The 18th SEFI Mathematics Working Group seminar

on Mathematics in Engineering Education

PROCEEDINGS

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European Society for Engineering Education
Europäische Gesellschaft für Ingenieur-Ausbildung
Société Européenne pour la Formation des Ingénieurs

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European Society for Engineering Education
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SEFI is an international non-profit organisation *(a so-called "aisbl")* established in 1973 in accordance with the Belgian Law. Founded by 21 European Universities SEFI is the first example of an association directly linking the institutions of higher engineering education, hence independent of national and/or community filters in establishing its policy, as an international forum for discussing problems and identifying solutions relating to engineering education.SEFI is the largest network of institutions of higher engineering education, linking individuals, associations and companies in Europe. SEFI´s mission is to contribute to the development and improvement of engineering education in Europe and to enhance the image of both engineering education and engineering the profession in society.

SEFI objectives are to
− act as an interlocutor between its members, other societies and international bodies
− contribute to the development and the improvement of higher engineering education
− improve communication and exchange between professors, researchers and students in Europe
− offer expertise relating to the situation of higher engineering education in Europe
− preserve the diversity of courses and of teaching methods
− promote cooperation between industry and those engaged in engineering education
− provide appropriate information to its members
− serve as a European Forum for higher engineering education players and policy makers and contribute in formulate ideas and positions on engineering education
− pursue a lobbying role to ensure influence in engineering education in Europe
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Since its foundation, SEFI has done its best to develop a European dimension in higher engineering education according the intentions of its founders and regularly publishes reference documents and position papers on specific aspects of the education of engineers.

All SEFI's activities are undertaken in the view of core values as:

- Creativity and professionalism: in reaching the highest professional quality possible whilst encouraging creativity in our thinking, in our doing, in our learning and in our working
- Engagement and responsibility: in achieving our aims and objectives and fulfilling our mission for the benefit of higher engineering education in Europe
- Respect for diversity and different cultures: in cooperating with different regions all over the world, with specific social and economic settings, with different educational environments,and with different ways of thinking and communicating
- Institutional inclusiveness: in involving all higher engineering education stakeholders, at individual, institutional, organizational and governmental level
- Multidisciplinarity and openness: in being open minded and promoting exchanges across all engineering disciplines
- Transparency: in cooperating in a good and open spirit
- Sustainability: in working efficiently and effectively with technological achievements and with available environmental, economic and human resources, to the benefit of future generations

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18thSEFI MWG SEMINAR PROCEEDINGS

Introduction

SEFI's Mathematics Working Group was established in 1982 under the co-chairmanship of Professor D.J.G. James (Coventry University, England) and Professor K. Spies (University of Kassel, Germany), and succeeded in 1990 by chairman Professor L. Råde (Chalmers University of Technology, Gothenburg, Sweden) and vice-chairman Dr. F.E. Simons (University of Eindhoven, Netherlands).

The aims of SEFI MWG, stated 34 years ago, remain unchanged:
- to provide a forum for the exchange of views and ideas among those interested in engineering mathematics
- to promote a fuller understanding of the role of mathematics in the engineering curriculum and its relevance to industrial needs
- to foster cooperation in the development of courses and support material
- to recognise and promote the role of mathematics in continuing education of engineers in collaboration with industry.

To fulfil these aims and maintain international participation 17 seminars on mathematics in engineering education were held by the SEFI MWG since 1984, when the first event took place at the University of Kassel, Germany. The 17thseminar in 2014 was held in Dublin, jointly at the Dublin Institute of Technology, the Institute of Technology Tallaght and IT Blanchardstown. The third edition of the SEFI MWG curriculum document "A Framework for Mathematics Curricula in Engineering Education" launched in 2013 was the highlight of this seminar, where further discussions on the concept of mathematical competencies and other important issues in the mathematical education of engineers continued.

The Steering committee of the Mathematics Working Group, in charge of choosing an attractive seminar venue from those offered by representatives of the volunteering technical universities, was pleased to move the 18th SEFI MWG event to Sweden, Gothenburg, as the third seminar organized by Chalmers. Thanks to professors Lennart Råde (1987), Carl-Henrik Fant (2002) and Tommy Gustafsson, Chalmers became the most hospitable SEFI MWG university in Europe.

"*The decline in mathematical knowledge, especially at the school-university interface has worsened considerably since the Group was formed. The reasons behind this are complex, but pressure on the curriculum, and a wide variety of social and cultural changes, now being felt internationally, are believed to play the part. The role of the Group is to advice upon how to cope with this within the engineering curriculum.*" was concluded by Dr. Mike Barry, University of Bristol, in his report from the 11th SEFI MWG Seminar on Mathematics in Engineering Education, 9-12 June, 2002 in Chalmers,

available at the MWG website. Undoubtedly, the important topics and main problems faced in mathematical education of engineering students seem to be more persistent than expected.

The 18th SEFI-MWG European Seminar on Mathematics in Engineering Education organised by Chalmers in June 27-29, 2016 is aimed to provide a forum for the exchange of views and ideas amongst participants interested in engineering mathematics, in order to promote a fuller understanding of the role of mathematics in engineering curriculum, and its relevance to industrial needs and continuing education of engineers in the economic, social and cultural framework of Europe. The overarching theme of the seminar is the concept of mathematical competencies reflected in the following themes:

- Transition to higher education for traditional and adult learners
- Learning mathematics through project work
- Mathematical competencies in web-based learning scenarios
- Using technology to improve mathematics education

The rich programme of the seminar comprises 3 plenary keynote lectures: Professor Tom Lindstrøm from the University of Oslo: "Teaching mathematics to students who are not primarily interested in mathematics", Professor Jana Madjarova from Chalmers University of Technology, Gothenburg: "Making the Right Choice", and Professor Mogens Niss from the Roskilde University, Denmark: "Competency based curricula in mathematics". Special guest is Professor Ann-Marie Pendrill, the director of the Swedish National Resource Centre for Physics Education, Lund University, discussing "Mathematics for carousels and roller coaster: Challenging project work for engineering students".

SEFI MWG seminars are traditionally focused on guided discussions among participants during special discussion sessions. Proposed topics include:

- Mathematical competencies
- SEFI Mathematics Working Group Seminars – mission and future

A great response to the seminar call for papers, represented by 51 accepted high quality papers with direct relevance to the seminar themes, resulted in very promising programme including extensive poster session with 18 presentations and 31 paper presentations related to important topics in mathematical education of engineering students. The paper presentations are divided into several topics, most of them in parallel sessions, such as *mathematical competencies, technology, assessment, modelling, transition, motivation of students, activation of students.*

All accepted contributions are included as full papers in the proceedings that are freely available at the SEFI MWG webpage, to provide a summary of the topics dealt with at the seminar. This is fully justified by the group´s main objectives to sustain the accumulative process of gathering published materials and reports related to identified

important topics in mathematical education of engineers for building up a sound body of knowledge in this field.

Finally, the author would like to thank all members of the SEFI Mathematics Working Group Steering committee, the language editors, and the local organizers for doing the language check and editing of the proceedings for the benefit of all potential readers.

Bratislava, June 2016   Daniela
Velichová

CONTENT

KEY NOTE LECTURES

SPECIAL GUEST

Papers presented in the seminar sessions

Papers presented in the poster sessions

The 18th SEFI Mathematics working Group Seminar
27rd – 29th June, Gothenburg, Sweden

KEY NOTE LECTURES

Teaching mathematics to students who are not primarily interested in mathematics

Tom Lindstrøm

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Abstract

Most of the students enrolled in mathematics courses are not primarily interested in mathematics. They are there because they have been told that they need to take mathematics in order to become engineers, physicists, chemists, biologists, computer scientists, economists etc., and they seem to regard this advice with increasing scepticism. There are several challenges in teaching students of this kind. The most obvious one is perhaps how to motivate them, but there are deeper, underlying questions on how to teach them and what to teach them. As far as I know, there are no magic solutions to these questions, but I'll try to make the questions a bit clearer and present a few insights that might be helpful.

The motivation challenge

We have all been there. Looking up at the students after having presented a key argument or a key calculation in a way that made us feel that this time we really got it right, we find them fast asleep or busy texting. "Don't they have any sense of importance and significance?" we may ask ourselves (and even them if we are brave enough), but it doesn't help much.

Unmotivated students are clearly not a new challenge to frustrated teachers, but there are reasons to believe that the problem is increasing. The most obvious reason is that we recruit other kinds of students than before. When I was young, going to university was a deliberate decision – it was something I did because I was interested in the subjects I wanted to study, and I could just as well have stayed at home and found a job at the bank or some other local firm or institution. These days many students go to university because they see no other option. They have been told that higher education is important, and they have had to find a subject to study although there aren't any they are particularly interested in. One sometimes has the feeling that they have used the method of elimination to find their subject or profession – instead of looking for what they are most interested in, they have chosen what seems least uninteresting (or what pays the best).

Students of this kind may be as bright as the ones we used to get, but they raise the challenge for mathematics teachers to a new level. Our standard strategy has been to try to motivate students by relating mathematics to their main discipline, but if they are no more interested in engineering than in mathematics, engineering students may not find this approach particularly convincing. For obvious reasons, mathematics courses are usually put in the first semesters of educational programs, before the students have had any real experience with their chosen profession, and this aggravates the problem for students who don't have a clear idea of what this profession is all about. It's not uncommon

these days to hear of engineering students who don't understand why they have to take mathematics classes (can't everything be done by computers nowadays?), or biology and geology students who are frustrated because they also have to study chemistry.

In some countries (and Norway is definitely among them), the problem may to some extent be explained by the way the high school system is set up. In the old days, you applied to integrated programs in, e.g., the natural sciences, the social sciences, or the humanities. These days the high school system is more like a "smörgåsbord" where students (within certain constraints) can help themselves to whatever dishes they find most appetizing. This means that half the students in your mathematics class will not be taking physics, and that there may even be physics students not taking mathematics (although most schools will try to stop them). This system forces the subjects to be taught in isolation, and many students do not get to see how the sciences build on each other and inspire each other. Ironically, the universities have in the meantime moved in the opposite direction replacing their smörgåsbord by integrated study programs.

There is an important aspect of motivation that we teachers may have a tendency to overlook, and that is the connections between motivation and success – or rather between demotivation and lack of success. It's obviously much easier to get demotivated by something you do not understand. Many student claims for lack of motivation are really expressions of frustration: "I can't do this – and what the heck is it good for anyway?" This observation links the motivation problem to another important problem – the transition problem between high school and college.

The Transition Problem

In 2012 the transition problems from Norwegian high school to engineering colleges and university programs in the natural sciences were seen as so serious that The Norwegian Association of Higher Education Institutions appointed an expert group to look into them. From the very outset, mathematics was seen as the major problem. The first task of the group was to (all translations are mine):

"Examine the introductory mathematics education at the institutions with the aim of seeing how the subject is taught and learned. Earlier surveys of the mathematics teaching is to be looked into in order to analyze the connections between failure in mathematics and drop-out."

Needless to say, the committee didn't solve the problems, but they did (among other things) conduct a survey where students and teachers in colleges and universities were asked about their experiences with mathematics, and particularly about the transition from secondary to tertiary education. The response rate was not overwhelming but, as it was a nationwide survey, we still got almost 3000 student responses.

To begin with the good news, the students were on the whole well satisfied with their mathematics instructors, although they did score them somewhat higher on mathematical expertise than on pedagogical expertise. What was more disconcerting, was the big gap

the students experienced between high school mathematics and college mathematics. In one sense this is surprising as Norway has national school curricula and national exams in mathematics. There are only a few textbooks in use, and everybody should be able to figure out what is in the high school curriculum and what is not.

The survey shows quite clearly that the problem is not that colleges and universities don't know *what* is taught in high school, but that they don't know *how* it is taught. The gap between high schools and colleges is not in content, but in culture. Let me illustrate by a few typical student responses:

"In school mathematics is taught in terms of methods, i.e., the students learn how to compute, they don't learn mathematics. That's why the transition to college is so big. For the first time students have to learn mathematics, a totally new subject for them."

"High School = memorizing formulas, not understanding.
College = understanding that math is essential for making your own formulas, but you also need to practice the methods."

"In high school there was little mathematics teaching, only solving concrete problems. Hence you didn't get an understanding of the principles behind the topics.
In college the focus is on general principles and theorems, and it's hard to solve the problems from the information you get in the lectures.
It's a very tough transition from high school where you are only used to problems very similar to the examples. In college you have to think outside the box."

The students don't quite agree on where to put the blame, but most of them want high school to become more like college (but we should keep in mind that these are the students who survived the first semester in college!):

"My experience was that the mathematics level in high school was too low. I never really learned to read mathematics. In high school it was enough to thumb through the pages and do the problems (which you could do by looking at the examples)."

"Have more focus on abstract thinking earlier. Mathematics isn't about pushing the buttons on a calculator, something we spend far too much time doing in high school."

"My experience is that high school gives you formulas and theories for free, but at the university you have to understand them from rock bottom (formulas are derived, you work for implicit and explicit understanding and for practical applications). This is backward. Understanding should come before practical applications."

And the connection to motivation:

"The transition is much too big. Either high school has to step up its game, or universities have to take into consideration what the students really know/don't know from before. For me it was a great transition to go from star student in high school to flunking the first math course at university. I got depressed and lost all motivation for higher education."

The cultural gap between high school and college is impossible to see from the national high school curriculum, which actually puts a lot of emphasis on higher-level skills. It's definitely more noticeable from the textbooks with their emphasis on worked examples rather than explanations, but it would be unfair to put all the blame on the authors alone – they are reacting to a double pressure from an oversized curriculum and a political demand to get more students through high school.

I am sure this situation is not unique for Norway – in fact, everyone's instinctive reaction when things don't work out, is to "teach for the test" by emphasizing the mechanics of the problems that are likely to be on the exam. What should we do about it? I don't think lowering our standards is a good idea unless we really think we are aiming too high, but many of us can probably do a better job at helping the students with the transition by being more sensitive to the cultural change. And I should add that the committee appointed by The Norwegian Association of Higher Education Institutions did a little more than just conducting a survey – they initiated a national program for producing videos that (hopefully) will be able to help students with the transition.

How do we teach?

If you take lunch with the mathematics instructors at a Norwegian engineering college, especially one that provides three-year technical education on the bachelor level, you may hear exchanges along the lines of the discussion above. On the one hand you have instructors who only want to burden their students with what is absolutely essential for their engineering courses, and who aim to give them just enough to be able to handle the formulas they meet there. On the other hand you have instructors who believe that even in technical colleges, mathematics is a subject in its own right and should be taught with sufficient depth that the students can adapt what they learn to new situations. If you want to typecast, you can picture the first instructor as a seasoned teacher with a degree not in mathematics, but a neighboring field such as physics or electrical engineering, and the second instructor as a relatively recent Ph.D. in mathematics.

There are several constraints that make the discussion less obvious than it may at first appear to a professor of mathematics. One is student quality and background. Not only is there a limited supply of students with a strong background in high school mathematics, but in addition the colleges have an important mission in helping technical workers in industry upgrade their competence. These workers often have a detailed knowledge of industrial processes, but they lack a good background in mathematics, and what they have is either twenty years old or the result of a crash course. Another restraint is time. In addition to the calculus and the linear algebra we teach to our university students, many engineering students also need a quick introduction to other topics such as Fourier analysis, differential equations, statistics, and discrete mathematics – and in less time than what we give to the university students.

It's clear that under these circumstances we cannot give the engineering students what many of us would think of as a "full" mathematics course – we have to skip (many) proofs

and the careful build-up of the theory. Still I think it's important that what we teach makes sense and that it provides the student with a connected and coherent picture of the subject. It is almost impossible to remember and use a collection of formulas and techniques that do not connect, but it is often possible to provide a coherent picture of a mathematical topic without going into formal proofs. If you want an example, think of the theory of power series including Taylor series and termwise integration and differentiation. It takes a lot of time to explain this theory with full proofs as you have to look into several aspects that are not really needed for applications, but it is perfectly possible to give a coherent informal presentation (including caveats!) in much less time.

Whatever you choose to do, there is one golden rule: Don't cheat! Don't call something a theorem if it isn't, and don't call something a proof if it isn't. An informal explanation is fine, but don't confuse the students' sense of logic by calling it a proof. And please don't pretend that something is obvious when it isn't: if you present termwise differentiation of power series as a straightforward generalization of termwise differentiation of finite sums, the students will be utterly confused when they get to Fourier series.

Worst of all are explanations that seem intended to *convince by confusion*. The classical example is the treatment of separable differential equations where

$$Q(y)y' = P(x)$$

is rewritten as

$$Q(y)\,dy = P(x)\,dx$$

and then an integral sign (one with respect to y and one with respect to x) is added on either side. It is possible to make sense of this in terms of increments of y and x, but I would like to see the freshman student who thinks that way; the great majority of students just accept the totally unwarranted manipulations as a proof – or (perhaps better!) accept that they don't understand. The irony of this example is that it takes about one and a half line to provide a correct explanation. Meaningless symbolic manipulations turn many students off mathematics, and especially the students we can least afford to lose – those who really want to understand.

What do we teach?

If you look at a standard calculus text, it looks exactly as it did twenty years ago except that it is a hundred pages longer as it includes exercises tailored to graphic calculators, symbolic calculators, Maple, Mathematica, Matlab and whatever. The computer challenge is definitely here, but what is our reply?

What we now teach our first-year students, was cutting edge research in the days of Newton and Leibniz. That we now can teach it to students of eighteen or nineteen, doesn't mean that mankind has become all that much brighter – it just means that we have found more efficient ways to teach calculus and that we have removed a lot of material that we

no longer deem relevant. The computer revolution calls for something similar if we aren't going to drown our students in a combination of old-fashioned and ultramodern skills. What should we no longer deem relevant?

This isn't an easy exercise at all. You may think that integration by parts in an outmoded technique as we now have free computer systems that can do any integral solvable by the method in the fraction of a second, but integration by parts is still an essential tool in more advanced parts of mathematics such as Fourier analysis, ordinary and partial differential equations, and distribution theory. What happens to student understanding if you throw out elementary calculations and just concentrate on advanced applications? To put it more generally: how much hands-on experience with the methods of elementary university mathematics is needed to be able to handle the challenges of advanced mathematics? Or more brutally: what have you really understood if you let a computer system solve your problem for you? I don't think anybody knows the answer. The best we can do is to experiment, evaluate, and compare.

At the University of Oslo, the experiment is a project called Computing in Science Education (CSE) that has been running for a little more than ten years. It started with an integrated first semester for students in mathematics, physics, and parts of computer science, chemistry, and geophysics. In the first semester, the students take three courses – a (fairly theoretical and traditional) calculus course, a course in (Python) programming, and a course in modeling and computations that is meant to form a bridge between mathematics and programming. The idea is that these three courses will form a basis to build on in later semesters. The physics department is probably the department that has taken the challenge most seriously, creating a sequence of courses where traditional problems solvable by analytical methods are replaced by more complex and realistic examples that will only yield to a numerical approach. Also in the mathematics department we have had to rethink our introductory courses, reducing the emphasis on some traditional techniques (such as methods of integration) and instead putting more emphasis on numerical methods. It's important to note that this hasn't in any way trivialized the mathematics courses. Quite the contrary; we now teach the completeness of $\mathbf{R}^n$ and Banach's Fixed Point Theorem (for closed subsets of $\mathbf{R}^n$) to second semester students as a basis for iterative procedures. But in all honesty I should admit that we have not yet been bold enough in reducing traditional material, and that the courses are threatening to burst at the seams.

Still I think it is fair to say that CSE is a success. The science study programs that were not initially part of the project are eager to join, although some of them only want a reduced package where the programming course and the course in modeling and computations are merged into one, and some (especially biology) want to develop their own versions. Perhaps more importantly, the Ministry of Education wants to extend the project to other universities and colleges; see University of Oslo (2011). The project has so far produced at least two international textbooks, Langtangen (2011) and Malthe-Sørenssen (2015), and one associated text in Norwegian, Lindstrøm and Hveberg (2015), and more are being developed.

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Making the Right Choice

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Abstract

In this presentation we focus mainly on the criteria universities in general, and Chalmers University of Technology in particular, use to select new students. We discuss why upper secondary school marks may not be the best criterion for admission, and what other methods of choice we have tried. More specifically, we discuss our experience of the admission test in mathematics and physics, which exists since 2007, and which is used for several programmes at Chalmers and at the Royal Institute of Technology.

We also discuss a related topic which has figured rather frequently in Swedish media lately, namely the weaker backgrounds of newly admitted students nowadays, compared to earlier. Chalmers has a long history (more than 40 years) of submitting students to a diagnostic test at the very beginning of their studies. We discuss the outcome and give some statistics. Our conclusion is that the level of mathematical abilities and knowledge of newly admitted students at Chalmers now indeed is lower than it was some 20 years ago. The last topic also touches a hobbyhorse of most universities, namely the wish to gain influence over upper secondary school education.

Introduction

There may be a few genuinely bad educational programmes, but there certainly is no such thing as an unconditionally good one. The success of a programme is dependent on many things, such as the good choice of contents, good planning and good leadership, but also on the qualities of the teachers and last, but not least, on the qualities of the students. Again, there is no such thing as an abstract good student. A student may be a genius in music, and completely unsuitable for engineering studies. This is why governments and universities spend a lot of effort and money on developing systems and processes which ensure, or at least make it probable, that the right students choose and are chosen for each educational programme.

The process of entering university resembles a competition, particularly when there is some kind of written exam involved. However, what matters most here is not the competitive element, but the flow of information submitted by each side of the process. The young applicant has chosen a programme which hopefully will lead him/her to a degree and a successful career, and the university wishes to choose a student who, again hopefully, will study hard and graduate within reasonable time. When both sides choose wisely, it may be called a union made in heaven. Unfortunately, this is not always the case. A wise choice is dependent on information from both sides being sufficient and trustworthy. The applicant's choice is based on our presentations of the university programmes, and the choice of the university is based on the applicant's marks or on some kind of test results. There is however a variety of problems bringing a degree of uncertainty into the picture. The two most important ones are probably the virtual

impossibility to explain the contents and the level of an educational programme to a person who has not been exposed to it, and the virtual impossibility to find absolutely trustworthy criteria for admission.

The two extreme systems of admission are *(i)* allowing everybody who so wishes to start, and *(ii)* spending a large effort and a lot of money on an elaborated process of elimination. In the first case governments and universities are aware of the fact that a large number of students will quit within a semester or a year, leaving those who are interested and motivated enough to finish their studies. In the second case there is a wish to see all students who enter get a degree. In fact, there are two sides to the coin. In the second case the students have invested so much work in the process of admission that they are reluctant to quit, even if they should not have picked the right subject. Most countries choose a system in between, with admission based on upper secondary school marks, on a written and/or oral exam, or on a combination of both. Illogically enough, without much being invested in the selection of students, expectations are often quite high as to their results.

The Flow of Information from the University to the Presumptive Students

Each and every year all potential students receive a vast amount of information, digitally or printed on paper. Some of the information is a matter-of-fact description of the university programmes, some is (here comes a quotation) "pimped" in an attempt to make university education appear more modern and appealing to the young people. It is not easy to choose. Even if the description of a certain programme is adequate and relevant, chances are it does not mean the same to the person who wrote it and to the person who reads it. It is indeed difficult (impossible?), to give a fair explanation of the contents of a five year programme to a student who has not even begun his/her studies. The choices young people make are therefore based not only on their interests, but also to a large extent on trust – they trust a certain university, a certain teacher, or a certain older and more experienced friend. There are, of course, other factors as well, as the prestige of the university and the programme, the prospects of a good career etc.

The Flow of Information about the Presumptive Students to the University

The main criteria for selection of students in Sweden are, somewhat simplified, twofold: the marks from upper secondary school, and the results of the so called "högskoleprovet" (a test, which is not dedicated to any specific subject). Statistics for the programme Engineering Physics at Chalmers show that high marks from school are no guarantee for successful studies. The cut-off level for Engineering Physics is very high, around or above 21.5 (out of a maximum of 22.5). Even if marked by the same teacher, a difference of 0.1 or 0.2 at this level is totally insignificant. Knowing that teachers can be generous or stingy, and knowing that the governmental criteria for marking are open to interpretation, makes many university teachers suspect that as a criterion marks are not completely reliable. The correlation between successful studies and results on "högskoleprovet" is non-existent. This is one of the reasons we have been using a third method of selection since 2007, namely the Mathematics and Physics Admission Exam. We started with only

two programmes at Chalmers. Currently the test is used for at most one third of the places on five programmes at Chalmers and three at the Royal Institute of Technology, Stockholm.

The Mathematics and Physics Exam – an Admission Test

The Mathematics and Physics Exam is constructed by JM (mathematics) and Martin Cederwall[1] (physics). The main goal of the test is and has been to provide an additional tool of selection, allowing motivated and talented applicants with comparatively low marks from school to compete for some of the places at our programmes. Unlike the school marks, it gives us information of the student's abilities at the time he/she applies, rather than at some point a few years back.

Our plans were to use the admission exam not only as means of gathering information about the applicants, but also to make it part of the flow of information from us to the applicants, to their teachers, and, eventually, to all future applicants, not only those who take the exam. We had hopes that the test would be widely seen as a signal of what we expect our students to know and to be able to do when they come to us. The test would thus counteract the tendencies of weaker backgrounds. For several reasons (there are probably more of them than we know of) it has not quite worked out that way.

The structure of the test in mathematics[2] is

A. 20 multiple choice questions, worth 1 point each;

B. 10 answer-only questions worth 2 points each;

C. One (fairly standard) problem with full solution, worth at most 5 points.

This structure was chosen since

(i)   Demands on full solutions vary.

(ii)   One person must be able to mark a large number of papers.

(iii)   The marking scheme needs to be robust.

In the literature on the subject there are many pros and cons multiple choice questions. For various reasons, we made the decision the pros were heavier than the cons. Also, probably against most recommendations, the mathematics part uses the option "none of the above". However this option is used with care and discriminates between "another numerical answer", "cannot be determined", "there is no solution", "the object does not

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[1] professor at the Department of Physics, Chalmers

[2] The structure of the physics test is similar, it is however somewhat smaller.

exist", etc. There are also some multiple choice questions without this option, which could be reformulated as "eliminate the wrong answers".

The answer-only questions can be divided into two parts – fairly easy, or at least predictable ones, and small problems, which demand a higher level of understanding. The answers are hard to guess. These questions are tough on those who tend to make many minor mistakes and on those who do not read properly. A difficult matter to resolve each year is how much variation to allow in the answers.

The full-solution-problem is fairly standard, although not very easy.

The most frequent mistakes are in the areas

- Absolute values

- Signs

- Inequalities

- Geometric sums

- Detecting false roots

The Diagnostic Test

For the last 43 years all new students at Chalmers have been submitted to a diagnostic test during their very first day at the university. The test was introduced by Rolf Pettersson, who managed and analysed it for 40 years. It consists of nine simple problems, chosen out of a pool of 30. The test confirms what university teachers have had reasons to suspect for a long time, namely the drop in backgrounds of students compared to earlier years. Here is a comparison between the results on an identical test, given in 1992 and in 2013.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	
				Results	(percentage of correct answers)					
1992	77	74,1	60,1	45,2	57,1	26,6	36,9	54,6	34	(934 st)
2013	52,5	47,2	26,8	20,3	21,5	4,8	15,7	17,8	13,4	(1233 st)

To give an idea of the contents of the test, the problems considered: 1. fractions; 2. a quadratic equation; 4. the logarithmic laws; 5. & 6. trigonometry; 7. a simple derivative; 8. the equation of a straight line in the plane.

The table speaks for itself.

Conclusions

An attempt to evaluate the admission exam was made in 2009. It turned out however that the number of students admitted based on the exam, and who would not have been admitted based on their school marks, was at the time too small for a serious statistical analysis. Still, one thing was clear. The prediction value of the admission exam was much better than the one of school marks in so far as there were no students with very good results on the test and poor results in their further studies. After seven more years, and given the wider range of applicants who take the test, we plan another evaluation in the near future.

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Competency based curricula in mathematics

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Abstract

This paper focuses on presenting and utilising the notion and rationale of mathematical competence and mathematical competencies as put forward in the Danish "Competencies and the Learning of Mathematics" project. The main points are how competency based mathematics curricula can / should be formulated, how teaching / learning environments and arrangements can be established so as to help foster and further students' development of mathematical competencies, and what appropriate modes and instruments of assessment of mathematical competencies may look like in different contexts and settings. Finally, issues and challenges to the implementation of competency based approaches to teaching and learning of mathematics will be discussed.

Introduction

There are two very different - yet by no means incompatible - conceptualisations and views of what the intended, or desired, outcomes of mathematics teaching and learning should be.

The first perspective – (a) – places emphasis on knowing mathematical facts, such as concepts, definitions, theorems and other results, including formulae, methods, procedures, and algorithms.

The second perspective – (b) – places emphasis on the ability to do mathematics, i.e. to act within and by means of mathematics in different sorts of settings and contexts.

Whilst it is possible - in principle, albeit less so in practice - to know mathematical facts without being able to do mathematics, it is hardly possible to be able to do mathematics without knowing at least some basic mathematical facts. Hence the relationship between the two perspectives is somewhat asymmetrical.

In lots of extra-mathematical fields that make substantial use of mathematics, including engineering, mathematics has two different – yet again by no means incompatible – roles to play.

(i)   To assist in establishing or formulating the very concepts, notions and terms in the extra-mathematical field at issue (velocity, population density, concentration, elasticity, flow, stress, torque,...). This involves prescriptive mathematical modelling.

(ii)   To assist in drawing inferences about situations, contexts, systems, interventions etc. This involves descriptive mathematical modelling.

In some cases role (i) predominates. In other cases role (ii) is of particular importance. For role (ii), the ability to do – enact – mathematics, i.e. perspective (b), is essential.

A characteristic aspect of engineering – and especially of engineering science – is the design of technical systems to possess certain desired properties. So, a crucial component of the science aspect of engineering is to account for the degree of fulfilment of these properties with respect to a given design. This accounting process usually involves mathematics in role (ii), and hence perspective (b) of mathematical mastery. In other words, engineering cannot make do with perspective (a) only.

What we have just looked at is a specific instance of a general issue: What does it mean to master mathematics? This is the subject of the next section.

What does it mean to master mathematics?

In many "traditional" curriculum documents and syllabi in secondary and tertiary education, perspective (a) is predominant, albeit often complemented with the most basic associated skills concerning procedures and routines, aspects which belong to perspective (b). However, as we have already seen, this is far too restrained when it comes to capturing what is involved in mastering mathematics, not least as regards mathematics in highly mathematics dependent fields.

If we look at different curricula and syllabi in different segments and levels of the education system that define the framework for mathematics teaching and learning in terms of content, one may well get the impression that what is called mathematics is a completely different animal in different contexts. In primary school, the content is focused on natural numbers and geometric shapes and objects and their mensuration. In lower secondary school, the content is typically focused on topics such as fractions, symbolic expressions and elementary algebra, linear functions and equations, coordinate systems and graphs, more geometry, and perhaps some descriptive statistics. In upper secondary school, the content typically includes new kinds of algebraic or transcendental functions, trigonometry, analytic geometry and vectors, probability and statistics, and perhaps some introductory calculus. In tertiary institutions, depending on the mathematics programme at issue, we encounter an enormous variety of abstract and linear algebra, vector spaces, multivariate calculus and/or analysis, ordinary and partial differential equations, number theory, metric and topological spaces, Fourier and/or functional analysis, measure and integration, discrete mathematics, probability theory and stochastic processes, axiomatic or differential geometry, to mention just a few. The intersection across educational segments, levels and contexts of all these disciplines and topics is very small indeed. It mainly boils down to the content of primary school mathematics. If it is justified to use the same term, "mathematics", for all these very different animals, the commonality that justifies this use must lie somewhere else than in content cum procedures and routines. But where?

This, then, constitutes our first challenge: We have to capture what it means to master mathematics in ways that are independent of educational segments, levels, and contexts, as well as of specific mathematical topics. To assist us in this endeavor, we may seek inspiration from an analogy: mastering a given language. Linguistic mastery – or, equivalently, possessing linguistic competence in a language – has four main constituents.

The ability to understand and interpret what other people (want to) say when they *speak* in the language at issue, in different registers and genres. The ability to understand and interpret what other people (want to) say when they *write* in the language, in different registers and genres. The ability to express oneself in *oral speech*, and the ability to express oneself in *writing* in the language, again in different registers and genres. The same constituents are on the agenda both with young children and with professors of literature, even though they do of course talk and write about very different things. Needless to say, all of this presupposes a solid knowledge of the vocabulary, the orthography and the grammar of the language, but linguistic competence goes far beyond such knowledge. You would never consider someone linguistically competent who only knows a list of correctly spelled words, lots of declinations and conjugations and grammatical rules. Similarly, you would never consider someone mathematically competent who can only cite lists of definitions, theorems, formulae, and rules without being able to enact them in different sorts of contexts. What, then, are the counterparts in mathematics of linguistic mastery?

Mathematical competence and mathematical competencies

In the Danish KOM Project (the acronym KOM (Kompetencer og Matematiklæring) stands for Competencies and the Learning of Mathematics) we defined mathematical competence as follows (Niss and Jensen, 2002, p. 43; see also Niss and Højgaard 2011):

> "*mathematical competence* means to have knowledge about, to understand, to exercise, to apply, and to relate to and judge mathematics and mathematical activity in a multitude of contexts which actually do involve, or potentially might involve, mathematics." [Translated from Danish by MN].

The project further identified eight overlapping constituents of mathematical competen*ce*, which we decided to term mathematical competen*cies*, defined as follows:

> "a mathematical competen*cy* is insight-based readiness to act purposefully in situations that pose *a particular kind* of mathematical challenge." [Niss and Jensen, 2002, p. 43, translated by MN. Italics added.]

These eight competencies are: the mathematical *thinking, problem handling, modelling, reasoning, representation, symbols and formalism, communication*, and *aids and tools* competencies. They are well-defined and distinct, yet overlapping. They all pertain to enacting – doing – mathematics in different kinds of situations and contexts. Each competency possesses a duality between *the receptive* side (being able to follow, understand and evaluate other people's exertion of the competency) and *the constructive* side (being able to activate and exert the competency oneself). One important reason why the competencies overlap is that focusing on the enactment of one of them typically invokes several of the other competencies as "auxiliary troops".

The competencies can be visually represented as the petals of a flower, the *competency flower* (see Figure 1). The colour of each petal is most intense in the middle and fades

away towards the boundary. Since all eight petals have a non-empty intersection, the same is true of any subset of them.

In mathematics education, the competencies can be used in at least different three ways. First, as a *normative tool* for defining and specifying curricula (which competencies should be pursued where, when and how?). Secondly, as a *descriptive tool* for investigating, characterising and analysing existing curricula and actual mathematics teaching and learning at any level (which competencies are actually being pursued in Curriculum C or in Teaching T?). Thirdly, as a *tool for assessment and metacognition* for students and teachers (are we reclaiming new land with regard to competency C_k, and are there aspects of competency C_n which need particular attention in the future, (k, n = 1,2,…,8)?
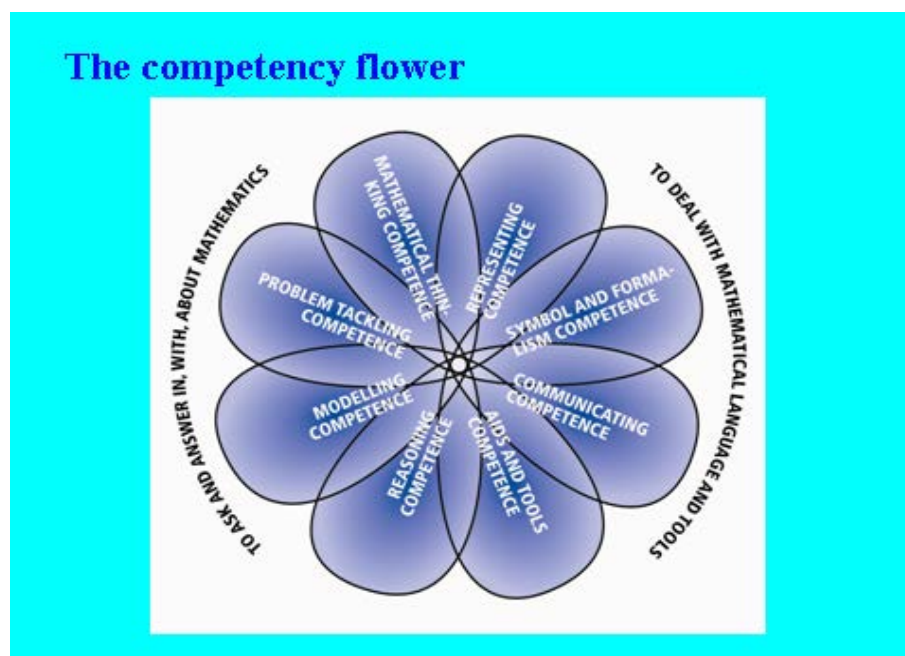
Figure 1: The competency flower

Since the competencies, by their very nature, are not defined in terms of mathematical content, it is natural to ask what their relationship with subject matter/content knowledge then is. The answer is that their relationship is that of a matrix organisation as illustrated by the following table:

	Competency 1	Competency 2	…	Competency 8
Maths topic 1				
:			maths topic i × competency j	
Maths topic n				

Table 1: Content by competency matrix

Cell $i \times j$ then represents the interaction between topic i and competency j, which takes two different forms. The role played by competency j in dealing with topic i, and the role played by topic i in the enactment of competency j, depending on whether the competency or the topic is the primary focus of attention.

How are competencies developed with students and how can they be assessed?

For those of us who teach mathematics, a key issue is how the mathematical competencies can be developed with students, or – differently put – how teaching and learning can be orchestrated so as to foster and further students' possession of them.

It is a general observation that there is no royal road to guaranteed success of any given scheme of mathematics teaching, because the local socio-economic and cultural boundary conditions, contexts, circumstances and situations, together with the characteristics of the individual students, co-determine what can be achieved in terms of learning outcomes. Of course this does not mean that there is nothing we can do to further our intentions and come closer to our goals. What it does mean, however, is that all the factors just listed have to be taken seriously into account when designing teaching and learning environments and orchestrating and implementing teaching. This is also true when it comes to mathematical competencies.

First of all, if mathematical competencies are to be pursued this pursuit has to be a primary goal in and of itself, not one which is secondary to other goals, such as obtaining content knowledge of particular mathematical topics. Competencies are usually not developed solely by way of osmosis. This means that the pursuit of mathematical competency development has to be prioritised in curriculum documents, syllabi and in the plans for teaching. Simply put, the development of competencies has to be explicitly placed on the agenda of mathematics teaching and learning. This further requires that teachers and lecturers are consciously aware of the nature of the competencies, and of how they differ and how they overlap.

In order to develop secondary or tertiary students' mathematical competencies it has proven fruitful to begin by introducing the students to the very notion of competency and the definition of each of them and then ask them to identify the competency content and demands in a variety of mathematical activities, situations and tasks, thus making the competencies a meta-cognitive tool for the students. One reason for this approach is that an individual mathematical competency is rarely activated in "splendid isolation". Dealing with mathematical issues normally involves several interwoven competencies at the same time. It is therefore important to design – both as introductory activities and as arrangements and activities for subsequent phases - an array of teaching and learning situations and tasks that draw on different combinations of the competencies. In some situations, perhaps the modelling, the problem solving, and the symbols and formalism competencies are predominant. In other situations, the thinking, the representation, the reasoning, and the communication competencies are particularly manifest. In still other situations, the problem solving, the representation and the aids and tool competencies may well be the crucial ones. Let us agree to call this approach to staging the development

of competencies in integrated contexts *holistic*, since it preserves the complexity and the interwovenness of the competencies which is characteristic of authentic mathematical activity.

Despite the fact that competencies are typically activated in clusters, it has proven useful to students' development of the competencies to also invite them to work, from time to time, in somewhat stylised and semi-artificial situations, in which one of the competencies is the prevalent one. For instance, the thinking competency is in focus in situations where students are asked to state conjectures or to propose possible generalisations of claims, whilst the problem solving competency is in the centre when students are invited to come up with, say, three different ways of solving a geometry problem, whereas the representation competency is predominant if students are asked to identify the information gains or losses in a translation between two different representations of the same mathematical entity, and the communication competency is in focus if students are asked to explain a mathematical model and its outcomes to different kinds of audience. We call this approach, in which we try our best to single out an individual competence and disentangle it as much as we can from the other competencies, *atomistic*.

Experience suggests that alternating sequences of the holistic and the atomistic approaches to competency development tends to yield promising outcomes. This also seems to be "the good" answer to the assessment challenge. Students should be assessed by qualitative means in both holistic and in atomistic settings. Again, as always the specific assessment instruments to be adopted have to fit the conditions, the context and the situations at issue. So, instead of simply importing and transplanting specific assessment modes and instruments from outside, it is more advisable to look for inspiration and then make suitable modifications locally.

Conclusion

This paper has provided a brief introduction to the notions of mathematical competen*ce* and competen*cies* and their role and use in mathematics teaching and learning. When it comes to implementing this approach in mathematical teaching and learning practices it was argued that the first thing to do is to explicitly place these notions on the agenda as primary goals of mathematics education. It has further been argued that there is no on-size-fits-all scheme of implementation, since local conditions and circumstances have to be taken into account when designing teaching and learning environments. There are challenges to competency based mathematics curricula. Authorities and educators have to understand the approach and its implications. Balances have to be struck between competency and content foci. And professional development of educators is a must.

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Mathematics for carousels and roller coasters: Challenging project work for engineering students

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Abstract

An amusement park is full of examples that can be made into challenging problems for students, combining mathematical modelling with measurement in the rides. For many years, the new students in the engineering physics program at Chalmers have visited the Liseberg amusement park, with group assignments, to be presented in written reports, as well as in oral presentations to student groups who have worked on other rides. The students have experienced the weightlessness in free fall and the large forces in roller coasters loops. They have observed the Coriolis effect using a little pendulum in slow carousels. They have also solved relatively straight-forward problems, such as working out and measuring periods in pendulum rides and train speeds in roller coasters. In addition they have investigated more complex questions such as: Does it matter what seat you chose in a roller coaster – and if so, how much? Can any speed difference between different trains be detected? How much mechanical energy is lost during the ride? What temperature do the brake fins reach after stopping the train? The authenticity of the tasks in enjoyable situations often leads to inspiring and enlightening discussions.

Introduction

Motion in three dimensions is common in engineering applications. Describing three-dimensional motion including combinations of acceleration and rotation is a challenge where mathematics plays a central role. For several years, I had the privilege of running problem-solving classes during the two first-year mechanics courses in the spring, for engineering physics students, who I had met already in their introductory course. As the dynamics problems got more complicated, with situations where intuition was of little help, I observed, as the course progressed, how students, one by one, learned to trust the mathematics and display happy confidence when they were able to solve these challenging problems.

As students struggle to visualize generalized 3D motion, including rotating arms, links, shafts, cranks and disks, real-life concrete examples can be useful, such as rotating pendulum rides (Pendrill and Rohlén, 2011). The Star Flyer ride Mechanica (Figure 1), which opened at Liseberg in 2015 would have been another excellent example.

Not long ago, measuring rotation was a specialized task, requiring equipment not generally available. At that time students were provided with prerecorded data and corresponding graphs for amusement park assignments, to enable connections between angular velocities describing rotations in a comoving system and the real-life experience of this motion (Pendrill and Rödjegård, 2005). Today's smartphones include 3D accelerometers and gyros, giving students easy ways to measure acceleration and rotation,

using e.g. the app Physics Toolbox Roller Coaster (Vieyra, 2016). The interpretation of the data offers good practice in identifying coordinate axes (Pendrill and Rohlén, 2011).
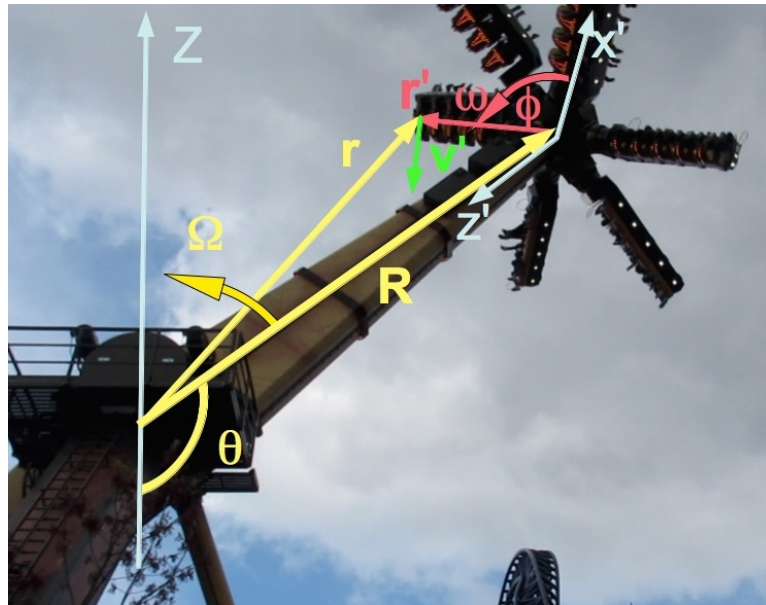
Figure 1: Coordinate axes and angles that may be used to describe the motion of the Star Flyer Mechanica at Liseberg.

Traditional amusement-ride related textbook problems include free fall, circular motion, pendula and energy conservation in roller coasters, where the moving bodies are typically considered point-like. However, an amusement park can offer many more examples that are useful in engineering education, many of them with a strong mathematical content. Below, we first give a few examples of assignments used and then discuss student conceptual challenges and evidence of involvement and learning.

Forces in circular motion

Students are familiar with centripetal acceleration, and understand the forces acting in an ordinary carousel moving in uniform circular motion in a horizontal plane. If the motion is, instead, in the vertical plane, such as in Ferris wheels and Flying carpets, most of the new students can work out the forces at the top and bottom, but lack systematic ways to describe what happens half-way up or down, or in more generally chosen points. These rides offer opportunities to connect the mathematical description with the experiences of their bodies in the ride. Textbooks often fail to make this connection – forces usually act on inanimate objects.

Circular motions in vertical planes come in many different versions. In the ferris wheels and flying carpets, your own body does not rotate, but the experience differs depending on your own orientation relative to the plane of motion. If your own body also takes part in a rotation that brings you upside down, it becomes even more obvious that our bodies are not point particles. This rotation and the associated forces also show up in electronic

data collected during a ride, inviting students to relate the data to the experiences of their own body.

Roller coaster loops are a special type of circular motion, although these are not strictly circular, as discussed in more detail by Pendrill (2005a). The motion is also not uniform, since the speed depends on the altitude.

Motion in a system rotating relative to an inertial system leads to the Coriolis effect. This can be observed even in a small children's carousel – or even better in a slowly rotating observation tower – by bringing a small, soft object on a string, as a miniature "Foucault pendulum" (Bagge and Pendrill 2002, Pendrill 2008a)

Energy conservation and modelling

Classical roller coasters start with a lift-hill providing initial potential energy, and the continued ride involves an interplay between potential and kinetic energy. Many modern roller coasters instead provide initial kinetic energy during a "launch", which uses, e.g. electromagnetism or hydraulics. In any case, the speed at a given point can, to a good approximation, be obtained from the elevation difference between that point and the highest point, (or a recent high point where the velocity is known or can be measured).

One assignment for the students has been to estimate losses during the ride in the wooden roller coaster Balder at Liseberg, where the train passes three times at approximately the same location, but at different heights, with about 250 m track in between. When all group members use their phones to measure the time for a train to pass, it can also be used as an exercise in estimating measurement uncertainties, and considering if the precision is sufficient to note any difference between different trains in the ride.

Another aspect of energy conservation is the braking at the end, where the kinetic energy is converted to thermal energy. Students have been provided with IR photos of the brass sheet responsible for the magnetic braking in the roller coaster Kanonen and provided with data enabling them to compare the measured heating with calculated values, based on measurements taken on the ride (Pendrill et al, 2012). It is also possible to model the distribution of the heating of the different parts of the brake.

Energy conservation provides a hint about the difference between different seats in a roller coaster: Although the whole train moves with the same speed at any given time, the riders are in different locations, making the front and back of the train move fastest over the top, whereas the middle moves faster in a valley, as discussed in more detail in Pendrill (2013) for the case of roller coaster loops. E.g. estimating the location of the centre-of-mass is a useful exercise in integrating over a circular arc.

Pendulum rides.

Playground swings are easily accessible pendula, where the body can experience the forces connected with the changing motion. Even as simple motion as a pendulum brings surprises when accelerometer data are collected – only the axis in the direction of the

chain is significantly larger than zero, which is usually not the acceleration discussed in the mathematical solutions for pendulum motion. Amusement parks offer pendulum rides (and rides with circular tracks) with much larger radii, and students sometimes express their delight that the formula for the period for a mathematical pendulum gives such good results.

In some rides, students may also investigate the effect of the maximum angle during the swing.

Combined circular motions

Simulating the motion of a classic teacup ride can be a purely mathematical task. The principal rotation of the "tea table" is 8 rpm clockwise, in the Liseberg version, with a faster rotation of the "tea trays", about 20 rpm counter-clockwise (relative to the table). The relevant distances can be measured or provided. During a visit, observations of the motion of a rider can be compared to the simulated motion. In addition, velocity and acceleration in the different parts of the ride can be calculated. Since the whole motion happens in a horizontal plane, the acceleration can also be measured with a smartphone app or by measuring the angle to the vertical of a soft object held in a string.

A similar motion pattern – but at a higher elevation – is found e.g. in the new FataMorgana ride at Tivoli gardens. In Octopus or Scrambler rides a motion up and down is added to the combined circular motions, leading to more complicated simulations.

Some newer rides add a circular motion to the swinging in large pendulums. This type of motion was studied by Pendrill and Rolén (2011) and is well suited for data collection using smartphones, using e.g. the app Physics Toolbox Roller Coaster (Vieyra, 2016).

Planning for conceptual development

Although the student group enters the engineering physics education with consistently high scores on the Force Concept Inventory ("FCI", Hestenes et al, 1995) - around 80% average for all cohorts where I have administered the test, many concepts are not yet stable (Pendrill, 2005b). The tests also included a few additional questions e.g. about forces in circular motion in a vertical plane. Most of the students have visited Liseberg or another park many times before staring their engineering education - although most often without making the connection to physics.

The course group of around 120 was divided into 4 "classes", each with 5 groups of about 6 students. Preparations and final presentations were done in the classes of 30 students, where each group had different assignments. During group discussions, the tutors find students coming to terms with e.g. "centrifugal" forces. As they write the reports before their oral presentations, they process forces in 2-3 dimensions in more detail, and tutors have many opportunities to discuss with students and help them discover and sort out conceptual difficulties. The students also have opportunities to discuss their findings during the amusement park visit. We always give feedback before the presentations, aiming to ensure that the presentation to other students is conceptually correct. Post-test

scores on the FCI after the introductory course (but before the mechanics courses) have been around 90% (i.e. a "normalized gain" around 50%).

The www page tivoli.fysik.org/english/articles includes links to articles with more information about the project.

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The 18th SEFI Mathematics working Group Seminar
27rd – 29th June, Gothenburg, Sweden

PAPER PRESENTATIONS

Turning a standard statics task into a mathematical modelling opportunity: The case of the „tumbler task"

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Abstract

The mathematical modelling competency is an important constituent of the overall mathematical competence the mathematical education of engineers should strive for. It is not clear where in the engineering curriculum learning opportunities for acquiring this competency are or should be provided. Investigations have shown that in application subjects often only a small part of the competency is addressed but important steps like making assumptions, simplifications, interpretation or validation are not properly covered. In this case study we consider a specific task from a textbook on engineering statics, the "tumbler task", in order to demonstrate how a very restricted task can be transformed into a modelling challenge. We also report on a student project where one direction was followed in more detail.

Introduction

In the SEFI Mathematics Working Group's Curriculum Document (Alpers et al. 2013) the mathematical modelling competency is listed as one of eight competencies making up the overall mathematical competence. One might assume that this competency is mainly acquired in application subjects like statics where mathematical concepts are used to create models and to solve applied problems in such models. Yet, the empirical study by (Gainsburg 2013) has shown that neither in the mathematical education nor in application subjects mathematical modelling was properly addressed. The investigation by (Alpers 2015) has demonstrated that only a few steps of the mathematical modelling cycle described in (Blum & Leiß 2007) really occur in statics tasks presented in two widespread textbooks. A typical task in (Hauger et al. 2012) is the "tumbler task" where the tumbler is already idealized as an object consisting of a hemisphere and a cone on top and the question of whether it will work is also clearly stated by asking when the center of gravity is below the separating plane. In this contribution this task is used as an example for showing how a standard statics task can be reformulated as a project task that offers many learning opportunities regarding the different steps in the modelling cycle. One particular reformulation of the tumbler task was given to a group of students as a mathematical project. They had to create a tumbler design environment in Maple® and produce some physical examples. We report on the results.

The modelling cycle

According to (Kaiser & Brand 2015), more elaborated specifications of modelling competencies are often based on the so-called modelling cycle which is a well-accepted idealisation of the modelling process. Figure 1 depicts an example of such a cycle set up by Blum and Leiss (2007). When working on solving a realistic problem, one first tries

to understand the problem situation and where the problem really is (1). This leads to a "situation model". Then, the situation is structured and simplified (2) to capture only presumably essential features which results in a "real model" where the problem is formulated. Then, in a process of mathematisation (3), the real model (and problem) is turned into a mathematical model (and problem), and the problem is solved by working mathematically (4). The results are interpreted in the real model (5) leading to "real results". These are then investigated regarding their sense-making in the situation model, i.e. they are validated (6). The results are then exposed to those who are interested, i.e., they are documented and presented (7).

| 1. Understanding |
| 2. Simplifying |
| 3. Mathematising |
| 4. Working mathematically |
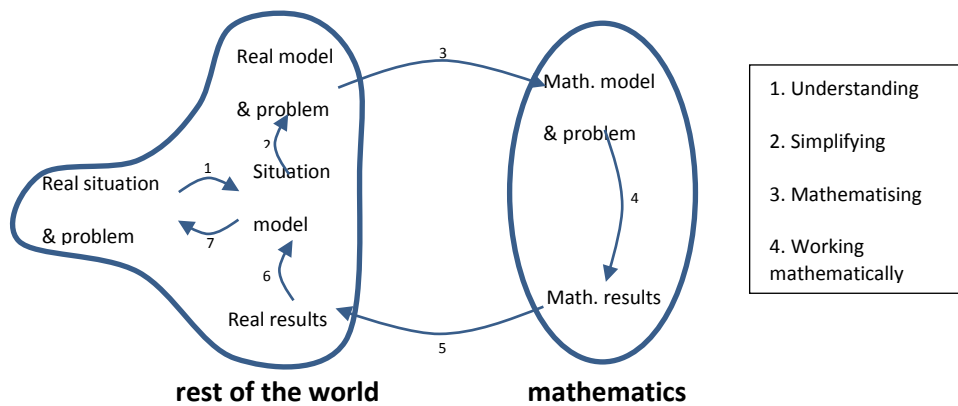
The original tumbler task

The original tumbler task taken from (Hauger et al. 2012) reads as follows: *„A homogeneous tumbler consists of a hemisphere and a cone on top. The centre of gravity must not lie above the separating plane for the tumbler not to tip over. If radius r is given, what is the maximum height h?"* (translated and picture redrawn by B.A.).
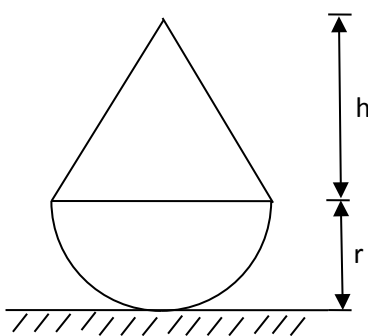
In this task formulation, the real model is already geometrically provided and the problem is directly given by stating that the centre of gravity should lie below the separating plane. The code word "homogeneous" allows to use the geometric centre of the body. The mathematical model can be looked up in the corresponding text book which contains formulae for computing the centres for a hemisphere and a cone as well as a formula to compute the overall centre of a body from its parts (the origin is the centre of the sphere and the vertical axis is called z-axis):

$$z_h = -\frac{3}{8} \cdot r, \quad z_c = \frac{h}{4}, \quad V_h = \frac{2}{3}\pi r^3, \quad V_c = \frac{1}{3}\pi r^2 h, \quad z_t = \frac{z_h V_h + z_c V_c}{V_h + V_c}$$

Therefore, mathematisation is reduced to finding the right formulae. The remaining mathematical problem then consists of solving the inequality $z_t \leq 0$ for h which yields $h \leq \sqrt{3} \cdot r$. No interpretation is required and validation simply means to compare the result with the one given in the book.

