Proceedings

SEFI MWG Seminar

Wismar 2010

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Social Dimension of Web 2.0 in Engineering Education: Students' Views

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Abstract

Contemporary engineers need to become more cognizant and more responsive to the emerging needs of the market for engineering and technology services. The social dimension of Web 2.0, which penetrates our society more thoroughly with the availability of broadband services, has the potential to contribute decisively to the sustainable development of engineering education. However, the success of the social dimension of Web 2.0 in engineering education requires student engineers' views on needs in the social dimension of Web 2.0 to be considered. Analysis of the needs of engineering students in the social dimension of Web 2.0 was undertaken alongside the efficient incorporation of the social dimension of Web 2.0 in the curriculum of engineering science. The study was conducted in the frame of the Fifth Baltic Summer School *Technical Informatics and Information Technology* at the Institute of Computer Science of the Tartu University, August 7-22, 2009, Tartu, Estonia. The results of the empirical study reveal that the student engineers' views on their needs in the social dimension of Web 2.0 have changed after the efficient incorporation of the social dimension of Web 2.0 in the curriculum of engineering science. The conclusions suggest the following hypothesis for further studies: in order to develop the use of the social dimension of Web 2.0 by student engineers it is necessary to promote student engineers' use of the social dimension of Web 2.0 for organizational and professional purposes, as well as to create a favourable learning environment which supports learners' needs in a multicultural environment.

Introduction

Web 2.0 is jointly formed by four dimensions, namely, the infrastructure dimension, the functionality dimension, the data dimension, and the social (or socialization) dimension. Socialization, described as taking software or even user-generated content and sharing or jointly using it with others, covers the aspect of user-generated content as it occurs in blogs or wikis, in tagging as well as in social bookmarking (Vossen, 2009).

The typical social dimension of Web 2.0 techniques and technologies, where the increased data exchange within the system is no longer a limiting parameter with the current developments in the infrastructure, includes "social software", namely, Skype, the eBay seller evaluation, the Amazon recommendation service, or Wikipedia, etc. It also includes online social networks, namely, a blog, or Facebook or MySpace for mostly private applications, LinkedIn or Xing for professional applications, or Twitter for both (Vossen, 2009) and they have found widespread acceptance in the community.

The aim of the following paper is to analyze student engineers' views on their needs in the social dimension of Web 2.0.

State-of-the-Art

The methodological foundation of the present research on the student engineers' views on their needs in the social dimension of Web 2.0 within engineering education is formed by the System-Constructivist Theory based on Parson's system theory (Parson, 1976) where any activity is considered as a system, Luhmann's theory (Luhmann, 1988) which emphasizes communication as a system, the theory of symbolic interactionalism (Mead, 1973; Goffman, 2008) and the theory of subjectivism (Groeben, 1986). The application of this approach to learning introduced by Reich (Reich, 2005) emphasizes that a human being's point of view depends on the subjective aspect (Maslo, 2007): everyone has his/her own system of external and internal perspectives (Figure 1) that is a complex open system (Rudzinska, 2008) and experience plays the central role in a construction process (Maslo, 2007). Therein, the subjective aspect of a human being's point of view is revealed to be applicable to the present research on the student engineers' needs in the social dimension of Web 2.0 within engineering education.

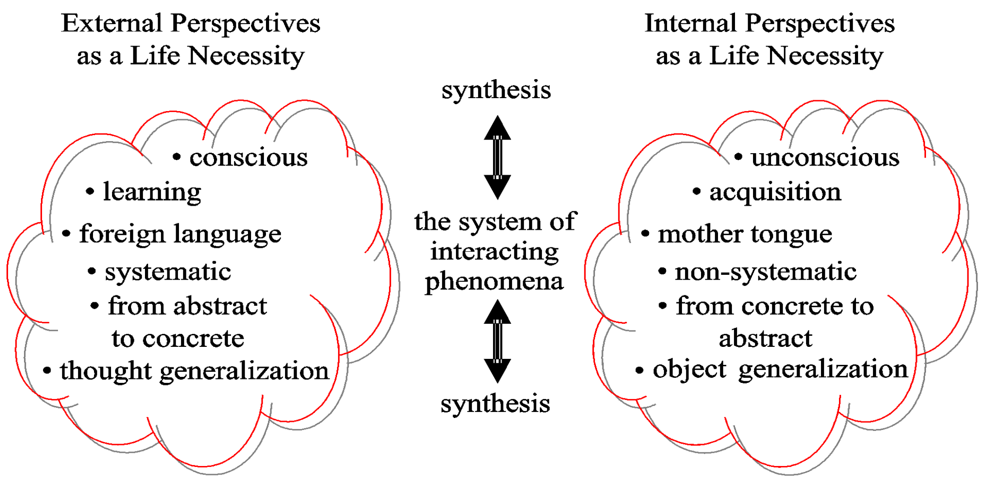

Figure 1. Developing the System of External and Internal Perspectives

Research Methodology

This study is oriented towards the revealing of efficiency of use of the social dimension of Web 2.0 within the Baltic Summer Schools *Technical Informatics and Information Technology* in 2009.

The sample of the present empirical study involves 22 participants of the Fifth Baltic Summer School *Technical Informatics and Information Technology* at the Institute of Computer Science of the Tartu University, August 7-22, 2009, Tartu, Estonia.

All 22 participants of the Fifth Baltic Summer School *Technical Informatics and Information Technology* have a Bachelor or Masters Degree in different fields of Computer Sciences and working experience in different fields. The aims of the Baltic Summer School *Technical Informatics and Information Technology* are determined as preparation for international Masters and Ph.D. programs in Germany, further specialization in computer science and information technology and learning in a simulated environment. The Summer School *Technical Informatics and Information Technology* contains a special module on Web 2.0.

The module on Web 2.0 examined the advantages and problems of this technology, which makes new social communication forms possible, namely, architecture and management, protocol design, and programming.

Explorative research has been used in the study (Tashakkori and Teddlie, 2003; Mayring, Huber and Gurtler, 2004). The study consisted of the following stages: exploration of the contexts in the use of Web 2.0 through analysis of the documents, analysis of the students' feedback regarding their needs (content analysis), data processing, analysis and data interpretation (Kogler, 2007) and analysis of the results and elaboration of conclusions and hypothesis for further studies.

Analysis of the student engineers' views on their needs in the Web 2.0 within engineering education is based on needs analysis of three levels, namely, individual needs, organizational needs and professional needs, where regular needs analysis of students' needs becomes a means of development of students' use of the social dimension of Web 2.0 (Lūka, 2008). Moreover, needs analysis serves as a basis for designing (Surikova, 2007) the following questionnaire:

- Question 1: Do you know the word *Web 2.0*?
- Question 2: Do you know the basic idea of Web 2.0?
- Question 3: Have you already used Web 2.0, namely, Facebook, Twitter, Wikipedia, etc?
- Question 4: Do you think Web 2.0 requires a lot of profound knowledge, namely, math, physics, etc?
- Question 5: Do you think Web 2.0 is useful for your individual needs?
- Question 6: Do you think Web 2.0 is useful for your organizational use?
- Question 7: Do you think Web 2.0 is useful for your professional use?

The evaluation scale of five levels for each question is given, where "1" means "disagree" and a low level of experience in use of Web 2.0 technologies and "5" means "agree" and high level of use of Web 2.0.

Findings and Discussion

The participants' use of Web 2.0 was evaluated by the participants themselves on the first day of the Baltic Summer School, namely, August 7, 2009.

The analysis of the survey (Fig. 2) reveals the following: the use of Web 2.0 by the Baltic Summer School (BaSoTi) participants is heterogeneous and the participants consider Web 2.0 to be most useful for their individual needs as revealed by responses to question 5.

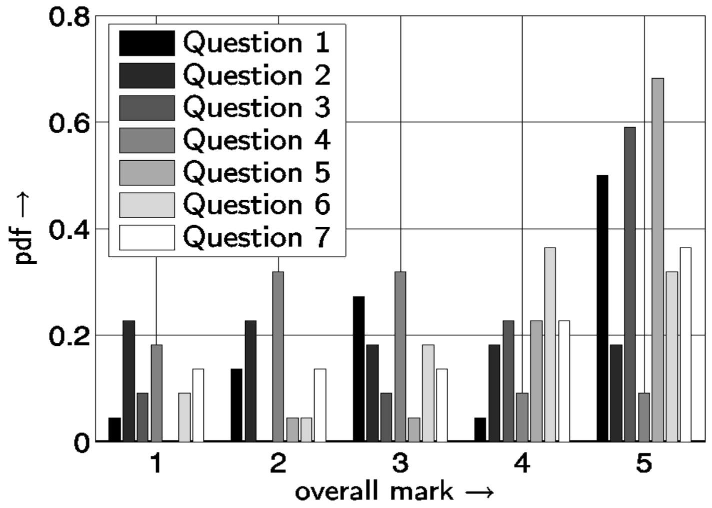

Figure 2. PDF (probability density function) of the BaSoTi participants' evaluation on August 7, 2009

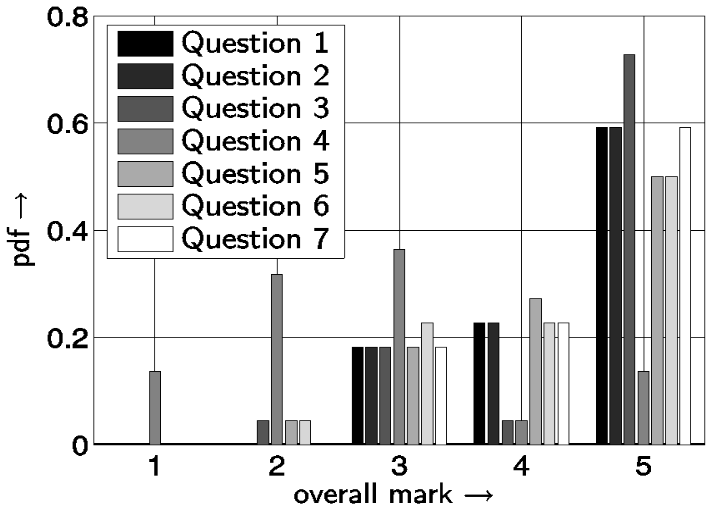

Figure 3. PDF (probability density function) of the second BaSoTi participants' evaluation on August 11, 2009

Hence, the use of Web 2.0 by the BaSoTi participants is provided by the knowledge the participants obtained in their Bachelor or Masters studies in different fields of Computer Sciences and by their working experience in different fields thereby putting the emphasis on developing the internal perspective.

Between Survey 1 and 2 of the participants' experience in use of the social dimension of Web 2.0, teaching/learning activity involved courses in Technical Informatics and Information Technology (German and English), pre-conference tutorials for introduction into advanced research topics, attendance at the conference *Advanced Topics in Telecommunication*, tutorials and practical tasks, language training for talking and presentation (optional in English or German), leisure activities and social contacts and practical work at an IT company.

Then, the analysis of the second survey (Fig. 3) reveals that the participants' experience in use of the social dimension of Web 2.0 has become homogeneous and the participants have put the emphasis on use of Web 2.0 for professional needs as revealed by responses to question 7.

The summary of results of the two surveys of the participants' experience within the Baltic Summer School 2009 demonstrates the positive changes in comparison with Survey 1:

- the level of the participants' experience in terms of use of Web 2.0 has been enriched;
- the level of the participants' experience in terms of knowledge of basic idea of Web 2.0 has been improved;
- the level of the participants' experience in terms of use of Web 2.0 for individual needs decreased, thereby developing the system of the external and internal perspectives;
- the level of the participants' experience in terms of use of Web 2.0 for organizational and professional needs increased, thereby developing the system of the external and internal perspectives.

The results reveal that the level of the participants' experience in the use of the social dimension of Web 2.0 has been enriched. The comparison of results of Survey 1 and Survey 2 of the participants' experience in use of the social dimension of Web 2.0 emphasizes the decrease of the number of participants' who have obtained the low and critical level of experience and the increase of the participants' number who have achieved the average and optimal level of experience.

Conclusions for Education

The emphasis of the System-Constructivist Theory on the subjective aspect of a human being's point of view and experience that plays the central role in a construction process does not allow analyzing student engineers' needs in the social dimension of Web 2.0 objectively: human beings do not always realize their experience and their wants in the social dimension of Web 2.0.

The recommendation here is that the role of educators in mathematical education at tertiary level is as mentors for student engineers' self-discovery and self-realization; to motivate student engineers, to stimulate their interests, to help them to develop their own structure and style, as well as to help them to evaluate their performance and be able to apply these findings (Maslo, 2007) to improve their futher use of the social dimension of Web 2.0.

The research results could be particularly useful for educators in mathematical education at tertiary level who could enable new specialists to act in a multicultural environment.

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The SEFI Math Working Group – Resources and future plans

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Abstract

This contribution gives an overview of the resources the working group currently provides via its website. Moreover, it states future plans for organizing an accumulative process such that we can offer a consistent, state-of-the-art body of knowledge and experience to those who are concerned with the mathematical education of engineers. We also present a concept for improving information collection and dissemination in Europe.

Introduction

SEFI’s mathematical working group (MWG) was established in 1982. Its goals comprise the creation and collection of useful material for the mathematical education of engineers as well as the provision of a forum for the exchange of views amongst those interested in engineering mathematics. The group pursues these goals by organising a biennial seminar and by producing a curriculum document and additional material. The next section gives an overview of the history and current state of the curriculum document and states some directions for further development. Then, we describe our list of “important topics” and the material we have produced or intend to produce in relation to these topics. Finally, we outline our concept of a network of national contact persons which should serve to improve the collection and dissemination of information all over Europe.

The Curriculum “Mathematics for the European Engineer”

Soon after constituting a working group for mathematics education of future engineers (so called SEFI MWG) the working group started discussions on what is the core of mathematics knowledge and capabilities of a European engineer. During the 6th Seminar in Balatonfured in Hungary in 1991 the round-table discussions brought together delegates from West and Middle and East Europe. The effort was crowned in 1992 when “A Core Curriculum in Mathematics for the European Engineer (eds.: M. J. D. Barry and N. C. Steele)” was published. It represented a guideline syllabus and perception of mathematics that engineering students in Europe should study. It contained:

1. Core Zero – information about the prerequisite base of high school mathematics that a student entering engineering education should have.

2. Undergraduate Core – divided into mathematical analysis and calculus, linear algebra, probability and statistics, and discrete mathematics.

The main focus was on analysis and calculus – it formed half of the undergraduate core. The fact that statistics was included into the core was one of the new features that marked out this document from previous work on the same topic. The other new features were infusion of numerical methods into all levels, stressing the importance of modeling skills and expanding the role of using computers in teaching mathematics.

The Core Curriculum in Mathematics for the European Engineer was translated first into French and German, later also into other European languages.

Since 1992 so much had changed not only in mathematics education that a revision of the material was necessary. First of all, the rapid changes in computer technology led to computer algebra systems like Matlab, Mathematica and Maple which became used for education of future engineers. Moreover, increasing numbers of students entering higher education and decreasing mathematical abilities and skills of students influenced the mathematics education in a significant way. In 1998 at the meeting of SEFI MWG in Finland the problems with decline of mathematics abilities amongst students who obtained reasonable entry qualifications formed a significant concern. All the problems mentioned above resulted in rethinking of the material adopted in 1992. Also, it became widely accepted that such material should emphasise learning outcomes rather than only state topics to be covered. The new discussions led to the material called "Mathematics for the European Engineer: A Curriculum for the 21st Century (eds.: L. R. Mustoe and D. A. Lawson)" which was published in 2002. The material was meant to be a benchmark for higher education institutions for their mathematics education.

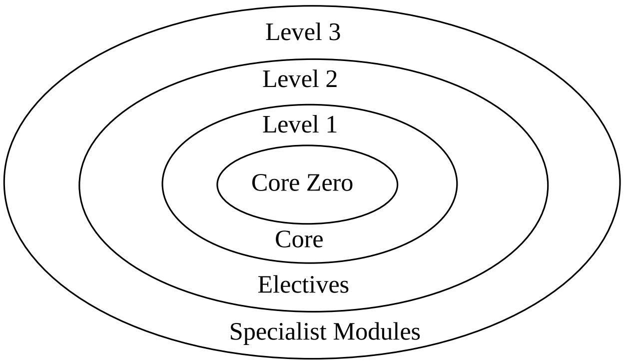

Figure 1. Organisational model underlying the curriculum.

The material had a structure similar to the previous one but was enlarged in the following sense. It contained (cf. Figure 1):

1. Core Zero – the knowledge and abilities the students entering the higher education should have. It contains topics from algebra, analysis and calculus, discrete mathematics, geometry and trigonometry, statistics and probability.

2. Core One – the material which is understood as a basis for all engineers even though the difference between different branches of engineers (especially computer science engineers) is mentioned and understood. It is considerably enlarged with comparison with the Core Curriculum from 1992, it contains geometry (which was missing in CC 1992) and to all parts the same importance is given.

3. Core Two – more advanced topics in mathematics (“Electives”). This is not meant as necessary for every engineering student but serves as a source for advanced material to different engineering branches that are more mathematics-based.

4. Core Three – special topics usually taught within engineering courses rather than in special mathematics ones (“Specialist Modules”).

The text is accompanied by two chapters “Background” and “Overview” that give principles and backgrounds of the whole material.

“Mathematics for the European Engineer: A Curriculum for the 21st Century” was published at the same time when the Bologna process started to be implemented within Europe – first in the Middle and East European countries, later in the western part of the continent. Unfortunately, the Bologna process often led to a decline of mathematics education for engineering students because of the splitting of long programmes into two self-contained parts each of which does not allow for a sufficient mathematics background. This was true especially for countries from Middle and East Europe.

Alongside with changes caused by the implementation of the Bologna process and rapid technological changes, there have been significant social changes particularly in terms of the availability of higher education, the impact of the internet on student approaches to learning, the attractiveness to teenagers of certain academic disciplines. As a consequence of these developments, what mathematics to teach to both undergraduate and graduate engineering students and how to teach it are even more pressing problems of considerable importance than previously. It is even more important than in the past that mathematics education should enable engineering students to communicate their ideas in an unambiguous and understandable way and should equip them with the analytical skills they will need as practising engineers. It is clear that mathematics is more than just a collection of tools that can be used to solve certain well-defined problems. Mathematical thinking and modeling give engineers the ability to approach new problems with confidence. So one of the future tasks for SEFI MWG is to rethink the “Mathematics for the European Engineer: A Curriculum for the 21st Century” once more. As a means for discussing social and didactical topics to be integrated in a new edition, the group set up the list presented in the next section.

Important topics

The MWG Steering Committee has identified several topics and questions which they consider as important for the mathematical education of engineers. The Steering Committee intends to provide a short state-of-the-art summary for each topic which gives, in particular, hints to former seminar papers that provide a relevant contribution to the topic (cf. the proceedings (Demlova, 2004), (Demlova/Lawson, 2006), (Alpers et al., 2008)). In this sense, the description is meant to be a gateway to further material. Moreover, the summary should also serve as a reference for contributions to future seminars such that contributors can specify in what respect they corroborate, challenge or extend the available results. This would help to update the summaries continuously and to organise an accumulative process instead of "re-inventing the wheel" all the time. So far, short summaries have been written up for three topics: activation of learners, use of technology, and modelling competences and their teaching. These can be found on the group's website.

In the sequel, we list the topics identified and provide a short explanation. A description of some of these topics in German can be found in (Alpers, 2007).

- Transition from school to university: What are the most important problems when students enter university and by which measures (before, during and after studies) can these be tackled?

- Contents and learning outcomes: What should be the content and the learning outcomes of the mathematical education of engineers? A comprehensive answer is given by the curriculum document but the document needs extension, particularly with regard to the higher-level learning goals relating to understanding.

- Higher-level learning goals: Which higher-level learning goals (e.g. deeper understanding, modelling competences, …) are important and how can they be achieved?

- Assessment: Which forms of assessment are in use and which are adequate for assessing certain learning outcomes? There was already a study on this topic (Lawson, 2004) but there are still open questions particularly regarding the assessment of learning outcomes related to mathematical understanding.

- Activation of learners: How can we organise the learning process such that the students are more actively involved in the process and not just "listeners"?

- Use of technology: What are the risks and benefits of using technology in the mathematical education of engineers?

- Modelling competencies and their teaching: Working with models (setting up models, solving problems within existing models, interpreting the output of such work) is essential for engineering work. What are the required modelling competencies and where and how should they be taught?

- Integration of mathematics and engineering subject education: How can we integrate the mathematical education and the education in classical engineering subjects (like mechanics or control theory) such that students are enabled to use mathematics for formulating and solving problems in application subjects?

- Students' attitudes towards mathematics: Which attitudes to mathematics do engineering students have and how can we ensure that students see and use mathematical concepts and procedures as problem solvers?

- Mathematics at the workplace: Since, in many educational systems mathematical education intends to qualify students for certain job profiles, it is important to know more about the mathematical qualifications engineers need for their daily work.

- Mathematical needs in continuing engineering education: What are the mathematical qualifications which are definitely *not* part of the engineering education but which might be needed later on for certain job profiles?

- Mathematical education research: What kind of didactical insights does professional didactical research relating to mathematics education provide (didactical principles, identification of competencies, learning and teaching styles etc.)?

Everyone working in the field is encouraged to contribute to these topics by writing a paper for one of the seminars, by writing a summary for one of those topics where there is none so far, or by sending comments on existing summaries.

National contact persons

The SEFI MWG disseminates information on results regarding the mathematical education of engineers via its website and via the bienniel seminar. The seminar is also the way to learn about new activities and projects in European countries. Although there seems to be a "kernel" of people who attended many seminars, attendance varies (probably also due to financial support and varying travelling costs) and many countries are not very well represented. And even when there is a participant from a certain country, one could hardly call her/him a representative for that country. This results in an information collection and dissemination process which is not very systematic.

SEFI itself has a system of so-called national correspondents (see www.sefi.be) who act as local contact persons for SEFI and provide SEFI with information on engineering education from their region. A similar system seems to be reasonable for the field of mathematics education since this is still large enough for covering lots of regional and national initiatives. The profile for the role of a national contact person might be stated as follows: The person should be willing to keep contact with national or regional circles interested in the mathematical education of engineers. Keeping contact means that the person should inform national or regional circles about developments within SEFI MWG and also provide SEFI MWG with information on national or regional initiatives. The person should particularly advertise the MWG seminars in order to increase participation especially from countries that are not represented so far. A further task consists of keeping the group's mailing list up to date such that we can also reach people who are active in advancing the mathematical education of engineers but not yet known to us.

Conclusions

SEFI's Mathematics Working Group provides the community of people involved in the mathematical education of engineers with useful information on curricular and other issues considered as important. The group tries to organise an accumulative process of gathering and organising this information by updating its curriculum document and giving short state-of-the-art summaries of important topics which serve as gateways to further reading. The group will try to improve its information collection and dissemination process by setting up a network of national contact persons who provide information on national and regional activities and spread information on the group's activities and resources.

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Cognitive structures and students' understanding of mathematics

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Abstract

Learning is a complex process. It does not mean just writing on an empty blackboard or substituting old knowledge by new information, but it is connected with cognitive structures. During the learning process cognitive structures are built and new information is integrated within already existing manifold structures. These structures have a strong influence, if and how information is understood.

Up to now most of the existing studies have dealt with physics. But are these structures also important in mathematics? Investigations reveal that these structures do exist in mathematics, and they have a very noticeable effect.

In this paper the application of the concept of cognitive structures to mathematics is presented and various examples are used to show their effects. It is also proposed how this can be taken into account to develop effective teaching, so that the learning process is facilitated and persistent mistakes can more easily be removed.

Introduction

Year by year lecturers are confronted with the situation, that at any time different students have different levels of understanding and that their knowledge grows with different tempos. Moreover, typical mistakes occur and are quite resistant even to good teaching. The reason is that learning does not mean to write on an empty blackboard or to substitute old knowledge by new information. Learning is a process, which starts at the knowledge already existing and develops gradually, until the new information is understood and therefore new knowledge is gained.

Investigations show that all existing knowledge and the information connected with it form cognitive structures. These structures have a strong influence on the learning process and the understanding of information. They must be taken into account to develop an effective teaching.

The first part of this paper explains the properties of cognitive structures and their influence on the understanding of mathematics. In the second part examples of cognitive structures and possible ways of dealing with them successfully are presented.

What are cognitive structures?

All knowledge we have is not retained in our mind as isolated information pieces, but cognitive structures are built. These structures are connected to the context, in which they are formed, so that associations and logical connections to other subjects exist, even to those completely different from the actual topic. During the learning process new information is integrated into the existing structures and these structures are extended. The effect of cognitive structures depends on the context in which they are formed first, on the time at which this took place and on properties such as strength, relation to other structures etc. Niedderer (1996) and Niedderer and Petri (2001) have discussed the effect of cognitive structures in the context of physics. But cognitive structures are also relevant in mathematics. Much experience exists already in statistics: Konold (1995) discusses some typical mistakes and the related misconceptions, Prediger (2008a) investigates in detail the prevalent misconceptions of probability, the resulting mistakes and the connection with cognitive structures. The situation in calculus is different. Up to now the knowledge of cognitive structures in calculus, their properties and effects is much less than in statistics. Prediger investigated the effect of some structures on fractions (2004) and on calculus (2008b). But there is great need of more expertise and experience here.

Therefore the following paragraphs discuss the effects of several different cognitive structures on mathematics, especially calculus.

The contexts in which the cognitive structures relevant for mathematics are formed, are
- mathematics at school or university
- science of nature (physics, chemistry etc.), engineering, computer science
- language and the arts
- everyday life

The times at which structures are formed, are quite different. In the following they are divided into three time segments:
- very early time, primary school
- time of middle and late school years
- recent, already at university

General properties of the structures are for example strength, degree of activation, i.e. always activated because of fundamental character or just sometimes, and the possibility to extend them, i.e. the structures are very rigid or relatively easy to extend.

When the knowledge and principles connected with the already existing structures can successfully be applied in the context of new information and new problems, the problems can be solved correctly and the structure is extended and its strength increases. But when the application is not possible, the understanding is hindered and mistakes occur. These mistakes are typical for the special conflict and difficult to remove.

The longer these structures have already existed, the more connections to other topics and their structures are formed and the greater their influence is. Therefore most of the problems in mathematics are found in calculus (e.g. numbers, functions) and statistics and the calculus of probability. Of course these effects can also be found in other topics in mathematics, but because those are new for the students at university, there are less conflicts with old cognitive structures.

Teaching and cognitive structures

The strong influence of cognitive structures on the learning process shows that being informed about their existence and their properties is an important basis for good teaching.

First the existing structures have to be identified. How is this possible? The learning history gives important information concerning the already existing knowledge of the subject considered, knowledge of related subjects and the school or university attended. Very important indicators are typical mistakes or wrong concepts. They become apparent in calculations, explanations or arguing. Special phrases or sentences used by the students are a rich source of hints.

Next the properties of the structures should be investigated. One should know: Which knowledge is retained there? Where does it come from? In which area do they yield correct results? Where is the boundary? Where are conflicts between old and actually taught information? Is it a strong structure and therefore easily activated? How is the structure activated, i.e. is the context most important or do special words act as signals? Which typical mistakes can occur?

After these investigations the existing cognitive structures can be taken into account, and a more effective teaching can be developed. Usually this is possible on two ways: First the expansion of the existing structures can be facilitated by using explanations and exercises, which start at the existing structures and lead step by step to the new knowledge. Second it can be demonstrated that the knowledge and methods connected with the old structures cannot be applied in the new context and therefore new information and methods are necessary.

Examples for Dealing with Cognitive Structures

In the following, five different cognitive structures are discussed as concrete examples. Every structure is characterized by the time it is formed and the context.
1) Old structure, built until the age of 10, context of calculus
2) Structure formed between the age of 12 and the age of 18, context of calculus
3) Very old structure, formed during the whole life, context of everyday life
4) Structure formed until the age of 14 or in the present, context of physics
5) Expanding an existing structure, context of calculus

1) Old structure, built until the age of 10, context of calculus

a) When students have to determine the poles of a function or when they have to use l'Hospital's rule, mistakes like $\frac{6}{0}=0$ or $\frac{6}{0}=6$ occur.

This problem has several causes. When division or when calculations using the number zero are taught, explanations may be used, which are applicable to divisions like $\frac{6}{2}$, but not to $\frac{6}{0}$.
Sentences used in the context of divisions can result in both mistakes mentioned above. The first explanation usually applied to the division by zero is "6 EUR are distributed among 0 people, so nobody gets anything." This causes the mistake $\frac{6}{0}=0$. Another explanation results in the rationale "6 EUR are distributed among 0 people, so there are still 6 EUR afterwards." and, therefore, in the mistake $\frac{6}{0}=6$.
As soon as students are able to describe a division in a way applicable to $\frac{6}{0}$, they get the correct result and can give a correct explanation.
The mistake $\frac{6}{0}=6$ also has its origin in an inadequate use of a rule taught in primary school. When calculations using the number zero are taught, the pupils learn how to add or subtract zero.

In this context the sentence "Zero means to do nothing" may be used (Didaktischer Informationsdienst Mathematik 1987). An application to division yields the wrong result $\frac{6}{0} = 6$.

The usual way to deal with this problem is discussing fractions, whose numerator is constant and whose denominator decreases, for example:

$\frac{6}{1/10}, \frac{6}{1/100}, \frac{6}{1/1000}, \ldots, \frac{6}{0}$.

This explanation is understood by the students, but it does not really help, because the existing cognitive structure is not taken into account.
A way out of the dilemma is the usage of the following adjusted explanations:
$\frac{6}{2} = 3$ - How often do I have to add 2 to get 6? How often can I give 2 people 1 EUR until 6 EUR are spent? Three times.
$\frac{6}{0}$ - How often do I have to add 0 to get 6? How often can I give 1 person 0 EUR until 6 EUR are spent? Infinitely often.
After these explanations even weak students have responded: "Oh, that is logical!"

b) Decimal numbers are used in many subjects, e.g. calculus, numerical methods or computer science. Again and again students state that the number 0.251421 is greater than 0.26.

