the following question: “A space shuttle captures a satellite in one attempt with 90% probability. How high is the probability that the satellite is captured after five attempts?” Receiving an answer like 45% shows that the student has absolutely no idea what (s)he is calculating here.

In the following we will report on another form of error which was, somehow, unexpected and, much worse, cannot be considered as a single case.

The exercise

“Consider a parametrized curve defined by the vector function $\vec{r}: [1,2] \to R^3, \vec{r}(t) = \left(t, \log t, 2\sqrt{2t}\right)$.

(a) Verify that $\|\vec{r}'(t)\| = \frac{t+1}{t}, \forall t \in [1,2]$.

(b) Calculate the length of the curve.”

The errors

In (a), many students didn’t manage to derive $\log(t)$ or $2\sqrt{2t}$. This is not good, but maybe not particularly worrying. The problem turned much more serious when a significant number of students tried desparetely to correct the errors from the differentiation, concluding that

$$\sqrt{1+t^2} = 1+t..$$

And the problem repeated itself for (b). Many students did know – to our surprise – correctly the formula for the length of curve:

$$L = \int_1^2 \|\vec{r}\,'(t)\|\, dt = \int_1^2 \left(\frac{t+1}{t}\right) dt.$$

But they didn't know to integrate $\frac{t+1}{t}$, up to solutions of the form$\int_1^2 \left(\frac{t+1}{t}\right) dt = \left[\frac{t+1}{t}\right]_2^2$.

The problem

The mentioned errors were made by the majority of the students, thus they are not related with individual deficits. After a short reflection one realizes that the errors don't concern the material of Calculus II, but that of Calculus I. In fact, the part of Calculus II, the formula for the length of curve, was known by most of the students.

The worrying question is now whether a student who is doing such mistakes should pass the exam or not. In a first approximation it appears to be clear that a student, calculating $\sqrt{1+t^2} = 1+t$ should not pass. The student, however, could argue that (s)he knows the material of Calculus II, and material of Calculus I is not supposed to be examined ("again" - in fact, to enter Calculus II it is required that the student already passed the Calculus I exam). As much as one is inclined to dismiss such an argument, as it would be absurd to let a student pass who is doing such mistakes, there is a fundamental obstacle: even forcing the student to *repeat* Calculus **II** is not particularly meaningful. The material of Calculus II (s)he does know! This knowledge is just useless as (s)he is unable to use it meaningfully having forgotten "everything" of Calculus I.

The situation

The given example is only one example; I could provide others. As for the "calculation" of $\sqrt{1+t^2}$, there is a significant numbers of errors which show that knowledge of Linear Algebra (a course which the students also already passed) or even high school mathematics is completely lacking. Thus, not even sending back the students to Calculus I – an idea which would turn out unfeasible in any case – would solve the problem. The students have simply no concept of the cumulative nature of mathematical knowledge. Apparently, the implementation of the Bologna process at European universities vindicate the students when it is required that every lecture is examined and graded separately, causing the impression that after passing an exam, it is possible to simply shelve – or even unlearn – the "acquired" knowledge. The gravity of the situation can be illustrated of an absurdity experienced by a colleague at a technical university in Germany (showing also that the described problem is, by now means, a local or Portuguese problem, but rather a universal one): an exam for a higher number of students had to be distributed over three lecture halls; in one of these halls the fire alarm started during the examination. After consulting the legal department, it was possible to repeat the exam two weeks later only for those students which were in this hall. Of course, the other students complained that, thus, these students have two more weeks to learn; but these students also complained, as they "would have to memorize the material of the lecture two weeks longer than their colleagues"!

Solutions?

It is not easy to see what one can do about this situation, if not either sending back students to repeat more elementary courses (an unfeasible idea, unfortunately) or really letting them fail "en masse", despite the fact that they were able to learn the material which was the topic of the course in a narrow sense.

The "conservative solution", adopted by my Faculty, is to let them repeat Calculus II as often as they finally pass the exam. This solution is unsatisfactory, in particular, because it gives the impression to the student that, at the end, the part concerning calculations (in the given example, the derivation of $\log(t)$ or $2\sqrt{2t}$.) is "more important" than the theoretical results (here, the formula for the length of curve). In consequence, they focus nearly entirely on calculating examples rather than understanding of the material proper.

Taking also the bad experiences with other problems, as the total lack of understanding in the mentioned R^2 drawing for a R^3 problem, into account, we like to put here a radical change up for discussion in the setup of Math courses for Engineers.