Potential for extended modelling opportunities

In this section we present three ways to reformulate the tumbler task such that more sub-processes of the modelling cycle are addressed. We also briefly outline which processes are concerned and how student work might look like. The fourth reformulation was given to students as project work and will be described in the next section in more detail.

1. Analysis of existing tumblers

One could give students real physical tumbler objects like the ones shown in the left picture and simply ask: *How and why do they work, how do you expect them to behave?* The first two examples are toys whereas the right ones are kitchen utensils: an "egg" made of stainless steel which can be used to remove bad odours from your hands after having cut onions; and a salt shaker. Being confronted with such a task, students first have to clarify the situation (1 in the modelling cycle.). What does it mean for a tumbler to work properly: Should it stand up in any "extreme" case or only if pushed moderately? Should it be possible to reorder the batch in the second example arbitrarily? In the second step students are to make assumptions and simplifications regarding geometry and material in order to set up a real model (2). For the mouse tumbler, the shape should be simplified and an assumption on the weight of the lower part should be made which also holds for the "odour egg". The lower part in the batch-of-discs toy might be simplified as the lower half of an ellipsoid, and here one might ask up to which point the "stand-up-property" is still guaranteed. Also assumptions on the density of the wood used in the wooden objects must be made or the weight could be measured. For the salt shaker object one might assume at first that the content does not move. Then students should find formulae for the simplified shapes and for compositions as in the original task (3). They should investigate whether the object stands up again after having been deflected. Except for the second object (batch of discs), this is similar to the original task since the lower parts have the shape of a hemisphere but one has to take into account that – except for the batch of discs – the objects are not homogeneous (4). The interpretation of the results seems to be easy at first sight: the result either says that the tumbler will or won't work. An interesting point here could be the interpretation of a result which is near the borderline between success and failure. In this case one should be very careful in interpreting the result given the assumptions made and the inaccuracies in the production of parts (5). Regarding validation one has the advantage of having a real object such that one can make experiments. So, one could check the result

for the batch-of-discs toy and possibly realize that the ellipsoid model is too coarse. Moreover, the results on the centre of gravity can be checked experimentally. Alternatively, one can also validate the computations by using a CAD programme which nowadays offers this functionality (6). Finally, in a project report students have to explain and justify their reasoning which addresses the final sub-process (7).

2. Design for stand-up in any position

One might also give the students the following task: *Design a tumbler that can be pushed arbitrarily and yet returns into an upright position (a "nasty-kid-tumbler"). Make it as simple as possible.* Here again, students first have to clarify the situation: What does "from any position" mean with respect to a tumbler? Will there only be one position (stable equilibrium) to which the tumbler always returns? Searching the web for existing tumblers might lead to the so-called Gömböc designed by two Hungarian scientists who first answered the question whether there exists a homogeneous convex body with only one stable and one instable equilibrium in the positive (Varkonyi & Domokos 2006). Another formulation that needs clarification concerns the "simplicity". This can be interpreted with respect to geometry, material or production of the part (1). In order to start with a simple geometry, students could come up with a shape like the one given in the original task. This would constitute the real model. Students could then proceed as in the original task to see under which conditions the tumbler stands up again when rotated by no more than 90°. But since it is required that the tumbler should return from any position one also has to consider the case where the tumbler lies on a surface line of the cone. One can then demonstrate mathematically that independent of the height of the cone the tumbler will never stand up again (4). How can this result be interpreted? It is clear that the result is consistent with the reports of the Gömböc since otherwise this would not have been an open problem for many years. The question that comes up then is: How can one modify the simple design such that the tumbler does the job without losing too much simplicity? (5) One can choose different directions here, e.g.: retain the shape but use different materials for the cone and the hemisphere; retain homogeneity but change the shape such that it is no longer convex. The latter option will be addressed in the next section. If one wants to retain the simple shape one has to investigate different densities of the materials used for cone and hemisphere such that the tumbler works. This is a non-trivial mathematical task resulting in the statement that the density of the cone must not exceed about 5% of the density of the hemisphere to make the tumbler work. Students can then search for materials that could be used, for example Balsa wood and steel. As to validation, the remarks made in 1 also hold (6).

3. Investigation of motion behavior

Another direction for investigation which is mathematically more challenging is provided by the following task: *How does the tumbler move and how can this be influenced?* One could even add a "market analysis" facet by asking: *How can you achieve a motion behavior which young children like?* In these questions, again a clarification of the task is required (1). What are the "characteristics of the motion behavior? The answer could be the frequency or the damping behavior or the amplitude when being pushed. In order to determine such characteristics of the motion behavior of a given tumbler students have

to set up a real model (2): Here, the tumbler can be reduced to its center of gravity and the geometry of the lower part (usually a hemisphere); for simplicity reasons one should neglect the rolling resistance at first. The motion is a combination of a translation and a rotation in the plane. In books on dynamics one can find a way to mathematise the situation (3) by setting up the motion curve of the centre of gravity and then setting up an equilibrium equation using d'Alembert's forces and moments of inertia. The motion curve is a so-called shortened cycloid and one ends up with a second order non-linear differential equation for the rotation angle of the tumbler which can only be solved numerically (4) (see e.g. (Ucke & Schlichting 2013) for the differential equation). Plotting the numerical solution provides an oscillating function with constant amplitude which makes sense since friction was neglected (5). For validation one could measure the oscillation period or even video the motion of a real tumbler and investigate it with video analysis software (6). Alternatively, a simulation in a CAD programme can be used.

Student work on the tumbler design project

The task given to a student group in the third semester (after having taken Mathematics I,II and Engineering Mechanics I, II) reads as follows (for a more comprehensive description of the project set-up see Alpers 2002): *Create a Maple® Worksheet in which a tumbler with a hemisphere as lower part and an arbitrary upper part can be designed and where one can determine whether it stands up again. Design some examples, transfer the designs to a CAD programme and have them produced.*

The students were familiar with tumbler toys, clarification was required regarding the requirements for a design environment. Since in the lectures preceeding the projects Bézier curves had been introduced it was a rather obvious decision to use these such that the user of the worksheet can set up and modify models with nearly arbitrary shape which are rotationally symmetric. They knew from their previous statics course that the position of the centre of gravity was essential for achieving the stand-up property and that it had to be below the separating plane such that the gravity force acting on the centre generates a moment that brings the tumbler into an upright position again. The students looked up the formula for computing the centre of gravity of a hemisphere, so the remaining task was to find the centre of gravity for the upper part which was generated by rotating a Bézier curve. Since they only found the integral formula for computing the volume and the geometric centre of a body generated by rotating a function graph, they computed about 10 points of the Bezier curve and let Maple® set up a spline function. Alternatively, they could have set up an own formula (which can also be found in some formularies but which does not seem to be wide spread) using modelling with differentials and using $dx = \frac{dx}{dt} \cdot dt$. They then used their own worksheet to design and investigate tumblers; they discussed with the author who supervised the project which ones might be interesting. It was decided to design one

that would not work (but very close to the border; the first one from the left in the picture above); one rather "ordinary" where the upper part is similar to a cone (second one); one that looks rather clumsy but snuggly (fourth one); and one with a considerable height (third and fifth one). The latter one should also be designed such that it stands up in any case, i.e. even when the tip touches the ground in order to make it really safe. This idea was brought in by the author, so some scaffolding might be necessary to point the students to interesting aspects. The students then computed several points of the upper part and imported them into a CAD programme where they reconstructed the body. They did not use the CAD facilities to validate their computations in Maple®. The CAD programme is able to generate CNC-code for milling or turning machines or STL files for 3D printers. Here the students (and the author!) learned about the intricacies of real production. The original idea was to use a milling machine but since the body could not be fixed properly, a turning lathe was used instead and even there it was quite hard to produce the "small diameters". They also used a 3D metal printer (fifth object in the picture) but as the photo already shows a considerable post processing effort is necessary to produce a smooth surface such that the object behaves properly. In the end validation was performed by testing the real object. The validation was successful except for the high object that was supposed to stand up in any case but when the tip touched the ground it did not stand up again. A look at the worksheet results showed that in theory it should but the centre of gravity was just 0.1 mm in the "safe" region. So, given the inaccuracies of production the lesson learnt was that there always should be a sufficient safety coefficient when one wants designs to work not only in simulations but in reality.

Conclusions

In this contribution we showed how a standard statics task like the tumbler task which offers just a few learning opportunities regarding mathematical modelling, can be reformulated such that many sub-processes of the modelling cycle can be addressed. The work of a student project group on one variation of the task demonstrated that students' thinking processes are really concerned with many parts of the cycle. Such projects can also serve to create a strong relationship between mathematics and engineering mechanics and hence let students experience mathematics as an integral part of their engineering study course and not just as a stumbling block at the beginning.

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Do you really know what resources your students use to learn mathematics?

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Abstract

The present paper is an investigation of the resources that engineering students use when they study mathematics. Our results showed that students use mostly resources provided by their institution and we conjecture that they choose these resources because of their exam-driven learning approach.

1. INTRODUCTION

Studying the role of tools in human practices is not a recent topic as early pioneers such as Lev Vygotsky underlined the importance of signs and tools as mediators of learning since the early 1920s (Vygotsky, 1978). Although historical records suggest that using tools has always been inseparable from expressing and doing mathematics, the use of the term "technology" is mostly used for referring to electronic devices (such as a calculator) something that manifests a kind of historical amnesia since the use of physical objects (i.e. tools) for the teaching and learning of mathematics has a longer tradition (Roberts, Leung, &Lins, 2013).

Students nowadays have access to a plethora of digital/online resources that they can use alongside more "traditional" ones (such as textbooks, lecturers or their own notes) and thus blend their learning. The notion of blended learning (BL) has a long tradition and according to Sharma (2010) there are three main definitions of BL used in education: the first considers BL as a combination of face-to-face and on-line teaching; the second as a combination of technologies; while the third as a combination of pedagogical methodologies. In this paper, we are mainly referring to BL as a combination of technologies or better as a combination of resources since each era has its own technologies not only digital ones.

In its current state, our literature review has revealed a lack of empirical studies adopting a holistic and thus a "blended" approach. Most of the literature is either focusing on few of the resources that students are provided with (by their institution or lecturer) or on digital/online resources only. In this way, a large portion of the resources that students are familiar with and may use to support their learning of mathematics is omitted. Conole, de Laat, Dillon and Darby (2008) studied the use of digital, online and hardware resources by undergraduates across disciplines by surveying 427 students and making a series of in-depth case studies (interviews and diaries). They found that although students access and use common hardware/software resources, the ways and frequency of use differ. Overall, students use extensively the web as an information source; they use communication technologies to a great degree; many of the reported resources are used

for assignment preparation and that students are beginning to use resources beyond their institution's virtual learning environment (VLE). Inglis, Palipana, Trenholm and Ward (2011) used behavioural data in order to determine how frequently a cohort of 534 engineering and mathematics undergraduates use three resources provided to them: online lectures hosted at the university's VLE, live lectures and the mathematics learning support centre (MLSC). They found that none of the students were making above average use of more than one resource: students were making heavy use of only one resource or weren't using any of them at all. Rønning's (2014) survey is among the small number of studies adopting a "blended" approach by acknowledging a variety of resources that students may use. In his survey, 662 engineering students were asked how frequently they use certain resources for one module (Calculus I). Besides digital or online ones his questionnaire also included resources such as the recommended textbook and human resources such as the drop-in centre and lectures. The results showed that more than 80% of the students attend lectures on a regular basis and almost 90% use extensively the textbook. Rønning concluded that although students are provided with a large variety of resources (e.g. video recorded lectures, theory on the home page of the module) they prefer to use more frequently the traditional ones (textbook, lectures).

Based on the literature review, we were interested in identifying what kind of resources undergraduates use when studying mathematics, which resources they use the most and why they use certain resources.

2. METHODS

In order to answer the above research questions, we analysed data from a survey and follow-up individual interviews with students. Both the survey and the interviews are part of a larger case study that follows a parallel mixed methods design (Teddlie and Tashakkori, 2009). This paper presents findings from investigations into the resources used by second year engineering students at Loughborough University. Loughborough has one of the largest cohorts of engineering students (over 3000 undergraduates) in the UK and is a leader in the provision of Mathematics Support. It has also led on significant projects producing high quality printed material (e.g. the HELM project) and so students had a great deal of choice in available resources.

The design of the questionnaire and the interview protocol was guided by Activity Theory (AT) (Leontiev, 1978). AT is not a theory in a broad sense (e.g. a predictive theory) but rather a theoretical framework aiming at studying different forms of human practices in everyday life circumstances by accounting for both the individual and her/his social context at the same time (Kaptelinin & Nardi, 2006). AT was particularly helpful in designing our approach and gathering data but space limitations restrict us from discussing further this aspect of the study.

During November-December of 2015, a paper-based questionnaire was administered to four different groups of second year engineering students and in total 201 completed it. Here we only report on two parts of the questionnaire related to the resources that undergraduates use: the first was asking students to identify how often they use a list of

15 resources on a 6-point Likert scale (1/Never, 2, 3, 4, 5, 6/Always) with two additional open ended items for other resources not listed in the questionnaire; the second, was asking them to identify which five of these 15 resources they use the most and rank them in a descending order (top-5 list of resources). The 15 resources listed in the questionnaire were identified through the literature review, five in depth interviews with engineering and mathematics students conducted during the spring semester of 2015 and the resources that Loughborough University offers to students. The list was carefully generated in order to include all the possible resources that students have at their disposal ranging from physical artefacts (such as the HELM Workbooks or students' written lecture notes) to human resources (such as lecturers or the Mathematics Support Centre) and digital/online resources (such as Wolfram Alpha, YouTube) or "LEARN" -the university's VLE which hosts a variety of resources such as lecture slides, problem sheets and past exam papers). In this way, the list encompasses a great variety of resources available to students and thus reflects -to a certain degree- their reality as learners when it comes to the resources they use when studying mathematics.

All students that completed the survey were invited to participate in interviews and in total 6 of them volunteered. The interviews were semi-structured and lasted for approximately 60 minutes each. Students were mainly asked to describe how they use their top-5 list of resources which included different resources for each one of them.

3. RESULTS AND ANALYSIS

3.1. What kind of resources do students use and how frequently?

The results of the survey are presented in Figure 1, where all the resources are listed in a descending order according to their mean (x). By using each resource's mean, we categorised them into three main groups: resources with a mean greater than 4.5 were

Figure 1: Resources used by undergraduate engineering students (mean and grouping)
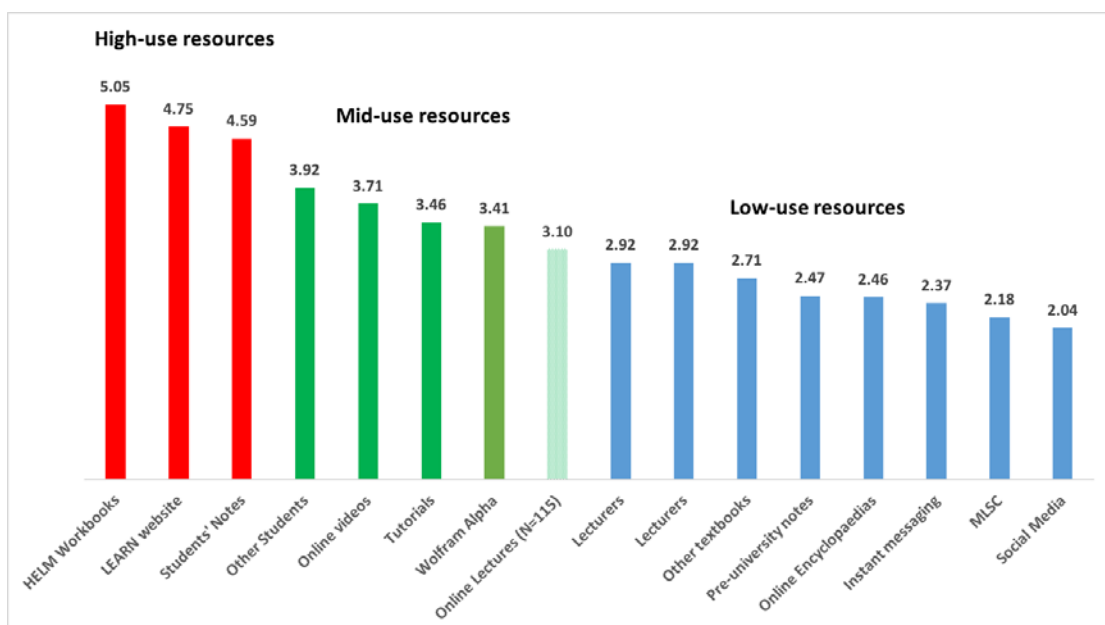
characterised as *high-use* resources (x ≥ 4.5, red-coloured bars), those with a mean between 3 and 4.5 were assigned into the *mid-use* group (3 ≤ x < 4.5, green-coloured bars) while resources with a mean between 1.5 and 3 were put into the *low-use* group (1.5 ≤ x < 3, blue-coloured bars). We didn't create a fourth group named "no-use" for resources with a mean smaller than 1.5 since there were no resources with such a mean. Finally, we can see in Figure 1 that all but one of the high-use resources are the ones provided to students by their university and that certain "external" resources populate the mid-use range (online videos, Wolfram Alpha).

3.2 Which resources do students prefer to use the most?

The resources that students reported as using the most (top-5) and their rank are presented in Figures 2a-2e. Each figure corresponds to the five ranks of the top-5 and for each rank

Figure 2a: First most used resources (5 resources)
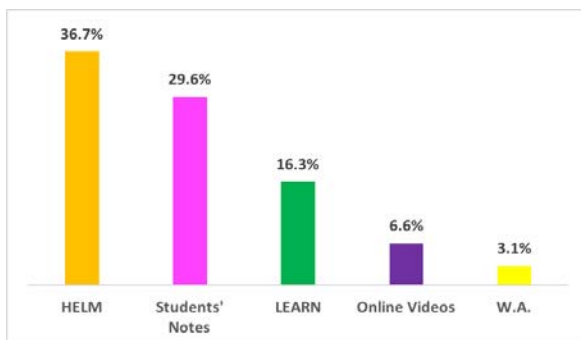
Figure 2b: Second most used resources (8 resources)
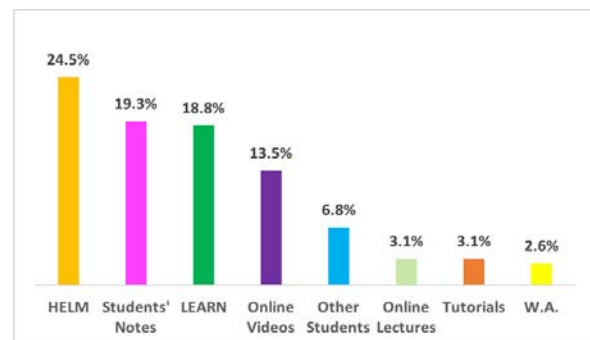
Figure 2c: Third most used resources (10 resources)
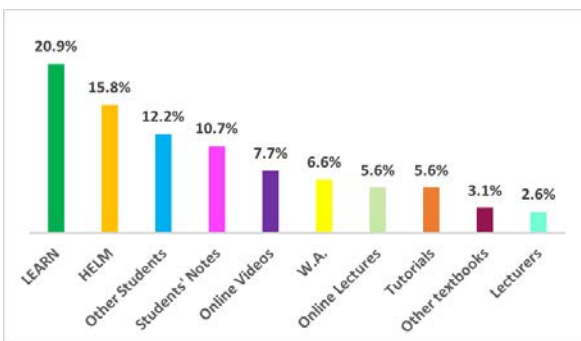
Figure 2d: Fourth most used resources (11 resources)
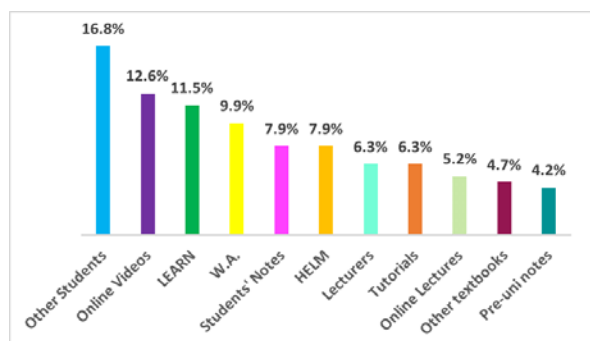
Figure 2e: Fifth most used resources (12 resources)
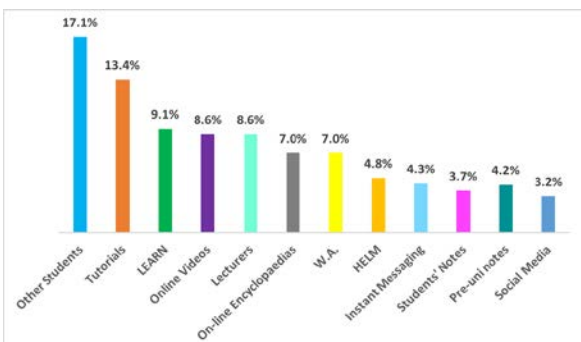
we have included only those resources that their cumulative percentage is just above 90%; in other words for each rank we have acknowledged the variability of resources for 90% of the students. For example, figure 2a shows the resources that students reported as their first choice: 36.7% of them reported the HELM Workbooks, 29.6% their own written lecture notes (Notes), 16.3% the LEARN website, 6.6% online videos and 3.1% Wolfram Alpha (WA). As shown at Figures 2a-2e, there are certain resources that appear at all ranks and these are: HELM Workbooks, the LEARN website, students' own written notes, online videos, Wolfram Alpha. We can also notice that as we move on from the first to the fifth choice we have an increased variability of resources (for example, 90% of first choices consist of 5 resources while 90% of fifth choices consist of 12 resources). Moreover, human resources (such as lecturers, staff at tutorials, other students, the MLSC) populate places at lower ranks and they have low percentages of use: for instance, "students" as a resource appear for the first time at the second place of the top-5 and have a 6.8 percentage; tutorials appear for the first time at the second place with a 3.1 percentage and lecturers appear for the first time at the third place having a 2.6 percentage. Finally, only 115 students had access to online lectures and therefore the figures for this resource would likely to have been higher if all students had access.

## 3.3 Why do students use certain resources?

From our work in section 3.2, we see that certain resources appear across all ranks of students' top-5 list. Interviews with the students enabled us to identify some of the reasons for their choices and helped us to interpret the results of the survey. The HELM Workbooks are used as a starting point, a resource that in some cases "*kind of replaces the lecturer*". Students also use them for finding additional problems to solve or for enhancing their lecture notes. Their structure enables them to keep on track and have the required amount of knowledge for a topic. The LEARN website is appreciated because of its ease of access and the resources hosted there. Students use it for organising their student life (check timetables, identify key dates such as for a test, see the topics covered each week) but mostly when preparing for a test or exams. During such periods they access past papers and their full solutions and some of them even time themselves when trying to solve them, replicating in a sense the exams environment. It seems that students use the LEARN website mostly because of the resources hosted there which are used to "*find out what* [they] *need to know*" and "*what* [they]*'ve got to do*" but "*rarely use*[d] *as a way to learn maths*". Students' own written notes are a very personalised resource that corresponds to their learning needs and styles. The way they are constructed depends heavily on the way their lecturer presents the material during a lecture. Some of the students enhance their notes with content from other resources (e.g. HELM Workbooks or YouTube) or they make condensed versions of them that mirror the material that they are allowed to have during exams (for example the formula book provided by the university).. Online videos are used when other resources cannot offer a satisfactory explanation of a topic and/or when seeking for an alternative explanation than the one presented by a lecturer. Finally, W.A. is used as a contemporary calculator for solving large equations or calculating large integrals and/or for checking results when solving problems. None of the students reported using it for other purposes or using W.A.'s other features.

4. DISCUSSION

Our survey results showed that students use mostly resources provided by their institution and that students also use resources "external" to their university although to a lesser degree. After interviewing the students we found that the reasons for using these resources are very much linked to their goals. AT asserts that goals can be seen as the short-term "reasons" for which a subject acts and thus uses a tool/resource. Our analysis showed that students' goals are mostly exam-driven: it seems that students adopt a strategic learning approach and it is this approach that drives them to these resources.

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Adaptive Teaching of Mathematics for Engineering Students

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Abstract

Adaptive teaching comprises several key ideas: adaptive use of tools to achieve teaching goals, adaptation to the knowledge level of students, adapting course settings and teacher attitude, and evaluation that supports adaptive teaching. This paper explains the need for adaptive teaching of mathematics to engineering students, the theoretical background for this approach and illustrates the concepts on two mathematical courses taught to computer science students in their first year.

Introduction

One of the common misconceptions in teaching of mathematics is that since the content of the basic mathematical courses that engineering students need to take does not change, there is no need to change the current mathematical courses. We claim the opposite - that the approach to teaching mathematics should be very adaptive for several reasons. One reason is that the cohort of incoming students vary – both in attitude to mathematics, e.g. apprehension, fear, etc. and in their mathematical background. This might be due to the fact that the secondary school curriculum changes, the interest in different types of colleges varies and as a result we teach "different" students. Also, the fields they are preparing for are constantly developing (especially so in Computer Science). It is our task to help students learn and understand these topics to an adequate degree of detail that they will need for their future study and professional work.

Therefore, we have put the emphasis on teaching mathematical thinking, identification and formulation of questions and on problem solving strategies, instead of teaching just exact mathematical theories, facts and formulas. At the same time, we try to instil the notion that mathematics is one of the most helpful tools they will use in the future. The learning should be based on the students' previous life-experience including their learning-experience and directly relevant to their primary field of study and interest rather than being a disconnected theoretical area. We should also adaptively use technology already familiar to the students that supports our teaching of mathematics instead of becoming another topic to learn.

In this work, we discuss these principles, provide their theoretical background and illustrate them on examples taken from undergraduate courses Mathematics for Computer Science and Discrete Probability.

Theoretical background

Our motivation for adaptive teaching is based on the concept of scheme-oriented education and the fact that it is not enough to teach only mathematical content, we need to teach mathematical thinking (Cardella 2008) and build intuition. "A scheme, in psychology and cognitive science, describes an organized pattern of thought or behaviour. It can also be described as a mental structure or pre-conceived ideas, a framework representing some aspect of the world, or a system of organizing and perceiving new information. [Schemes] influence attention and the absorption of new knowledge: people are more likely to notice things that fit into their scheme, while reinterpreting contradictions to the scheme as exceptions or distorting them to fit." (Gerrig 2012).

The concept of building mathematical knowledge as a network of interconnected experiences, concepts, methods and analogies was developed by Hejný et al. (2012). It shows a deep understanding of the psychological background of education in abstract sciences. The latest and most broadly used application of this research is a new method for teaching mathematics at primary schools known as Scheme-oriented education. It is widely and successfully used in primary education in Czech Republic and can also be applied in other stages of education.

A related concept is the theory of generic models (TGM) developed by Vít and Milan Hejný (2006). Its premise is that there is a universal rule that governs the understanding of mathematical ideas, which happens in several stages. The first stage is motivation – interest in learning about the given concept. The second stage is the stage of isolated models – first encounters with the concept that are not interconnected. The first abstraction lift is the stage, which creates a generic model from the cluster of isolated models and allows practical solutions of a broad range of problems. For general use, it is sufficient to learn the mathematical concepts at this level of understanding. Once the generic models are established, it is possible to start building the mental schemes described in the Scheme-oriented education and build abstract concepts - the second abstraction lift. The last stage is automatisation of usage of the given concept and its incorporation into the structure of other mathematical knowledge. The last two stages are typical for deep professional study and use of mathematics.

The theory of generic models also takes into account the historical development of the individual areas in mathematics. It turns out that not respecting this progression and skipping certain periods is a cause of forming formal, unusable mathematical knowledge, which fuels a negative attitude towards mathematics and its usage.

When working with our students, we have realized that they perceive mathematics as a self-contained, compact and formal theory, which does not have overlaps into their computer science knowledge. Moreover, the teachers who themselves were either mathematicians or computer scientists did not use or know such connections of the two disciplines. This resulted in the notion that students waste time learning theoretical facts, which they cannot interconnect and use in their major. When exposed to mathematical problems in programming class, their active interest tends to turn to passive acceptance of facts.

Originally, we hoped to merge the first-year mathematics and computer science courses into one, where we would teach the students to think in both areas simultaneously and connect the concepts. For staffing reasons, this did not work out, but we have implemented the overlap between mathematics and computer science in two courses (1 semester each).

In accordance with the above mentioned theories of Gerrig and Hejný, we have decided to teach the students how to discover the relations and connections among mathematical concepts and

their computer science analogies on a smaller scope of facts. Instead of teaching recipes and methods, we have given the students the opportunity to look for, express and defend their own solutions. The knowledge and experience from problem solving defended in front of the classmates were not only more durable but also more elastic than those classically learnt. The students were able to transfer them to other areas and reuse them. Often, their mathematical expressions and formulations were not polished, sometimes not even precise. It has taken a few weeks to clean up all the inconsistencies, however when based on the students' own experience and generalisation, the gained concepts were much more strongly anchored and more usable.

Description of the courses

In the following sections, we describe our empirical experience gained over 2 years of teaching the 12-week course Mathematics for computer science (MCS) and 5 years of teaching its 12-week sequel: Discrete probability (DP). The content of both of these is influenced by the MIT course Mathematics for computer science (Rubinfeld, 2005).

Typically, we teach around 250 first-year computer science students taking these courses every year. Every week they attend a plenary lecture and then they attend recitation classes in sections of 15-20 students. The recitation classes have two parts: the first one involves problem solving with pen and paper and on the board (2 hours), the second part involves solving illustrative problems on the computer (2 hours for MCS and 1 hour for DP). The students also report that, on average, they spend around 2 hours per week preparing at home for the in-class quizzes.

Adaptive teaching

1. Adaptive use of tools to achieve teaching goals:
In order to show to the computer science students the usefulness of mathematics, connections to their field of study and to make it more interesting for them, every week we included problems to be solved on computers. The majority of the work was done during the class and what was left, was finished at home. Absorbing the new mathematical concepts by using them to create small applications allows the students to adjust the learning rate and connect the new experiences with the mental schemes in the computer science area they already have.
The problems illustrated the mathematical ideas that were taught. One such example is coin-changer. When teaching combination theory and generating functions, we have asked the students to compute how many ways there are to change a given sum of money into available coins (€2, €1, 50c, 20c, 10c) by both a brute force calculation and compare and the theoretical result gained using the convolution of generating functions.

In the first year we found out that when they were asked to use either Java (the programming language they are taught in the first year) or another programming language they might know from high school, the problem solving typically involves issues around programming that should be addressed in the Informatics course and the mathematical problem is just an afterthought. Therefore we have switched to using Excel, i.e. a tool, which is not new for majority of them, which they are comfortable using and thus they can focus on the mathematical problem at hand. Some of the problems had to be reformulated in simpler form (e.g. the coin changer only used €2, €1 and 50c coins), but they had to still come up with an algorithm and use Excel functions to implement it.

Another advantage of Excel is that it allows us to revisit the same concept from different perspectives and compare the results in one place. Consider binomial distribution. We have used simulations in Excel to calculate a probability density function of a random variable with

binomial distribution (elections in 1000 villages, each of which has 10 inhabitants and each person votes for candidate A with a probability *p* and for candidate B with probability 1-*p*). We then used the theoretical formulas they have learned in the lecture and compared the results. Finally we used the built-in Excel binomdist() function which gave the same results as the theoretical formula. As expected, we found that once the students have the experience of generating the distribution themselves, they understand the theoretical formulas better and do not use the built-in formulas as a black box.

2. Adaptation to the knowledge level of students: Hetero- vs homogeneous sections:
We measure the level of knowledge on entry and grades from the prerequisite classes. This information is used to re-group the students at the beginning of the semester. With the number of first-year students we have, there are typically 15-16 sections of 15-20 students that have the recitations together. We aim to have the schedule done in such a way that there are always at least two parallel sections. At the beginning of the semester, we give them an entrance test and then we divide them in 2 (or 3 if there are 3 parallel sections) new sections according to the results – alpha and beta. This way we get more homogeneous sections. The test also includes a question on their personal preference, but we have seen that in the vast majority of the cases this preference corresponds with the results we get from the other questions. When it does not, the concerns are addressed individually.

We have faced criticism that in this kind of "separation of good students from the bad", we do not teach everybody the same amount of material, that the "smart students do not pull the weak students forward". Our experience is that by adapting the content of the class to the actual level of the students, we teach as much as they can absorb (in both sections) and in the long term teach them more (in both sections) than we would "at the average level". While it is true that the classes are structured differently, surprisingly there might be fewer problems discussed in the alpha section. This is due to the fact that in beta sections, more easier problems are needed to let the concept sink in and build confidence into mathematical calculations. In the alpha section, fewer problems suffice for this purpose and it leaves plenty of time to examine these problems from various what-if perspectives and dig into more sophisticated questions. As explained by TGM, the beta groups collect the isolated models, while the alpha groups examine the properties of generic models.

We emphasize, that the requirements during the semester and at the exam, as well as the study materials, were the same in both groups, The main difference and advantage of this approach was in the pace and type of work done in the groups. The students who are on the same level can much better share and exchange experience gained when solving problems. The students in beta groups gained more confidence, more attention of the teacher, both of which helped them to master the material. The students in alpha groups were not demotivated by too easy problems and slow pace. They could focus on deeper understanding and gain the fulfilment from intellectual progress.

3. Adapting course settings and teacher attitude
It had been shown that besides the grit and positive life-attitude (Duckworth 2009) the most effective teachers are those who get the most ideas across, those who are able to "kindle a fire" instead of "fill a vessel", are those who set big goals and are perpetually looking for ways to improve their teaching (Farr 2010). They are always in the middle of blowing up their classroom structure because they have an idea how it could be working better. They constantly reevaluate what they are doing. This kind of adaptive attitude can be illustrated by our approach to grading the problems solved using computers.

We wanted the students to work on those problems, so we gave them points for that. Every week, they could earn 5 points, 2 of which were for the computer problems. It turned out, that students spent time at home to work on these problems but then did not spend time to prepare and study for the class. Therefore, the next year, we have decided to only give points for in-class quizzes. As a result, the students lost interest in working on the computer problems. So we had to again modify the grading for the next year. The computer problems did not have any points assigned, but only the students who submitted correct solutions to the computer problems received their full scores from quizzes. Incorrect or not submitted computer problems resulted in 50% reduction of quiz points. We are prepared – should the necessity arise – to change this again to achieve the goal that the illustrative computer problems are done and at the same time, the students dedicate enough focus to the mathematical content.

4. Evaluation that supports adaptive teaching
In both courses, 60% of the grade is for the work done during the semester. The weekly in-class approximately 10 minute quizzes comprise a theoretical question that checks basic orientation in the latest lecture and a problem such as those that were discussed in previous recitation. The lecture materials, theoretical questions and sets of problems are available to the students in advance through an electronic system and the course evaluations confirm that the students use them when preparing for the quizzes.

The initial preparation and ongoing updates of the materials require significant time investment from the teachers, but it is crucial that the content, difficulty level and focus of the questions and problems lead to the true, appropriate and natural building of the desired mathematical concepts. Moreover, the students gain working habits and monitor their effects weekly by the quizzes. They are also up to date with the course material and the instructor can build on actual understanding of previous concepts.

The most important gain both for the teacher and the student is the in-time feedback. It allows the teacher to monitor and adjust the pace and difficulty of the in-class topics, focus the content and recognize and address issues as they arise.

The energy used for weekly testing pays off during the final exam. We only allow students who have gained at least 50% of points during the semester to take the final exam. This exam consists of a set of multiple choice theoretical questions generated from the weekly sets (answered on the computer) and only students who pass this part take a written exam that comprises problems from the sets available during the semester. The active role of the teacher is needed only at this stage, where it is focused on the students who have demonstrated sufficient both long-term and current knowledge. This system allows effective and objective examination and grading and may be adjusted annually as necessary. The perception of the students is that the system is reasonable and justly evaluates their knowledge, work efforts and contributes to their education.

Benefits and Conclusions for Education

To summarize, the main benefits of adaptive teaching are
- Continuous feedback, both for students and teachers, both in terms of grades and knowledge
- Work plan is available for students, continuous work throughout the semester
- During the semester, the students are up-to-date with the material and it is easy to build on previous topics
- Also weaker students can advance and master the material
- Better students can work on more difficult problems and are not held back by too slow pace

- Students are more involved, see the meaningfulness of their work and feel the responsibility for the outcomes
- Student-teacher relationship is based on their common educational goal

The teaching of mathematics should primarily develop logical thinking. This is not only an overused phrase but also an actual requirement our graduates have to meet when entering the workforce and the managers of respected IT companies that employ hundreds of our former students emphasize it at regular meetings. By helping the students to develop this quality we help them to be successful and find fulfilment.

By logical thinking, we mean the ability to create rich mental schemes in mathematics and abstract concepts, which can only arise from well-built generic models. Without building this ability, we produce useless and sometimes even harmful stacks of "mathematical knowledge". In practice, this means to respect the actual level of students' understanding in the individual areas and starting from there. It also means using activities with appropriate level of difficulty and allowing students to revisit the same concept repeatedly, ideally in different contexts. Some activity, initiative and responsibility needs to be left to the students, but in their first years of study it is helpful to offer a work plan, support and guidance.

Achieving this imposes fairly large requirements on the teachers. An important part of their success is the ability, willingness and intention to continuously adjust and re-evaluate the content of their work based on the understanding and learning of their students.

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Using Maple as a tool while studying calculus

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Abstract

A traditional calculus course for engineering students was reformed in 2003 by introducing Maple. Students were advised to use the program for visualisation and computation. They brought their personal laptops during lessons and examination. The course was launched within a department where the tradition is *not* to use digital tools in basic courses. Integrating the use of Maple turned out to change the course in several ways, some intended and some not foreseen. Inquiries among students indicate positive but varying attitudes toward computer support. Video-recordings of sessions of cooperative learning show in what ways Maple influenced the mathematical content of students' discussions. This contribution presents some findings from the course.

Introduction

Since the mid 1980's the use of computers and other digital tools in mathematics education has attracted a growing interest among mathematicians in Sweden. The issue became particularly topical in the late 1990's. In 1998 the National Agency for Higher Education (1999) organised a national conference on the use of computers in undergraduate mathematics education. According to an inquiry study presented at the conference by Bergsten (1999) almost all mathematics departments in Sweden had introduced computer use into some courses at that time, most often as laboratory work. The intention was to develop students' understanding of mathematical concepts and/or let them use the computer as an aid for computation and modelling. Bergsten found that the results in general were positive. However, he concluded that a number of issues appeared as problematic, such as lack of teacher resources and technical facilities, weak connection between the learning goals and the computer work, lack of time or rather competition for time within the course. A few years later and partly inspired by the discussions during the conference, a calculus course with a more intense use of the computer was developed at the engineering faculty of Lund University. In this case the method was a complete *integration* of the computer into the entirety of the course.

The background and the reasons for the reform are presented below. The next section describes two theoretical perspectives that contribute to the understanding of the intervention and its outcome. The reformed course with its content, the teaching and the role of Maple is presented in the next section, followed by a section on the results. A discussion concludes the paper.

A new engineering program with emphasis on computer skills

The engineering faculty in Lund (LTH) admits almost 1000 students every year to over 15 programs leading to an MSc in Engineering in five years. The basic mathematics courses are identical for all MSc engineering programs: calculus of one variable, linear algebra, calculus of several variables and finally statistics, in total 34.5 ECTS-points. A new program, Environmental Engineering, was launched in 1998. The steering documents emphasize high computer skills as one central goal (Reistad, 2004). The program management strongly encourage all departments to integrate the use of computers into their courses. In order to facilitate computer work all students in the program could borrow a personal laptop from the university. Due to financial reasons students have had to use their own computers since 2014.

A few years after the start of the Environmental Engineering program it became apparent that the course on calculus of several variables was causing problems. With a large number of dropouts and failed results at examination it stood out as an obstacle for further studies. The program management found the situation alarming and turned to the mathematics department to discuss modifications of the teaching. In this case, with access to a personal laptop for all students at all times, it was natural to pose the question: Would it be possible to enhance students' learning and remove some learning obstacles through the support of a suitable computer program?

In 2003 the department decided to design and try out a reformed course on calculus for several variables for the Environmental Engineering students with integrated use of Maple. Formally, the course remained the same. The reformed course was made permanent and taught every year until 2013. The next year a heavy saving program at the faculty forced the department to put down the course and let the students study together with students from another engineering program in a "traditional" setting.

Theoretical perspectives

Buchberger (1990) addressed the question whether students should implement by hand algorithms which computers can perform faster and more efficiently. He proposed a theoretical model that avoids a simple yes/no answer and captures two ways to teach with the use of symbolic computations systems. In a White-Box (WB) phase, students are taught concepts and theories that underlie an algorithm, study all steps of the algorithm and perform hand calculations until these become routine work. They may profit from doing experiments using the software with the algorithm during this work. Students may also rely on the computer program for algorithms belonging to areas they have already studied. In a Black-Box (BB) phase, when the students have learnt the algorithms and the underlying concepts, they are encouraged to let the symbolic system perform the algorithms in order to save time.

The WBBB model does not problematize? capture? the interaction between the student and the computer program. A theoretical framework that helps to clarify the interaction is instrumentation theory applied to teaching with the use of technology (Artigue, 2002; Drijvers et al., 2010). Here an important distinction is made between the tool in itself and the tool used for a certain purpose. A physical or symbolic *artifact* (object), which is intentionally used for some purpose, is viewed as an *instrument*. The *instrumental genesis* is the process during which the student develops her/his interaction with the computer program so that the program becomes an *instrument for learning*. The student gets used to the representations of mathematical objects in the program and learns techniques for communication with the program (the instrumented techniques). The process also includes thinking about possible uses, constraints and limitations of the program. A result from research is that the process is essential and time-consuming (Artigue, 2002). Teachers have to take the instrumental genesis into account when planning and implementing students' use of computer programs.

A reformed course: content, teaching, the use of Maple and examination

The standard course on calculus for several variables at LTH covers functions, vector valued functions, continuity, partial derivatives, differentiability, optimisation, partial differential equations, multiple integrals, line integrals and applications. When taught in a course of no more than 6 ECTS-points during 7 weeks the pace will obviously be fast. The emphasis is on

concepts and methods, but theory is included and students are supposed to be able to produce some definitions, theorems and proofs at examination, besides solutions to problems. Students spend a lot of time doing relatively simple but time-consuming calculations, essentially the same as they learn in one variable calculus and linear algebra (differentiation, finding primitive functions, computing integrals, solving equations and systems of equations etc.) but with some additional techniques.

For the reformed course two main learning goals were added. One was to learn the basics of Maple and the other to learn to use Maple critically for mathematics learning. Maple was used for visualisation of surfaces, graphs of functions, level curves, direction fields etc. Maple was also used for all relevant computations. Since Maple was "added" to the course, something had to be left out. Obvious choices were to reduce the demands on computational fluency (by hand) and leave out exercises to sketch curves and surfaces in three dimensions. Also, theory for line integrals was treated more briefly.

The teaching was radically changed when the course was reformed in 2003. The reform reduced the total number of teaching hours per student from 39 sessions of 2*45 minutes (25 lectures, 14 problem sessions) to 28. A small number (7) of lectures are combined with supervised lessons (21) with co-operative work. The lectures introduce and outline main ideas but all details are left to students' own reading. The students bring their laptops to the co-operative sessions. The sessions are guided by step-wise instructions for students' work with theory and problem solving. The instructions include a few main learning goals for each session, also goals related to Maple. Recommendations for out of class preparations for the next session are also found in the instructions.

Maple is introduced during the first week with connection to the study of quadratic surfaces. The students soon get acquainted with the command structure and input alternatives (using icons or sequences of symbols) in Maple. New commands are introduced successively during the whole course in conjunction with relevant topics. In principle the use of Maple follows the WBBB model. Students learn to handle new algorithms by hand first and then move on to use Maple. Hand calculations are restricted to simple cases without heavy computations. Algorithms from calculus of one variable and linear algebra are used directly with Maple. The interactive graphical tools are used to investigate curves and surfaces. The students realise through using the tools that scaling, choice of window, discretisation and interpolation influence the graphical outcome. They gain experiences of the strengths and limitations of the program. The recurrent use of Maple during the course gives the students a confidence in handling Maple. To a certain extent, each student has a choice to do calculations by hand or to use Maple when solving problems and exercises.

The form of examination is the same as earlier, a written exam with mainly problems to solve. The students were allowed to bring and use their laptops with Maple during the examination. The examination consisted of two parts during the initial period, the first with free use of Maple, and the second with no aids. However, after some years with this model, it was found that the second part was superfluous. With carefully chosen and formulated problems it turned out to be possible to let students have access to Maple during the whole examination and still let them demonstrate their knowledge of relevant theory, concepts, algorithms and methods. This simplified the administration of the exam. Most years the students delivered their examination papers in handwriting.

Methods and findings

The reformed course did not stay exactly the same through the years (2003 – 2013). Several teachers taught the course either as lecturer and supervisor at co-operative sessions or as supervisor. The teachers shared their experiences. The course was developed and adjusted according to these experiences, to each lecturer's views and to the students' opinions.

The course was evaluated every year with an inquiry to the students in connection to the exam. Examination results were analysed in depth in 2005. In 2013 a number of groups were video-recorded during lessons with co-operative work. Each group was recorded during one session of 90 minutes. The findings presented here are mainly based on the inquiries and analysis of the recordings but also on the author's experiences from developing the course and sharing experiences with other teachers.

Examination results were greatly improved with the reformed course. Two groups of students were compared. One group of students had followed the traditional course without Maple in 2000 – 2002 and the other followed the reformed course during 2003 – 2005. Attendance rose markedly in the reformed course and the proportion of students who passed the exam at the (first) regular opportunity increased from 32% to 70% of the registered students. Two factors influenced the rise, a larger proportion of students showed up at the exam and the proportion that passed increased (Larsson & Werner, 2006). The outcome was a result of the whole reform with the introduction of Maple and the new teaching. The relative importance of the interventions is not clear.

The students found the instrumented techniques with Maple easy to learn. This may be attributed to three factors: (1) the students' computer competence, acquired from earlier courses, (2) the similarities between Maple symbolism and syntax and common mathematical symbolism and syntax and (3) the time allocated to Maple in the course. The attitudes were mostly positive. According to the 2013 inquiry 89% of those who responded agreed with the statement "*The use of Maple has made the course more fun and interesting*". A couple of comments illustrate what the students have in mind.

"I find that the use of Maple contributed to better understanding through the 3d diagrams."

"Fun and instructive to use Maple in order to make the problems more understandable."

"The use of Maple made this course the most interesting mathematics course I have studied."

"Maple was a very good tool, even if it was a bit scary in the beginning, since I had no experience of programming."

The attitude varied more than expected when choosing between Maple or paper and pencil. Many students chose to use Maple as much as possible and some developed the instrument further and found new commands and options on their own. Other students preferred to leave out Maple whenever that was an alternative. It made them feel safer about their own learning and was more in line with earlier mathematics courses. Now and then students encountered minor problems related to Maple, most often the syntax. Most of these problems were solved among the students during the co-operative work but sometimes they caused frustration and the students lost time. Some students expressed uncertainty about the role of Maple in the examination:

"Am I supposed to know how to do this by hand, or only with Maple or should I master both methods?"

as one student formulated the worries. Some students questioned the examination in relation to Maple and called for another form, more adapted to the reformed course.

A new balance emerges when students solve tasks with Maple and students' interpretation of mathematical concepts is affected. Two examples will illustrate this.

1. A distinction is made in Maple between expression and function. The formalism helps students realise why it is important to distinguish between f and $f(x, y)$ and to grasp the concept of a function. They realise that the use of the symbol $f(x, y)$ in mathematical literature is handy but sometimes ambiguous.

2. One type of task in the course is to compute the integral of a function f over a domain $D \subset R^2$. One situation is that f and D are defined in Cartesian coordinates or geometrically, where the definitions transform to simple expressions in polar coordinates. The technique for solving this type of task is to (1) sketch D and realise that polar coordinates may be relevant, (2) find the definitions of f and D in polar coordinates, (3) transform the integral, (4) find the integration limits and rewrite it as iterated simple integrals, (5) compute the integrals and (6) evaluate the result. Often step (5) is the most time-consuming. With Maple one important step is added, namely to let Maple plot f over D. Step (5) is done by Maple. Varying the function f and inserting the new expression into the Maple command easily create new tasks of the same type. The integrals for different $f:s$ may be estimated and compared with help of the Maple diagrams of the graphs of the functions. The interpretation of integrals as "volumes with signs" will be enforced and step (6) enriched. This last example illustrates that focus is less on computation and more on visualisation and conceptual knowledge.

Discussion

The course was successful and the main goals were reached. Maple was well integrated into the course and students valued the freedom to choose if and when to use Maple. For some students the form of examination probably was the reason to use paper and pencil when that was an option. One major drawback at the start was the lack of suitable literature, which inspired three of the teachers to write a book on the topic with Maple integrated. The frames created by the traditions of the institution limited the development of the reformed course in several ways. One example was the requirement that the course should formally stay the same. The reason behind is the policy that basic courses must be identical for many engineering programs in order to facilitate a change of program. The result was a compromise that, for instance, prevented a rational form for the examination of the Maple course. More important is the view within the institution of what constitutes legitimate mathematical knowledge, what types of tasks and what techniques students are supposed to master. The main question is whether certain knowledge about a system like Maple is valuable in itself and adds to desirable mathematical competence. The answer ought to be based on an understanding of the contributions of the instrumental genesis to the mathematical knowledge. This understanding is probably best acquired by teaching a course resembling the reformed.

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Embedding mathematics content within the electronics courses for engineering students

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Abstract

This paper reports students' working methods and performance following the introduction of integrated approaches for teaching mathematics within the programme for electronics engineering students. In 2014 the programme was revised with the aim of strengthening students' performance in the bachelor programme in electronics through the integration of mathematics with the student's technical subjects. The intention is that this will contribute to a better learning processes and increased understanding of mathematics. Although the revision has only been in place for one and a half semesters, feedback and results so far indicate that students work harder and perform better than in previous years.

Introduction

Mathematics used to be taught in common courses for all engineering fields. This way mathematics was taught in general with few examples from each field. Topics taught in the first semester may not be relevant at all or will not be applied until maybe the fourth or fifth semester. This can demotivate students since they struggle to see the relevance of the mathematics they study. Large calculus courses had over 450 students attending. It was in fact already taught in an innovative way using technology such as interactive simulations and digital assessment tools (for an overview see Brekke (2015)). Using this kind of technology student performance did improve, but with only one lecturer teaching was characteristically one way communication from lecturer to the students. The idea of this project came after several talks with lecturers teaching subjects in electronics. The schedule of the mathematics courses never seemed to quite fit the schedule of the electronic courses. For instance complex numbers was taught in mathematics after it had been met in the introduction course in electronics. This was also the case for several other topics. I realized this could be done in a better way. With a background of running several online courses in pre-Calculus and pre-Physics we had experience of the possible to use technology in teaching and learning (for an overview see Brekke (2014)). Works from Wiemann, Perkins, & Adams (2008) also suggest that interactive simulations can be unique and powerful educational tools. However they should be carefully designed, tested and used in pedagogically effective ways. In January 2014 we applied for funding of this revision and received in total 370.000 NKr (about 40.000 Euro). Planning during 2014 led to a redesign of the bachelor programme in electronics, to optimize the connection between mathematics courses and electronic courses. This was a rather large task for the coordinator of electronics, but lecturers within electronics were positive and eager to help out with this task. After the bachelor program was settled, the format and content the mathematics courses was designed in the spring of 2015.

The first cohort of students following this new programme started in the autumn of 2015. The new programme differs from the previous with the following significant changes for the mathematics component.

- Mathematics 1 runs over the first two semesters (previously only the first). Mathematics 2 is taught in the third semester (previously in the second). A new textbook with many examples taken from electronics has been introduced.

- No lectures, students watch prerecorded videos, and meet the teacher 4 to 6 hours weekly for seminars and practice.

- Grading is through digital assessment throughout each course and through digital exams. Six digital tests, set throughout the course, a midterm exam and a final exam contribute to the grade (formerly there was only one final written exam).

Mathematic course description and setup

To get a bachelor degree in electrical engineering at the University of Agder (UiA) it is necessary to successfully complete two mathematics courses *Mathematics 1 – Electrical Engineering* and *Mathematics 2 – Electrical Engineering*. Each course gives 10 ECTS credits. These two courses cover all topics described in the learning outcome for engineers set by *The Norwegian Association of Higher Education Institutions*. Each course is based on five essential blocks where students get information, do their work and find different ICT – tools that are necessary.

- We use a learning management system (LMS) in which students will find all relevant information like weekly tasks, dates for tests and exams, links to videos, notes from video recordings, links to simulations and a link to the assessment tool. Implementation has been carefully planned and students have been instructed in how to use the teaching/learning resources introduced.

- The textbook for this course is Mathematics for Engineers (Croft and Davison, 2015). This book has a lot of examples from electrical engineering.

- Video resources are composed of videos recorded by a team of lecturers from different universities and university colleges in a UiA TV studio for MatRIC TV (MatRIC 2015) and short videos recorded by myself at UiA.

- Computer aided assessment tool MyMathLab (MML) from Pearson is used. Students do their homework, tests and exams with this tool.

- Interactive simulations using SimReal. SimReal is a free interactive learning tool in Mathematics and Physics (developed by Per Henrik Hogstad at University of Agder). Brekke & Hogstad (2010) give an overview of how SimReal was introduced into my teaching.

The LMS is the heart of each course, where students find whatever they need to do and work with. It is our experience that engineering students in particular like to have a "to do list". The more structured and strict plan they are given, the more effort they will make. We try to focus on mathematics with lots of examples from electronics and engineering in general. Students can choose to do their homework with exercises from the textbook or log in to MML and do their exercises on-line. In general homework in MML had an average of 36 exercises each week. Students will find information about how to use MML both in text and video resources. The schedule for all tests and exams are also available from the LMS. Direct links to relevant video lectures can be found. Links to interactive simulations with video support can also be accessed from the LMS.