This mistake has its root in primary school. When natural numbers are learnt, the phrase "the more digits, the greater the number" may be formed. While this is true for natural numbers, it does not work for other numbers, especially for rational numbers.

In this case confronting the wrong sentence and giving a few explanations is already effective.

2) Structure formed between the age of 12 and the age of 18, context of calculus

A very common problem concerns functions (Bellmer, 2009). Students do not define a function to be a special relation, but they consider a function to be equivalent to its representations, equation or graph. This problem only occurs in calculus and not in algebra, where the students use the different representations of a relation very well and without confusing the relation with its representation.

This can be explained as follows: When functions are taught at school, they are defined as unique relations. But this is mentioned nearly never again afterwards, and all the time calculations are carried out or graphs are plotted. Therefore over the years the original definition is forgotten, and the students think, that a function is equivalent to an equation or a graph. Moreover the graph is thought to be a rigid object, and its construction by using the equation to determine the coordinates of different points is completely forgotten.

Hard work has to be done to solve this problem, because the structure is very strong and rigid. The concept of a relation must be taught again and every representation of a function has to be discussed in this context very often. A lot of special teaching and exercises are necessary here. But finally there are positive results.

3) Very old structure, formed during the whole life, context of everyday life

The meaning of the derivative of a function being the rate of change at one point is not understood by a significant percentage of the students.

This misunderstanding is caused by a conflict with the situation in everyday life. In everyday life the change of the temperature or of the amount of money on a bank account is determined without mentioning the time interval in which this change takes place. So the change of temperature is given in °C or the change of money in EUR. When students apply this to mathematics, they consider only the values of the function itself to determine the rate of change and do not use the derivative.

An effective way out is teaching the difference quotient first and discussing it using suitable examples and exercises and then leading the students step by step to the concept of the differential quotient. The graphical differentiation of given graphs is also a very good exercise. A training like this is very effective and sustained success can be achieved.

In the last few years this is the usual way at German schools, and the pupils' understanding of the derivative as the rate of change at one point has become much better and hence they are much better prepared for university.

4) Formed at the age of about 14 or in the present, context of physics

Some students confuse the terms "concave" and "convex" curvature of a function.

This confusion has its origin in a conflict with a mnemonic trick used in physics. When the different types of lenses are introduced, the mnemonic trick "One can pour coffee into a concave lens" (German: "In eine konkave Linse kann man Kaffee gießen") is used. The strength of this structure depends on the person’s knowledge of physics, especially optics.

New mnemonic tricks, which should help to identify the curvature correctly, only work well when there is no interaction with the cognitive structure mentioned.
A way out is an adjusted mnemonic trikc which uses the already existing one: "In math the coffee is poured in from below." (German: "In der Mathematik kommt der Kaffee unten 'rein.") This funny sentence is very effective.

5) Expanding an existing structure, context of calculus

When a function f, its first derivative f' and its second derivative f'' are discussed, students find it difficult to recognize on which level the considered term or graph has to be interpreted.

This difficulty arises because students are used to deal with only one function. Therefore they interpret every graph or term presented as a representation or description of one function, the function f.

Very good exercises to train switching between different levels of interpretation are:

a) Given the graph of f (or f') the corresponding graph of f' (or f) has to be constructed or has to be identified among a number of graphs. The construction or choice has to be explained in detail.
b) One single graph is presented three times, and the graph represents first f(x) as well as second f'(x) and third f''(x). Then a special point x_0 is marked. At best the signs of f, f' and f'' are different, i.e. $f(x_0)>0$, $f'(x_0)<0$ and $f''(x_0)>0$ (or vice versa in every case). Then the following questions are asked:
- What is the sign of f?
- What is the monotone behaviour of f?
- Is the slope of f positive or negative at the marked point?
- Is $f'(x_0) >0$ true or $f'(x_0) <0$?
- Does the slope increase or decrease at x_0?
- Is $f''(x_0) >0$ true or $f''(x_0) <0$?
- What is the curvature of f(x)? Is f(x) convex or concave?

These exercises are very effective. Even a very good student sighed: "Oh, my head!"

Conclusions

Cognitive structures have a strong influence on the learning process in mathematics and the understanding of new information. Therefore further investigations are planned to identify cognitive structures, so that they can be taken into account and an effective teaching can be developed. As a result the students' learning is facilitated and persistent mistakes can more easily be removed.

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Improving Core Mathematical Skills in Engineering Undergraduates

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Abstract:

A large number of engineering undergraduates begin their third-level education with significant deficiencies in their core mathematical skills. Every year, in the Dublin Institute of Technology (DIT), a diagnostic test is given to incoming first-year students, consistently revealing problems in basic mathematics. It is difficult to motivate many students to seek help in the Maths Learning Centre to address these problems. As a result, they struggle through several years of engineering, carrying a serious handicap of poor core mathematical skills, as confirmed by exploratory testing of final year students.

In order to improve these skills in engineering students, a pilot project was set up in which a "module" in core mathematics was developed. The course material was basic, but a grade of 90% or higher was required to pass the module. Students were allowed to repeat the module as often as they liked until they passed. An automated examination for this module was developed on WebCT, and a bank of questions created for it. Initially, this project was piloted in the third-year Ordinary Degree mathematics module in Mechanical Engineering in the DIT, where it proved very successful.
Subsequently, the pilot project was extended to five Ordinary Degree engineering programmes in the DIT, across three different year-groups. Full results and analysis of this extended pilot will be presented, including the responses to in-depth interviews carried out with a selection of the students involved.

1. Introduction

Many students upon entry to third level engineering programmes have problems with core mathematical skills. This has been borne out in the results of diagnostic tests carried out in many third level institutions, both in Ireland ([1], [2]) and in the U.K. ([3], [4]). These problems with core concepts can lead to comprehension difficulties in numerous modules, both in mathematics itself and in related subjects. In recent years, this has been exacerbated by the fact that students are being recruited from an increasingly diverse student body. The past academic years of 2008 and 2009, in particular, have seen the return of a large number of students to full-time education after many years in employment, due to adverse economic conditions. In this paper we discuss the maths diagnostic test carried out in the Dublin Institute of Technology (DIT) and the deficiencies in students' core mathematics revealed by this test. We then outline the details of a pilot project carried out to address these deficiencies. The results of several focus groups are presented. The maths diagnostic test was also given to a selection of fourth year students and the results of this test are shown. Finally we outline future work we intend to carry out on this project.

2. Core Skills Initiative

Research conducted by the Dublin Institute of Technology (DIT) Retention Office showed that a student's mathematics grade in the Irish Leaving Certificate (the final examination in the Irish secondary school system) is a key determinant in that student's progression through engineering programmes [5]. As a result, a mathematics diagnostic test has been given to first year students for several years now and a Maths Learning Centre (MLC) has been set up in the Dublin Institute of Technology.

2.1 Mathematics Diagnostic Test

The DIT Mathematics Diagnostic Test showed marked deficiencies in core mathematical skills [6]**.** The test consists of twenty questions (ten paired questions) on basic topics such as algebra, fractions, indices, trigonometry, the equation of a line, logs, quadratic equations, simultaneous equations and basic differentiation. In 2006, the mean mark obtained by first year engineering students was 55% across all programmes. More worryingly, this mean dropped as low as 29% in some programmes. A large spread was seen within most programmes, with many students scoring significantly lower than the mean mark.

2.2 Core Skills Assessment

It was decided to set up a core skills assessment in mathematics, similar to that already in existence in the Institute of Technology Tallaght, Dublin [7]. This consisted of a multiple-choice quiz on WebCT, based on a randomised question bank. The material covered by the test was basic but the pass mark was set at 90% for Third years and at 70% for first years. The questions used were based on those already in use in the DIT Mathematics Diagnostic Test. Students were allowed to re-sit the assessment as frequently as required until they passed. Ideally a pass in this module would be compulsory for progression to the next year of the course, but this is not yet the case.

3. Pilot Project

In Ireland, students who have not achieved 55% or more in Higher Level Leaving Certificate mathematics are not eligible for the four-year Honours Degree engineering programmes, but instead may enter into a three-year Ordinary Degree programme. Upon successful completion of this, they may then enter into third year of the Honours degree. The pilot groups chosen for this study are first year ordinary degree students in Mechanical and Building Services, first year preliminary engineering, second year ordinary degree in Manutronics and third-year students in the Ordinary Degree in Mechanical Engineering in DIT.

Course	Year	Leaving Certificate Points
Preliminary Engineering	First	290
Building Services Engineering	First	150
Mechanical Engineering	First	315
Manutronics Automation	Second	150
Mechanical Engineering	Third	305

Table 1:List of Courses included in the Pilot Project. In the Irish Leaving certificate, six subjects are included for the purpose of calculating points. A maximum of 100 points can be attained in any one subject.

3.1 Project Overview

The "core skills assessment" was worth 10% of the mathematics module. In the first instance, the students sit the Mathematics Diagnostic. There are two different marking regimes depending on what year the students are. Third years had to achieve a score of 90%. Those who scored 90% received nine marks out of ten, whilst those who scored less than 90% received no marks and had to take the core skills assessment at a later date. For first and second year students a sliding scale was used, namely 70%=4/10, 80%=6/10 and 90-100%=10/10.These students continued to sit the core skills assessment on a monthly basis until they achieved the required pass mark. After their first attempt, students were given access to a WebCT site with resources tailored for each question and were also encouraged to attend the MLC. After their second and subsequent attempts, special classes on problem topics were provided. At the end of the year, students were asked to fill in a reflective online survey on the core skills assessment, and selected students took part in Focus groups to discuss the project.

3.2 Evaluation of the Mathematic Diagnostic Test/Core Skills Assessment

An evaluation strategy was devised in order to enhance and develop the diagnostic test and the way in which it is implemented in, and integrated into, the modules. The evaluation is essentially a comparison between aims and objectives of the development and implementing the test and the reality of the students' learning and development. However there was also a particular need for formative evaluation in order to discover areas where improvements can be made to the diagnostic test itself and its use within the engineering programmes. It was also the authors' intention to obtain reliable and triangulated data that would inform the subsequent changes and refinements, and minimise the occurrence of intuitive decision-making. The evaluation combines both qualitative and quantitative research methods in order to ascertain the effectiveness of the diagnostic test and to determine where improvements can be made. The methods of data collection are questionnaires, focus groups, diagnostic test results, the amount of attempts made by the students, and attendance at tutorials. It involves focus groups with different cohorts of students using the diagnostic test and hence a comparative analysis of the following groups is possible:

- Level 7 engineering first year students
- Level 7 engineering final year students
- Level 8 engineering first year students
- Level 8 third year students
- Mature students
- Preliminary engineering students

The evaluation will run over a complete academic year so that improvements to the test and its implementation can be made before the start of the next academic year. As this paper was written just before the end of the academic year, the evaluation process has not yet been completed in full, with only two focus groups carried out and not all quantitative data analysed, and therefore the next section presents preliminary findings.

The focus groups consisted of qualitative questions regarding the students' perceptions and opinions of the maths diagnostic test and the way in which it was implemented within their modules. They were carried out by an experienced education researcher who did not teach any of the students and was not known to the students. Analysis of the focus group data led to the following conclusions:

Positive Aspects

1. The students were able to describe the positive effects the diagnostic test had on the development of their mathematic abilities. They identified not only the ways in which their mathematical ability had developed but the role that the diagnostic test had played. They gave concise examples of difficulties they had in mathematics prior to the test and described how these were remedied once identified through the results of the test.
2. The students were clearly aware of the formative nature and purpose of the diagnostic test even though their final mark in the test did contribute to the overall module mark. They were also very cognisant of the need for the test to contribute to the module mark and the motivation associated with this.
3. The students supported the high pass mark and expressed their belief that it is this pass mark coupled with having to achieve this mark to get any mark that ensures the effectiveness of the test. It should be noted that a significant number of the preliminary engineering students felt that the pass mark of 70% was too low. This issue will be investigated further when all the data are obtained.
4. The students appreciated the chance to take the test multiple times and could clearly articulate the formative effect this had on their learning experience and development.
5. The importance in engineering of the mathematics examined by the test was evident to all students but particularly to the students in the latter stages of their engineering programmes.
6. The quality of the mathematics online notes and the ‘special’ tutorials outside of timetabled hours was commended by the students and described as “professional”, “effective” and “concise”.
7. Confidence in their mathematics ability was perceived as being positively affected by the test (although it should be noted that a number of students said the result after their first attempt was disappointing and had a detrimental on their confidence).
8. The students appreciated the time, effort and commitment of the staff involved in the implementation of the diagnostic test.

Development Aspects

1. The diagnostic test could provide more specific feedback to the students. The students felt the effectiveness of the test could be improved if the result of the test was not just a mark but it also suggested how the deficiencies could be rectified. For instance, the test could direct the students to a particular set of notes, chapter of a book or an online resource. In addition, if the lecturer noticed that a significant number of the students had difficulty with the same section, a tutorial could be run soon after the test to address that specific issue.
2. It was also suggested that similar diagnostic tests could be developed for specific elements of the mathematics modules. In that way, the full diagnostic test could identify areas of difficulty; the student then addresses this difficulty and can then complete a diagnostic test which only examines that particular area. The mark for this ‘smaller’ test would not count towards the final module mark and the student would still have the opportunity of retaking the full diagnostic test.

3. The students also expressed the view that a more advanced test could be developed for the latter stages of the engineering programmes, and for the students who excel in the diagnostic test on the first attempt.
4. All of the students expressed the opinion than the effectiveness of the test could be improved if its purpose, and the most effective way of using it, was clearly communicated to the students at the start of the process and again after the first attempt at completing the test.
5. It was suggested that greater links between the mathematics being developed within the maths modules (including the diagnostic test) and the other modules within the programmes may also improve the student mathematics ability.

Results of Diagnostic tests

As a first step, all of the students in the pilot project were given the DIT Maths diagnostic exercise. This test was also given to the First year Honours Engineering class. These are the students who, in the main, have done higher level Mathematics for their Leaving Certificate, and provide a benchmark for the level of maths required to complete an Honours degree in Engineering. We can see that the majority of first year honours students (69/87) have a mark of over 70% in the diagnostic exercise. Improving their core mathematics is clearly not a priority when we compare the test with the marks of other classes, and they are better than the marks of the Third year students in this pilot, who have already completed two years of mathematics at third level.

Course Code and Name	**Mean**	**Over 70%**	**Over 90%**
DT025 1st Year Honours	80%	69/87	17/87
DT020 Preliminary	48%	8/36	0/36
DT005/1 Building Services	65%	14/29	4/29
DT006/1 Mechanical	61%	30/72	11/72
DT003/2 Manutronics	45%	2/10	0/10
DT006/3 Mechanical	75%	16/23	7/23

Table 2: List of Courses tested and marks received in the first test

Overall improvement

Throughout the lifetime of this Pilot we have seen a systematic improvement in the core mathematical skills of the students as measured by the Maths Diagnostic test and core skills assessment. To illustrate this point we look at a case study for first year Preliminary Engineering from October 2009 up to the time of writing.

Preliminary Engineering (36 students)	Mean	More than 90%	More than 70%	Less than 70%
First Attempt	54%	1	10	25
Christmas 2009	65%	6	14	16
April 2010	73%	7	18	11

Table 3: Grades of preliminary engineering students in the core skills assessment

The results above show a systematic improvement in the results of the students. On the first test only 1 out of 36 students achieved a mark of over 90%, by the time of writing that had increased to 7. More importantly 25 out of 36 failed to achieve a mark of 70% in their first attempt. This number has now been reduced to 11, with several opportunities remaining to complete the test.

3.3 Reflective Online survey

At the end of the semester all of the students in the project will be asked to complete an online survey to get their feedback on the pilot project.

3.4 Sample Group of Final Year Students

Finally, it was decided to test a small subgroup of final-year students who had already completed an Ordinary degree and subsequently continued into the Honours degree programme. Forty Eight students volunteered to retake the diagnostic exercise. These students only had to take the test, no credit was awarded to them irrespective of how well or badly they did.

Final Year Engineering (48 students)	More than 90%	More than 70%	Less than 70%
Overall (48)	24	41	7
Ordinary Degree (23)	10	16	7

Table 4: Results of Final year Students

Twenty four out of forty eight scored more than 90% while 41 scored more than 70%. Seven of the forty eight students scored a mark of less than 70%. Of these forty eight students, twenty three of them came from an ordinary degree background, all seven students who failed to score more than 70% on the diagnostic test had come through from an ordinary degree, and three of these had failed to score 50%. Given that these students volunteered to do the test, there may be significantly more students in final year who still lack many core mathematical skills. These results show us that action needs to be taken early in the education of student engineers, doing ordinary degrees to address this problem.

4. Conclusion and Future Work

4.1 Conclusion

The introduction of the core skills initiative there has produced a systematic improvement in the core mathematical abilities of the students. This is evident both from the results of the students and the feedback we are getting from the focus groups. The results of the small group of final year Honours degree students who took the assessment have shown that there may be a significant number of students who struggle with basic mathematical concepts throughout their entire degree. Such problems are clearly endemic and will persist if not tackled in a consistent manner. The core skills assessment is one such way to encourage students to seek help to address these deficiencies, and it is extremely important that this work be rolled out across all first year courses in engineering.

4.2 Future work

The core skills assessment will now be introduced to all first year classes doing an Ordinary degree in Engineering. A full analysis of all the results of the tests will be carried out at the end of the academic year. The feedback from the focus groups and the online survey will be used to improve the process in the coming year. A more advanced version of the test is also being developed for students in the later years of the programme.

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Use of Technology for Mathematical Education

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Abstract

This paper reflects on the situation in the Republic of Macedonia regarding mathematical education. The Macedonian Government project "Computer for every child" is a part of the initiative "Education and Training for Everyone", based on the National Program for the development of education (2005-2015). For the purpose of this project, a supply of 17,818 personal computers and over 200000 other computational appliances was provided for the primary and secondary schools in Macedonia. The ambition is to achieve 1:1 ratio of computers to pupil in the public schools. This is the largest project in the country's 18 years of existence. In order to actually realise this project, this research aims to address the needs for developing tools for educational instruction, training, guides for software usage and educational materials in digital and printed form for mathematics.

Introduction

The dedication towards the European integration in the modern educational processes means to prepare the professors working in the secondary schools, who can successfully get involved into the European labour market; information exchange through the information-communication technology and internet systems; and increased cooperation with the educational institutions in Europe and the world. "The National Program for the development of the education in the Republic of Macedonia 2005-2015" includes a component dedicated to the development of secondary education. According to its content, the key issues that need interventions in the secondary education are: the differences between the urban and the rural environments, as well as the divergence between developed and not developed municipalities; the education of the teaching staff and improvement of their qualifications; the dissimilarities between the study conditions and the availability to the educational institutions.

The influence of the information-communication technologies in the education is very complex. There are many questions that have to be answered according to the existing society conditions. Expert teams need to work on the following aspects: the participants, the resources and the complete organisation of the educational process. The current position related to the implementation of the information-communication technologies in Macedonian educational institutions requires financial investments in planning, organising and educating the potential users.

The objectives of the research on the theme "Use of technologies for Mathematical Education" are to:

- identify the level of computer implementation during the mathematics classes in the secondary education in the Republic of Macedonia;
- determine whether and to what extent high school students and professors use information technologies for different purposes in the mathematics curricula;

- recognise the needs of the mathematics professors in high schools for teaching materials in electronic version and training about easier usage of the information technologies;
- set up recommendations for further perspective and development in Macedonian higher education.

Method of Investigation

The basic method for this assessment is a comparative analysis of statistical data gained by surveys focused on two target groups: one of them consisting of 80 students from high schools in the capital city Skopje, who were participating at the Macedonian Regional Mathematical Competition which took place in March 2010, and the other consisting of 10 high school professors – tutors of the students participating at the same competition. As an instrument for the research, a suitable questionnaire for each of the two groups was created and the questions can be compared at an appropriate level.

All high school students who were participants in this research procedure are from one city in the country. These students are those who show their main interest in mathematics compared with the rest of the school subjects and achieve the best outcomes on the regular mathematical classes throughout the whole school year. Therefore the results obtained cannot be related to the entire student population in all high schools in the country. The sample of 10 mathematical professors who work in the high schools only in the capital city can not be generalised for all the other secondary schools in the country, as well. We must have in mind that this teaching staff from Skopje has the best conditions and opportunities to follow the new methods and educational trends involving implementation of the information technologies, compared with the high school teachers in the other cities in Macedonia.

Approach of the Students

The first reaction of the students, when they were told that they will a part of scientific research and when they were given the questionnaires, is a surprise, because they are finally given a chance to state their own opinions. The real reason why they are involved into this research is to obtain relevant information on the one hand, and to present them an idea on different possibilities about computer implementation in their active learning, on the other hand. Namely, several questions are referred to the student's participation during the classes, doing the homework and working on project tasks.

Another important item here is that the students who will take part into the investigation will have diverse perspectives when they go to the universities, especially outside the borders of the country. They will have a vision for their expected challenge, where the computer is a simply irreplaceable tool in everyday learning. Moreover, they will be aware of the opportunities that computers offer and they will appreciate the initiative about increased level of computer implementation in their education.

Findings and Discussion

This section explains the essential findings concerning the use of computers in gaining the necessary mathematical knowledge in the Macedonian education process.

In the first survey given to the students, 90% of the respondents answered the first question “Do you know how to write a word document with mathematical formulas on a computer” posititively and none of the professors answered negatively. The second question was: “Do you use Microsoft Equation (Microsoft Office) or Formula (Open Office, Org Word Processor)?”; again, almost all students and professors answered the same, which is that they both use Microsoft Equation (Microsoft Office). If we are aware of the fact that Microsoft Office is supported by the Windows Operating System and only the Operation System Linux is installed in all the computers that the Government bought for the purpose of the project “Computer for every child”, this means that computers in the schools are not exploited at all for this kind of an activity.

We obtained quite an interesting situation on the third question “Do you use computers during the mathematics classes?” where 67% of the students answered negatively and 33% answered positively, while the statements of the teachers are opposite. This gives us enough reasons to believe that the professors use the computers only for presentations or some other similar activities which do not involve students into practical use of the computers. Thus, they do not actively participate in the study process, but only observe what they are being taught.

In contrast to the previous three Yes/No questions, the fourth one “How often do you use the computers during the mathematics classes?” has four optional answers: every week, few times weekly, few times in a semester and do not use at all. 49% of the students answered that they do not use the computers at all, 21% answered that they use them few times in a semester, 16% few times in a week and only 9% once in a week. On the same question only 11% of teachers answered that they do not use the computers at all, which is again quite opposite of what the students have said and 45% of the teachers answered few times in a semester. This is one more confirmation to what we have assumed previously that the teachers use the computers mostly for individual activities in screening some information, without any student’s active contribution. As to the rest of the teachers: 11% answered that they use the computers few times in a week and 33% once in a week. Expecting this kind of result we have asked, only the teachers, one more question in this context, which is “Do you use the computers for teaching new material or for exercises more?”, 57% of their answers are that they use the computers mostly for teaching new material, while the rest of them, 43%, said for exercises.

The next question is “Do you give homework to the students that needs a computer support?” Most of the teachers, in fact 89%, answered negatively. This result is confirmed by the students’ answers where the majority of 65% answered with ‘No’. This kind of responses is again understandable, because most of the students do not even have computers at home and their teachers are very aware of this fact. Another assignment, except the homework, that can be done at home is a project activity, which according to the curricula, is done once in a whole school year. Consequently, teachers ask the students to take advantage of the information technologies more often while preparing the project. As we mentioned, 11% of the teachers give homework seeking computer support, while 50 % of them give project assignment to be done with the aid of a computer. The student's 85% positive answers verified this same conclusion.

The following question is related to another area of the educational process, which is evaluation of the obtained knowledge. Thus, on the question "Have you ever used the computers for evaluation of the student's knowledge?" 87% of the teachers answered negatively, which is confirmed by the students' 69% negative answers. This means that either the professors are not familiar with the advantages of this kind of examination and they are simply not trained enough or they are convinced that this method is not good enough for evaluation of mathematical knowledge.

Furthermore, the professors were asked whether they would use educational materials in electronic version if they were offered the opportunity. There were unanimous in their positive answers. They were asked if they needed training sessions for computer education and implementation in their work. 78% of them were honest and admitted that they need this kind of help. They are also persuaded that the Ministry of Education and Science may contribute much in additional education of the teachers, but the Union of the Mathematicians in Macedonia may facilitate this as well.

The last question, intended for the students, is about their future aspiration in enhanced computer implementation during the educational process. 69% of the students answered that they are willing to see many prospects that computers can offer and that this will make their education to a great extent easier.

Finally, we will discuss some limitations and advantages of this study. In fact, we must have in mind that only students who are truly devoted to mathematics were involved with the surveys. The real conditions in the classroom must be adapted for the majority of the students who, certainly, are not very interested in mathematics. Moreover, these students live and learn in the capital of the country, where the study circumstances are much above the average level compared with the rest of the cities.

Beside the explained limitations, there are also clear advantages of this study. Up till now, we did not have any evidence about the actual situation and the utilisation of computers in the high schools. Currently, it is obvious what exactly needs to be done, when and how.

For a mathematician or a mathematics educator, it is clear that the computer cannot completely replace the chalk and the board. But, the computer may perform as a very efficient tool in implementation of innovative methods that can may mathematics easier to be understood.

Conclusions for Education

Improvement of the education process is a crucial issue in any developing country in its commitment towards European integration. It is not enough to invest only in countless computers and technical appliances. We must rather accomplish something essential and specific in each area of the education process. Especially when mathematics is being observed and now that we have this verification about the current situation, we may derive several conclusions, which are as follows. There is an evident student aspiration in the direction of learning by means of effective methods and techniques. It is our duty to offer them a variety of digital materials and computer interactive programs. This new approach of daily learning, doing homework and working on practical projects in the area of mathematics is a step forward on the way to their sophisticated education. At the same time, from the teacher's point of view, it is obvious that they also manifest a need of assistance. According "The National Program for the development of the education in the Republic of Macedonia 2005-2015", the Ministry for Education and Science, actually the Biro for development of the education in Macedonia, has to provide trainings for advanced teacher education and growth in their working career. According to the teachers' answers on the questionnaires in this research, the Union of the Mathematicians in Macedonia may contribute a lot, too. In this context we refer to creating e-books, e-libraries, guides for software usage and other digital materials (all in the native language), which can be achieved by joint and cooperative effort of all previously-mentioned participants in the education process. It is our belief, also, that experts and foreign experience from the developed education systems in Europe may provide a great support towards achieving these goals. Meanwhile, either the teachers will have to apply the existing EduBuntu applications for mathematics (such as: GeoGebra, Dr. Geo, Kig, KmPlot, Kpercentage, KBuch, etc.), for the Linux Operating System more intensively, in order to give a visualised aspect of mathematics and to lend a hand of the students, or they will have to create their own applications. The second suggestion will certainly require financial support, too. Whatever is decided, implementation of any of these applications during classes, means that students may enthusiastically participate in gaining new mathematical knowledge. Furthermore, they will have a superior foundation for their further higher education at the universities within the country, or abroad.

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One-variable Calculus: A Spanish overview in accordance with the EHEA

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Abstract

In this contribution, after providing some ideas about how to put into practice the "Bologna Process" in Spanish Universities, we analyze two different examples of its implementation. These experiments involved students of Calculus I from two Spanish Engineering Schools belonging to the Polytechnic University of Madrid and the University of Salamanca. Here we shall describe the experiments, the results, and the reactions of students and teachers.

Introduction

Adapting to the European Higher Education Area (EHEA) implies the development of a new teaching and learning model, with active methodologies and learning based on competencies. The deadline for the introduction of the new curriculum is the academic year 2010-2011. Accordingly, different approaches to this implementation certainly merit consideration. In our experiments we designed the courses bearing in mind the following considerations:

- Teachers and students. The actors involved in the educational process must profoundly change attitudes concerning mathematics courses in accordance with the EHEA. Teachers must endeavour to monitor students' work and to draw up a high-quality and realistic plan for the course.
- Competencies. Competencies must be defined for each subject in order to determine what will be required of students. A competence is a bundle of skills, knowledge and attitudes that must be supported by learning goals, which are more specific and measurable. In a Calculus I course for Engineering, the focus is mainly on generic competencies, such as self-learning, critical thinking, teamwork, and problem solving, and on the specific competencies that involve the use of knowledge and general techniques for differential and integral calculus, sequences and series, to solve engineering problems with the help of mathematics software when needed.
- Material. Teachers must design and select high-quality material. This material might include: A textbook to be used as a study guide for the course, self-assessment tests, worksheets, and the development of projects and drafting of papers (for working in groups).

- Technology. CASs (Computer Algebra Systems) such as Derive, Maxima, Mathematica… and Learning Management Systems (Moodle and other portals) should be implemented.
- Learning. Face-to-face learning, e and b-learning will be combined according to teachers' and students' needs and possibilities.
- Assessment. Teachers must be consistent and design a "*continuous assessment*" plan based on students' daily work. One-minute papers, marks awarded for answering questions, projects, periodic exams (approximately one per month) and the final exam will all be taken into account in the final grading.

A Course of Mathematical Analysis for Computer Engineering

At the Polytechnic University of Madrid, following a pilot project aimed at adaptation to new methodologies and assessment techniques (see Garcia (2009)), we have designed a course of Mathematical Analysis and have implemented it in the *Computer Engineering* degree.

This is a first-year course to which 6 ECTS credits have been allocated, and which involves around 156 hours of students' work. These hours are equally distributed between face-to-face activities and other activities that do not involve teacher-student interaction (see table 1). The number of students involved in the experiment was 73, divided into three groups of 30, 28 and 15, respectively.