Calculations

How do you calculate the limit of a sequence? There are several, quite elaborated techniques which Mathematicians developed over the centuries, some of them not easy to perform. As a matter of fact, I noticed that for practically all examples – given by colleagues for courses where I was just teaching exercise classes – Google led me with one click to the solution. Without having ever done a real statistical analysis, I would assume that probably more than 90% of the calculations required to solve exercises at University can be solved almost immediately, and without any particular programming knowledge, by computer help. Of course, it might be a different case in "real world situations", where the involved functions may significantly differ from the cases discussed in a general Calculus II course. If so, it is advisable that a company hands over such problems to trained mathematicians in any case; if not, companies will definitely make use of the computer help instead of expecting their employed engineers to solve math problems by hand.

Let us give another, somehow positive, example here from a course on Discrete Mathematics. Essentially all students were knowing the Euclidean algorithm to compute the greatest common divisor; as the exam was without pocket calculator, however, half of the students made, at least, one calculation error when they had to calculate the gcd for two, let say, 3 digit numbers. I don't consider this worrying in any way, as – like for the formula of the length of curve – the primarily objective of the lecture was to teach the Euclidean algorithm. That many students cannot perform any longer a series of elementary calculations by hand, does not corrupt the knowledge of the algorithm, as in any reasonable situation where they would have to use the algorithm, they could use a pocket calculator.

“Math light”

In view of the preceding considerations we would propose a form of teaching and examining Math which “abstracts” from the calculation part, a part which could reasonably handed over to computers – or being looked up in formulary. As a matter of fact, most of the formulas needed by an engineer can be looked up in Bronshtein and Semendyayev (1964) and, if not, they could be found today in the internet. We consider it has much more important to teach the students the *meaningful* use of these formulas than the memorization. Of course, both would be better – but it is just our experience that this ideal appears to be unrealistic. Accepting this reality, let’s try to build it in: all kind of computations which can be handed over to computers – and which are *not* part of the proper material of the course – should not be part of the evaluation. In the example under discussion, this means that the student may have access to the derivatives of $\log(t)$ and $2\sqrt{2t}$, etc. (but, in Calculus I, where these derivatives are part of the objective, of course, they would still have to calculate them by themselves).

In some exams this is already established by allowing the students to use formularies with this kind of information. We think, however, of going a step further: the objective of the exam should not just whether they can simply *use* the formulas to obtain a correct numerical result, the objective should be to test whether they are able to *find* the relevant formula (not in their mind, but somewhere else) and, more importantly, whether they are able to *understand* (the use of) such formulas. Thus, a shift has to be made away from numerical calculations towards *interpretations,* avoiding such absurdities as believing an R^3 calculation could take place in the plane.

I call this “Math light” as, in consequence, students might – intentionally – not be able any longer to perform calculations by themselves, but just with help of computers (and/or formularies). In fact, the situation is not quite different from the discussion of use of pocket calculators in elementary and/or high school, where this is problem is largely lamented. Still, I would definitely prefer students which are able to understand and interpret calculations made with computer aid over those which could – by the skin of one’s teeth – perform such a calculation but would fail to interpret the result. In fact, the latter ones will probably also not be able to make meaningful use of computer aid even if it would be to their disposal.

Recalling the error from the statistics exam, one has to admit that the meaningless results may occur easier if one uses computers (or pocket calculators), not only by invoking a wrong formula but even just by mistyping a number. Thus, it is even more important that the *plausibility* of a obtained computer result has to be clear immediately. The lack of being able to realize that a numerical result is implausible – if not senseless – in a given context, cannot be tolerated at any rate.

Final remarks

Here it is not the place to propose any practical scheme to implement the idea of “Math light” (which, for sure, will not be easy at all); my objective is just to point to the need of such a shift in view of the unsustainable situation which I encountered when I found myself in the Calculus II lectures.

I never put forward the idea of “Math light” in our Faculty, as it would require an administrative structure is open to considering implementation of changes. I’m also not aware of tests in this direction at other universities – which most likely exist. In any case, a “test run” of such a fundamental change would require, in my view, the involvement of companies which have to employ the students at the end of the day – and I might be quite wrong in the impression what these companies need and expect from our students. But, on my experience, I have doubts that they could be happy with the Calculus II skills of the average student as (s)he leaves our Faculty at the moment.