Four hours of seminar and two hours of practice with teachers was scheduled each week. The seminars replaced the ordinary lectures. Only when special issues appear does the teacher used the whiteboard and do some exposition. In this scenario the teacher has an informal setting with

the students. Teachers can challenge them and they can challenge the teacher in a completely different way to an ordinary lecture. By having seminars instead of ordinary lectures the teacher talks with the students and not only to them. There is time for discussion and this appears to help students achieve a better understanding of mathematics.

To set the grade in Mathematics 1 six digital tests set throughout the course count towards 40% of the grade. One midterm and one final digital exam contributed 30% each. All three parts must be passed to set a grade. Students were allowed to take each test on two occasions, while they only have one try on exams. Tests are open for 5 days and they get the same type of problems on their second try but with different values. This is done deliberately to make them work with their errors. All digital tests and exams had in general 30 exercises each. One mandatory task was given in computational mathematics. This was an open exercise in which students needed to solve integrals using the trapezium rule and Simpson's rule by computer simulation and programing. Students could choose any program to do this and could, if they wanted to, work in groups. The way this course was set up students seemed to encourage students to be more and more self-driven throughout the course. They attend seminars whenever they need help. They tend to sit in groups in different place around Campus.

Approach of the Students

The first implementation of Mathematics 1 is still running and we did not know what to expect from our students. At the start of the semester students were encourage to give immediate feedback if things needed to be changed. At any point the lecturer would give "regular" lectures if that was desired. Up to now that has not happened. The feedback from students has given us a great deal of information about how students approach this course. Most students prefer doing exercises in MML. This is probably because it the best way to prepare for tests and the exam in which the same assessment tool is used. Students are encouraged to work in groups. The course ends in May 2016 so we do not know the end result yet, but so far (by mid-April) the results of student performance are far better than anticipated.

Results

At the start of semester in august 2015 39 students registered. By April 2016 32 students remain. That means an 18% drop out rate, which is close to the failure rate for the previous course. Table 1, show the results on tests so far for the remaining 32 students.

Test nr.	Date	Average score: (%)
1	17.09.15 – 22.09.15	92,8
2	15.10.15 – 20.10.15	92,3
3	12.11.15 – 17.11.15	94,2
4	02.02.16 – 08.02.16	87,5
5	23.03.16 – 04.04.16	83,8
Average score all 5 tests:		90,2
Midterm exam:	4 hours 03.12.15	86,9

Table 1. Results from tests and midterm Exam so far in Mathematics 1.

As can be seen from Table 1, scores are very high with an average score of 90% on tests. Why are these scores so high? Since students are allowed to take each test on two occasions, scores will improve. This course runs for two semesters which allows more time on each topic. About 60% of the curriculum in Mathematics 1 should be familiar for students with Upper secondary

or pre-Calculus courses. Tests are limited to two or three chapters taken immediately after their work on these subjects. It appears that these students put more effort into their work than we have previously experienced. On tests all aids are allowed. This should help them perform better. If we look at each test on its own, the latest one shows an average just over 80%, which is closer to what expected. The reason for this may be that this includes new or more difficult topics.

Looking further behind the results we find that 6 out of 32 students did not do any homework in MML at all. The average score of these 6 students on tests are 87% and on the midterm exam an average of 84%. That means it is possible to just work in a traditional way with problems from the textbook and still get a good grade. The workload for those who did homework in MML shows an average of 66% of completed material. Which means about 24 exercises each week, which is rather more than the 10-12 that used to be the case in the previous implementation of the course.

The task in computational mathematics showed use of several different computer programs. No preferences were given to students beforehand. Seven used Geogebra, 6 used Matlab, 3 used Maxima, 1 used Maple and 1 group used Python. Geogebra which is very common in High schools could not solve all exercises, students then solved this task numerically by hand. Not all delivered a high quality result, but it was a very good practice for them.

On 1st. of October 2015 I published the first short-videos especially made for this course. The videos follow quite strictly the themes in the textbook. We monitored the number of plays up to the midterm exam two months later, and it showed 601 plays in this period. That is in average 10 plays each day. The highest number of plays in one day is 49.

Findings and Conclusions

Up to now it seems that Mathematics 1 has been a success both in terms of student performance and feedback. When grades are high positive feedback could be anticipated no matter what is done, but this group of students seem to be very positive towards this way of teaching. In December 2015 students returned their midterm evaluation without any teacher present. In these evaluations students discussed 5 key points:

1. Information and structure: *"Very good information, LMS gives you everything"*

2. Academic content: *"The course is relevant and can be connected to ELE100 (Introduction course in Electronics)"*

3. Teaching: *"Very happy with the short video, they are simple and thorough, easy to rewind if you need to listen repeatedly"." MML is very good and helps you solve problems if you are stuck. Great with immediate feedback when finished". "Good distribution and dissemination of tests throughout".*

4. Student efforts: *"Requires a lot of work with tasks, using formulas and rules we learned, therefore using MML really helps, there you can take tests and get feedback right away, it motivates you to work harder".*

5. Proposals for changes: *"None, very happy with this first semester".*

One thing that we are particularly happy with is the way students worked with the tests. Students were told that within these exercises they will find similar tasks that will appear on their exam.

This seems to motivate them to work harder. Student's help each other to perform better as this feedback shows:

"Tests that give grades are really a dream for students. When we are presented with these tests we have a lot more respect for them. Since there are two attempts both me and many others in the class use the first attempt to see what we actually do know. After our first attempt we go through what was wrong together with others. With this we also ensure that all is doing well. This contributes to an environment where people work together to understand mathematics. Since test tasks are related to exam, students work harder to perform better. The thing with these tests is personally one of the best experiences I have seen."

It seems that these students reflect more about why they need maths and that they try to understand maths. It is not easy to prove that these students actually do get a better understanding of maths, but we do get feedbacks from them in a way we have never seen before, like:

"The subject is relevant and teaching is very suitable for what we need to learn as engineers".

After the last test on integration this was one of the feedbacks:

"This test was absolutely fantastic and I learned a lot from it. Arguably the best test until now. 6 Hours of math can be tiring, but it feels rewarding at the end of the day!"

Feedback from one group doing the task in computational mathematics wrote this:

"We decided to do the assignment using Python program code together with Sympy and Matplot libraries. The reason for this is because we wanted to become more proficient with Python programming and because we felt it gave us a deeper understanding of how a program like for instance Matlab or Geogebra actually works. We feel like this was a great exercise for understanding how mathematical functions operate in digital tools. It also made us more steadfast in our integrating!"

All of this feedback reveals that at least some of the students are trying to not only solve problems but also understand what they actually do. And I believe this improves their understanding of maths. This leaves a lot of issues that are up for discussion. Is this way of teaching doable for every teacher? I don't think so; one needs a certain level of digital competence. Is this way of teaching suitable for all students? I think it is perfect for engineers, and should be possible for others to. I think it really comes down to the teacher and how it is presented for the students. We will continue to monitor this project and there is still unanswered questions left to be discussed.

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Lesson Moodle for a self-directed learning of mathematics

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Abstract

Moodle (Modular Object-Oriented Dynamic Learning Environment) is a course management system to promote interactive and collaborative learning activities. At Coimbra Institute of Engineering (ISEC), the project e-MAIO (Interactive Online Learning Modules) collects and describes a set of interactive tools for teaching, and learning, mathematics to engineering students through a technology-enhanced environment on the Moodle platform.

This paper aims to present the activity Lesson, available in e-MAIO, one of the most complex and interesting activities in Moodle. This activity enables the formation of theoretical thinking, based on reflection, analysis and planning which leads to mental and intellectual development of students. The students get self-directed learning, taking the initiative and responsibility for their learning. Therefore, they can guide and evaluate their own learning activities, which can be realised anytime and anywhere. A pilot student project was performed. Together with the activity lesson, the results obtained by the students are presented.

Introduction

The e-mail, the internet, virtual platforms such as Moodle and other communication tools are considered to be differentiating factors in organisational forms with respect to traditional teaching methods, so the recourse to its use has been increasingly enhanced, requiring teachers to be in a continuously changing teaching model (Rasteiro 2013). In this sense the use of Information and Communication Technologies (ICT) represents added value to the teaching/learning process, particularly with regard to the flexibility and access, making physical presence not mandatory for the monitoring of the material, the improvement of communication channels and the increase in collaborative work, which are critical engines of a more and more frequent interaction in the current job market. No less important, due to continuous determination in developing the higher education quality of training intensified attention is given to school failure, making the promotion of success a fundamental objective of the institutions. In the last decade, although many studies have been seeking to understand the reality of academic failure in higher education, leading to investigations that seek to know, in depth, how learning takes place in the students' level of education, some analysis on the relationship between teaching methods and how students learn (Grácio et al, 2007) needs to be developed. On the other hand it is important to contextualise these concerns, particularly in the case of teaching and learning mathematics. The authors, mathematics teachers in various degree courses taught at the Coimbra Institute of Engineering (ISEC) are confronted with an increasing lack of ability in understanding mathematical concepts, uninterested and consequent neglect of students in relation to the mathematics courses. This situation, compounded by the difficulty shown by students in elementary and basic concepts, essential to successful integration in the syllabus, inevitably leads to high failure rates and subsequent concern of the teachers.

A learning environment can be considered as a unique and unrepeatable space built by teachers, based on their views of the educational process and mastery of its knowledge, in particular in relation to the use of ICT as continuously adjustable to the interaction between its actors (Marin, 2009).

The activity Lesson available in e-MAIO enables the formation of theoretical thinking, based on reflection, analysis and planning which leads to mental and intellectual development of students. The students get self-directed learning, taking the initiative and responsibility for their learning. Therefore, they can guide and evaluate their own learning activities, which can be realised anytime and anywhere (Núñez 2011).

The Lesson is a set of pages with information and leading questions, with the main objective to teach and test students' knowledge. The lesson can contain a set of alternative paths, providing different content, depending on the student's answers. This feature allows the creation of adaptive content, being especially useful in contexts of heterogeneity of the prior knowledge or the students' past learning process.

In this Lesson, student learning is self-directed and can be repeated several times to strengthen the knowledge and the understanding of the student. It can be performed anywhere, anytime and it can contain materials of any discipline, so it is very flexible. The Lesson is an excellent activity to present the material to students in a structured and interactive way so it can be used as an open educational resource to all students with audio, video, animations, text, questions and images.

In order to reflect and analyse the skills acquired by students when using the Lesson activity and to define strategies for their development, a small pilot study was conducted. Initially a first test about the subject of the Lesson was made before the students are involved in the learning process. At the end, the students take the same test to be able to assess the knowledge and skills obtained during the Lesson. Furthermore, an inquiry and an interview were made to collect some conclusions and suggestions in order to improve the development of such activities (González 2013, Lima 2014).

Method of Investigation

The e-MAIO (Interactive Online Learning Modules) project, Figure 1, describes a wide range of interactive tools developed to complement some mathematics courses at ISEC. The interactive modules that integrate the e-MAIO project are intended to support the teaching and learning of mathematical content beyond the classroom. This project was motivated by the desire to implement some innovative and attractive tools but also because we believe that their use leads students to act responsibly in their learning process.

Some of the most important features of e-MAIO are: to complement the traditional teaching, develop autonomous and collaborative learning, to minimise text, in order to give the student an overview of the concepts, rather than providing lengthy descriptions of procedures or theories that can be found in books or class notes (Caridade 2013).
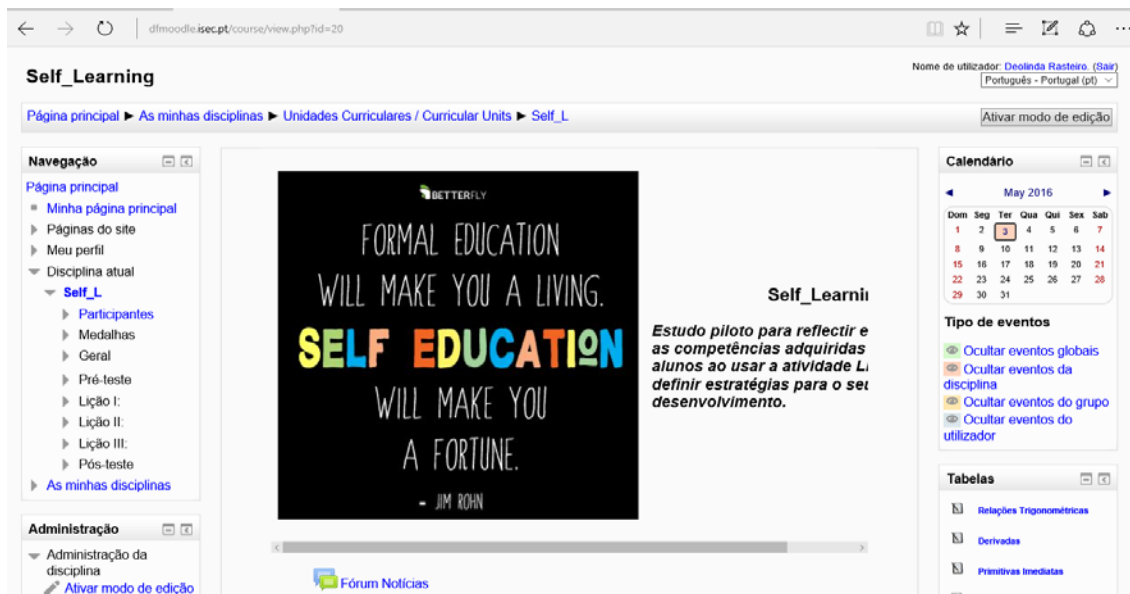
Figure 1. Study environment.

The pilot study that was conducted, named Self_Learning, has three lessons related to antiderivatives, Figure 2. The different processes of integration presented are: rules for basic functions (Lesson 1), techniques for rational functions (Lesson 2) and integration by substitution (Lesson 3). There is a set of supporting concept guides and an extended set of solved exercises to help the student, together with online tests to assess the student's understanding with automatic feedback and formative assessments exercises where the student is guided to the problem resolution, via comments and suggestions in the resolution.

Figure 2. Study Lessons Presented

The study starts with an assessment (pre-test with 5 questions related to the 3 different techniques) performed by students. Afterwards the student has to follow by himself the contents lessons. To have some self-containment Lesson 2, other than basic mathematical concepts, only refers to knowledge acquired on Lesson 1. Similarly, Lesson 3 only needs concepts of Lessons 2 or 1. When solving the proposed exercises students are, in case of failure, directed to the corresponding previous lesson in order to complete the missing knowledge. At the end the

students perform the same assessment, (final test) to test the success obtained in the learning process.

The investigation questions that the authors want to get answers for are:

Q1: If the only available method for learning was the-Maio platform, were the lesson contents enough?

Q2: What are the main advantages of this type of learning?

The sample

Since this was a pilot study the sample only involved 13 students. These are students from the first year of Electrotechnical Engineering graduation. Their mean age is 21.
During the time that the study was conducted students were not allowed to have in-class lectures.

Findings and Discussion

Comparing the results obtained in the two proposed tests, Figures 3 and 4, we can observe that there was a knowledge improvement although it was not a completely effective learning.
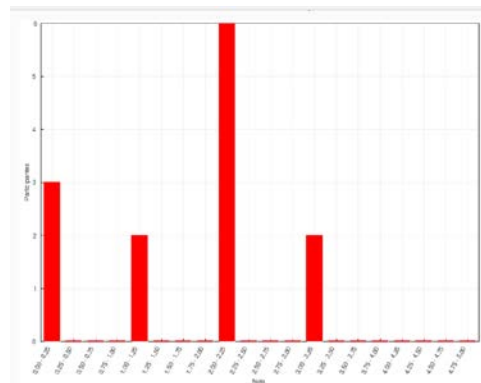
Figure 3. Pre-test results   Figure 4. Final test results
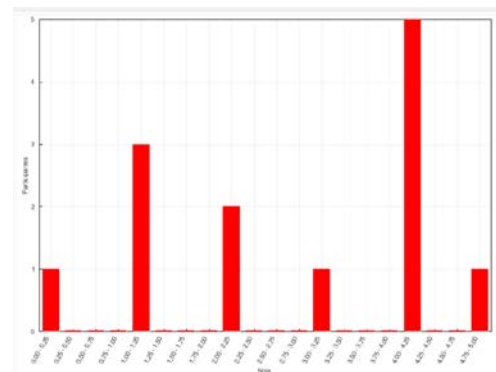
The median pre-test result is 30.8%. Considering that the median value for the final test was 53.8% we may confirm the previous observation.

There are still students who did not learn the material. The reason may be insufficient information or lack of interest. To clearly identify the reason, a questionnaire and an interview was made after the final test.

When answering question Q1 students refer to the type of learning only as a complement to in-class lectures. The importance of the teacher figures still high. The suggsetions received were: insert video lectures, have doubts explained via skype which suggests that the one-to-one explanation is really a need for this type of student.

Related to question Q2, the students' opinion is that this type of learning method although flexible and very important, because it permits access everywhere and whenever they want, is not the best for them. They feel the need for the teacher's presence in order to better understand the contents which, in the authors' opinion, is closely related to the lack of self responsability and autonomy.

Conclusions for Education

Before designing and implementing a teaching method one should perform a small investigation concerning the type of students involved. There is not only one efficient method to teach a subject. A teacher has to be also someone who understands the type of students in front of him and should guide them through the adequate style of learning. In order to do that there are learning styles that may be evaluated before starting the learning process (Dias 2014). An investigation performed in 2012 by (Bigotte, E *et al, 2012*) showed that most of engineering students have a visual style of learning.

An ICT platform should, in most cases, complement in-class lectures.

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Attachments:

Questionnaire performed by the students during the interview.

Questionário

A - Características Pessoais

Nome completo:

Idade:

B - Competências do aluno

É trabalhador-estudante?

Além do estudo que efetuou, já utilizou outras plataformas de ensino? Quais? Onde?

Classifique os seus conhecimentos de informática:

C - Operação e organização educacional

A existência de sessões presenciais é muito importante nas disciplinas do seu curso?

As atividades existentes neste estudo são relevantes para a sua aprendizagem?

As lições apresentadas são úteis na aprendizagem dos conteúdos?

Os exercícios resolvidos/propostos durante as lições são úteis na consolidação da aprendizagem dos conteúdos? Permitem avaliar os conhecimentos sobre os conteúdos?

Os exercícios de autoavaliação propostos são úteis para averiguar os conhecimentos adquiridos em determinado tema?

Os docentes estão sempre disponíveis para apoio ao processo de autoaprendizagem?

Se não existissem aulas presenciais, acha que a quantidade e tipologia de exercícios existentes neste estudo seriam suficientes para consolidar a matéria dada nas aulas teóricas?

Existe algum material didático não usado que seja útil para a consolidação da sua aprendizagem? Como, por exemplo, a elaboração de vídeo-aulas.

D - Benefícios

As lições são claras e exigem pouco esforço para se lidar com elas?

Podem ser utilizadas em qualquer local? A qualquer hora?

Este tipo de ensino pode ser aplicado a outras disciplinas do curso? Quais?

Indique, no seu caso particular, qual a principal vantagem deste ensino.

E - Acesso

O acesso é rápido?

Está sempre disponível?

De que local acedeu mais vezes a este estudo? Com que frequência?

Encontra alguma dificuldade no acesso? Qual?

F - Opinião

Dê a sua opinião sobre o estudo que realizou? Algumas sugestões, ou ideias …

F - Opinião

Dê a sua opinião sobre o estudo que realizou? Algumas sugestões, ou ideias …

A project-based-learning approach to teaching second-order differential equations to engineers

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Abstract

In an attempt to increase engagement in a third-year mechanical-engineering mathematics module, a series of group-work sessions were introduced on a single topic. Students were asked to use the solution of a second-order differential equation (previously introduced in lectures) to solve a problem in which they must design a simple spring-damper system for one of a lorry, digger, truck, tractor, car, motorbike or pogo stick. This project was worth 10% of the module and done in groups of four. The groups gave an oral presentation of their results and a written report was handed up for grading. A detailed description of the implementation of this assessment is given along with an analysis of how student performance in questions on similar topics in the terminal examination for this module.

Introduction

Mathematics as a discipline presents challenges for a large number of students, not least in terms of engagement and relevance (Hidi and Harackiewicz 2000). Since September 2012, incoming first-year students to higher education in Ireland have studied a revised mathematics curriculum (Project Maths) in second-level (Jeffes et al 2013). This new approach to the teaching and learning of mathematics in Ireland aims to situate mathematics in everyday contexts where possible, so that students will be better able to understand the uses and relevance of mathematics. However, these students entered into a higher education system that was accustomed to a mathematics curriculum that had not changed substantially in almost fifty years prior to this. Much of the material taught in the early years of mathematics is not explicitly mapped at that point to modules or applications in later years, making it difficult for students to understand the importance of what they are learning at this early stage in their careers. It is commonly the case that, in later years, lecturers will refer back to material covered in first year and show students how they will now use this mathematics in more advanced applications. However, from the point-of-view of student engagement and retention, this seems to be done at too late a stage, and needs to be dealt with in early years instead. This is particularly important in engineering as it relies heavily on mathematics throughout the degree programme. Successful service-teaching of mathematics relies heavily on a "sufficient supply of discipline related problems" (Yates 2003). This changing mathematical landscape in Ireland provided the motivation for the development of the project-based-learning approach described in this paper.

Engineering mathematics is generally either taught by engineering lecturers (who are usually not experts in mathematics) or by lecturers from mathematics departments who are not embedded within the students' home departments, and naturally may not be experts in the overall discipline being studied by the students. As a result, the balance between theory and practical applications is often skewed (Sazhin 1998, p.145). Lecturers may forget the significance of students "getting a feeling for the importance of the subject" (Rota). The level of interaction between mathematics lecturers and staff members in the students' home departments can vary widely from institution to institution.

In Dublin Institute of Technology, students are offered two main routes to obtain a Level 8 engineering qualification: via direct entry onto a four-year Honours degree programme (Level 8) or alternatively through a three-year Ordinary degree programme (Level 7) followed by a transfer into third year of the Honours degree (Llorens et al. 2014, Carr et al. 2013). A recent study of the First Year Experience (FYE) in the eight third level institutions in the Dublin Regional Higher Education Area (DRHEA) found that one of the key problem areas identified by academics across all eight institutions was lack of "student engagement" (Roper et al 2013, Cusack et al 2013). This lack of engagement often results in poor performance and ultimately impacts upon retention. This project is thus an attempt to evolve the teaching of engineering mathematics at Level 7 to both improve the engagement of students in engineering mathematics classes and to provide a deeper understanding of the material, which may ultimately help these students to progress onto a Level 8 degree programme if they so desire.

Background

There exist many examples in the literature on the need to make the mathematics we teach to engineers more applied. The development of such examples and projects can be challenging for those teaching engineering mathematics as they may not be engineers or familiar with all aspects of engineering. However, there is a full spectrum of initiatives available in the literature, from improved examples in the classroom (e.g. Helm, Young et al 2012, Robinson 2008) to teaching the material via problem-based learning (e.g. Rooch et al. 2012). We now provide a brief overview of some of these approaches.

Helping **E**ngineers **L**earn **M**athematics (HELM) is a major curriculum development project undertaken by a consortium of five English universities - Loughborough, Hull, Reading, Sunderland and Manchester - led by Loughborough. This project provided a huge list of engineering mathematics resources including a set of good examples of engineering applications of mathematics (http://helm.lboro.ac.uk/). Although this work is of a high standard, many of the examples contained therein are more relevant to later years of a Level 8 engineering programme, and the examples suitable for Level 7 students are limited in number.

Young et al (2012) at the University of Central Florida developed a bolt-on single-credit module called "applications of calculus", taught in parallel with their calculus modules. This has been shown to be effective in terms of retention of students within STEM subjects, although there are no projects introduced, simply a range of problems completed in class that are relevant to applications.

Robinson (University of Loughborough) used sports-based group projects for undergraduate students in sports science (Robinson 2012). These projects consisted of teamwork, use of software and application of mathematics to realistic problems. This not only improved engagement, it also introduced a range of important skills for engineers, such as technological and communication skills as well as collaborative and analytical techniques.

Rooch et al. (2012) developed a series of projects (ribbed cooler and a Segway) for teaching mathematics to first year engineering students in Germany. However, the mathematics required is quite involved in each case, meaning that they must supply the students with a number of different formulae in order to allow them to complete the projects. This is a common difficulty when designing "real-life" mathematics projects for students to attempt, due to the scaffolded nature of mathematical knowledge.

Within Dublin Institute of Technology itself, some example of project-based learning already exist. For example, design projects were introduced into first-year physics lab sessions for

engineering students. These projects relied upon material covered in mathematics, physics and mechanics modules, bringing them all together in a single design project (Sheridan et al. 2010). A range of other variations exist between teaching applications and full problem-based-learning from Verner et al. (2008) and Mills and Treagust (2004), through to Abramovich and Grinshpan (2008).

Aims and scope of project

Within Dublin Institute of Technology, we wish to move towards a more student-centred-learning approach for the teaching of mathematics across all three years at Level 7. To do so, it has been decided to first pilot this technique in the third year of the programme and then work backwards to first year, once initiatives have proven to be successful. Much work has been done on using project-based/application-based learning as a method for teaching mathematics in higher education, but in the main, these modules have many mathematical pre-requisites, so they are in essence only suitable for later years of a programme and/or re essentially being "bolted on" (Young et. al 2011) to pre-existing modules. Similar work has been done in the third year of the Level 8 degree programme to teach mathematical modelling with good success (Keane, Carr and Carroll, 2008), so it was of interest to introduce it at a similar stage in a Level 7 programme and monitor its impact. Given that the standard of first year in a Level 7 programme is not high enough for many of these existing resources to be used, by trialling this approach in third-year, we can learn valuable lessons before considering earlier years.

The aim of this work is to use a hybrid approach to "project-based-learning" where a significant amount of the pre-requisites is taught over several weeks in a more standard approach and then a realistic project is introduced that consolidates the material that has been covered in class and provides an opportunity to learn applications of the material.

The objectives of the project were to improve engagement and ultimately retention of students; to give students a deeper understanding of the material; to introduce problem-solving, teamwork and communication skills; to move towards a more student-centred environment within the existing structure of lectures and tutorials; and to create a series of resources that could be used by lecturers teaching at Level 7.

Project overview

A series of two-hour group-work sessions were introduced, focused on the topic of second-order differential equations. Following a number of standard lectures, students were assigned to groups of four to work on a short project together during the group-work sessions, with additional work to be completed outside of class time. The project asked students to use the solution of a second order differential equation (previously introduced in lectures) to design a simple spring-damper system for a vehicle from the following list: lorry, digger, truck (large), truck (small), motorbike, motorbike (scrambler), bus (large), bus (small), moped, quad bike, tractor, tractor (seat), car (large), car (small), pogo stick, racing bike or standard bicycle. No two groups were assigned the same vehicle, and the different masses, number of wheels involved and type of damping required meant that the projects were sufficiently different that each group had to work independently on their solution.

The project was worth 10% of the students' final grades for that module, with the marks awarded per group. At the end of three weeks, each group presented their solution to the class during a ten-minute presentation slot, as well as handing up a short (four-ten page) report. The variation in report-length was chosen to allow students to include detailed diagrams and additional information where needed. The mixture of assessment methods included within the

project gave students the opportunity to display their skills in a range of areas, while providing them with useful practice of presenting technical data in a clear and coherent manner. Students were obliged to attend all the presentations given, which also provided a valuable opportunity for peer-learning, as they heard how different groups had approached a similar problem, and allowed for some class discussion about optimum approaches afterwards.

In order to design a simple spring-damper system, students needed to first consider the mass of the vehicle, calculate an appropriate spring constant, and decide on what type of damping would be ideal for the vehicle in question. For example, the damping needed by a scrambler motorbike is different from that of a family car, where a smoother ride would be required. This was a multi-layered problem, which allowed students to investigate a number of areas in greater depth, considering aspects relevant to the generation of the second-order differential equation. Once they had solved the differential equation, they were then required to sketch the analytic solution by hand, to investigate if the resultant sketch resembled the type of damping they hoped to produce. If so, they then needed to plot a graph of the analytic solution and relate these back to the original problem, giving an interpretation of their results.

Analysis of exam paper questions

Judging the success of a project-based intervention such as this is difficult, although student engagement with the project was high and their reaction was universally positive. However, in previous years, examination questions based on realistic uses of second-order differential equations were extremely poorly answered, or, in many cases, not even attempted, despite similar questions having been addressed in lectures. There was no choice given in the examination and so these questions were compulsory but they were still avoided by students. Therefore, an analysis was done of a similar question from the terminal examination paper sat by this year's students, all of whom had the benefit of having completed their short project on this area. A total of 35 examination papers were analysed, with two separate parts of one question considered. The question asked:

> A spring dashpot system has a damping force resulting from the dash-pot $F_d = -kV$ and the restoring force from the spring $F_s = -nx$. This system is represented by the differential equation below
> $$\frac{md^2x}{dt^2} + \frac{kdx}{dt} + nx = 0$$
> (a) Find the general solution to this equation if $k^2 = 4mn$ and thus sketch your solution.
> (b) Explain your sketch and how it relates to what is happening in the damping system. You should illustrate what is happening by using an engineering example of where this may be used.

For part (a), 11 students (31.5%) answered correctly, showing they were able to derive an analytical solution and sketch the required graph. A further 11 students (31.5%) made a reasonable attempt but derived the wrong analytical solution and drew the wrong graph (even for the solution they derived). 5 students (14%) derived the wrong solution but knew what the correct graph should look like and drew this. The final 8 students (23%) were unable to make a proper attempt at this question.

For the descriptive answer to part (b), 11 students (31.5%) gave an entirely correct answer and 5 students (14%) made a reasonable attempt at an explanation. However, 19 students (54.5%) gave a poor explanation or did not attempt to give any.

When each student's project mark was plotted against their performance on this exam question, the result was statistically significant, with a Pearson correlation of 0.453 found with $p=0.006$. Similarly, when each student's overall performance was plotted against their performance on this exam question, another statistically significant correlation was found, with a Pearson correlation of 0.71 with $p=0.000$. While this is not surprising, showing that the most capable students performed well on all components of the assessment, it does contrast with previous years, when even strong students avoided or did poorly on the differential equations question in the examination.

Conclusions and future work

The introduction of a short project-based-learning element into a mathematics module for third-year Level 7 mechanical engineering students was well-received by students and resulted in greater engagement with examination questions on the same topic in comparison with previous years when this material was taught in a standard lecture environment. Student performance on these examination questions was also improved, though the strongest students overall performed the best on these questions. Through the inclusion of a presentation element within the project assessment, the "active verbal involvement" of students advocated by Kwon (2000) was addressed, allowing students the opportunity to explain and justify their thinking, which is particularly important for engineering students while studying mathematics. Although it is challenging to develop projects of this type, the aim was to create a scenario that would be "experientially real to students and…take into account students' current mathematical ways of knowing" (Rasmusen & Kwon 2007). As a first foray into project-based-learning for mathematics for third-year Level 7 engineering students, it was a success and now the focus will shift to attempting to do similar in earlier years of the programme, where it is hoped that it will be equally well-received by students and provide a positive contribution to the teaching and learning of mathematics for Level 7 engineering students.

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CalculEng – An On-Line Tutorial Tool to Assist the Teaching and Learning of Calculus

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Abstract

Development of proficiency in mathematics is an essential aspect of many programmes of study. This applies both to specialist mathematics students, and students of many other disciplines, especially engineering. Students' knowledge of and expertise in mathematics (or lack thereof), at least at an elementary level, can have a major impact on many other areas of their studies and their subsequent career prospects. However, mathematics is an area which many students find difficult, particularly those from "non-traditional" academic backgrounds, including disabled and mature students, and they often do not realise its relevance and importance to their other courses, nor do they (or can they) devote as much time or effort as they perhaps ideally should, and face to face tutorial support is often limited. Although there have been previous attempts to develop on-line tutorial exercises, with automated marking and feedback, to assist students with their mathematical studies, most of these have either been restricted to multiple choice or numerical answer questions, or have only addressed the most elementary of topics – notably simple algebra, functions & graphs and trigonometry.

In this paper, we describe our efforts to remedy this situation. We have produced a set of resources, called *CalculEng*, in the form of structured calculus exercises, including applications, which can be delivered on-line. These were developed using the QTI framework, and allow algebraic input from the student, which is checked for consistency with the "correct" solution and the outcomes of anticipated "common errors", using a computer algebra engine. The students' responses are automatically marked, and intelligent, relevant feedback - based on the mistakes made by the student - provided. We discuss the design and implementation of resources we have produced, their relevance to various degree curricula, and their evaluation when used on a number of first year mathematical modules within undergraduate engineering degrees. We also describe the reflections and opinions of the students participating in the evaluation, both with respect to their experiences of using *CalculEng*, and regarding their attitudes and preferences towards studying mathematical topics. Analysis of these comments and feedback should facilitate improving *CalculEng* to make it better-suited to students needs and study styles.

Introduction

Development of proficiency in mathematical techniques, especially differential and integral calculus, is an essential part of the education of engineers. However, mathematics is a subject which many students of engineering find particularly difficult, and some also do not appreciate why mathematics is so relevant to their other studies. Teaching staff are faced with the problem of teaching a quite comprehensive syllabus of material, often in a rather short amount of classroom time and with strictly limited resources. Class sizes are often very large, and the amount of staff time available for face to face tutorial support may be low. Mathematics is, for many people, best learned through examples – ideally with the students practising problems under supervision of, and receiving feedback from, experts. However, limitations on staffing, resources and tutorial time frequently mean that opportunities for students to practise in this way are severely restricted.

In this paper, we describe our approach in an effort to solve these problems. We have produced a set of resources, called *CalculEng*, including a variety of exercises, on single variable differential and integral calculus and their applications in Engineering. These can be delivered

on-line, or using a Virtual Learning Environment such as Blackboard or Moodle. They are in the form of structured questions developed using the QTI framework (QTIWorks, 2015), and allow algebraic input from the student, which is then checked for consistency with both the "correct" model solution and with the outcomes of anticipated "common errors", using the *Maxima* Computer Algebra System (Schelter, 1998). The students' responses are automatically marked, with the aid of *Maxima*, and intelligent, relevant feedback - based on the mistakes made by the student - provided.

Principles of the *CalculEng* System

Although there have been previous attempts to produce on-line resources and "self-test" questions for engineering mathematics, most of these have only either provided multiple choice or numerical questions without detailed feedback to students - e.g. *Mathletics* (Greenhow, 2008) – or are subscription services, in some cases tied to the purchase of particular textbooks – e.g. *MapleTA* (MapleSoft, 2015) and *MyMathLab* (Pearson Education, 2014). There are a few systems which are free to use on line, allow algebraic input and provide reasonably detailed feedback to students – such as *MathDox* (Cohen et al 2006) or *CALMAT* (Gorman et al 2009), although most materials for the former are only available in Dutch, and for the latter are at a rather elementary level more appropriate to pre-University studies.

The *CalculEng* system (Davis et al 2015a) has been produced to support students with their mathematical studies, in-course assessments and improve their progression. This system offers students a set of exercises on elementary differential and integral calculus and covers material relevant to a good range of engineering topics, including problems on engineering applications. These on-line materials are designed using the Question and Test Interoperability (QTI) specification, which is widely being used to represent on-line questions and assessments. Each question is encoded using QTI XML code (Neve et al 2012), in which the question, and the dynamic behaviour of the question, are described. The QTI provides a programming facility, which allows the tutor to author the mathematical exercises, encoded using XML, and writes the mathematical equations and formulae by employing MathML. These mathematical exercises developed using the existing QTIWorks system, hosted at the University of Edinburgh, U.K., which allows the questions to be linked to the freely available *Maxima* Computer Algebra System, check the questions and student's responses for mathematical consistency with the correct solutions.

CalculEng system provides a set of structured exercises, which allow the students to enter their answers in a window (provided especially for their responses) in ASCII-based mathematical format, rather than just making a selection from a list of choices or entering a numerical value. Some of the aspects of these questions, such as specific parameters and coefficients in the equations and formulae, are written to be generated randomly and, by so doing, enable students to develop the ability to recognise the same problems when expressed in different forms. Moreover, the basis of the system is that each question can identify a student's error via a set of rules, which are encoded in XML (Neve et al 2012). The system allows the student's answer to be checked against a list of perceived "common errors" for that type of problem and then provide feedback, tailored to the particular type of mistake made. Therefore, the system provides readily available support, informs students of their mistakes and includes the facility whereby they can request a hint and/or the full solution.

Students are able to use multiple-section structured questions on the application of calculus to engineering problems, with detailed feedback on each step being provided. In these multi-section questions, feedback is revealed to the student in a step-by-step process. Further technical details of how the questions are encoded can be found in Davis et al (2015b).

Example of a Typical *CalculEng* Exercise

A simple example on integration by parts, with a student's response and feedback is illustrated in Figures 1 and 2. In this example, the student's answer contains an error which had been anticipated by the question's setter, and the *CalculEng* system provides feedback appropriate to that particular error. In addition, students are able to request a hint or the full "model" solution, by using the *Show Hint* or *Show Solution* buttons, and then *CalculEng* will provide what was requested.

Integration by parts 1

This assessment item is being delivered using a set of default 'delivery settings'. You can create and use your settings when logged in QTIWorks system account.

Use Integration by Parts to find:

$$\int (-4ze^{2z})\,\mathrm{d}z$$

Hint: When using the integration by parts formula, define the term which is easier to integrate as $\frac{\mathrm{d}v}{\mathrm{d}z}$.

Input Maths: -2e^(2z)+e^(2z)

✓ I have interpreted your input as: +C

$-2 \cdot e^{(2z)} + e^{(2z)}$

Incorrect answer.

In the first term of your answer z is missing. You should have multiplied $-4z$ by $\frac{e^{2z}}{2}$.
Please see the solution.

Show Hint

Show Solution

SUBMIT RESPONSE

Reset | Reinitialise | Finish and review | Exit

Figure 1. Example *CalculEng* question, with feedback appropriate to the student's answer.

Some further *CalculEng* examples are given in Davis et al (2015a,b). The complete set of resources currently available cover all the main elementary techniques of differentiation (e.g. product rule, quotient rule, etc.) and integration (including by substitution and by parts), and applications to tangent and normal lines, rates of change and optimisation problems.

Pilot Study - Evaluation of *CalculEng* by First Year Undergraduate Students

It was originally planned to evaluate *CalculEng* in use by having it tried out by several groups of different types of first year undergraduate (Aeronautical, Automotive, Civil and Mechanical) Engineering degree students studying Mathematics, including both differential and integral calculus, as part of their curriculum. Delays in getting the resources ready during 2014-15, the scheduling of parts of some modules across only parts of the academic year, plus a local restriction on surveys of students due to National and intra-institutional Student Surveys, resulted in the initial evaluation only being performed on two groups of first year BEng Civil Engineering students, each of approximately 40 students, towards the end of the Spring terms in 2015 and 2016 respectively. These periods coincided with a large number of other deadlines for their in-course assessments, with the consequence that only a total of 20 students completed the evaluation survey and, of these, 6 admitted to never having used the *CalculEng* system. The survey took place after the students had been given the opportunity of using the *CalculEng*

system during three 2 hour supervised practical sessions. They were then given a questionnaire regarding their views on their mathematical studies, their confidence with mathematical topics and their opinions on their experience of using *CalculEng*. The questionnaire they were asked to complete can be found in Davis et al (2015a).

Use Integration by Parts to find:

$$\int \left(-4ze^{2z}\right)\mathrm{d}z$$

Hint: When using the integration by parts formula, define the term which is easier to integrate as $\frac{\mathrm{dv}}{\mathrm{d}z}$.

Input Maths: -2e^(2z)+e^(2z)

I have interpreted your input as:

$-2\ e^{(2z)} + e^{(2z)}$ +C

To integrate this expression use **Integration by Parts:**

$$\int \left(u\frac{\mathrm{dv}}{\mathrm{d}z}\right)\mathrm{d}z = uv - \int\left(v\frac{\mathrm{du}}{\mathrm{d}z}\right)\mathrm{d}z$$

- First define the terms:
  For $\frac{\mathrm{dv}}{\mathrm{d}z}$ choose the one which is easier to integrate

$$u = -4z \qquad \frac{\mathrm{dv}}{\mathrm{d}z} = e^{2z}$$

- Find derivative and integral accordingly:

$$\frac{\mathrm{du}}{\mathrm{d}z} = -4 \qquad v = \frac{e^{2z}}{2}$$

- Substitute into the formulae and then integrate the second term:

$$\int\left(-4ze^{2z}\right)\mathrm{d}z = -4z \times \frac{e^{2z}}{2} - \int\left(-2e^{2z}\right)\mathrm{d}z = -2ze^{2z} - \left(-e^{2z}\right) + \mathrm{C}$$

Therefore:

$$\int\left(-4ze^{2z}\right)\mathrm{d}z = e^{2z} - 2ze^{2z} + \mathrm{C}$$

Reset and play again   Reinitialise and play again   Exit

Figure 2. *CalculEng* question, with complete "model" answer shown at the student's request.

As can be seen from the results in Figure 3, 50% of the respondents claimed they regularly used web resources for their out-of-class mathematical studies and 85% practiced exercises set in class, either individually or in groups of peers, with 45% preferring to use textbooks. The majority of the students (70%) carried out between 1 and 3 hours of individual study for the module per week, with only 20% spending 4 or more hours per week studying for this module outside of class. Since this 30 credit module has approximately 100 hours of scheduled class time, this figure contrasts with the University's expectation that students put in 200 hours of individual study for the module, equivalent to approximately 8 hours per week ! We did observe marked differences between the responses of the 2014-15 and 2015-16 year groups, but since both groups were rather small it is difficult to infer anything of statistical significance from those differences.

Students' opinions on *CalculEng* (see Figure 4) were rather mixed. Although quite a lot of the students expressed "Neutral" (neither agree nor disagree) responses to the questions - roughly half the students with regard to *CalculEng* helping their understanding of Calculus or it ease of use – more believed that it would improve their understanding (35%) rather impair it (18%), and more thought it would assist their time management when solving mathematical problems (47%) than believed it would make that worse (24%). However, equal numbers found it easy to use (24%) as did not, although no students claimed it was very difficult to use, whereas 6% found it very easy.

(a)

(b)

(c)

(d)
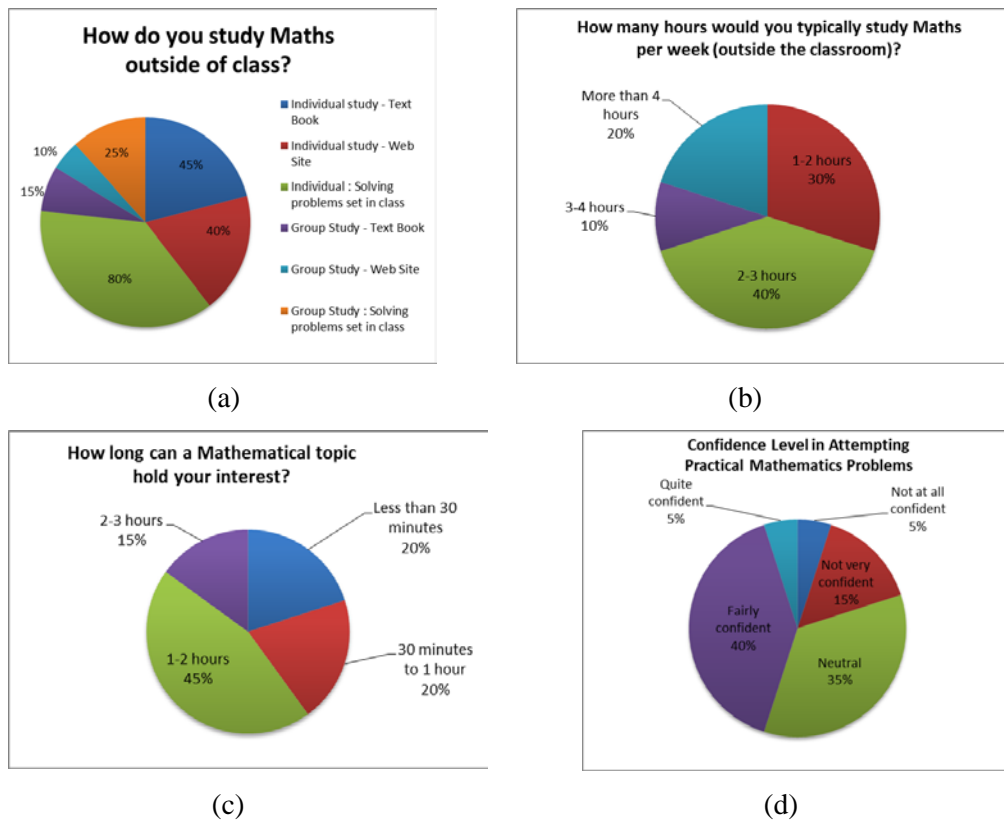
Figure 3. Summaries of the students' approaches to various aspects of their mathematical studies. Note that in (a), each student was allowed to select more than one answer, so the proportions do not sum to 100%.

It should be born in mind that these survey results were based on very small samples of students, and came after those students had only had a limited opportunity to use the system. It is intended to review the on-line tutorial materials provided in *CalculEng*, and the questions asked to the students, before carrying-out a more comprehensive evaluation of the system during the 2016-17 academic year. This follow-up study should also allow the students more time to become familiar with *CalculEng* and the facilities it offers before they are expected to evaluate them.

(a)

(b)

(c)
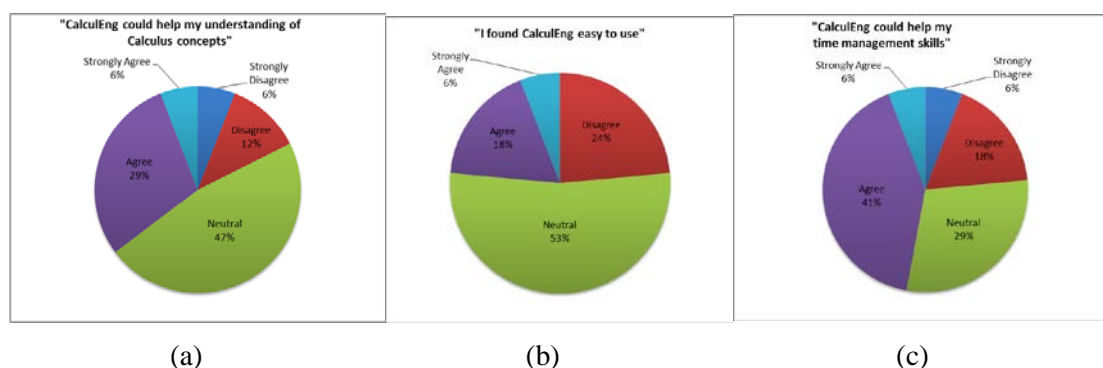
Figure 4. Summaries of the students' views on their experience of various aspects of *CalculEng*. In (b), "time management" refers to this in relation to solving mathematical problems.

Conclusions and Further Work

We have described the development and preliminary evaluation of our on-line *CalculEng* system to assist engineering students learn elementary calculus. Initial results of this are quite promising, but not yet conclusive, regarding its value. We plan to extend this work by including a wider range of Engineering Mathematics topics, improving the system's design and ease of

use for both students and teachers, carrying out a more comprehensive evaluation and analysis of students' use of the system – similar to that carried out by Hunter et al (2013) for their *Nooblab* system for the teaching and learning of computer programming - and, if possible, automatically identifying further "common errors" made by students through use of "Machine Learning" approaches (Mitchell 1997).

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On Transition from High School to University - Experience from FEE Czech Technical University in Prague

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Abstract

The lack of graduates in STEM fields, mainly in engineering disciplines is a well-known situation in many countries. One of the many problems that causes this situation is the high failure rate of first years students entering HE in engineering. At the same time, the above mentioned failure rate is strongly influenced by the admission procedure that a HE institution adopts. This contribution brings experience gained at Faculty of Electrical Engineering, CTU in Prague in order to find a "good" admission procedure.

Introduction

According to OECD (2013) during the last two decades access to higher education increased considerably; between years 1995 and 2011 entry rates increased from 39 % in 1995 to 60 % in 2011. Engineering education has a special role concerning this phenomenon. The number of students entering engineering has remained nearly the same, according to OECD (2013) on average only 15 % of those who progress to HE enter engineering, manufacturing, and construction programmes. In many countries, including Czech Republic, the number of institutions that provide bachelor study programmes has increased rapidly. For example, in the Czech Republic this number has more than doubled. At the same time in some countries the number of students completing secondary schools has dropped (for example in the Czech Republic this number has reached a minimum this academic year). Moreover students entering HE institutions to study engineering have different backgrounds; not all of them come with sound mathematical skills (not speaking about understanding of basic mathematical concepts). All this leads to two main problems

1. What policy to adopt for admission of students to engineering studies.

2. What strategy to use to help students who had enrolled into engineering studies to overcome difficulties with mathematics in the first year of study.

The above problems are strongly interrelated, but in this contribution we will focus on the first problem and present experience gained at the Czech Technical University, mainly at the Faculty of Electrical Engineering.

Admission procedure at Faculty of Electrical Engineering, CTU

Historically, there was only one study programme at FEE called Electrical Engineering and Informatics (E&I) leading to master diploma (its duration was 5 years). After changes caused by the Bologna declaration this programme was divided into two separate programmes: a bachelor and a master one (3 years and 2 years). Since the bachelor programme had to bring self-contained education, a new strategy was adopted – five new programmes were accredited in 2009/10 instead one programme with five branches. This coincided with first serious

decrease of interest in studying FEE. Hence unlike the E&I programme four of the new programmes decided not to have an entrance examination and accept every student who had applied. These programmes were Power Engineering and Management (EEM), Communication, Multimedia and Electronics (CME), Cybernetics and Robotics (CYR), and Software Engineering (SI). The only programme where an entrance examination was maintained was Open Informatics (OI), a programme with large number of applicants. In the academic year 2010/2011 CYR also declared itself as an "exclusive" programme and return to the practice of testing at least a part of the applicants. In academic year 2012/13 all five different programmes had returned to "classical" admission procedure.

The admission procedure that was adopted in 2012/13 is the following: An entrance examination from mathematics consisting of 15 multiple choice tasks, 10 small ones (1 point) and 5 bigger ones (2 points). The parts of secondary mathematics that are covered in tests are: quadratic equations, inequalities with absolute value, goniometric functions, elementary functions, goniometric equations, complex numbers, sequences, basic geometry including analytic geometry, and combinatorics. The test is the same for all programmes, while the admission requirements vary from a minimum of 8 points to enter EEM, CME, and SI to 11 points for CYR and OI.

The requirement to successfully pass the entrance examination is exempted for those applicants who satisfy one of the following conditions:

1. Their average grade from at least 3 mathematics courses was not worse than 1.5 (1 is the best, 5 is the worst) during their secondary studies.
2. Their average grade from at least 3 physics courses was not worse than 1.5 during their secondary studies. (This does not apply for computer science programmes OI and SI.)
3. They passed the state school leaving examination in Mathematics.
4. They were successful in some national competition, e.g. Mathematics, Physics, or Programming Olympiad.
5. They score high in national SCIO mathematics exam (among the top 10 %).

For the purpose of this contribution we will choose two of the programmes, CME and CYR. The programme CYR declares itself as exclusive, unlike CME that has presented itself as a standard bachelor programme. At the academic year 2009/10 both programmes exempted entrance examination for all applicants; in 2010/11 CYR returned back to the strategy that the exams was exempted only under the above conditions.

	2009/10		2010/11		2011/12		2012/13		2013/14		2014/15	
	CME	CYR	CME	CYR	CME	CYR	CME	CYR	CME	CYR	CME	CYR
# appl.	577	372	904	389	808	320	350	323	295	291	308	320
# exempt	577	372	904	130	808	117	92	165	98	148	79	133
# exams	0	0	0	182	0	162	191	121	158	108	182	137
# did not attend	0	0	0	77	0	41	67	37	39	35	47	50
# enrolled	354	239	458	161	446	163	126	169	121	152	137	155

Table 1: Admittance

The next table shows the number of students who enrolled in the first year and did not succeed to continue to second year. More precisely, the number of students not successful in the first semester, the number of students that failed to continue their study in second year, and finally the number of students that continued in the second year.

	2009/10		2010/11		2011/12		2012/13		2013/14		2014/15	
	CME	CYR	CME	CYR	CME	CYR	CME	CYR	CME	CYR	CME	CYR
# enrolled	354	239	458	161	446	163	126	163	121	152	137	155
# failure 1st sem	223	103	279	21	286	43	65	61	52	39	53	43
# failures 1st year	243	128	279	22	309	56	72	84	64	55	69	57
# in 2nd year	111	111	179	139	137	107	54	79	57	97	68	98

Table 2: First year

One can observe from the above tables that there was no decrease in the number of applications for CYR after reestablishing the entrance examination, while the number of students enrolled at CYR decreased by 32.6 % in 2010/11, and remained nearly the same for the academic years 2012/13 – 2014/15. On the other hand, the number of students who enrolled in CME for the academic years 2010/11 and 2011/11 increased with respect to 2009/10 by 129.4 % in 2010/11 and by 126.0 % in 2011/12. After the entrance examination was reintroduced in 2012/13 there was a drop to 35.6 % of the 2009/10 enrollment or to 28.3 % of the 2011/12 figures. From the academic year 2012/13 CME has become a smaller programme than CYR with respect to the number of students

The decrease of the number of students who enrolled into programmes was caused by several reasons. Let us mention some of them:

1. Decrease of students finishing secondary education by more than 30 %
2. The capacity of HE institutions in popular fields (law, medicine, economics, humanities) remained unchanged
3. Decrease of interest in study engineering programmes
4. Increase of the number of HE institution where engineering and computer science can be studied with less expense (Prague is considerably more expensive to live in than other towns)
5. Reputation of FEE CTU as a difficult school.

Failure rates

A comparison of failure rates at CME and CYR in the academic years 2009/10 until 2014/15 can be seen from the table 3. The third row shows the number of students who had only gained at most 5 credits (out of 30); requirement for passing to the second semester is at least 15 credits. The fourth row shows the percentage of the number of students who enrolled in 1st year but failed to continue after the 1st semester and the fifth row shows the percentage of enrolled students who failed to continue to the second.

	2009/10	2010/11	2011/12	2012/13	2013/14	2014/15

	CME	CYR	CME	CYR	CME	CYR	CME	CYR	CME	CYR	CME	CYR
# enrolled	354	239	458	161	446	163	126	163	121	152	137	155
# failure 1st sem	223	103	279	21	286	43	65	61	52	39	53	43
# stud. 0-5 credits	155	53	216	14	185	23	30	21	27	14	26	19
% failures 1st sem	63	43.1	60,9	13	64.1	26.4	51.59	37	43	25.7	38.7	27.7
% of failure	68.6	53.6	60.9	13.7	69.3	34.4	57.1	52	52.9	36.2	50.4	36.8

Table 3: Failure rates

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The table shows a very high failure rate in 2009/10 (68.6 % for CME and 53.6 % for CYR). In 2010/11 the reintroduction of the entrance examination at CYR led to a sharp drop in the failure rate. On the other hand CME, which had no entrance examination had only minor drop. In the following years, the failure rates at CYR increased back to approximately 34 – 37 % and remained stable. This is, however, considerably lower than in 2009/10. Similarly, the reintroduction of entrance examinations for CME in 2012/13 led to a decrease of the failure rates by approximately10 percentage points to app. 50 %.