A pattern similar to that provided by Miguel (2006) was followed for the drafting of teaching plans. A modularized system was defined, each module being based on a number of educational activities in accordance with the learning goals, and the time required for the completion of the activities was estimated. After, following the instructions in the European Commission (2009), the estimated time established for non-contact activities was contrasted with the time it took students to carry them out. Table 1 shows the distribution of the number of working hours for the different types of activities, including tutorial activities and assessment.

Methodology	Hours for face-to-face activities	Hours for non-contact activities
Teaching and learning concepts	26	30
Problem-solving	26	30
Laboratory sessions	14	6
Team work	2	10
Tutorial Activities	4	
Assessment	6	

Table 1. Temporal distribution of Educational Activities

The face-to-face activities were carried out in explanatory lectures and more participative ones, complemented with workshops and practical classes. The workshops were devoted to solving problems in the traditional pencil and paper way, and to laboratory sessions solving designed worksheets using Maxima.

An extensive corpus of material, including a reference text, self-assessment questionnaires, and instructions for the various learning activities, was developed for non-contact work. Moodle was used as a learning tool and a means for on-line teacher-student communication. Indeed, most work materials were delivered through this platform. As regards assessment, students were offered the possibility to choose between the following:

1. A continuous assessment model based on different learning activities and a team project or paper, as well as four assessment tests to be taken during the course.
2. A end-of-course exam, in which the whole subject would be assessed.

Almost 30% of the students chose the single-exam option, considering the effort involved in the other option excessive. It is worth mentioning that none of these managed to pass the subject, while among those who took the continuous assessment option the percentage of success in passing was 71%.

An example of a b-learning activity: Numerical Integration

In our Mathematical Analysis course, we suggested a group activity in which students, working in groups of two or three people, were expected:

1. To autonomously learn two algorithms on numerical approximations of integrals (Composite Trapezoidal rule and Composite Simpson's rule) and write a theoretical report, using Garcia (2007) as a reference.
2. To program the appropriate functions to implement these algorithms using a CAS (Maxima).
3. To test the programmed functions by means of a comprehensive test battery, including both explicitly defined functions and functions whose values were known only through a table of points.
4. To model and solve an engineering problem (different for each group).

The aim of this activity was to put into practice the generic competencies of teamwork, the use of technology, self-learning and problem-solving, and also the specific competence of the ability to solve mathematical problems that might arise in relation to engineering by applying knowledge of integral calculus and numerical methods. As a result of this, students should be able to achieve the following Learning Goals: the ability to construct a mathematical model to solve an engineering problem, the use of appropriate mathematical language to describe algorithms and define concepts, and an understanding and the application of concepts relevant for problem-solving using algorithmic skills and CASs.

The results were as follows:

- Acquisition of competencies: The results were reasonably satisfactory. It should be borne in mind that numerical integration algorithms were not addressed in any of the explanatory lectures, despite which considerable learning was achieved since the students proved capable of applying self-acquired knowledge and the teacher's tutorials when necessary.

- Assessment: The final grade of this activity involved 15% of the student's final grade. 45 students carried out the work, of which 26 achieved a grade above 5 points (out of a maximum of 10). The average was 6.3, the median 6.8, and the standard deviation 2.23. A histogram displaying grouped relative frequencies is shown in figure 1.
- Workload: It was estimated that each student should devote around 10 non-contact hours to carrying out this project and all of them were asked to state the amount of time spent on it. After processing the data reported by the students, the result was an average time length of 8.7 hours with a standard deviation of 4.09. A histogram of these data is shown in figure 2.

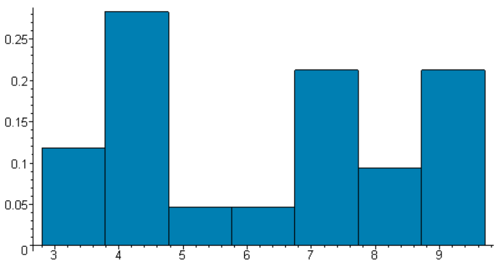

Figure 1. Grades for the teamwork

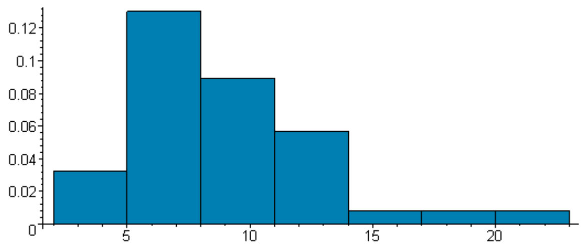

Figure2. Hours for the teamwork

A Calculus course for Construction Engineering

A new degree in Construction Engineering has been implemented at the Polytechnic School of Zamora (University of Salamanca) for the academic year 2009-10. The subject of the first semester is called Applied Mathematics I and its contents are based on a standard One variable Calculus course, the competencies to be acquired being those mentioned above.

Students were provided with all the required administrative documents through a teaching guide drawn up by the Polytechnic School of Zamora (2009). 135 students enrolled for this academic year, being divided into two groups (mainly for explanatory lectures) and into four groups for laboratory practice, where the CAS Mathematica was used. The scarcity of lecturers appointed for teaching tasks (only one teacher) prevented a greater subdivision for the performance of other tasks. In all, the teaching hours amount to 45 in-class lessons for large groups and 10 laboratory hours using Mathematica. The number of ECTS allocated to the subject is 6 and the estimated workload for each student is 150 hours.

Students begin with different levels of initial mathematical knowledge, because in Spain university studies can be accessed in different ways, involving a considerable amount of work performed via face-to-face tutorials at the Mathematics Centre, Rodriguez (2008), aimed at filling in gaps in students' knowledge that would have been detected by means of an initial assessment test carried out on the first day of class.

The in-class activities were carried out using different teaching materials, including text books, Garcia (2007), and practical guides for Mathematica, which were made available to students through the http://portalevlm.usal.es/ portal and which are the product of the activity developed within the European EVLM project, Rodriguez (2007). All the teaching materials included in the portal's database and in the students' book were used freely by all students, also as a complement to non-contact activities when seeking information for carrying out the assigned tasks, both individual and in groups. In addition to interesting links and information about activities for the subject, the portal contains a blog referring to the subject, through which tasks related to the field and suggested by the teacher and/or student groups were carried out, also offering the possibility to perform on-line enquiries using the Virtual Mathematics Centre linked to the portal.

The methodology used was eminently practical, the teaching process being focused on problem-solving. Here there were very few theoretical explanations and these always took place in activities carried out in large groups.

The materials used in the practical classes with Mathematica were established by the teacher responsible for the subject and included the following: representation of functions, Taylor polynomials, interpolation polynomials, numerical methods for solving nonlinear equations, exact and approximate numerical integration, and first-order ordinary differential equations. Some of the students' papers, made in groups of a maximum of three, consisted of the documented elaboration of processes using Mathematica, including the description of the algorithms used.

The assessment process consisted of examinations (one every month) involving the solution of 2 problems, with the possibility of using reference books, which accounted for 40% of the final grade. In addition, students handed in a total of 9 papers, drafted individually or in groups of a maximum of three, accounting for 60% of the final grade. Retake activities focused on an additional test for students who had not achieved the established goals. The assessment method used evidently involved a huge amount of correcting for the only teacher of this course (more than 1000 papers).

This continuous assessment method (papers+exams) has contributed to the spectacular improvement in students' performance. Upon comparing the results of former teaching methods, where assessment consisted of a single end-of-course exam, with the results of this academic year, it is seen that the number of students who have followed the subject has increased considerably (30% with the previous teaching method as opposed to 90% with the current one), and the percentage of success has increased slightly (70% for the students who took the exam under the previous teaching method as opposed to 75% of the students assessed under the current one). This conjunction of percentages means that the number of students who have had to retake the subject has decreased noticeably. In this sense, a deeper study of what the methodological change implies will be required when more data on the new degrees become available.

Conclusions and suggestions for the future.

Based on our experiences, we shall offer some conclusions and ideas for the design and greater chances of success of mathematics subjects in Engineering adapted to the EHEA.

Autonomous learning is appropriate for algorithms and processes once the basic concepts have been understood.

Writing algorithms with accuracy and implementing them on Maxima or a CAS contributes to the development of important competencies for a student of Computer Engineering.

Students' daily work contributes to an improvement in the level of knowledge and understanding of mathematical concepts, and at the same time a much better performance is achieved.

In the second experiment the results have been better. Note that in this case the students have not the opportunity of pass the subject with a single end-of-course exam.

The teacher's workload may well be excessive if the number of students assigned to his/her class is high. This will harm the continuous assessment process once the teacher's initial enthusiasm has worn off.

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The Use of Technology in Mathematics - approaches used and lessons learnt at ITT Dublin.

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Abstract

Whilst addressing the mathematical learning needs of students pursuing technical courses is multi-faceted, this paper will describe several uses of technology made by Mathematics staff in ITT Dublin to support students learning of mathematics and it will also address the lessons that have been learnt in how best to use these technologies. The first example of technology described is the use of the Calmat mathematics learning environment as a tutorial support tool, as a complete learning environment for different student cohorts and as a mechanism to facilitate the re-engagement of mature students with mathematics. Secondly, the use of Maple TA to construct an alternative approach to engage first year marketing students with mathematical learning is described. Finally the use of an e-book as a part of a preparatory mathematics course is outlined. In terms of lessons learnt from these uses of technology to support mathematics the following are addressed: the promotion of active learning, the need to use the same technology differently depending on the profile of students being supported and the importance of providing suitable cognitive scaffolding so that students reflect appropriately while using technologies so as to optimise their mathematics learning.

Introduction.

The Institute of Technology Tallaght is located in South Dublin County and was established in 1992. The Institute caters for a student population of approximately 3,700 full- and part-time students and offers a wide range of programmes from Higher Certificate, Ordinary Degree and Honours Degree to Masters Degree and Doctoral level. The mathematical learning needs of students studying on technical courses in ITT Dublin are diverse and in particular there is a significant percentage of students in year 1 groups facing challenges in mathematics. With this in mind the ITT Dublin Mathematics staff engage in a range of teaching and learning strategies to promote student engagement and active learning in mathematics. Spreadsheets and computing environments such as Matlab are incorporated as part of the learning in mathematics modules on technical courses. Also, Moodle-based quizzes are used to emphasis the learning of key mathematical techniques as described by Marjoram et al (2008). However, in addition to these uses, technology has been utilised in ITT Dublin to create learning environments which replace or act as a complement to the traditional lecture delivery mode for Mathematics courses, and which encourage active learning by the student. This use of the Calmat mathematics learning environment, Maple TA and an e-book to create three such environments which increase student engagement are described in the sections following.

1.0 Use of Calmat with coupled Primer and Mathematics 1 courses.

The Calmat mathematics learning environment, developed in a project at Glasgow Caledonian University, has been in use in ITT Dublin since 2001. At first it was used only as a tutorial support tool. In 2002 it was decided to set up the entire semester 1 mathematics module, Mathematics 1, so that it could be delivered completely in a computer laboratory using Calmat. The Mathematics 1 course through Calmat was used at IT Tallaght with one group of full-time Mechanical or Electromechanical engineering students in each of 2002, 2003 and 2004 whilst the other groups followed a traditional lecture-driven delivery. The Semester 1 course through Calmat was also used for a group of part-time students in those years. The level of engagement and enthusiasm for this mode of learning using Calmat for this course was markedly different between the full-time and part-time students, with the full-time students expressing a dislike for the structure but the part-time students enjoying and thriving in the self-paced course embedded in Calmat.

In 2004 the School of Engineering at IT Tallaght won significant funding from the Higher Education Authority Retention Fund for the design and delivery of a flexible Higher Certificate in Electronic Engineering (FLASHE) aimed at meeting the changing requirements of our part-time students, Robinson et al (2007). Key to this was the aim to bring prospective students back in contact with mathematics in a way which would boost their confidence in their mathematical ability. With this in mind, two mathematics courses are offered to the incoming FLASHE students; a pre-entrance Primer enabling mathematics course and the normal Semester 1 Mathematics course. From the experienced gained in using Calmat to facilitate delivery of Mathematics 1, the desire was to have students re-engage with a mathematics learning environment which is different to the 'chalk / talk / get left behind' they last experienced. The linked pair of Primer mathematics and Mathematics 1 courses were designed to be delivered using Calmat only.

At the beginning of the FLASHE program, after a session introducing the students to the basics of calculator manipulation, they take a short Calmat-generated diagnostic test which benchmarks where they are mathematically. The test covers the very basics of number, basic algebra and properties of linear data. If the students achieve a high 80% threshold (note this threshold was 90% for the first cohort) on this diagnostic test they can (if they want) proceed directly to the Semester 1 Mathematics course, as this has been set up to be delivered using Calmat also (see Figure 1).

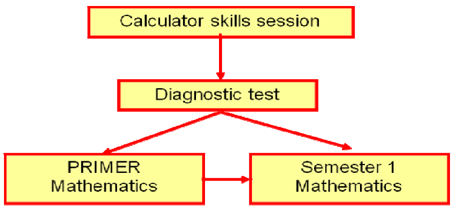

Figure 1: Flow diagram of how students enter onto Mathematics modules initially

The Primer course is of 24 hours duration structured into two hour sessions over a 12 week semester. The students start with concrete concepts in number manipulation, are introduced to a simple level of abstract symbol manipulation in the brief introduction to algebra, and finally study graph manipulation, a topic that will have easy application to engineering. The course has four tests – one every three weeks. This is used both to show the students how they are doing (they get instant feedback from the Calmat system) and to pace students through the material. Unfortunately Calmat is not web-based but nonetheless students were also given a copy of Calmat on disc to do two further hours per week work at home. The students are asked to complete a reflective diary entry after each Calmat session. The Primer course is optional, no credit is awarded for it and a poor mark does not stop progression to Mathematics 1.

The Mathematics 1 course is also entirely done within Calmat in the same computer laboratory as the Primer course. This means that students need not be split up for their mathematics courses and one mathematics lecturer can be assigned to the whole group. There are two assessment modes for the Semester 1 course. In the first mode students study the material and do four tests. These count for 30% of their total mark and they do the exam in the normal way, counting 70%. In the second mode students must complete 70% of all Calmat tutorials set (148 tutorials comprising 3-4 questions each), must get 100% on 24 Calmat Assessments (which may be retaken and comprise 6-12 questions on a lesson) and, finally, must get more than 70% on each of four tests (15-20 questions on groups of lessons). Completing all these requirements gives them at minimum B+ grade and they need not sit the end-of-module written examination.

1.1 Student engagement and performance in Primer/Mathematics 1

Table 1 shows the number of students who started work on Mathematics 1 or Primer material in a particular academic year beginning in September. The Primer students would typically finish Mathematics 1 after two semesters e.g. Primer students in September 2005 should finish by May 2006. Access course rules allow students to start their second year without finishing year 1 and several students labelled as "not complete" in the September 2008 cohort are still in the college. Primer students of the September 2009 intake are not yet finished but eight are currently doing the Mathematics 1 course.

	No. of Students **starting.**		No. of students **passing** Mathematics 1				
Start date of student on Mathematics 1 (S1) or Primer Maths (P) :			**Without** sitting exam		**Having** sat exam		
	S1	P	S1	P	S1	P	**Commentary on discrepancy between number starting and passing**
Sep-05	**6**	**25**	**4**	**12**	**2**	**7**	3 didn't complete , 2 deferred, 1 didn't register.
Sep-06	**16**	**13**	**15**	**12**	**0**	**0**	2 didn't complete or sit paper
Sep-08	**15**	**11**	**12**	**6**	**0**	**0**	1 P left, 7 not complete yet
Sep-09	**17**	**12**	**14**	**---**	**1**	**---**	2 failed exam Jan 2010, 8 P following S1 (May2010)

Table 1: Performance of students on Mathematics 1

As can be seen from Table 1 above the optional nature of the Primer course is becoming an issue. Several students in 2008 and 2009 have insisted on doing Mathematics 1 immediately, even though they got less than 80% on the diagnostic test and were advised to sit the Primer course. This could be a factor in the non-completion rate for the 2008 intake as the three of that Semester 1 group who have not completed did not reach the 80% diagnostic test mark. These three students also made no attempt to complete their work in the second semester of that academic year but attended the Semester 2 mathematics course.

1.2 Conclusions on using Calmat for FLASHE mathematics

As will be noted later in the paper in the context of Maple TA, the key change that the use of a computer based environment such as Calmat is that it allows lecturers to alter their role with the students from one of 'sage on the stage' to one of a 'guide on the side'. The experience of the mathematics lecturers using this Calmat scaffolded approach to Mathematics on the FLASHE program since September 2005 is that adult learners seem willing to accept a Calmat-based, assessment-driven course as it has the flexibilities they require and seems to address some of the anxiety issues they face coming back to learning mathematics. Concern prompted by the student performance issues seen in 2008 and 2009 has led to the optional nature of the Primer course for those students get less 80% on the initial diagnostic test being under review.

It is also worth noting that there was concern in the recent past regarding the viability of using the Calmat platform to deliver these courses as the Calmat group at Glasgow Caledonian University which supported the platform was terminated in 2005. However Calmat Version V.0 in both Student and Institutional edition has recently become available free under a Creative Commons Share-alike License. Its next, web-based, reincarnation, MELA, which was not fully developed before the Calmat project was discontinued, has also been made available along with its source code. These will be available initially through the FETLAR project and the UK JORUM Library. Unfortunately, Calmat Version V.0 only runs on 32 and 64 bit machines so its lifetime is limited, though it will still be useful for several years in many schools, homes and colleges. The FETLAR project plans to re-use many of its materials, especially assessment materials.

2.0 The use of Maple TA with year 1 Business Mathematics

Maple TA is a web-based package for testing and assessment, which can be used as a teaching tool in class and can also be accessed remotely off-site. Instructors create their own content consisting of a database of questions on which assignments, exercises and tests are built and tailored to the specific needs of the course being delivered. Students access the various assignments for practice and credit. One of the many strengths of Maple TA is that it allows the instructor to design a question template using parameters whose values can be randomly-generated, enabling students to try many different versions of the same basic question. In addition to the wide selection of inbuilt question types, the underlying Maple symbolic manipulation engine can be harnessed to create an extremely wide variety of questions of all levels of complexity. Maple TA records all results obtained by students accessing the exercises earmarked for credit and this information is easily transferred to a spreadsheet.

The pilot project which introduced Maple TA into ITT Dublin began in March 2008, preceding course delivery to first year students in the Department of Marketing in the winter semester of 2008/2009. The first phase of the project was the time-consuming, labour-intensive preparatory work of constructing the database of questions to accompany the material for the thirteen week Business Mathematics 1 module undertaken by the marketing students. Over 100 questions, many involving random parameters, were created for the five topics: Calculus, Statistics, Regression and Correlation, Probability and Discrete Probability Distributions. During course delivery a series of 21 exercises was subsequently built from the database of questions, together with three tests interspersed at key points of the semester. The exercises could be repeated as often as desired and the results obtained contributed to the 30% of the overall mark allocated to continuous assessment for the course. The remaining 70% could be obtained in two possible ways. The first option was the traditional end of semester written exam. The second option was allowing students who had reached a threshold of 50% on the three Maple TA tests taken during the semester to pass without taking the written exam at the end of the module.

Several reasons prompted the use of Maple TA as a tool to aid the delivery of this course. Firstly, noting the experience gained from using Calmat for the delivery of the Primer/ Mathematics 1 courses, it was recognised that using computer-based exercises has the potential to encourage students to engage with the course material and learn actively, rather than to remain passive in a traditional lecture. Similar to the Calmat Primer/Mathematics 1 course, active learning is also encouraged by the fact that students earn credit for all of the work they undertake on the system. Secondly, independent learning is facilitated by the opportunity to repeat, or revise, exercises previously attempted and by the option to remotely access assignments outside normal class times. Thirdly, on completion of an exercise, or a test, students can obtain instant feedback on their performance. The instructor can also view this information and can assess individual, or class, progress. As Maple TA does the tedious work of marking, the instructor can reduce, or eliminate, this work. (It is worth noting that Maple TA allows the instructor to grade students' work by hand in cases where it is necessary, or appropriate, to do so.) Fourthly, students are required to attempt the Maple TA exercises on a frequent basis, which incentivises attendance since students need to keep up to date with the material required to attempt each exercise as it arises.

2.1 Student engagement and performance in Business Mathematics

Delivery of the Business Mathematics module to the students in their first year on the Department of Marketing programmes began in September 2008. There were two groups who took the course, each consisting of approximately 35 students. Of those who completed the module, 58% qualified for an exemption from sitting the module written exam in January 2009. All students were later asked to provide feedback on their experience of using Maple TA. In particular, the students were asked to rate the overall helpfulness of the package for learning, rate its usefulness for testing, rate the opportunity for more practice and rate the facility for instant feedback. Table 2 below shows the percentages of students who gave an above average rating for these properties.

Helps Learning	Good For Testing	More Practice	Instant feedback
71%	62.5%	83%	91%

Table 2: Above-average survey responses on the use of Maple TA

Significantly, the option to access Maple TA remotely was considered to be important by 92% of the students surveyed. Overall, only 17% of students felt that Maple TA compared poorly with the learning experience of traditional lectures. In a straight choice between the two options 48% favoured Maple TA, compared with 30% for the more traditional, lecture and paper-based approaches to learning and assessment. Comments submitted on the survey were largely positive about Maple TA and supportive of its use as a teaching tool. Students were less positive about the use of Maple TA for testing. The main concern expressed was that partial credit for answers was not given. There was a perception that answers were completely right and received full marks, or were completely wrong and received 0 marks. (In reality almost all test questions were divided into parts so that partial credit was given.) A linked concern was the need for absolute accuracy in entering answers; for example, if a capital letter was accidentally entered instead of a small letter the computer graded the answer as incorrect, whereas a human marker would be more forgiving. Only 54% of students regarded Maple TA as an improvement on traditional paper-based testing.

In addition to its general popularity amongst students, it was interesting to observe that similarly to the Calmat-based Primer / Mathematics 1 courses, class dynamics were changed by using Maple TA. It was clear that students often came to see the instructor in the Maple TA sessions as a collaborator helping them to negotiate the challenges presented by the computer, hence fostering a more cooperative environment and improving the working relationship between student and instructor. However, practical and educational challenges also arose. The delivery of lecture material relevant to upcoming Maple TA assignments was more difficult in the computer laboratory as many students were regularly distracted by the opportunity to use web browsers etc. A practical solution to this was to cover lecture material before computers were even switched on or to complete the Maple TA session and switch machines off before covering new material. When tackling computer-based assignments, students were more inclined to look for individual attention, asking questions of the instructor in preference to reading the notes and summaries provided before launching the assignment. For this reason Maple TA is probably better suited to smaller groups where the instructor has the opportunity to devote more time to individuals. A further issue was the retention of concepts/techniques practiced during Maple TA sessions. In many cases the exercise was successfully completed but the student quickly forgot the basic theory underpinning the problems. In many cases learning seemed superficial rather than deep, pointing to the need for careful use of such packages. It would seem prudent to ensure that students first think about what they will do in a computer-based exercise then carry out the exercise and, finally, reflect later on what they have undertaken on the computer.

In terms of students' performance, the use of Maple TA for the Business Mathematics 1 module brought a marked improvement. In January 2008, before Maple TA was introduced, 39% of students recorded a failing F grade for the module at their first attempt. In 2009, using Maple TA for assessment, 27% recorded an F grade. Also, the average mark increased from 40.8 in 2008 to 51.6 in 2009 and an overall upward shift in the grade distribution was evident. Since two distinct testing methodologies were applied, it is not possible to conclude that student learning has been improved by the use of Maple TA. However, there is much to recommend its use as a valuable teaching tool and to encourage further study to assess its impact. To this end Maple TA is now been used for a second year in succession in the Department of Marketing where its continued use is envisaged. It is also worth noting that as the result of staff training on the package of the pilot programme on Maple TA in ITT Dublin and the positive results to date, there are plans to also use Maple TA for the first year Electrical Science module in the Department of Electronic Engineering.

3.0 Use of an e-book

In 2008 the Department of Lifelong Learning at ITT Dublin identified a need to provide a mechanism for Mathematics revision for mature students who expressed an interest in a range of level 7 degrees in technical areas at the institute. Unfortunately the 12 week Primer model was not an option as these students would not have registered yet with the institute and so could not have access to the necessary computing facilities needed to run the Primer model using Calmat. As a result of this a short 'blended' course was designed and delivered. The course has a four-week duration with the students having two contact hours each week, complemented by two hours of self directed learning. It is structured around an e-book package by Croft and Davision (2009). The e-book package consists of a text book complemented by a lecturer-customised course, hosted at the MyMathLab website, to which the students are given an access key. The main aspect of the website used in this instance is the customisation of the exercises and materials from the textbook to correspond to the topics to be covered during the preparatory mathematics course. The most useful feature for the students' self-directed learning is the facility to do the questions on the website in a tutorial mode which displays whether each student input to an answer is correct and provides hints and answers as required by the student. In this way the use of the e-book provides the independent learning aspect in between classes and also provides the potential for students to continue working on their mathematics revision during the summer months before September entry to a degree course.

There are several advantages for the college in using the e-book to structure this course. Firstly, it is web based so there is no need for potential students to have access to college computers, removing any registration / computer logging-on issues. Secondly, ITT Dublin now has a Mathematics Preparation package ready to give to potential mature students who express anxiety about mathematics as an impediment to starting a course. Finally, the cost of the technology is transferred to the student.

3.1 Student engagement

The first time the course was offered 58 students enrolled and received the e-book pack. Of these 23% made active use of the web exercises aspect of the e-book (over five hours of usage) with another 17 % making some use of this aspect (between two and five hours) and 23% of the students making a limited use of this aspect (between one half and two hours). The maximum number of hours a student made of the e-book aspect was 17. It is worth noting that that students continued to use web exercises aspect of the e-book materials during the summer months of 2009 after the four-week course was completed. Some students who had attended the preparatory Mathematics course and are now on year 1 engineering programmes at ITT Dublin were surveyed in December 2009. These students expressed the view that the e-book combination of traditional text book and web-based exercises had been an invaluable aid to getting them ready for their courses.

Concluding comments

As seen from the three individual uses of technology to support mathematics at ITT Dublin presented above, the first lesson learnt is that active learning is promoted. Secondly, assigning credit to work completed in the computing environment incentivises the student. Thirdly, the same technology may need to be used differently depending on the profile of students being supported. Also practical considerations such as time constraints and access to computing facilities may require different software to be used to create the appropriate learning environment. The principal characteristic shared by all three platforms used at ITT Dublin is that they change the dynamic between instructor and student creating the opportunity for a greater collaborative approach to the learning of Mathematics. Finally, the provision of suitable cognitive scaffolding so that students reflect appropriately, and so optimise their mathematics learning while using technologies, has been identified as a key area for future work in ITT Dublin.

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Teaching Unusual Applications in Engineering Mathematics: Students' Attitudes

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Abstract

This paper presents results of three studies on using an innovative pedagogical strategy in teaching mathematical modelling and applications to engineering students. The studies analyse engineering students' attitudes towards non-traditional, for them, contexts in teaching/learning of mathematical modelling and applications: environment, business and epidemics. The first study deals with using differential equations in the environment and ecology. The second study deals with using linear programming for maximizing profit and minimizing expenses for a company. The third study deals with using differential equations for making a prediction of the number of cases in an epidemic. Analysis of students' responses to questionnaires, their comments and attitudes towards the innovative approach in teaching are presented in the paper.

Introduction

Many researchers and practitioners consider skills in mathematical modelling to be different from skills in mathematics. "Model building is an activity which students often find difficult and sometimes rather puzzling. The process of model building requires skills other than simply knowing the appropriate mathematics" (George, 1988). Some relationships between students' mathematical competencies and their skills in modelling were considered in Galbraith & Haines (1998) and in Gruenwald & Schott (2000). Obviously, there is a link between mathematical modelling and solving application problems. We support the view that solving application problems can be considered as a subset of the mathematical modelling process which can be described as "consisting of structuring, generating real world facts and data, mathematising, working mathematically and interpreting/validating (perhaps several times round the loop)" (Niss, Blum, and Galbraith, 2007, pp.9-10). Our perception of application problems again is similar to that expressed by Niss, Blum, and Galbraith (2007): "Standard applications: Typified by problems like finding the largest cylindrical parcel that can be shipped according to certain postal requirements, standard applications are characterised by the fact that the appropriate model is immediately at hand. Such problems can be solved without further regard to the nature of the given real world context. In our example, this context can be stripped away easily to expose a purely mathematical question about maximizing volumes of cylinders under prescribed constraints. So, the translation processes involved in solving standard applications are straightforward, that is, again, only a limited subset of the modelling cycle is needed (p.12).” An interesting question is how students in general and engineering students in particular react to the context of a modelling task. Do they value and learn modelling skills if the context is unusual for them? Can they rely on their past experience and common sense if the context is unfamiliar for them? Engineering students’ attitudes towards environmental and ecological applications of mathematics were investigated in Klymchuk et al (2008). The role of entrepreneurship in engineering education was studied in Gruenwald & Krause (2006). In this paper we present the summary of the study by Klymchuk et al (2008) and two more studies dealing with unusual contexts for engineering students. We also discuss possible implementations for the education of engineering students. We agree with Kadijevich who pointed out at an important aspect of doing even simple mathematical modelling activity regardless of the context by first-year undergraduate students: “Although through solving such … [simple modelling] … tasks students will not realise the examined nature of modelling, it is certain that mathematical knowledge will become alive for them and that they will begin to perceive mathematics as a human enterprise, which improves our lives” (Kadijevich, 1999).

The First Study

The first study was conducted with 2 groups of students. The first group consisted of the first-year engineering students studying mathematics courses at Wismar University of Applied Sciences, Technology, Business and Design, Germany and Auckland University of Technology, New Zealand. The students did a project on ecological and environmental applications of mathematics. On the one hand, the context was not directly related to engineering. On the other hand, chances are that most of the graduates in engineering will be dealing with mathematical modelling of the environmental systems in one way or another in their future work because nearly every engineering activity has an impact on the environment. The total number of students who completed the project was 147 in both universities. The number of students who answered the anonymous questionnaire was 63 so the response rate was 43%. Participation in the study was voluntary. After completing the project the students were asked to answer the following two questions:

Question 1. Do you find the project to be practical?
Question 2. Do you find the project to be relevant and useful for your future career?