When complaining about the performance of my students, I should add a proviso here: we have, of course, also good students which have no problem in solving properly the exercises of the exams. When I referred to a majority of students which have problems to do so, there will be a statistical effect which may influence the perception: good students pass only once, while bad students return again and again to the same course and are, therefore, counted much more often than the good ones. In Portugal, there is no limit in attempts to pass a lecture – at least, no such limit is enforced at my Faculty – and the extreme case I encountered was a student being subscribed for the 24^{th} time in Calculus II (but it was not an outlier, the next one was subscribed for the 22^{nd} time). These repeaters overshadow the good students by far. But even without going for the numbers, the question is what one should think of students which pass the Calculus II exam, still doing the described errors but collecting just enough points in other questions, and eventually receives a Engineering Diploma?

To avoid any risk of misunderstanding: I would never ever suggest “Math light” for the training of Mathematicians proper. It is just one of the task of a Mathematician to be constantly aware of the technicalities which are behind computational tools – let it be a pocket calculator or a supercomputer.

References

Bronshtein, I.N. and Semendyayev, K.A. (1964). *A Guide-Book to Mathematics for Technologists and Engineers.* Pergamon Press and Macmillan.

Lipschitz, R. (1986). *Briefwechsel mit Cantor, Dedekind, Helmholtz, Kronecker, Weierstrass und anderen.* Vieweg.

Acknowledgements

Research supported by the Portuguese Science Foundation, FCT, through the project UID/MAT/00297/2013 (*Centro de Matemática e Aplicações*) and the project *Hilbert’s 24th Problem* (PTDC/MHC-FIL/2583/2014).

A New Tool for the Assessment of the Development of Students' Mathematical Competencies

Yannis Liakos[1], Svitlana Rogovchenko[2], and Yuriy Rogovchenko[1]

[1]*Department of Mathematical Sciences, University of Agder, Norway*
[2]*Department of Engineering Sciences, University of Agder, Norway*

Abstract

We look at the assessment of mathematical competencies of undergraduate biology students. Mathematical modelling tasks with biological content were introduced to engage students more actively into learning mathematics. Profiles for individual learners were created using five basic families of mathematical competencies. Sixteen mathematical competencies in five families were coded in transcripts of video recordings and students' writings in a reliable unambiguous manner; competencies frequency and intensity were recorded. These data were analysed with a new assessment tool suggested by the authors to monitor students' competencies development.

The construct: mathematical competence

In broad sense, competence is a theoretical construct defined as a "complex ability (…) that (…) [is] closely related to performance in real-life situations" (Hartig, Klieme & Leutner, 2008). Seven facets of competence and its assessment are identified: complexity, performance, standardization, fidelity, level, improvement, and disposition (Shavelson, 2013). Competence cannot be observed directly, but can be inferred from individual's performance on sample tasks; it can be improved through learning and deteriorates through forgetting (Shavelson, 2013). Remarkably, "competence is not the same as academic knowledge and (…) academic competence is not the same as professional competence" (Oser, 2013). We discuss only cognitive (knowledge and skills) component of competence; metacognitive and non-cognitive components are out of the scope of this paper.

Mathematical competence means "the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role. (…) A mathematical competency is a clearly recognizable and distinct, major constituent of mathematical competence" (Niss, 2003). Three mathematical competencies frameworks are often used in the research literature: "Principles and standards for school mathematics" (NCTM, 2000), Danish KOM-project (Niss, 2003), and Mathematical Competencies: A Research Framework (MCFR) (Lithner et al., 2010). One more framework focuses on mathematical giftedness and uses nine component mathematical abilities whose combination can lead to high achievement in mathematics (Krutetskii, 1976).

Research Setting and Data Collection

This research is aimed at increasing biology students' motivation for mathematics using mathematical modelling activities. The first author prepared teaching materials and conducted teaching once a week during one semester complementing regular lectures and seminars in a standard mathematics course MAT101 for the first year students in natural sciences. The topics discussed in the complementary sessions: periodic functions (2 sessions), exponential growth and regression (2 sessions), population dynamics (2 sessions), integrals and modeling (2 sessions). Twelve out of about a hundred biology students enrolled in the course participated in additional sessions providing a purposeful random heterogeneous sampling which mirrors characteristic features of the larger sample to "add credibility to the results of a larger study" (Teddie & Tashakkori, 2009).

An important feature of our approach is the use of "embedded assessment" where "opportunities to assess student progress and performance are integrated into the instructional materials and are virtually indistinguishable from day-to-day instructional activities" (Wilson & Draney, 2013). Becoming an integral part of the teaching and learning process, embedded assessment can be viewed as "assessment for learning" (Black, Harrison, Lee, Marshall, & William, 2003). Using mathematical problems with biological content, we assure that the assessment is curriculum dependent for its full and meaningful embedding into teaching and learning (Wolf & Reardon, 1996).