Advantages and disadvantages

There are two main approaches to admission procedure which can be taken by a university/faculty.
1. Accept all applicants who want to study and consider the first semester as a "prolonged" admission procedure.
2. Admit only those who have shown ability to successfully study on the programme

Of course, it is difficult to follow completely the second strategy in the Czech Republic; indeed, by the Czech University Act any student who has successfully passed the school leaving examination can apply to any HE institution, and at the same time there is no limit to the number of applications to different HE institutions that one student can make. Moreover, a university/faculty does not know whether an applicant really wants to enrol or if he/she will choose other school. During the last 4 years, FEE has adopted a strategy which is "between" the both above mentioned ones – not everybody is accepted but on the other hand the requirements for admission are not too restrictive. Let us summarize advantages and disadvantages of this strategy (with respect to the full acceptance used in academic years 2009/10 – 2011/12).

Advantages
1. From table one it is seen that in 2011/12 the CME had more than 800 applicants who were accepted. Only a little more than half (55.2 %) enrolled in the first year of study. Moreover, only less than 36 % of them continued their study in the second semester. In first year tutorials were organised in groups of 25 students, this meant demand for 6 more teachers who were not needed in the summer semester. From 2012/13 approximately only 1 extra teacher was needed to organize the teaching.
2. The percentage of students that were/are interested in their studies has increased considerably. Figures cannot be shown but teachers experience justifies it.
3. FEE has gained the reputation of being "a good choice", a faculty where a student gains very good education.

Disadvantages
1. FEE might lose students that do not have sound skills (entrance examinations test mainly skills) from their secondary education.

2. Considerable decrease in number of students. This has affected mainly programmes that for couple of years admitted all applicants.

To overcome the above disadvantages, in 2015/16 the faculty introduced a second round of admissions that took place at the beginning of September. Before entrance examinations an intensive five day mathematics course was organised. Each day was devoted to one or two mathematical themes from the tests. In the morning, a 2 hour lecture was followed with a 2 hours tutorial. After a lunch break students worked individually and a teacher served as an advisor. Of the attendants at the course 27 applied for admission and 20 were successful. In the second semester 17 students continued their studies into the second semester.

Conclusions

The experience gained at Faculty of Electrical Engineering, CTU in Prague shows that the more advantageous strategy is to admit those applicants that have shown some willingness to study and some knowledge of basic mathematics skills. For those with deficiencies faculties should provide some additional help.

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Blended Learning for Mathematical Preparation Courses – Video Based Learning and Matching In-Class Lectures

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Abstract

Many universities offer preparation courses in mathematics for students in STEM fields. Within the project viaMINT, located at the *Hamburg University of Applied Sciences*, such courses are developed based on the concept of blended learning.

In the online learning environment viaMINT, students first take an online placement test that generates recommendations on which topics they should concentrate on. The online modules are video based and include online exercises that focus on providing practice. This way fosters the competency to deal with the symbolic, formal and technical elements of mathematics. Other mathematical competencies, such as problem solving, modelling, communicating and reasoning, are promoted in the corresponding in-class lectures, where the students also benefit from the opportunity to socialize. Teaching methods, such as group work, peer instruction and other activating methods, are applied to train not only the mentioned mathematical competencies, but also social and linguistic skills.

The first in-class lectures offered as part of this blended learning scenario were taught in September 2015. In the first evaluation, the concept was approved of by the students. In the spring of 2016, the in-class courses were expanded and further evaluations were conducted. This paper will present the concept of blended learning, in-class lectures, as well as results of the evaluations and the experiences gained so far.

Introduction

In order to close the knowledge gaps between high school and university, many universities provide preparation courses in mathematics and other topics. Attendance courses, as opposed to online courses, often result in a crash course, which may last one to two weeks before the first semester begins. In these crash courses, all topics are presented in the same way to every student regardless of his or her individual prior knowledge or mathematical strengths and weaknesses.

Online courses can lessen some of these restrictions through a longer time frame or individual learning possibilities that will be described later on. But online courses also have their weaknesses. Mathematical competencies, such as arguing and reasoning or devising strategies are better developed in a social environment. Feedback also clearly shows that students appreciate having attendance courses as well as the online courses as they allow them to meet their fellow students and discover the university.

To get the best out of both formats, viaMINT[3] offers a blended learning format which combines the advantages of online learning with the benefits of attendance courses.

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[3] https://viamint.haw-hamburg.de

Blended Learning Concept

The blended learning approach in viaMINT is based on concepts of the Inverted Classroom model (Loviscach, 2013). To allow every student to work on his or her individual knowledge gaps, it additionally takes into account the heterogeneity of the student's prior knowledge.

Firstly, the students work autonomously in the online environment on topics recommended based on the results of an online placement test (see Figure 1). Each topic is accumulated and clustered in an online learning module and can be processed by students at their own pace. The subsequent matching in-class lectures do not simply replicate the content covered online, but extend and deepen the students' mathematical knowledge and capabilities by means of various learning methods e.g. group work, modelling tasks or exercises connecting different mathematical topics.
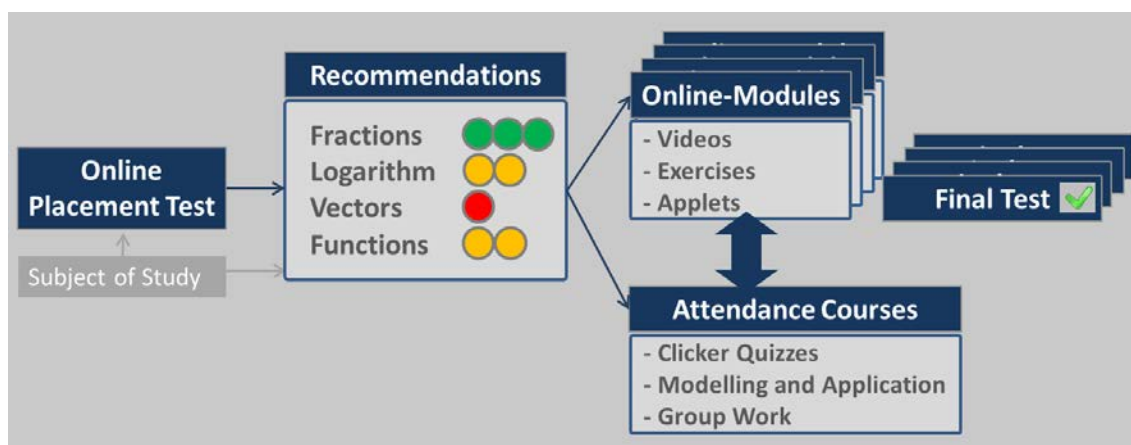

Figure 1: The blended learning concept of viaMINT, containing an online placement test, recommendations and online modules as well as corresponding attendance courses.

Ideally, the students should complete the online learning modules before the beginning of the attendance course. The attendance course is conducted according to a timetable with one mathematical topic per day. Each day is divided into a morning and an afternoon session and is thematically self-contained. Some afternoons are reserved for the students to revise the corresponding learning module of the topic which is to be discussed the following day. To enable the students to participate selectively according to their needs, a schedule showing the topical structure of the course is announced before the start of the attendance course. Students can access viaMINT even before they are enrolled. They are informed about the online learning opportunities and the dates of the attendance course about 2 to 4 weeks in advance via e-mail.

Online Learning with viaMINT

viaMINT is an interactive, video based, online learning environment that offers individual learning opportunities (Landenfeld et al. 2014; Landenfeld, Göbbels & Hintze 2016). Initially, the students are recommended to take an online placement test that focusses on school knowledge and automatically generates recommendations on which topics the student should focus on. At the moment, the online test consists of six questions per topic, e.g. *Logarithms,* which is a compromise between test length and test validity.

The online learning modules are structured in chapters and subchapters in which the content is explained in short videos. The videos mostly use a screen capturing format and are ordered in a sequence with different learning elements, such as interactive questions and online exercises

for self-assessment purposes. The questions provide extensive feedback on how the students can adjust their approach to reach the correct conclusion. A final online exercise, offered with each learning module, provides feedback on mastery of the content. The students are supported in their learning organization by means of a "personal online desk." It provides an overview of their learning process by using different categories (recommended modules, modules in progress and completed modules) to organize the modules based on test results and by features like progress bars and badges indicating learning achievements (ibd.).

viaMINT is based on Moodle[4]. The online exercises are realized using STACK/Maxima[5], a Moodle extension (Sangwin 2013). The diversified questions provide instant feedback. They not only aim at fostering understanding, but also at training routines and applying calculation methods to various problems. In terms of mathematical competencies (KMK, 2003; Niss, 2003), they primarily support the use of symbolic, formal and technical language as well as using and switching between different representations. Mathematizing or modelling and problem solving can be implemented in the assessment system only to a limited degree, arguing, reasoning and communicating are better trained with face-to-face learning methods. Due to this, the online modules are complemented by an attendance course with matching class lectures.

Concept and Example of In-Class Lectures

Each classroom lecture typically starts with a multiple-choice quiz using an audience response system (clicker) as a 'warm-up.' The questions refer directly to the content of the associated online module. While some of the clicker quiz questions simply ask for factual knowledge, others are comprehension questions which give stimuli to further discussions. Especially for these questions, the method of peer instruction is applied (Mazur, 1997). Thus, the questions offer an opportunity to communicate, argue and reason mathematically. The quiz helps the students recall different aspects of the topic and also helps the teacher to identify and correct misconceptions.

To foster the further development of mathematical competencies like modelling, problem posing and solving and, in general, mathematization, the quiz is followed by open application and modelling tasks that have students working in different teaching methods. For example in the class lecture on *Vectors,* the students work in groups, each on a different modelling task like 'How is it possible to sail against the wind?' or 'How does a rear reflector work?' The modelling tasks refer to the students' everyday lives and aim at transferring mathematical knowledge to real-world problems by the use of different strategies. The goal of the group work is to deepen the mathematical knowledge by asking questions, explaining and discussing. At the end of this

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[4] Moodle is an open source online learning platform: https://moodle.org/.

[5] STACK is a System for Teaching and Assessment using a Computer algebra Kernel:

http://www.stack.bham.ac.uk/

Maxima is a Computer Algebra System: http://maxima.sourceforge.net/

three hour lecture, the groups display their results on posters in a Gallery Walk. Thus, they not only have to choose an appropriate way to present their work, but also have to explain their work to the other students and argue about different solution processes or further issues. This social and communicative involvement is one of the additional benefits of the attendance courses.

Currently, the topics *Vectors*, *Powers and Roots*, *Logarithms*, *Equations and Inequalities* are implemented in this way and serve as pilot in-class lectures. In order to have manifold and application-oriented lectures, the learning method in the second half of the lecture is varied according to the different topics. In *Logarithms*, for example, the students get the chance to discover 'How does a slide rule calculate?' as a hands-on exercise and in *Powers and Roots,* scientific notation of numbers is explained to show the relevance of the topics to their following studies (see Table 1).

Lecture parts	Intention	Examples from *Vectors*	Examples from other topics
clicker quiz	recapitulation and comprehension questions about the online module	'Are light and wind vectors?'	mathematical puzzle
activating methods, experiments, modelling tasks application in different collaborative social forms	mathematizing, strategic thinking, communicate, argue and reason mathematically	Modelling about 'How is it possible to sail against the wind?' or 'How does a rear reflector work?' presented in Gallery Walk.	'How does a slide rule calculate?' (*Logarithms*) Working with Scientific notation (*Powers and Roots*).

Table 1: Structure and exemplary content of the attendance course

Evaluation and First Experiences

The first in-class lectures offered in this blended learning scenario were realized in September 2015 and spring 2016. About 150 students attended these courses. In qualitative evaluations (questionnaire and oral feedback), 80 participants and the lecturer evaluated the blended learning format with a focus on the in-class lectures.

Feedback showed that the presentation of the timetable in advance is very important for the students in order to prepare questions regarding content covered in the online modules. As oral feedback, the students requested a time slot for their prepared questions in each session.

The qualitative evaluations showed that students who had worked with the online modules extensively in advance appreciated the quizzes because it exemplified their improvements. Students who had not worked with the online modules thoroughly beforehand realized that extensive preparation is important in order to benefit from the in-class lectures. Therefore the clicker quiz provides a motivation to work with the online modules extensively in advance.

Some students were unaware of the blended learning concept before the beginning of the in-class lectures. They participated in the in-class lectures during the day and worked with the online modules in the afternoon/evening. They reported that this was hardly feasible due to the

length of the online modules and their extensive content. Therefore it is very important to inform the students at least two weeks before the beginning of the in-class lectures about the blended learning concept and why it is necessary to work through the online modules beforehand.

The in-class lectures demand active collaboration and commitment from the students. To prepare the Gallery Walk, students had to develop strategies to solve the problem as a group and then explain their results in a comprehensive way to students from other groups. When developing the concept for the in-class lectures, we were sceptical how the engineering students would react on these cooperative tasks and teaching methods, because typical university courses are still lecture-centred. But students' and lecturer's feedback showed that most students appreciated the group work phases in the in-class lectures and considered the phases as beneficial for a deeper understanding: due to the complexity of the tasks, students found it helpful or even necessary to work cooperatively to develop effective solving strategies.

Conclusions and Outlook

Evaluations clearly show that the students appreciate the blended learning concept of viaMINT. They also show how important it is to ensure that they understand this concept in advance, as the success of the concept depends on the knowledge the students gain through the online modules. For this, earlier announcements of the blended learning preparation courses especially the online courses are planned, e.g. an automated e-mail, which is sent immediately after the submission of the application. Further topics will be included in the blended learning format and the evaluations will be enhanced as well.

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On three alternative approaches to teaching mathematics

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Abstract

This paper analyses three common suggestions for how to change the way we teach mathematics: *a problem-solving approach*, *a modelling approach,* and *mathematics as an artistic subject*. They are all based on a critique of the traditional mathematics education and argue for replacing the focus on *answer-getting* with an educational system that rather values *sense-making*. A main difference between the three approaches is which kind of mathematics, from *pure* to *applied*, we should focus on to best motivate and teach students.

Introduction

Mathematics is one of three core subjects in Swedish schools, and we put a lot of time and effort into teaching all children mathematics. However, many pupils and adults do not understand the purpose of studying mathematics and why we all need it. Issues regarding mathematics education are debated at all levels of society, in national politics as well as in local newspapers, and most people seem to have an opinion about what the problem is or what we should do about it.

This paper is a summary of the results of a Master thesis by the first author, to be published in 2016. The thesis analyses the debate on mathematics education using the three questions "Why should we study mathematics?", "What should students learn?" and "How should we teach mathematics?". It is based on experiences from the Swedish educational system, interviews, and a review of the mathematics education research literature, which serves as a base for the categorisation of both *critique* of the current state of mathematics education as well as *proposals* of what should be done to solve the problems.

Based on the questions "Why?", "What?", and "How?" we have chosen to focus on three common propositions for how to change mathematics education to increase pupils' motivation and knowledge: *a problem-solving approach*, *a modelling approach* and *mathematics as an artistic subject.* All three have in common the critique of the traditional approach to mathematics education, but they differ in their ideas for what this approach should be replaced with. In this paper we will outline all the different approaches, and finally discuss how they relate to each other. While we have chosen, in this paper, to focus on these three common propositions, we note that there are yet other suggestions which emphasize other aspects such as e.g. general education or computation.

The traditional approach

Many pupils experience mathematics as a subject where the most important skill is to be able to remember rules and formulas as well as possible. Magdalene Lampert describes this as an *answer-getting* process rather than a *sense-making* process (Green 2014). When faced with a task or a problem, rather than analysing and thinking about ways of solving it, pupils experience that they are expected to just apply the formulas they have most recently been shown. Research has shown that pupils' focus on remembering an abundance of formulas impairs their problem-solving ability (Boaler 2009).

Another characteristic of the traditional approach to mathematics teaching is that pupils work individually and in silence. Instead of talking about problems and ideas for how to solve them, the pupils are often expected to learn by listening to the teacher, reading, and practicing in their textbooks. There is a significant difference between listening to someone talk about mathematics, and talking about it yourself or explaining it to others (Boaler 2009). When hearing someone explain something it is often easy to agree that it makes sense, but in order to really understand, talking and discussions are crucial. According to Lampert (1990), the traditional classroom environment often fosters pupils who do not want to explain their thoughts out of fear of being wrong or of getting into discussions about more complex problems.

A problem solving approach

One of the most widespread views about how we should teach mathematics is that we should focus on problems and problem solving, rather than practicing rules and doing standard calculations. The idea that problem solving is at the heart of mathematics, and that it should be the main focus of mathematics education, has been expressed within mathematics education at least since the end of the 1970's (Schoenfeld 1992), and probably far earlier than that. Problem-solving is often considered important for pupils' ability to learn the contents of mathematics courses, and for helping pupils to understand how mathematics can be applied in other subjects and in real life situations (Boaler 2009).

When distinguishing problems from standard tasks, many highlight as characteristic of a problem the absence of a method that will lead to the solution, for example Posamentier & Krulik (2008) define a problem like this:

"In essence, a problem is a situation that confronts a person, that requires resolution, and for which the path to the solution is not immediately known." (2008, p. 1)

Posamentier & Krulik (2008) claim that working with problem-solving strategies is an important tool to make pupils aware of the methods they use to solve problems, both in school and in everyday life. We often encounter tasks that we are familiar with and know how to solve, but to practice our ability to solve other more complicated problems, studying problem-solving strategies is an efficient method. Mathematics courses in schools are often planned and connected by how the contents of the different courses relate to each other, but we often neglect the fact that similar strategies can be used throughout most parts of mathematics. By focusing on strategies and their general applicability we can help pupils see the purpose of practicing problem-solving, and there is a better chance that they will learn to see mathematics as a fascinating subject (Posamentier & Krulik 2008).

One of the most common and important aspects of a problem-solving approach is that it is a method that advocates changing the conventional mathematics learning environment, where pupils work individually and silently, into a place where collaboration and discussions are encouraged (e.g. Boaler 2009 and Hagland, Hedrén, & Taflin 2005). One argument for this way of studying mathematics is that it resembles how more experienced people work with the subject. This brings legitimacy to the method in two ways, it indicates that it is an efficient method, and it is also an approach that is used outside of school (Lampert 1990).

A modelling approach

A "models and modelling"-focused mathematics education is a platform where pupils need to work in groups, discuss their ideas, and analyse their results while working on realistic problems

that they find meaningful (Lesh & Doerr 2003). Teaching modelling and problem solving in combination can be very effective both when it comes to teaching specific abilities and knowledge, and as a way of helping students understand the purposes and applications of mathematics (Wedelin & Adawi 2014).

In their different forms, modelling approaches to mathematics education are to some extent similar to the problem-solving approach, since mathematical modelling also revolves around solving problems, but both the starting point of the problems and the methods are different. In traditional, as well as in problem-solving mathematics education, pupils usually have to try to make meaning of abstract tasks or problems themselves. A modelling approach starts out at the other end, and makes mathematical problems out of real world situations. Learning how to create models to describe or analyse real-world situations can help pupils become better problem solvers in everyday life and provide them with tools that help them handle and understand complex situations (Lesh & Doerr 2003).

Mathematical modelling resembles the way mathematics is used in many different disciplines, such as engineering and economics, connecting mathematical knowledge to other subjects that pupils study in school and which they might work with in the future (Ang 2001). This way of adapting mathematics education to methods used in other disciplines differs from the, perhaps more common, opinion that mathematics education should be more concerned with the objectives of mathematicians, such as for example prioritising how to formulate formal proofs and similar activities.

Learning mathematical modelling involves learning how to use mathematics to describe and analyse problems or situations in non-mathematical contexts. Doing so requires simplifications, guesses, and assumptions which are somewhat unusual mathematical methods to most pupils since they generally consider mathematics a subject that requires great accuracy and exactness (Schoenfeld 1992). Many engineering students know much more mathematics than they are able to use since they have never practiced the applications of it in their mathematics classes (Wedelin & Adawi 2014).

Mathematics as an artistic subject

If we are unable to convince pupils of the necessity of mathematics, another way to motivate them is to convince them that mathematics is interesting and fascinating in itself. In his critique of the American K-12 learning standards, Paul Lockhart (2009) argues that pupils who consider math class stupid and boring are correct. What Lockhart means is that mathematics on the one hand, and the subject taught in mathematics classes on the other, are two completely different things. While he argues that mathematics is an art form that should encourage creativity and arouse curiosity, he asserts that mathematics education engages in a subject that rather values skills such as discipline and precision. Instead of arguing that the study of mathematics is necessary for our everyday lives, we could argue that it is important for our imagination and creativity, or just fun. Lockhart (2012) points out that the physical reality we live in and mathematical reality are two different things, and that we should point this out to the pupils as well.

Summary of results and discussion

Figure 1 illustrates that the three approaches all argue for a shift from answer-getting towards sense-making. The three approaches differ in their views on the subject and how we can best motivate students.
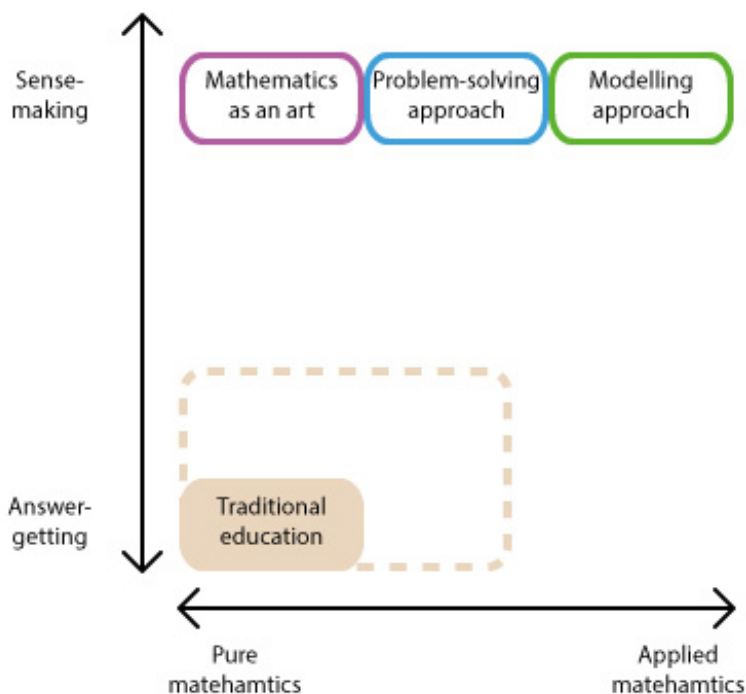
Figure 1. Conceptual framework part 1.

As figure 1 shows, the three approaches present three different ways to present the purpose of studying mathematics, from focus on the exploration of the mathematical world to focus on the applications of mathematics in many different situations.

Figure 2. Conceptual framework part 2.

Figure 2 highlights similar and different characteristics of the three approaches based on the questions why, what, and how we should teach and learn mathematics.

Given these different approaches, we can conclude that there is no universal consensus regarding how we should resolve the problems with mathematics education. However, there is significant agreement on some points, in particular that mathematics education should be an investigative and creative subject where students get to explore, rather than just read and practice what mathematicians have discovered before them. Then, to make sure that any agreed changes would actually be carried out, it is also important that teachers and schools are provided with appropriate resources and help to implement such changes, since lack of time and support often results in staying with what you are used to.

Much of the frustration among students and teachers could stem from the fact that mathematics education is claimed to be important, but at the same time the learned skills are never or seldom used in practice. Maybe the traditional mathematics education could be compared to the idea that it is important to know how to play football, but that we actually just practice running and the theories of penalty shooting and never actually get to use footballs or play any matches. If we continue to argue that mathematics is important, we should try harder to prove it to students and to ourselves. Otherwise maybe we should agree that mathematics is not so important, but perhaps a fun and interesting activity.

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Using Electronic Exams to Provide Engineering Mathematics Students with Rapid Feedback

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Abstract

With increased sophistication and capabilities of e-Assessment systems it would seem that online examinations will become standard practice, particularly for numerate subjects, in the near future. We have gained valuable experience in this area through running online mathematics examinations for over 300 engineering students in January 2015 and January 2016. We used the fully algorithmic open-source Dewis e-Assessment system to run the online examinations. One of the major advantages to running exams in this way, as opposed to running a traditional paper-based exercise, is that students are able to receive rapid feedback on their work, because their submissions are marked immediately. It also enables academics to quickly identify those students that are at risk on the module, enabling them to specifically target such students early on in the year, at a point where interventions are likely to yield positive results. We have seen improvements in module pass rates since introducing the online exam.

Introduction

We report on our experiences of running an on-line exam in January, under controlled conditions, to assess all students taking the Level 1 Engineering Mathematics module, at the University of the West of England, Bristol (UWE). Engineering Mathematics (EM) is a compulsory 30 credit module taken by students on a range of awards; Mechanical Engineering, Aerospace Engineering, Automotive Engineering, Electronic Engineering and Robotics. The module is taught over the whole academic year; in the first semester the focus is on the consolidation of existing mathematical techniques, with new material being introduced in the second semester.

E-Assessment has been used on this module for many years and our assessment strategy has evolved from being merely summative to also include formative assessments that give high quality feedback from which students actively learn (Greenhow and Gill, 2008). Using e-Assessment for coursework has become standard practice in many institutions (Sangwin, 2013) and it seems likely that online examinations will become standard practice, in the near future (Kuikka et al, 2014). We use the Dewis system for all the e-Assessments on this module.

Dewis is a fully algorithmic open-source web-based e-Assessment system which was designed and developed at the University of the West of England (UWE). It was primarily designed for the assessment of mathematics and statistics and supports a range of inputs, such as numeric entry, algebraic entry, matrix entry, multiple choice and radio selection. An example of an e-Assessment question used for EM is illustrated in Figure 1 together with the full feedback received. Using an algorithmic approach enables the separate solution, marking and feedback algorithms to respond dynamically to a student's input and as such can perform intelligent marking. In addition, the Dewis system is data-lossless; that is, all data relating to every assessment attempt is recorded on the server. This enables the academic to efficiently track how a student or cohort of students has performed on a particular e-Assessment (Walker, Gwynllyw and Henderson, 2015). Recent developments include using embedded R code to facilitate the assessment of students' ability to perform involved statistical analyses (Gwynllyw, Weir & Henderson, 2016) and using Dewis to automatically mark computer code (Gwynllyw, 2016).

The system is currently used in the fields of Business, Computer Science, Nursing, Engineering, Statistics and Mathematics. Implemented for the first time in 2007 the system is now well-established and in 2015/16 within UWE and partner institutions, Dewis was used for formative and summative tests to support over 3,500 students involving more than 50,000 assessment attempts.

Figure 1. An example Dewis question, together with feedback and marking bespoke to the random parameters used in this question. This question illustrates partial marking; the method of solution is correct, but the implementation was not.

Prior to the 2014/15 academic year, the controlled element of assessment took the form of a single end of module paper-based exam. However, in order to better prepare students for the rigour of an end-of-module examination, a January on-line exam, run under controlled conditions was introduced. The module is assessed through exams and coursework and the coursework element comprises a suite of e-Assessments and a single Matlab assignment. The rationale for including an on-line exam, on top of the summative coursework e-Assessments, is that it forces the students to demonstrate their understanding under controlled conditions and is early enough in the module that students who have fallen behind can be efficiently be identified.

Online exam

The number of engineering students at UWE has increased substantially over the last few years, due in part to the success of the Bloodhound project (2011). In the 2014/15 and 2015/16 academic years there have been 316 and 340 students enrolled on the module, respectively. Due to the lack of available computers, the January online exam was delivered in two sessions. Approximately half the students were timetabled for the morning session and the other half for the afternoon. For each separate run of the exam we fixed the parameters of the questions, in order to ensure fairness and also that, at the start of the exam, students could be given a hardcopy of the specific questions that they were attempting. Students valued this, as some found it easier to work from a paper copy than from the screen. In 2014/15 each exam version contained 14 questions, whilst in 2015/16 each exam version contained 15 questions. Both exam versions contained a mixture of input types. Students were accommodated in at least five computer labs

and each room contained two invigilators and a member of academic staff involved with the module who could field any possible Dewis-related or mathematical queries from students.

In the computer labs, the computers were set up so that only the relevant Dewis examination link was available and each computer was allocated a unique examination key. When instructed to do so, students logged in to Dewis using their standard UWE access details and also their computer's examination key. At this stage, the computer's examination key was linked to the student which provided an additional security feature, ensuring that only students within the room were able to access the exam. At the start of the exam, students were directed to "Start the online exam" and at this point, Dewis displayed a persistent green top horizontal bar (as shown in Figure 2) containing details of the exam version and student's identity. This display was intended to facilitate the invigilation process. On submission of the assessment, the colour of the horizontal bar changed to pink. Students were allowed to submit their answers ahead of the two hour limit and, in such cases, they were instructed to ask an invigilator to view the pink bar before leaving the exam room (as shown in Figure 2). The invigilators commended this process as it provided them with an easy visual check of the student engaging with the Dewis system and also of the status of their assessment attempt.

Each student's attempt was marked instantly on submission. However disclosure of the mark to the students was delayed for a few days in order to perform an evaluation of the examination. This process of reviewing and reflecting on the examination included an analysis of the complete spread of marks over both morning and afternoon sessions. The official submission was online but students were given exam booklets to write in their workings and these booklets were collected in after the exam. A sample of 30 of these paper-based booklets was marked by hand in the traditional way and the mark that the paper copy would have received versus the official online mark for this sample is shown in Figure 3. In nearly all cases, we found that there was very little difference between their online mark and that which would have been awarded had paper-based marking been employed. The biggest differences occurred when the student did not enter their workings on paper, choosing instead to enter the answer directly on the computer, resulting in a higher online mark. For the cases whereby the paper-based mark was higher, it was due to method marks being awarded. Following the e-Assessment evaluation some of the questions were improved, such as Question 9, as shown in Figure 1. This now gives method marks for correctly identifying the method to obtain the desired (derivative) answer.

(a)

DEWIS Examination: [em_10am] [morning1516] [student_id]

University of the West of England

DEWIS e-Assessment @ UWE

UFMFJ9-30-1 Engineering Mathematics January Exam 2016

(b)

DEWIS Examination: [em_10am] [morning1516] [student_id]

University of the West of England

DEWIS e-Assessment @ UWE

UFMFJ9-30-1 Engineering Mathematics January Exam 2016 - **Result**

You have now successfully submitted all your answers.

Please do not shutdown this browser nor leave the examination room until an invigilator has seen this page.

For reference, below is a list that summarises all your submitted answers.

Figure 2. The top horizontal bar was a visual aid for invigilators. (a) Green indicated that students were currently taking the exam; (b) Pink indicated that the student had submitted their answers.

Once we had completed the review process, students were able to log back into the appropriate version of the exam and view their submission to get full bespoke feedback for their attempt as well as their mark. Having prompt access to the marks breakdown meant that all failing or non-submitting students were identified early and invited to attend additional sessions, held during Semester 2, in which the module team were able to provide targeted support.

Results

In Table 1 we show the average exam mark and pass rate for the exams held over the last four academic years. Prior to 2014/15 the controlled element comprised solely a traditional paper-based exam held at the end of the academic year and during this time the pass rate and average marks have been disappointingly low. However, since the introduction of the online exam, there has been a marked increase in both measures. The results for 2014/15 show the aggregated exam mark (75% summer written exam, 25% online January exam), whilst the results for 2015/16 only include the results from the online January exam.
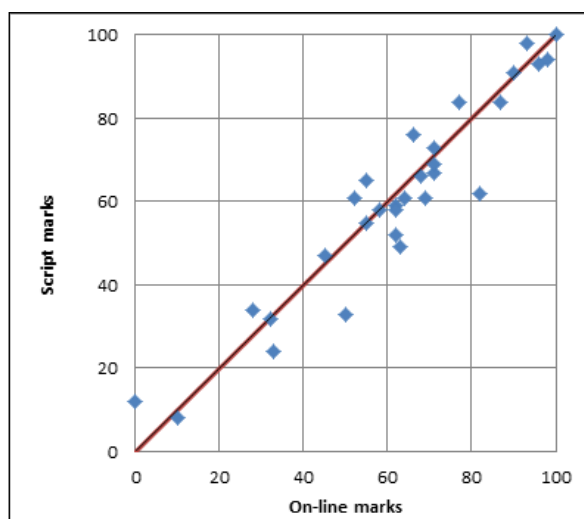
Figure 3. A comparison between the mark that the paper-based exam booklet would have been awarded compared to the official on-line mark for a sample of students (blue squares). The red line represents $y=x$.

Exam and year	Number of attempts	Pass rate	Average Mark
Summer 2012-13	248	66.6%	49.7
Summer 2013-14	284	62.7%	50.4
Overall 2014-15	**298**	**80.1%**	**66.5**
January 2015-16	**313**	**73.9%**	**60.0**

Table 1. Comparison of exam marks and pass rates, over four academic years.

In Figure 4 we show a scatter plot of the summer written exam marks against the January on-line examination marks for 2014/15. We can see that there is a close correlation between the two sets of results and we conclude that the January online exam is a good predictor of success in the module.

Discussion

The questions for the January exam are taken from the Semester 1 weekly Dewis practice tests and we have observed increased engagement in these since 2014/15. The results for January exam in 2015/16 are not as good as for 2014/15. The exam contained one more question and students took longer on average on the exam; an average of 85 minutes, compared to 66 minutes in 2014/15. Also, the number of students submitting in the last 10 minutes of the exam increased from 13 to 60, so it seems that the difficulty level was higher. On a more positive note, the number of students viewing their feedback also increased, with 77% of students viewing it in 2015/16. This is much higher than the 10% that is typical for paper-based work. Research (Race, 2014) shows that feedback has to be quick to be effective, while students still remember clearly the work they were engaged in and online exams are one way of achieving this.
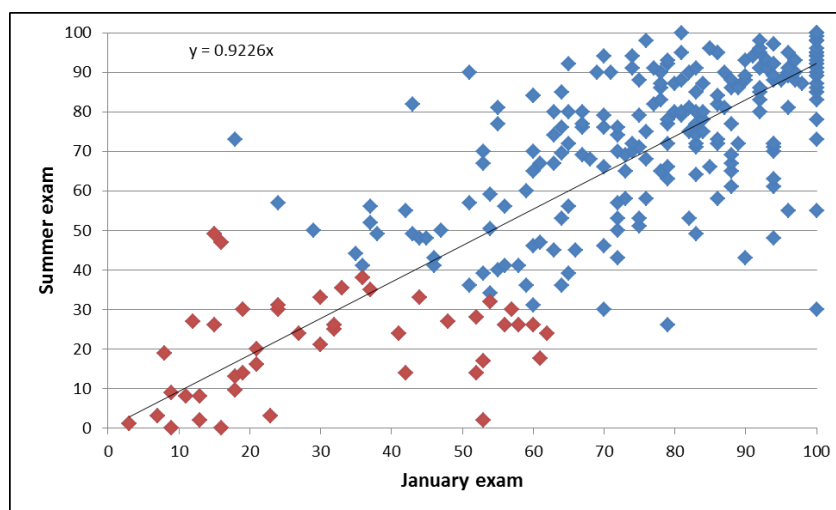
Figure 4. Examination Results for the 2014/15 academic year showing the correlation between the written summer exam results and the on-line January exam results. The blue/red squares indicated students who passed/failed the module and the black line (y=0.9226x) is the line of best fit through the origin.

Considering the 2014/15 results we conclude that the January exam gives a good indication of a student's current performance and future success on the module. It is also early enough to identify those students who require extra support and get them back on track with the module.

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Evaluating the design of a course in mathematical modelling and problem solving from the students' perspective

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Abstract

Mathematical modelling and problem solving constitute central aspects of everyday engineering work. But how can we design courses that help students develop these key skills? In this paper we describe the design and evaluation of an inquiry-based course in mathematical modelling and problem solving for second-year engineering students. The course was evaluated using a qualitative research approach based on reflective reports submitted by the students at the end of the course. The students report a significant development in their mathematical modelling and problem solving skills, including metacognitive skills, and their belief system. The authentic learning environment contributed to these learning outcomes. The problems in conjunction with several other aspects of the course created this authentic environment. We discuss a framework for designing authentic learning environments and its significance for engineering education.

Introduction

Engineering workplace problems are complex and ill-structured in nature and answers are seldom right or wrong (Jonassen 1997). Mathematical modelling constitutes a central aspect of everyday engineering work (Gainsburg 2006). Perlow and Bailyn (1997) added that the 'real' work of engineering is mathematical modelling, theoretical analysis and conceptual problem solving. It is therefore important for engineering students to learn *mathematical modelling* - that is, creating mathematical representations of real-world problems, and *problem solving* - that is, the process of approaching ill-structured problems. Gainsburg (2006) noted that engineers often do not recognise how the mathematics they use to deal with workplace problems relate to the mathematics that they were taught. Jonassen et al. (2006) argue that classroom problems, in stark contrast to workplace problems, are often routine and well-structured. Research indicates that learning to solve well-structured problems does not necessarily lead to an ability to successfully deal with real-world problems (Jonassen, Strobel & Lee 2006).

In an attempt to mitigate these issues, the second author of this paper has developed a course in mathematical modelling and problem solving for second-year engineering students at Chalmers university of Technology. The course is divided into six modules, where each module deals with a certain class of models and applications (e.g. dynamic modelling, statistical modelling). The dominant pedagogy underpinning the course is *inquiry-based learning* (Prince & Felder 2006) and the course is based on a collection of 30 small but reasonably realistic problems that the students solve in pairs. With several smaller problems the students get to practise and get feedback on the different stages of the problem-solving process several times during the course. The teacher mainly facilitates the learning process by asking the right question at the right time. For more detail description of the course, see Wedelin and Adawi (2014).

The aim of this study is to evaluate the design of this course from the students' perspective. More specifically, we are interested in what the students learn – the range of learning outcomes and the aspects of the learning environment that contribute to these learning outcomes.

Method of investigation

The empirical data consisted of reflective reports submitted by 160 students after completing the course. The reports were mostly written in pairs, with the exception of a few written in groups of four. As a part of the report, the students were asked to describe what they have learnt in the course and what aspects of the learning environment that contributed to those learning outcomes. There were otherwise no strict criteria for writing the reports, so the students were free to write what they experienced as most important for them.

The data was analysed using a *general inductive approach* for qualitative analysis (Thomas 2006). The analysis process is iterative, where segments in the texts (in this case, the reflective reports) are coded, sorted and sifted, and refined until a number of clear themes emerge.

Below, we first briefly describe the major learning outcomes of the course and then the aspects of the course that the students perceived to have contributed to the learning outcomes. We draw on a framework for *mathematical thinking* (Schoenfeld 1992) to better understand the students' learning outcomes and a framework for designing *authentic learning environments* (Herrington & Oliver 2000) to better understand how the course contributed to those learning outcomes. These two frameworks were not applied a priori but were used to get a deeper understanding of the findings.

The learning outcomes as described by the students

The learning outcomes could be grouped into two overarching themes, problem solving and mathematical modelling, and we will discuss these themes in terms of Schoenfeld's framework for mathematical thinking. Schoenfeld argues that an individual's mathematical behaviour can be described by four factors: 1) *resources* – the mathematical knowledge base; 2) *heuristics* – the problem solving strategies; 3) *self-regulation* – the ability to monitor and control the use of resources and heuristics; and 4) *beliefs* about oneself and mathematics. These four factors will determine how successful a person is in solving a problem.

    a. Problem Solving

Almost all students mentioned that the course had developed their problem-solving skills. In terms of resources, the students felt that they already had enough mathematical knowledge for this course. But since they were used to solving well-structured problems, they lacked the experience of solving ill-structured problems.

Even though the students were familiar with some basic *heuristics* for problem solving (e.g. plotting a graph) most did not know when and how to use them. During the course, the students developed several strategies for approaching non-routine problems and learned how to use them appropriately, e.g. *dividing the problem into smaller parts, discussing with peers* and *re-formulating the problem*. The students also mentioned *understanding the problem* as one of the most important learning outcomes. By trying and failing to solve several problems during the course, they realised the importance of understanding the problem. In the same way, they came to understand the importance of *exploring alternatives*.

Lack of effective self-regulation skills explains why the students had difficulties in employing heuristic strategies as well as understanding the problem and exploring alternatives. Self-regulation consists of *active monitoring* and *controlling* one's thinking process. These twin processes are important aspects of *metacognition*. By the end of the course, many students said that they have developed much stronger metacognitive skills.

The students reported several examples of non-constructive beliefs, shaped by their past experiences, such as 'mathematics is not important' and 'it's too hard to be solved by me'. These kind of beliefs typically override self-regulation and hinder the problem-solving process. At the end of the course, the students demonstrated more constructive beliefs and positive attitudes. They now felt more confident and had begun to see themselves as 'independent problem solvers'. Moreover, many students emphasised that the course had given them a taste of their future profession as engineers.

b. Mathematical modelling

At the beginning of the course, the students had a very vague understanding of models and modelling. During the course, they started to realise the purpose of modelling and developed a better understanding of when and why it is appropriate to use a certain modelling approach. The students learned the importance of *being thorough and rigorous* throughout the whole process as well as the importance of *testing the quality* of the solution.

Again, a lack of effective self-regulation severely hindered the modelling process. The students mentioned examples such as 'not being careful', 'not asking the right questions when stuck' and 'rushing into solving the problem'.

The students reported a new-found appreciation for modelling. They described it as a creative process where there is 'no right or wrong' and certainly no 'failure'. Many students mentioned that they have become more confident in their modelling abilities. They also wrote that their experience of modelling in the course will be beneficial in solving future problems as an engineer.

For a more detailed discussion of the students' learning outcomes related to problem solving and mathematical modelling, see Wedelin et al. (2015).

Aspects of the course that contributed to the learning outcomes

There were four aspects of the course that contributed to the learning outcomes: the *problems used in the course,* the *teaching and supervision,* the *course aim and philosophy* and the *course design and structure.*

a. Problems used in the course

According to the majority of the students, the problems used in the course contributed most to the learning outcomes. Three mentioned characteristics of the problems stand out: *variation in context and complexity, ill-structured nature* and *problems as an initiator to learn and explore.*

The problems in the course vary both in terms of complexity and context. Many of the students experienced the complexity both as challenging and motivating. Through the careful variation in context, the students mentioned that they learned different modelling approaches.

The students mentioned that the ill-structured and real-life nature of the problems demonstrated the relevance of mathematics for their future work. They strongly appreciated the opportunity to learn to deal with the kind of 'real' problems they might encounter as an engineer.

Since the problems are open-ended and there is not just one correct way to solve them, the students felt that they were given an 'opportunity to be creative' and that it was possible to 'explore one's imagination to the fullest'. They felt that once they started exploring and solving the problems, they started to think and learn.

b. Teaching and supervision

The students wrote that the introductory lecture helped them to better understand the problem - solving process and how to approach different types of problems. Moreover, the follow-up lecture at the end of each module 'closed the learning loop' for that module. The follow-up lectures helped the students to see how the problems could be solved and to reflect on that week's work, get a wider perspective and resolve misunderstandings.

According to the students, the supervision sessions provided great guidance throughout the entire course. The Socratic supervision style was highly appreciated, as asking questions instead of giving answers forced the students to reflect on the problem solving process in a deeper way. They wrote that this supervision style 'gives you confidence in your own ability' and 'helps you to become an independent problem solver'.

c. Course aim and philosophy

Many students wrote about how the course was different from other mathematics courses they had taken. One of the biggest differences that they mentioned was how this course encouraged them to apply the mathematical methods and concepts they knew. In contrast to many other courses, this course was able to 'bridge the gap between theory and practice'.

The students also mentioned that 'thinking outside the box' was encouraged throughout the whole course. Since the course does not have a final exam, there is a focus on the students' problem-solving *process* rather than the *product,* which gave them a great sense of freedom. The students felt that this course focused on 'actual learning' rather than learning for the exam.

d. Course design and structure

The use of *pair work* was highly appreciated, as it helped to generate new ideas and provide different perspectives on a problem. Many students noted that 'it is a good way to do reflective thinking' since discussions 'make the problems clearer' and sometimes help to 'make connections to problems solved before'.

Many students reported that the *module-based structure* of the course helped them to be more systematic and organised. They felt the course structure provided a 'good disposition…starting with the slightly simpler models and building up to the more complex ones'. The students also appreciated the weekly reports. They felt that writing the reports provided an 'opportunity to reflect', and that these 'detailed reports' will be useful as a 'future reference' in the workplace.

A framework for designing authentic learning environments

It is interesting to compare the aspects of the course that the students believed contributed to the learning outcomes with a framework for designing *authentic learning environments* proposed by Herrington and Oliver (2000). The framework consists of nine elements:

1. Provide authentic contexts that reflect the way the knowledge will be used in real life
2. Provide authentic activities
3. Provide access to expert performances and the modelling of processes
4. Provide multiple roles and perspectives
5. Support collaborative construction of knowledge
6. Promote reflection to enable abstractions to be formed

7. Promote articulation to enable tacit knowledge to be made explicit
8. Provide coaching and scaffolding by the teacher at critical times
9. Provide for authentic assessment of learning within the tasks.

The focus on dealing with real-life problems in the course rather independently provides an authentic context for the students. Solving ill-structured problems with no straightforward answer serves as an authentic activity. The lectures, especially the follow-up lectures where the teacher gives his own solutions and perspectives, not only provide access to expert performance and modelling of processes but also promote reflection since the students can compare their own problem-solving route with that of the teacher. The use of pair work supports the collaborative construction of knowledge. The students articulate their learning in discussions with peers, the supervisors and in the report writing. Coaching and scaffolding are prominent in the supervision sessions. Since the students work in pairs, peers also sometimes take on the role of coaching and scaffolding. Assessment, mainly of a formative nature, is naturally embedded in this wide range of activities rather than shoehorned into a final exam, which is closer to authentic assessment of learning.

We conclude that the course matches well with this framework for authentic learning environments, even though the problems of the course are smaller and simplified compared to full real-life engineering problems.

Summary and conclusion

We have described a qualitative study evaluating the design of a course in mathematical modelling and problem solving. In the study, we focused on the students' learning outcomes and what aspects of the course that contributed to these learning outcomes. The students report better use of heuristic strategies in problem solving and mathematical modelling. They also report that they have developed more effective metacognitive or self-regulation skills, together with more constructive beliefs. On a general level, the authentic nature of the learning environment contributed to these learning outcomes. The students emphasised the importance of using authentic problems in conjunction with an authentic setting for learning.

As alluded to in the introduction, engineering education has been criticised for failing to help students to develop core competencies and to experience the relevance of what they are learning. This study offers a description of an alternative and, through the eyes of the students, highly effective learning environment. We note especially that mathematics is not a subject that is known for providing courses with authentic learning environments, and the fact that this course is so different may explain why the students report so significant learning outcomes, despite having a long history of learning mathematics in school and at the university.

Based on how the course design matches with the framework of Herrington and Oliver (2000) for authentic learning environments, we also believe that our findings support the general idea of more authentic learning environments as described by this framework, and that this holds great potential for improving engineering education. So we think that both the course as an example in the area of mathematics, and this instructional framework, can make the implementation easier for other engineering educators who wish to adopt this kind of teaching

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A calculus course in knowledge feedback format

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Abstract

This presentation describes the format of a calculus course (taking place 2015) with the main intention of interchanging two different elements during the course. The first is examination in small format, and the second is an examination-free opposite: maximal openness towards student's mathematical problems in the spirit of "It's better to make the errors now than at examinations". One purpose here is for the teacher to increasingly recognize students' actual mathematical difficulties, and modify lectures. Also, students solved problems during lectures every week (examination-free), hence regularly practiced verbal presentation and dialogue in mathematics.

The calculus course: the practicalities

This paper starts with the practical description of the course and its design, while underlying theory is presented in the concluding section "Theoretical rationale for the course design".

The course "Analysis 2" is a basic calculus course at the Blekinge Institute of Technology. Its 8 weeks contain 3 weeks about integrals, 3 weeks about ordinary differential equations, and 1 1/2 weeks about Taylor series, including applications of the concepts. The immediately preceding course "Analysis 1" covers numbers, the function concept, different function classes, limits, derivatives and applications. Both courses are mandatory in five civil engineering programs at Blekinge Institute of Technology, Sweden. The course entails for each student 8 hours of lectures each week, and 4 hours of problem solving sessions.

The examination of the course is in two parts: a written exam and a project. A student must pass both. In the written exam, 50% of the points are needed to pass. The teacher has some liberty in how to design the project – this design is important for the case reported in the present paper.

It should be pointed out that the last of six tasks in the written examination is slightly non-conventional. It asks for an account of the relations between concepts in the course. An example of such a task is the following: "6. How are differentiation and integration related? Are there elementary functions that do not have an elementary primitive function, although they are integrable? What is the difference between a definite and an indefinite integral? Which are the main methods to calculate a primitive function? Is there any differential equation that can be solved by integration?" The more correct, complete and relevant the answers given, the more points awarded.

Main aims and design of the course

The main goal of the course design is, of course, to use the 64 lecturing hours to maximize students´ learning of the course content. This is attempted by the following basic aims, which, if achieved, obviously enhance each other.

Aim 1. Students actually study and learn *during the course*, instead of postponing to the week before the exam. Part of this is to increase students' weekly work time.
Aim 2. The teacher should *continually learn more about the students' ways of presently understanding the content*, to be able to make lecturing maximally efficient.

Aim 3. To make it as easy as possible for students to *voice what they don't understand*.

Aim 1: Learning during course
The main design feature for Aim 1 is to include "Project tests" every second week, in total 3 tests. A certain total number of points in the Project tests are required to pass in the project part of the examination.
No points can be carried from any part of the course to the written exam. In this sense the written exam is independent.

The Project tasks are not far from the tasks of the written examination, however mostly requiring less calculation and merely testing the new concepts that have appeared during the previous two weeks.

Aim 1 is also supported by a weekly "Guide". This is one page describing 1.) the new topics of the particular week, 2.) what students are supposed to learn this week, 3.) six "Student's tasks" and 4.) six "Teacher's tasks". Among the Student's tasks some are easier, some harder – this provides a choice for the students (see next section).

Aims 2 and 3: Teacher-student subject communication
The intention of the planning around the Project tests is to achieve maximal openness around the mathematical concepts and ways to calculate, according to "It's better to make the errors now than at examinations". This points towards Aim 2 and Aim 3.
Lectures were Tuesday 8-12 and Friday 8-12.
At the Tuesday lecture, new concepts are described geometrically and intuitively without proofs, finishing with that the six Teacher's tasks are calculated by the teacher.
Friday lecture 8-10 is the Students' Session. Then students are expected to solve the Student's tasks on the whiteboard. The lecture room is divided in 6 parts, and students who choose to sit in part x are supposed to be able to solve task x. So students had three days from Tuesday to choose a task to focus on.
At Friday lecture 10-12, the teacher adds mathematical comments that have been collected during the students' session, and provides proofs about the concepts that the students just have been engaged with. This may also extend to the next Tuesday 8-10.
For students, there is thus a clear line from Teacher's tasks to Student's tasks to Project tasks, that can be expected to motivate students to participate. If successful, it helps all three Aims. Mathematical errors that become visible during the Students' Session are not graded, but may help to avoid similar errors in the coming Project examination. For errors to become visible, it is probably important that students' performance at the lecture is not graded.

Weekly quiz

By Thursday night a quiz on the course web site had to be completed. This quiz asks yes-no questions about the ideas of the material of the week. Immediately after a yes-no answer, the student receives the correct answer and a comment on it, before clicking on the next question. The following is a rather typical part of a quiz. True or false?

1. The differential equation $xy'' - (\cos x)y' + y = 0$ is linear.
Answer: True. *Comment*: But it does not have constant coefficients, since x and cosx depend on x.
2. The differential equation $y = 1/y'$ is linear.

Answer: False. *Comment*: Not even after multiplication into y'y = 1 does it have the form f(x)y' + yg(x) = 0, as it should for linear equations of the first order (for some functions f(x) and g(x)).

3. Linear differential equations can only be solved if the coefficients are constants.

Answer: False. *Comment*: With the integrating factor method, many first order linear differential equations can be solved that do not have constant coefficients.

4. The integrating factor method is based on both the chain rule and differentiation of a product.

Answer: True. *Comment*: The derivative of $e^{F(x)}$ is $f(x)e^{F(x)}$, here the chain rule is used. Collecting the two terms on the left side of the differential equation after multiplication with $e^{F(x)}$ into one, before integration, is differentiation of a product, but in this case used in reverse.

5. No differential equation is both separable and linear.

Answer: False. *Comment*: The differential equation y' = y, for example, has both properties. In the form y' – y = 0 it is linear, and in the form y'/y = 1 the variables are separated. So it can be solved by both methods.

The quizzes focus on the connections between the main ideas, while the Teacher's tasks and Student's tasks focus on calculation. The quizzes avoid too narrow a focus on calculation during the Friday lecture.

For each completed quiz, a student gets one point regardless of the number of correct answers. A student passes the project if the sum of quiz points and project task points is high enough. The pass/fail limit is determined so that a student who has all quiz points needs 50% of the project task points to pass the project. This means that if some quizzes are skipped, a student must meet a higher requirement on the project tasks than normal.

Another reason not to assign points to quizzes according to the number of correct answers is that they represent new facts that cannot be expected to be understood yet. The quizzes are more an interactive repetition and learning occasion, than a test. Their importance is underlined, though, by the points assigned on their completion.

Comments about the realization

A central experience during the realization was that feedback to students in the students' session almost always could be given both mathematically accurately, relevantly and encouragingly. This helps a student's overall participation in the course. Below are some examples of such feedback.

Student: Makes a common student error.
Teacher: "You are really not alone in this error. And it is very good that it appeared before the exam!". Followed by a specific explanation.
Implicit conclusion: The student has indirectly helped also other students.

Student: Misses an important argument in the calculation.
Teacher: "You may think this, but you need to write it down also, otherwise you lose points unnecessarily. When grading I can only consider what you have actually written down".
Implicit conclusion: It is clearer for a student how to handle an exam.

Student: Hesitates a lot halfway through the solution.

Teacher: "You have made a good start, if you want I can finish?"
Implicit conclusion: The activity is only becoming complete knowledge, a process that a course is all about.

For the sake of the students that are listening to a student presentation, dialogue and commenting are important so the solution becomes complete and relevant.
When the student has left the stage, the teacher can always ask: Are there other ways to solve this problem?
Implicit conclusion: The student's solution is a valid contribution to the set of solutions, sometimes the only one. A deeper implicit conclusion to this question, about the nature of mathematics, is that methods of solution are what matter most. One can argue that theorems contain parts of methods that need to be combined somehow, and/or information about which methods have any chance of succeeding. .

Measurable course results

In the written exam, 98% of the students participated, and 52% passed. This percentage for Analysis 2 usually lay around 40%. Colleagues regard the written exam as slightly more difficult than usual, or at least not easier. On the students' course evaluations, the course was rated good or very good in all respects.

In total 48 tasks were solved by 12 different students during the course's lectures. This develops a mathematical competence that was not graded.

The university teacher Johan Silvander was present during one Friday lecture. As a part of a university pedagogical project, he evaluated the activity as follows:

> A student from each group showed the solution of their specific exercise on the whiteboard. The teacher is encouraging students who have not been in front of the whiteboard before to show the solutions. The teacher is helping the students by putting leading questions, if needed, and explains to the audience what is taking place on the whiteboard. If some steps are not included by the student these are filled in by the teacher at an appropriate time. The students in the audience were active and asked questions or proposed solutions. Questions and proposals that could have been considered as "stupid" were handled in a way that enlightened the student and saved the student's face. Since the same basic methodology was used to solve all of the exercises the teacher pointed that out during all the exercises. The teacher took care to inform the students about their relevance by always showing all the steps of the methodology even if more advanced students had omitted a step due to "This can easily be understood".