Brief statistics and common students’ comments are presented below.
Question 1. Practical? Yes – 48%. Selected students’ comments were: “The models describe the real world”, “A good way of increasing students interest in the subject”, “It was so helpful for my other subjects”, “I didn’t realize modelling is used for fishing quotas. It also helped me realize the effects of sneaky illegal fishing (which most of us have done)”.
Question 1. Practical? No – 52%. Selected students’ comments were: “It is not possible to calculate the nature”, “It did give a practical situation but you bearly think about that at all when doing the assignment”.

Question 2. Relevant for your career? Yes – 35%**.** Selected students' comments were: "Mathematics is the base needed to go into the Engineering World, so it will help a lot", "In engineering, we will be dealing with these kind of situations", "We are more motivated to solve such real problems than working with dry examples", "Everything you learn is bound to be beneficial at some point".
Question 2. Relevant for your career? No – 65% . Selected students' comments were: "I don't see how it relates to mechanical or electrical engineering" (most common comment), "I don't compute formulas, I have to calculate beams…".

The second group consisted of a mixture of year 2-5 engineering students studying the experimental course Mathematical Modelling of Survival and Sustainability at Wismar University. The students solved a number of ecological and environmental models in their individual and group projects among models in other contexts. After completing the course the students were asked to answer the question "Do you think this course is suitable for engineering students and if so, why?" There were 25 students in the course. The response rate was 100%. Participation in the study was voluntary. All 25 students answered 'Yes' to the above question. The main two reasons were:

- Improving knowledge in mathematics, Matlab and mathematical modelling that is useful for engineering – 23 (92%). Typical students' comments were: "You consolidate your mathematical knowledge", "Raise knowledge about differential equations and especially how to build them", "Increasing skills in Matlab", "In my opinion many problems or predictions in the 'engineering world' could be handled/solved with the techniques that you can learn here", "Because you learned how to put some problems into a mathematical system", "To see new ways (models)", "In the course you can better make a statement for normal problems about the life", "Because I could improve my understanding for differential equations", "The mathematical models all around us and the true way for an engineer is to understand how a model from the nature reacts if you change one parameter".
- Practical and interesting – 10 (40%). Typical students' comments were: "To get practical problems", "It is very important to use practical part in the course as it is done here to help students to understand what are they going to do in their future jobs", "Of course it deals <u>not</u> with typically engineering problems but after all it was an interesting subject", "Engineering students can apply their knowledge and broaden their horizon", "It is nice to see we can use differential equations in other areas", "I think that every subject which has a lot of practical things is very useful. This mathematical course was very useful for me and I think, that in our university everyone must study mathematics in this way".

There was a big difference between the students' responses in the two groups about the relevance of the suggested context of applications. Only 35% of the students in group 1 (first-year students) indicated that the environment/ecology context is relevant for their future career whereas 100% of the students in group 2 (year 2-5 students) commented that the course was suitable for engineering students. One of the reasons for such a difference might be the difference in maturity. Another reason might be the difference in students' mathematics background. It was reflected in the exam performance. The pass rate of the first-year students in their maths courses was around 50%. The pass rate of the year 2-5 students in the modelling course was 100%. Moreover, all 25 students from the second group received excellent or very good final grades. From informal talks to the students of the second group we received a strong indication that their enthusiasm and positive attitudes towards the course significantly contributed to their high performance in the course and very positive attitudes towards the unusual contexts. They were mature enough to value the new knowledge in mathematics and modelling they received from the course that can be applied in engineering (92%). They also enjoyed the practicality of the course that enhanced their problem solving skills (40%).

The Second Study

The second study was conducted with 2 groups of engineering students – Bachelor and Masters – studying an Operations Research course at Wismar University. The students studied a variety of linear programming models dealing with maximizing profit and minimizing expenses. On the one hand, the (business) context of the course was not directly related to engineering. On the other hand, chances are that most of the graduates in engineering will be dealing with mathematical modelling of optimization problems in one way or another in their future work because nearly every engineering activity has commercial implications. The course did not require any special knowledge from economics or business studies. It was based on a spreadsheet modelling and had many real practical applications from the business world. Excel was used as the main program being a standard for small businesses and the chosen program of the course textbook. Apart from using the Excel Solver tool the students learnt many useful Excel functions thus mastering their spreadsheet modelling skills. There was much attention put into enhancing students' generic mathematical modelling skills while doing sensitivity analysis and discussing assumptions and limitations of each model. After completing the course the students were asked to answer the question "Do you think this course is suitable for engineering students and if so, why?"

There were 16 students in the first group. All 16 students answered "Yes" giving the following reasons:

- Benefit for the future work - 15 (94%). The typical students comments were: "Because in all jobs in engineering you must maximize the profit and minimize the cost", "Companies want engineers who are trained in financial questions of small companies", "It is also useful for independent engineers to optimize their own profit", "In small firms engineers are responsible for calculating the costs and make sure the company makes profit. And also in bigger firms engineers are used for management positions thus a basic idea of business and optimization is very useful", "To know how to maximize the profit is very important for me. This could be a help for us if we have our own firm later on".
- Enhancing the skills in Excel - 7 (44%). The typical students comments were: "It is good to know how Excel works", "It is very good because the skill to handle Excel

is very useful. We use Excel very often in our mechanical engineering", "To learn to work with Excel and to use its 'hidden' functions".

There were 17 students in the second group. All 17 students answered "Yes" giving the following reasons:

- Benefit for the future work - 8 (47%) The typical students comments were: "Students learn to fix problems in a way that is quite often used in 'real life' business", "It is good for me to know how to optimize a problem and it will make my chances bigger on employment market", "It is very suitable because it teaches to implement the problem which is similar with the problem in a company".
- Enhancing the skills in Excel - 10 (59%). The typical students comments were: "This course shows me new ways to work with Excel and solve mathematical problems", "Excel is software that almost every company owns. For that reason it is quite useful to know this program very well", "Learning basics and extended applications in Excel".

All 33 students in both groups indicated that the course was suitable to engineering students. They appreciated the practical nature of the models, the opportunity to enhance their problem solving, modelling and computer skills. Bachelor students saw more benefits from the course for their future job than Masters students: 94% versus 47%. One of the reasons for this difference could be that some Masters students were planning to do PhD study so the relevance of the course to their employment was not a priority at that stage.

The Third Study

The majority of the participants in the third study were first-year engineering students from Wismar University with several second and third-year students majoring in applied mathematics from Auckland University of Technology. 90 questionnaires were distributed over 2009 and 2010. 48 responses were received so the response rate was 53%. It was a self-selected sample. The study was about differences in prediction from 3 familiar models (linear, exponential and logistic) in an unfamiliar context – an epidemic of an infectious disease. One of the questions in the questionnaire was about students' attitudes towards the unusual context: "Are you interested in learning more about epidemic modelling and possibly doing research projects in this area (e.g. modelling of the spread of swine flu)? Why?" The students' answers are below.

Yes – 4. An interesting topic (4), important for the science of viruses (2). No – 39. Not my area of interest (18), lack of time (8), this is only making me panic (3), don't see the point of playing with numbers or equations which are not correct (2), it is going to have too many factors and the predictions may not be that reliable (1), not enough knowledge in mathematics (1).

Very few students (9%) reported that they were interested in doing a research project in epidemic modelling. However, in spite of the unfamiliar context most of the students did very well in answering the other questions from the questionnaire – on the reasons for differences in predictions from the three models and the ways of improving the accuracy of the predictions as it was reported in (Narayanan, 2009).

Implications for Education

The majority of engineering students participated in the three studies valued practical aspects of application problems in spite of the non-traditional contexts. They also valued problem solving and modelling skills they learnt that could be applied to problems in an engineering context. This was more the case for more mature students. The main lesson for us as lecturers was: students' feedback should be taken into account when designing curricula for their study.

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Promoting active learning in Mathematics - a 'Problems First' approach.

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Abstract
The engagement of students in active learning in mathematics is an ongoing challenge. As a sudden move from a traditional mode of mathematics delivery of a complete module to an enquiry based approach for the module appears daunting, an approach of ongoing incremental change in delivery is under investigation at the ITT Dublin. This paper describes a '*Problems First'* project in which mathematics lecturing staff identify a section or aspect of a Mathematics module which is suitable for modification of approach from a traditional one to one in which the students encounter sets of mathematical problems first. The students attempt these problems using appropriate materials, aids to reflection and other inputs. This is followed, as needed, by some lecture input to ensure that the necessary learning outcomes have been achieved.

The *'Problems First'* project was implemented on three modules. In the first module, a mathematics module in the first year of a degree in mechanical engineering, problem sheets were given to the students first for the first six weeks of the module. These problem sheets were augmented by notes given to the students and by input from the lecturer. As well as developing the materials to enable this approach to be undertaken, a reflective diary template for the lecturer was developed and maintained for this section of this module.
In the second module, a first year mathematics module on a pharmaceutical science course, the implementation consisted of an unseen problem being given to small groups of students early in the semester. The students were directed to online preliminary notes and library books and were encouraged to discuss the problem with the lecturer at any time.
In the third module a '*Key Skills Testing in Mathematics'* Moodle-based testing approach was used to reinforce key mathematical skills needed for a particular semester of engineering and is implemented as part of a Mathematics module for a 3rd year electronic engineering course. An additional reflection sheet was deployed to help students reflect on, and take action on, key areas highlighted for them by their performance in these '*Keyskills'* tests.

In this paper the implementation of this 'Problems First' approach in these three modules is outlined. Also presented in this paper is the student feedback relating to the facilitation, documentation and learning outcomes of the project as well as any effects on group dynamic during the project**.** Finally, the effectiveness of this approach is examined by comparing student performance in continuous assessment and end of semester examination in these modules in January 2010 with the performance data for similar students from previous academic years.

Introduction
The work reported in this paper is the result of ongoing efforts to improve teaching and learning in ITT Dublin with particular emphasis on a move to more inductive teaching learning as outlined, for example, by Prince and Felder (2007). As part of this work projects are funded internally to encourage staff to engage with a process of change of teaching methodology by embracing, in the first instance, small incremental changes in approach which to greater active learning using techniques such as those as suggested by Felder and Brent (2009), for example. In particular the authors of this paper were awarded a grant for a *SIF CONTINUE Innovations in Teaching, Learning, and and/or Inclusive Education Project*

in 2009 entitled *Problem First Learning in Mathematics*. As part of this project mathematics staff sought to identify areas or aspects of an existing mathematics module that could be modified in such way as to improve student engagement with the module, increase student reflection on their learning and hence lead to improved learning. Initially staff engaged in the project discussed which aspects of the 3 modules would be modified and also designed a questionnaire for the evaluation of the student experience of this modified approach. The details of the implementation and outcomes of this project in each of the 3 modules are outlined in the following sections.

1. Problems first approach in Mathematics 1 of a Mechanical Engineering Degree.

The first module for which this Problems First approach was adopted was the Mathematics 1 module in the first semester of a level 7 degree in Mechanical Engineering (see The National Framework of Qualifications). At the SEFI 14 (MWG) conference Ole Christenson (2008) outlined the benefits for mathematics learning when certain modules in Aalborg University were delivered in a PBL mode at his plenary talk. During a workshop at the same conference Christenson also outlined to the third named author how other mathematics modules at Aalborg University are arranged to increase student engagement. He described how the materials to be studied are introduced via problem sets given to the students to work on first and how this is followed with a subsequent lecture session to deal with any issues arising and to recap the material. Following on from this, the lecturer for the ITT Dublin Mathematics 1 module, being constrained by logistical reasons from the PBL approach for the whole module, undertook to use a similar problems first approach for the six review sections of the module. This review material constitutes 40% of the overall course and aims to achieve a thorough revision and consolidation of key basic mathematical topics that have been encountered by students prior to entry to higher education. To do this the students were first given a problem sheet for each review chapter. These were augmented by notes given to the students and where necessary by input from the lecturer. This in essence reversed the standard approach of lecturing the topics first and then giving problem sheet work. In most scheduled class sessions the students worked on problems with the lecturer helping the students on an individual or small group basis when his help was sought. Certain class sessions were devoted to recapping and summarising key concepts that had arisen from the review problem sheets. As well as the development of the materials needed for this approach, a reflective diary template for the lecturer was developed and maintained. This reflective diary was completed after each session and captured information such as the topic being explored, the lecturer reflections on the mood of the class, interaction in the class, what worked well in the session, new insights gained regarding the teaching/learning of the topic and finally one thought to carry forward.

A much higher degree of active engagement and questioning by the students occurred in the mathematics sessions in the problems first mode. It was also observed that the level of student engagement continued throughout the rest of the semester during the sections of the module that were delivered in the traditional ‘lecture first/problems later’ mode. At the end of the semester a student survey was conducted regarding the implementation of the Problems First approach using a 4 point Likert scale: Agree Strongly, Agree, Disagree and Disagree Strongly (Carlisle and Ibbotson 2005). The replies to the 12 questions are outlined in Table 1 below. The replies illustrate that the most students agreed or agreed strongly in most statement categories. In particular there was strong agreement regarding the materials used, the ease of asking questions and confidence in answering exam questions in these topics.

Survey Statement	Student responses (n =21) for Mathematics 1(Mechanical)				Student responses (n=24) for Pharmaceutical Science			
	Agree Strongly	**Agree**	**Disagree**	**Disagree Strongly**	**Agree Strongly**	**Agree**	**Disagree**	**Disagree Strongly**
The structure of the problems first approach in maths 1 enabled me to take more responsibility for my own learning in maths.	29%	67%	5%	0%	26%	57%	9%	9%
The structure of the problems first approach in maths 1 was interesting and stimulated learning in maths.	5%	76%	19%	0%	8%	46%	33%	13%
I would be confident in answering exam questions in these topics.	19%	71%	10%	0%	13%	46%	17%	25%
I felt comfortable asking questions relating to the materials.	52%	48%	0%	0%	21%	58%	13%	8%
I found the materials presented to be easy to use.	38%	52%	10%	0%	4%	52%	35%	9%
I found the materials presented to be easy to follow.	62%	33%	5%	0%	0%	29%	58%	13%
Overall I achieved an improved understanding of the topics covered.	24%	67%	10%	0%	8%	54%	21%	17%
I was able to demonstrate a good understanding of the topics covered.	0%	84%	16%	0%	19%	33%	38%	10%
I believe that the problems first project approach in Mathematics helped create an atmosphere of active learning in mathematics.	19%	74%	5%	0%	20%	35%	35%	10%
I believe that the problems first project approach in Mathematics helped to improve a group dynamic (ie working with others) in mathematics classes.	24%	53%	16%	5%	35%	40%	20%	5%
I believe that the problems first project approach in Mathematics helped the lecturer provide more one to one help than he could in a traditional lecture structure.	14%	79%	5%	0%	32%	37%	32%	0%
I believe that the problems first project approach in Mathematics made it easier for me to ask questions in Mathematics lectures.	14%	84%	0%	0%	10%	55%	35%	0%

Table 1: ‘Problems First’ Evaluation Survey responses for Mathematics 1 (Mechanical) and Pharmaceutical Science Mathematics 1 cohorts.

To measure the effectiveness of the approach comparisons of student performance was undertaken, comparing the 2009 ‘Problems First’ group with the students from the previous three academic years. The prior mathematical attainment of the students in the two groups was benchmarked using the Mathematics grade (points) achieved by the students for their end of second level Leaving Certificate examination (for further detail on points see Central Applications Office website). The average Leaving Certificate Mathematics points score for the students in 2009 cohort was 38.24 points as compared to 42.5 points for students in the previous 3 years. It is to be noted also that the module Mathematics 1 was delivered by the same lecturer in each of the first semesters in the previous three academic years beginning in September 2006, 2007, 2008 and that the standard of continuous assessment components and the end of semester examination for Mathematics 1 in these years and for the 2009 ‘Problems First’ group were equivalent (as far as humanly possible). Finally it should be noted that all questions on the end of semester examination are compulsory.

The effectiveness of this changed approach on student learning is considered by comparison of two measures. Firstly, the improvement score in student marks between a one-hour diagnostic test (administered at the first lecture) and an equivalent diagnostic test re-take (administered after the review material has been completed) was undertaken. Secondly, the student performance on the end of semester examination in the two groups was compared. The average scores for student groups using these measures are presented in table 2 below.

	Average improvement between Diagnostic test and CA test results (%)	**Average end of semester examination result (%)**	**Number of students taking examination.**
2006, 2007, 2008 Students	40.10	50.92	48
2009 Students	25.57	58.14	21

Table 2: Mechanical Engineering student performance comparisons.

In conducting the statistical analysis of these scores a one–way ANCOVA (analysis of co-variance) was used. This was done to seek to remove the extraneous variability that arises from pre-existing individual differences in so far as these differences are reflected in the co-variate *mathematics leaving certificate points* and to the adjust the means of the *examination performance* and *the improvement on initial test scores* measure to compensate for the fact that the student cohorts might have started out with different mean levels of ability as measured by the *mathematics leaving certificate points* co-variate. The comparison of the improvement in student marks between the diagnostic test and the test re-take after the review material was a source of concern to the lecturer as it indicated that although in 2009 the students still improved their initial test scores the average improvement was less than that of the previous cohorts. A statistical analysis (general linear model with year as factor, leaving certificate mathematics points as co-variate and diagnostic to re-take test improvement score as response) showed that the variation in test improvement scores was significant at p=1% level. The comparison of the end of semester examination performance was a source of relief as it indicated that the 2009 cohort results were better on average than the examination performance for the previous years. A statistical analysis (general linear model with year as factor, leaving certificate points as co-variate and examination performance as response) showed that the variation in examination performance was significant at p=1% level.

The measures of effectiveness considered give considerable food for thought regarding using this Problem First approach. Given that the lower average improvement in student mark between the between diagnostic test and the retake test suggests that the implementation of the 'Problem First' approach in this module does not in the short term lead to improved learning of the review material, it would be unwise to claim that the this approach alone contributed to the overall improvement in effort and examination performance. However, as stated earlier, an improved level of engagement by the 2009 student cohort in comparison with previous years was observed by the lecturer throughout the semester. This commitment to work was very evident in the continued efforts of the students late in the semester. In particular statistically significant improved performance in examination questions for topics covered later in the semester was observed. For example, for the examination question on the last topic covered in the semester, there was an average improvement in mark from 36.20 % for previous years to 55.5% for 2009, which when analysed indicates a statistically significant improvement at a p =1% level. Therefore on balance the observed change in student engagement with the overall Mathematics 1 module and consequent improvement in end of semester examination performance has convinced the lecturer teaching the module to repeat the Problems First approach, subject refinement of the materials and approach being used for the review section of the module. The insights recorded in the reflective diary maintained by the lecturer during the initial implementation will be key in informing how the Problems First approach to this module should be modified.

2. Problems first approach in Mathematics 1 of a Pharmaceutical Science Degree.

The second module for which this Problems First approach was adopted was for a first year mathematics course for a level 8 degree in Pharmaceutical Science called Mathematics 1. The topic chosen for the project was the use of calculus to find and classify the critical points of a function, as this was one that students of previous years on the same course were reluctant to engage with. Results from the preceding year show that of the students who did attempt the question, the average score was 5.145 out of 20. A second motivating factor was that, due to course restructuring, the allocated hours for this module had been reduced by one hour per week, while the number of students in the class doubled. It was wished to offer the students an opportunity to engage with the lecturer outside of traditional teaching hours. It was hoped this would lead to a more general increase in lecturer/student interaction outside of the traditional teaching structure.

At the start of term the students were arranged in groups of three or four students. Each group was given an unseen problem to find and classify the critical points of a different function. The problem given was expressed using mathematical terms that were not explained and that most students would not have seen before. The duration of the project was 10 weeks. The students were assured that they did not need any previous knowledge to approach the problem. They were instructed to make use of any tools at their disposal (library books, the internet, friends etc). They were also given a series of five introductory tasks to complete. These tasks were designed to give the students a feel for the problem, to lead them to the starting point of the project and to allow them to check any results gleaned from differential calculus (the main part of the project) against the findings that would result from completion of these tasks. Finally, the students were encouraged to interact with the lecturer on a regular basis throughout the duration of the project. They were reminded regularly that the lecturer would be available for discussion before or after lectures, or at any other suitable time that could be arranged throughout the semester. Projects were marked in a two-stage process. The projects were first submitted to be marked and at this stage mistakes were outlined but not explicitly corrected. The projects were returned to the students who were then encouraged to re-submit the project in order to receive extra marks (half of any marks lost).

As in the Problem First project undertaken with the mechanical engineering group, at the end of the semester the students were surveyed using the same questions and Likert scale. The replies of the Pharmaceutical students to the 12 questions are outlined in Table 1 earlier in the paper. The replies illustrate that most students felt that they were able to take responsibility for their own learning and that they felt comfortable asking questions related to the material. There was strong disagreement that the problem presented was easy to follow. Over the 10 weeks of the project the lecturer noted that there was an overall reluctance by students to engage with the process, and some of the very weakest students found that they were totally lost. This was especially true in the case of several weak students who were in a group whose other members were of a higher academic standard. Instead of the group bringing the weakest member up to the level of the group, the weak member was marginalised and in the case of one student asked to be excused from the project altogether. Some preliminary work on group dynamics and group problem solving will be required in the future. Conversely, several very weak students engaged fully with the project and established a rapport with the lecturer that was of benefit in other areas of the course, and this lead to discussions with the lecturer on an ongoing basis regarding areas of the course outside of the remit of the project.

To measure the effectiveness of the approach comparisons of student performance was undertaken, comparing the 2009 'Problems First' group with the students from the previous academic year. Two measures were considered: firstly the performance of the students on question 5 in the end of semester examination (which dealt with finding and classifying critical points) and secondly the students overall grade on the end of semester examination. The average scores of the two student groups using these measures are presented in table 3 below.

	Average end of semester Question 5 result (%)	**Average end of semester examination result (%)**	**Number of students taking examination.**
2008 Students	5.154	41.79	26
2009 Students	5.67	34.7	21

Table 3: Pharmaceutical Science student performance comparisons.

A statistical analysis (a two sample t-test) showed that the variation in question 5 scores and in overall grade was insignificant at the 5% level. While it is disappointing that no statistically significant improvement was seen, it is perhaps of comfort that no deterioration was seen given the decrease in teaching hours on the module and the increase in class size. The lecturer teaching the module believes that the project was of benefit, both in an academic sense in terms of enabling the students to take more responsibility for their own learning, and also in a broader sense in that it enabled students to develop a range of skills that would not necessarily be encountered within the traditional lecture based delivery of material.

The main lesson learned from the project is that the students completed the introductory tasks almost universally and to a high standard. While these tasks were designed to bring the students to the beginning of the project, most students did not complete the rest of the project (the un-scaffolded main aim) to the same degree of precision. Another lesson learned was that most students failed to engage with the project until very shortly before the deadline (despite constant reminders and weekly encouragement). It was only at the arrival of the deadline that the majority of students finally engaged with the project. In future implementations of this approach an official weekly support session will be timetabled and there will be an intermediate deadline early on in the process. Also the final deadline will be earlier in the semester (three weeks before the end of term instead of one week) so that students have sufficient time to correct any mistakes after the first stage submission.

3. Reflection Sheets as part of *Key Skills* testing on an Electronic Engineering Degree

For the last three years, Engineering students in years 1, 2, and 3 in IT Tallaght have been given "Key Skills" tests as part of their mathematics continuous assessment. Key Skills tests last one hour and have 15 questions each, most of which are multiple choice. The topics included in the tests have all been studied in earlier semesters. Usually a topic is included because it is considered to be essential for the current semester's mathematics and other modules. However, sometimes a topic is included in the test because it is not covered in current semester course work but is considered to be essential in future, such as basic calculus. The questions are generally straightforward, such as identifying the equation of a straight line from its graph, solving a linear equation, adding two algebraic fractions, or performing a differentiation or integration. As the tests are based on material from earlier semesters, the first Key Skills test is given in Semester 2. In common with the other initiatives described in sections 1 and 2 of this paper the intention is to use problems from an early stage in the semester to prompt student engagement with necessary learning (in this case revision) in mathematics.

The tests are delivered as quizzes in Moodle. Questions are drawn randomly from question banks developed by the authors covering specific topics. Each semester's test is constructed by choosing those categories considered appropriate for that semester, many of which will be carried from one semester to the next. 15% of the semester's total mathematics marks are assigned to Key Skills. However, as the intention is to encourage real competence, students receive none of this 15% until they reach the threshold of 10 correct answers. The assignment of marks against test performance is as follows:

Number of Correct Answers	0 – 9	10	11	12	13	14 – 15
Percent awarded out of 15	0%	6%	7%	9%	10%	15%

Given the marking structure, students are offered several (usually 6 or more) opportunities each semester to take a test. However some students exhibited no evidence of having worked on the test topics between tests. When asked about preparation for the next test, some students admitted having no record of the question categories where they gave an incorrect answer, although their performance in each individual test was readily accessible through Moodle. To address this problem, a "Key Skills Reflection Sheet" was introduced in September 2009 for third year Electronic Engineering. The sheet identifies the question categories, both with a short description and a sample question drawn from a practice test:

Question Type: Rearrange a formula

Q.6 Make z the subject of $r = \frac{1}{x + \sqrt{z}} + 2$.

Correct	
Incorrect	

Action

Figure 1: Example of reflection sheet practice test question.
(As the question order is randomised, question categories cannot be identified by number)

Students must then mark on the sheet those question categories in which they gave an incorrect answer. This is best done directly after each test. The sheet must be returned to the lecturer prior to the next test with actions filled in against some or all of the incorrect answers. Recommended actions include attendance at mathematics support sessions or use of online resources targeted at the problem category. The quality of information recorded on the reflection sheets varies. Some students are careless even about recording which questions they got wrong and give minimal or overly general entries as "Actions", e.g. "studied" or "revised". Others fill in the forms very conscientiously and give detailed "Actions", e.g. specific online resources used.

To measure the effectiveness of this approach the performance of students in the 2009 cohort (who have used the reflection sheet) and those in prior years (who did not have the reflection sheet) was considered. (It is to be noted that as yet, data exists for relatively small numbers of students.) As the reflection sheet is aimed at modifying student behaviour between consecutive tests, only the performance of students who took multiple tests is considered. (Note: Students who achieve 14 or 15 correct answers on their first attempt receive full marks for the Key Skills element of continuous assessment and do not resit; some students choose not to resit even though they have not received full marks.) Data for Semester 5 only is included in this study. The number of students in third year electronic engineering who took multiple tests in semester 5 prior to the introduction of reflection sheets was 44. This year, there were 19 "repeating" students all of whom needed to fill in and return a reflection sheet in order to be allowed to resit.

The key result which is evident to date is that there has been a significant increase in the proportion of students who reach the threshold level of 10 correct answers. The evidence is particularly strong when considering the students who failed to reach the threshold level on their first attempt, whom we refer to as sub-threshold students (see Figure 2).

	All Repeating Students			Sub-Threshold Students		
	Passing Threshold	Not Passing Threshold	p-value	Passing Threshold	Not Passing Threshold	p-value
With Sheet	18	1		13	1	
No Sheet ('07 & '08)	30	14	0.0198	14	14	0.0061
No Sheet ('08 only)	17	7	0.0504	9	7	0.0296

Figure 2: Fisher's Exact Test: 2 × 2 Contingency Table of Passing Threshold of 10 Correct

Note that a separate analysis is required for 2008 students only as the 2007 class was offered the opportunity to compensate for poor key skills marks through their performance in other elements of the mathematics module, leading to a less uniform engagement in the process. For the sub-threshold students, the evidence is still clear that there has been a significant improvement in performance since the reflection sheets were introduced.

Based on the evidence to date it would seem that the reflection sheet should be viewed as an essential element in implementing the problem driven key skills testing. The combination of using reflection sheets and offering no compensation for poor key skills performance seems to offer the best approach. Efforts to improve the quality of students' entries under "Actions" will be the focus of attention in the next phase of development in the use of these sheets to aid student reflection.

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Simulation programs for teaching quantum mechanics

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Abstract

Fundamentals of quantum mechanics are part of most physics curricula for engineering students. For the teacher this is a non-trivial didactical task, since the mathematical tools necessary for a deeper understanding are much too advanced and the phenomena themselves are mind-boggling. Simulation programs can be useful here by illustrating the behaviour of quantum mechanical systems and allowing a hands-on approach. Several example applets will be presented that solve the two-dimensional Schrödinger equation to help understand notions such as uncertainty, the tunnel effect and energy eigenstates.

Quantum mechanics for engineering students

Fundamentals of quantum mechanics are a standard part of the physics education for engineering students. Since the mathematical tools necessary for the description of quantum systems are too advanced, teachers have to resort to some basic ideas and presentation of results without derivations. Furthermore, due to the often perplexing quantum phenomena students have problems in creating any intuitive pictures or in making connections to their knowledge about "classical" systems.

To remedy this situation several authors have produced tools to simulate and visualise quantum systems, among them two very comprehensive packages: Physlet Quantum Physics (Belloni (2006)) and the Visual Quantum Mechanics project (Thaller (2000)). The first provides a large number of simulation programs ready for interactive use in the form of applets embedded in web pages and the second consists of an overwhelming set of animations describing mostly two-dimensional systems on a high level.

The two-dimensional examples are especially very helpful since they can be compared to standard wave tank simulations. Studying a properly chosen set of examples the students can understand both where classical pictures still work and where they have to be modified by new ideas, most importantly the uncertainty relation. Unfortunately, most of the Physlets present one-dimensional systems – the few exceptions relying on analytical solutions – whereas the animations from Thaller (2000) are not interactive. This restriction is due to the large amount of computation necessary for a numerical solution of the Schrödinger equation.