Data include video recordings of participants, researcher's observation/field notes and students' written material obtained using *Livescribe 3* smart pens and notebooks. One camera was recording the "focus group" and a *GoPro* camera with a panoramic view was used to record all interactions in the classroom. Sessions were taped with minimal disturbance to students so that "the effect of video becomes negligible in most situations after a certain phase of habituation" (Knoblauch, Schnettler & Raab, 2006).

Mathematical Competencies Framework and Scaling

In 2016, the first author conducted a "pilot study" to test the functionality of the KOM mathematical competencies framework (Niss, 2003). Not surprisingly, in several episodes competencies significantly overlapped because "the competencies are closely related - they form a continuum of overlapping clusters - yet they are distinct in the sense that their centres of gravity are clearly delineated and disjoint" (Niss, 2003). To minimise possible complications with coding, we retained only five basic groups of mathematical competencies out of the eight suggested in KOM. All sixteen competencies in five groups are described below. *Thinking/acting mathematically:* pose questions that are characteristic of mathematics; understand and handle the scope and limitations of a given concept; attack mathematical problems. *Mathematical modelling:* assess the range and validity of existing models; interpret and translate elements of a model during the mapping process; interpret mathematical results in an extra-mathematical context and generalize solutions developed for a special task or situation; criticize the model by reviewing, reflecting and questioning results; search for available information differentiating between relevant and irrelevant information; choose appropriate mathematical notation. *Representing and manipulating symbolic forms:* choose a representation; switch between representations; manipulate within a representation. *Reasoning and communicating:* understand others' written, visual or oral information with mathematical content; follow and assess chains of arguments put forward by others; express oneself in oral, visual or written form in mathematical context; provide explanations or justifications to support own results and ideas. *Aids and tools:* know different tools and aids for mathematical activity and their properties; use appropriate aids and tools to develop insight or intuition.

Data analysis of three sessions suggested that not only the frequency of activation of a competency should be recorded but also its intensity at each activation instance. Furthermore, for each competency, three perspectives are considered: task solving vision (T.S.V.), use of mathematical language/vocabulary (M.L.V.), and independent thinking (Ind.T.). The first perspective relates activation of a competence with the depth of student's understanding of the steps towards solution. The second puts in the spotlight the use of appropriate mathematical language needed to activate competencies in written and oral communication since insufficiently developed mathematical language can be the reason for the overall deceleration of mathematics learning (van der Walt, Maree, & Ellis, 2008). The third perspective monitors student's independent thinking measured by the extent of instructor's prompting. Bernstein (1967) emphasised the importance of reducing alternative actions during skill acquisition; less scaffolding means more stimulation for the independent work and a higher competency intensity.

We rate competencies intensity by the evidence of understanding of mathematical content: C1 - little or no evidence, C2 - occasional, B1- limited, B2 - basic, B3 - substantial, A1 - full, A2 - in-depth, A+ - exceptional. The hierarchy of qualitatively distinct levels of performance with a clear description of students' abilities/skills is needed for the construct validity of our assessment tool (Kane, 2001). Our scaling relates two facets of mathematical competence: "performance – a capacity not just to "know" but also to be able to perform" and "level – the performance must be at a "good enough" level to show competence" Shavelson (2013). The evidence for cognitive validity is achieved through the analysis of the "logical link between the interpretive claim and the nature of the assessment – the characteristics of the task, response demands, and scoring system" (Ruiz-Primo, Shavelson, Li, & Schultz, 2001).

Assessment Tool

To design an assessment tool for students' mathematical competencies development within and across semester cohort, we adopt operationalist view defining measurement as "a procedure for the assignment of numbers to specified properties of experimental units in such a way as to characterize and preserve specified relationships in the behavioural domain" (Lord & Novick, 1968). Transcripts and students writings were coded and rated each time the competency was activated resulting in a large number of sets of heterogeneous data for each competencies frequency and intensity, for all students and sessions. We converted data into quantitative (the value 1 is assigned to C1, 2 to C2, 3 to B1, etc.) to spot the trends in competencies development. The intensity of one competency, reasoning, is presented in Figure 1 for two students, M and J. We see that the competency has been activated for M 26 times in all seven sessions and its intensity never dropped below the green "developing" competency intensity strip and often jumped above it, even to the highest level 8 (A+). For J, the reasoning competency has been activated 23 times in all sessions but the last one, the intensity was dropping below the green strip but sometimes jumped up, at most to the level 7 (A).