> When introducing a new concept the teacher showed how it is used to solve an engineering problem. The methodology was explained in a parallel manner by showing the generic solution on one whiteboard and altering to a specific problem on another whiteboard.

> The take-aways from this session were:
> - Letting the students show and explain their knowledge.
> - Guiding the students to the answer and handling the students with respect.
> - Using different levels of abstraction when introducing the students to a new concept/methodology.

Theoretical rationale for the course design

In Wittman (1995), mathematics education is described as a design science, as is engineering, architecture, business and medicine. Here the most essential question is how to design artifacts that are successful, which may be an engine, a building, an enterprise, a medical treatment method, or a course. All these areas are strongly dependent upon knowledge of different sciences, of course, but the main point is that in design sciences *creative design has a decisive importance*. In the case of education: how a course is designed is adamant in the courses' result. The questions "What?" and "Why?" may be enough in basic research, but must in a design science lead to an answer to "How?".

The course is in the spirit of Donald Schön's *reflective practice* (Schön, 1983). This means professional development in the sense that professionals learn from their own professional experiences, and engage in a process of continuous learning.

The idea of reflective practice can be taken further when the profession is the teacher's. In this case there are always learners to engage in dialogue, with the aim of reflective practice and continuous professional development. Then professional development becomes entwined with the learners' learning, and thus to the immediate goal of the education.

The willingness of students to interact and share the teacher's knowledge and skills is discussed in Kindberg (2013) from a perspective that is both pedagogical and rhetorical. The teacher's ability to construct a safe and open learning atmosphere, which is also a rhetorical issue because of its focus on *verbal* communication, is essential for the students' learning outcome.

In the special case of mathematics education, verbal reflection is scarce by tradition, despite the subject's fundamental dialogical nature (Lennerstad, 2008, Ernest 1994). One can argue that mathematics problem solving, and mathematics in general, has a characteristic and important vagueness, making mathematics in need of verbalization. This is expressed by P. Ernest in the preface of Rowland (1999):

> Precision is the hallmark of mathematics and a central element in the "ideology of mathematics". Tim Rowland, however, comes to the startling conclusion that vagueness plays an essential role in mathematics talk. He shows that vagueness is not a disabling feature that detracts from precision in spoken mathematics, but is a subtle and versatile device which speakers deploy to make mathematical assertions with as much precision, accuracy and confidence as they judge the content and context warrant.

Teacher's pre-grading test

Mathematics is not empirical, but mathematics education is very much so. A course is scientific not only if the theory described is correct and scientific. A course is scientific if also the students' participation is considered and evaluated as empirical material, and used to enhance the quality of the course on an ongoing basis.

Before grading a written exam, it is possible and fruitful for a teacher to predict the mean number of points that the students will receive for the different tasks on it. This may, however, not be very difficult. The pre-grading test is harder: to predict, before grading, *which kinds of errors will be most common and which will invoke the largest losses of points*.

A teacher that does well in a pre-grading test has a good understanding of some aspects of the students' mathematical difficulties. For the sake of the next edition of the course, it is a very

important type of teacher competence, in particular if it inspires an improved course design to meet observed mathematical needs.

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UniDoodle: A Multi-Platform Smart Device Student Response System – Evaluated in an Engineering Mathematics Classroom

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Abstract

Most of the existing student response systems, such as Clickers, have limited input capabilities, whereby students can only input a numerical, and in some instances, a textual response. Hence, mathematical equations, diagrams, etc. are all beyond the capabilities of such systems. This paper proposes and presents a novel multi-platform smart device-based student response system, called UniDoodle, that allows for a more generic and flexible input. This system consists of a student application that allows for freeform input, through sketching capabilities, a lecturer application that allows easy viewing of multiple sketch-based responses and a cloud-based service for co-ordinating between these two applications. In essence, students can now respond to a question posed by the lecturer using sketches and, hence, mathematical equations, circuit diagrams, graphs, etc. are all possible on the UniDoodle system. In addition, the lecturer can now gain a richer and more useful insight to the students' understanding of the relevant material. This new system currently operates on any device that has an iOS (iPads, iPhones) or Android (smart phones and tablets) operating system. This paper also evaluates the UniDoodle system in a large class of first year Engineering Mathematics students. Details of the UniDoodle system, the evaluation process and the feedback obtained are all presented within.

Introduction

Student response systems (SRSs) are slowly becoming more commonplace in the classroom today. These systems exist under many different guises in the research literature, including clickers (Barber and Njus, 2007), classroom response systems (Roschelle *et al*, 2004), audience responses systems (Miller *et al*, 2003) and voting machines (Reay *et al*, 2005). The literature also clearly documents how such systems offer many pedagogical benefits ranging from improved student learning to better classroom interaction (Caldwell, 2007, Moredich and Moore, 2007, Auras and Bix, 2007). However, to date, these systems only allow for limited input capabilities, whereby students can only input a numerical, and in some instances, a textual response. Hence, mathematical equations, circuit diagrams, sinusoidal responses, annotations of diagrams, etc. are all beyond the capabilities of existing SRSs.

This lack of freeform input is a significant drawback for the Science, Technology, Engineering and Mathematics (STEM) disciplines where equations, circuits and diagrams are important aspects of the student learning experience. For example, consider the solving of an algebraic equation, the designing of a circuit, the sketching of a mathematical function, presenting the forces of a moving object on a free body diagram, the minimisation of a Boolean function using Karnaugh Maps, sketching the root locus of a control system, etc. The list of such examples is endless and it is very important that students of STEM disciplines, in particular, can carry out such fundamental processes and methodology. In order to capture the real-time feedback of the students' grasp of this information it is necessary for a SRS to facilitate freeform input.

Here, we propose the use of a novel multi-platform smart device-based student response system, called UniDoodle, that allows for a more generic and flexible input. Students can now respond

to a question posed by the lecturer using sketches and, hence, mathematical equations, circuit diagrams, graphs, etc. are all possible with this system. In addition, the lecturer can access a richer and more useful insight to the students' understanding of the relevant material. This new system operates on any device that supports either the iOS (iPads, iPhones) or Android (smart phones and tablets) operating system.

The UniDoodle system was evaluated by a large class of first year Engineering Mathematics students. Both student and lecturer feedback was obtained at the end of the evaluation period. The evaluation process and a summary of the feedback will be outlined later.

UniDoodle – a brief overivew

UniDoodle is a recently developed, multi-platform, smart device based student response system that provides a freeform-style input using sketch capabilities. It builds on previous work in the area by the lead author (McLoone et al., 2015), and now operates on all devices using either the iOS or Android operating system. This system consists of a student application that allows for freeform input, through sketching capabilities, a lecturer application that allows easy viewing and editing of multiple sketch-based responses (see Figure 1) and a Google-app engine cloud-based service for co-ordinating between these two applications.
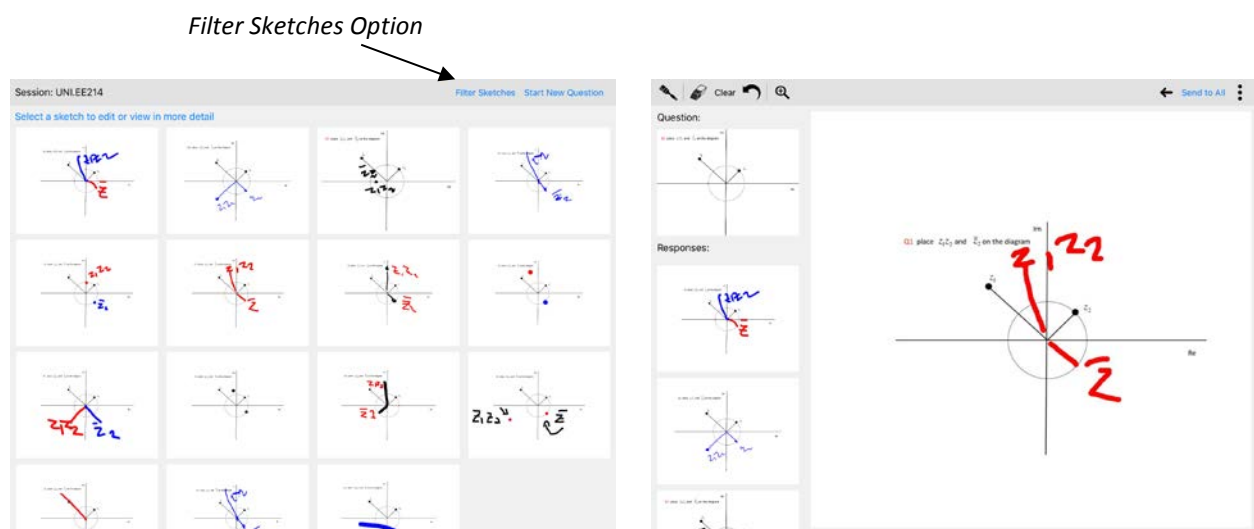
Figure 1. UniDoodle Viewing (left) and Editing (right) Teacher App.

Overall, the system works as follows – firstly, the teacher poses a question in class (this can be a pre-prepared question in the form of a template or a new on-the-spot question); secondly the students receive this question (in the case of a template question) on their device and can now respond appropriately using the in-built sketch capabilities (equations, diagrams, graphs, and annotations are all possible); thirdly, the teacher receives the student responses in a neat and concise format on their own device (typically a tablet); and lastly the teacher can point out and respond to any obvious errors that students may have made. In addition, the student responses can be viewed via the overhead projector allowing all students to see all the submitted responses. This offers students a level of peer learning as they can now see where other students are making mistakes, if any, and what those mistakes are.

Several new and important features exist in UniDoodle in comparison with the work carried out by McLoone et al. (2015). Firstly, it supports the preparation of questions in advance of class through the use of templates. Teachers can now pre-prepare questions and load them on the cloud-based database. Furthermore, templates can be arranged into various folders (see Figure

2). For example, a teacher could have a different folder of questions for each subject or class they teach. Secondly, diagrams can be created on a PC using any suitable drawing package, saved as jpegs, and uploaded to the database. This allows for more detailed and precise diagrams to be used as the basis of questions. Finally, UniDoodle includes a filter feature that allows the teacher to remove any undesirable student responses (see Figure 1). This quick and easy to use feature means that unwanted images can be deleted before the responses are shown live to the whole class.
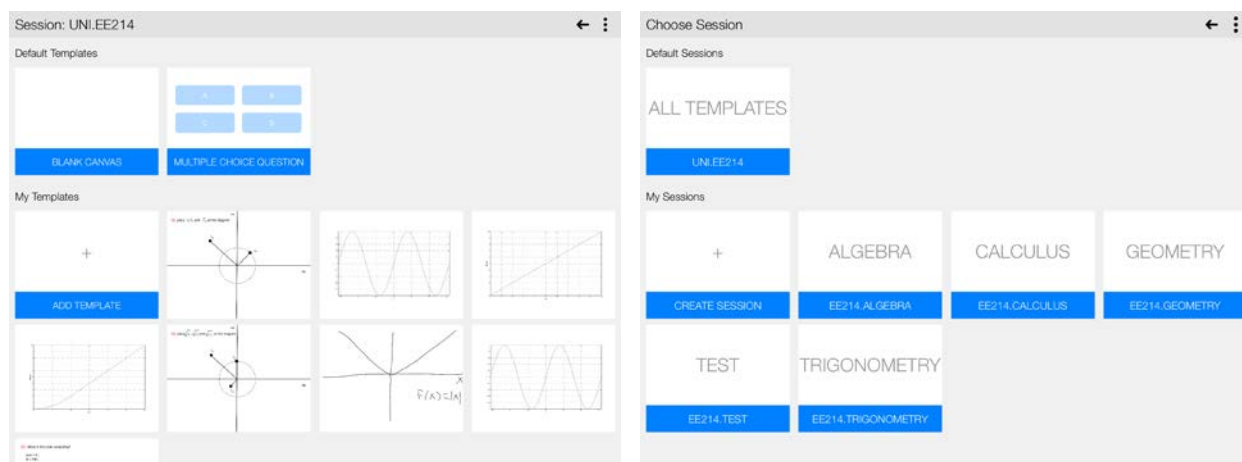
Figure 2. UniDoodle Template Development (left) and Management (right).

Educational Context & UniDoodle Evaluation

The UniDoodle response system was trialled in two first year modules which are offered by the School of Electronic Engineering in Dublin City University (DCU) to all their first year engineers. These consist of students taking the Common Entry, Electronic and Computer Engineering, Mechatronic Engineering, Mechanical and Manufacturing Engineering and Biomedical Engineering programmes. The modules were EM122 Engineering Mathematics II and EM114 Numerical Problem Solving for Engineers. The former covers traditional topics in calculus, matrices and complex numbers and has 165 registered students, while the latter has 151 registrations and introduces numerical techniques for approximately solving a range of practical problems based on the material covered previously in EM122 and elsewhere.

Both modules contained effectively the same cohort of students and consisted of approximately 20% female students and 80% male students. This represents a typical breakdown on an Engineering class in Ireland. International students made up about 10% of the class.

The UniDoodle system was trialled in 4 distinct 1-hour stand-alone sessions, between February and April 2016, as follows:

Session 1: Students had to investigate basic concepts in functional analysis. In particular students were asked to sketch, for example, trigonometric functions and investigate the difference between abs(sin(x)) and sin(abs(x)). They were also asked to estimate the derivative of a given function, based purely on the shape of its graph (no functional description, values or other information was given).

Session 2: Students had to explore more complicated topics in calculus, such as for example, identifying the derivative of abs(sin(x)) and sin(abs(x)) (two functions that had been explored in Session 1). In addition, applications of calculus were explored with students being presented with sketches of a particle's acceleration against time and asked

to produce sketches of its speed and position. Importantly the functional description of acceleration was not given. This means that the student could not simply integrate to find the desired solutions and instead had to consider how the shape of the curve yields the required information.

Session 3: Students had to explore topics from complex numbers, namely the polar form of a complex number and its relationship with the number's position on the complex plane. Students were challenged to compute powers or square roots of the given complex number. Again specific information about the arithmetic value of the complex number was not provided. The unit circle was provided for reference.

Session 4: This was a pre-exam session where the multiple choice questions from the previous year's exam paper was reviewed. Students primarily used the multiple choice facility on UniDoodle to submit their answers and engaged in peer-learning to review the submissions and decide, as a group, on the correct answers.

At the end of the trial period, students were asked to complete detailed survey forms regarding their views on a range of aspects relating to the UniDoodle response system, including (i) usability, (ii) learning using the system and (iii) engagement in the classroom. In addition, the lecturer of the two modules was asked for his personal thoughts and opinion on the use of the UniDoodle system. It is worth noting that the lecturer had used the previous system (McLoone et al., 2015) and this allowed him to express relevant views on some of the new features of the UniDoodle system, as outlined in the previous section.

Findings and Discussion

In total, 98 survey forms were completed and returned at the end of the evaluation period. Due to space restrictions, a selection of the student feedback is given in Table 1. The feedback shows that the majority of students found the UniDoodle response system easy to use, found it an effective means of interacting with the lecturer during class and helped them be more active in class. As expected, the students noted that the anonymity of responses meant that they were more likely to respond to questions.

Interestingly, approximately 50-60% of the students felt that using UniDoodle helped improve their understanding of key concepts and found that the feedback offered by the lecturer focused them on what they needed to learn. Less than 10% felt that this was not the case while the remaining students responded with *not sure* in this regard.

Finally, most students found it useful to be able to draw sketches using UniDoodle, although a few noted, via additional comments, that some of the drawing aspects could be improved. These included providing a keyboard input for text input, a thinner pen size option for neater sketching and the inclusion of a library of basic geometry shapes for convenience.

Statement	Average rating (1–5)	Std. dev.
The use of UniDoodle helped me to be active in class.	4.0	0.9
The fact that my answers were anonymous encouraged me to submit my responses in class.	4.5	0.7
UniDoodle makes me think more about the course material during my lectures.	3.5	1.1

I found it useful to be able to draw sketches with UniDoodle.	4.0	0.8
I found this method of interaction between students and lecturer effective.	4.1	0.8
I would recommend that the lecturer continue to use UniDoodle.	3.7	1.1
The use of UniDoodle allows lecturers to identify problem areas.	3.8	0.9
UniDoodle allows me to better understand key concepts	3.6	0.8
The feedback provided by the lecturer after completing a UniDoodle question helped me focus on what I should be learning in the course.	3.9	0.8
It was easy to use UniDoodle.	4.4	0.8
I rarely had to seek help to use UniDoodle.	4.2	0.9

Table 1. Student feedback on UniDoodle (1 to 5 represents strongly disagree, disagree, not sure, agree and strongly agree respectively).

The lecturer, who had used the previous version of the SRS, reported that UniDoodle constituted a great improvement. In particular, he found that the extension of the system to iPhones and iPads meant that coverage within the classroom was now effectively 100%. He also noted that since the assessment was not summative, there was no issue with any students who did not have a device (or had forgotten one) as they could pair up with colleagues who did.

The lecturer also felt that the ability to filter responses was a great innovation. The lecturer started each session with a blank template and an invitation to the students to draw a picture of whatever they felt like. Ostensibly this was framed as an invitation to re-familiarise themselves with the basic functionality of the system, while in reality it was intended to quickly address and temper the temptation for students to abuse the system by submitting inappropriate images. The filter facility meant that such submissions could be removed without drawing attention to them and, in practice, students who were intent on disruption quickly realised that there would be no opportunity to do so.

The lecturer found that the template functionality offered a fantastic off-line way of producing good quality questions. However, noting that he occasionally asked on-the-spot questions in the classroom, he added that useful future innovations may include the ability to render simple geometrical figures, text and mathematical content in real-time. Interestingly, this echoed some of the comments given by the students themselves in relation to improving the sketch capabilities on UniDoodle.

As for its use in class, the lecturer found that the students greatly enjoyed using UniDoodle. "*The shift from passive observers in a lecture to active participants is something that they really respond to.*" In addition, he felt that the anonymous nature of the interaction is vital, noting that it was "*astounding that one can ask the simplest question in class to a resounding silence, while a question posed on UniDoodle can receive dozens of responses – many of which can be wrong, but at least have been volunteered*".

Overall, the lecturer observed that the adoption of the UniDoodle system is very worthwhile, but that it was not a trivial undertaking. Devising exercises which make appropriate use of the system requires time and practice. Furthermore, given that he had to quickly examine up to 100 submissions for each question, he found that he had to carefully think about the types of answers that he was likely to receive, as well as common mistakes that were likely to present themselves. This would then allow him to react quickly in the classroom and identify submissions that were worthy of feedback.

Concluding Discussion

Here, we have presented UniDoodle, a student response system that offers a flexible input in the form of sketch capabilities, works on both iOS and Android based devices and supports the creation and management of both pre-prepared and on-the-spot questions. Evaluation of this system in a large first year Engineering Mathematics classroom has shown that the students generally find it easy to use, are more active in class as a result, and feel that it improves their understanding of important concepts covered in class. The lecturer also found the use of UniDoodle beneficial and noted that the additional functionality relating to template questions and filtering responses was innovative and extremely useful.

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Models of re-engaging adult learners with mathematics.

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Abstract

So called 'Maths anxiety' can be a key inhibitor for some adult learners considering higher education. The Institute of Technology Tallaght Dublin hosts the 'Centre of Expertise for Adult Numeracy/Mathematics Education' mathematics research group which is a hub of EPISTEM, formerly known as the *National Centre for Excellence in Mathematics and Science Teaching and Learning* (NCE-MSTL). Members of this group identified the key issues that informed the design of enabling-mathematics courses for adults returning to higher education; how to structure their re-engagement, and how to adapt the re-engagement process to practical time constraints.

This paper outlines how insights from the successful *Primer Mathematics* module for mature students taking the FLASHE (FLexible AccesS to Higher Education) Higher Certificate in Electronic Engineering at IT Tallaght were used to identify the key elements of a model to shape such courses.

How these key elements were used to design and implement a new module in preparatory mathematics for students entering all modules in IT Tallaght, as part of *Certificate in Preparatory Study for Third Level*, is explained. The positive impact of this new module on student learning is evaluated using student feedback. The paper concludes with a discussion of the emergent strong, positive correlation between favourable progression rates for those who completed the module, compared with those who did not.

1.0 Introduction

The Institute of Technology Tallaght (IT Tallaght), which is located in South Dublin County, caters for a student population of approximately 2,900 full-time and 1,450 part-time students and offers a wide range of programmes at all levels from Higher Certificate to Doctorate. The college is committed to, and facilitates, lifelong learning by encouraging a continuing process of skill acquisition through the modular provision of programmes in Accumulated of Credits and Certification of Subjects Scheme, (ACCS) mode. As a consequence IT Tallaght has developed a depth and breadth of experience in helping adult learners to re-engage with education. Central to this is the recognition of the role that adults' mathematical abilities, and their perception of such abilities play in their lives and decision-making regarding entering higher education. According to the National Development Plan in Ireland, the lack of appropriate mathematical knowledge skills and competence may have a critical impact on a person's employability and re-employability in the long term (NDP_CSF_Information_Office, 2007). Similarly, longitudinal studies in the UK tend to confirm the correlation between low levels of numeracy and poor long term employment prospects, socio-economic deprivation and wellbeing, (Bynner & Parsons, (1997, 2005); Williams et al, (2003)). The confidence of learners in their mathematical abilities, together with their attitudes, beliefs, and feelings regarding mathematics is thought to construct a significant psychological barrier to embarking on a technical course of the type provided in IT Tallaght (Coben (2003)). The array of potentially inhibiting effects may be compounded by negative attitudes to mathematics and math-anxiety persisting from childhood experiences (Klinger (2005, 2006). The awareness that such anxieties can exert a constraining influence on adults' life and education choices, shapes the module design and implementation strategies developed by the IT Tallaght mathematics teaching staff, originating with the *Primer Mathematics* module.

2.0 *Primer Mathematics* module - key elements of models for re-engaging adult learners with mathematics

In 2004 the School of Engineering at IT Tallaght secured funding from the Higher Education Authority Retention Fund for the design and delivery of a flexible *FLASHE (FLexible AccesS to Higher Education) Higher Certificate in Electronic Engineering* aimed at meeting the requirements of up-skilling part time students in response to an identified skills shortage. As part of this a *Primer Mathematics* module for adult learners intending taking this course was designed by the first and second named authors (Robinson et al., 2006). In particular, student feedback via their reflective diaries, in conjunction with student completion and progression rates suggested that the *Primer Mathematics* module provided a vehicle enabling the students to reconnect with mathematics in a way that boosted their confidence in their mathematical ability (Robinson et al., 2007). A review of the successful implementation of this module identified four key, underpinning elements:

1. Creating a different learning environment;
2. Promoting active learning;
3. Allowing time for practice and not addressing all perceived deficits in pre-entry mathematics learning;
4. Promoting reflection.

2.1 Creating a different learning environment
The *Primer Mathematics* module was lab-based and used the CALMAT learning software package, (Glasgow Caledonian University). This enabled a learning environment which was different to the 'chalk/talk/get left behind' experience which the prospective students may have had previously in mathematics classrooms, and changed the role of the mathematics lecturer to facilitator of learning.

2.2 Promoting active learning
The resulting structured learning environment allowed students to progress at their own pace and promoted active learning via the assessment-driven (and success driven) course tailored to their individual needs.

2.3 Time for practice and not addressing all perceived deficits in pre-entry mathematics
Primer Mathematics was timetabled into two hour-long weekly sessions spread over a 12 week semester covering four basic areas: number, algebra, data presentation and linear laws rather than trying to address all self-perceived or reported deficits in mathematics learning. No effort was made to cram too much content into any particular session as this might overwhelm students in the process of building or re-building their confidence. Importantly this scheduling allowed students the time to practice between sessions.

2.4 Promoting reflection
The students were asked to complete a reflective diary entry after each session in response to prompts such as, what they have just covered, what they found difficult, what they feel they need to work on, how they felt about the *Primer* course, their work and how things were going. The expectation was that students would develop good learning habits by reviewing their learning and perhaps gain insight into how their own mathematics anxiety might inhibit their learning. The accumulated content of the reflective diaries provided valuable insights.

3.0 Learning from student reflections on *Primer Mathematics*

The term 'mathematics' can evoke recollections of school curricula and may be accompanied by negative feelings and anxiety, rooted deeply in the early years and persisting beyond the classroom (Klinger, 2008). The analysis of the reflective diary entries echoed this research locally and although the *Primer Mathematics* module was successful, members of the research group recognised that a deeper investigation of issues associated with mathematics anxiety experienced by adult learners and the associated strategies for dealing with it was needed. The clear intention was to learn from the experience of the students and incorporate that learning into the design of any future mathematics intervention for adults considering higher education. This was achieved by the mathematics staff in consultation and collaboration with the Adults Learning Mathematics an International Research Forum. As part of this work the approaches outlined in *Maths Study Skills* (Bass 2008) were recognised as potentially adaptable for the needs of adult learners in Ireland. In particular it was decided that such an approach to dealing with maths anxiety explicitly at the beginning of any future such module would be adopted.

4.0 *Preparatory Mathematics* module – applying the key elements derived from the *Primer Mathematics* model for re-engaging adult learners with mathematics

An unprecedented economic crisis engulfed Ireland in 2008 which led to an associated rapid rise in unemployment, from 4.7% in 2007 to 12% in 2009 (Eurostat, 2016). This presented challenges that were different to those of the skills shortages in engineering for which the FLASHE programme had been designed. The higher education providers in Ireland were tasked with providing interventions for newly unemployed adults who did had not anticipated being prospective students until the economic crisis. As part of the response to this need in January 2009 IT Tallaght designed a *Certificate in Preparatory Study for Third Level* (a 10 credit, National Framework of Qualifications Level 6 Special Purpose Award for *potential* students returning to various courses) which would enable unemployed people to examine what up-skilling options were available to them in IT Tallaght. The mathematics staff designed a 12-hour mathematics component entitled *Preparatory Mathematics* which built on the four key elements identified in the review of *Primer Mathematics*, and informed by the accumulated knowledge regarding student self-perception, confidence and anxiety related to mathematics learning. The ethos and strategy underpinning the presentation of *Preparatory Mathematics* prioritised the building of student confidence and mathematics self-belief over the twelve contact hours rather than solely addressing any perceived student deficits in mathematical content. The overall aims of the mathematics component were explicitly stated as seeking to provide students with:

- insight into their own **mathematics self–image** and an opportunity to **build confidence** in their mathematical ability
- **basic foundation** for the basic type of mathematics that is needed for courses in higher education in IT Tallaght.

Detailed description of learning outcomes, content and other materials associated with *Preparatory Mathematics* can be accessed via the appropriate links contained in the teaching exemplar document (O'Sullivan, 2015).

The *Primer Mathematics* approach could not be used in its original form due to practical constraints, viz.,

- the time available, 12 hours, was insufficient for *Primer Mathematics*
- it was focused on engineering, whereas *Preparatory Mathematics* required a more general audience for a broad variety of courses
- CALMAT could not be used as a platform for a variety of technical reasons.

Notwithstanding these differences, the design team used the four key aspects identified previously to guide the development of *Preparatory Mathematics* in creating a different learning environment, promoting active learning, allowing time for practice and trying not to focus on remediating deficits, and finally promoting reflection. Figure 1 provides an overview and is followed by a description of how each of the key elements was adapted for the course.
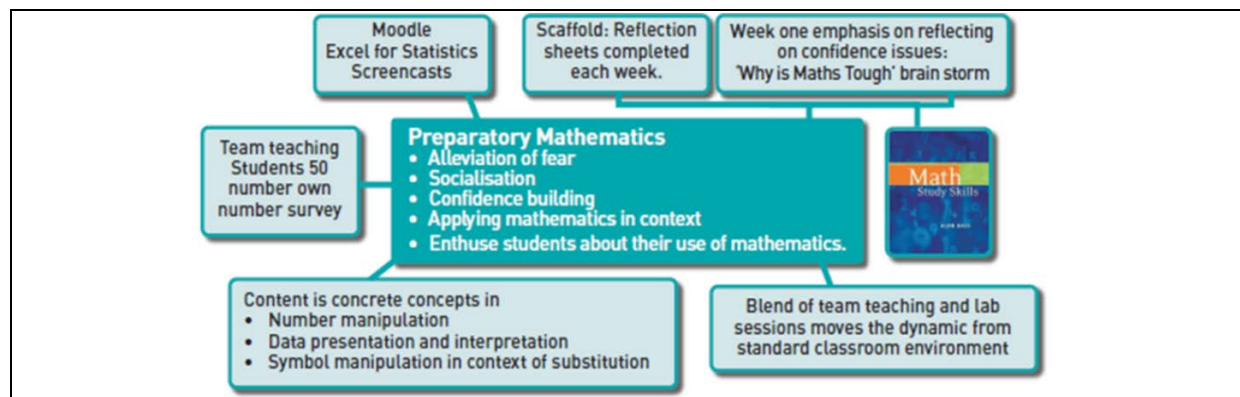

Figure 1. Overview of *Preparatory Mathematics* intervention

3.1 Creating a different learning environment

a. Team teaching

A key element of the component was the creation of the dynamic feedback environment that contributes significantly to the impact on individuals. Nine out of the 12 contact hours take place in a lecture hall but during the sessions, one of the two lecturers leads the learning whilst the other lecturer monitors the students learning continuously and intervenes with individual learners as needed. The second lecturer also plays a key role in acting on feedback immediately as described below.

b. Lab based environment for mathematics

The key element of the laboratory based environment that was so successful for the *Primer Mathematics* course is maintained by having three of the sessions based in a computer lab with the students engaged in guided learning regarding using spreadsheets for statistics, again changing the lecturer's role to one of a 'guide on the side'.

c. Building mathematical confidence session

A key additional innovation not present in the *Primer* module was the design of the first session to address maths anxiety explicitly. Even though there are 12 hours available overall, the first session of the module deals with mathematics anxiety and confidence building, and has been reported by students as a key element of the module. Using ideas based on *Math Study Skills* (Bass, 2008) students investigate the concept of maths anxiety and identify strategies they can employ to reduce this and build confidence in their mathematical ability as follows:

Students are formed into clusters of three or four and are asked to appoint one person to record the result of their deliberations and a second person to report these back to the whole group. The students are told they are going to be posed a question and they have 2 minutes to consider their own response in silence and then will have a further 5 minutes to formulate a list of three to five responses from the group. The class is then given the query: *Why is maths so tough?* After the 8 minutes allotted, each group calls out their responses and the lecturers compile a cumulative list on the whiteboard. Typically, there are about 30 separate items on the list, including, having to remember formulae, getting left behind, being consumed by 'maths with letters', maths not being useful in the real world, and so on, Table 1. The lecturers then remove the concerns stating how the module is going to run and, by emphasising the importance that will be placed on responding to

students feedback dynamically, they re-assure students that "we will be starting from the very beginning" and that the connection between mathematics and their own lives and interests will be shown by a statistics project. The list of concerns can be reduced to one comment which is usually about learning requiring effort, which it does.

Table 1: Reported students' concerns regarding mathematics learning

Comments to prompt query *'Why is Maths Tough?'*	Frequency of response*
Formulas	9
hard to understand/complicated	9
stressful	5
lack of application	5
too many methods/sections	4
algebra	4
boring/don't like	3
teaching methods comments, time consuming, big numbers, theorems	2 each
trigonometry, statistics, graphs, imaginary numbers	1 each

* Responses from 10 groups, with 4 to 5 students in each group (2011 cohort).

3.2 Promoting active learning

Towards the end of the first session the lead lecturer takes a kinaesthetic approach to the discussion of measures of central tendency and deviation by creating a 'human' bar chart based on shoe size of the people in the room, allowing a practical demonstration of measures of location and dispersion (0'Sullivan 2015). This exercise is always enjoyed by students. Following on from this students are informed that they will be given a data gathering task of selecting a sample of 50 measurements from any population of their own choice, e.g. sports data, technical data, financial data, sales data, population/census data. The students are told that the data can be obtained by conducting their own survey or from the web, a book, magazine etc. This statistics project been particularly successful for the students on *Preparatory Mathematics* as they make connections between the own lives/interests and mathematics, and see the potential relevance to learning and applying more mathematical ideas in their future. Subsequent mathematics lectures are structured around the use of 'gap' notes with the emphasis on students 'doing' mathematics rather than copying mathematics from the whiteboard.

3.3 Time for practice and not addressing all perceived deficits in pre-entry mathematics

In designing *Preparatory Mathematics* it was considered crucial that the sessions be spaced across the 6 weeks of the Preparatory course schedule so as to allow students time to practice and develop their mathematical skills, rather than the sessions being crammed into an intensive set of sessions over consecutive days. During each lecture session students are assigned work to have completed before the next session. The students are assessed using assessment techniques (project and formal tests) that are typical of other IT Tallaght mathematics modules so that students learn to engage with doing mathematics outside of lecturer-led sessions.

The material in the course has been carefully chosen in four basic areas: number, data manipulation, algebra and linear laws. This is to ensure that students start with concrete concepts in number manipulation as well as data presentation and interpretation. In addition, students are introduced to a simple level of symbol-manipulation in a brief introduction to algebra that will help them to see its easy application to their later studies regarding linear law manipulation. It is important to note that learning outcomes associated with number, data

manipulation are prioritised and the allocation of hours to content may be adjusted accordingly. There being little benefit in rushing to complete the content, emphasis is placed instead on students achieving success and *building confidence* as a consequence.

3.4 Promoting reflection

From the insights provided by the students on the *Primer Mathematics* module previously, it was clear that the reflection was a powerful tool for learning. On that module student reflection was only assessed at the course end and an opportunity for more timely reaction to the reflections was missed. Therefore, although the tool for reflection was not altered, how it was used was modified as follows. Students taking *Preparatory Mathematics* are expected to engage in a process of reflection which is now part of a weekly feedback loop:

(i)   Students are asked to complete a reflection sheet after each session which uses two prompts:
   - *My reflections……… * e.g., key points learned, high point of the class, low point of the class, my mood, my learning, ……….)
   - *What have I learned from these reflections?   For example* one thought to carry forward, aspects of learning of topic which are still of concern, a question to ask for the next class and so on.

(ii)   At the start of the next session, these reflection sheets are collected by the lecturers. Immediately one of the two lecturers reads and summarises the reflections so that any issues that are raised are dealt with at some stage during that session. In this way students experience an immediate constructive reaction to their feedback in 'real-time' and so are part of a mathematics learning environment that is different to a 'chalk/talk/get left behind' environment that they may have experienced in their past.

5.0 Impact of Preparatory Mathematics

IT Tallaght has offered the Preparatory Certificate since 2009. The very positive impact of the *Preparatory Mathematics* module has been demonstrated in two ways viz., positive student feedback coming from the reflection sheets and progression rates for adult learners.

Positive student feedback:

The first session of the module deals explicitly with mathematics anxiety and is designed to be a reassuring introduction to Mathematics for students who have not studied the subject for many years. The impact of this first session is illustrated in the next weeks reflection sheets and shown in Figure 2, is a tag cloud generated by Wordle © (http://www.wordle.net/), from key words extracted from student reflections written after their first *Preparatory Mathematics* class in 2013 (N=40) and 2015 (N=40). Word size in the tag cloud directly reflects the frequency of the key word in the students' responses.

Figure 2. Student Feedback - first session

It is interesting to note the prevalence of optimistic comments in contrast with the relatively minor mention of 'confident'.

At the end of the module the students completed reflection sheets for the overall module. The impact of the module on learning is illustrated in Figure 3, a second tag cloud of key words extracted from student reflections written after they completed *Preparatory Mathematics* in 2013 (N=40) and 2015 (N=40).

Figure 3. Student 'Overall' Feedback - after last session

The reduced range of key words in the responses in the overall feedback suggests a convergence or coalescing of language, most likely through discussion amongst the class as to their experiences. The two main additions in the overall feedback are 'Confident' and 'Learning'. The prominence of 'confident' is particularly significant given that the aim of this enabling mathematics module is to bring prospective students back in contact with mathematics in a way which will boost their confidence in their mathematical ability.

Progression rates for adult learners:

In discussing the effectiveness of any intervention it is important to consider indicators of impact on student learning in the longer term. For Preparatory Mathematics, the progression rates for adult learners who have completed the module and have progressed to courses in IT Tallaght have been examined. The progression rates for adult learners who have completed the module are consistently higher for those who took the module than those who did not, and for the overall student cohort, Table 2. While these progression rates may be attributable to other factors, perhaps other aspects of the Preparatory Certificate or the prior educational achievement of these students, it is reassuring that they are better.

Table 2: Comparison of rate of progression to Year 2 after Year 1 (in next two academic sessions) for students who completed Preparatory Mathematics with all full-time mature students and all full-time students

Academic Year	2009/10	2010/11	2011/12	2012/13
Students who completed Preparatory Mathematics as part of the Preparatory Certificate	83%	75%	78%	95%
All full-time mature students	68%	69%	69%	58%
All full-time students	75%	66%	75%	63%

In conclusion, the evidence to date from both the *Primer* and *Preparatory Mathematics* courses suggests that IT Tallaght can be confident that using approaches underpinned by the 4 key elements is demonstrably worthwhile for the students involved. That there is strong evidence to suggest that even minor adjustments can have an impact on the learning outcomes for students taking *Preparatory Mathematics*, encourages the ITTD Mathematics Research Group to pursue continuous improvement through reflective practices similar to those recommended to students.

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Case study: Acquisition of Mathematical Industrial Engineering competences during the first year

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Abstract

The Department of Applied Mathematics from the University of Salamanca is responsible for teaching mathematics to almost all the engineering students at this university. In the case of Engineering Degrees of the Industrial Engineering branches the competencies included in the curriculum should incorporate the requirements of the Order CIN/351/2009. On the other hand, from the Mathematics Curricula in Engineering Education document, by the SEFI's Mathematics Working Group, we consider 8 competencies that should be acquired by engineering students learning mathematics. In this paper we present a case study developed with the first year students from industrial engineering degrees, and how we incorporate and integrate both sets of competencies.

Introduction

In Spain, the Engineering Degrees of the Industrial Engineering branches allow students to practice in the Industrial Engineering profession. These studies must meet the requirements regarding a specific technology: Mechanical, Electrical, Industrial Chemical, Textile, or Industrial Electronics, established in Order CIN/351/2009, of February 9, 2009. This Order established by the Ministry of Science and Innovation from the Spanish government enumerate the competencies that should be acquired by the undergraduate students. The professional powers as set by the Order CIN depend on the Ministry of Industry, and not on the Ministry of Education. This has an important consequence: the professional responsibilities of Technical Industrial Engineers and Industrial Engineers are not established by the Ministry of Education but by the Ministry of Industry. The current law conforms the profession of Industrial Engineering as a regulated profession whose professional practice requires the official Graduate degree.

The Royal Decree 1393/2007, of 29 October, establishing the planning of official university studies, structures the university education in three cycles: graduate, master and doctoral degrees. The finality of the degree level is established as "to provide the student with a general training, in one or more disciplines, in order to carry out professional activities". In particular, Article 12.9, about the guidelines for the design of the graduate degrees states (translated into English): "In the case of studies that qualify for the practice of regulated professional activities in Spain, the Government will establish the relevant conditions that should be fulfilled by the curriculum, which must be designed according to the applicable European regulations. These curricula should, in any case, be designed to make possible to obtain the necessary skills and competencies to practice this profession."

From the first degree course students at any School of Industrial Engineering should include two competencies which are basic competencies in the Order CIN: (a) Knowledge of basic and technological matters that will enable them to learn new methods and theories, and equip them with versatility to adapt to new situations, and (b) The ability to solve problems with initiative, decision making, creativity, and critical thinking, and also to communicate and transfer

knowledge, skills and abilities in the field of Industrial Engineering. The Order CIN describes the mathematical competence included in the curriculum as the ability to solve mathematical problems that could appear in engineering context about: linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial differential equations; numerical methods; numerical algorithms; statistics and optimization.

The publication of the results of the Danish KOM project by Niss (2011), and the Framework for Mathematics Curricula in Engineering Education by the SEFI's Mathematics Working Group in 2013 (SEFI MWG 2013) allows us to integrate the mathematical competencies within the undergraduate degrees' curriculum with the competencies detailed by the Spanish Order CIN. For our students from the undergraduate degrees in Electricity, Electronic and Mechanical Engineering we proposed the elaboration of a cooperative and collaborative team work where they should acquire the 8 competencies adapted to engineering context: (1) thinking mathematically; (2) posing and solving mathematical problems; (3) modelling mathematically; (4) reasoning mathematically; (5) representing mathematical entities; (6) handling mathematical symbols and formalism; (7) communicating in, with, and about mathematics; and (8) making use of aids and tools.

Therefore, we must consider the purpose or rationale that requires the teaching of mathematics in the environment in question. Engineering is characterized by a deep knowledge of the principles on which its action is based and their ability to calculate, that is, to predict behaviors and get solutions to problems with minimal cost. The good mathematical training of an engineer is recognized in its ability to pose first and, then solve, mathematical models of reality. To achieve this training, we have to make an adequate selection, arrangement and presentation of contents.

Methodology and Assessment during the course

As is well known, an engineer is required to use mathematical tools, like models, software, or several calculations for solving problems in professional or academic projects as well as to communicate and present mathematical content (Alpers, 2002). To involve students in their own learning process we proposed 2 different activities to help the acquisition of the competencies detailed in the Order CIN. The first one is to use a CAS (usually Matlab® or Mathematica®) as a tool to solve engineering mathematical problems. As García (2014) and Martín del Rey (2013) suggested: the use of CAS in all learning and assessment activities has the potential to positively influence the development of competencies. The second activity is to develop the understanding of an engineering problem by means of a team work (including the final public presentation), since teams are at the centre of how work gets done in modern life (Kozlowski and Ilgen, 2006). As this proposal was developed with students from the 1st course, they do not have a lot of engineering knowledge. Some of the topics suggested were (amongst others):

- electrical circuits;
- a falling parachutists;
- hydraulic system;
- building shake.

All of the students should address a mathematical problem for each of the cases above: system of linear equations, a differential equation that is solved exactly, a differential non linear equation, or a matrix diagonalisation, respectively. Our main goal is to integrate the mathematical competencies with the competencies required for getting an Engineering

Graduate Degree. In the case of the hydraulic system for example, students should, first of all, get the equation that models the discharge of water from a tank, considering the height of water and the diameter of the tank, the input and output flow, the velocity of the jet, and the Bernoulli's principle to get a differential non-linear equation that could be solve by the different numerical methods that were detailed in theory classes, as the Euler's or Runge-Kutta's methods. From the hydraulic system they arrive at the solution of a mathematical problem to get the physical and engineer results.

The proposed team work should include, apart from an introduction, a summary and the references, an application to engineering, or a problem solution with a CAS, or the history of the mathematical formulation. It is really interesting to make students pay attention to quality assurance and not only the content, and also to work in teams which will be essential in their future professional career, as was mentioned by Kayes (2005). We have analyzed the results in two ways: the students' assessment using a rubric-survey where each student has to assess the rest of their classmates (the public exposition, the originality, the actuality, the use of technological tools, the relation to engineering subjects, etc), and a general overview of the 13 groups paying attention to the acquisition of competencies.

To assess the team work during the course we have proposed a rubric that should be filled in by each student. This rubric includes 5 general issues that are related to the 8 competencies from the framework for mathematics curricula:

1. Presentation of the work done (quality, including the replies to questions submitted by the rest of the classmates). For the presentation students should be familiar with the mathematical environment, handling mathematical symbols and formalism, and be able to communicate in mathematical language.

2. Originality and authenticity (the topic is explained clearly and accurately, included key issues). This includes the identification of the problem and the proposal of the possible solutions making use of the suitable aids and tools.

3. The importance of the topic of the work (is it relevant for the society?). This point is related to the importance of mathematics, if it is part of the daily life or not. This also includes the mathematical thinking competency as mathematics can contribute to engineering work as Alpers and Demlova stated (2013).

4. The topic is related to the mathematics contents of the course. Students should be able to reason mathematically; and also to represent engineering problems and entities mathematically.

5. Integration of the team members (contribution to teamwork) which also includes thinking mathematically. It should be taken into account that failures of team leadership, coordination, and communication are well documented causes of the majority of air crashes, medical errors, and industrial disasters, so the team tasks are great important as Kozlowski and Ilgen (2006) stated.

With this rubric we evaluated the acquisition of mathematical contents and competencies. Furthermore, the team work competence has taken on greater significance as some studies developed by Torrelles (2015) show that teamwork competence is not fully acquired by workers in the Spanish companies.

Some results and Discussion

In this study the contents and competencies acquisition were evaluated during the 2015/2016 course. A total of 34 students participated, in groups of 2-3 students. We collected 234 answers from students that participated in the assessment session. The distribution of the values for the 13 teams is presented in Table 1, where for each of the evaluated items the marks go from 1 to 5 (Likert scale).

	1	2	3	4	5
Presentation	1,7	7,3	19,7	41	30,3
Originality	2,1	8,1	19,2	38,5	32,1
Importance	35	18,8	20,1	9	17,1
Rel. to maths	9	8,5	33,3	25,6	23,5
Integration	9	6,8	27,8	23,9	32,5

Table 1: Distribution of the values after the assessment process.

We appreciated with these results that students, in general, presented well done work, concerning the originality and authenticity, they worked properly, explaining all the steps accurately, they have prepared their work according to the mathematics topics included in the course syllabus, and they worked in integrated groups. What is more remarkable is that they think that mathematics and the topics covered by the team work and classes were interesting. In some cases they argued that they did not read nor listen news about them.

Conclusions for Education

The competencies from the Order CIN are incorporated into the 8 competencies included by the SEFI's Mathematics Working Group in 2013 in the proposed Framework for Mathematics Curricula in Engineering Education. Students developed their team work during 3 weeks of the course where they learn mathematical contents and where they also acquire mathematical competencies. We have developed a rubric including this competencies acquisition. What we appreciated after this experience is that students saw the relation of mathematics with real life and their engineering studies. They used to think that the mathematical procedures are different in mathematical classes from the ones in real engineering problems.

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Future Mathematics project – using technologies to improve mathematics teaching and learning in engineering studies

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Abstract

This paper presents a project that aims to develop methods to better exploit the ICT in teaching and learning engineering mathematics. The project's main focus is to improve innovative ways of learning and teaching of mathematics through digital contents and with the help of new technologies. Theoretical research carried out in the field of mathematics online pedagogy during the project will provide a collection of pedagogical approaches, best practices and useful resources for designing and implementing web-based teaching and learning of mathematics. A comprehensive framework for mathematics learning and teaching via the web will be created. This versatile repository will include learning resources which respect the digital learning possibilities and it will bring together technological innovations and best practices for mathematics learning.

Introduction

Mathematics skills and knowledge create a theoretical basis for understanding different engineering disciplines. In fact, a profound basic knowledge and skills of mathematics enable engineers to find new and better solutions for today's and future challenges and to make new technological innovations. Unfortunately, various studies in recent decades have shown that mathematical competence in Europe has weakened. The lack of mathematical proficiency is already causing problems in engineering mathematics and other courses in European HEIs. In fact, this seems to be a global problem, and the learning outcomes of Eastern European countries have been weaker than expected, especially in mathematics, after they moved to the Western European model of education (e.g. SEFI 2002). (James et al. 2008, Lawson 2003).

Compounding the issues, the weakened mathematics skills, large and heterogeneous study groups and decreasing resources have raised problems encountered especially in engineering studies during the past decades. Various studies and projects related to the methods of mathematics learning and teaching have remarked that these issues have been present for several years now. Additional courses, support materials and various tools for mathematics learning and teaching have been developed. However, a comprehensive framework for engineering mathematics learning, that effectively utilises technology and digitalisation, respects the 21st century skills and is easily accessible, is not available. These results contrast with education globalisation and universal access to resources and contents available on the web all around the world.

Overall, the importance of online contents will continue to grow in the future. In higher education, the accessibility of studies is also one major factor in the future. In fact, the European Union has as its stated ambition the goal of 40 % of all young people having graduated from higher education by 2020. This percentage will require large-scale exploitation of online contents. Moreover, in the Agenda for the Modernisation of Higher Education, it was stated

that to achieve this goal, the focus will be on the quality of teaching and learning (Report to the European Commission 2013).

FutureMath project

Future Mathematics (FutureMath) is a three year project that began in September 2015. The project is EU-funded under the ERAMUS+ Programme. The project involves four partners from different European countries. The partners are Tampere University of Applied Sciences (Finland), Slovak University of Technology in Bratislava (Slovakia), Technical University of Civil Engineering Bucharest (Romania) and Technical University of Madrid (Spain).

The FutureMath project aims to respond to the requirements of modern society and to make mathematics learning and teaching more digitalised, effective and accessible. Additionally, the aim is to explore and develop the most motivational, learner-centred methods, techniques and resources for engineering mathematics learning and teaching with the help of technology. Furthermore, the objectives are to pay attention to the different learner types, individual learning solutions, flexibility, effective feedback and assessment. By these means, it is expected to improve the efficiency, accessibility and quality of mathematics teaching and learning on a European level which is one of the four common objectives of EU Strategic Framework of Education and Training 2020.

Project outcomes

The main outcomes of the project are needs analysis, online pedagogy, Mathematics Learning Platform (MLP) and learning resources. This section describes in more detail these main outcomes and the current status of the outcomes.

Needs Analysis

Since the educational institutions are very diverse and have to deal with very different and heterogeneous student cohorts, and since the different branches of engineering have a variety of mathematical requirements, it is important to have a comparative analysis of existing mathematics education at European technical universities.

We will rely on analysis categories stated in the document published in 2013 by SEFI 'A Competence-based Framework for Mathematics Curricula in European Engineering Education' to give answers to essential questions on the relevant content of mathematics courses for engineers aimed at higher-level learning goals of conceptual understanding, mathematical thinking and competencies. Moreover, at the beginning of the project we conducted a questionnaire to discover what is already taught, how the material is delivered, the types of problems encountered and how the students see what the mathematics teaching and learning should/could be. Summary of discussions at the 17th SEFI Mathematics Working Group Seminar, Dublin, June 2014 together with the SEFI curricula publications and results of the analysis presented here will help us to establish what kinds of materials we should produce and how effective they could be.

In the questionnaire students were asked to select which teaching methods they perceive as modern. From Table 1 we can recognise their opinion about the most modern teaching and learning methods. Nowadays students think that the modern ways to teach and learn mathematics are computer presentations, short video/screencasts, utilising smartphones and

tablets, etc. In contrast, teaching with transparencies and making notes were seen as the least modern ways to teach and learn mathematics.

Table 2. Modern teaching and learning methods

Teaching with computer presentations using data projector
Using short teaching videos/screencasts
Utilising smartphones/tablets for learning purposes
Utilising symbolic calculator / Use of simulations/demos
Writing on whiteboard / Operating in Moodle or other learning environments / Teaching using dynamic mathematical software applets
Working in groups
Making notes
Teaching with transparencies and overhead projector / Use of virtual labs

In the following question, students were asked which teaching methods are currently used in their university mathematics courses. Responses to this question are presented in table 2. The order of answers in this table is exactly upside-down to what the students think are modern teaching methods.

Table 3. Methods that are in use

Writing on whiteboard/blackboard
Making notes
Teaching with slides and overhead projector
Utilising symbolic calculator
Operating in Moodle or other learning environments
Using short teaching videos/screencasts
Teaching with transparencies
Utilising smartphones/tablets in learning purposes
Use of virtual labs

The questionnaire also contained a question where students were asked to select the five methods, which are the most important from their point of view in terms of learning. The most important way of learning was very teacher-centred as the top one was 'Doing exercises during classroom sessions' followed by 'Personal guidance of lecturer during classroom sessions' and 'Teaching with whiteboard / Using on-line instructional resources'.

Figure 1 represents student replies on the question where they were asked to select the option that best represents their opinion concerning mathematics learning and teaching. From this figure we can perceive that using modern technology in mathematics learning increases motivation and students wish to have more alternative learning and teaching methods and more online content.
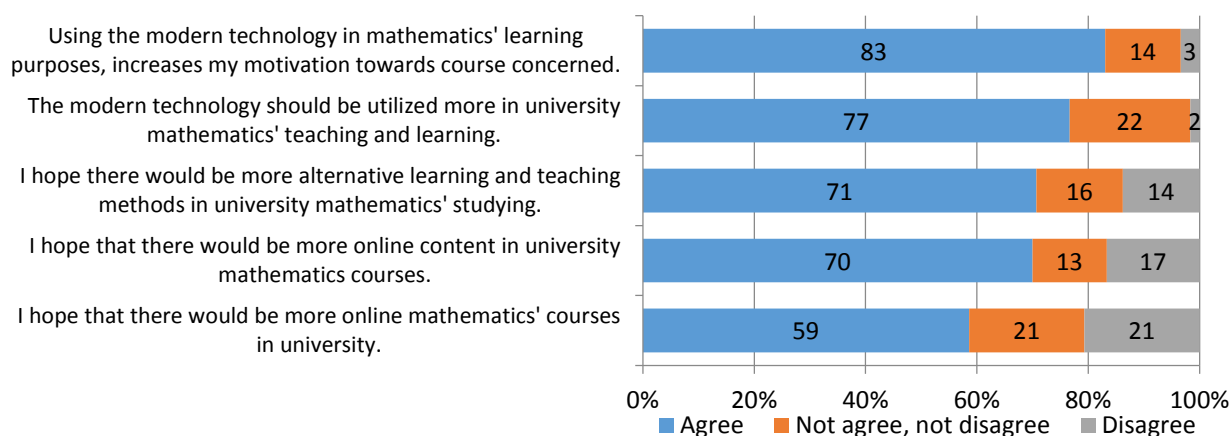
Figure 1. Student's opinion regarding mathematics learning and teaching

Mathematics Online Pedagogy

One main goal of the Future Mathematics project is to develop an online pedagogy of engineering mathematics that encapsulates pedagogical resources for mathematics online learning, teaching and assessment by providing effective teaching and learning methods and strategies for ICT-supported mathematics learning.

The mathematics online pedagogy combines best practices for meaningful utilisation of, for example, ICT-tools, learning environments and mathematical software in a mathematics learning and teaching context. Furthermore, the online pedagogy provides a collection of pedagogical approaches, best practices and useful resources for designing and implementing web-based teaching and learning of mathematics.

Previously, we have selected the trends that will be further explored during the project lifetime. The trends are: flipped classroom, learning analytics, short videos / screencast, gamification and online assessment.

Flipping the classroom practically means that the common practices – teaching in the classroom and making homework - are inverted. The concept of flipped classroom is quite flexible. While one may assign a short video or theory slides for homework, the other may allow watching videos or reading notes during classes (Herreid and Schiller 2013). It appears that flipped classroom increases its popularity also in the field of higher education. Flipped classroom has been described as follows: it belongs to "a part of a larger pedagogical movement that overlaps with blended learning, inquiry-based learning, and other instructional approaches and tools that are meant to be flexible, active, and more engaging for students. (Johnson et al. 2014)". It is not only the online videos that make the change: it is more the way that they are integrated in the overall picture. (Tucker 2012)

In gamification, the central idea is to motivate and engage the learners in non-gaming environments by using elements of games i.e. game-based mechanics, aesthetics, game thinking, problem solving, competition and cooperation. In game-based learning a learner can study the topic concerned by playing a game. Thus the games are integrated as a part of learning process in some specific topics, skills etc. Both gamification and game-based learning can include competition and co-operation among other learners.