But, with better algorithms and the always growing performance of standard PCs the situation has changed. As part of the PhysBeans project (Junglas (2008)) programs have been developed that compute the solution of the 2d Schrödinger equation in "real-time", i.e. as fluently-running animations with variable parameters. The example applets presented below will show how they allow a hands-on approach to several standard topics of a quantum mechanics course.

Simulating the Schrödinger equation

The Schrödinger equation describes the time evolution of the wave function $\psi(x,t)$ of a particle in $\mathbf{R}^n$ in a potential V(x). In properly chosen units it reads

$$H\psi(x,t) = i\frac{\partial}{\partial t}\psi(x,t)$$

with the Hamilton operator

$$H = \frac{1}{2}\left(\frac{\partial^2}{\partial x_1^2} + \ldots + \frac{\partial^2}{\partial x_n^2}\right) + V(x)$$

One is mainly interested in solutions $\psi \in L_2(R^n)$ with an $\mathbf{L}_2$-norm of one. The square of the absolute value $|\psi(x,t)|^2$ can then be interpreted as the probability distribution of the particle.

The algorithm for the numerical solution of the Schrödinger equation used in the applets is based on the method given in DeRaedt (1994). To get rid of reflections at the boundary it has been augmented by a PML boundary layer (Antoine (2008)). More details about the implementation are given in Junglas (2009). The visualisation of complex wave functions is a non-trivial task, which is discussed extensively in Thaller (2000). Its author favours a clever combination of amplitude and phase representation. The interactivity of the programs presented here allows for a more straightforward approach: the user can switch between several representations (real/imaginary part, absolute value, square of the absolute value, phase) in order to get the most information from the wave function.

Example Applets

The following programs are all built up from the same elements:

- a display showing the two-dimensional wave function for immediate qualitative understanding of the phenomena,
- a graph of the wave function along a line in the x-y plane, allowing accurate “measurements”to be made or, alternatively, values of some observables as functions of time.
- a set of input elements for changing physical or visualisation parameters and control of the simulation.

The free particle

The first example shows the behaviour of a Gaussian wave packet with given initial position, velocity and width. This is as close to the intuitive picture of a slightly “smeared” particle as one can get. The time evolution shows mainly the classical behaviour: the packet moves along a straight line with the given velocity. Quantum mechanics shows up in the spreading of the wave function which is due to the range of velocities that are inherent in the Gaussian. The spreading gets faster with smaller initial width. In a hands-on session the students are asked to quantify this behaviour, which leads to a first glimpse at the Heisenberg uncertainty relation.

The uncertainty relation for a free particle

After the notions of position and momentum uncertainty have been introduced, the next program displays their explicit values as functions of time for the Gaussian wave packet. Here the students can experiment with the packet to find the minimal "combined" uncertainty $\Delta x \cdot \Delta p$ and thereby reproduce the Heisenberg relation. One could use additional programs with different initial functions to show the distinctive minimisation properties of the Gaussian wave packet.

Another point to notice here is the behaviour of the displayed values when the wave function leaves the simulated area: the uncertainties drop to values that are much too small, which violate the uncertainty relation. This is an artefact of the numeric algorithm, which can only use the part of the wave function still inside the computed area.

This observation can be a good starting point to discuss intrinsic limitations of simulation programs. On the other hand it can be too demanding for students who already struggle with the peculiarities of quantum mechanics. For that reason it might be a good idea to introduce a stop time and to constrain the parameters of the program accordingly. Due to the modular structure of the programs and the availability of the source code this can be done easily (Junglas (2008)).

The double-slit experiment

The famous double-slit experiment is especially bewildering when done with single photons. This applet shows what happens with a single electron – modelled as usual by a Gaussian wave function – that is going through a double-slit (cf. Fig. 1).

The resulting simulation shows a wealth of features: first one notices an interference pattern in front of the slits that is due to reflections. The students are already familiar with this behaviour from a previous example. Next one sees two "bubbles" coming up immediately behind the slit, which is more or less what one would expect. But after a while a third bubble appears in the middle, which together with the other two develops into the well-known interference pattern at the screen.

To understand more clearly where this central part comes from one can study the example again, this time looking at the real part of the wave function instead of the absolute value. The emerging pattern should be familiar from classical wave theory: it resembles the diffraction pattern stemming from Huygens' principle and shows how the central maximum is produced by "bending around the corner".

Many more interesting features can be studied by varying the parameters. Most important probably is the increase of the number of maxima with rising particle velocity, which is best seen in the backwards scattering. Classically this comes as a surprise, but a moments (quantum-mechanical) thought reveals the cause: With rising velocity – and therefore momentum – the Compton wavelength of the electron

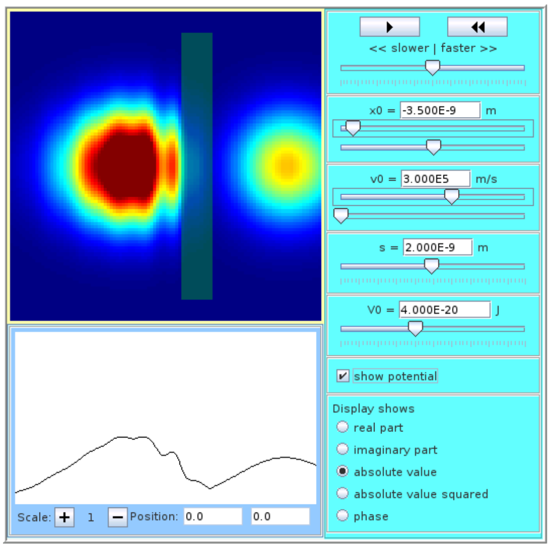

Figure 2: Potential barrier

decreases, which leads to smaller scattering angles, just as in classical wave theory. Again, this can be seen directly in the real part image.

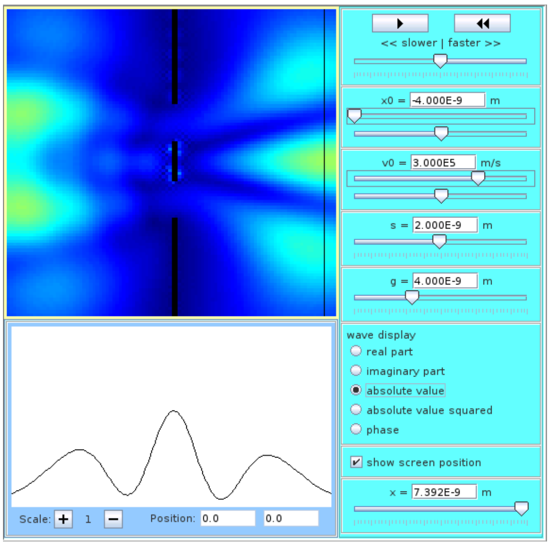

Figure 1: Double slit

Particle running against a potential barrier

The next example deals with a particle running against a potential barrier (cf. Fig. 2). The simulation shows the splitting of the wave function into a transmitted and a reflected part. Thinking of EPR and entangled states it might be tempting to expand here on the interpretation of a “bilocalised” state, but this is probably too advanced and causes more confusion than it provides deeper understanding. Therefore the situation is interpreted only in a statistical way.

In exercises students try to “measure” the transmission coefficient as function of the incoming velocity or the potential. The results are compared to the classical situation, which leads to the discovery of quantum tunnelling.

Eigenfunctions of the 2d Coulomb Hamiltonian

Understanding the eigenstates of the hydrogen atom provides the basis of the modern understanding of atoms, including the whole of chemistry and material sciences. Therefore it is one of the major goals of an exposition to quantum mechanics. Unfortunately it is not so easy with its complicated scheme of quantum numbers and complex spherical harmonics.

Here, the two-dimensional Coulomb potential comes in handy: Its eigenfunctions are much simpler and easier to visualise, but share many features with its famous 3d-cousin. The energy level is given by a “main quantum number” n, the quantisation of the angular momentum L_z leads only to one additional quantum number m with simple eigenfunctions and $m < n$.

These properties can be studied with an applet, which allows the user to enter values for n and m and displays the corresponding eigenfunction (cf. Fig. 3). The restriction $m < n$ is not enforced by the design of the user interface; instead, an invalid choice produces a blank screen in order to draw the student's attention to the restriction. In addition, one can enter an arbitray real value for the central charge. This is a simple way to scale the image and to provide a better resolution of the central parts.

The time evolution shows the constance of $|\psi|^2$, the images of the real or imaginary part simply rotate – at least for some values of n and m. Unfortunately one easily runs into numerical problems here: if the function extends to the border, one gets boundary reflections; if it extends into the centre, all kinds of artefacts show up, since the grid is too coarse to cope properly with the 1/r potential. Again, this can be turned into an advantage by making the students aware of the limitations of the simulation.

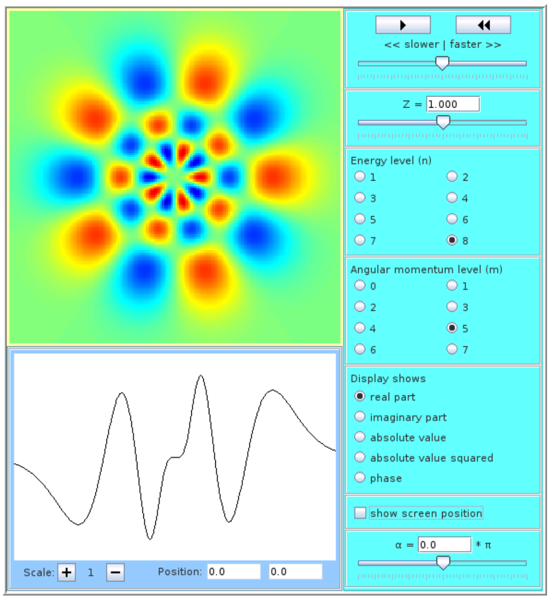

Figure 3: Coulomb eigenfunction

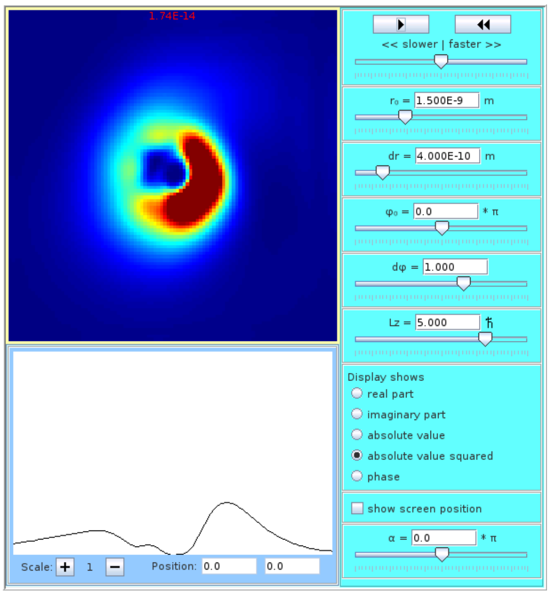

Figure 4: Coulomb “particle” state

“Particle” states in the 2d Coulomb potential

The last simulation (cf. Fig. 4) tries to transfer the notion of a “classical particle” to the Coulomb case. Preliminary experiments with a Gaussian initial state show a rather complicated behaviour, which bears very little resemblance to a classical picture. If one uses a Gaussian spreading in polar coordinates instead, the results are more amenable to a quasi-classical interpretation.

The default parameters define a state that is concentrated around a given radius and spreads over a rather large angle. Furthermore it has a given mean angular momentum, which corresponds roughly to that of an eigenfunction that is concentrated in the same radial region. Starting the simulation one sees mainly a rotation around the centre, combined with a spreading in angular direction, until almost a whole annulus is filled up. This could be interpreted vaguely as a classical circular motion combined with a spreading caused by the uncertainty in L_z. The students can work this out further by measuring the orbital period and compare it with the given value of L_z.

For larger values of L_z the wave function rapidly moves outwards, like a classical particle would do. For much smaller L_z it approaches the centre, where strange things happen, only partly because of the inaccurate simulation. If one tries to concentrate the particle in a radial or an angular direction, it blows up rapidly – the uncertainty relation takes its toll!

Conclusions

The simulation programs presented here demonstrate the general behaviour of wave packages and illustrate some basic notions of quantum mechanics. Furthermore they provide a set of examples on which a first intuition about quantum behaviour can be based. Finally they show how previous knowledge about classical behaviour (of particles *and* waves) can be put to good use if one augments it by the uncertainty principle.

An important feature is the two-dimensionality of the examples: On the one hand the corresponding animations are more easily interpreted as the graphs of one-dimensional functions, on the other hand they allow for generic 2d-examples like the double-slit or the Coulomb problem.

The set of examples will be expanded in the near future to cover the standard situations discussed in introductory courses. Further extensions will be the simulation of the measurement process (or, better, models thereof), the addition of electric and magnetic fields and of the electron spin. All programs are provided under standard open source conditions and can be downloaded from the PhysBeans homepage (Junglas (2010)).

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Modelling by Differential Equations – from Properties of Phenomenon to its Investigation

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Abstract

The Panevezys campus of Kaunas University of Technology comprises the Business and Administration Faculty and the Faculty of Technology. They host students of business administration, civil engineering, electrical engineering, management and mechanical technology. Students of all specialities need mathematical knowledge for the solution of practical problems and for research from specific domains, such as mechanics, control, techniques, physics, economics etc.

The paper discusses methodological issues of teaching ordinary differential equations to the bachelor and master students from the Faculty of Technology and bachelor students from the Business and Administration Faculty.

Our experience of many years shows that students understand the theory of ordinary differential equations better if it is illustrated with applications. For building up models we use mechanical and economical problems and solve the inverse problems for ordinary differential equations as well. Our opinion is that inverse problems promote better understanding of the idea of modelling idea.

Introduction

The Panevėžys campus of Kaunas University of Technology consists of the Faculty of Technology and the Business and Administration Faculty. General higher mathematics topics are covered during two terms, 3 hours for theory and 2 or 3 hours for tutorials per week. It is possible to introduce first and second order ordinary differential equations (ODE). The level of proficiency of different mathematical topics slightly differs among different specialities. Engineers of all specialties will need mathematical knowledge for the solution of practical problems and for research in the fields of mechanics, control, techniques, and physics. Students of business and economics should master skills that will be useful in economics, statistics and econometrics. For students of both specialities we point out that differential equations are tools for solving real-world problems that have been discussed in lectures of other subjects.

During tutorials we firstly master finding solutions of particular ODEs. After that it is easier to draw attention to the applications of the ODE. Generally, modelling provides a chance to develop practical skills of applying mathematics in particular domains (Gershenfeld, 1998).

To build a model, student must make many decisions. Therefore it is convenient to divide the whole modelling process into separate phases (Shier & Wallenius, 1999).

We usually follow five steps when demonstrating investigations of applied problems by means of differential equations.

1. Analysis of the real-world problem. Usually it is a problem that students have already considered in engineering or economics lectures. This involves evaluation of the major known properties of phenomenon and the formulation of what properties should be investigated by modelling using an ODE as a tool.
2. Set up a mathematical model, i.e. write down a differential equation of the model. Our opinion is that in this step it is important to point out how the differential equation describes the known properties discussed in the first step and what fundamental laws of nature and economics should be taken into account (Самарский, 2001). Attention should be drawn to the fact that the ODE is an expression that includes first or higher order derivatives. Therefore it represents the rate of change of the properties of the system or phenomenon under discussion. This is a good point to review the mechanical and economic meaning of the first and second order derivatives.
3. Solve the ODE. Here one can often but not always focus solely mathematical knowledge.
4. Analysis of the solution of the ODE. This step generally remains mathematically oriented and it includes analysis of the solution's closed-form expression if it is available. The analysis of the solution may include a discussion of the expression, evaluation its analytical properties, pointing out how the solution depends on boundary or initial values, plotting the graph etc.
5. Reformulate the mathematically discovered characteristics of the solution into engineering or economical language. This is a good place to compare the new properties and knowledge about the problem revealed by modelling with already known facts, to predict the behavior of the device or economical process and to check if it is what the engineers expected (Dym, 2004). This step also contributes to the validation of the model.

Practical examples

For example we consider the determination of the expression for demand for some good, depending on income, when the elasticity is constant.

1. Analysis of the problem. This problem comes from the microeconomics. Constant elasticity functions are often discussed in economic theory and they are considered as simple functions. But the analytical expression is not always given. So it is interesting to find this expression and to investigate properties of it from the mathematical viewpoint.
2. The definition of elasticity of demand (D) with respect to income x is $D'x/D$ and the requirement to be constant yields $\left(D'x/D\right)' = 0$.

3. This is a second order ODE and there are no difficulties in showing its solution is $D = c_2 x^{c_1}$ with arbitrary constants c_1 and c_2.

4. When performing the mathematical analysis of the function *D,* it is essential to point out that $\left(D'x/D\right) = c_1$. Therefore graphs of the demand function *D* with various elasticity c_1 values including $c_1>1$ and $c_1<1$ should be plotted.

5. The main result is that the demand of constant elasticity is a power function. The demand *D* has neutral elasticity, i. e, when $c_1=1$, only when $D=c_2x$ and in this case the demand is proportional to income *x*. But the linear function of general form $y=kx+b$ can not have the constant elasticity property.

Instructors of applied mathematics are typically more concerned with direct problems. In real life, the problem of interest to engineers and scientists is often an inverse problem. For instance, in many practical situations it is necessary to find coefficients of the ODE, parameters of the source term or parameters of the boundary values. The engineer seeks to determine these unknown parameters from collected measurements or other accessible knowledge about the solution of the given ODE.

Our experience shows that students understand the theory of ODE better if this theory is also illustrated with inverse ODE problems. In addition, considering inverse problems also promotes better comprehension of the essence of ODE models. The theory of inverse ODE problems nowadays is well developed, and has wide applications due to the accessibility of powerful computers.

When the educator introduces such problems, the major difficulty that students face is that of cause and effect, i.e. the direct problem and the inverse problem switch places with one another. Therefore the educator must pay special attention to this peculiarity and explain it.

As an example, we consider one problem that reveals methodological aspects of solution of the inverse problem.

We start from the boundary value problem that describes the transversal deflection $u(x)$ of the heterogeneous beam when it is supported at points $x=0$ and $x=L$.

$$\begin{cases} \dfrac{d^2M}{dx^2} = F(x), \\ M(0) = M(L) = 0, \\ D(x)\dfrac{d^2u}{dx^2} = M(x), \\ u(0) = u(L) = 0. \end{cases} \tag{1}$$

where $F(x)$ is a distributed load, $M(x)$ is bending moment acting in the beam cross section, $D(x) = E(x)I(x)$ is flexural rigidity of beam, $E(x)$ is Young's modulus of the beam material, $I(x)$ is the second moment of the cross section area.

The first two equations of (1) have analytical solution

$$M(x) = \int_0^x \int_0^v F(t)dtdv - x \int_0^{x/L} \int_0^v F(t)dtdv\,, \tag{2}$$

and provided that $D(x) > 0$, we obtain the solution of problem (1)

$$u(x) = \int_0^x \int_0^v \frac{M(t)}{D(t)} dtdv - x \int_0^{x/L} \int_0^v \frac{M(t)}{D(t)} dtdv\,. \tag{3}$$

Hence we have reduced the problem (1) into boundary-value problem for the linear non-homogeneous differential equation

$$\frac{d^2u}{dx^2} = P(x),\ u(0) = u(L) = 0 \tag{4}$$

where $P(x) = M(x)/D(x)$. Assuming $u_1(x)$ and $u_2(x)$ are linearly independent solutions of the homogeneous problem. Then the Green's function of the problem (4) is given by expression

$$G(x,s) = \frac{u_1(x)u_2(s)}{W(s)}\left[1 - H(x-s)\right] + \frac{u_1(s)u_2(x)}{W(s)} H(x-s) \tag{5}$$

where $W(s) = \begin{vmatrix} u_1 & u_2 \\ u_1' & u_2' \end{vmatrix}$ is the Wronskian and $H(x)$ is Heaviside's step function. Then the solution of the problem (1) is

$$u(x) = \int_0^L G(x,s)P(s)ds \tag{6}$$

Now we shall solve the inverse problem, i.e. we seek $P(x)$ for given $u(x),\ F(x),\ x \in (0,L)$. Notice that function $M(x)$ becomes known when the problem (2) is solved. Hence one can derive the bending stiffness

$$D(x) = \frac{M(x)}{P(x)} \tag{7}$$

and then , assuming that the second moment of the cross section area $I(x)$ is known, one can also derive Young's modulus distribution

$$E(x) = \frac{D(x)}{I(x)}\,. \tag{8}$$

The problem (6) can be solved by applying numerical quadrature formula. The interval $(0,L)$ is divided by points $s_i = iL/N$, $i=\overline{0,N}$ into N equal subranges $S_i=(s_i,s_{i+1})$, $i=\overline{0,N-1}$ and then

$$\int_0^L G(x,s)p(s)ds=\sum_{i=0}^{N-1}\int_{s_i}^{s_{i+1}} G(x,s)P(s)ds \approx \sum_{i=0}^{N-1} P\left(\overline{s_j}\right)\int_{s_i}^{s_{i+1}} G(\overline{s_j},s)ds \quad ,$$

where $\overline{s_i}=\left(s_i+s_{i+1}\right)/2$ are middle points of intervals S_i. If $u_i=u(\overline{s_i})$ are measurements then the solution of the inverse problem (6) is the solution of linear system

$$AP=U\,, \tag{10}$$

where

$$A_{ij}=\int_{s_i}^{s_{i+1}} G(\overline{s_j},s)ds \;\; , \; U^{\mathrm{T}}=(u_1,u_2,...,u_N)\,, \; P^{\mathrm{T}}=(p_1,p_2,...,p_N), p_i=p(\overline{s_i})\,.$$

Now the educator can consider quite important real-world engineering problems. Beams made of materials like concrete, show degradation of flexural stiffness during their service life due to mechanical and environmental loadings. Using the method outlined above, we can investigate how the material parameters (Young's modulus and flexural stiffness) change.

Finally, we note that, from the expression of the ODE it is sometimes possible to draw conclusions about which real-world problems and systems this ODE can serve as a model. For instance, when covering the first order differential equations, we consider the equation $y'=k_1y-k_2y$. Educators can draw attention that the equation models general process changes that are influenced by two factors corresponding to terms k_1y and k_2y. The first term increases the changes while the second contributes to the decrease.

Conclusions

1. Modelling skills should be taught by practically building models in concrete domains including data acquisition from these domains. Models become more meaningful when students collect their own data.
2. The background of successful modelling with differential equations is the knowledge of the meaning of derivatives.
3. The consideration of inverse ODE problems contributes to better understanding of the essence of modelling by differential equations.

4. When a particular differential equation is given it may be useful to discuss what processes and what relationships between properties of the process it can describe.
5. Modelling teaches students to apply mathematical concepts to solve real problems.

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About the Activity of the Network HU-0028 in the Central European Exchange Programme for University Studies

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Abstract

This paper reports on the activity of the Network HU-0028, Active Methods in Teaching and Learning Mathematics and Informatics, a project that is originated in the cooperation of some of those SEFI partners, who are also from CEEPUS (acronym for the Central European Exchange Programme for University Studies) countries. The network activity involved initially seven partners from five countries, and grew to 22 partner universities from nine countries. The success of our networking has many reasons, involving the well-balanced partnership, the joint programmes, the fact that the network activity is combined with other European projects, like Comenius, Erasmus, Leonardo, organizing regularly summer universities, and intensive courses, etc. Joint research and conference participation, scientific cooperation in other domains are useful outcomes as well. During the ten years of activity about 230-250 teacher grants, and 850-900 student grants have been available and were used by the partners. In 2006 the network activity was highly appreciated by the CEEEES Ministers' Prize of Excellence awarded in Ljubljana to the network coordinator, the author of the present report, accepted as a poster presentation to the SEFI seminar.

What is CEEPUS?

CEEPUS is an acronym for "Central European Exchange Program for University Studies". a cooperation form created initially in 1994 by six of the Central EU states Austria, Bulgaria, Hungary, Poland, Slovakia, and Slovenia, Csernovitz et all (2005) involving by now 15 countries altogether, including Albania, Bosnia-Hercegovina, Croatia, Czech Republic, Macedonia, Montenegro, Romania, Serbia, Prishtina/Kosovo.

The main objectives of CEEPUS are to

- contribute to merging the European Higher Education Area and European Research Area
- use regional academic mobility as a strategic tool to implement Bologna objectives
- enable cooperation with South and Eastern Europe, the Ukraine and Moldavia

The structure of CEEPUS is based on the CEEPUS Agreement of the 15 partner countries, signed by the ministers. Due to the continuous changes in the European higher education this agreement was adapted to the actual challenges, while the CEEPUS II agreement was signed by 9 countries in Zagreb 2003, and revised by the Joint Joint Committee of Ministers, the highest forum of CEEPUS on March 16, 2007.

The 16th Meeting of the Joint Committee of Ministers has made up CEEPUS III Agreement signed by all Contracting Parties. The agreement will enter into force on May 1, 2011, when probably the Republic of Moldavia will join the programme.

The structure of CEEPUS is based on lean management. The highest-ranking decision making CEEPUS body is the Joint Committee of Ministers that meets once a year and takes all strategic decisions. Coordination, evaluation, program development and advertising are the main tasks of the Central CEEPUS Office (consisting of only two persons). Each country has a National CEEPUS Office in charge of national implementation. In order to avoid setting up new administrative bodies, the National CEEPUS Offices are integrated into already existing structures, usually national agencies. All CEEPUS activity is organized on-line, the participants create their own personal desktop – this has several functions, possesses different level attributes – and almost everything is managed in this way: handing in mobility applications, accepting or refusing mobility of partners, planning of the network activities.

The exchange of students and teachers is organized in networks by the accredited universities from the CEEPUS countries, the minimal requirement for a network is to have at least 3 partner universities from two countries. One of the partners is acts as network coordinator, its university is called coordinating university. The network identifier reflects the country of the coordinating university, e.g. HU-0028 is our network, coordinated by the University of Miskolc, Hungary. The number of partners is not limited; our network is rather large, we have 22 partner universities from nine countries. The CEEPUS year is between 1st September and 31st August the next year. Each year the network activities have to be revised, by 15th January, new network applications can be handed in, the running networks must apply for prolongation of their activity. The networks selected for support will receive their quotas of mobility months, the only ‘internal currency’ of the programme. The mobility awarded to the networks, is expressed in this term of months, a grant which is paid monthly by the hosting partner. The amount of the grant is linked to the local living expenses, and differs for students and teachers. For the latter the grant sometimes is related to the teacher’s position. The travel expenses are not supported by CEEPUS, in some countries the home universities give travel grants for outgoing mobility, in others the participants have to find other forms, or support it from their own resources.

Partners of the network HU-0028

UNIVERSITY OF MISKOLC co-ordinator of the project, UNIVERSITY OF ELBASAN, UNIVERSITY OF APPLIED SCIENCES VIENNA, ANGHEL KUNCHEV UNIVERSITY OF ROUSSE, PAISSI HILENDARSKI UNIVERSITY OF PLOVDIV, CHARLES UNIVERSITY PRAGUE, CZECH TECHNICAL UNIVERSITY Faculty of Civil Engineering, CZECH TECHNICAL UNIVERSITY Faculty of Electrical Engineering, UNIVERSITY OF DEBRECEN, UNIVERSITY OF MISKOLC, COMENIUS Teacher Training Faculty, SZENT ISTVÁN UNIVERSITY, Faculty of Education, Szarvas, UNIVERSITY OF PÉCS, LUBLIN UNIVERSITY OF

TECHNOLOGY, NORTH UNIVERSITY OF BAIA MARE, PETRU MAIOR UNIVERSITY TG MURES , TECHNICAL UNIVERSITY OF CIVIL ENGINEERING BUCURESTI, UNIVERSITY OF NOVI SAD, Faculty of Civil Engineering Subotica, UNIVERSITY OF NOVI SAD, Faculty of Science, UNIVERSITY OF NOVI SAD Hungarian Language Teacher Training Faculty Subotica, SLOVAK UNIVERSITY OF TECHNOLOGY, The CATHOLIC UNIVERSITY IN RUZOMBEROK, and the UNIVERSITY OF TECHNOLOGY IN KOSICE

Aims and objectives of the network

The classical academic methods, like lectures and practical classes do not seem to be efficient any more; in consequence, one of the main aims of mathematical education is to find the proper active methods in teaching and learning Mathematics and Informatics, in order to face the challenges of the Bologna process, to follow the rapid development of educational technology.

We planned our activity around four main objectives.

1. To increase Mobility and develop Joint programmes

The network is aimed in the development of a joint curricular environment of the Active Methods in Teaching and Learning of Mathematics and Informatics, in a framework of partner institutions, universities and other higher educational institutions throughout the Central and Eastern Europe, to offer a larger variety of special educational programs joining the existing possibilities of partners and enhance the student and teacher mobility especially beside partners and beyond, to share resources of teaching materials (in printed or electronic form) available at the partner institutions via information communication technologies, using the excellent opportunity of the European Virtual Laboratory of Mathematics (EVLM) Leonardo Project developed by part of the partners.

2. To introduce ITC and CAS in Mathematics Education

The partners develop the methodology of computer algebra based learning and project teaching, and provide expertise consultancy and training in mathematics for university students, teachers, researchers and scientists within this framework trough the partnership, and using the framework of our CEEPUS network and EVLM. Cooperating partners utilise and test available e-learning sources as mathematical electronic courses, electronic books and other learning materials. They are aimed in organizing the summer university Computer Algebra Driving Licence and offer the possibility for student to take part due to the facilities offered by CEEPUS

3. GeoGebra Institutes

As a new feature of our activity we plan to initiate international teacher training and a new cooperation in using GeoGebra, developing and sharing Geogebra teaching materials. We will organize the training of the trainers on national level based on the international expertise of our partners. Most of the partners have been involved with the Geogebra activities, and more institutions are partners in the national Geogebra Institutes, to be set up in the last months, like in Slovakia, Hungary, Romania, Czech Republic, Bulgaria and Albania. The partners actively contributed to the translation in to the National languages, even the Albanian and Romanian versions were initiated by the partners of our network see the http://www.geogebra.org/en/wiki/index.php/Main_Page, or the GeoGebra pages of the partners, e.g. http://www.uni-miskolc.hu/evml/geogebra/.