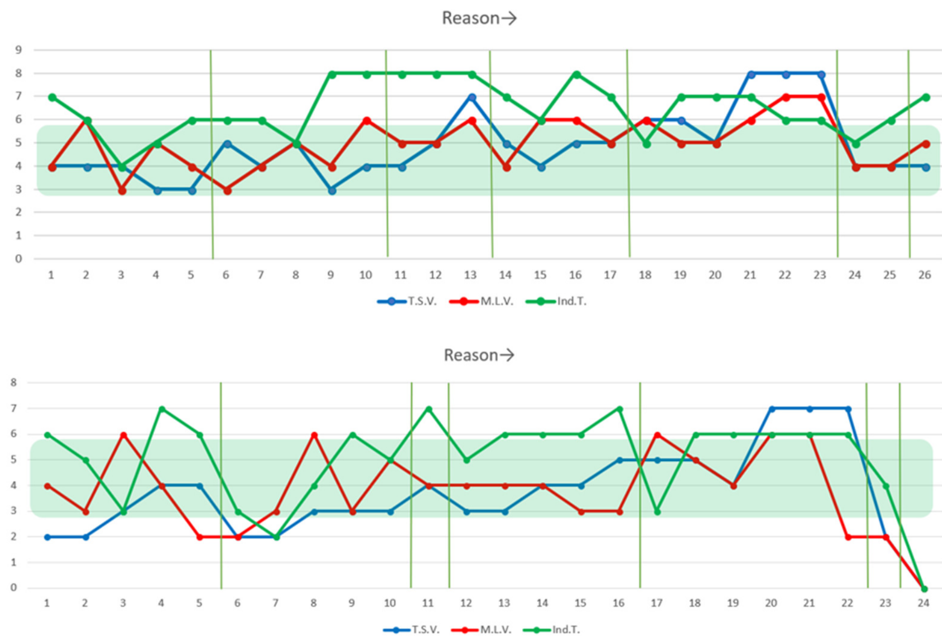


Figure 1 Development of the reasoning competency for students M (upper diagram) and J

To classify the learning progress, we introduce four key indicators: *total progress indicator* (TPI), the difference between the final and initial intensity values, *winding number* (WN), the total number of intensity value changes (slope changes) between two successive instants, *intensity spans 1 and 2* (IS1 and IS2), the difference between the highest (respectively, lowest) and the final intensity values. Using key indicators, we define five learning types: progressive, persistent, unsteady, alternating and transient in Table 1.

Learning type	TPI value	WN value	IS1 value	IS2 value
Progressive (Pr)	Large	Any	Small	Large
Persistent (Pe)	Small	Any	Small	Small
Unsteady (U)	Small	Low	Small	Large
Alternating (A)	Small	Any	Large	Large
Transient (T)	Small	Low	Large	Small

Table 1 Classification of learning types

Not surprisingly, learning type changes for teaching blocks with different topics (separated in Figure 1 with vertical lines), see Table 2. Classification of learning types allows to compare the individual competency development through the topic/semester or compare learning types of two students. For example, we observe from Table 2 that M performs better with mostly persistent or progressive type of learning whereas J's learning type is mainly unsteady and is not even classified in the last block.

TPI, WN, IS1, IS2 and learning type for student M in four teaching blocks (1, 2&3,				
Ind. T.	-1/2/1/-2 Pe	2/2/0/-3 U	3/6/1/-1 Pe	1/0/0/-2 Pr
T.S.V.	-1/1/1/0 Pe	2/4/0/-4 Pr	¾/0/-3 Pr	0/0/0/0 Pe
M.L.V.	0/4/2/-1 Pe	3/5/0/-3 Pr	3/5/0/-2 Pr	1/1/1/-1 Pr
TPI, WN, IS1, IS2 and learning type for student J in four teaching blocks (1, 2&3, 4&5,				
Ind. T.	0/3/1/-3 U	4/3/1/-5 Pr	2/0/0/-2 U	-4/1/4/0 n/c
T.S.V.	2/1/0/-2 Pr	1/0/0/-1 Pe	2/0/0/-2 U	-2/0/2/0 n/c
M.L.V.	-2/3/4/0 T	¾/1/-3 U	-1/2/1/0 Pe	-2/1/2/0 n/c

Table 2 Learning types for M and J

Changes in the learning type between teaching blocks indicate that activation of a competency in a new learning environment may follow a different pattern; tasks should be carefully designed to avoid dramatic drops in competencies intensity up to a complete failure to activate a competency.