Mathematics Learning Platform (MLP)

Mathematics Learning Platform (MLP), a comprehensive framework for mathematics learning and teaching on the web, will bring together technological innovations and best practices for mathematics learning. The aim of MLP is to make mathematics learning more motivational, interesting and to increase accessibility and the alternative modern methods for mathematics learning. The MLP environment was designed to be attractive to students. The MLP will support the teaching and learning activities and, as a web-based resource, it will be accessible from any region in the world with common devices: laptop, tablet, smartphones, etc. The MLP will support a variety of activities which reinforce the teaching/learning activity.

Mathematics Learning Resources

One key output of the project is a collection of Mathematics Learning Resources (MLRs) such as e.g. short video lectures, podcasts, vodcasts, personalised learning materials, lecture materials, online learning materials, stack exercises, online assessment components, authentic learning modules, online resources for learning etc. Thus the MLRs encapsulates a vast variety of ICT-based learning and teaching resources. These resources will be tested with pilot courses and updated based on the evaluation reports from piloting. All the learning resources and materials developed in the project will be freely available under the idea of Open Source or Open Educational Resource (OER).

Conclusions and future plans

FutureMath is a transnational project aimed to investigate current trends and needs for changes in mathematics education for engineering students in the environment of information society, where *information* of one kind or another, is usually transmitted mostly by electronic signals. Expected project outcomes are to develop a functional mathematics learning platform MLP equipped with a collection of novelty learning materials suitable for the needs and expectations of students. The MLP has been successfully designed in an appropriate and attractive way and it is capable of supporting modern teaching/learning resources. Besides, it can be accessed from anywhere with a variety of modern devices which are familiar to the students. At the moment we are working with the online pedagogy, and we have started the planning phase for development of learning materials. At the end of this year we will start the implementation phase, while the first test courses will take place in autumn 2017.

According to our questionnaire students seem to regard as modern ways to teach and learn mathematics those utilising ICT. Additionally, students think that the most important ways of learning are very teacher-centred. Compounding the student's expectations and results of various studies and projects related to the methods of mathematics learning and teaching there are clear indications that a comprehensive framework for engineering mathematics learning, which effectively utilises technology and digitalisation, respects the 21st century skills and is easily accessible, is not available. These issues have been noticed for several years now. The project presented here helps to bridge the gap and addresses the recognised needs by bringing modern methods, pedagogical approaches and technological innovations for engineering mathematics learning into the same place.

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Computer Aided Assessment in Mathematics Courses for Engineers

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Abstract

This paper is based on an on-going project for modernising the basic education in mathematics for engineers at the Norwegian University of Science and Technology. One of the components in the project is using the computer system Maple T.A. for handling students' weekly hand-ins. Successful completion of a certain number of problem sets in Maple T.A. is required for being admitted to the final exam. This also gives partial credit towards the final grade. In this paper I will look at possible consequences for the students' learning process from using Maple T.A.

Introduction

The computer aided assessment system Maple T.A. is used as a platform for weekly problem sets in calculus courses for students in the Master of Technology programmes at the Norwegian University of Science and Technology. Maple T.A. is based on the kernel of the CAS Maple and can evaluate answers given in numerical or algebraic form provided the Maple syntax is adhered to. It can also be used for multiple choice questions. The use of Maple T.A. is part of a larger project for innovative education in mathematics (Rønning, 2014, 2015). Maple T.A. has been used in education for many years and there are several reports on experiences from using it, e.g. (Brito et al., 2008).

Our students are given one problem set in Maple T.A. each week, 12 sets altogether per course. To be accepted for the final exam at least 6 out of 12 sets have to be passed (4 out of 6 problems correctly solved). Passing 10 or more problem sets gives 10 % credit towards the final exam. It is therefore to be expected that the Maple T.A. tests are highly prioritised in the students' work and that they will spend much time and effort in passing them. In addition to the Maple T.A. tests the students get four problem sets in each course as classical written hand-ins, graded and commented on by teaching assistants. Two out of four hand-ins must be passed to be admitted to the exam, and passing all four will give another 10 % towards the final grade. In addition to the compulsory problem sets the students get a number of "recommended exercises" each week. These are to a large extent more routine type exercises, meant as an introduction.

In this paper I will address the following question: *In what ways can the use of Maple T.A. be seen to influence the students' ways of working with mathematics?* The data I will draw on are collected from surveys and interviews of students.

Theoretical framework

I will analyse students' work with the Maple T.A. problems using Activity Theory. Leon'tev (1979) has described this on three levels; an *activity,* which is driven by a *motive* (level 1), is seen as consisting of *actions* that are *goal*-related (level 2). Actions are mediated by *operations,* which are subject to certain *conditions* (level 3). Engeström (1999) has developed the *expanded mediational triangle* (Figure 1), which contains the elements of an *activity system.* In this system the activity is seen as the *subject's* work on the *object* under the influence of the

mediating artefact. The motive of the activity is represented by the *outcome.* However, the activity is influenced not just by the mediating artefact, but also by certain *rules* that apply and by other actors in the context, (*community*)*,* and by how various actors contribute to the outcome, *division of labour.*
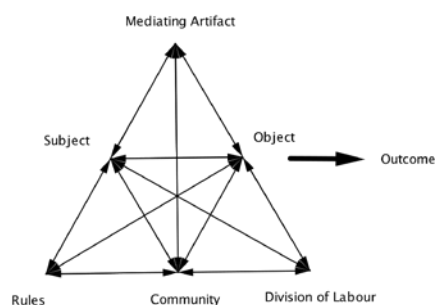

Figure 1. Engeström's model of an activity system

I will consider Maple T.A. as a *mediating artefact,* mediating between the students (subject) and the mathematical problems that they are working on (object). In this process there are a number of other factors that influence the students' work and these factors constitute the lower part of the mediational triangle.

Student experiences with Maple T.A.

The Maple T.A. problems can be answered by entering a number or an algebraic expression. Multiple choice questions are also possible. An example from Calculus 2 is shown in Figure 2. Answers must be entered following the syntax of Maple.

Let the function f be given by $f(x,y,z) = \dfrac{2xz}{5y+2}$.

Compute the third order partial derivative $\dfrac{\partial^3 f}{\partial y \partial z \partial x}(x,y,z)$.

The answer should be an expression in x, y and z.

Figure 2. Example of Maple T.A. problem from Calculus 2

Each test is open for a certain period of time and within this period the students can make an unlimited number of attempts at completing the problems. A correct answer is indicated by the displaying of a green circle with the mark □ inside and an incorrect answer by a red circle with the mark ×. There is no indication of the reason for an error. For each student slightly different problems are generated but the variation consists mostly of varying parameters in the problems.

Since starting to use Maple T.A. in 2013 as part of a reform of the basic courses in mathematics, surveys have been conducted to monitor the students' experiences with the various parts of the reformed structure. In addition students have been interviewed in focus groups. One interview during Calculus 2, spring 2015, addressed in particular the Maple T.A. problems. Every year approximately 1600 students take Calculus 1 (one variable) and around 1200 students take Calculus 2 (several variables). The Calculus 1 survey for autumn 2015 had 799 respondents and the Calculus 2 survey for spring 2016 has, at the time of writing, been completed by around 500 students.

Before 2013 the weekly problem sets were given as paper based hand-ins and they were compulsory for being admitted to the exam. It was well known that students to a considerable

extent copied solutions from each other to get enough hand-ins approved of. It was hoped that the feature of randomising variables in Maple T.A. would reduce or eliminate the copying. However, this did not happen. Since the randomisation mostly has to do with changing the value of parameters, one can produce a general answer and share this with fellow students. For the problem shown in Figure 2 the randomisation could be to vary the numbers, so that by solving the problem for the function $f(x,y,z) = axz/(by + c)$, any solution can be obtained by substituting values for a, b and c. Indeed, students have established Facebook groups where general worked solutions to the Maple T.A. problems for each week are published.

That this is widespread is documented by the surveys where 40-50 % of the respondents agree completely or partially with the statement "I copy some problems every time". For the written hand-ins the rate is lower. In particular, those answering "completely agree" are considerably fewer for the written hand-ins, 15 % versus 27 % in the on-going survey of Calculus 2. When asked about the perceived learning outcome, there is considerable difference between Maple T.A. problems and written hand-ins. For Maple T.A. around 60 % agree completely or partially with the statement "I learn a lot by doing these problems". The corresponding figures for the written hand-ins are more than 90 %. The main criticism against Maple T.A. is "I miss getting information about what I have done wrong" (82 % agree) and the main positive effect is that they get immediate feedback, which is appreciated by almost everybody.

From open-ended questions in the surveys and focus group interviews I have obtained more detailed insight into certain aspects of how Maple T.A. is used and perceived. Students report that they work with problems in a slightly different way in the Maple T.A. environment compared to the written hand-ins. Some of these differences are described below, documented by student utterances, which are also used as headings.

Hunting for the answer

In Maple T.A. the only thing that counts is the correct answer. The students know that only the answer is evaluated and that nobody will look at how they arrived at it. In addition they will get immediate feedback on whether the answer is correct or not. This makes them pay less attention to writing out the solution process carefully. "When you do it on paper you do it more properly" as one student said. When you solve Maple problems "it is like scribbling because you can just sit there and test out solutions". On the written hand-ins, however, "you have to trust what you have done". Students report that, in particular, when they don't get the correct answer at the first attempt, they start trying out different ways of solving it, somewhat at random, because they don't know what causes the error. The error could have to do with the mathematics but it could also just be a syntax error. Then "you start to hunt for the correct answer".

It is more important when a person looks at the solution

Students report that they learn a lot from the written hand-ins. This includes both learning from doing the problems (almost 90 % agree) and learning from the feedback (close to 60 % agree). Important reasons for this can be that the students want to have feedback not only on the answer but also, and perhaps mainly, on the process: "I want feedback on my thinking". "When I write the solution for a written hand-in I include more explanations and that is also helpful for myself. It becomes clearer then." Because they know that a person will look at and evaluate what they have done, they take greater care in formulating the solution, which in itself is helpful for their own learning process.

I lose confidence in myself if I get it wrong

The students explain that the main thing is to pass the exam and this can be taken as an important explanation for the desire for "feedback on the thinking". They know that at the final exam they get credit both for the correct answer and for "the thinking". And obviously, correct thinking is necessary in order to get the correct answer. So when Maple T.A. responds with "Incorrect solution" this is frustrating because the student does not know *why* the solution is wrong. "I lose confidence in myself if I get it wrong. Then I start asking questions about whether everything I have done is wrong and that makes me uncertain". The fact that the feedback from Maple T.A. provides no details, it shows only correct or incorrect, may make the Maple T.A. problems very sensitive to level of difficulty. In the beginning there were many complaints against the level of difficulty and although a lot of effort has been made to make the problems easier, also by including hints, still about 65 % of the students agree with the statement "The problems are too difficult". Statements from interviews indicate that this may be connected to the feedback from the system being just correct/incorrect. "The written hand-ins could be more difficult because then you get feedback from the teacher. If the Maple problems are too difficult you don't know what you have done wrong". This is experienced as frustrating and leads to loss of confidence. "It is like a roller-coaster. In the end you just sit and guess". Getting "harsh comments" on a written hand-in is not perceived as bad for one's self-image because then the comment is addressed to a particular part of the solution and being corrected is then seen as helpful because it is valued as helpful for being able to pass the exam.

We do Maple first because we have to

The recommended exercises given every week are meant to be rather easy, intended to become familiar with the material. For these exercises worked solutions are provided, written or presented in the lecture room, but they are not handed in and there are no requirements for the students to do them. The Maple T.A. test, however, has to be completed before a certain deadline and therefore the work with this is prioritised. This has as a consequence that less attention is paid to the recommended exercises. Even if the students know that it would be a good idea to do those first, the fact that the Maple T.A. test has to be completed makes them start with this. This in turn has the effect that students start to copy. "You copy solutions to the Maple T.A. test to get it done away with and then you do the recommended exercises". That the copy rate is lower for the written hand-ins than for Maple T.A. can be explained by the fact that it is much easier to copy Maple T.A. since only the answer is needed. The written hand-ins are considered more important for the learning process and therefore copying does not help, and besides it involves much more work. Comparing solutions on written hand-ins, however, is more widespread, and accepted among the students.

An Activity Theory approach

In an educational context it is certainly possible to see activity on many levels. Here I will see the *activity* as participating in a mathematics course (Calculus 1 or 2) and for the students the main *motive* is passing the exam. This activity can be seen as consisting of several *actions,* one of them being the Maple T.A. tests. The students' work with these tests is directed by the *goal* of passing the test. To pass the test they have to complete the problems. These are the *operations,* which are constrained by *conditions,* such as giving the answer in a form that Maple T.A. can recognise.

In Table 1 I have identified important elements in the activity system and mapped them to the various nodes in the mediational triangle. This is inspired by Jaworski et al. (2012) who have

used Activity Theory in a university setting to analyse a situation seen both from the students' and the teachers' point of view.

Nodes in the mediational triangle	
Subject	Students
Object	The Maple T.A. problems
Mediating artefact	The Maple T.A. system
Rules	Requirements for passing the test Min. number of tests required Tests count towards the final grade Deadline The Maple T.A. syntax Feedback only on the answer
Community	Other students Teachers
Division of labour	Students and teachers have different roles Compulsory work is done first Facebook groups for worked solutions

Table 1. The elements of the mediational triangle for the students

In my analysis I see everything from the point of view of the students, based on what they report in surveys and interviews. From the point of view of the teachers the main motive of the activity can be viewed somewhat differently, namely that the students should learn mathematics. More precisely, the teachers want the students to develop good conceptual and procedural knowledge in mathematics (Hiebert & Lefevre, 1986). The teachers also want to maintain high requirements for passing the exam, or at least for getting a good grade. Therefore, since the Maple T.A. tests count towards the grade the problems cannot be *too* easy. The typical training exercises are placed among the recommended exercises. However, the students do the Maple T.A. problems first (division of labour) because they are compulsory (rules). When the problems are seen as too difficult, copying from worked solutions comes in as another element in the division of labour, necessary to comply with the rules of the system.

The mediating artefact is a computer system, not a person. This affects the relation between the subject and the object, as expressed by the statements "it is more important when a person looks at the solution" and "I lose confidence in myself if I get it wrong". It is also a feature of the mediating artefact that it only evaluates the answer. This feature can also be seen as being part of the rules. That the students do not get feedback on the process or on their thinking leads to the situation of "hunting for the answer". This can be seen as encouraging guessing or trying out different solutions until you get it right. On the other hand, with written hand-ins "you have to trust what you have done", and therefore the Maple T.A. system may encourage a way of working with mathematics that is not in line with the teachers' motive for the activity.

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Learning mathematics through classroom interaction

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Abstract

We have used flipped classroom for three years running in our module on *Discrete Mathematics* for the undergraduate programme in computer engineering. Online video lectures free up classroom time for active learning. The success of flipped classroom depends on what replaces the lectures. Debating exercises and solutions is generally more rewarding than individual problem solving. Even though mathematics is not usually seen as a 'chatty' subject, there is a strong theoretical basis to increase all forms of social interaction also in mathematics education, from informal chatting to more formal discussion. Feedback is a well-known bottleneck when students work individually with exercises. Through dialogue, the participants can get prompt feedback both from peers and from tutors. Discussion may also help to develop the necessary vocabulary and mathematical language. In this paper we report on a detailed survey conducted in the 2015 class. We will give an overview of different learning activities used, and evaluate them in terms of the student survey and their theoretical justification.

Introduction

Flipped classroom is a hot topic at all levels of education. Modern consumer-end computers have made anybody a potential film and video producer, and many teachers and lecturers take advantage of the new technology to move their lectures from the classroom to the web. Thus class time is free to be used for other activities. "In a flipped classroom, the information-transmission component of a traditional face-to-face lecture (…) is moved out of class time. In its place are active, collaborative tasks" (Abeysekera and Dawson, 2014). In other words, *active learning* is an inseparable part of the flipped classroom. Thus there are as many approaches to flipped classroom as there are to active learning. Finding the optimal approach for a given subject, in a given culture, for a particular class of students, is non-trivial. At NTNU Ålesund we have used flipped classroom in the module on *Discrete Mathematics* in the third semester of the computer engineering course. In this paper we will discuss a variety of learning activities used in this module over three years. The analysis is based on a quantitative survey carried out with the 2015 cohort. A qualitative report has been made previously (Schaathun, 2015).

Literature review

Higher education is dominated by *teaching by telling*, or *the 'transmission method' of teaching* (Sotto, 2007). During a lecture, information is *transmitted* from the teacher to the student. Exercises are often used in addition to lectures, but mostly for rehearsal which assumes that the students have already learnt the material from transmission. Many voices are speaking up against the traditional lecture (e.g. Mazur, 2009). The alternative is *active learning*, which includes any instructional method that engages students in the learning process (Prince, 2004). The most extreme forms of active learning, *learning by doing*, are dominated by self-directed discovery on the part of the students, popular under different names (e.g. discovery learning, problem-based learning). Yet there is no empirical evidence to support it (Mayer, 2004). It is however important to observe that active learning also includes less obvious activities like *teaching by questioning* (Mazur, 2009) or inclusive discussions in groups or in class.

Active learning is motivated by the constructivist position that knowledge is constructed by each learner. It is well known in cognitive psychology (Anderson, 2015) that deep cognitive processing is necessary to commit new information to long-term memory. This is often

modelled as organising the information into schemata. Mayer (2004) speaks of the *constructivist fallacy*, where the popular interpretation of constructivism fails to distinguish between cognitive activity and behavioural activity. The former is essential for learning, while the latter is neither sufficient nor necessary.

Cognitive Load Theory can explain why overdoing active learning fails (Clark et al., 2005). New information has to be processed in working memory before it can be committed to long-term memory, and working memory has very limited capacity (Anderson, 2015). Complex problems requiring complex information to solve, can only be managed when the necessary information has been encoded in a schema. Many authors have argued for small manageable exercises which the students can solve without excessive cognitive load (e.g. Colburn, 1822 and Sotto, 2007). Abeysekera and Dawson (2014) argue that the flipped classroom helps students manage cognitive load, because they can regulate the pace of the videos watched.

The module design

The module is worth 10 ECTS credits and run for fourteen weeks including two weeks of exam revision sessions, using
- **Classroom sessions: Two-hour session three times a week.**
- **Video: Lectures are given on video. Typical length is 3–7 minutes long for videos created 2015, while older ones tended to be longer, up to about 15 minutes.**
- **Web pages: Simple static web pages make all teaching material available, including exercises, reading lists, video, et cetera.**

Two alternative textbooks were suggested, but neither gives a complete coverage. The syllabus was defined by the videos and the exercises, with emphasis on the latter.

The assessment is a single written exam at the end of term. However, to be allowed to sit the exam, the student has to complete a compulsory assignment. Such assignments are common in similar modules in Ålesund. The primary intention is to force students to work steadily throughout the semester. The compulsory assignments follow a format suggested by Kristina Edström in her keynote at *MNT-Konferansen* in Bergen March 2015. One of the three weekly class room sessions is used for a student-led tutorial. Typically, six problems are assigned, and the students have to prepare to *present* solutions to as many as possible. At the start of the session every student ticks which solutions they can present on a class list. For each problem one student is drawn randomly to present. Each student is required to have at least 40% ticks over the semester to sit the exam. If a student is caught bluffing, being drawn to present something he is clearly not prepared for, all ticks that day are cancelled.

The other two classroom sessions are used in a flexible way, allowing for improvisation. The starting point is always a set of exercises. Originally, in 2013 the default was individual seat work, although the students were free to collaborate if they pleased. The module convener was available to answer questions. Over the years, we have made a gradual shift towards more group work and plenary discussion. In the 2015 delivery, the session would always start with a plenary discussion either about new material in the videos or about the first problems assigned. Discussing the solution of an exercise in full class would aim to use student input as far as possible, step by step through the problem. Thus an idea to approach or a single step could be heard and praised. Students who cannot see the full solution, can still get positive feedback and see how *their* idea can be brought forward to a complete solution. When several solutions are possible, each one can be heard.

At any time, when appropriate for the exercise or step at hand, the class can switch to group or individual work. Individual work is suitable for computations which the students should know but need more practice to automatise. Group work is very suitable to discuss different solution alternatives. If the class is stuck on a piece of theory, a mini-lecture can be inserted to fill the gap. It is important to note that the activities in the classroom are decided on the spot, based on gut feeling and a good rapport with the students.

Evaluation

The data were gathered in the form of paper questionnaires which were handed out and completed in one of the last student-led tutorials. Because of unusually poor attendance this week it was repeated a couple more sessions, including the following student-led tutorial. We received 26 completed questionnaires. A total of 32 students sat the exam. A few students occasionally attended class and may have completed the questionnaire without sitting the exam. The questionnaire was anonymous, without any question which could reveal identities.

		1. How do you think flipped classroom works?				
		Very well	Well	Neither	Badly	Very badly
2. How much do you learn from flipped classroom compared to traditional methods?	More	1	7	1	-	-
	Same	-	5	2	-	-
	Less	-	1	2	3	3
	Don't know	-	1	-	-	-

Table 1: Overall attitude to flipped classroom.

The overall attitude to flipped classroom is shown in Table 1, based on two slightly different questions. A broad and generic definition of flipped classroom was given in the questionnaire, but the particular activities in *Discrete Mathematics* must be expected to influence their understanding of the concept. With more than half the respondents in favour of flipped classroom, we have a positive bias, but there is a very significant minority who are not well served by flipped classroom in the current version. Six students (23%) think that flipped classroom works 'badly' or 'very badly' (in the following referred to as the negative group), and nine students (one third) think they learn less.

Table 2 gives the student view on the amount of each activity in the classroom. We observe that both plenary and group discussions are well received, and this is interesting because we have used such discussions much more than what is common in higher education mathematics. Other questions confirm the positive attitude to discussions. More students are positive to group discussions (16/26) than plenary discussions (9/26).

	Too much	Appropriate	Too little	Don't know
Presentation of theory	-	15 (2)	10 (3)	1 (1)
Plenary discussions	5 (3)	21 (3)	-	-
Group discussions	6 (3)	17 (1)	3 (2)	-
Individual exercises	2 (1)	14 (2)	9 (2)	1 (1)
Student presentations	11 (4)	13 (1)	-	2 (1)

Table 2: How do you rate the amount of each activity in the classroom?
Numbers in parenthesis include only students negative to flipped classroom.

When we look at the two most traditional learning activities, namely theory presentations and individual exercises, we find the students split almost in half. The majority think the amount of these activities is appropriate, but about 40% think we have too little.

We also asked the students how much they learn from each activity. To be able to judge the answers on a one-dimensional scale, we assign a score from 0 to 3 to the answers nothing, something, much, and very much, excluding «Don't know». Table 3 gives the average score per student. The global average score is $\bar{x} = 1.459$ and the standard deviation is $\sigma = 0.15$.
Two activities stand out. Reading the textbook is perceived as significantly less effective than any other activity. The teacher presenting solutions on the blackboard is similarly significantly more effective than anything else. The overall impression is that most students learn something from every activity, and few learn very much from anything. Fourteen students did not tick any activity where they learn a lot.

	Very much	Much	Something	Nothing	Don't know	Score
Theory on blackboard	3 (0)	12 (3)	9 (3)	1 (0)	-	1.68
Theory on video	3 (0)	8 (1)	12 (4)	2 (1)	-	1.48
Plenary discussions	1 (0)	8 (1)	15 (4)	1 (1)	-	1.36
Group discussions	2 (0)	12 (2)	11 (4)	-	-	1.64
Individual exercises	4 (1)	10 (1)	11 (4)	-	-	1.72
Own presentation	-	6 (1)	14 (1)	5 (4)	-	1.04
Solutions by students	1 (0)	4 (1)	16 (3)	4 (2)	-	1.08
Solutions on video	3 (1)	12 (2)	8 (2)	2 (1)	-	1.64
Solutions by teacher	8 (3)	9 (2)	8 (1)	-	-	2.00
Read textbook	1 (1)	2 (0)	13 (1)	5 (2)	4 (1)	0.72

Table 3: How much do you learn from each activity?
Numbers in parenthesis include only students negative to flipped classroom.

The most controversial activity in our scheme is the student presentations (compulsory assignments). Table 2 splits the class in half, where one half thinks there are too many presentations while the other thinks the amount is appropriate. In Table 3, the student presentations have scores more than two standard deviations below average. However, this negative attitude to student presentations is not as clear in other questions we have asked. The negative group is consistently less favourable to student presentations. Four of the six negative students think there are too many student presentations, and all six agree that they learn more from written assignments than from blackboard presentations (four strong agree and two weak). The non-negative students are almost balanced on the same question.

There are many plausible reasons why students dislike the student presentations. A majority of students think it is difficult to speak in front of the full class, and it is also important for them to choose when to do compulsory assignments. On these two questions, the negative students do not differ significantly from the rest.

Discussion

These empirical results do not endorse a pure implementation of flipped classroom. Rather, they point to a range of techniques and learning activities which can contribute to an effective module delivery. The results are consistent with observations made by others. In particular, Cognitive Load Theory (CLT) gives a good foundation to understand the results. The traditional 2h lecture gives the students a lot of information with little time to process it. Without time to build schemata, most of the transmitted information is lost. Thus, from a CLT perspective, the

problem is not the transmission mode itself, but the duration of each transmission session. The flipped classroom does not solve this problem, unless the students, on their own initiative, put in the time and effort to process new information between videos and before they come to class.

In *Discrete Mathematics* we now interleave transmission teaching and active learning in the classroom, and thus we have moved beyond the pure flipped classroom with which we started in 2013. An overwhelming majority of students, consistently over several overlapping questions, indicate that this transmission teaching is very important for learning. The interleaving achieves two things. It breaks up transmission teaching which would otherwise overload working memory, and gives essential scaffolding during active learning. This is also supported by a study of a module in *Microcontrollers* on the same degree programme (Schaathun and Schaathun, 2016). While a large minority of students want more transmission teaching in the classroom, no learning activity is a popular candidate for reduction.

Both group and class discussions are used in class. There are many reasons why we think this is important. The plenary discussions form important feedback to the teacher. The in-class mini lectures are improvised in response to questions from students. Sometimes it allows the selection of examples best aligned with the previous knowledge. For instance, introducing relations between sets, when we saw that the students had done relational databases in another module the previous week, we focused on this as an example instead of the example which had been prepared, based on object-oriented programming. It is a welcome observation that most of the students approve of the time spent on discussions. We do not have data to analyse why the students are happy with the discussions, whether they enjoy discussing in itself or if they see how the discussion supports and enables other activities.

Theoretical foundation for the use of discussion is found in Vygotsky's work and social constructivist theory. Most learning occurs through interaction, and social interaction is a form of scaffolding. When students present a solution verbally, it is possible to give instant feedback. Instead of individually completing a false solution, the class can stop at the first mistake and discuss why it is wrong and how to get it right. Plenary discussions have the advantage of involving the more knowledgeable teacher, while peer discussions allow more students to take an active role. Foldnes (2016) has also shown that group work is much more effective than individual seat in class. Discussions can also develop the verbal language to discuss the subject matter. It is known (Holm, 2012) that many people who struggle with mathematics lack the language to explain their problems and ask the right question. In a famous experiment, Vygotsky has shown that children talk constantly during problem solving, and preventing speech also prevents the solution. Similarly, inner speech is widely recognised as a key tool in mathematics, and the development of inner speech goes through outer speech (Holm, 2012).

It is not seen in the quantitative data, but in the student-led tutorials we have observed that the development of language leaves a lot to be desired. Even comparably highly skilled students would tend to copy their notes onto the blackboard and fail to explain what they do verbally. This is disconcerting, since it will be difficult for these students to use their knowledge on a professionally applied problem where they need to collaborate with others. Maybe this calls for developing the discussions further with the express goal of training verbal maths skills.

An instructional technique which we have only started to explore is worked examples (Clark et al., 2005), which is essentially a problem with a detailed step by step solution. Worked examples can be used in two different ways. In the inductive approach (Colburn, 1822; Hendrix, 1961) examples are given as the first step of the instruction cycle. General rules and principles

are then induced from the examples. In the more traditional, deductive approach, rules and principles are presented first, and examples are introduced later as special cases deduced from the rules. Hendrix (1961) considers the inductive method to be one type of learning by discovery. Worked examples and sample solutions emerge as the one activity which gives the most learning in our study. Qualitative feedback in class also confirms that the students want more solutions. However, most of the examples are used in a deductive fashion; only in 2015, we started to experiment with an inductive approach. Thus while the worked examples are confirmed to be important, we do not have data to compare inductive and deductive use thereof.

Both CLT and the inductive method motivate reconsideration of the use of solutions. Naïvely one might think that solutions are required primarily for difficult problems which few students can solve on their own. Maybe what we really need is a large number of worked examples based on simple problems which everybody understands, to allow students to build up schemata before they try to solve difficult problems.

Conclusion

Teaching is a design problem which cannot be solved by a simple recipe. It is an art form which depends on a close understanding of both the learners and of the subject matter. Cognitive psychology and educational research provides very useful background knowledge to guide the design, but it all has to be interpreted in the concrete context of a specific subject and real students. This paper should mainly be read as an example of *one* approach to delivering a higher education module in mathematics. It is an approach which depends on a range of constituent techniques, and student responses indicate that every technique contributes to the learning. We have not yet found the optimal mix of these techniques, nor do we expect a universally optimal mix to exist. However, we do believe that just lectures and individual exercises make too small a toolbox for a successful maths teacher. We hope that this paper can motivate other teachers to expand their educational toolbox, with new tools and activities from here or elsewhere. We are not writing this paper because our three years of experimentation in *Discrete Mathematics* has given any final answers. Rather we hope to persuade the reader that it is worth taking some years to experiment with new techniques.

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Teaching modelling and problem solving with a varied set of realistic problems

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Abstract

We outline how mathematical modelling and problem solving can be learned together, by using a varied set of small and reasonably realistic problems, thereby providing familiarity with a wide range of problems that can be remembered as cases, and making problem solving patterns visible. We also briefly discuss how such teaching develops a more complete set of skills, with fewer undesirable side-effects than traditional mathematics teaching with very small artificial problems, where components of knowledge or single competencies are considered in isolation.

Introduction

Between courses of mathematics, which mostly engage in mathematics itself, and traditional engineering courses which are often concerned with already existing models and methods, other more elusive competencies such as mathematical modelling and problem solving may slip through the cracks, especially since they are difficult to teach within the traditional scope of the well-defined. We here refer to problem solving in the sense that no specific method is known in advance.

Modelling problems are typically ill-structured (Jonassen, 1997), and thus provide an excellent opportunity to teach both mathematical modelling and problem solving in a way that is relevant to engineers. For example, modelling problems are not precisely defined, they depend on the real-world context, and there may be many different solutions. At the same time, they evoke many classical aspects of mathematical problem solving, which become natural to consider at the same time.

To do this, we will here discuss and reflect on a multi-problem approach based on *realistic problems* and *variation*. My views are based on several years of teaching a second-year course in the Chalmers software engineering programme and related programmes, based on a set of about 30 reasonably realistic, highly varied and challenging problems, solved in pairs (Wedelin and Adawi, 2014).

A particular reason for highlighting this approach is that it can be seen as an intermediate step between how traditional mathematics education mostly uses numerous very small and artificial exercises, and how on the other hand both mathematical modelling and real-world problem solving is often taught in the form of projects with a single large real-world problem - see e.g. Alpers (2001) for an account of how a larger project can be designed to provide a rich experience.

Before coming to our course, students have taken all the mathematics courses within their engineering programme, and mostly know mathematics in the form of components of knowledge and skills applicable to specific kinds of problems, like solving equations. The students typically know more mathematics than they can use, due to limited experience of real problems, modelling and problem solving. In problem solving, students often apply a

"cookbook" approach, by superficially identifying the problem at hand with some specific problem type they are familiar with and then solving it with the corresponding known method. If this fails they may quickly conclude that they are unable to solve the problem, and that they need more mathematical knowledge. Other related beliefs include that there is a single correct way to solve every problem, and that every problem has a single answer. For an account of student challenges in the beginning of our course, see Wedelin, Adawi et al. (2015).

Small but realistic problems

In our approach, we use size-limited but still reasonably *realistic problems.* These are problems which share important properties with larger real-world problems, large enough to be remembered as a case, but which are also limited in size so that we can have many of them in the same course - the problems we use typically require from one to several hours of student work.

One way to create such problems is to take real-world problems and simplify them, or to take parts of real-world problems while still providing some context so that the source of the problem can be understood. An important consideration (Wedelin and Adawi, 2015) is that the solution to the simplified smaller problem should still be of practical or theoretical interest (this is the fundamental property of any real problem!). The simplified problem should also be sufficiently challenging both to understand and to solve, so that you work with it in a way similar to how you work with full real-world problems - requiring investigation, exploration, communication, and - as appropriate - the use of computational tools. Clearly, this depends not only on the problems themselves, but also on *not telling* the students in advance how to approach them.

Such problems normally provide multiple learning opportunities. The problem, say for example a curve fitting problem together with some appropriate context, has a value as a simplified example of a real problem. If no method has been provided it also invokes problem solving aspects such as how to search for the functional form and estimate parameters. Students may themselves discover the iterative search process known as the modelling cycle. They need to think about how to determine the fit, and when it is good enough. The solution itself can be further discussed and put in perspective in relation to the situation from which the curve fitting problem emerged.

The problems used in the current course are all available on the course home page (go via the author). Some problems are also discussed in Wedelin and Adawi (2015).

Creating familiarity with realistic problems through variation

A major benefit of multiple varied problems is that they outline a statistical distribution of cases, providing a familiarity with the character of problems in some area of interest, in our case mathematical modelling across different applications. In addition, the problems, seen as cases, illustrate how these different problems can be approached, and how familiar components of mathematical knowledge can be used. Neither large numbers of purely artificial problems, nor single large real-world problems are able to provide this kind of familiarity, including the possible sense of relevance and judgement that can be expected to follow.

There is a connection here to *case-based reasoning* and how experienced problem solvers exploit past cases in complex ways when they encounter new problems (Kolodner, 1993). Most fundamentally, there is a connection to how encountering such *variation,* and learning to handle it for known problems of today, is the key to being able to handle variation in future unknown

problems (Bowden and Marton, 2004). In fact, the whole field of problem solving is ultimately a consequence of the never-ending variation of real problems, which is also why the usefulness of the cookbook approach is fundamentally limited.

So the theoretical basis is clear, with the main keywords being *realistic* and *variation* -ensuring that the experience gained will both be relevant and sufficiently rich to be useful in the future.

Making problem solving patterns visible through variation

Problem solving in any area is a complex skill, and problem-solving patterns are by their nature descriptive, elusive and not very well-defined. For example, it is often useful to split a problem in parts, but how to actually do this varies considerably between different cases - so this is not an algorithm and hardly even a method. At the same time, to be aware of this possibility and to be able to see when this can be done - or at least search for a way to do it - is essential in order to be a successful problem solver.

Problem solving therefore needs to be learned through one's own experiences, and a sufficiently large set of reasonably realistic problems becomes an essential base for seeing and learning common recurring patterns in modelling as well as in other important aspects of the problem solving process (here freely organized along the lines of Schoenfeld (1985)):

- How to associate with *existing knowledge* and see when known concepts and methods can be used as components in a solution. A general familiarity with the typical problem solving situation can also be seen as a form of knowledge.

- In classical problem solving, there are many techniques known as *heuristics* that experienced mathematicians and engineers will typically acknowledge. However, these are descriptive and not possible to describe in detail since they constitute perceived recurring patterns that take very different concrete forms in different cases. Still, when you have seen for yourself in several problems how a solution can be split in parts, this will naturally occur to you in certain new situations and based on your experience you will have some idea of how to go about doing this. Or when it has been pointed out to you that, in both this and this problem, a key aspect was to change the representation - or to consider a special case - then both the usefulness of such heuristics and some idea of when and how to use them becomes more clear.

- Overall *control strategies*, where you begin by carefully investigating the problem to understand it better, how you need a meta-cognitive awareness to monitor and control your overall problem solving behaviour and taking appropriate strategic decisions. For example when you have seen a number of times how you have failed by not sufficiently understanding the problem, by not starting with something simple, or by not being able to change your mind about the approach despite lack of progress, this comes clear to you as being important. In addition, well-known schemes such as Polya's problem solving phases, the modelling cycle, or one's own versions of these, begin to make sense and can be developed through the successive experience of grappling with many problems.

- To develop a well-calibrated set of *beliefs and attitudes* requires a form of generalization which also benefits from exposure to multiple problems. In many ways students here need to *relearn* a whole new set of beliefs and attitudes compared with what they are used to. There is often not a single "right track" to "the right¨ solution - especially not in modelling. It is not

important that everything is right from the beginning, only at the end, leading to a very different and self-correcting way of working, compared to the conventional expectation in mathematics education that you should be right already from the beginning (which kills creativity).

We emphasize that the varied problem set is necessary not just to understand what these problem solving patterns are, but also to gain the desired experience in when to apply the different techniques.

Of course, in our case all of this also includes the techniques and considerations of modelling, and not just of purely mathematical problem solving. Modelling problems have some additional characteristics, and these are naturally learned in the same way. This includes how you need to use the situational context beyond the actual problem statement, make suitable simplifications etc.

Additionally, the set of realistic problems highlights many complementary aspects of thinking and working mathematically, including mathematical reasoning and working practices that are not really developed until students are given sufficiently realistic and challenging problems. These range from fundamental aspects such as knowing if you actually understand something or not, to more technical aspects such as making proper distinctions between variables, constants and names. With respect to working practices you need to be able to work with pencil and paper as a tool to investigate and explore, to do the same with computational tools (and sometimes with your own programming), and to learn the importance of being careful.

Other aspects of the learning environment

For most students, the learning needs to be supported by a suitable learning environment. In the course we give, this is done with a *cognitive apprenticeship approach* (Brown, Collins et al., 1989), where we make the thinking of both the student and the expert/teacher visible. This is adapted to the constraints of a large course, where we provide as much individual group supervision as possible, together with collective feedback, where reflection is an important part. See Wedelin and Adawi (2014) for more details. Important aspects of the learning environment include:

- Supervision. Extensive supervision is provided - discussions within the group and with supervisors is considered as an essential part of the course. Compared to a project with a large problem, the smaller problems give a more controlled environment, and the problems are easy to efficiently supervise. We mostly supervise by asking questions, and by reminding about general problem solving techniques.

- Reflection and feedback. Based on the students' own first-hand experiences, problem solving patterns are explicitly discussed throughout the course. Since we use many smaller problems, we can provide short and repeated feedback loops on the whole problem solving process.

- Course philosophy and culture. With a few exceptions, we do not bring in new theory, but focus on using the mathematics students already know. We do not require that students solve the problems completely, but rather encourage them to do their best - otherwise we would never be able to give students such challenging problems.

Discussion

In this paper we talk about *competencies* such as modelling, problem solving, reasoning and so on. However, just as is the case with problem solving techniques, these are not complete and well-defined concepts, but rather incomplete and simplified descriptions of what students need. They can be most helpful to provide some structure that we can talk about, but we cannot in words fully characterize what can be learned through one's own complex practice with real problems.

In particular we wish to make note of the limitations of teaching pieces of knowledge or competencies as separate components. This is otherwise the most common form of teaching in mathematics, usually with the help of small artificial problems. However, this usually fails in teaching how components should be combined, and is insufficient in terms of a true ability to think and work mathematically. Additionally, students will in this way not learn more than what we as teachers already know, except for undesirable side effects, especially in beliefs and attitudes that we are often not aware of.

Instead, we have in this paper highlighted the approach of using small *realistic problems* and *variation*. An important property of such teaching is that it inevitably trains a more complete range of competencies, and generally tends to set things right, also in ways that we as teachers may not be aware of. This is, of course, also true for larger projects, but we have found that the approach suggested here has significant complementary value as an intermediate step.

Compared to what our students have previously experienced, this kind of teaching strikes a fundamentally different balance between learned knowledge and independent thinking, in favour of the latter. After taking our course, most students express and demonstrate a fundamental change in their ability to think and work mathematically, considering it as one of the most important courses in their education.

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Applied Problems and Use of Technology in Basic Courses in Probability and Statistics – A Way to Enhance Understanding and Increase Motivation

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Abstract

Several authors have reported problems on service courses in basic probability and statistics: students may lack motivation, find the theory difficult or boring, others see no applications for the results. To remedy these problems we have developed a learning environment where two important components are applied problems and use of technology. However, the mere existence of applied problems and technology in the course does not automatically imply increased motivation or enhanced learning. Technology is helpful for the students if it is used to achieve learning goals, is integrated with the needs of the students and is aligned with the rest of the course. Real-life data and problems evoke interest if the students perceive that they benefit from the task. We give two examples, one where active work with applied exercises and projects give a more positive attitude towards the subject, the other where aligned web-based test and exercises increase the result on the final exam and also indicate a better conceptual understanding.

Introduction

Most engineering students have a compulsory course in basic probability and statistics; for some students, it is their single course on the subject. Petocz and Reid (2005) reported problems on these service courses: students may lack motivation, find the theory difficult or boring, or see no applications for the results. At the department of mathematical statistics at Lund Institute of Technology (LTH), we have during several years developed course material and an active learning environment in order to reduce these problems. Two important components in this setting are applied problems and the use of technology in various ways.

Neumann et al. (2012) report how important it is for students' motivation to work with real-life data. Believing that motivated students are better learners than unmotivated ones, we have developed applied exercises and projects for different groups of engineering students, Zetterqvist (2010). However, presenting applied problems in the course does not automatically increase motivation, it is *how* the students work with real-life data that is important. According to Biggs and Tang (2011) students are motivated if they perceive their task reasonable and beneficial in some way. It may be that they find the task helpful for understanding the theory, for the exam, or for future working life.

Mathematics software has been used for decades in probability and statistics education in order to analyse and visualise data and for simulation and illustration. Nowadays, many courses (including ours) also include several other uses of technology. Some examples are video clips presenting theory and solutions to exercises, web-based exercises and tests or 'applets' where the students are able to explore the theory interactively. Chance et al (2007) give an overview of the role of technology in improving student learning in statistics. But using technology does not automatically mean an enhanced student learning. Price and Kirkwood (2011) argue that

technology is helpful for the students if it is used to achieve learning goals, is integrated with the needs of the students and is aligned with the rest of the course.

Here we present two examples where problems with low motivation, slow starters and misconceptions were reduced by using applications and technology in an aligned way. The methods of investigation are presented in the next chapter. For each example we present the problem, our action and the results of our investigation. Our experiences are that active work with applied exercises and projects give a more positive attitude towards the subject. Investigations also showed that introducing a web-based test increased the final exam result and also indicated a better conceptual understanding.

Method of Investigation

How do we know that one group of students have an enhanced learning compared to another group? Or that the groups differ when considering motivation? Since we have no possibility of making a controlled experiment, we are forced to compare results from different years. We measure an interesting variable before and after a change in the course and try to keep other factors as constant as possible. As an example, overall satisfaction with the course may depend on the lecturer so we have the same course coordinator and lecturer for the course during the period studied.

Finishing a course at LTH, a student has the possibility to answer a web-based course evaluation questionnaire (CEQ). On average, the response rate is 50%. In this questionnaire, the student has to take a stand on statements that are presented. Two examples of statements are "I'm overall satisfied with the course" or "The course is important in my education". The student answers on the scale -100, -50, 0, 50 and 100 where -100 means "do not agree at all" and 100 means "totally agree". In this paper we present the result from different statements and we then use the distribution of the CEQ-value or the average CEQ-value.

We recorded the proportion of students attending the ordinary final test and the proportion of students passing the test. The results on parts of the final test are recorded for different years (before and after a change). We have also studied students' solutions from comparable questions on two exams in different years to look for changes in misconceptions.

Example 1: "The problem with unmotivated students"

The course for civil engineers (90 students) was not working well in the beginning of the 2000s. The reasons were several: frequent changes between lecturers and course coordinators resulted in a lack of continuity, the course was given over 3/4 of a semester of the second study year and was outcompeted by two parallel courses. Many students never "got into the course"; those who did the final test had low points. There were several compulsory computer exercises, using Matlab, both for illustrating the theory and analysing data but many students came unprepared and were "ticking off" the moment. The results from the project were of poor quality. There were several real-data sets and applied problems on lectures, exercises and projects but still the students thought the course was irrelevant for their education.

In 2007 and 2008 an optional test was given half-way in the course, resulting in an increase of the pass rate but not in the attitude. When the programme of civil engineering in 2009, after our urging, decided to concentrate the course in time to 7 weeks and also move it to the third year of studies, we also decided to act. We made a rearrangement of the computer exercises in Matlab, introducing a number of "mini-projects", where each student worked with two - one on distributions and the other one on regression. The mini-projects used relevant real-life data and

the problems were written with open-ended questions, simulating a situation where the students acted as consultants answering a client. Two examples are: "Should we complain to the manufacturer about our bearings?" or "Is there a relationship between the price on my real estate and the distance to the railway?" We scheduled time for guidance and discussions of the corrected reports and the focus on the exercises was put on methods and techniques, being able to answer the questions in the mini-projects in a correct way. Simulations and illustrations of the theory were also included, in the same amount as before, but now mainly using scripts in Matlab where the students could interactively explore the theory. We also made connections between lecturers, exercises and computer exercises in order to have a better alignment between different parts of the course.

The change in 2009 produced a further increase in the pass rate but also a dramatic change in the attitude towards the course. Figure 1 shows the proportion of students attending and passing the final test during the period. Figure 2 shows the average CEQ-value for the two statements "I'm overall satisfied with the course" and "The course is important in my education". During the studied period 2006-2011, the course had the same lecturer and course coordinator.
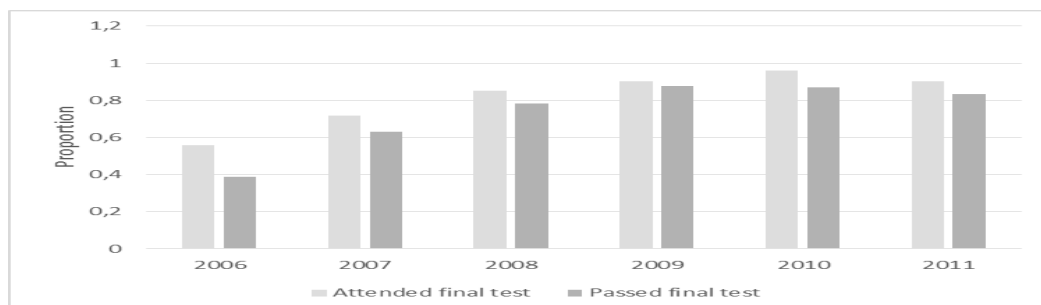

Figure 1. Proportion of students who attended and passed final test, 2006-2011
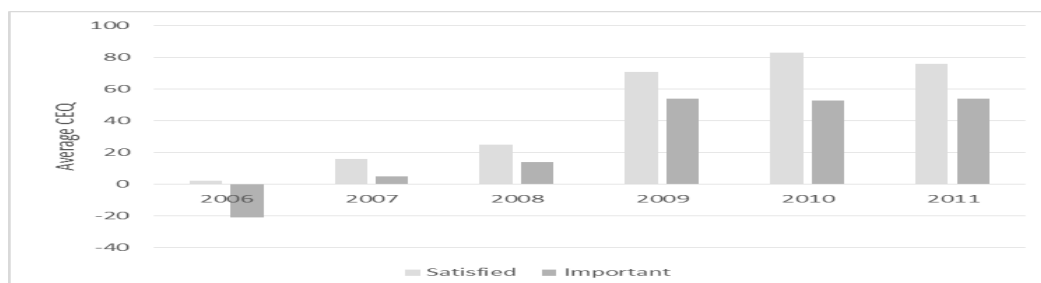

Figure 2. Average CEQ-values for questions about satisfaction with, and perceived importance of, the course, 2006-2011. Number of students who answered these questions are approximately 50 each year.

We also asked the students how they perceived the mini-projects and computer exercises. Figure 3 shows a typical answer from a year.
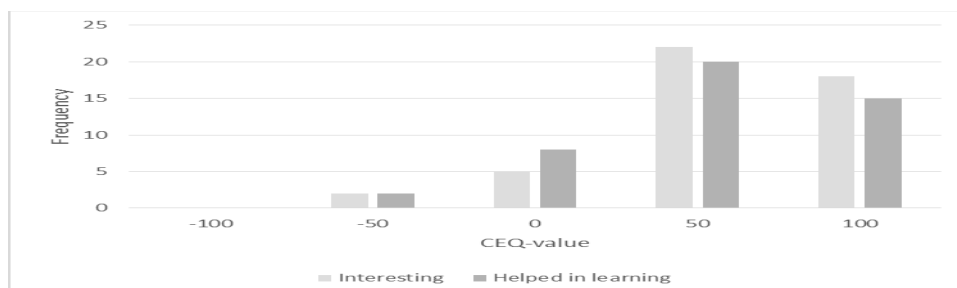
Figure 3. CEQ-distribution for statements: "Mini-projects were interesting" and "Computer exercises helped me a lot in the learning"; -100 do not agree at all, 100 totally agree.

Example 2. The problem with slow starters and misunderstood concepts

Typically, a course in mathematical statistics at LTH, starts with a number of basic concepts in probability that are fundamental for understanding the subject. These concepts are non-trivial and not easily understood, but necessary for the second part of the course. We wanted to speed up the learning of these concepts and ensure that most of them are understood when starting the second part of the course. In the second part there is a further number of methods and concepts that are often misunderstood and treated more like a black box by the students. It is often the case that students can perform the calculations correctly but show grave misunderstanding of the concepts.

We developed questions and introduced web-based exercises and tests. The system Maple T.A. was used. The different courses used the exercises and tests in slightly different ways, but all of them had a compulsory test half way in the course on basic concepts in probability. We compared the results on the final exams in the course for mechanical engineers in two years, one year without the web-based test and one year with the test. We looked at questions on the final test where knowledge on basic probability are tested, the maximum score on this part of the test is 28 points.

Year	Average points	Standard Deviation	Number of Students
2013 (no web-based test)	20.9	6.3	107
2014 (with web-based test)	23.5	4.2	128

Table 1: Result on the probability questions for mechanical engineers.

There is a significant difference between the expected number of points between the two years, a 95% confidence interval for the expected difference is (1.3, 4.1).

In the small course in biostatistics (25 students), web-based exercises and test were also used in statistical inference in the second part of the course. Here we also compared the results on final exams two different years (with or without test) but now looking for conceptual misunderstandings. When comparing two questions that were directly comparable on the two exams, we found that the number of students who could set up the correct hypotheses was very significantly increased when using Maple T.A. There also seems to be an increase in the number of students who could correctly motivate and calculate a confidence interval for a proportion; however the total number of students is too small to draw any statistically significant conclusions.

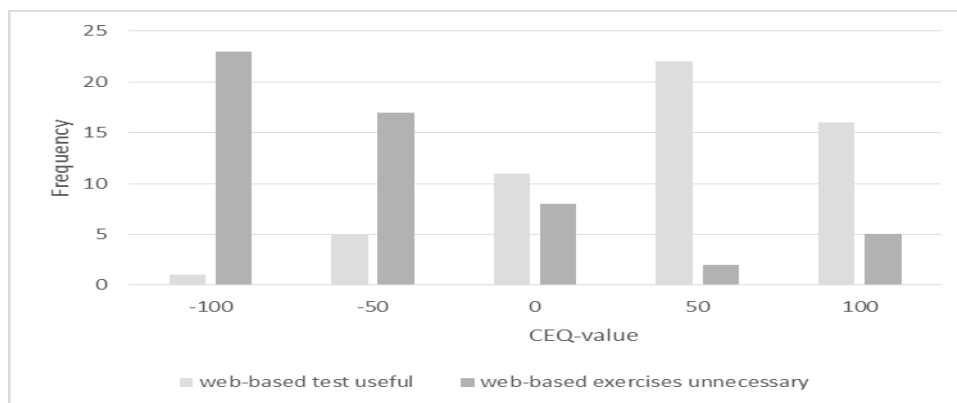We also asked these students how the perceived the web-based test and exercises, see Figure 4.

Figure 4. CEQ-distribution of the two statements: "The web-based test gave me a good idea of my knowledge and what was expected of me" and "The web-based exercises felt unnecessary"; -100-do not agree at all, 100-totally agree.

Findings and Discussion

Both examples illustrate how using applied problems and technology in an aligned way may increase motivation and enhance student learning. In example 1 (unmotivated students) the mere existence of computer exercises with real-life data and problems to work with were not enough to evoke interest. But presented in a setting where the students worked in a "consulting role", it did. We moved focus on the computer exercises from ''copying Matlab commands'' and looking at (not so interesting) data sets to using the computers as a natural tool for reaching the answer on interesting questions. We showed the students that by using their knowledge on basic probability and statistics, they are able to solve problems relevant in a future profession. Interestingly enough, the amount of theory presented and illustrated on the computer exercises was almost the same as before but "rearranged" and presented using scripts in Matlab where the students could interactively investigate the theory. Connecting the computer exercises and use of Matlab with ordinary exercises and lectures also made an aligned appearance for the students. The concentration of the course and that the students are a bit older (and wiser?) may also have some effect on the results.

Example 2 (slow starters and misunderstood concepts) shows how useful web-based exercises and tests can be for the students in their learning. We found that introducing a web-based test on basic probability, three weeks in the course, not only speeds up the learning, it also results in a small (but significant) increase in the result on the final test eight weeks after the web-based test. Based on a small investigation on biostatistics students, we also found indications that web-based exercises and tests in the second part of the course increase the conceptual understanding of hypothesis and confidence intervals at the final test.

The students seem to appreciate both web-based tests and exercises. The latter are training students' conceptual understanding and the students benefit from them most if the web-based exercises are well integrated with the ordinary paper-and-pen exercises. Ideally, in an exercise lesson, the student should alternate between the two types of exercises, a strategy which nowadays is facilitated by students' increased use of computers and tablets in lessons.

Our approach has been to use real-data exercises and projects in order to show the students how their knowledge in probability and statistical reasoning can be used in other courses, in everyday situations or in a future profession. The developed material, real-data problems, web-based exercises and scripts in Matlab for exploring theory, are presented in Zetterqvist and

Lindström (2016). Many exercises in this material are specific for each student programme; for examples, exercises suitable for environmental engineers, exercises suitable for mechanical engineers and so on. We think the effort put in the developing process is rewarded by an increased motivation and a more positive attitude towards the subject.

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POSTER PRESENTATIONS

Using Simple Tests to Identify Students Needing Support in Engineering Mathematics

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Abstract

Within large first year classes it can be difficult to identify, at an early stage, those students who are struggling to adapt to studying at a University. In mathematics, many universities adopt a policy of early diagnostic tests to try to identify those struggling with the background material, however this can often identify those who have been away from formal learning for a long time who will not have problems adapting to study within a university environment. In addition, any support provided to students may not be directly related to their studies and such support is often not taken up by those who need it, but instead taken up by those who do not.

This paper presents an approach using simple tests which may be repeated weekly to identify those students who are struggling. To complete the first year mathematics course, a student must pass five such tests covering the basics of algebra, the use of calculators, vectors and matrices, differentiation and integration. In each test ten multiple choice questions are asked and a student is deemed to have passed the test when they correctly answer all questions at a single attempt. In advance of the tests, students are shown an example test and told that each of the 10 questions will be on the same topic as in the example. They are therefore able to revise the basics before the test, know the topics that will be present in the test and know that they cannot try to be strategic in their approach by avoiding what they perceive to be hard topics. Students who fail to achieve 100% at each attempt, are given feedback on their attempt, encouraged to seek further support during tutorials, and required to take the test again the following week. Experience has shown that at each attempt between 60% and 80% of students pass the test. Within two tests it is therefore possible to narrow down the list of students struggling with the basics to about 10% of the class.