4. To combine within the partnership and beyond more EU educational projects

To increase scientific cooperation, especially involving our PhD students, correlating the organization of scientific conferences, joining the partners in other projects, we plan to continue the joint research activities, reflected by joint publications, even by creating a new Electronic Journal on Teaching of Mathematics in Rousse, based on the CEEPUS cooperation.

Instruction activity

We plan to continue to offer regular and special courses, seminars, consultations according to the specialities, human resources, and study programs of each partner. We plan to continue the joint programme in Computer Science between 2 or more partners of the network, and to initiate a joint MSc programme in Informatics for Economy, and PhD programmes as well. We plan computer algebra summer universities and intensive courses, and study excursions as well. We plan the continuation of the development of the European Computer Algebra Driving Licence, ECADL, to be included as a 30 credit module for BSC and MSC curricula of most of partner universities. As a new feature of our activity we plan to initiate international teacher training and a new cooperation in using GeoGebra, developing and sharing GeoGebra teaching materials. We will organize the training of the trainers on national level based on the international expertise of our partners. Beside courses offered in English, German and French we offer courses in Czech, Hungarian, Romanian, Slovak, Polish, Slovenian, Serbian in case our exchange students are speaking the given language, and there is an interest to take part in these courses.

The European Computer Algebra Driving Licence is a complex study program including:

Basic level: General introduction of Computer Algebra software, comparison of them, knowledge for the handling of basic commands, help files, and special packages. Presentation of Maple, Mathematica, Derive, MuPad, MatLab

Intermediate level: Using the special packages of the CA software, and programming in Maple and Mathematica.

Advanced level: Proficiency in using the general CA software for Applications of Mathematics according to speciality the faculty.

Some of the partners have participated in a Leonardo project a European Virtual Laboratory of Mathematics (see http://www.evlm.stuba.sk/) developed for national centres in Hungarian, Bulgarian, and Czech as well. This project can be used for enhancing individual learning, to include modern CAS technology in teaching, and through the on-line consultation to find support even from abroad.

More partners are directly involved with the activity of the International GeoGebra Institute, and the community of GeoGebra users, and members, or initiators of the accredited National GeoGebra Institutes. Our network contributed to the rapid extension of the GeoGebra software and its introduction is university level teaching of Mathematics.

CEEPUS Ministers's Prize of Excellence

The prize is awarded once a year to one of the about 50-60 CEEPUS active networks, based on the analysis of their results and activity for at least the last four years of activity. The award was initiated in 2002 by the Joint Conference of Ministers. In 2006 the network activity was recognised by the CEEEES Ministers' Prize of Excellence awarded in Ljubljana to the network coordinator, the author of the present report.

Impact of the network

Most of the partners had been co-operating before the creation of the network, but the collaboration has got new dimensions due to the regular visits, a possibility offered by the CEEPUS networking in the previous years. The chain of Computer Algebra Driving Licence Summer Universities which involved and plans to involve students and teachers beyond the CEEPUS partnership as well is innovative. The relatively large number of partners is an advantage for the network, as we can offer study exchange for our students in most countries in Central and Eastern Europe, and cover a large area of subjects. We extended our cooperation to the activities of Leonardo pilot project the European Virtual Laboratory of Mathematics Project No. 2006 - SK/06/B/F/PP – 177436 - see web page: http://www.evlm.stuba.sk/. Our partnership includes three of the partners who made up the EVLM project, and we wish to extend the dissemination of the results to our larger partnership in CEEPUS cooperation. The students have been included in co-operation due to this form of activity as well. They participated actively in the preparation of the social program of summer universities and intensive courses. They are involved with the creation of a special students' web page for keeping contact with their colleagues. The most important activities combined with the CEEPUS network activity are: the cooperation in the SEFI- MWG, Socrates projects, Tempus projects, bilateral agreements due to the cooperation, and many conferences such as MicroCAD in Miskolc, each Spring in March, the Conference on the History of Mathematics and Teaching of Mathematics, Miskolc, 2004, 2006, organised in cooperation by project partners at Tg Mures in 2008 are seen as an important scientific outcome of our network, involving many of the students in partnership. Based on the SEFI-MWG European Seminar on Mathematics, Miskolc, 2000 we used the partnership to organize a computer algebra workshop as a satellite for the SEFI-IGIP Joint Annual Conference, 2007 Miskolc. Our partners do take part regularly in ICAM in Baia Mare each September. We plan to continue the joint research activities, reflected by joint publications, even by creating a new Electronic Journal on Teaching of Mathematics in Rousse, based on the CEEPUS cooperation. Most of the partners have been involved with the GeoGebra activities, and more institutions are partners in the national GeoGebra Institutes, to be set up in the last months, like in Slovakia, Hungary, Romania, Czech Republic, Bulgaria and Albania. The partners actively contributed to the translation in to the National languages even the Albanian and Romanian versions were initiated by the partners of our network see the http://www.geogebra.org/en/wiki/index.php/Main_Page, or the GeoGebra pages of the partners, e.g. http://www.uni-miskolc.hu/evml/geogebra/.

Conclusions for Education

Our chances and concerns about how to fulfil the tasks of planned network activities are summed up in the following **SWOT** Analysis. **Strengths :** Great variety of teaching programmes and excellent staff of partner universities, the dedication of partner coordinators, combining the networking with more cooperation forms, organizing Summer universities and planning more joint programs. Most of the partner coordinators are members of decision-making bodies/are scientific leaders of the partner universities, organisations. **Weaknesses:** Uneven level of the teaching requirements in different partner countries, inefficient communication in a few cases, especially with some of the partners we lost during the project. The size of the network makes difficult to organize full coordination meetings. **Opportunities:** Well-established and well-structured groups beside the partners, ranging from technical universities to faculties of teacher training and science. Excellent cooperation due to a large variety of complementary projects, cooperation forms beside the partners, e.g. ECADL EVLM, GEOGEBRA. The activity of our network is attracting new partners every year – for the next period 3 new partners joined our network. **Threats:** Difficulty in offering a greater number of subjects for visiting students: mathematics is usually done in the first year (ineligibility of first year students). The clear joint programme regulations in some of the partner countries are missing. **Results:** In 2006 the network activity was highly appreciated by the CEEEES Ministers'Prize of Excellence During the ten years of activity about 230-250 teacher, and 850-900 student grants have been available and were used by the partners.

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A Never Ending Story – Mathematics Skills & Deficiencies of Engineering Students at the Beginning of their Studies

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Abstract

Monitoring the mathematics knowledge and skills of students at the beginning of their studies has a long tradition at the University of Applied Sciences Esslingen. After establishing the Bologna two-cycle system it seemed appropriate to investigate in detail the mathematics level of first year students. In the winter term of 2008/09 an entry cohort of 670 students at the beginning of their studies in engineering participated in a mathematics test. The results were analyzed in detail with respect to (1) the various admission certificates and (2) standard test theoretical criteria (index of difficulty; discriminatory power). The main results confirm investigations in the former Diploma-degree courses: (1) An arithmetic mean of only about 50 % of the possible maximum score, (2) a significant difference of the test score between the holders of a 'general admission certificate' ('Allgemeine Hochschulreife') and those with a 'specialized certificate' ('Fachhochschulreife') and (3) a spread of the test scores over five standard deviations.

Introduction

The Association of German Engineers (VDI, Koppel, 2010) states a severe lack of engineers in Germany (in the near future of about 35,000 per year). Upcoming retirements in the work force and the demographic development in Germany have led to a 'cry for more engineers' to be graduated from the system of higher education. At the other end an investigation by the Higher Education Information System (HIS, Heublin et al., 2009) reports on increasing drop-out rates in engineering in the newly established Bachelor degree courses. Drop-out in engineering studies is due to a large extent to failure in the major course assessment in the first year of study, i.e. basically in mathematics and natural sciences. In this climate it seems appropriate to monitor and analyze the mathematics knowledge and skills of an entry cohort in engineering fields of study.

Admission Certificates to Higher Education in Germany

The German Gymnasium awards the general admission certificate ('Allgemeine Hochschulreife – AHR' or 'Abitur certificate') as the traditional entry qualification for university studies. Graduation from Gymnasium used to be after a total of 13 years of general schooling; in some States at present this has been now shortened to 12 years. Special types of Gymnasium are the 'Fachgymnasien', which prepare for special fields of university studies; i.e. a Technical Gymnasium for engineering studies.

A large variety of specialized certificates of admission ('Fachhochschulreife – FHR') allow students to enter courses at Universities of Applied Sciences. As a general rule this certificate is issued by passing the final examination after grade 12 of a 'Fachoberschule' (a special type of upper vocational (high) school). In the State of Baden-Württemberg the majority of specialized admissions is via a 'Berufskolleg' (which is a specialized preparatory school for students with an apprenticeship in a related craft or trade).

Engineering in Higher Education

Two types of institutions offer studies in the field of engineering, namely (1) the (Technical) Universities and (2) the Universities of Applied Sciences. Both types of institutions offer Bachelor and Master degrees in a two-cycle system. Ph.D. degrees are awarded by the universities only. For consecutive Bachelor-Master courses of study the maximum (standard) duration is limited to ten semesters yielding two (6+4) and (7+3) Bachelor/Master models. Two out of three engineers in Germany graduate from a University of Applied Sciences.

A mathematics test for first-year students upon entry

The test was not part of the admission procedure and was given to the students in the first week of classes. The test consists of 31 items covering the various fundamental areas: algebra (12), trigonometry (8), elementary functions (5) and analytical geometry in a plane (6). The test is standard multiple choice format offering one correct answer and four distractors. An alternative 'don't know' option is included to avoid mere guessing. The test is based on mathematics skills taught in middle school grade 10. An example, including the psychometric analysis is presented in Figure 1.

$\sin(90^0 - \alpha) = ?$	$\sin\alpha$		a
	$-\cos\alpha$		b
	$-\sin\alpha$		c
	$\cos\alpha$		d
	None of the above		e
	Don't know		z

The index of difficulty gives the fraction of correct answers over the number of participants [p= .44 or 44 %].
The discriminatory power compares the answer of an individual for a given item (correct/wrong) with the total test score of this person; i.e. it discriminates according to the overall performance [rit=.38]. (For best discrimination a value of about .5 is desired.)

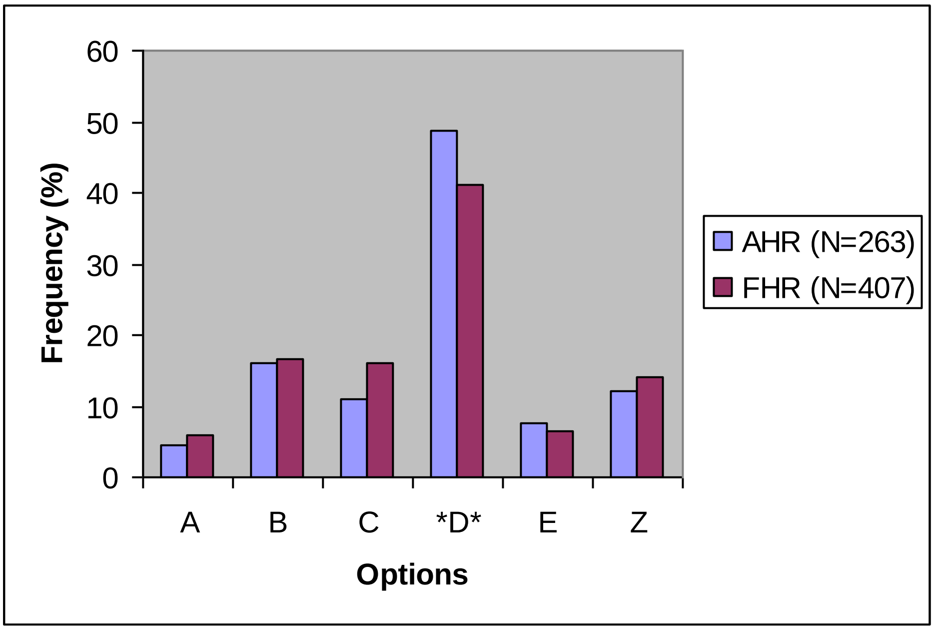

Figure1: Example for a test item (trigonometry): psychometric analysis and relative frequencies for the answer options.

The Bachelor degree courses in Esslingen requiring mathematics in the first-year curriculum are: Natural Sciences (AN); Management (BW); Automotive Engineering (FZ); Information Technology (IT); Mechanical Engineering (MB); Mechatronics and Electrical Engineering (ME); Building Services, Energy and Environmental Engineering (VU); Engineering Management (WI).

In the winter term 2008/09 a total of 760 students at the beginning of their studies in engineering participated in the test. For the analysis presented only those 670 students who have been enrolled for the very first time in the German system of higher education were considered.

Results

The results itemized for the different admission certificates are given in Table 1.

	N	M	SD
Allgemeine Hochschulreife –AHR			
Gymnasium (Normalform)	153	19.22	5.79
Fachgymnasium	90	15.67	5.45
Sonstige	20	14.90	4.61
AHR – all	263	17.67	5.87
Fachhochschulreife – FHR			
Berufskolleg	266	14.42	5.48
Fachoberschule	24	17.38	5.90
Berufsfachschule	66	13.45	5.35
Meister/Technikerschule	24	14.71	5.19
Sonstige	27	13.41	7.28
FHR – all	407	14.39	5.64
AHR & FHR – all	670	15.68	5.95

Table 1: Test results according to the certificate of admission to higher education.
[The German nomenclature was kept to illustrate the variety of certificates]
(Z=31 items; N: number of participants; M: mean; SD: standard deviation).

Figure 2 gives a graphical representation of the score distributions for the admission certificates AHR and FHR.

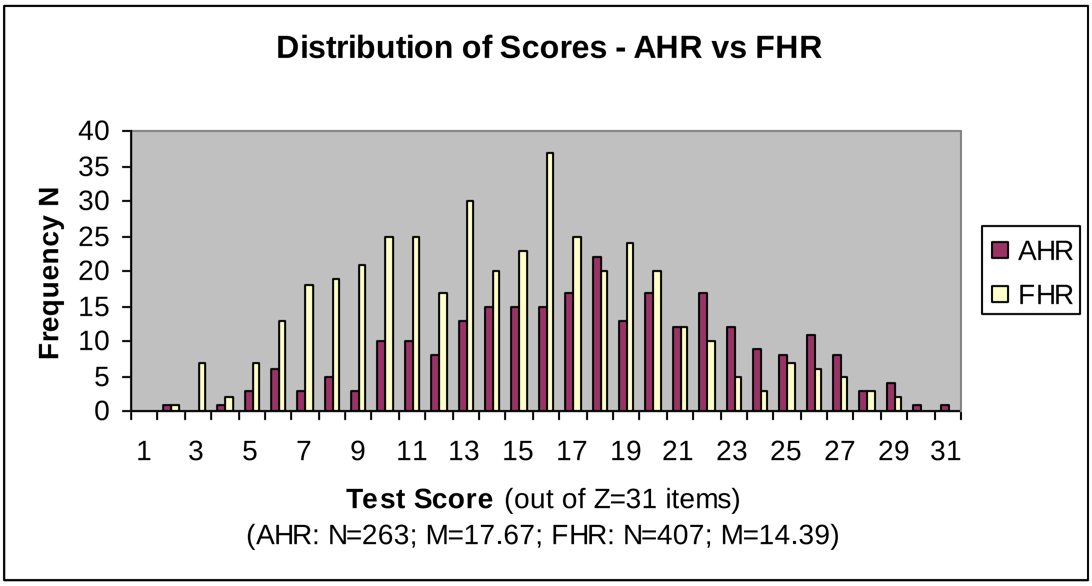

Figure 2: Histogram of the test-score distributions for holders of a general admission certificate (AHR) and a specialized admission certificate (FHR), respectively.
(Z=31 items; N: number of participants; M: mean; SD: standard deviation).

The results of the test confirm strongly earlier findings by Kurz (1988) offered then to entry students in diploma-degree courses:
(1) The arithmetic mean (about 50 % of the possible maximum score) is low as judged from the expectations of the college teachers in charge of the first year courses.
(2) The test scores cover a range of about five standard deviations, and this is regardless of the admission certificate.
(3) The difference between the two groups of admission certificates AHR and FHR is significant at the p< .01 level (two sided t-test). This difference, however, can be attributed at least partially to of the secondary educational system allowing different curricula in the variety of paths to higher education.
A detailed analysis of a trigonometry item was already introduced in Figure 1. The analysis of all 31 items is available upon request from the author.
The superiority in mathematics knowledge and skills of entrants with a general certificate holds for all courses of study in Esslingen (see Figure 3). With the exceptions of 'mechatronics and electrical engineering' (n.s.) and 'mechanical engineering' (p<.05) the differences are significant on the level p<.01. Consequently, any teaching/learning method relying on homogeneity is bound to fail.

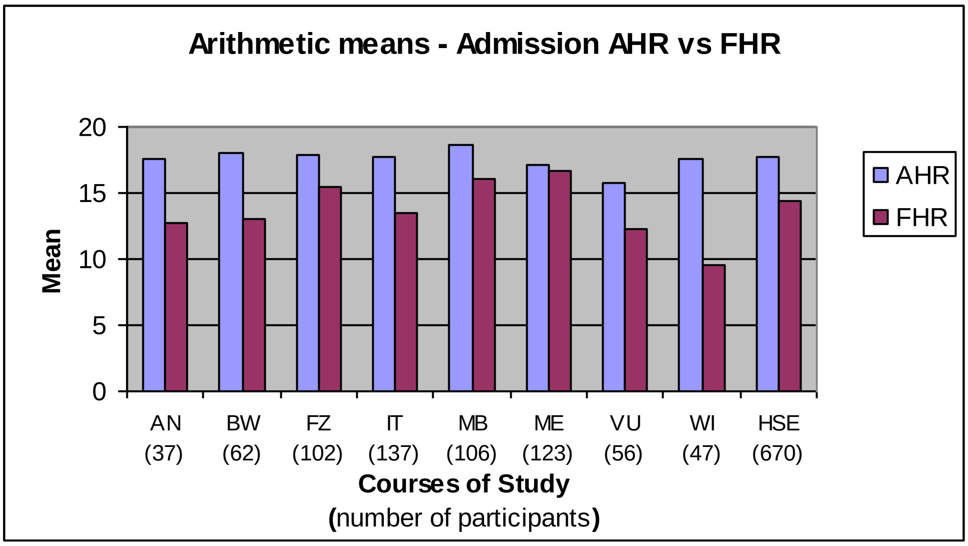

Figure 3: Mean test scores for AHR and FHR admission certificates for courses of study in Natural Sciences (AN); Management (BW); Automotive Engineering (FZ); Information Technology (IT); Mechanical Engineering (MB); Mechatronics and Electrical Engineering (ME); Building Services, Energy and Environmental Engineering (VU); Engineering Management (WI).

Conclusions and Future Activities

The reported results indicate a pattern unchanged for decades: *mathematically ill-prepared students for study courses in engineering.* In the federal structure of Germany education is the responsibility of the States i.e. 'Ländersache'. 16 State ministries run by ministers of different political parties guarantee the least common denominator and mutual dead locking. With the increasing financial squeeze the institutions of higher education can only try to make the best out of the situation.
In engineering and natural sciences most institutions offer remedial courses before the commencement of classes. In Esslingen this has been a tradition for about 25 years (Kurz, 1990);

but a one week crash course can never lay the sound mathematical foundation required for the rigour of an engineering programme. Unless significant changes are made, the drop-out rate of about one third could not be reduced drastically.

A ‘preparatory semester’ to guarantee a more solid base in mathematics (and physics and chemistry) for engineering studies is currently under discussion to address the basic problem of the mismatch on the transition from secondary to tertiary education.

The State of Baden-Wurttemberg will – starting in the academic year 2011/12 – require specific assessment tests for admission to higher education field. In the field of engineering a dominant role is attributed to mathematics. Mathematics knowledge and skills as part of the admissions procedure will hopefully force applicants to remedy deficiencies before entering higher education. A three-step procedure is intended: (1) stating the mathematics requirement for engineering studies, (2) pre-university mathematics courses and (3) successfully passing the admissions test procedure.

In the US the College Board offers the Scholastic Aptitude Test (SAT, 2010) which is accepted nationwide for admission. The test assesses reading, writing and mathematics. Given that in Germany education is a State issue, it seems to be an impossible task to define nationwide standards of entry qualifications to different fields of study.

Another activity in the State of Baden-Wurttemberg is the task force COSH (‘Cooperation Schule/Hochschule’ – ‘Cooperation secondary and tertiary level’). A group of colleagues from both levels pushes a better preparation in mathematics for engineering studies and managed to start a dialogue between the two state ministries responsible for the secondary level (‘Ministerium für Kultus und Sport’) and tertiary level (‘Ministerium für Wissenschaft, Forschung und Kunst’). These developments give some hope for improvement.

Finally it would be timely for the SEFI WGM to revitalize and update the Zero Core Curriculum Mathematics for the European Engineer (1992, 2002).

Mathematical skills upon entry and study success

An ongoing study in Esslingen in mechanical engineering monitors the study process and progress of two cohorts at the beginning of their studies (the last diploma group and the first Bachelor group). Predictors are the final grades of secondary education (used for the admission procedure) plus (1) a mathematics test (similar to the described one) and (2) field specific engineering test (ITB Consulting) offered at the commencement of classes. In the first part of the survey these predictors were correlated to the interim results of the (then) pre-diploma examination (issued then after four out of eight study semesters). A combination of the secondary school leaving certificate and the marks of the mathematics test are the best predictor of a successful study (Hell et al., 2008). Preliminary results indicate that this high correlation also holds for the final graduation marks. Final results will be available after the examination period of summer term 2010.

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Modeling toward numerical solution of an applied engineering problem

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Abstract

Some of the difficulties in the teaching/learning process, namely in the area of the numerical solutions of differential equations, are very well identified. Real life engineering problems can usually be described by sets of differential equations that are mathematical approximations of the physical reality. Writing a differential equation model to represent a physical problem can be a very demanding task to freshmen students. Some difficulties also arise in solving the final mathematical model. This can be facilitated by using some sort of software. With a group of students on the MSc in Industrial Engineering Management, an experiment has been done on the knowledge application to be gained in the Ergonomic Workplace studies area, with regards to motivating and raising awareness of students to this area. The main difficulties reported by students will be highlighted and they will be used in order to improve this teaching/learning methodology.

Introduction

The modelling and simulation of physical systems are very common in solving engineering problems, as well as other areas of knowledge. It is essential that the future professionals in these areas understand the concept that lies behind the solution. The concept of a differential equation is, in most cases, always present since it mathematically translates a physical reality: differential equations (DE) in physics with the fundamental dynamics law, the differential calculus concepts such as the work of a force, mechanical, potential, kinetic energy, in describing the motion of a particle, amongst others. The DE together with boundary and initial conditions are a way to model many of these systems. Therefore, to be able to understand the meaning of the differential equation, students must be well prepared in order to be able to solve these types of problems. Thus it is necessary to find ways to lighten the difficulties in the learning of this concept from when it is introduced, in secondary education, until their use at university level (Teixeira et al, 2007; Leão et al., 2008). Martínez-Torregrosa and co-authors (2005, 2006) state that, in Physics, students deal with the calculus in a superficial and algorithmic way, in other words, mechanical application of rules regarding meaningless symbols. There is a predominance of techniques over other categories that develop the concepts, contributing to a poor understanding of differential calculus. Studies by Kwon et al., (2007) related to the students' retention of mathematical knowledge and skills in DE, state that students where conceptual understanding is achieved retain the knowledge for a long period of time. In differential calculus teaching the algebraic approach is predominant in relation to geometric and numerical approaches. Algebraic procedures are presented as the most used by several generations of teachers since they are easier to implement and consume more accessible resources (Teixeira et al., 2007). Nevertheless, other approaches should be seen as complementary as they allow a better mathematical understanding either in approach or in the models described by DE. These approaches are often made using tools such as graphic/algebraic calculators and didactics software. Most authors support that computers are an auxiliary tool in the construction of knowledge (Wagner et al., 2007; Brandão et al., 2008). Santos and co-authors (2006) claim that "... *interactive animations fit in the concept of computational tools that are able to assist in the construction of knowledge*...".

In higher level, the use of new technologies is more common given that there are problems that cannot be analytically solved. There is numerous software that can be used to solve academics problems in the mathematics and physics areas: dynamic geometry (Geometric's Sketchpad, Cabri, Cinderella), algebraic calculation (MatLab, Maple, ClassPad, Mathematica) or modelling (Modellus, Stella). Several studies discuss the students' performance when using activities of modelling and computer simulation for learning Physics (Brandão et al., 2008). Simulation and computational modelling can be considered a complementary tool to the techniques used in the traditional teaching. According to Brandão et al. (2008), the process of modelling is a key role in finding answers that help to understand the world that we live. The scientific modelling is a process of creating models in order to understand the reality. A conceptual model is a simplified representation of a system or phenomenon and accepted by the scientific community. In Physics, the traditional methods are inadequate since the complex concepts are difficult to visualize when presented orally or in textual form (Santos et al., 2006). With the continuous change of technologies and curriculum contents updating, it is essential to use different teaching methods to meet the learning needs. The combination of teaching/learning strategies leads to better results in learning as they interact with different realities. Teachers often use different strategies that come together to their own way of teaching but also to the content of classes and students.

Thermal comfort software as stimulus for the self-learning of DE

One way to teach the essence and the estimation of DE solutions is through the development of software to determine the numerical solution of practical problems that are described by them in conjunction with a set of additional conditions (boundary and/or initial conditions). When properly implemented, this set of equations allow us to know diverse characteristics like the temperature distribution in a particular field, concentration of a particular element, etc. Notice that the development of this specific software is often the only way to model a given reality. In this paper, the development of free software that can model thermal comfort will be described and discussed as a teaching/learning tool in the DE subject. The study of the thermal environment of today becomes increasingly important to create conditions for the performance of activities in a created space. Thermal Comfort is a compulsory topic in the 4th year of the Masters in Industrial Engineering and Management (MIEM) (miegi.dps.uminho.pt) included in the Ergonomic Study of Workplaces UC.

Several softwares have been developed for determining the thermal comfort for both hot and cold environments. Although there exist several models, it is not known which one is the best to simulate a particular environment (Haslam and Parson, 1994). However, when the thermal comfort of the human body, the environment and clothing, are included in the thermal comfort model, it becomes much more complex. So, normally simplifications are used: Indices of Thermal Stress, PMV (Predict Mean Vote) and PPD (Predict Percentage of Dissatisfied), which provide general information and the statistical level (Silva, 2009).

The design of good software brings several benefits, not only in business but also in academia. Because software is so widely used its interface is extremely important, besides being appealing to use, its ease in applying it is also relevant. Most of the existing software is developed for research use (Xu and Werner, 1997) or with business characteristics. Due to the lack of thermal comfort software of an academic nature, ConfTermal was developed at the University of Minho (Ferreira, 2005). The development and use of software ConfTermal helps students to strengthen the mechanisms of self-learning in the context of DE. The mathematical model implemented in this software is a system of ordinary DE describing the thermal variation of the human body over time, for a set of environmental clothing type conditions. The student has the opportunity to experience, in a quick and a convenient way, the result of changing an initial or boundary condition, to include an additional term in the differential equation.

CONFTERMAL Software brief description

The software ConfTermal V2 incorporates the mathematical model of thermoregulation in the human body, which is divided into 16 parts (Arezes et al., 2006). The user, or the student, should identify the model physical parameters, namely size and body weight, clothing parameters, physical activity and temperature. To control the numerical system of equations, students should choose the numerical parameters, namely, time and integration step, the stopping criterion and also the initial values of temperatures in various parts of the human body. The main screen of the software interface ConfTermal V2, Figure 1, is divided into two main parts: one on the left side with the drawings of the human body and on the right side the data input, with the legend of colours and a panel where final numerical results are provided. The two drawings of the human body allow viewing of the temperatures before and after simulation, respectively.

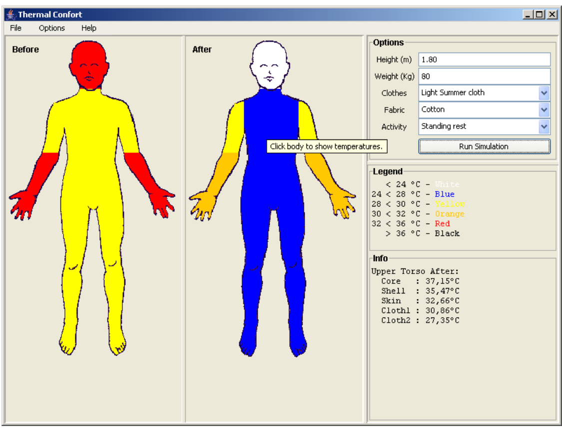

Figure 1. ConfTermal V2 software main menu.

Use of CONFTERMAL as a teaching tool

During the development of Conftermal as educational software, several usability tests were performed in order to test the interface. Several groups of students according to level of education: from high school to postgraduate students, not forgetting the professional technical school students (Arezes et al., 2006; Leão and Teixeira, 2008). Using two standard questionnaires (Software Usability Measurement Inventory, SUMI, and Questionnaire for User Interface Satisfaction, QUIS), it was possible to identify areas for improvement and get a product with good acceptability, meeting the expectations of students and professionals in Comfort areas.

With the results obtained, the objective of the research team is to continue the study, finding innovative ways in the DE teaching/learning methodology. In order to create a bridge between the theoretical concepts already acquired, and the practical problems of engineering, the subject of Thermal Comfort was given as a practical application to students in 2nd year of MIEM under the teaching of numerical solution of a DE system.