Conclusions and Discussion

Undoubtedly, our assessment tool has certain limitations since “it is never a question of whether the models are true; it is a question of whether they provide adequate approximations of performance, which fit our current understanding of learning in the domain and prove useful for their intended purposes” (Mislevy, 2016). Good performance tasks are not easy to design, they should be rated by experienced assessors, and the scores should be correctly interpreted to make competence claims that are then used to make decisions. We suggested the tool and a novel classification of learning types and look for the feedback. We believe that our tool allows to follow students’ competencies development fairly well and hope to improve it through further testing.

References

Bernstein, N. (1967). The coordination and regulation of movement. Pergamon Press, Oxford.

Black, P., Harrison, C., Lee, C., Marshall, B., & William, D. (2003). Assessment for learning. London: Open University Press.

Hartig, J., Klieme, E., & Leutner, D., eds. (2008). Assessment of competencies in educational contexts: State of the art and future prospects. Göttingen: Hogrefe & Huber.

Kane, M.T. (2001). Current concerns in validity theory. Journal of Educational Measurement. 38(2): 319-342.

Knoblauch, H., Schnettler, B., & Raab, J. (2006). Video-Analysis. Methodological Aspects of Interpretive Audiovisual Analysis in Social Research. In H., Knoblauch, J.R., Schnettler, & H., Soeffner, eds. Video analysis methodology and methods: Qualitative Audiovisual Data Analysis in Sociology (pp.9-26). Frankfurt: Peter Lang.

Krutetskii, V.A. (1976). The psychology of mathematical abilities in schoolchildren. Chicago, IL: The University of Chicago Press.

Lithner, J., Bergqvist, E., Bergqvist, T., Boesen, J., Palm, T., & Palmberg, B. (2010). Mathematical competencies: A research framework. In C. Bergsten, E. Jablonka & T.

Wedege, eds. Mathematics and mathematics education: Cultural and social dimensions. Proceedings of MADIF 7, the seventh mathematics education research seminar, Stockholm, January 26-27, 2010 (pp. 157-167). Stockholm: SMDF.

Lord, F.M., & Novick, M.R. (1968). Statistical theories of mental test scores. Reading, MA: Addison-Wesley.

Mislevy, R.J. (2016). How developments in psychology and technology challenge validity argumentation. Journal of Educational Measurement, 53(3): 265–292.

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagatsis, & S. Papastavridis, eds. 3rd Mediterranean Conference on Mathematical Education (pp. 115-124). Athens, Greece: Hellenic Mathematical Society and Cyprus Mathematical Society.

NTCM. (2000). Principles and standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Oser, F. (2013) "I know how to do it, but I can't do it": Modeling competence profiles for future teachers and trainers. In S. Blömeke, O. Zlatkin-Troitschanskaia, C. Kuhn, & J. Fege, eds. Modeling and measuring competencies in higher education: Tasks and challenges (pp. 45-60), Rotterdam: Sense Publishers.

Shavelson, R.J. (2013). An approach to testing and modelling competence. Educational Psychologist 48(2): 73-86.

Ruiz-Primo, M.A., Shavelson, R.J., Min, L., & Schultz, S.E. (2001). On the validity of cognitive interpretations of scores from alternative concept-mapping techniques. Educational Assessment 7(2): 99-141

Teddie, C., & Tashakkori, A. (2009). Foundations of mixed methods research: Integrating quantitative and qualitative approaches in the social and behavioural sciences. Los Angeles, CA: Sage.

van der Walt, M., Maree, K., & Ellis, S. (2008). A mathematics vocabulary questionnaire for immediate use in the intermediate phase. South African Journal of Education, 28: 489–504.

Wilson, M., & Draney, K. (2013). A strategy for the assessment of competencies in higher education: The BEAR assessment system. In S. Blömeke, O. Zlatkin-Troitschanskaia, C. Kuhn, & J. Fege, eds. Modeling and measuring competencies in higher education: Tasks and challenges (pp. 61-79), Rotterdam: Sense Publishers.

Wolf, D., & Reardon, S. (1996). Access to excellence through new forms of student assessment. In J.B. Baron, & D.P. Wolf, eds. Performance-based student assessment: Challenges and possibilities. Ninety-fifth yearbook of the National Society for the Study of Education (Part I, pp. 52-83). Chicago: The University of Chicago Press.

Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. Journal of Child Psychology and Psychiatry, 17: 89–100