An added benefit is that the tests provide early events for monitoring attendance and those students failing to engage are quickly identified. Personalised emails are sent to each student after each test detailing their test performance, or inviting them to identify problems they are having. The paper concludes by correlating the test results with overall performance in the first year engineering mathematics course.

Introduction

One of the major challenges in devising a support scheme for students struggling with mathematics is that often resource is taken by students who do not need the support, while those that do require support are often those with poor engagement and do not immediately recognise that they are struggling. Traditional support mechanisms such as tutorials are generally attended by those who are engaged and putting on extra tutorials general misses those students who really need the help. Allied to this, is that many students who struggle with mathematics in the yearly years of university have fundamental weaknesses in their underlying mathematical skills and it is these insecure foundations which are the primary cause of students failing to progress in mathematics (Engineering Council, 2000).

When marking mathematics exam papers it was noticed that many students were able to recall and use the new concepts that had been taught, but were making errors in simple mathematics that was assumed to have been covered by students before they arrived at university. Often

marks were being taken away from students for making errors where no marks would have been gained by getting it correct, for example, correctly factorising a quadratic polynomial.

An approach that is often taken is to have a diagnostic test, administered either in paper or online, at the beginning of the course and an approach sometimes linked to this is to have an online quiz that students must get a threshold mark in by the end of the year. These approaches, while giving useful information on the general level of ability of the class, and possibly guaranteeing a certain level of core knowledge by the end of the year, do not clearly identify those students who are not engaged with their learning, or who are struggling with the basics (LTSN MathsTEAM, 2003).

Many universities, in recognising the difficulty of mathematics for some students, have set up Mathematics Support Centres (Matthews *et al*, 2012) but often these centres are used by self-referring students rather than ones who have been specifically identified as needing help.

To try to address these weaknesses, a series of simple tests were introduced to cover the fundamentals of each area of mathematics covered in the year 1 mathematics course. Initially two tests were introduced but this has been expanded to five tests over the years. As the concepts are so fundamental and at such a low level, the requirement is for each student to achieve 100% in each of the five tests. Tests are timetabled for one hour but students can usually complete the test in between 10 and 20 minutes. Students who are struggling usually take longer but time taken to complete the test is not a strong indicator of success.

The approach detailed is used for all first year students in Engineering at the University of Glasgow, a class size of up to 400 students. Before the test students are directed to an example of the test on the course Moodle. Each question in the example mirrors a question in the test, usually with only the parameters changed. The first sitting of each test is taken in a normal lecture period, and marked within 3 days.

A typical profile of results for one sitting of a test taken by 288 students is shown in Table 1. This result is typical of the outcome for a test and shows that almost all students get the majority of questions correct, but that some have serious weaknesses in the basic mathematics.

Mark	100	90	80	70	60	50	40	Total
Number	209	63	12	1	2	0	1	288
Percentage	72.6%	21.9%	4.2%	0.3%	0.7%	0.0%	0.3%	100%

Table 1: Typical profile of marks for a test.

Students may take the tests as many times as necessary to achieve 100%. In practice, between 60% and 80% of students pass the tests at each sitting. Taking a figure of 70% pass rate means that, for a class of 400, 120 have to take the first resit, 35 have to take the second resit with only 10 failing three times. It is these 10 individuals in whom extra help can be focussed with the dual aims of helping the student understand the issue with which they are struggling and allowing a personal relationship to develop between the student and someone on a position to help and advise them.

Content of the Tests

The tests comprise 9 or 10 questions about the fundamentals of mathematics. Calculators are allowed in all except the first test, and all except the second test are multiple choice with one correct answer out of 5 options.

The five tests used for 2014–2015 were:

1. Algebra

No Calculator, 10 multiple choice questions. Example questions are:

factorise $x^2 - 5x + 4$; solve the equation $x^2 - 20x + 84 = 0$; evaluate $\sqrt[3]{-216}$; simplify $\frac{5\sqrt{20}}{2\sqrt{15}}$, expressing your answer in surds (i.e., using square roots).

2. Calculator Test

Calculator required, 10 questions. Example questions:

Evaluate $\cosh 2.1$; evaluate $\sinh^{-1} 7.5$; evaluate $\sin^3 x$ or $\sin x^3$ where $x = 1.9$; find 2 solutions between 0 rad and 2π rad of $x = \cos^{-1} 0.35$.

3. Functions, Vectors and Complex Numbers

Calculator required, 10 questions, multiple choice. Example questions:

If $y = f(x) = x^2 + 2$, the value of $f(x + 2)$ is …; find the argument of $z = 3 - 2j$ (answer must be in radians); find z^3 when $z = 3e^{-j\pi/5}$; determine the magnitude of $\bar{A} = 3\bar{\imath} - 2\bar{\jmath} - 2\bar{k}$.

4. Differentiation and Matrices

Calculator available, 9 questions, multiple choice: Example questions:

Differentiate $3t + 2f$ with respect to f, t is a constant; differentiate $\ln x$ with respect to x; evaluate the determinant of a 2-by-2 matrix; multiply together two 2-by-2 matrices.

5. More Differentiation and Integration

Calculator available, 10 questions, multiple choice. Example questions:

Differentiate a sum of two functions; differentiate a product of two functions, differentiate using the chain rule, from values of the first and second derivatives of a curve at a point, identify a

point of inflection, maximum, zero crossing or a vertical asymptote; integrate $3t + 2f$ with respect to f, t is a constant; integrate $1/x$ with respect to x, evaluate $\int_1^4 x^2 dx$.

Main reasons for Student Failure

There are three main reasons why students fail to get 100% in the tests:

- Lack of attention to detail;

- Rushing the question paper;

- Fundamental problems or misunderstandings.

Most students who fail for the first two reasons usually complete it successfully the second time and only fail one or two of the tests. Those students who fail for the third reason are the ones who is often difficult to identify by other means and some students, despite apparently having the mathematical; qualifications, really struggle with basic mathematical concepts and operations.

Following a test, each student receives a personalised email stating either:

1. that they have passed the test;

2. that they have failed the test and identifying which questions they got wrong, or;

3. that they failed to attend the test and reminding them of the next opportunity to take the test.

The emails also remind students of the help available to them in tutorials and the Student Maths and Statistics Support available in the Student Learning Service.

The above approach enables identification of students with poor fundamental mathematical skills and at present they are encouraged to see help. At later stages of re-testing students are marked when they hand in their test and if they fail, they are given an immediate opportunity to review their answers. Those failing again are given individual tutoring in the questions they have repeatedly failed. This is usually only necessary for a few students each test. In this way all students who remain engaged will eventually pass all five of the tests. Students who do not engage and do not complete all of the test do not receive a grade for their first year mathematics course and therefore do not continue to subsequent years of their degree programme.

For each student, the number of resits required to pass all five tests (missing an opportunity to take a test is considered a failed attempt) has been evaluated and this has been compared with the outcome of the end of year exam in Engineering Mathematics, and the resit exam. The outcome is summarised in Table 2.

Number of resits required to pass 5 tests	Number of Students	Passed Exam First Time		Passed at resit	Total Passeed	
		Number	Percentage	Number	Number	Percentage
0	116	104	89.7%	10	114	98.3%
1	58	45	77.6%	11	56	96.6%
2	46	41	89.1%	1	42	91.3%
3	21	13	61.9%	4	17	81.0%
4	16	10	62.5%	3	13	81.3%
≥5 (range 5–16)	27	6	22.2%	7	13	48.1%
Total	284	219	77.1%	36	255	89.8%

Table 2: Comparison of number of resits required to pass all tests with overall success in the course

The conclusion that can be drawn from Table 2 is that support currently provided to students who are able to complete the tests with one or zero resits gives a high pass rate amongst those students. The overall pass rate tails off as the number of resits required to pass the tests increases. Where a student requires an average or one or more resit per test then it can be seen that the success rate of these students, even after the resit, is less than 50%. Fortunately, there are relatively few students (approximately 10%) who fall into this category and therefore the additional help required can be targeted towards these students.

Conclusions and Further Work

The approach of using a series of simple tests to identify weaknesses in fundamental mathematics has proved useful. The number of students failing to pass the test each time quickly identifies those students who require additional help.

Extra help for these students is currently made available through course tutorials and also through the Mathematics Support Centre of the University. At present attendance is not compulsory but consideration is now being given as to how students may be required to seek help with their maths.

Feedback from students has general been positive, particularly from later year students looking back at their year 1 experience. One comment received is typical of many:

> "I never understood, and hated, surds at school and would always avoid them in questions, but they are actually really easy … the 100% test forced me to learn how to do them as I couldn't hope to get by on other questions".

Areas for further development include investigating the use of multiple randomised paper versions of the test at one time (to avoid copying by some students), or a move to online tests with individualised questions and opportunities for immediate feedback and personalised support.

This paper presents results from the 2014–15 academic year and further research into earlier years would provide more certainty over the relationship between the number of resits required to pass the tests and success in the course. Additional research is also required to look at degree outcomes of students and how they relate to their performance in the first year Mathematics course.

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The Hamburg Online Math test MINTFIT for prospective students of STEM degree programmes

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Abstract

MINTFIT is a joint project of HafenCity University Hamburg (HCU), Hamburg University of Applied Sciences HAW), Hamburg University of Technology (TUHH) and University of Hamburg (UHH) together with the Ministry of Science, Research and Equalities (Behörde für Wissenschaft, Forschung und Gleichstellung, BWFG) to support high school students and others interested in STEM studies. The MINTFIT Math Test is a diagnostic online test which gives its participants the opportunity to check if their math skills are sufficient for a successful start into the first stages of a STEM degree programme.

Introduction

Many first-year students of STEM degree programmes have difficulties with mathematics at a university level. Mathematics is a common reason why students drop out of university in the first semester. Daily experience shows that many first-year students lack basic skills in high school mathematics and that their problems with new content is due to this fact. In order to point out this problem as soon as possible – before they start with the first semester – and to help them to address any identified shortcomings, a concept of a diagnostic online test (the MINTFIT Math Test) and online mathematics bridging courses (OMB+ and viaMINT) has been developed at and with the support of universities of Hamburg within the scope of the MINTFIT project.

The Transition from High School to University

In Germany, the subject matter taught in high school differs between the federal estates. Additionally, there is a huge variety of ways to achieve a university entrance qualification. Consequently, the level of knowledge of the students in a math course at a university varies broadly. As a lecturer, it is difficult to identify which subjects can be assumed to be known. The fact that students come with different backgrounds in mathematics, because they achieved their university entrance qualification in different ways and in different parts of the country, is one of the reasons for the problems that arise at university.

In 2014, a group of high school teachers, professional school teachers and university professors of institutions in the German federal state Baden-Württemberg published a paper which caused a stir in the community of math teaching university staff. This group, which calls themselves "cosh" (short for "Cooperation Schule-Hochschule", in English "cooperation high school university") defined a set of competencies and associated problems that should be taught at high schools in Baden-Württemberg and which has been accepted as a minimum of assumed knowledge at the start of a STEM degree programme, see Cooperation Schule-Hochschule (2014). Therefore, if a high school student or a person with another educational background decides to begin a STEM degree programme in Baden-Württemberg, he or she knows which

subjects have to be mastered - and especially where he or she has to put some effort before the first semester starts.

Shortly after the publication of this cosh catalogue, many universities in Germany decided to follow this position, originally made only for a part of Germany, and to assume the knowledge defined in the cosh catalogue for students of the first semester of their STEM degree programmes. For example, the universities of the group TU9 (a group of nine big technical universities in Germany) now accept this cosh catalogue, and each of the universities in Hamburg which offer a STEM degree programme. These are the HafenCity University Hamburg, the Hamburg University of Applied Sciences, the Hamburg University of Technology and the University of Hamburg. For the history of the cosh group and the cosh catalogue, see also Dürrschnabel and Wurth (2015). For an alternative overview compare also Schramm (2015).

The MINTFIT Math Test

MINT in German is an abbreviation of "Mathematik, Informatik, Naturwissenschaften, Technik" which translates to "mathematics, computer sciences, natural sciences, technology". It is the German equivalent of STEM.

The MINTFIT Math Test is a free diagnostic online test for high school students and those interested in STEM degree programmes. After finishing the test, the participants instantly get feedback in which subjects of high school mathematics they should still put some efforts before beginning their university studies. Furthermore, two online mathematics bridging courses – the OMB+ and viaMINT – are presented, the recommended chapters in both courses are highlighted. The OMB+ was developed with the support of the HCU, TUHH and the UHH. viaMINT is a product of the HAW.

The MINTFIT Math Test is originally in German, but will be available in English hopefully from August 2016 on. It is accessible via www.mintfit.hamburg.

The Structure of the MINTFIT Math Test

The MINTFIT Math Test consists of the two separate tests Basic Skills I and Basic Skills II. Basic Skills I includes questions dealing with fractions and exponents and is based loosely on the subject matters taught in junior high school. Basic Skills II tests the skills in more advanced areas such as differential and integral calculus. It is based loosely on the subject matters of senior high school. Together, the two tests cover the subject matters defined by the cosh group in the cosh catalogue. Each of the separate tests should be finished in 45 minutes. This is just a recommendation, because there is no time limit. The test result and the suggestions which skill areas to review do not take the time taken into account.

Basic Skills I consists of 22 questions, Basic Skills II of 14 questions. For each area such as fractions or differential calculus, two questions are randomly drawn out of a pool of questions. The test runs on the free and open source software course and learning management system Moodle. Many questions are written using the Moodle Plug-in STACK. This Plug-in allows random generation of a huge variety of versions of one question within structured templates.

All questions were developed so that they can and should be solved by doing the calculations on paper and sometimes also by mental arithmetic. It is pointed out in the information that neither a calculator nor a formulary should be used. Since the test is designed to be taken at

home, it relies on the honesty of the participants. Participants can choose the order and the times when they want to complete the tests. Before starting with the first of the two separate tests, they have to complete a short preparatory section consisting of four tasks. These serve to show how to enter mathematical expressions. The tasks in the tests themselves are designed such that the input is as simple as possible. During the entire tests, a symbol key is available on the edge of the page displaying how to input mathematical expressions.

In both tests, there are different types of questions. There are arithmetic problems, where the capacity to do calculations is tested. There are also multiple choice and true or false questions, and as well questions where rules or laws such as the laws of logarithm need to be entered.

The Feedback for the Participants

The results of a test are displayed immediately after participants complete this test. At the top of the results page, a medal is shown which expresses a feedback in a visual way. The medal is available in gold, silver, bronze and blue. It is a face either smiling or with a neutral expression, depending on how good the result was. This medal is called "Plietschi", "plietsch" meaning "clever" in Low German language which is typically spoken in Northern Germany. A text describes the result and suggests how much effort participants should put in their studies of mathematical basics. The score in the form of the percentage of maximum available points is not displayed, because it had a discouraging effect on the participants at an early stage of the development of the test.

For each question, there is a standard solution shown as well as the solution that the participant gave. Additionally, if it is mathematically and technically possible, there is a specific feedback for wrong solutions indicating which mistakes were made. With the help of decision trees, the system gives these specific feedbacks as well as partial points for partially correct answers or consequential errors.

On the subpage "Persönliche Übersicht" ("Personal Overview"), participants can see their test results broken down to the specific areas. For each area, the percentage of achieved points is displayed as well as a visual feedback in form of zero to four golden stars. If participants chose to repeat the test, for each area the best result of all attempts is shown. On this page, there are also listed the corresponding chapters of the two online mathematics bridging courses OMB+ and viaMINT. With a click on the logo of one of these courses, participants can (after accepting with another click) create an account on the chosen learning platform. The results of the test are then sent to the chosen platform, and the recommended chapters are highlighted on the learning platform. Participants can work either on one of the platforms, or on both at the same time.

For the future, it is planned to implement the other way of information transport – if a participant choses one platform and passes the final exam of the recommended chapter, the learning progress will be shown as well at the MINTFIT test page in the Personal Overview.

The Learning platforms OMB+ and viaMINT

The Online-Mathematik-Brückenkurs+, OMB+ (Online Mathematics Bridging Course OMB+), is a joint project of 13 German universities and the company integral learning GmbH. It has been developed in the years 2013/2014 and has been accessible online since November 2014. Authors of the HafenCity University Hamburg, the Hamburg University of Technology

and the University of Hamburg were involved in the development. The OMB+ offers its participants the opportunity to repeat and complement high school mathematics. It addresses those interested in STEM degree programmes and covers the subject matters defined in the cosh catalogue. The approach is text oriented, but there are many questions and interactive elements. Many examples, with standard solutions which can be uncovered step by step, are presented as well as a huge amount of training questions. 31 German universities, the Deutsche Physikalische Gesellschaft DPG (German Physical Society) and the NRW StudiFinder use and recommend the OMB+. The OMB+, originally only in German, has been available in English since March 2016. Further chapters, covering topics such as. stochastics and supplementing educational videos, are currently in production. See www.ombplus.de.

viaMINT is an online learning platform for bridging courses developed by Hamburg University of Applied Sciences (HAW) and funded by the Bundesministerium für Bildung und Forschung BMBF (Federal Ministry of Education and Research). In viaMINT, first-year students can find different bridging courses on one common learning platform. The mathematics bridging course has been available for several semesters. The physics course is still being developed. viaMINT is a video oriented approach with supplemental exercises. It includes numerous examples, animations and interactive applets that serve as visualization. Supplementary material such as the formula sheet are included to support sustained learning. Students using viaMINT work in a personalized learning environment, the "Persönlicher Online-Schreibtisch" (Personal Online Desk). The Personal Online Desk supports organized study by visually indicating the study recommendations on the basis of an entrance test as well as the corresponding learning progress. Fitting the specific needs of each individual student, viaMINT offers different learning opportunities e.g. a "Detailed Learning Track" and a "Short Learning Track". As further supplement, custom-fit courses with on-site attendance are held at the Hamburg University of Applied Sciences. viaMINT is available in German. An English translation is in progress. A distinction for different degree programs is scheduled. A more detailed description of viaMINT is available in Landenfeld et al. (2014) and Landenfeld et al. (2016). See viamint.haw-hamburg.de.

Additional Use of the MINTFIT Math Test

At the HafenCity University the MINTFIT Math Test and the bridge courses OMB+ and viaMINT are recommended for a parallel use in the first year courses in engineering mathematics. It is mandatory to pass the classroom online test, but students can repeat it as often as necessary. The actual lecture can so be concentrated on university mathematics. First results show a strong correlation between passing the test and success in the final examination. Additionally, in Hamburg there are special rules for persons without a German "Abitur", but three years of vocational experience. They can apply for an examination to be accepted as a student for a particular programme. Following the idea that the most important obstacle for a successful STEM study programme is mathematics and that the minimum competencies are defined in the cosh catalogue, we use a variant of the MINTFIT Math Test as the major part of the examination. The candidates can prepare themselves using the mintfit.hamburg portal and get an immediate result.

A Look into the Future

From summer 2016 there will be an on-site attendance bridging course "Math Camp" for all those interested in a STEM degree programme at one of the universities in Hamburg. It will be separated in two parts, one covering the matters of Basic Skills I, the other part covering Basic

Skills II. Each part will be taught at two levels. After taking the MINTFIT Math Test, participants will get a recommendation which course to attend and at which level. After these courses, there will be another offer: the "Free Practice", a course over half a year, starting weeks before courses of the first semester begin and ending when courses at the universities end. In these courses, students or even high school students will get the opportunity to work with tutors on their basic skills using exercises from the OMB+ and viaMINT.

The MINTFIT Math Test will be continuously complemented with new questions. A similar concept of online test and online course for physics is scheduled.

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Designing and implementing interactive modules of learning mathematics for engineering students

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Abstract

The teaching of mathematics has to be adapted to the new kind of engineering students in order to guide them in their own learning process. Today, in higher education, it is common to use support tools like e-learning to increase the interaction between teacher and students, allowing more flexible learning. In order to develop online interactive modules of learning mathematics that can motivate students in their learning process and increasing the levels of success e-MAIO (Interactive Learning Modules Online) was created.

e-MAIO is a set of interactive online modules for learning mathematics where students can develop autonomous and collaborative learning and where they can use the computer to build knowledge, allowing responsibility in the individual learning process. Its design was made with a single objective: to capture the various learning styles and respect the form of learning of each student, and to do it we used a variety of activities and materials. Students can access e-MAIO during classes, at home or anywhere at any time. The activities presented are simple and concise with several examples solved so that students become familiar with this type of environment. This learning modules, supported by Moodle platform, have been used in a b-learning system for Calculus I course in Electrotechnical and Electromechanical engineering in the Coimbra Institute of Engineering. This project was motivated by the desire to implement some tools for innovative and attractive learning, but also because we believe that its use leads students to act with more responsibility in their learning process.

Introduction

The project e-MAIO (Interactive Learning Modules Online) of the Coimbra Institute of Engineering (ISEC) was created to develop and implement new ways and solutions to teach and learn mathematics in higher education (Figure 1). This platform explores new educational techniques as well as motivating and encouraging students through the use of different types of interactive resource adapted to the differences of each student. It is common knowledge that today's students feel a lack of motivation and poor preparation in mathematics, which makes their integration in an engineering course difficult. Using this platform, we intend to support students in their learning, expecting a decrease on their lack of interest and an increase on success in the mathematics discipline.

This paper demonstrates how important is the project e-MAIO in teaching and learning of engineering students. The aim of this paper is to evaluate the learning process in e-MAIO, because it allows us to verify relevant issues, check the results achieved and identify any necessary changes.

Figure 1. Interactive Learning Modules Online (e-MAIO).

Course Methodology

The course Calculus I has been developed on the platform e-MAIO over the past two years. This is a six-month course available to all students of Electrical and Electromechanical Engineering in ISEC, to support the classes. It is intended to promote and develop self-study, cooperation between students and teachers and motivate students in the learning process.

The online course is designed to be different from a traditional course. So it becomes a challenge to find the pedagogical model that is best suited to develop an online course (Anderson & Dron, 2011). A model that meets the needs of students, not limited to the simple availability of content on the online platform, but looking for solutions and strategies capable of enhancing increasingly innovative technologies, but above all, to design environments that enable development autonomous and collaborative learning (Anderson, 2008). As recommended in the literature, the activities and the materials used in the platform must be diverse, thus seeking to correspond to the different learning styles of students (González et al., 2013). We take in consideration the principles of instructional design in order to create a virtual learning environment according to the pedagogical models during the preparation of the course (Caridade & Faulhaber, 2013). It is also important to provide the contents in different methodological sequences to yield the best results for the greatest number of students.

The course available in e-MAIO is organised into four topics. In each topic, there are a number of lessons according to the themes of content and self-assessment tests to evaluate training and testing of knowledge obtained. It also has tables and study guides that are always available for consultation throughout the course; these are very useful as support materials. In each lesson, students may evolve in the study at their own pace. Students study the syllabus followed by examples and interactive applications, watch the videos or do the exercises and applications, as often as they want. At the end of each lesson there are self-assessment tests that allow students to have access to their knowledge (Figure 2). These tests evaluate the skills of the student and identify their mistakes, contributing to the clear perception of the level of student's knowledge on the subject. The tests are performed in a pre-defined period of time; they have the question's value and can run an unlimited number of times. When the student does not obtain the exact solution of the exercise and misses the answer, the platform shows all the steps to follow for the correct resolution of the exercise.

At the end of each module there is a final test that will be included in the student's evaluation (Figure 2). The final test is performed online during a period of time and pre-defined duration. This test may be performed only once and has exercises randomly chosen from a database of

questions. Each test question is displayed on a page with the corresponding value and the time remaining to finish the test. The student can solve the questions in any order and can move to any questions by using the test navigation bar.

Figure 2. The three lessons of topic II (exponential and logarithmic function); the self-assessment test and the final test.

The development of a new type of training requires monitoring and evaluation so that the consolidation of practice occurs in a reasoned and consistent form (Gomes, 2009). In this sense, this initiative was followed and monitored since its inception. The continuous assessment of the course itself allowed the decisions to be taken to be adjusted and changed.

Study description

During the 2013-14 school year, 68 students of Electrotechnical and Electromechanical Engineering from ISEC accessed the e-MAIO, followed the lessons, solved the proposed exercises, trained with the self-assessment tests and took four tests online which were included in their evaluation.

To assess the effectiveness of the platform e-MAIO as a b-Learning system and students' interest in its use, we made an online questionnaire. This questionnaire was structured into two parts (characterisation and skills of the students, design, benefits and access to e-MAIO), based on information obtained in the pre-survey, with 26 volunteer students from the previous year. The results were analysed by using SPSS version 21.

Presentation of results

- Characterisation and skills of the students

Most of the students are male, with an average age of 24 years, one third of students are working students, 66% are residents from outside Coimbra city. From the 68 students, 20 (29%) are Electromechanical Engineering and 48 (71%) are Electrical Engineering students.

Of the 68 students, 16 (24%) prefer a classroom learning style, 1 (2%) an e-learning style and 51 (75%) a b-Learning style, verifying that the e-Learning style is not significant (Figure 3). Analysing the preferential learning styles vs student age, the youngest group (under 28 years)

preferred b-Learning style (in 57 students, 54 prefer b-Learning) as opposed to the older ones who prefer the classroom style (11 students, 1 prefers b-learning).
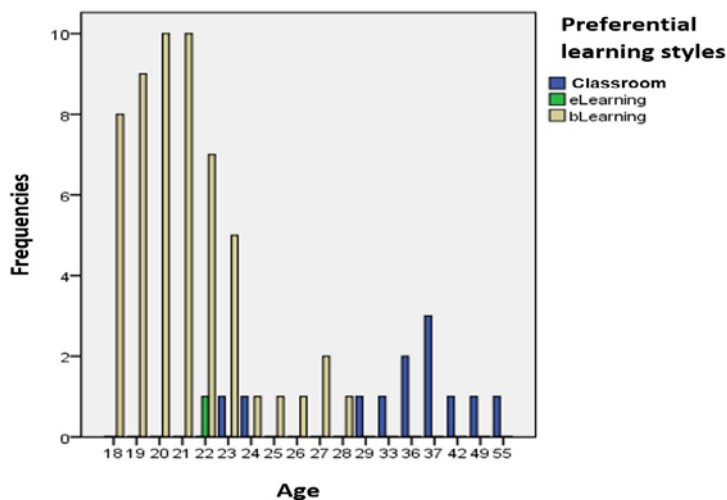

Figure 3. Preferential learning styles versus student age.

Regarding the computer skills of students, 2% are poor, 37% reasonable, 48% good and 13% very good (98% of students have some computer). Regarding the use of e-mail and internet, 55 students (81%) use email and 66 (96%) use internet often or always, only 11 students answered rarely use e-mail and 3 use the internet.

- Design, benefits and access to e-MAIO

All students agree or strongly agree that the activities of e-MAIO are relevant for learning, and students who prefer a classroom learning style have similar responses to those who prefer the b-Learning. The student's opinion is also important concerning the proposed activities on e-MAIO and if the materials available are sufficient. The questionnaire analyses the students who agreed that the activities available in the e-MAIO are relevant and useful in their learning. With regard to the lessons, 62 students agree (91%), 18 strongly agree that are these are useful in clarifying knowledge. Most students agree (35, 52%) and 30 strongly agree (44%) that the exercises solved in e-MAIO are useful in consolidating the course content. With regard to self-assessment tests proposed during the lessons, only 3 students have no opinion, 37 agree and 28 strongly agree that these tests are important for evaluating the acquired knowledge about a particular subject. According to the opinion of the students, the connection established between the e-MAIO and classroom is made clearly (84% agree and 16% have no opinion) and that teachers are always available to support the learning process (87% agree and 13% have no opinion).

The questionnaire is intended to analyse the benefits that students can have when using e-MAIO. According to this response, 93% of students accessed more than 2 to 3 times per week (16% over 5 times per week) and 7% only once per week. The access to the platform is made via the computer for 66 students (97%) and via the tablet for only 2 students (3%). It should be noted that students already have five contact hours per week of Calculus I, therefore, the use of the platform is for a pointer to the success of the course. Regarding the interest in using this type of learning, students are satisfied with the e-MAIO (average between strongly agree and agree) and they expressed interest in using this learning style on other courses.

Student's accessed e-MAIO from ISEC (4%), house (90%) or other place (6%). 91% of the students (39.7% totally agree and 51.5% agree) agree that the access to e-MAIO is fast and the remaining 9% neither agree nor disagree. In the opinion of students, obtained through an open response, the advantages of e-MAIO are essentially in the form of access and presentation of content available there. In their opinion 56 (82%) were in favour of developing other courses on e-MAIO (48.5% strongly agree and 33.8% agree) and 12 (18%) neither agreed nor disagreed. This reflects the benefit that this type of platform offers.

Conclusions

The use of a b-Learning system in e-MAIO arose from the interest in promoting the mathematics learning in a motivating way for students of engineering. Thus, it aims to improve the teaching and learning of mathematics practices for engineering students in ISEC and eventually extending to other courses (Lopéz-Pérez, Pérez-López & Rodrígues-Ariza, 2011). to obtain the students' opinion about the learning environment and its way of evaluating. The study allows the description of the profile of the student who uses e-MAIO and his satisfaction in learning mathematics content from the use of this platform. As in other studies (Martinho & Jorge, 2012, Lima, Cabral, & Pedro, 2014), the relative flexibility of time and place of access to e-MAIO was one of the aspects most valued by the students. It is clear that students recognise as a positive aspect the access to a large and varied number of support materials and notes. The contents, structure and course design are identified as important contributions to the satisfaction of students by promoting improvements in their studies and enabling online learning. The results indicate that students felt motivated in using b-learning and are receptive to the introduction of this learning style to other courses.

In the next school year, we pretend to develop other activities such as video lessons, allowing a different way of learning and acquiring new skills. It is also intended to apply the evaluation model described by Machado & Gomes (Machado & Gomes, 2013) using the dimensions: subject (students and teachers); structure (pedagogical and organisational) and technology (supporting technologies, infrastructure and support services), to identify different factors in the evaluation. It is also suggested as future work a satisfaction questionnaire to students and teachers (Rodrigues & Monteiro, 2013) as a way to quantify the satisfaction of all involved in the process of teaching and learning, identify problems and carry out the necessary modifications to the proper functioning of e-MAIO.

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Preparing students in second level education system for Engineering Mathematics

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Abstract

For many years now we have known about the significant deficiencies in the basic mathematical skills of many engineering undergraduates. Mathematics diagnostic tests in the UK, Ireland and Portugal have shown these shortcomings.

Many third level institutes throughout Europe have introduced high threshold tests in basic maths in the early years of third level Engineering. In this paper we discuss a joint project between several third level institutions in Dublin and the Further Education sector to introduce a new 1 year Mathematics course aimed at students who have left school without a higher Level mathematics qualification, but who wish to start a STEM discipline at college and at honours degree level. Currently, most colleges in Ireland will only accept a passing grade in Higher level honours mathematics as an entry requirement for a STEM discipline. This 1 year module will focus on mathematics for STEM disciplines in particular and should form a viable alternative to the 2 year Leaving Certificate qualification for acceptance onto a STEM course for non-traditional students..

This paper will focus on an automated testing component of this module which has been designed in collaboration between third level and Further Education colleges. We discuss the particular implications of this work for Engineering Mathematics at third level.

Introduction

Over the last 20 years students who enter third level engineering programmes have struggled with core mathematical skills (e.g. converting units, indices, dealing with fractions and basic algebra). We have seen this in the results of diagnostic tests carried out in many Higher Educational establishments, both in Ireland (Carr, Bowe, and Ní Fhloinn 2010; Cleary 2007; Faulkner, Hannigan and Gill 2010; Marjoram, O'Sullivan, Robinson and Taylor 2004) the United Kingdom (Lawson, 2003) and in Portugal (Carr et al. 2015) to name a few countries. This marked increase in poor mathematical competency has been due to a mixture of effects.

In an effort to tackle this problem in higher education high threshold "technique-mastering courses" have been used extensively but mainly with first year students in third level (Croft et al, 2001, du Preez, 2004, Marjoram et al, 2008, 2013, Carr et al 2013a). Students are given multiple attempts to achieve a mark of 80 or 90%. In this project we discuss a collaboration whereby we attempt to introduce such an approach into the teaching of mathematics in the Irish further education sector

Overview of Further Education in Ireland

In Ireland any education that occurs after second level but that does not form part of the third level system is known as Further Education. This sector is very diverse encompassing Post Leaving Certificate (PLC) courses, Vocational Training Opportunities Schemes (VTOS) (second chance education for the unemployed), programmes for early school leavers, adult literacy and basic education. There are approximately 250,000 students in this sector including 34,000 studying on PLCs (Irish Times, 2014). A fully comprehensive overview of the Irish Education system can be found at
 https://www.education.ie/en/Publications/Education-Reports/A-Brief-Description-of-the-Irish-Education-System.pdf   About 6000 students who completed PLCs proceeded to third level in 2014 with a target of 9,000 to proceed in 2016 (Irish times, Sept. 2014).

Most stakeholders view Further Education systems as having two main objectives, to prepare people for the labour market and also to increase social inclusion levels (ERSI, 2014 p. 33). However there does appear to be diverse views among stakeholders on which the two objectives should take priority, with some feeling that its primary role 'is ultimately to provide the economy with the skills and expertise…. that is needed' while others argue that the Further Education sector role is in 'developing the human person' (ERSI, 2014, p.34-35).

As with the sector itself, the possible pathways of progression from Further Education is very diverse. There are several issues with gaining accurate progression statistics, for instance people can be classed as both employed and in third level education or there is often no progression data available (ERSI, 2014, p. 94). However it does appear that main progression routes for students on Further Education programmes are (not given in order) progressing to employment, progressing to third level, progressing to another Further Education programme and leaving to become unemployed.

Many prospective students who graduate from further education are ineligible for enrolment on STEM programmes because they don't have a mathematics qualification that is accepted for access. For example, a HC3 (or higher) in the leaving certificate mathematics paper is a typical requirement for engineering programmes

There are good indications of a demand for another kind of alternative route to meeting the mathematics entry requirements for access to STEM programmes in HE. Effectively, if a student does not get a C3 or above in Higher Level Mathematics while at school there are few other entry paths (if any) to an honours degree at an Irish university in a STEM discipline.

Historically many of these students would struggle to cope with the demands of STEM courses at honours degree level. In an effort to plug this gap Quality and Qualification Ireland (www.QQI.ie) formed a group of experts to develop a new standard entitled 'Mathematics for STEM'.

Project maths

Internationally there has been a trend towards more problem-centered mathematics instruction (Conway & Sloane, 2005). Significant changes have been made to the second level mathematics curriculum with the introduction of 'Project Maths'. This new curriculum places greater emphasis on student understanding of mathematical concepts, enabling students to relate mathematics to everyday scenarios with increased use of contexts and applications. The goals of project maths are "strikingly similar to the goals of the reform movement led by the National Council of Teachers(NCTM) in the US" (Lubienski, 2011). These reforms in the US are described in detail in NCTM(2000).

Overview of STEM level 8

In an effort to better prepare students who enter STEM courses at third level from the further education sector and to realign the teaching of maths in further education in conjunction with the changes at second level an expert group was formed for the design of a special purpose awards. The module consisted of 6 sections: 1. Numbers 2. Sets Theory and Logic 3. Algebra 4. Functions and Calculus 5. Geometry & Trigonometry and 6. Probability. As part of the assessment it was decided to include a high threshold multiple-choice/numerical input section worth 20% with a pass threshold of 80%. This test will consist of 14 questions across the following learning outcomes (Maths for STEM).

Numbers

- Master the operations of addition, multiplication, subtraction and division in the N, Z, Q, R, domains. Represent these numbers on a number line. Understand absolute value as a measure of distance on the number line.

- Be able to make basic calculations without any errors, with and without the use of a calculator. Verify the accuracy of these calculations using estimates and approximations.

- Convert fractions to percentages, and numbers to scientific notation and calculate percentage error.

- Solve practical problems by choosing the correct formula(e) to calculate the area and perimeter of a square, rectangle, triangle, and circle, giving the answer in the correct form and using the correct units.

- Solve practical problems by choosing the correct formula(e), to calculate the volume/capacity and surface area of a cube, cylinder, cone, and sphere, giving the answer in the correct form and using the correct terminology

- Solve problems using the rules for indices and the rules for logarithms.

- Demonstrate a fundamental understanding of binary numbers. Represent a number as a binary number. Perform binary addition. Convert from binary to base 10 and base 10 to binary.

- Understand the concept of a complex number and illustrate their representation on an Argand diagram, be able to add, subtract and multiply complex numbers and calculate and interpret the modulus of a complex number.

Algebra

- Distinguish between an expression and an equation.

- Evaluate, expand and simplify algebraic expressions.

- Transpose formulae and perform arithmetic operations on polynomials and rational algebraic expressions.

- Multiply linear expressions to produce quadratics and cubics.

- Reduce quadratic expressions to products of linear expressions through the use of inspection to determine the factors. Use this to solve quadratic equations.

- Solve linear inequalities.

Operation of assessment tool

The assessment is operated on Moodle. Moodle is free, is already in use in much of the further education sector and is usually compatible with existing IT structure. The short questions will be a combination of MCQs and short answer question. Each question category has a set number of questions and each student is randomly assigned a different question. Normally the assessment will last 1 hour but the individual centres will have flexibility around this. Grades and feedback are generated automatically. Where a student gives an incorrect answer they are directed to online learning resources such as Khan Academy. The pass mark for the test will be 80%, and repeat tests are relatively easy to schedule.

Populating the Central Bank of questions

The unique feature of this project is that lectures in the Higher Education sector will collaborate with teachers in the further education sector to populate the question bank. Documentation has already been produced on all aspects of question creation and quiz creation and management. Training sessions for teachers at FETAC level with their third level colleagues is to follow.

This unique collaboration will run from September 2015 and will set a template for further collaboration between Higher Education Institutes, further education and second level teachers in Ireland

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Assessment of asylum seekers' and immigrants' mathematical competence

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Abstract

There has been an influx of Near East asylum seekers to Finland during the latter part of the year 2015. A substantial number of them are highly educated. Helsinki Metropolia University has started services for asylum seekers and immigrants that aim to assess their prior learning and competence. They will also receive guidance and counselling about suitable higher education possibilities or career paths. The first pilot group consists of asylum seekers who have studied engineering or related topics. The assessment includes a mathematical test on basic engineering mathematics. The pilot group's test accomplishment is analysed and the improvement of the test process is discussed.

Background

The immigrant population in Finland has risen from 1 per cent to almost 6 per cent from 1990 to 2014. There were around 320 000 immigrants (foreign born people) in Finland in 2014. Asylum seekers have been a small minority of immigrants until 2014, when the count of them was 3651. The year 2015 saw a sharp rise in asylum seekers in Finland, as in most of Europe. In 2015, 32 476 asylum seekers arrived in Finland. Most asylum seekers in Finland in 2015 came from Iraq (63%), Afghanistan (16%), and Somalia (6%).

SIMHE

Helsinki Metropolia University of Applied Sciences has launched the Supporting Immigrants in Higher Education project (SIMHE) to start services for recognizing prior learning and competence of highly educated immigrants and for their guidance. The project is funded by the Ministry of Education and Culture. Metropolia is developing the new model together with the UniPID network that is coordinated by the University of Jyväskylä.

The aim of the project is to streamline the recognition of prior learning (RPL) of highly educated immigrants who reside in Finland with various statuses, and to guide them to higher education both on a national and regional level. The project seeks to establish a model that can be used by other higher education institutions in Finland joining the activities.

The need for recognising prior learning is acute particularly in the Helsinki Metropolitan area, where there are large numbers of immigrants and asylum seekers and the best employment opportunities. SIMHE-Metropolia concentrates on guiding immigrants and asylum seekers who have a degree or previous studies in higher education.

Services for highly educated immigrants will include recognition of prior learning through different methods, at first through a series of tests. Enrolment to the process will be through an e-form at Metropolia website. There will be "Guidance generalia" afternoons for immigrants and personal guidance and counselling by appointment.

Recognition of Prior Learning and Competence in Technology

The "SIMHE" operations will initially be mostly linked to the studies of technology. Currently the majority of asylum seekers are young men. A substantial portion of them possess training and work experience in the field of technology and they have good potential for continuing their higher education studies or finding work once their competence has been recognised. Metropolia's engineering training is the largest and most international in Finland which gives the UAS the resources to assess foreign engineering degrees and to arrange demonstration examinations for recognising competence. Recognition of Prior Learning, Competence and Experience testing days will be introduced to all fields of study at Metropolia from 8/2016 onwards.

The competence assessment will first focus on mathematical competence, digital fluency and skills in a field of engineering. The digital fluency test is a multiple choice test of basic concepts and methods in information technology. The test in engineering consists of analysing a case in some engineering field, a reflective task, and description of skills and competence acquired in the field. The cases are from ICT, civil engineering, architecture, electronics, mechanical engineering, process technology, electrical power engineering, building engineering plus an open case. We hope that competence assessment will help to determine the needs for subsequent supplementary training before entering further engineering studies.

The mathematical competence is assessed using a test comprising of 30 mathematics problems. The pilot test was a traditional written test, but subsequent tests could be delivered on computer using the STACK software. The problems represent difficulty levels from secondary schools to basic engineering studies. Problem topics are arithmetics, algebra, polynomial equations, other equations, functions, powers and logarithms, derivative, integral, and other (vectors and probability). The pilot test was offered in Finnish, English and Arabic. The Arabic version had the explanatory texts in Arabic and the mathematical expressions in normal Latin symbols and Western numbers.

There was an introductory day (8.4.2016) and the actual assessment took place on 20.4.2016 at Metropolia. The whole three-part competence assessment took 4.5 hours. There will be a feedback day 16.5.2016. The pilot group consisted of asylum seekers who estimate themselves that they have a good potential for engineering studies. A substantial proportion of asylum seekers state that they have a university degree, from bachelor to doctorate. Only a few participants of the pilot group have diplomas of their studies. English competence is variable.

Results and analysis of the mathematical competence test

The pilot group consisted of 31 asylum seekers, 29 men and 2 women. 29 participants of the pilot group speak Arabic as the mother tongue. The countries of origin were Iraq (26), Syria (2), Afghanistan (1), Belarus (1), and unknown (1). The age distribution was from 19 to 45 years, median at 30 years.
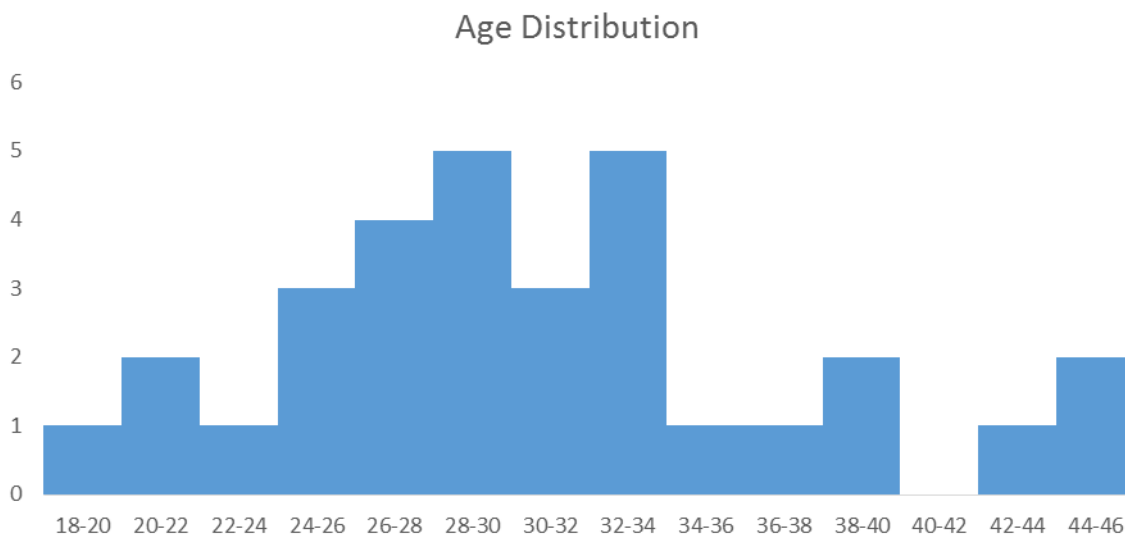
Figure 1 Pilot group age distribution

27 participants of the pilot group took the mathematical test in Arabic, 3 in English, and one in Finnish. The answers had to be written using Latin symbols and Western numbers. Only final answers were evaluated.

The maximum score for 30 problems was 180 points. The distribution of the results was from 0 to 92 points (from 0% to 51%). The average score was 22.6 points (12.6%) and the median was at 16.0 points (8.9%). There was practically no correlation between the age and the test score ($r = 0.08$).
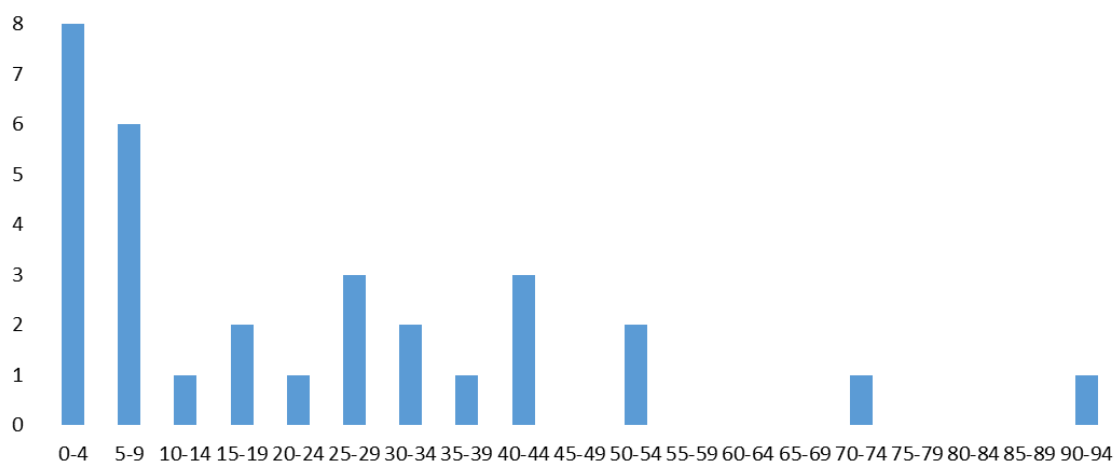

Figure 2 Mathematical test score distribution

The mathematical test scores were generally very low and only a few would be eligible for engineering studies without further preparatory training. There were several reasons why the test results were probably lower than the actual competence would yield. Many asylum seekers had completed or given up their studies several years ago. They had experienced stress during the months before and after arriving in Finland. There were also a few unexpected cultural differences in understanding the problems, e.g. a confusion between the multiplication symbol (5·3) and the decimal symbol (5.3).

Arithmetics and basic equations were the topics that were mastered best, but even their average scores were only around 30% of the maximum.
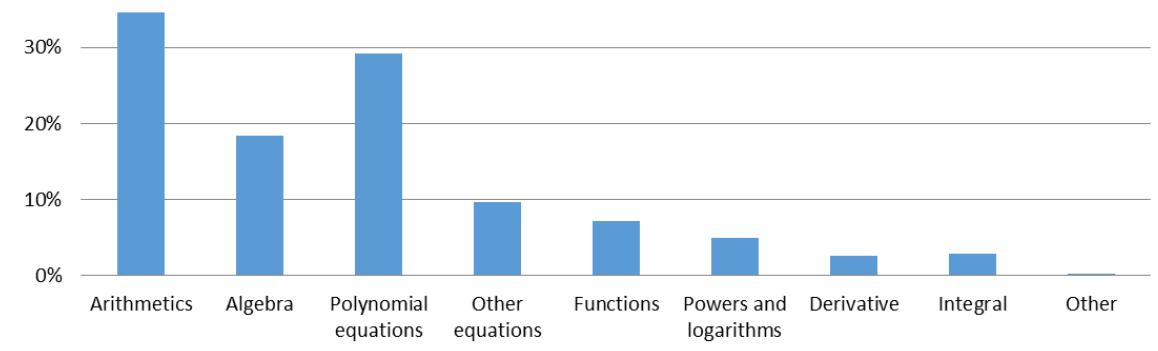

Figure 3 Relative scores for mathematical test topics

Possibilities to improve the mathematical test process

A fact is that basic mathematical skills, if not regularly used, are often gradually weakened after the end of studies. This fact was also supported by the feedback given by the participants of the test. Many participants explained that it has been a long time from their studies in the university, and that they don't remember all the rules needed to solve the problems. An easy and straightforward response to this would be to just observe the present situation and state that at least now skills are weak, or they do not exist at all, whatever has been the case in the past. However, if some kind of fundamental understanding on engineering mathematics has been achieved before, it can still remain at some level but lack of experience has significantly weakened the ability to use the understanding in practice and solve the given problems in limited time. If this is the case, possibly a short review of topics would significantly improve the test results. Review or training sessions before the test were suggested by two participants in their feedback. However, it can be asked if training and teaching before the test would change the original idea of testing existing skills and knowledge.

The number of problems in the test of mathematical skills was 30. In the pilot test there were few problems within the capabilities of an average participant, and only a couple of candidates were able to answer questions on more advanced topics, like differentiation and integration. This raises a question whether a smaller number of questions would be sufficient to get the desired information. The total time allowed for the test was four and a half hours, and many students used all or almost all the time available. In the feedback one participant suggested that test should be divided into three parts which indicates that test was found quite laborious. Reducing the number of questions in the mathematical part would decrease the required time without necessarily remarkably decreasing the value of the answers. Still there should be both very basic and more advanced questions on different topics, but the number of questions in each group could be smaller.

The test was meant to measure competence in engineering. Skills in mathematics are an essential part of this, but would it be better to test ability to solve mathematical problems relating to engineering applications, instead of purely theoretical problems? Possibly yes, but

this would expand some observed problems and complexities relating to case study questions beyond the mathematical part; the most remarkable of them probably being the fact that test participants have their education and experience on various disciplines. This could be considered by providing problems in various applications but still it would remain nearly impossible to cover all areas of technology and engineering. On the other hand, understanding of very basic applications can be expected not to depend on the specialisation area, especially if formulas or theory briefings are included in the task description. However, this would make the process considerably more complicated, but maybe it would give added value to the test and its results. Some participants complained that they had studied mainly administration and economics and therefore a few problems on percentages and interests could be added.

Feedback for the participants

Test participants will be given written feedback on the feedback day 16.5.2016. They will receive a document that describes what has been assessed and which assessment methods have been applied. The feedback will state what expertise and skills have been recognised from their test accomplishments. The feedback will also give informal recommendations how the test participant could improve his/her skills if he/she wishes to pursue studies in engineering or related fields at a university of applied sciences. The document is not an official certificate and it does not give access to university studies.

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Flipped classroom in interdisciplinary course

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Abstract

In our paper we report about a course on simulation of transport processes, which we offer for Master students of process technology, life science engineering and energy technology. The learning objective of the course is that students:
- learn or repeat the basics in using simulation tools.
- learn to combine specialized knowledge with mathematical concepts to simulate a technical process.

Simulation technologies are addressed in several courses and classes during the Master studies at the Faculty of Engineering. Quite often it is possible to understand difficulties arising during a simulation run by looking at the mathematical fundamentals. Our course is a hands-on-training. Students will practice implementing abstract formulas as program code and analyze problems which arise, from an engineering and a mathematical viewpoint. We like to guide the students to explore the connection between the application subjects and the underlying mathematical questions in a quite natural way. Up to now the mathematical models of the technical processes and the repetition of the mathematical basics were presented as talks. Consequently only a little time remained for the discussion. We have seen that repetition needs more time than scheduled, e.g. to understand the way to describe the first derivative of a function on an interval by a system of linear equations. By using elements of e-Learning (e.g. flipped classroom) we try to extend the time to prepare topics, which is needed during single face-to-face-sessions. This has to be done by the attendees in a self-organized way. Furthermore we hope to end up in more discussions during the attendance phase for a better understanding.

Introduction

The idea of the course "Simulation of transport processes" was already born in 2012. Two of the authors wanted to develop a joint course, which combines engineering and mathematical aspects. Three goals have been formulated:
1. Students complete simple simulation tasks without using ready-made simulation tools.
2. The preparation of the mathematical model is part of the simulation task.
3. The realization of the simulation will act as a guide to mathematical questions, which can be explained and understood using basic knowledge which students already have from their Bachelor studies.

Up to late 2013 we were focused on the conception of the contents. A pilot had been planned as a face-to-face lecture in a computer room on two weekends. Even though we did not offer any credits, in total about 30 students have attended the two pilot courses. Ongoing from this and considering the feedback we received, we next developed a course as a module for the master studies of process technology, life science engineering and energy technology where students may earn credits. In the first part of the paper we relate our experiences and the room for improvements which we identified in the two courses run after the two pilots. Our ideas to improve the didactical concept we explain in the second part of this paper.

Key concepts of the course

In general we discussed two applications in our course:
1. Rankine cycle (static state),
2. Heat transfer (static, transient) in a bar (1d), a cross-section of a tube (2d) or a spherically-shaped shell (3d).

The task for the Rankine cycle was to determine the efficiency of the process based on given parameters like pressure upstream and downstream of the pump, the temperature before and boiler the vessel, and the assumed state changes in the pump, boiler, turbine and condenser. The aim for the heat transfer was to plot and compare the solutions for the different cases.

Altogether we planned six sessions according to the following schedule:
1. Repeat the basics of using Matlab (one session)
2. Preparation of the mathematical model to describe the Rankine cycle, implementation in Matlab (two sessions)
3. Usage of simulation tools (PDE-Toolbox, DiffpackSE) (one session)
4. Simulation of a static heat transfer process (one session)
5. Simulation of a transient heat transfer process (one session)

Already at the beginning of the Bachelor degree courses the students had taken first steps in using Matlab but these skills were not actively available. A refresher in using the software, even on a low level, is essential. Each attendee could work on a single machine and had to write program code for themselves. In this state teamwork of two or three students was encouraged to discuss the model equation itself or difficulties in implementing it. As supervisors we joined the groups or went to the people who worked alone to discuss problems and challenges. By doing so we were able to establish a very individual teaching-learning-relation. Repeating already known topics and checking them out directly at the computer combined with the opportunity to ask questions (and get hints or answers) was perceived as very fruitful by the students. At the same time we as supervisors had to be very flexible and to be able to switch instantaneously to the questions of the individual person or groups. Difficulties which arose more often were discussed within the whole group.

The model of the Rankine cycle was prepared as a handout and additionally explained as a talk. The attendees had not repeated the model equation before the session took place. The process itself was known from their studies. The Rankine cycle was described by the steps boiler, turbine, condenser, and pump. For each step a function had to be implemented, where unknown parameters (e.g. energy input) had to be computed from known ones (e.g. inflow and outflow pressure).

Example: Step Pump

Given are the data $p_{in} = 4$bar, $p_{out} = 64$bar and $T_{in} = 293,15$K. Using the isentropic compression, the outflow temperature T_{out} and the needed pump power P_{in} has to be determined. To carry out the calculation by hand, we used a simplified model: the changes in enthalpy we modeled by using constant thermal capacities.