The methodology used was to use the software ConfTermal V2 and to compare the final results with the results obtained by implementing the system of DE in Matlab (commercial software, Figure 2). This project work was then distributed and prepared by groups of 4-5 students. Some of the subroutines needed for the application were available. The different groups of students had the opportunity to experience the application of learned concepts on initial value problems and understand the variation of temperature in different parts of the human body over time when subject to a specific set of environmental conditions, clothing (dressed or naked and a particular kind of clothing), metabolism, as well as to understand the influence of physical factors and numerical parameters (step of integration, integration time and numerical method used). At the end, a written report, for each working group, was produced, where students reported beyond the physical problem, explained the simplifications to the physical model, discussed the possibilities of real analytic and numerical resolution of this system of equations. They also explained chosen parameters, obtained many results that they had to analyze and understand and construct tables and graphs to facilitate exposure of the results (Table 1). With the validation of the results using the software, students could understand not only the validity of this graphical interface, a practical application of DE, but also the reason of having an educational tool in this area.

```
T0=[36.71 36.89 36.89 35.53 35.53 35.53 35.53 35.41 35.41 35.81 35.81
35.81 35.81 35.14 35.14 36.96 35 35 34 34 34 34 35.3 35.3 35 35 35 35
35.07 35.07 35 33.62 33.62 33.25 33.25 33.25 33.25 35.22 35.22 34.10
34.10 34.10 34.10 35.04 35.04 34.58];
T0(47:61)=28;
T0(62:76)=27;


dt=10;



Tf=3600;

tspan=[0 1];
[t,T] = ode45('der',tspan,T0)
```

Figure 2. Example of MatLab commands to solve the DE system defined.

Table 1. Temperature values obtained by simulation

Layer	**Chest**	**Belly**	**Arm**	**Forearm**	**Hand**	**Thigh**	**Leg**	**Foot**
Core	37.15	37.15	35.25	34.36	32.88	35.76	35.74	35.13
Shell	35.47	35.47	34.51	33.56	32.12	34.6	34.59	34.34
Skin	32.66	32.66	32.99	31.81	30.55	32.39	32.39	32.69
R1	30.86	30.86	32.44	---	---	31.22	31.24	32.06
R2	27.35	27.35	28.61	---	---	27.79	27.81	27.69

This work is still under development and it is only possible to formulate a qualitative assessment of the impact of this form of teaching and learning in the area of DE. The enthusiasm shown by the involved students at the end of this module of the course of numerical methods was a necessary condition for defining an evaluation methodology.

Final comments and Future work

DEs are quite applicable in various areas of knowledge. Students normally have great difficulty in learning differential calculus and have difficulties in the mathematical translation of the physical problem. Several studies have been made to overcome these difficulties, independently of the level of the educational process and in the tools used for their learning. Concerning the learning tools, several authors advocate the use of computers for the construction of knowledge, alternatively to the methods used in traditional education. The informatics tools allow the connection with real problems and the successive construction of knowledge, independently and in accordance with the flow of the work. Most of all, in such a way, that can improve students' motivation (Leão et al., 2008).

The presented study describes a teaching/learning methodology that covers two important aspects in an engineering course. On the one hand, the practical component in the teaching of solving a system of DE involving an issue of the syllabus of the course and, moreover, its numerical solution was done using software developed by a team of researchers. This study, though exploratory, identified the students' adherence to this new methodology. To quantify and evaluate the students' acquired knowledge, as well as the process itself, it will be important to conduct a questionnaire amongst students, where they would be asked how implementing this methodology can help them to understand the meaning and how to solve differential equations. A complete study of this questionnaire will be done subsequently, giving to the research team, new opportunities for strengthening this approach.

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From differential equations to real-world problems in Control Engineering

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Abstract

This paper describes two approaches of interactive use of academic content, both with the main objective to complement the classes: (1) SimLab and (2) a streamed module. At the moment, the study of the use of the virtual laboratory SimLab as a tool in the students' learning process is being carried out. In the Control teaching/learning process, SimLab allows the user to choose the practical engineering problem and the control type to be implemented. The control problems are modelled by a set of differential equations. SimLab, as a didactic tool, allows testing of several numerical methods in the Differential-Algebraic Equations solution, building synergies between the different acquired competencies during the student's academic path. The theoretical pedagogical aspects of the module on differential equations were making accessible in streaming. The degree of satisfaction with this experience was significantly elevated.

Introduction

A web-assisted page can be considered as a complementary learning tool. It consists of coupling the class teaching with free teaching through the Internet, with animated case studies and interactive means, where students are active pieces in the learning process. It is an active learning, as instructional activities engage students in doing something besides listening to a lecturer and taking notes (Felder, 2000). Cognitive theory states that knowledge learned and applied in a realistic problem solving context is expected to be remembered and used properly when needed later. In fact, these problem-based learning/teaching strategies, case methods and simulations, are useful tools for effective teaching since students must become active participants rather than passive observers. Students must make decisions, solve problems and analyse the results achieved (McKeachie, 2001). The benefits of providing such a complementary learning are also: a higher rate of student approval, an increase in students' engagement, an improvement in students' responsibility and better communication between students and teachers (Leão and Soares, 2006).

Merging the ideas of Jurow (2007) and of Garrison et al. (2003), streaming presents a new way to acquire knowledge, which results in an opportunity to explore via interaction with technologies that can offer indisputable quality and up-to-date support. Streaming can be defined as the act of transmitting media content so that, at any time, it can used by a customer in real-time (Shephard, 2003). Developing academic content, which enables students to study in their own rhythm, anywhere and whenever they want, and that, simultaneously, appeals to their creativity and autonomy in search of alternative sources of information, which allows them to deepen their knowledge, and which creates a support structure that facilitates resolution and discussion (in the face-to-face environment) of these problems, was the desired goal of the implementation of the thematic modules in streaming (Santos et al., 2007, 2009).

In the following sections, a brief description of the two strategies developed to follow the trend of the new generation of students, by creating different virtual environments of learning is presented (Adams, 2006).

SimLab: a Virtual Lab for Control Problems

In 2009 SimLab (Lemos et al., 2008), a virtual laboratory for the modelling and simulation of real-world problems in process control engineering was developed (Figure1). One of the major goals of SimLab is to focus a student's attention on how numerical methods in the resolution of differential equations can be directly employed in Process Control real-world problems. In fact numerical methods are used in control simulations running in open and closed loop. All the available examples simulate different real systems: hydraulic, electrical (RL and RLC circuits of 1st and 2nd order), thermal (greenhouse, water heating) and mechanical (mass-spring systems, pendulum, parachutist problem, suspension, inverted pendulum), running in open and closed loop modes. Process and control parameters (hysteresis or PID gains) can be changed during simulation testing. Specific help routines with theoretical support on the subject being studied, regarding control as well as numerical integration methods, are also available. Due to the fact that control systems are governed by differential equations, the virtual laboratory also incorporates and permits the testing of several numerical methods (Runge-Kutta and Predictor-Corrector Methods) in the resolution of the corresponding differential equations. The simulation results are presented in graphical form (showing state and control variable) as well as in animated form (animated drawing of a water level control in two tanks, Figure 2). A set of multiple-choice questions was included allowing to the student assess their study of elementary concepts on Ordinary Differential Equations (Figure 3). This can be used not only as a self-evaluation process but also as a tool to identify learning difficulties.

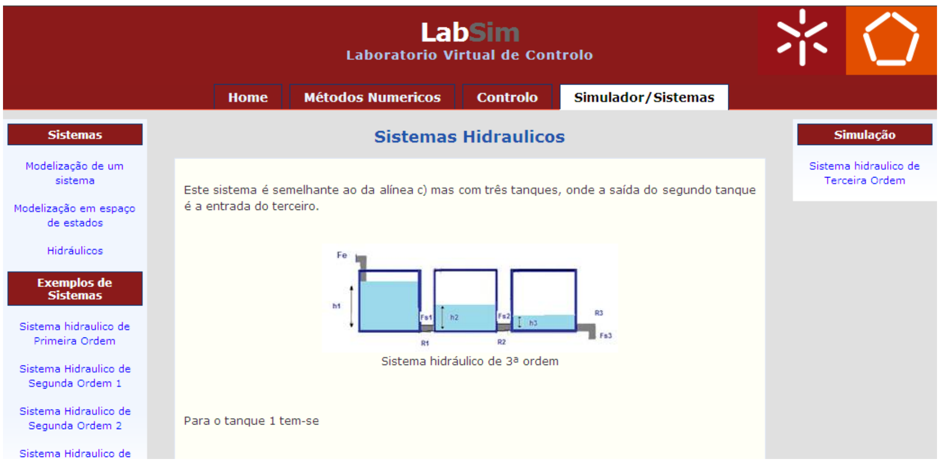

Figure 1. Web page for Numerical Methods and Control Simulator, SimLab. Available in: www.labsim.dei.uminho.pt (in *Portuguese*).

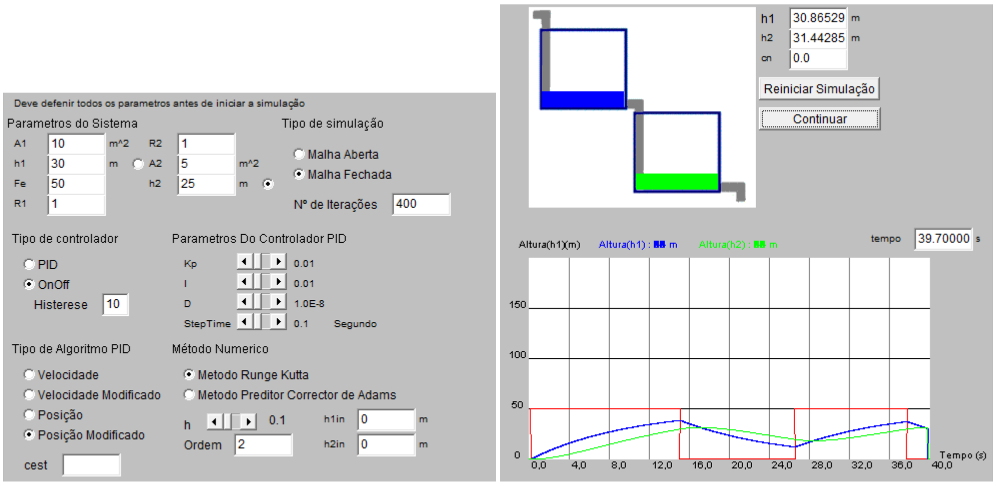

Figure 2. LabSim: closed-loop controller with On-Off of a second order hydraulic system controlling the height of tank 2.

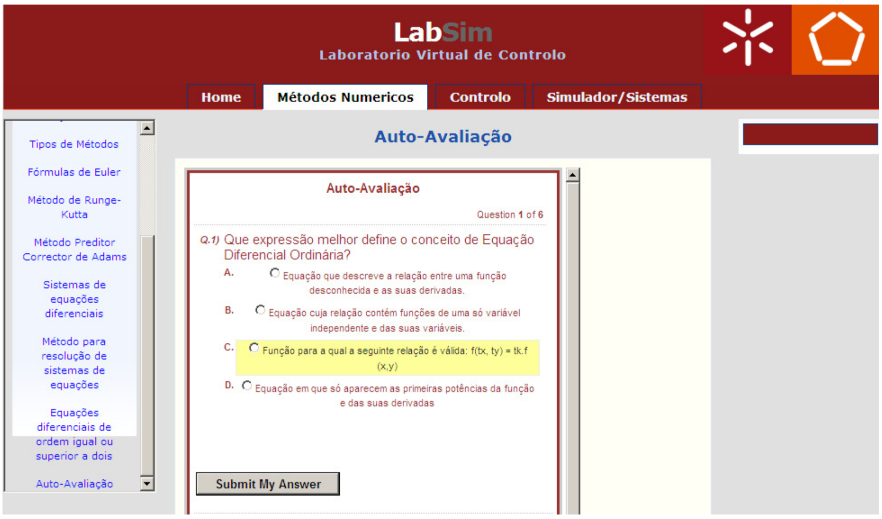

Figure 3. LabSim: multiple-choice questions for students to assess their study of concepts on Ordinary Differential Equations.

Streamed Differential Equations Module

Simultaneously, the theoretical pedagogical aspect of the module on differential equations was made accessible via streaming (Santos et al., 2009). The development of the module followed the structure presented in Figure 4: a) summary of contents; b) video stream creation with the teacher presenting the content; c) slides with audiovisual support; d) activities and bibliographic support. These activities were based on problem resolutions to be discussed in face-to-face classes. The selected contents were of a theoretical nature, based on two pre-supposed premises: they are characterized by the need to be transmitted one-way (belonging to the teacher) and secondly, they are vital because they form the knowledge base, about certain subjects that every student must obtain. The video capture was made by a webcam directly connected to an encoding system. This allowed compressing the video to a format that could be streamed. To ensure compatibility with Microsoft technology, WMV was used. The synthesis of the theoretical contents was compiled in Power Point, in order to enable the student to follow the presentation recorded by the teacher on video. The platform Moodle was the virtual classroom. The unit learning was kept on-line during the entire course unit.

Figure 4. Page of the module presented in streaming.

An interdisciplinary share experience was also implemented, involving two teachers from different departments teaching different learning units. However the units had a common scientific base, where some of the contents can be reused. The differential equations contents were reused in order to refresh the students' knowledge with respect to this subject in order to carry out a different curricular unit in a different year.

Findings and Discussion

At this moment the SimLab virtual laboratory is integrated into a platform for Web Assisted Laboratory for Control Engineering on-line education, named WALC, to facilitate the remote and virtual process, monitoring and control of the laboratories developed in this context. At the end of this semester, this platform will be tested in real context, analysing students' interactions in their assessment of the platform, namely motivation, technical and collaborative skills acquired. It is the team's belief that the students will be able to identify and follow their courses, interacting with the interface parameters, testing system performance, defining inputs and analyzing results.

For the streamed modules' analysis, the results were obtained based on the gathered information from students through the distribution of two questionnaires designed for this purpose. The first was applied before the presentation of streamed modules, with objectives to identify the characteristics of the audience who were to engage in the experience, and to collect relevant and necessary information to implement the modules, particularly with regard to the type of available resources, patterns of Internet use and analysis of motivations and expectations. The second questionnaire, delivered on the final exam day, permitted knowledge of the balance made by students against the use of content streaming, as well as evaluating the model of teaching/learning used, and identifying problems and potential that derive from this practice, seen as a complement to classroom learning.

Due to the extent of both questionnaires, only the main variables will be here discussed and shown. The learning experiences supported by this form of content (streaming) were a novelty for 89% of the students. Only a minority of 11% referred to having already had some contact with this kind of material. The students had no difficulty in accessing these materials, demonstrating a technological knowledge perfectly suited to the requirements of streaming contents. It was interesting to note the findings when the students were asked to describe the best ways of consolidating their study, if reading, listening, viewing images or through discussion and dialogue on the subjects of study. The Figure 5 illustrates the results obtained. Students indicated reading (34%) as the first choice activity to better assimilate a content followed by hearing the material (35%), viewing images (30%) and finally the discussion and dialogue on content analysis (32%). Naturally, this does not misinterpret streaming as a way of presenting content, because it can include any of the three options as most valued by students. However, it should be noted that there is also another dimension, important as a complement to streaming, which is the debate. Indeed, the debate and the generated discussion around the topics of study are critical for fostering dynamism and interactivity, so important to strengthen learning. The degree of satisfaction with this experience was significantly elevated (76% of the students mentioned that were enough and very satisfied).

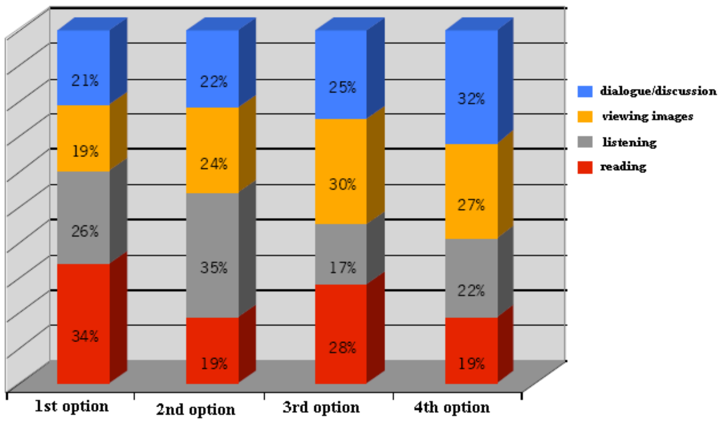

Figure 5. Options that reflect the students' best way to assimilate the contents.

Under the pedagogical point of view, the majority of students strongly agree that this form of content gives them the opportunity to study at times that they consider more convenient and to promote self-evaluation.

The perspective of teachers that participated in this experience, on the use of a teaching model based on streaming, was obtained via semi-directive interviews. The completion of these interviews was based on a screenplay of orientation, consisting of a set of questions previously developed for this purpose. Reflecting the views of teachers, it can be concluded that they recognized the potential value of streaming to the teaching/learning process. This technology, to be applied in the processing of content-specific nature, acts as a promoter of innovation in educational practice. It also provides, to the teacher and to the learning process, a better approach to the philosophy of the Bologna model, since it drives a process of transformation that reorganizes and reformulates time, spaces, contexts and strategies.

Acknowledgments

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What type of student avails of mathematics support and extra mathematics initiatives?

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Abstract: Most mathematics support initiatives aim to help struggling or at-risk students. However, studies show that students who avail themselves of mathematics support vary both in terms of their ability and their reasons for seeking extra support. It is also well known that a considerable number of at-risk students do not avail themselves of support services. To provide efficient and effective supports it is important to have a description of the type of students attending or not attending and the reasons why. We present an overview of the initial findings of two research projects on these topics. We will discuss preliminary findings from a study of repeat and non-repeat mathematics students which suggests several reasons (not necessarily related to ability) why students do not avail themselves of mathematics support. We will also present evidence that at-risk first year students are more likely to attend the Mathematics Support Centre than students who are not deemed to be at-risk. However, a significant minority of at-risk students do not avail themselves of the supports offered.

1. Introduction

The Mathematics Support Centre (MSC) at the National University of Ireland Maynooth (NUIM) is now in its third year of operation. The MSC was originally set up in order to provide support for at-risk students. The MSC is one of many supports in place to help students if they experience difficulties; others include weekly small-group tutorials, graded assignments, online courses and follow up workshops.

In the academic year 2007/2008 the drop-in centre had 2493 visits from 273 students. In its second year (2008/2009) there was a 93% increase in the number of visits to 4647 visits from 509 individual students. To date in 2009/2010 (with two weeks of operation remaining) there have been 6474 visits from 590 students.

As a result of initial investigations by Mac an Bhaird and O'Shea (2009) into the operation of the MSC evidence emerged to suggest that a small minority of at-risk students were not availing themselves of the supports and that students, other than at-risk students, were using the centre also. In this paper we will focus on first year students. We will present evidence which shows that the majority of first year students who attend the MSC are at-risk students. We will also discuss investigations into the possible reasons why first year students do not avail themselves of support. Several themes have emerged including fear, lack of awareness of the details of support services, embarrassment etc. This is also reported upon in more detail in Grehan et al

(2010). In this paper we will focus on their apparent lack of awareness when compared to their counterparts.

The dramatic increase in numbers attending the MSC has put the service under severe strain and the results and implications of our research are crucial to making our service more efficient and effective.

2. Related research.

Some recent research by Patel and Little (2008); Dowling and Nolan (2006) and Lee et al. (2008) highlights the benefits of mathematics support to students with weak mathematical backgrounds. Mac an Bhaird et al. (2009) discussed the impact of the MSC on the grades of first year students. In addition, Pell and Croft (2008) and MacGillivray (2009) have reported on the use of support services by students with strong mathematical backgrounds. Ryan et al. (2001) and Hannula (2006) have found that the fear of showing a lack of knowledge or ability negatively impacts on students' willingness to ask questions. This fear factor and many other reasons were also identified in a study of students at Loughborough University by Symonds et al. (2008).

3. Methodology

The data collected and analysed on the type of student who takes advantage of support comes from MSC attendance and registration forms, students' second level grades and Department of Mathematics diagnostic tests and end-of-semester exams. The data was analysed using SPSS.

The data presented on the reasons why students do not take advantage of support was collected from anonymous questionnaires and follow-up interviews. In October 2009 39 students who were repeating first year mathematics modules were identified, contacted and invited to participate in the study. Twelve students agreed and they were asked to complete a short questionnaire concerning their first year mathematical experiences. Seven of these students were also interviewed. The transcriptions were coded using grounded theory, Strauss and Corbin (1990). In February 2010, we contacted students who had passed first year mathematics modules despite having similar mathematical backgrounds to the students who had failed. Nine of these students were interviewed. The most recent interviews have not been completely analysed to date but initial findings will be reported on here.

4. Results

4.1 What type of student avails of support?

We consider the composition of the first year student groups that took advantage of the MSC drop-in services in 2008/09. The overall breakdown of attendances for 2007/08 was very similar and the breakdown for 2009-2010 is not yet available. In the year 2008/09, 54% of visits were by first year students registered for a mathematics module with the Mathematics Department. This group consists of Science students, for whom mathematics is compulsory, and Arts and Finance students who have chosen to study Mathematics as one of their three first year subjects. For the sake of brevity, we will refer to the Arts and Finance group as the Arts group, since they take the same modules. The remainder of visits is made up of second year and third year mathematics students and students who were not registered for a mathematics module or by a small group of students who were taking a pure mathematics module; a more complete analysis is available in Mac an Bhaird and O'Shea (2009). There is evidence that the MSC is being used by students who are not registered for a mathematics module. Many of these students are studying Engineering, Psychology, Geography, Sociology and Economics. Since we do not have access to these departments' records, we will not be able to include these students in the analysis that follows.

Table 1 shows the percentages of 1st year groups who attended the centre. It is clear that there has been a huge increase in the percentage of students attending. The increase in the attendance rates of the first year students is encouraging.

Groups	2007/2008	2008/2009
First Science	32%	61%
First Arts	34%	55%

Table 1: Percentage of 1st year groups attending the MSC.

We decided to consider the number of visits made by students from each group. Note that we did this for all year groups and the pattern of visits was not uniform, for example second arts students attended more regularly on average than the other groups, which is not surprising as they are a highly motivated group and usually do not fit into our at-risk category. We only considered students who took the final examinations and the data is reported in Table 2. In this table we consider the percentage of the group who made no visit, one visit, two to five visits etc.

Group	n	0 visits	1 visit	2-5 visits	6-10 visits	11-15 visits	16-20 visits	>20 visits
First Science	267	39%	10%	25%	12%	7%	2%	5%
First Arts	204	45%	15%	20%	12%	4%	2%	2%

Table 2: Numbers of visits to MSC in 2008/09 by 1st year group.

We then wanted to determine if the MSC was catering mostly to at-risk students.

The Mathematics Department administers a diagnostic test to every first Arts and Science student in the first week of term. This test has 20 questions and students receive 3 marks for a correct answer and -1 for an incorrect answer. Students who receive 20 marks or less are considered to be at-risk of dropping out or failing their examinations. In the Irish Education system, students take an examination called the Leaving Certificate at the end of their second level education. Mathematics can be taken at Foundation, Ordinary or Higher levels. Only students who have passed Mathematics at Ordinary Level (OL) or Higher Level (HL) may enter university. Students who have studied mathematics at OL are often disadvantaged compared to their peers who have studied HL mathematics. For this reason, the Mathematics Department also considers OL students to be at risk. An in-depth analysis of the breakdown of pass and fail rates within the HL and OL groups is available in Mac an Bhaird et al. (2009). Table 3 shows the percentages of first year students in these at-risk categories who attended the MSC.

Group	Attendance	Leaving Cert Level		Diagnostic Test	
		HL	OL	Pass	Fail
First Arts	Attended >1	48%	54%	53%	73%
	Attended >15	2%	6%	5%	3%
First Science	Attended >1	45%	62%	57%	62%
	Attended >15	1%	8%	5%	7%

Table 3: Percentages of at-risk first year groups attending the MSC in 2008/09

It appears that on the whole the attendance rate for students in the at-risk categories is higher than the rate for the students who are not considered at risk. However the differences are not very big and HL students and those that have passed the diagnostic test are still attending the MSC.

Additional data presented in Mac an Bhaird and O'Shea (2009) shows that in higher years, students who attend are less likely to be at risk, they also attend more often and it appears that they stay longer than the average first year visit to the MSC. For the most part they are students who are seeking higher grades in their exams.

However, it is clear from the data above that there are at-risk first year students who are not attending. This is supported by a similar outcome reported in the end of year (2008/2009) MSC anonymous questionnaire. It was completed by 446 students, 307 had attended the MSC and 139 had not. Amongst other questions, students were asked 'How often do you have difficulties with Mathematics?' and given the options: always, often, sometimes, rarely and never. The majority of students (both attendees and non-attendees) reported having difficulties with Mathematics. There was a significant difference (Fisher exact test, $p < 0.0001$) between the responses of attendees and non-attendees to this question. Attendees reported having difficulty more often than non-attendees. It is reassuring that students with difficulties are accessing support, however, there are a significant number of non-attendees with difficulties; 12% said they always had difficulties; 24% often and 39% sometimes. As the questionnaire is anonymous we can not determine how many of these are at-risk students. However, the following section confirms that some at-risk students are not attending the MSC.

4.2 Why do students not avail themselves of support?

A preliminary analysis of the interview and questionnaire data from the students who failed some mathematics modules in 2000/2009 has shown that the main factors in students' non-engagement with mathematics support were fear, lack of awareness of services, personal difficulties, and lack of personal motivation. Students were also reluctant to ask for help and feared embarrassment. Grehan et al. (2010) focus on the fears that students expressed and how these fears prevented them from engaging with mathematics during their first year at university. We found that this fear manifested itself in four different ways: fear of failure; fear of showing a lack of knowledge or ability; fear of being singled out; and fear of the unknown. Students also displayed a lack of awareness of services or structures within mathematics.

When we compare these comments to the comments of students with similar mathematical backgrounds who had attended the MSC regularly we see a marked difference. It is important to note that the students who failed had almost exclusively not engaged with supports whereas the students with similar mathematical backgrounds who passed had engaged.

The transition from second to third level is difficult for most students, and often they are not sure how the system works. The students who failed seemed to be wary of anything that they did not understand and were unwilling to try anything new. This was particularly true of their reaction to the MSC. Some students expressed reservations about attending when they were not sure what happened there.

> *You know, kinda nervous to go off somewhere you didn't understand, you know you didn't, stuff that you want to (inaudible) strangers of stuff that you did not understand. And you just kind of felt embarrassed about not knowing how everything was working.* ***(MSF2 on the MSC)***
>
> *Then second semester I went to the door, looked in and it was really, really busy and I just thought "hmmm, no!". And I turned around.* ***(FFF1 on the MSC)***

If we compare these responses to some from students who attended the MSC and passed first year, there is a stark contrast in their attitude towards support.

> *I thought, you know, that's a really good idea to help people so you're not on your own. 'Cause you know yourself, sometimes if you're struggling with a problem, you just look at a blank page, you know, you have to start something but sometimes somebody can just say, "have you considered this?", and it just sets you off doing the whole thing on your own.* ***(MFP1 on the MSC)***
>
> *Ciarán [MSC Manager] came in our first or second lecture and told us about it. And he told us the opening hours and I took them down straight away. Then after the first week I decided I better go. He said*

there was tutors there, willing to help, willing to help explain stuff and I thought "well, what have you got to lose?" ***(FSP3 on the MSC)***

So the difference in attitude towards engaging in support is quite different. There also seems to be a difference in their understanding of the reasons for trying mathematics problems. The students who failed do not seem to understand that you need to try problems to gain a better understanding and that asking for help is part of this process. This clearly ties in with their fear of failure.

*'Cause I wasn't going to lectures, when I was doing my homework I didn't have a clue so I was just like "I can't go to a tutorial 'cause I won't know how to do...", you know that sort of way? (***FFF2 on lack of engagement)***

If you hand in a bad homework the lecture can focus a lot more on you and you know it will make you feel, not stupid but if you hand in a bad homework, this is me personally, I'd be less inclined to go to the tutorial. I probably, coming back from being a 1st year just coming from secondary school I would have thought as well that if I went up to lecturers and said things, like "I haven't been coming to many of your classes, I haven't a clue what's going on". I would have thought I just be given out to or I dunno, I didn't know what way it worked you know? ***(MSF3 on lack of engagement)***

Students who engaged with support had significantly different responses.

It's fundamental, how can you learn anything if you're not gonna get help with it? I mean, you can improve on your own and I must say, when you're stuck on a problem and you solve it yourself it's a wonderful feeling. But unfortunately, it's all too rare. ***(MFP1 on engagement)***

No, like I'm one of few people who don't mind doing that, but like, most people wouldn't be like that, most people would not put up their hand and say, "that makes no sense! "You can't put a value on how important that is because if you don't seek the help then, you know, you're [in trouble], you're just digging a hole for yourself." They're too embarrassed that someone's gonna look at them and say, look at them like they're an idiot, whereas my attitude was, well I don't care if they think I'm an idiot. As long as they help me at the end of the day, I'll put up with that! ***(FSP3 on engagement)***

So there is a clear difference in the attitudes and behaviour of the two groups of students and a more in-depth analysis is underway. It is important to note that these attitudes do not appear to depend solely on ability. Some of the students who failed do not fall into our at-risk category.

5. Conclusions

Our analysis to-date has shown that the MSC is very well attended by all year groups and also by students who are not studying mathematics. The at-risk students in first year have good attendance rates as do the high achievers from the senior years. This shows that the centre is not viewed as a 'remedial mathematics' centre, but as a resource for the entire student body. However, a considerable number of at-risk students still do not avail themselves of support. Initial analysis of the interview and questionnaire data has shown many reasons for this lack of engagement and this requires much more in-depth investigations.

Analysis of the data allows us to tailor supports to the specific needs of different groups. For example, we have started a pilot mentoring system for students who are at risk and the Department of Mathematics sends out letters to all students who fail their module to encourage them to come and talk about their issues. A review of these initial interventions will take place in the summer 2010 to decide if they should be adapted on a wider scale.