The computation is done in the following steps:
$$s_{in} = s(H_2O_{liquid}, p_{in}, T_{in})$$
$$h_{in} = h(H_2O_{liquid}, p_{in}, T_{in})$$
$$s'_{out} = s_{in}$$

$$h'_{out} = h(H_2O_{liquid}, p_{out}, s'_{out})$$
$$h_{out} = h(\eta_{s,pump}, h_{in}, h'_{out})$$
$$w_{t,pump} = h'_{out} - h_{in}$$
$$T_{out} = T(H_2O_{liquid}, p_{out}, h_{out})$$

To implement this as a Matlab function, one needs to change one's point of view. This set of equations has to be interpreted as a vector valued function

$$\begin{pmatrix} P_{in} \\ T_{out} \end{pmatrix} = Pump(T_{in}, p_{in}, p_{out})$$

which is not given in a closed form. It turned out that this change of perspective was a real challenge for a lot of students.

The discussion about the quality of the model was launched by the students themselves. They saw the need for the material properties to be modeled more accurately. No assistance was given to solve this problem. In consequence the students have learned to find packages providing certain functionalities and to use them in their own programs.

Since the focus until now was on programming skills, we changed the focus in the second part of our course to numerical concepts. The heat transfer equation was in our opinion well suited to this purpose. Starting from a heat balance, a heat transfer model was derived:

$$\frac{d}{dr}\left(k \cdot r^{n-1} \cdot \frac{d}{dr}T(r)\right) = 0, \text{ für } r \in [r_0, r_1] \subset \mathbb{R}_0^+, \tag{1}$$

with boundary conditions

$$T(r_0) = T_0, T(r_1) = T_1,$$

and the dimension of the problem n (bar, tube cross section, spherically-shaped shell). The solution for the one-dimensional case can easily be computed by hand in analytical form. The students enjoyed the recap. Unfortunately we had underestimated the time they would need. Handling boundary conditions or actively solving some integrals could not be accomplished by all the students. As the next step we explained the idea how to describe a function in a discretized form on a subdivided interval as a vector. We ended up with the question of how to compute a function on discretized points from the derivative (known at these points) by assembling and solving a system of linear equations. However all this and, in particular, the relation between solvability of the problem and solvability of the system of equations, the students did not understand well. Therefore our learning objective in this session could not be achieved. The possibility to, need for and benefit accrued by comparing analytical and numerical results in certain cases could only be shown very generically.

We have stated in general, that we have well-chosen examples and a good presentation. The idea behind our course was well accepted but sometimes we lost contact with the learners. The students could not always access previous knowledge to make use of it in practice. It became abundantly clear that the attendees would need enough time to receive and process (new or old) information.

Rework the didactical concept

Since we held the course twice in 2014 and 2015 we saw that some didactical improvements were useful. Together with the "Innovations in Learning Institute (ILI)" we thought about possibilities available to us. A main issue was to give the participants enough time to repeat or to work on the subject-related content. Also the quality of the discussion during the attendance phase could be improved. To prepare the single face-to-face-sessions we will now create tasks which the attendees should complete in a given period of time. Guided by these tasks the students will hopefully refresh knowledge learned in previous courses. Therefore, we will utilize different features offered by the LMS ILIAS and hence all the materials will be provided on this platform. At the beginning we will schedule a questionnaire about the concept "simulation". All attendees of the course had already encountered (numerical) simulations. On this basis we hope to initiate a critical reflection on simulation in general.

The model for the Rankine cycle or particular mathematical aspects (e.g. solvability of systems of linear equations or how to solve them, how to integrate functions or find appropriate initial conditions) will be prepared as online learning modules inside ILIAS. The modules treating mathematical subjects will be completed by e-tests to give the learners feedback about their learning progress and to motivate them to close potential knowledge gaps. Even the STACK-type questions would be a good resource for us. These types of questions are now also available in ILIAS.

Let us discuss possible improvements concerning the Rankine cycle. The behavior of water and water vapor is highly nonlinear. For a better understanding of the Rankine cycle itself the material behavior of water and vapor was approximated by a simplified model. During the face-to-face session we will discuss the confidence range of such approximations and will be looking for a better solution to modeling the water behavior. In preceding courses the students discussed very briefly the disadvantages of the simplified models and ended up with a strategy to use steamtables. Now there it was only a small step to incorporate the library XSteam to realize a less sophisticated model. Therefore they will have to adopt the models for each process step (boiler, turbine, condenser, pump). Further they will have to change the already written programs to use the external library. Therefore the students have to understand usage of XSteam as well as its operation. This will be done in a combination of teamwork and a phase of discussion. To prepare the second session on the Rankine cycle the students will be encouraged to model the process steps boiler, turbine and condenser using the steamtables.

Another topic where we observed difficulties was the transition from a function $f : \mathbb{R} \to \mathbb{R}$, $x \in [a, b]$ via a discretized representation on a grid $a = x_1, \ldots, x_n = b$ to a representation as a vector. This part the students should explore with an online learning module. All questions that arise will be collected in a forum or wiki. Students and supervisors will be able to discuss questions that appear, already during the preparation phase. Driven by the example $f'(x) = g(x)$ for $x \in [a, b]$, the transition to a system of linear equations will be explained. This is a very crucial idea for the numerical simulation. Based on this example it is easy to explain how the relation of the existence of a unique solution f by providing a value $f(x_0)$ at $x_0 \in [a, b]$ and the unique solvability of the corresponding system of linear equations works. Analogously, the influence on the solution of the heat transfer equation (1) by the dimension of the considered problem can be shown in a nice way. In general we see that the affinity for mathematical subjects differs amongst the students depending on their degree programs.

We see a large tendency to trust numerical results blindly. For this reason we will give the students an idea about numerical difficulties, trouble arising from the software used and (mathematical) properties of the considered model and to distinguish.

Conclusion for education

Focused on the learning objectives we will divide our course into two alternating phases by using the flipped classroom methods: self-study phase (online) and phase of interaction and utilization (attendance phase). The potential arising from the flipped classroom is that the combination of online and traditional learning will activate the learners and will inspire a participation and interaction of peers and teachers (Behringer, 2014; Lehmann, Oeste, Janson, Söllner & Leimester, 2015). We expect positive effects on the learning outcome by using computer-assisted learning and teaching systems (Hattie, 2008). By providing teaching material e.g. as online modules we want to encourage the students to think about upcoming issues independently, refresh or acquire knowledge they need, look beyond their nose and find new relationships between seemingly unrelated material. Through all this we will strengthen cooperative learning (peer-to-peer interaction) and discussion. Through the enhanced scheduling in providing materials for repeating some issues (e.g. solvability properties or mathematical models) and the time when this is needed to solve certain tasks, we improve the communication during the attendance phase. In general interaction improves the individual learning outcome (Lehmann et al., 2015). Furthermore, we will establish a computer-assisted interaction between learner and learning environment by providing an online forum, e-assessment and online questionnaire. This will encourage better understanding, motivation and transfer by the learners (Niegemann, Domagk, Hessel, Hein, Hupfer & Zobel, 2008). The additional formative e-assessments will give the students a self-regulated monitoring of their learning success. They will get direct feedback about knowledge gaps, and therefore better learning effects may be expected (Ras, Whitelock & Kalz, 2016). Furthermore we as teachers will get feedback about the teaching/learning process and will be able to adjust our teaching methods according to the needs of the students. Hopefully, there will be a benefit arising from the change of perspective: the learners have to learn more independently (especially during the online phase) and will exit the role of passive recipient. At the same time the teachers will become pioneers on the "road to learning". We no longer just spread a lot of information around. Now we support the learners in acquiring new subjects, in talking about them and finally in using them. This "Shift from Teaching to Learning" is of crucial importance for our teaching concept.

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Enhancing Ability to Identify and Use Mathematical Concepts

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Abstract

The paper presents results of project work integration into regular teaching of "Basics of Numerical Mathematics" in bachelor studies with common conception of course, based on traditional lectures and exercises aided by CAS *Mathematica*. Project work and additional activities are discussed as possible tools for enhancing the ability of students to identify and use mathematical concepts in technical solutions.

Introduction

There is no doubt about the importance role of mathematical competencies for future engineers in their study and career. At times engineers are heard to say that they have never used the mathematics in the form it is taught at school, and this sometimes leads to opinions that there is no need for mathematical subjects anymore. However, such extreme but not rare reactions could hardly be admitted. When opening textbooks of technical subjects, problems are mostly expressed via formulas coming from standard parts of tertiary mathematics courses and undoubtedly, in some cases, specific parts are required. In relation to the latter, there are also opinions that a minimum amount of mathematics based on formulas could be taught by engineers directly within the technical subjects; after all, "nowadays everything is solved using a computer". Could such knowledge be sufficient for an engineer's career in 21st century?

With regards to time, content scope and overall importance there is an awareness that mathematics lost its previous prevailing position in the elementary and secondary school education system. Pedagogues had to accept the reality that there is not sufficient time and space for mathematical education, not even at technical universities. Students coming to study at technical universities have serious gaps in their entering knowledge and skills, and in addition, they have difficulties studying individually due to the increase in subject matter and are hardly able to create their own style of learning. Despite the fact that for many students solving specific transfer tasks is the maximum of a course they are able or willing to reach, working on higher aims of taxonomies of learning objectives, cannot be set apart. Besides various supporting forms, one of methods that can make mathematics more understandable, attractive and closer to students is project learning.

Project Learning at Bachelor Numerical Mathematics Course

Usually, in the project method, tasks for students are formulated in the form of complex, time demanding technical or other assignments. In order to affect the students' ability of identification of mathematical concepts and usage of numerical methods we turned this approach around and asked students to find a suitable technical problem for a given numerical method and to solve it.

In the first week of the course, the students were acquainted with the conditions of granting credits and the exam, which included preparation and presentation of the project. The lecturer carefully selected main mathematical project topics covering common themes of the subject:

equations in one variable, approximation, integrals, differential equations, and linear algebra problems. The task was to find an application of the determined mathematical topic in problems presented at courses that have been attended previously or at the same time - comprising mechanics, physics, mechanical engineering, thermodynamics, etc. They then had to describe the problem from both a technical and mathematical point of view, to solve the problem using the numerical methods given by the topic, and to compare and interpret the results. Each project had to be submitted in written paper form, as well as presented in front of the class. In the end, the students were asked to answer a questionnaire to analyse their opinions and attitudes.

The first time the method was applied it was as a pre-experiment with a special class of 77 students retaking the course (see details in Richtarikova, 2014). We decided on the form of group projects, where students were randomly selected into 16 groups involving ca 5 members, and the topics were randomly distributed among the working groups. It allowed us also to monitor social skills of students such as ability to work in pre-determined groups on a pre-determined topic, work allocation, division of roles, responsibility, etc. The results showed that half of the students were not able to create a satisfactory project; a great fraction (28%) of internet plagiarism; 10% of students participated at minimal rate and 13% did not manage cooperation and split into subgroups. Since cooperation with team members was the most demanding part of the project work (64%) and admittedly, almost half of the students (48%) would prefer to work on the project by themselves, we gave up the random manner of grouping of students in following performance. The most surprising finding was the meaningful detection of insufficient critical thinking and inadequate self-assessment.

The following two school years, we extended the experiment also for regular students and the project work was included into assessment for one of the course parallel groups. Each year, we made some slight modifications due to our previous findings. Students were provided with instructions on how to elaborate a project and use available consultancy time. In order to provide a maximally pleasant and encouraging atmosphere, we also did not enforce the random manner in assignment of numerical topics. We let students make groups consisting of 2-3 members on their own. For each numerical topic, we set the maximal number of groups and let groups make their choice. However, covering all topics was enabled by the assignment of the predefined maximal number of groups per topic. Additionally, preparing students for project work and initiating them to look at technical problems from a mathematical point of view, we asked them to go through materials of previously attended technical subjects to find in class as many problems as they could with at least the possibility of numerical resolution. To avoid plagiarism, students were instructed to cite all sources they had used in their work, and to focus on their own laboratory works or experience. The project work could increase or decrease the student's assessment up to 10%, which could make $\pm$ one grade.

Findings and Discussion

Overall, 61 seminar works were assessed, out of which 50 (82%) met basic requirements at least, and their assessment was positive or neutral. The assignment was not understood in three works as they were elaborated as surveys. Two of them did not contain any computational part and had to be worked out again. In 15 works (25%) students were not able to clearly formulate the goal or conclusion from a technical or mathematical point of view. Comparing shortcomings in numerical processing, the results in 2014 and 2015 were very similar. (Table 1.) Despite this, we registered a difference in the level of quality, when students in 2015 chose more adequate problems.

Year	2014	2015
Number of assessed works	36 (100%)	25 (100%)
Number of students involved in assessed works	85	54
Works that met all attributes	9 (25%)	6 (24%)
Works with minor shortcomings in numerical part	21 (58%)	14 (56%)
Works with major shortcomings in numerical part	4 (11%)	3 (12%)
Works with fundamental shortcomings in numerical part	2 (6%)	2 (8%)
Works with problems in formulation of goals and conclusion	8 (22%)	7 (28%)
Works, where the assignment was not understood (surveys)	2 (6%)	1 (4%)

Table 1. Project work assessment

The questionnaire survey was answered by 90 students in 2014 and by 59 students in 2015 (the number of students in experimental parallel at the end of semester). We were interested in opinions on how demanding, attractive and useful numerical mathematics (NM) is, and of the project work in particular.

The students were aware of the utility of numerical mathematics, 64% considered it to be very useful or useful and only 10% not useful. For 63%, the course was adequately demanding, and only for 15% it was demanding and for 5% very demanding. Not fewer than 30% of respondents regarded it to be interesting or very interesting.

To find a suitable application was the most demanding item (71%) in a project work, while numerical processing was in second place (30%). An assumed project development time of 7-10 hours was declared by 37% respondents, while a shorter time 3-6 hours was marked by 36% and a longer time 10-15h only by 15%. 32% of students enjoyed the work on project, and 27% of respondents felt intrigued.

The statement: *Looking for applications and project work helped me to better understand the role of NM and M* was strongly agreed or agreed by 52% of participating students.

Comparisons

As expected, the results and indicators in questionnaires of regular students had more positive values than pre-experiment observations taken on students repeating the course. For some of them the differences were substantial (number of satisfying projects 82% vs. 50%, awareness of numerical mathematics usefulness 64% vs. 33%, time spent on project work longer than 6 hours 56% vs. 31%, etc.). Surprisingly, we noticed as well the considerable differences between the outcomes of regular students in the first year (2014) and in the second year (2015) of the experiment. As mentioned above, the works in 2015 had a higher level of quality, students of this group exhibited higher awareness of numerical mathematics usefulness (78% vs. 55%), they were more prudent in seeking and selecting problems for their projects, they aimed for originality of their project in higher numbers (64% vs. 47%) they spent on project work more appropriate time (time less than 6 hours: 30% vs. 53%, expected time 7-10 hours: 54% vs.

26%), etc. Although they complained bitterly about the time spent on seeking applications (3-4 times during the semester 10-20 min: 50%, 20-30 min: 26%) no fewer than 64% strongly agreed or agreed with the statement that looking for applications and project work helped them to better understand the role of numerical mathematics and mathematics.
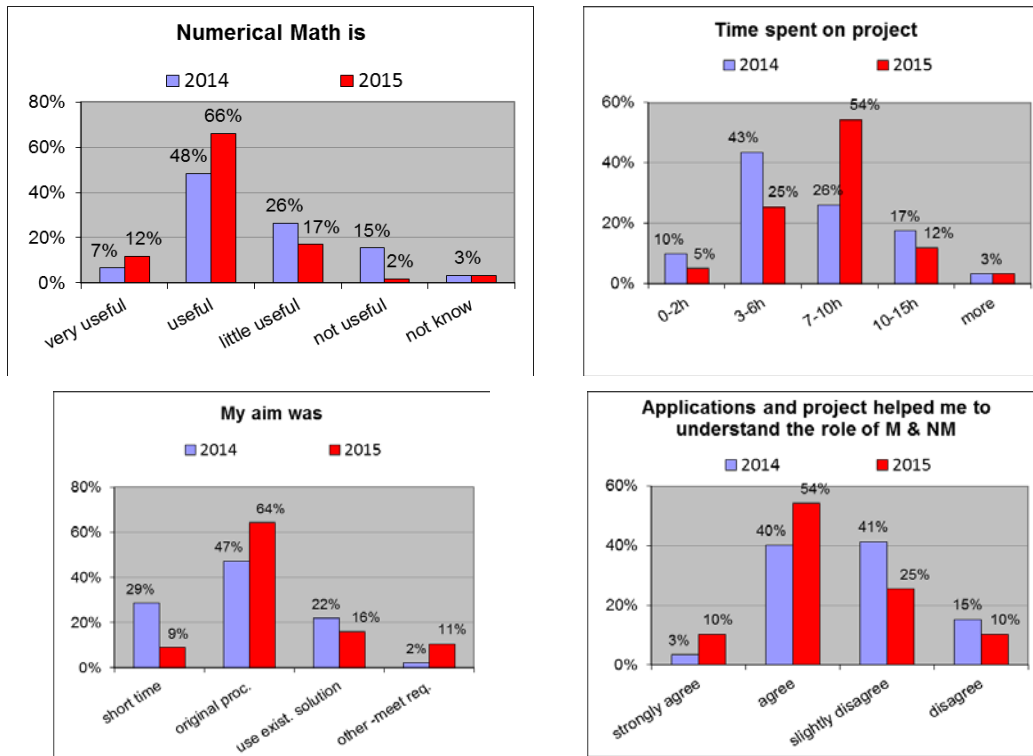
The project work was

Figure 1. Comparison of questionnaire answers in 2014 and 2015

included into assessment with an impact of $\pm 10\%$ in maximum. It could also influence the student's level of knowledge and skills; therefore we compared the exam results of the group working on projects (experimental group EG) in parallel with students without project work (control group CG). For this purpose, the standard rating of 6 classification grades (A, B, C, D, E (pass) FX (fail)) was extended by the seventh one (FN), when a student attended classes with respect to school rules but he/she did not attend the exam. Although there was no significant difference between groups EG and CG (Mann-Whitney test, $\alpha = 0.05$), in grades 2014 (Figure 1), we observe better results for EG through the difference between lower quartiles; at least 25% of students achieved grade A in EG, while in CG they achieved only B or better. In grades 2015 (Figure 2), we see a difference in the upper quartile: in EG at least 75% of students achieved grade D or better, while in CG it was FX or better.
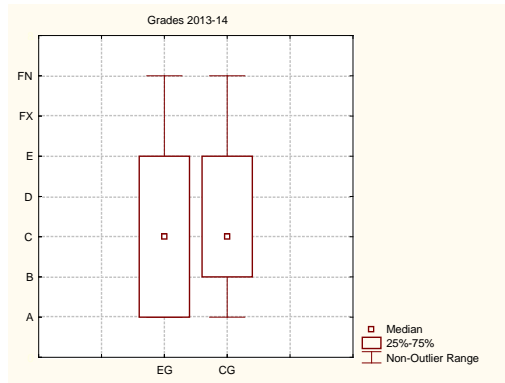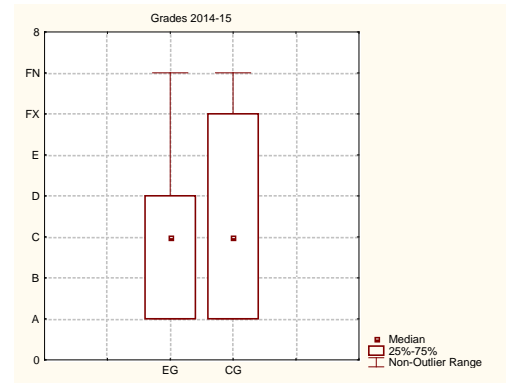
Figure 2. Box plot of grades 2014

Figure 3.   Box plot of grades 2015

Conclusions

Ability to identify and use mathematical concepts is one of the higher competences that a future engineer should reach. Development of this ability takes a long time and it is built step by step. Despite bachelor students at technical universities suffering from deficits in mathematical knowledge and skills from secondary school, we tried to find and verify the possibility of this ability development with a specific form of project work and with students searching suitable problems in previously or currently attended technical courses. Learning materials of technical courses preceding the numerical mathematics course operated with exercises that usually had an exact solution or if necessary, numerical methods were only mentioned, or most frequently, only the final numerical result was used. We hoped that students would uncover such cases, and would use them for their project work. Although, we demonstrated similar motivational examples in lectures, the students were not able to perform it. To avoid plagiarism, which was observed in abundance during the pre-experiment, students were instructed to focus on their own laboratory works or experience.

Several groups found the topic through their spare time activities and for computations they used available data published by a manufacturer or measured by themselves. Others handled examples with the exact solution from textbooks and demonstrated realization of numerical methods. There were also a few works that were inspired by model numerical examples in internet or in books and were presented with modified data. Focus on school problems led in some cases to simple tasks, where students were to solve, for instance, the integral of a polynomial using numerical methods. Nevertheless, good management and well prepared instructions with warnings could help create good background for formulating interesting project works, development of ability to identify and use mathematical concepts, development of students' critical thinking, and recognition that for engineers, mathematics is worth to study.

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MODEL OF PASSING MATHEMATICS COURSE AT THE UNIVERSITY

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Abstract

Some engineering students struggle to understand mathematics. In this article there are discussed the effects of mini-tests given in a period of each week during one semester of the course of mathematics are discussed. A model of students failing and/or passing the assignments is presented.

Introduction

The first year students of engineering at Aleksandras Stulginskis university are struggling to grasp the mathematics presented. According to the study conducted by Kaminskiene et al (2010), 54 out of 93 students passed and so 39 out of 93 failed the mathematics exam in the first semester of the first year studies.

Engineering students study mathematics for three semesters. In the first Autumn semester students study a course called "Analytic geometry and differential calculus" (8 credits; where 1 credit has associated with it about 25 – 30 academic hours). In this course the students are introduced to matrices, vectors, analytic geometry, limits and derivatives. In the second Spring semester students study a course called "Integrals and Differential Equations" (7 credits). In this course the topics deal with indefinite and definite integrals, applications of integrals and solving the first and the second order differential equations. In the third semester (Autumn) in second year students study a course called "Applied Mathematics" (4 credits), in which students are taught probability, statistics and apply these in the laboratory sessions.

In the Autumn semester of 2014, only 42 % students (34 out of 80 students) passed the "Analytic geometry and differential calculus" at the first sitting of the final exam, whilst another 27 passed the final exam at the second attempt. It was decided to have mini-tests during the Spring semester of the 2015 year, while the course "Integrals and Differential Equations" was being taught. A similar mini-tests was successful for dental students at Mahsa University, where they founda statistical difference between the 2nd and the 4th mini-tests; the 3rd, the 4th mini-tests mini-tests ($t = -3.948, DF = 31, \ p = 0.00$) and other tests out of 6 tests. Applying a similar methodology, the first year engineering students were given tests each week, followed by the 3 mini-exams and 3 homework assignments.

Methodology

In this paper 48 out of 80 male (there were no female students entering the course) engineering students were observed during the Spring semester of the 2015 year for the "Integrals and Differential Equations" course. 80 students studied this course in the Autumn semester. After the Autumn semester 32 students decided to end their studies, (a drop out rate of about 40 %).

During the course of the Spring semester, 8 tests were given: 3 mini-tests before the first mini-exam consisting of problems on indefinite integrals, 4 mini-tests before the second exam consisting of problems on definite integrals and their applications, and 1 mini-test before the third mini-exam consisting of problems on differential equations.

Practice lectures were scheduled once a week. Each week there were 3 academic hours. Students were given 5 problems to solve at the last hour of the lecture.

The aim of this article is to examine the dynamics of the grades during this one mathematics course in which the mini-tests approach was used. To do this logistic regression equation model was created.

Some assumptions were made in creating the model: a) attendance of student is not included, because previous data has shown that if a student did not attend the classes, the student failed the course; b) gender of a student was not included as a factor, because only male students took the course; c) the minimum for passing an assignment is 4.5 points (even though technically it is exactly 5 points, but depending on a difficulty and the number of credits for the course, the minimum bar is lower by 0.5 of a point); d) the formula for a practice grade is 30 % of the average of 3 mini-exams and 20 % of the average of the 3 homework assignments, where a student gets maximum points (10 points on a decimal scale), if they solve 50 % of the problems correctly; e) the variable is dichotomous – passing the course (1 - success)/failing the course (0 – failure); f) mini-tests were included in the grade of the practice course – therefore, they had an impact of passing the course as well as homework assignments.

The analysis was done using MS Excel 2013 and R programming language.

Results

Even though in theory there were 48 students out of 80 students participating, but some of these can be classified as nonresponsive students as they did not do any tests, or did not complete mini-exams). These 17 students had to be eliminated in order to do the analysis. In order to compare all three mini-tests with their corresponding mini-exams, there were eliminated. Therefore, 31 out of 48 students were used in the analysis (the response rate was 65 %, i.e, 65 % of responsive students).

Testing for equality of means (t-test for two tails) given that the variances are equal, we cannot conclude that mini-tests are effective way of passing the Mathematics course: $t_{obs}(-0.24) < t_{crit}(2.00)$ comparing mini-test 1 vs Mini-exam 1; $t_{obs}(0.49) < t_{crit}(2.00)$ comparing mini-test 2 vs Mini-exam 2; $t_{obs}(0.03) < t_{crit}(2.00)$ comparing mini-test 3 vs Mini-exam 3), which is in contrast to results reported in the articles concluding about the positive effect of mini-tests (Abdulhadi Laith M., Mohammed H. Abbas (2013)).

As the testing for equality failed to reject the hypothesis about the equality of means between the mini-test and mini-exam, it was decided to examine the dynamics of mini-tests and mini-exams. Here nonresponsive students were taken into account (meaning that the group being considered was 48 out of 80 students in total), because we are testing each mini-test vs mini-exam separately.

Since there were 11 assignments that each student had to do during the lectures, we look at them as weeks (there were a few weeks prior to the 3rd mini-exam, when there were no mini-

tests due to the fact that it was the most difficult topic – differential equations, which combines indefinite integrals and derivatives). In the Figure 1, it is shown how the number of students passing change during the course of 11 weeks. During the 4th, the 9th and the 11th weeks when mini-exams were held, the number of students that passed the mini-exams has increased compared to other weeks, when the mini-tests were held. During these weeks the number of students who did not complete the assignments decreased, whereas during the weeks with mini-tests there were an increase in students choosing not to complete these. This may be due to the fact that mini-exams have a direct impact to the practice grade whereas mini-tests only can be added to the mini-exams after the writing of the mini-exam.
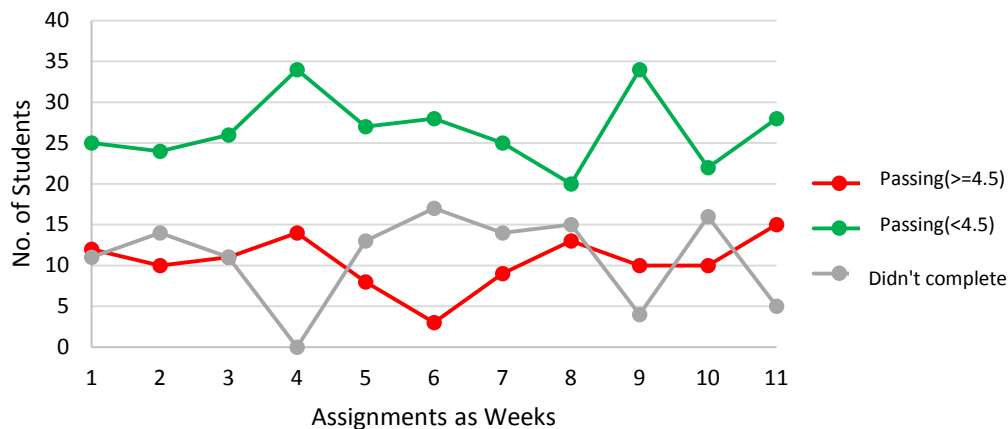

Figure 4. Number of Students passing (green line)/failing (red line) the assignments (mini-exams 1, 2, 3 take place in week 4, 9, 11, respectively, mini-tests take place on the other weeks) as mapped against time in weeks.

Since the study is concerned only with whether students passed/failed the course, a logistic regression analysis was conducted. In the Figure 2 it is shown that the more assignments students pass, the probability of passing all three mini-exams increases and the less assignments students engage doing the less likely they will pass the mini-exams. A weak correlation was found ($R^2 = 0.46625$). The explanatory variable was the number of mini-tests passed and the response variable was the binary variable of passing (1)/failing (0) of three mini-exams. The formula of logistic regression ($\chi^2(1) = 43.878, p\,(0) < 0.05$) is:

$$\ln\left(\frac{p_i}{1 - p_i}\right) = 3.26080 - 0.64712 \cdot no\_of\_mini\_tests\_passed_i$$

where p_i is the probability of the i-th student passing three mini-exams;

$no\_of\_mini\_tests\_passed_i$ is the number of mini-tests passed by the i-th student.

From the formula it can be seen that the Y-intercept is positive and the slope is negative. For each new assignment each week the odds of passing the 3 mini-exams increases by 0.52 times ($e^{-0.64712} \approx 0.52$).

If a student passes 5 mini-tests, then the probability of passing the 3 mini-exams will be approximately 0.506.
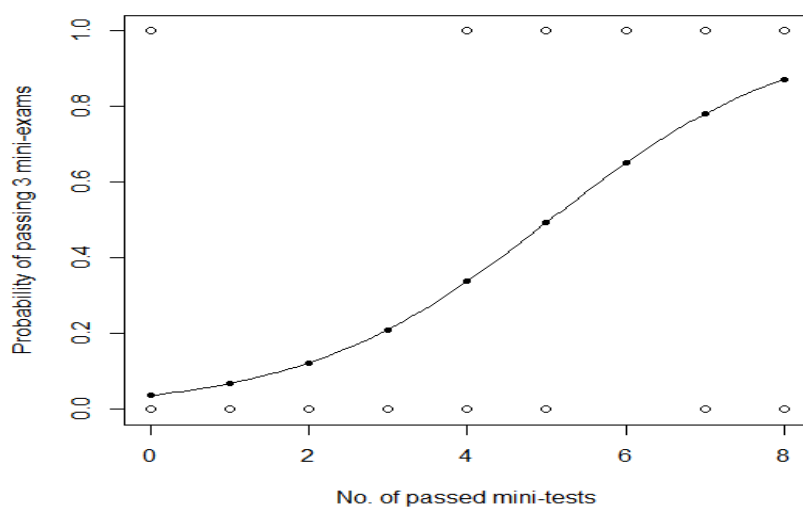
Figure 5. Probability of passing the practice part (if 3 mini-exams are passed for at least 4.5 points) of the Spring semester (empty circles show observation data).

Since the probability of passing the practice course was solely based on three mini-exams and only the number of passed mini-tests was considered of impacting the passing factor, it suggests that a student needs to pass at least 5 mini-tests in order to have a 50 % chance of passing all three mini-exams.

Conclusions

Although the hypothesis about the same means of mini-tests was not rejected, the mini-tests appear to help students to get better at mini-exams (the number of students that pass the mini-exams are higher than the number of students passing the mini-tests).

Secondly the data gathered regarding the number of mini-tests passed shows that the more assignments students pass, the better their results overall (as shown by the logistic regression equation).

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The Manchester Engineering Campus Development : with an angle for Mathematics

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Abstract

The University of Manchester is currently planning a major new building development that will house the four schools of Engineering and which will act as a focus for the teaching and research activities for these schools. The vision for the Manchester Engineering Campus Development (MECD) involves floor-space of 76 000 square metres spread across 8 floors. With completion expected in 2020, it will be home to 1300 staff and 7000 students.

One common factor possessed by all these schools is the requirement for high-quality mathematics teaching and the consequent role of the school of mathematics. The needs of the school of mathematics in teaching forms a key part of the design process in this new development.

The committee being consulted on matters to do with teaching consists of MECD project staff, technical staff (e.g. audio-visual) the Directors of Studies of the Engineering Schools (including the director of Foundation Studies) and the Director of Service Teaching in the School of mathematics. This committee considers such matters as lecture theatres, small group teaching and consultation rooms, computer clusters, study spaces and, of course, multi-function rooms. It considers such aspects as suitability for small/medium group mathematics tutorials as well as specialist engineering project group meetings. It considers constraints on the wifi-networks and projection systems relevant for the needs of mathematical software as well as the needs of engineers on their mainstream courses.

Once complete, MECD will be an inspiring home for the engineering students for their studies including the study of mathematics.

Historical Perspective

Settlement at Manchester dates back to Roman times but the significance of the city really took off at the time of the industrial revolution and the textile industry. Today Manchester is home to around half a million people with perhaps five times as many being within the larger conurbation. The University of Manchester employs 4 410 academics among 12 125 staff and provides for 38 590 students.

While the predecessors of the University of Manchester date back by various routes to 1824 and played a major role in the industrial heritage of Manchester, the years prior to 2004 were characterised by there being two universities i.e. the University of Manchester Institute of Science and Technology (UMIST) on the southern edge of the city centre and Victoria University of Manchester (VUM) about one mile to the south.

In 2004, the two institutes came together to form the University of Manchester to build on the heritage of the two predecessor institutes. The university consisted of several dozen schools arranged within four faculties including the faculty of Engineering and Physical Science.

The site of the new University measured more than one mile in extent and is shown in Figure 1 below (see MECD (2016) and note that east is at the top).

Figure 1: The Campus of the University of Manchester

Within the campus were two distinct areas.

- The Sackville Street area to the North-East, top left on this map and formerly the UMIST Campus.
- The Oxford Road area to the south, right on this map and formerly the VUM Campus.

Among the schools are the following engineering schools

- Mechanical, Aerospace and Civil Engineering (MACE). Formed from predecessor departments mainly at UMIST but partially at VUM. Currently based mainly in buildings 12 and 17 within the Sackville Street area.
- Electrical and Electronic Engineering (EEE). Successor to the corresponding department at UMIST. Based in Building 1 within the Sackville Street area.
- Chemical Engineering and Analytic Science (CEAS). Successor to departments at UMIST. Based mainly at Building 14 within the Sackville Street area but including the new James Chadwick Building at 89.
- School of Materials. Formerly part of VUM with links to UMIST. Formerly based in a building located between 89 and 23 (but currently in the process of demolition) and temporarily housed mainly in 99.

In addition, Foundation Studies educates students who, for various reasons, are not proceeding directly to the schools in the faculty but who require further teaching in mathematics, physics and other subjects before joining the first year (Steele, 2015).

The newly-formed school of Mathematics was relatively unusual in consisting of significant fractions from both UMIST and VUM. Formerly occupying towers at 99 and on the area now

occupied by 37, the School of Mathematics is now based in the Alan Turing Building at 46. Among the activities of the school of mathematics is the Service Teaching i.e. teaching of mathematics to students in various other schools including the Engineering schools and Foundation Studies referred to above. The Service Teaching has a dedicated director and administrator and involves many of the 80 academics.

The years following 2004 had seen a gradual trend of schools moving from the Sackville street area to the Oxford Road area. The last and most significant of these moves is to be the move of the four Engineering schools and Foundation Studies to a new dedicated area on the north edge of the Oxford Road area.

MECD: The Manchester Engineering Campus Development

An area (hatched) has been identified at the northern edge of the Oxford Road area. Formerly this was occupied by some student residences (now demolished) and by the original building for the school of materials (being prepared for demolition). Adjacent are the new James Chadwick Building (89), Oddfellows Hall (23), Manchester Business School East (26) and the Manchester Aquatics Centre.

This area is to house the schools of MACE, EEE, CEAS, Materials and Foundation Studies, to provide offices for the staff, research facilities and a contribution towards the teaching facilities including mathematics service teaching.

The largest building within MECD will be known as MEC Hall and will run for the length of the development (dark blue in Figure 2). With exceptions, the bottom three floors will contain public and teaching facilities while the top floors will contain staff offices and school facilities. The Upper Brook Street building (green) and York Building (red) will contain research facilities with some teaching facilities within the Upper Brook Street building. The development also includes the existing buildings of James Chadwick (orange), Oddfellows Hall (with a slight extension but retaining the heritage features, light blue) and Business School East (bottom, white).
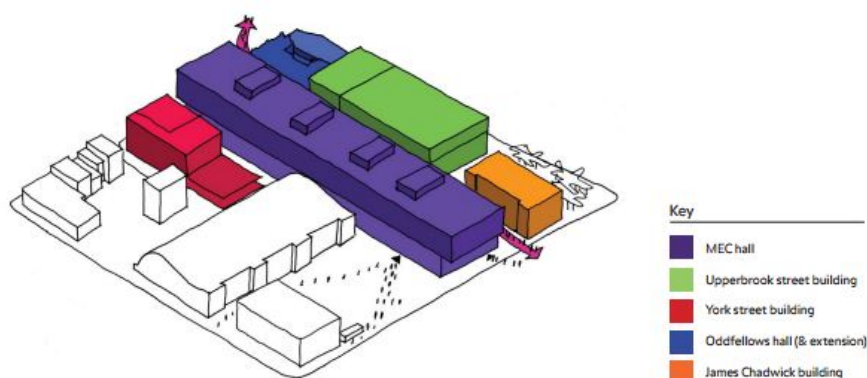

Key
- MEC hall
- Upperbrook street building
- York street building
- Oddfellows hall (& extension)
- James Chadwick building

Figure 2. The Buildings of MECD

Figure 3. Artist's Impression of MECD. Oddfellows hall in foreground with Upper Brook Street Building to the left and MEC Hall in the background.

Teaching in MECD

The University of Manchester has long operated a central system of allocating teaching rooms to classes rather than have teaching facilities allocated by schools. So, it is possible that some of the teaching to students in MECD schools will take place outside the MECD area and also possible that some teaching to students in other schools will take place within MECD. A timetabling exercise had considered the 2015/16 student data against planned teaching accommodation in 2020/21 allowing for expansion of student numbers and this had fed into the room multiplicities.

It is of course, to be expected that techniques in the teaching of mathematics and other disciplines to engineering students will change in the year preceding and succeeding the opening of MECD. Accordingly, a wide variety of room-types are planned. While, for some, the viewpoint is that large lecture-theatres do not have a future, there are enough supporters of large theatres to justify two large theatres. However, different types of room are planned i.e.

- Large Theatres of size 600 and 450
- Blended theatres of size 200 and 150
- Four Flat Teaching rooms of size 150
- Medium-sized rooms seating 80 (six rooms), 50 (fifteen rooms) and 34 (five rooms)
- Small teaching rooms of size 20 (35 rooms) and 10 (50 rooms)
- Computer clusters totalling 450 machines
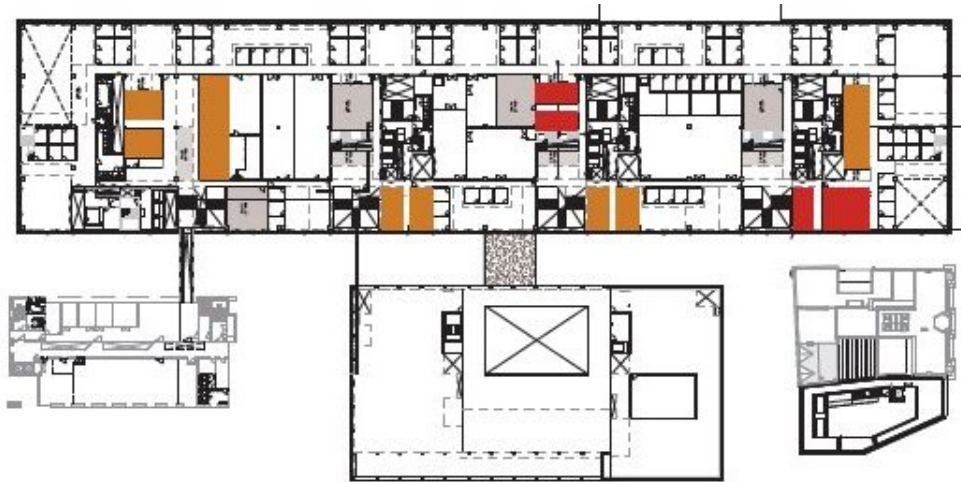- Private study space accommodating 2 000 students at any time.

Figure 4: Proposed location of 10 and 20 seater rooms on the third floor of MEC hall.

Stage 3 of the consultation had involved the sizes and numbers of the various rooms and, on the teaching side, had involved a committee consisting of MECD project staff, technical staff (e.g. audio-visual) the Directors of Studies of the Engineering Schools (including the director of Foundation Studies) and the Director of Service Teaching in the School of mathematics. However, during 2016, the consultation moved to stage 4 where wider teams of colleagues from each school including the school of mathematics were invited to regular meetings to discuss the facilities to be made available in the rooms in the various categories. As at all stages, colleagues from IT and Audio-Visual units were present to advise on relevant matters.

For many of the rooms being created, flexibility is paramount. For example, the small rooms (capacity 10 or 20) are part of the cohort of rooms available for classes but can also be used for staff meetings, consultations (including these involving maths service teaching), informal student meetings etc. They are also in a part of the building that will be open to students 24 hours per day e.g. open for student booking in evenings.

Similarly, some of the informal study spaces are to be set up in such a way that they can change to computer clusters and vice versa should technology (and indeed student attitudes) develop in an appropriate direction. Indeed, the direction in which technology may develop has been a great uncertainty in the planning of MECD. Where possible detailed decisions have been postponed but an infrastructure created which allows flexibility. For example, it is understandable that students may wish to bring devices to class to run mathematical and other software. While it may be that, at opening of MECD, most students will have devices capable of retaining charge all day, to cover all possibility, MECD will be built with extensive numbers of floor-boxes providing power. Similarly, high-density wifi will be prevalent with there being no student wire-access outside the computer clusters.

Decisions on the most appropriate surfaces and devices for displaying and recording mathematics and other types of material have been delayed pending development of such devices over coming years. Instead, the design includes ceiling hangings so that contemporary devices of whatever nature can be hung and thus made available. This may well apply to whiteboards (but probably not chalkboards) but also to successors at the time. It is likely to apply to devices used to display materials recorded elsewhere e.g. on a computer or visualiser. For many rooms, it should be possible to display material on different devices near different walls and, of course, to display the same material several times over. The groundwork has

therefore been laid for the display of a larger amount of material, something often necessary to build up a mathematical argument.

In recent years, students have been very appreciative of 'lecture capture', known unofficially as 'podcasting' whereby videos are made of the projector-feed and sound of lectures. MECD provides an excellent opportunity for advances in this technology e.g. combining several streams to produce an output, can be developed in new surroundings.

For mathematics lectures in certain modes, due to the main points being written on boards rather than through the projector, lecture capture can be less useful. Currently, experiments are being made combining direct videoing with lecture capture and such options will be catered for in MECD adding to the relevance of the facilities in MECD for the teaching of mathematics.

All potential users of the facilities advocated the use of sturdy desks rather than, for example, chairs with armrests. However, for mathematics and other disciplines, the larger the desk and the more space available to the student, the better. In the MECD discussions, the desirability of space for paper notes and an electronic device has been something very much to the fore.

Conclusions

MECD is planned to open in 2020 and will be a base for engineering students for their mathematics and other teaching.

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TeStED Project – Transitioning without A2 level mathematics

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Abstract

The research presented is the first stage of a project to support students entering STEM degrees. The study aims to investigate and address the mathematical difficulties that many students present transitioning to undergraduate Engineering courses. To this end data were collected to identify how age, gender, mathematical background, and preferred learning styles relate to outcomes on a mathematics diagnostic test. Both quantitative and qualitative methods of analysis were used to analyse the data. Our findings complement findings from previous research. We compared students with BTEC, GCSE and A/AS level qualifications, and related qualifications to study habits.

Introduction and background

In the UK, Advanced levels or A-levels were the traditional entry route for students to university degrees, usually taken over a two-year period from the ages of 16 to 18. However, universities have increased the number of admissions of students presenting alternative qualifications such as BTEC National diplomas to a variety of courses, including engineering degree programmes. This has presented challenges for university teaching and support structures. These structures, which range from pre-university short courses to staffed university mathematics support centres, are now common at many UK universities and provide a system of support for these and indeed all students who may struggle with the mathematical content of their course.

Looking particularly at alternative qualifications, Lawson (1995) found that there was little difference between students' overall end-of-year performance when presenting a BTEC or when presenting a failed A-level qualification (i.e. at grade N or U) upon entry. In addition, algebra skills, in particular, were found to be weak for all students. We wanted to re-investigate and refine the results on test performance and qualification in our current study, adding consideration of self-reported study habits and the effects of dyslexia and maths anxiety. We defined study habits in terms of resources used in preparation for assessments. This was in students' own time. Dyslexia relates to difficulties with phonological processing, working memory, processing speed and the automatic development of skills that may not match up to an individual's other cognitive abilities (see BDA, 2007). Mathematical anxiety is defined in terms of feelings of tension, or even dread, that results in an inability to manipulate numbers or solve mathematical problems (see Richardson and Suinn, 1972).

In this study we take a fresh look at diagnostic test results, and their relationship to study habits and students' previous mathematical experiences at school or college level. Our focus is to understand and find ways to help engineering students. In doing so we also seek to enable the dyslexic and/or maths anxious student. In the first stage of our research we aim to answer the following research questions:

1. What factors influence a student's choice of post-16 qualification in the UK?

2. Can we observe any trends in study habits? If so, how do these relate to prior qualifications and to diagnostic test performance?
3. What factors influence a student's performance on the algebra component of the diagnostic test?

Method of Investigation

To address our research aims the research team, consisting of a mathematician and two mathematics educators, collected data from (a) a diagnostic test that students took upon entry to the university and (b) a questionnaire about students' mathematical background and study habits. These formed the basis of the analysis of the first stage of the study. Additional data from (c) a screening test for dyslexia and (d) a questionnaire about mathematical anxiety is currently being collected. The final stage of the project will be the development and testing of a learning resource. We report here on the results of the quantitative analysis of the first stage of the study.

(a) The diagnostic test is a formative test taken by all engineering students in the first week of arriving at our university. The test covers number, algebra, and calculus.

(b) The questionnaire was handed out in the first two weeks of the semester as students arrived for their lectures. It was aimed at obtaining background data in relation to students' pre-university study, namely qualifications and study habits.

The research participants were all students enrolled for an engineering degree and in their first year of study. Three different cohorts of students took part in the study: material engineers, electronic, electrical and system engineers, and chemical engineers. 349 students completed the diagnostic test but not all completed the questionnaire. Subsets of the students were selected for analysis of each question to maximise sample size while controlling the possible biases caused by missing data.

Quantitative data from the diagnostic test and questionnaire were coded and entered into R for statistical analyses which we report in the next section.

Analyses of quantitative data

The analysis aimed to characterise the research participants in terms of their age, gender, qualification obtained, whether enjoyment of mathematics was a factor in deciding to continue with further study of mathematics, and whether the student had been encouraged to do so. In terms of demographics, there were 43 female participants and 226 male participants. The vast majority of students was aged 18-19 (235 participants), 26 students were aged 20-22 and 8 were aged 23 and over. Most students presented with A-levels at grades A and B in mathematics.

Factors that influenced a student's choice of post-16 qualification

We investigated which post-16 study route of mathematics among those available in the UK[6] was chosen by students who had entered university to pursue an Engineering degree. We

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[6]In the UK students aged 16 years complete their compulsory education with the General Certificate of Secondary Education (GCSE) which includes grades for several subjects including mathematics. Continuing into

distinguished between two types of qualification: the more academic qualification of A/AS levels (taken by 215 of the research participants), and the more vocational qualification of BTEC/MWD which also included GCSE qualifications when these were the highest qualification that the student held (taken by 34 participants).

Using regression analysis, we found that gender was not a significant predictor. But age, enjoyment of mathematics and encouragement were all significant with p-values p=0.003 for the age group 20-22 (Age2), p=0.054 for those aged 23 and over (Age3), p=0.038 for enjoyment of mathematics (EnjoyY) and p= 0.001 for having been encouraged (EncouragedY). Removing the variable gender we obtained a regression model of the form

$$\text{logit}(L) = -0.46 + 1.6 *(\text{Age2}+\text{Age3}) - 1.2*\text{EnjoyY} - 1.4*\text{EncouragedY} \qquad (1)$$

The regression equation shows that age was most strongly related to a student's choice of post-16 qualification with older students less likely to have an A/AS level qualification. Students who enjoyed the subject were more likely to take A/AS levels as were those who had been encouraged. We must be careful interpreting this, as enjoyment and being encouraged could be associated with performance at GCSE. This information was not provided by participants unless it was their highest qualification. For our study these results are important as a baseline for subsequent analyses of data from the stage two screenings and testing.

Trends in study habits and relationship to qualification and test performance

The questionnaire was aimed at eliciting students' responses in relation to study habits. Since the questionnaire was administered in the first two weeks of the university degree course, students' responses were taken to represents their (self-reported) study habits at school or college level, and not at university level. Statements were formulated with Likert scales and focussed on the use of printed and online resources as well as 'in-person' help, e.g. from peers or family. We obtained summary data about resource use and identified three distinct clusters in terms of study habits expressed.

The resource reported as *most frequently used in preparation for a mathematics assessment* was past exam papers. Over half the students reported that they used them every time that they studied for a mathematics assessment. The second most frequently used resource was printed notes (teacher's notes or own notes) followed by mathematics textbooks. Almost all students reported getting help from a friend. When considering *online* resources, in particular, we found that online videos were used most often, followed by 'other online written' resources (not including online textbooks). It is worth noting that the use of *printed* resources (such as past exam papers) considerably outweighed help from either online resources or help provided by another person.

We next explored *differences in students' study habits* using exploratory cluster analysis. We extracted three clusters of statistically different study habits as shown in the dendogram in Figure 1 (p<0.001). This represents the clustering of the students in terms of similarity of reported study habits, with the three large coloured areas representing the main three clusters. Participants who did not have A/AS levels are represented by the cyan labels under the dendogram, whereas students with A/AS levels are represented by the purple labels.

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a Sixth Form or College post-16, students can take Advanced levels (A- and AS-levels) or, alternatively, vocational or technical qualifications such as BTEC (Business and Technology Education Council) or MWD (an Engineering certification, 'Measurement while Drilling').

Study Habit Clusters
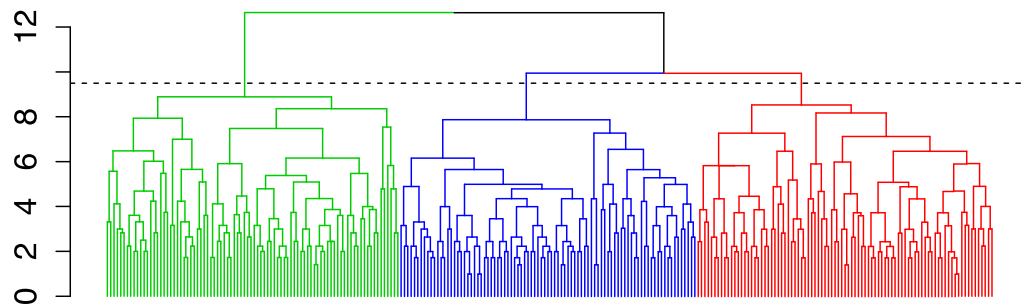
purple=higher qualification, cyan=lower qualification

Figure 1: Students' study habits

There were highly significant differences among the clusters in their use of printed resources and online resources with both p-values less than 10^{-16}. There was no significant difference in students' use of 'in-person' help (p-value of 0.1127).

The clusters can be characterised as follows. The green cluster represents the highest users of all types of resources. The red cluster used fewer resources altogether when compared with the other two clusters. The green and blue clusters used similar levels of 'in-person' help and printed resources, but the green cluster used more online resources. We can note from Figure 1 that this is the cluster containing the largest proportion of students without A/AS qualifications.

We also compared and analysed how *study habit clusters related to performance on the algebra part of the diagnostic test*. Using ANOVA, we found that when we controlled for the qualification, study habit cluster was not a significant predictor for performance, with p-value 0.24. This suggests that being successful or not in obtaining a high algebra test score did not relate to particular study habits. It indicated a preferred style of studying but this was not a predictor for algebra diagnostic test performance.

Trends in diagnostic test performance and prior qualification

This part of the analysis was aimed at characterising research participants in terms of their performance on the algebra part of the diagnostic test. To do this we coded for participants' A/AS-level qualification in terms of grade obtained as well as for BTEC and GCSE qualification (GrQualN).

The algebra part of the diagnostic test was marked out of a total of 21 marks, with a mean of 14.58 and standard deviation 4.98.

Using standard regression analysis we obtained the regression equation
$$y = 23.22 - GrQualN*(1.59 + 0.35*GenderM - 0.45*Age1 + 0.05*Print + 0.01*Help + 0.01*Online) \tag{2}$$

This shows that gender, age (where Age1 refers to the 18-19 age group), qualification with grade ('GrQualN') and use of resources which included printed and online resources ('Print' and 'Online' respectively) as well as in-person help ('Help') were all significant predictors of test scores in algebra.

When we considered test performance and qualification for different age groups (as shown in Figure 2) we found that older students tended to do better on the algebra test than younger students with equivalent qualifications.
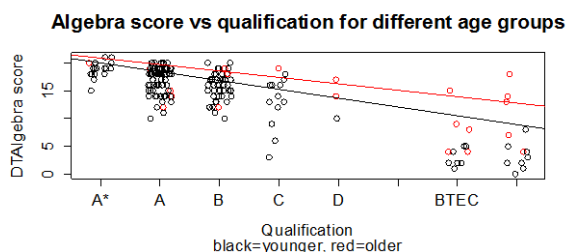

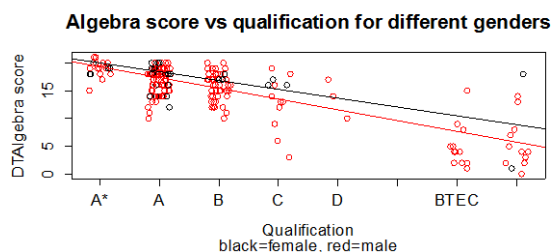**Figure 2**: Test score, qualification and age group   **Figure 3**: Test score, qualification and gender

When we considered test performance and qualification by gender (Figure 3) we found that female students tended to do better on the algebra test than male students with equivalent qualifications. Female students mostly presented A- and AS levels; there were no female students with a BTEC qualification, for example.

The regression equation (2) shows that resource use was significant, but the effect size was very small, less than 0.5 points mean difference in overall test scores.

We can see a substantial drop in algebra performance across the qualification levels with students presenting a BTEC or GCSE as highest qualification performing worse than students presenting an A-level at grade D.

Discussion of results

At this stage of our project we have results that warrant further discussion. We found that students with a BTEC qualification performed worse on the algebra diagnostic test than students with an A-level grade D, which confirms findings from previous research (Lawson, 1995). Thus after twenty years not much has changed in this position. Furthermore, detailed evidence in Gill (2016) points to a BTEC qualification as not equivalent to A-levels in terms of UCAS tariff. Suggestions are made that the current BTEC tariff of 360 points be made equivalent to an A-level tariff of 190-200. A re-evaluation of the BTEC qualification seems appropriate, as suggested by Gill (2016). In addition, students may be advised to enter for A-level mathematics since a grade D is a better predictor of performance and preparation for study than a BTEC qualification.

We also found that enjoyment of mathematics and encouragement were significant factors in the take-up of A-level mathematics. Age, too, was an indicator of choice of qualification. From this we concluded that students taking lower qualifications delayed going to university later in life.

We found three clusters of resource use. Within these clusters we found that resource use by A-level students was 'light' and based mainly on printed formats, while students with BTEC or similar qualification accessed online resources more, and resources in general more

extensively than any other group. This could be explained by the difference in style and demand of teaching and learning at schools and colleges.

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