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Control Theory – association of mathematics and engineering

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Abstract

In this paper a methodology of teaching a Control Theory at AGH UST and HVS Tarnow is briefly presented. Control Theory is an area associated with both a wide part of mathematics and a very important part of engineering. The course of Control Theory contains three parts: lecture, auditory classes and laboratory. The mathematical skills are crucial for automation engineers to project, implement and supervise real control systems in industry and life. The general view about the Control Theory course is illustrated by a simple example of development, tests and practical realization of a laboratory control system.

An introduction

Control Theory is anarea of technical sciences closely associated with both mathematics and engineering practice. This implies that mathematical skills are absolutely necessary for an automation engineer to solve all problems from areas of development, supervision and servicing of industrial control systems.
The following aspects will be presented:

- Areas of mathematics applied in Control Theory,
- The methodology of teaching Control Theory: lectures, auditory exercises, laboratories,
- An example of a real automation task solved by students during Control Theory course.
- Conclusions.

Areas of mathematics applied in Control Theory

Control Theory applies a number of mathematical tools, associated with different areas of mathematics. The most important are the following:

Differential equations

They are a fundamental tool to describe dynamic systems. Depending on the physical nature of the described plant they have different forms. For example systems with distributed parameters are described by partial differential equations, non-linear systems are described by non-linear equations. The differential equation is always a basis to build a model closely associated to Control Theory: state equation or transfer function. Transfer functions are calculated with the use of Laplace or “z” transforms.

Matrix theory

A number of fundamental properties of dynamic systems described by the state equation can be defined and analyzed with the use of matrix theory. For example: the form of the spectrum the state matrix determines the stability of the system, the rank of the controllability or observability matrices determines the controllability or observability of the system. A broad set of examples covering applications matrix analysis in Control Theory was presented by Mitkowski in 2007.

Polynomial algebra

Polynomial algebra is a main tool in the analysis properties of dynamic systems described with the use of transfer function. The stability of a system is determined by the localization of the roots of the transfer function denominator.

Complex numbers

Complex numbers allow us to describe the properties of dynamic systems from the point of view of frequency. This analysis can be done by using the idea of the spectral transfer function, which is obtained from the transfer function by replacing the complex variable "s" by "jω".

Functional analysis

Functional analysis is an advanced tool to analyze control systems for all classes of control plants, for example for distributed-parameter or non-linear plants. For example, it allows us to transfer a partial differential equation into the form of an infinite-dimensional state equation and to analyze properties of such systems. A broad presentation of applying functional analysis in Control Theory can be found in books by Balakrishnan (1989) or Mitkowski (1991). An example of using functional analysis to construct the control system for infinite-dimensional plant (a heat plant) can be found in paper: Mitkowski and Oprzedkiewicz (2009).

Interval analysis

This area of mathematics is a powerful tool to analyze control systems for plants with uncertain parameters. The basics of interval analysis were formulated by Moore in 1966. Applications of interval analysis in Control Theory were presented for example by Jaulin in 2001.

Numerical methods

The main area of application of numerical methods in Control Theory is simulations useful for the verification of results obtained with the use of "theoretical" methods. A typical approach is the use of simulations programming tools dedicated to this goal, for example suitable toolboxes of MATLAB.

The methodology of teaching Control Theory

The course of Control Theory run at AGH UST and HVS Tarnow consists of three parts.

The basis and crucial part of the Control Theory course is a lecture. During lectures there are presented main problems and methods of their solution with the use of particular mathematical tools. Lectures start with the presentation of mathematical models of real control plants. A most general mathematical model describing the dynamics of each dynamic system is a non-linear, non homogenous, time- variant, *n*th order differential equation of ordinary or partial type. In a lot of real situations this general model can be simplified and then it turns to the form of first order, linear, time-invariant matrix differential equation. This equation is called in Control Theory a state equation.

Additionally, with the state equation is associated an algebraic linear equation, describing the relation between a state of system (it is generally not available to observation) with an output of the system (only output is “measurable”). It is called in Control Theory an “Output Equation”. The two equations build the complete mathematical model of the each dynamic system. For linear, finite-dimensional, time-invariant system they have the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

In (1) $x(t) \in R^n$ denotes the state vector, $A_{n \times n}$ denotes the state matrix, $u(t) \in R^m$ denotes the control vector, $B_{n \times m}$ denotes the control matrix, $y(t) \in R^r$ denotes the output vector, $C_{r \times n}$ denotes the output matrix and $D_{r \times m}$ denotes the matrix of direct control.

The equations (1) can also describe infinite-dimensional systems, an example of such approach will be presented.

The equations (1) fully describe the behaviour and elementary properties of the system: eigenvalues of the matrix *A* determine the stability of the system. Stability is a fundamental property of each system. From a mathematical point of view a definition of stability is similar to the definition of continuity, the pair of matrices *(A,B)* determine the controllability of the system, the pair of matrices *(C,A)* determine the observability of the system.

An alternative (and more simple than state equation) model of the dynamic system applied in Control Theory is a transfer function. The transfer function describes only the observable and controllable part of the linear system and it is a description of “input-output” type. A transfer function can be obtained from the differential equation describing the dynamics of the system or from the state equation (1). It has the following general form:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + ... + b_1 s + b_0}{a_n s^n + ... + a_1 s + a_0} \tag{2}$$

where: *U(s)* denotes a Laplace transform of input, *Y(s)* denotes a Laplace transform of output, a_0 … a_n and b_0 ... b_m are real coefficients. Degrees of the numerator and denominator of the transfer function (2) must meet the assumption $m \leq n$ to assure the physical realization of the system.

The essential problem during construction of the both models is identification of their parameters. These are realized via suitable experiments on real plant and with the use of the least square method.

Furthermore, during lectures are presented the main properties of dynamic systems: controllability, observability, stability and mathematical methods of their testing for different classes of plants. Controllability and observability are discussed for the state-space equation only (the stability can be tested both for state space equation and transfer function) because the transfer function describes only the controllable and observable part of the system.

Then the idea of a closed-loop control system is introduced. This idea is fundamental in Control Theory. It was presented by a lot of authors; a good preview of classic results can be found for example in books: Franklin (1991), Grantham W. J. (1993). The general scheme of a closed-loop system is shown in Figure 1. The system contains a controller and a control plant. In Figure 1 *r* denotes a seat point, *e(t)* denotes an error signal in system, *u(t)* denotes a control signal and *y(t)* denotes a process value. The general idea of this system is to keep the process variable *y(t)* equal to seat point *r* independent of disturbances. This is realized by the controller: it is required to calculate such a control signal *u(t)*, which assures the difference between seat point and process value equals zero. The measure of this difference is the error signal *e(t)*. The general scheme shown in Figure 1 is applied in all areas of industry and life. It can be found in air conditioning system, cars, planes etc.

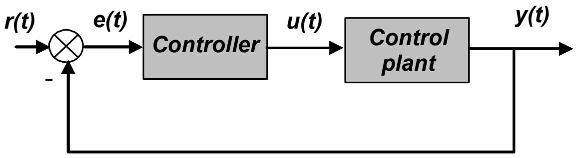

Figure 1. The closed – loop control system

Methods of projecting control systems dedicated to different plants with the use of different controllers are also presented during lectures. During the first level of study are presented most typical control algorithms: the relay controller and a PID controller. For PID are also presented tuning methods. During the second level of study are also presented methods of optimal control.

The next important part of Control Theory course are auditory classes covering problems of calculations associated to problems presented during lectures.

Classes start with the construction models of control plants: transfer functions and state equations for particular examples of real systems from areas of mechanics, electrical engineering, chemistry, etc. During classes models of real plants, available in the laboratory, are always analyzed. The starting point to construction of these models is always a differential equation describing the dynamics of the system. With the use of the differential equation are obtained state equations or transfer functions. Next for these plants are constructed control systems: the controller is proposed and the stability areas are estimated with use for example of the Hurwitz theorem.

The third part of the Control Theory course are laboratory sessions. During laboratories there are jobs possible for realization in the laboratory only (for example identification of control plant) and obtained results are tested during auditory classes. Verification of results is accomplished with the use of simulation tools (for example MATLAB/SIMULINK) or with the use of real control systems, containing real laboratory plant controlled by computer or industrial controller (for example PLC).

An Example

As an example consider a laboratory servo system with DC motor, shown in Figure 2. This system is analyzed and investigated during both auditory classes and laboratories during Control Theory course.

The DC motor via gearbox and load moves the output potentiometer. The input signal for the system (the control signal) is the input voltage *u(t)*, the output signal is the voltage *y(t)* determined by the position of potentiometer connected to the rotor.

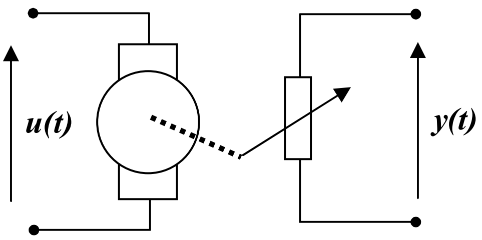

Figure 2. Laboratory servo system

The analysis of the above system during auditory classes starts with construction of the mathematical model for it. The fundamental model of this system is built by the following ordinary differential equation:

$$T_1\ddot{x}_1(t) + \dot{x}_1(t) = K_1 u(t) \quad (3)$$

where: $x_1(t)$ denotes the position of the rotor, T_1 denotes the time constant of the motor, $K_1 > 0$ denotes the coefficient determined by mechanical parameters of the motor, $u(t)$ denotes the input voltage, $y(t)$ denotes the output voltage, determined by the position of rotor: $y(t) = cx_1(t)$ ($c > 0$).

Equation (3) is the basis for constructing models useful in Control Theory. The first one is the state equation in form (1). For the considered case a state vector can be defined as: $x(t) = [x_1(t)\ x_2(t)]^T$, $x_2(t)$ denotes the velocity of rotor. Then the equations (1) become:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T_1} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_1}{T_1} \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} c & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \quad (4)$$

Another model of the system we deal with is the transfer function. It can be obtained directly from differential equation (3), or from state equation (4) and it is defined as follows:

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B = \frac{cK_1}{s(T_1 s + 1)} \quad (5)$$

The next step is to project for the DC motor a control system in according to scheme 1, whose job is to "trace" the changes of input voltage, which is the seat point $r(t)$. needs to project and tune a controller. The simple solution of this task is to apply a proportional controller with inertia, described by the first order transfer function $G_c(s)$:

$$G_c(s) = \frac{K_c}{T_c s + 1} \quad (6)$$

In (6) $K_c>0$ denotes the gain of controller, $T_c>0$ denotes the time constant of the controller. In a correctly constructed controller it should be much smaller than the time constant of the plant: $T_c << T_1$.

The closed-loop system containing both controller and plant has the structure shown in Figure 1. The transfer function of the whole control system is:

$$G_{cs}(s) = \frac{Y(s)}{R(s)} = \frac{cK_1K_c}{T_1T_cs^3 + (T_1 + T_c)s^2 + s + cK_1K_c} \tag{7}$$

The next problem is to tune the controller to the plant to assure the correct working of the system and assumed control performance. The fundamental property of each real control system is an asymptotic stability. The stability of the system described by the transfer function is determined by the localization of roots of the transfer function denominator in complex plane. This implies that the stability can be tested with the use of mathematical tools to localization of polynomial roots. The one of most typical is the Hurwitz theorem. The Hurwitz array for the closed-loop system described by (7) has the following form:

$$H = \begin{bmatrix} T_1 + T_c & cK_1K_c & 0 \\ T_1T_c & 1 & 0 \\ 0 & T_1 + T_c & cK_1K_c \end{bmatrix} \tag{8}$$

The system (7) will be stable, if all the sub-determinants of array (8) are greater than zero. This condition allows us to calculate the controller parameters K_c and T_c assuring the stability of the closed-loop system. For our example a condition for these coefficients was calculated from condition (8) with the use of sub-determinant H_2 only, because sub-determinant H_1 is always positive for positive values of time constants T_1 and T_c , and H_3 is positive if H_2 is positive. The sub-determinant H_2 has the following form:

$$H_2 = \begin{vmatrix} T_1 + T_c & cK_1K_c \\ T_1T_c & 1 \end{vmatrix} \tag{9}$$

From condition: $H_2 > 0$ we obtain at once the following relation for the controller's gain K_c:

$$0 < K_c < \frac{T_1 + T_c}{cT_1T_cK_1} \tag{10}$$

After the project the control system should be tested with the use of MATLAB/SIMULINK. A suitable model is shown in Figure 3.

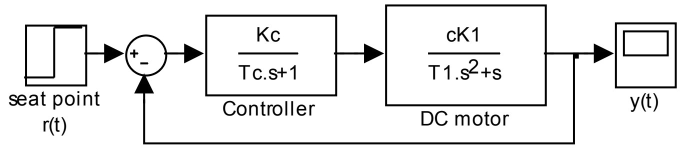

Figure 3. The SIMULINK model of the system

The project of the control system run by students is finished by tests with the use of real servo, shown in Figure 4. These tests are also run during laboratories, the controller is implemented onto a PC computer; during other courses it is also implemented onto PLC.

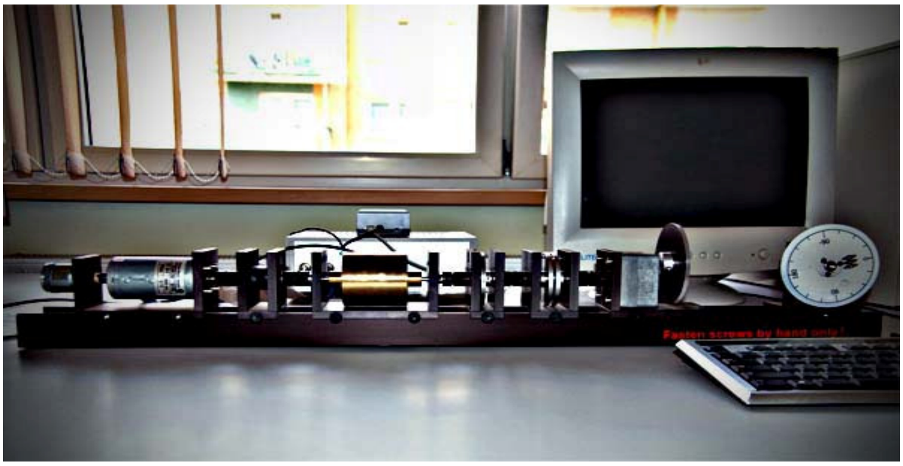

Figure 4. The real laboratory servo system

Conclusions

Themain conclusions from the paper can be formulated as follows:

- Control Theory is an essential area of applications mathematics in engineering and industrial practice. Each control system must be developed and tested with the use of suitable mathematical tools.
- Mathematical skills are crucial for the automation engineer during the project, verification and supervision of each control system in practice.

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What Do They Really Need to Know? Mathematics Requirements for Incoming Engineering Undergraduates

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Abstract

Engineering programmes in Irish universities have a minimum mathematics requirement for all incoming students. At least 55% in Higher Level Leaving Certificate mathematics (the final examination after secondary school) is required in order to enter an accredited honours engineering degree programme. However, in recent years, a wide variety of practices have developed to allow students who have not obtained the necessary mathematics grade a second chance to enter into such programmes. These practices range from one-off mathematics examinations offered by individual universities to summer schools or bridging years to allow students an opportunity to improve their mathematical skills, and the initiatives have been of mixed success.

Previous research undertaken in the Dublin Institute of Technology has shown that the greatest predictor of successful completion of first year for an engineering undergraduate is their incoming mathematics level. Therefore, with the ultimate aim of establishing the minimum level desirable for new entrants into engineering programmes, we compare the Irish situation to the approach taken in other countries with equivalent engineering qualifications. We explore some of the various "second-chance" mechanisms on offer in other countries, with a view to attracting as many students as possible into engineering programmes while maintaining necessary standards.

Introduction

Eight years ago, when the SEFI (MWG) produced the first revision to their report on a mathematics curriculum for engineers, they noted that *"in increasingly more countries, there is concern over the deterioration in the mathematical ability of new entrants to engineering degree programmes"* (SEFI, 2002, p. 3). Ireland is no exception to this experience, with studies into mathematical preparedness being undertaken as early as 1985, when Cork Regional Technical College concluded that their incoming undergraduates were deficient in basic mathematics (Cork Regional Technical College, 1985). The SEFI report goes on to state that:

> *"On its own this decline in key engineering mathematics skills amongst students who obtained reasonable entry qualifications would be a significant concern. However, it has been compounded by the trend across Europe to increase the numbers entering higher education. As a result, some students who are less qualified have started courses to which, previously, they would not have been admitted.*" (SEFI, 2002, p.4)

This trend has also been noticeable in Ireland, where overall enrolment in the university sector has gone up 10.8% in the past five years (HEA, 2009, p. 14). At 42%, the percentage increase in net graduation rates across higher level education from 2001 to 2007 in Ireland was just slightly below the OECD average of 47.4% (HEA, 2009, p.71). Across the university sector as a whole, 11.8% of enrolments last year were international students (HEA, 2009), further increasing the inhomogeneity of the student cohort.

In 2002, the Retention Office in Dublin Institute of Technology (DIT) completed a study (Costello, 2002) aimed at determining whether poor numerical skills on entry were a strong predictor of failure to pass first year engineering. The results unsurprisingly showed that those with low scores in this area were highly likely to withdraw before the terminal examinations, or fail to pass the year overall. In interviews conducted with some of these students, a significant portion professed themselves to be taken aback at the strong mathematical content of an engineering programme, and felt themselves to be completely unprepared for the level of mathematics they would be studying.

All of these facts point to an increasing challenge for third-level mathematics educators in relation to the mathematics entry qualifications necessary for incoming engineering students.

Mathematics Requirements for Engineering in Ireland

In the Irish education system, the final examination taken by second-level students is called the Leaving Certificate. Students take at least six subjects, with most taking seven or eight, as their best six results are counted for entry into third-level programmes. The Leaving Certificate Mathematics examination can be taken at three different levels: Foundation, Ordinary and Higher. Within each level, fourteen possible grades can be awarded, as laid out in Table 1 below. In order to pass a subject, a student must obtain 40% or higher.

Result, *r* (%)	**Grade**	**Result, *r* (%)**	**Grade**	**Result, *r* (%)**	**Grade**
$90 \leq r \leq 100$	A1	$65 \leq r < 70$	C1	$40 \leq r < 45$	D3
$85 \leq r < 90$	A2	$60 \leq r < 65$	C2	$25 \leq r < 40$	E
$80 \leq r < 85$	B1	$55 \leq r < 60$	C3	$10 \leq r < 25$	F
$75 \leq r < 80$	B2	$50 \leq r < 55$	D1	$r < 10$	NG
$70 \leq r < 75$	B3	$45 \leq r < 50$	D2		

Table 1: Percentage range for each grade awarded at Leaving Certificate (SEC, 2009a).

Generally, to qualify for any third-level degree programme, at least a pass in Ordinary Level mathematics is required, with Foundation Level mathematics not normally accepted. However, the minimum mathematics requirement for accredited Honours Engineering degree programmes is a C3 in Higher Level Leaving Certificate Mathematics (namely, 55% or higher) or its equivalent (Engineers Ireland, 2007). Students must also obtain sufficient "points" from their other subjects and satisfy various other criteria, but the specifics of this need not concern us for the purposes of this paper. In 2009, 16.2% of Leaving Certificate students took the Higher Level Mathematics paper, with 80.8% of those who took the examination receiving a C3 or higher (SEC, 2009b). This translates to 13.1% of the overall student cohort of 2009, a total of just over 6,800 students.

Current Practices for Non-Standard Entry in Ireland

For some years, alternative entry pathways into Honours Engineering programmes have been in place in Ireland. These generally consist of two main approaches: one is to require students to first complete an Ordinary Engineering degree (which takes three years) and then, provided their grades are sufficiently high, they can enter directly into third year of an

Honours programme and complete the final two years to attain their degree (National Qualifications Authority of Ireland, 2006, pp. 22-23). The other approach involves students taking five years instead of four to complete the degree, with the first year treated as a "foundation" year and progression being dependent again on sufficiently high grades (as is the case, for example, in Dublin City University or Dublin Institute of Technology).

However, in the past couple of years, a new trend has emerged in Ireland with a number of third-level institutions offering their own supplementary mathematics examination to students who have failed to reach the mathematics requirement of a C3 or higher for Honours Engineering programmes (Qualifax, 2010). Some of these institutions impose a lower threshold that students must have achieved in their Leaving Certificate mathematics before they can take the supplementary examination; others do not. One university offers a week-long summer school to help students to prepare for the examination, while stressing that this only serves as a final preparation, with students needing to do considerable work on their own in advance (NUIG, 2010). Other institutions employ an interview process to take into account students' motivation in combination with their examination results in order to determine if they are suitable candidates for an engineering programme.

Comparing the Irish Situation with Other Countries

Initially, the intention had been to compare the pre-requisites for incoming engineering undergraduates in Ireland with those of several other European countries, and to look at their approaches to determining alternative entry procedures. However, on closer inspection, the university entry systems in many countries appear to be too different from Ireland's for this to be a useful comparison. For example, in countries such as Austria, Belgium, France and Germany, final secondary school qualifications are sufficient for entry into most university programmes, with some notable exceptions in the French Grandes Ecoles (EUA, 1999, p. 35). Ireland is, however, a signatory to the Washington Accord for engineering. As a result,

> "*qualifications accredited or recognised by other signatories are recognised by each signatory as being substantially equivalent to accredited or recognised qualifications within its own jurisdiction.*" (International Engineering Alliance, 1989)

Therefore, when considering the approach taken by other countries in relation to the mathematical education of engineers, it is most appropriate to compare the Irish situation with those other Washington Accord countries with reasonably similar admission requirements. Australia and the United Kingdom are two such countries, and therefore we will now consider the situations there in some detail.

Engineering Mathematics in Australia

The Australian Mathematical Sciences Institute (AMSI) recently undertook a comprehensive, nationwide review of practices in mathematics education for engineering students (Broadbridge, 2008). Largely similar to the Irish system, Australian school students take five or six subjects in their final examinations, with almost all students opting to take mathematics, which can be studied at three different levels: elementary, intermediate and advanced (Henderson, 2009). However, enrolments are growing in elementary mathematics at the expense of the other two levels, with only 64% of high schools now offering advanced mathematics. In a situation that is mirrored all too frequently in Ireland,

> "*There is a widespread, perhaps erroneous belief that students can improve their Universities Admission Index (a national index that ranks students from their state assessments) by taking less demanding subjects.*" (Henderson, 2009).

The most dramatic difference between the Irish and the Australian situations is that, as a result of this drop-off in the uptake of advanced mathematics, a large number of universities in Australia have decided to drop the requirement for incoming engineering undergraduates to have this level of mathematics. Most universities stated a decline in mathematical preparation of students entering engineering programmes and attributed this to the lowering of entry standards (Broadbridge, 2008, pp. 5-6).

Streaming in first year mathematics is commonplace, with streams usually based on the mathematics level a student has studied in school. These streams are taught separately for the first year, but are expected to have attained the same level by the end of the year, and enter a common second year. Mathematics bridging courses are offered before the start of first year by a number of universities; the delivery of these varies from classes run over the course of two weeks, to online or flexible delivery programmes, which can be completed over the course of a year.

Engineering Mathematics in the United Kingdom

The final school examination for students in the United Kingdom (excluding those in Scotland) is the A-Levels. Students generally take between three and four subjects, with an A-Level in mathematics generally required for entry into an accredited engineering degree programme. One important difference between the U.K. system and the Irish and Australian systems is that, due to the smaller number of subjects studied at A-Level, a large proportion of students do not study mathematics; in 2009, 8.6% of the A-Level cohort for that year sat the A-Level mathematics examination (Joint Council for Qualifications, 2009). Thus, choosing to lower mathematical prerequisites, such as was done in Australia, would effectively involve accepting candidates who have not studied mathematics in at least two years (and even then only to GCSE level). This is not the same choice as lowering the acceptable level of mathematics studied to the final year of schooling. However, a good discussion about the arguments for and against relaxing the A-Level mathematics requirement for engineering is given in (Kent, 2003).

A large number of universities in the U.K. offer a “foundation” year (similar to that offered by some Irish universities) to students who do not have the appropriate background for direct entry into an Honours Engineering degree programme. In addition, many innovative approaches are used once students have entered into engineering programmes, to deal with the wide variety of mathematical backgrounds present; a number of such examples can be found in (LTSN MathsTEAM, 2002), but these are beyond the scope of this discussion paper.

Conclusions and Future Work

The situation for incoming undergraduate engineering students will of necessity vary from country to country, depending on the structure of final school examinations and also the entry process for third-level programmes. Ireland has a standardised admission process for third-level education, along with a controlled national prerequisite currently in place for mathematics for accredited engineering undergraduates. However, it is clear that the student cohort entering third-level education is increasingly diverse and therefore, this may necessitate alternative entry pathways. The aim is to provide opportunities to as many suitable students as possible, while maintaining standards within the programme.

Using this paper as a starting point, we hope to conduct a collaborative, national study of how students who enter engineering programmes using these new, non-standard entry mechanisms cope in their first year of university and beyond. The idea of providing such students with a summer school in advance of any supplementary examination seems to be one which much to recommend it, and if it proves to be more successful for students who take this route than those other take other alternative routes, we will investigate the possibility of developing a standardised, short "bridging course" to ensure that students entering engineering programmes through these routes have a solid base of core mathematical skills.

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Supporting the Composition of Micro-Modular Content to Mathematical Learning Modules

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Abstract

For reasons of authenticity, didactic concept and domain-specific content, lecturers at universities want to use their own manuscripts. Therefore, the common lecturer needs support in generating light-weight digital courses by enriching their manuscripts with reusable micro-modular learning objects. Large amounts of interactive visualizations and animations (Flash, Java applets etc) are reusable for this purpose. In this paper, we address this need, propose an easy-to use solution and show a first prototype supporting a social network of lecturers using multimedia learning objects particularly in mathematical learning modules.

Introduction

Originally, e-learning platforms (learning management systems, LMS) were developed for pure e-learning situations as typically met in companies, public institutions or distance universities. They provide most of the functionality needed by the course instructor and course participants, e.g. learning paths, wikis, calendars and forums. Nowadays these e-learning platforms have also emerged as a standard tool in universities with their typical mixed learning situations, i.e. digitally assisted attended course, so-called "blended learning", in order to increase learner support. Thus, several commercial (e.g. Blackboard (Blackboard Inc., 2010)) as well as open-source platforms (e.g. Moodle (Al-Ajlan and Zedan, 2008), LON-CAPA (Kortemeyer and Cruz, 2009)) have found their way into the service centres of universities.

Creation of digital courses, i.e. e-learning contents, is usually expensive and therefore the number of learners during the course live-time serves as an economic measure for the usage of a course. At Universities, most professors develop their own courses with a specific didactic concept for a specific application domain. Admittedly, the use of a digital course developed by a professor – author and lecturer in one person – is usually restricted to the lectures held by this professor and co-workers.

Without referring to actual data, fig. 1 shows the typically expected reuse statistics of digital content. Complete courses or course parts obey a strongly peaked distribution indicating the above described situation. Therefore, best candidates for maximizing the course usage are first and second year courses in classical slowly-evolving subjects with high numbers of learners. Experiences with such application situations are the topic of several recent e-learning publications (Jungic, Kent and Menz, 2006). In German Universities of Applied Sciences, student groups with the number of participants limited to 50 to 80 and several different lecturers for the same course are the central educational policy characterizing this type of universities. Thus, an increase of content usage requires other methods.

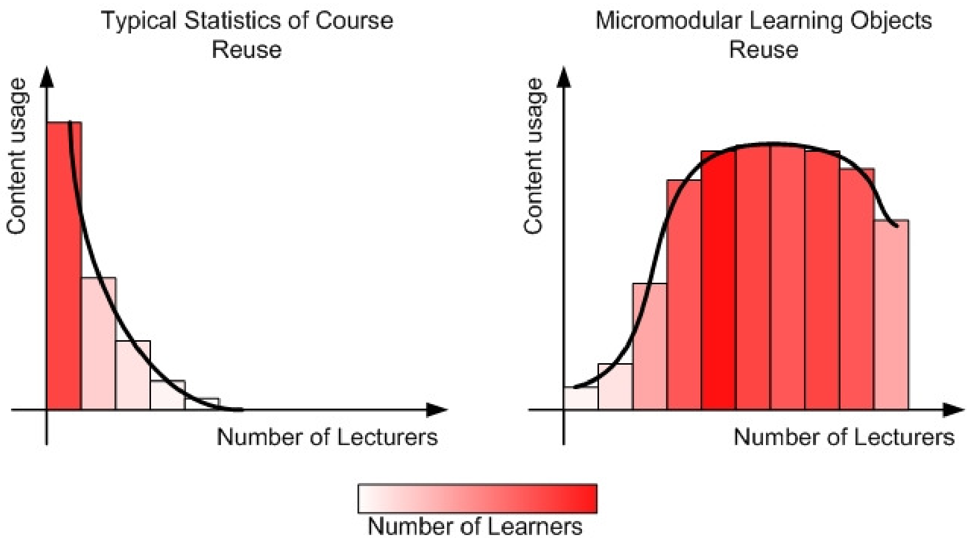

Figure 1. Typically expected reuse statistics of digital content broken down into lecturing persons using that content. White color code indicates a small number of learners while dark red indicates economically desirable high learner numbers.

Further increase in usage of digital content can be achieved by modularizing content into learning objects (LO) that have to be constructed with the intention of high reuse – a great domain for transferring architectural ideas from software engineering to content engineering. In fig. 1, the improvement of content reuse by modularization is shown in particular for micro-modular LOs like interactive visualizing gadgets typically implemented as flash or applet and wrapped with metadata for use in a content management system (CMS). Technically, course creation tools have to be distinguished from content creation tools like text, image or flash creators. Course creation tools generate the course view and provide means for navigating through the course contents. They are often called Learning CMS (L-CMS) and provide course contents as LO repositories for online publishing. While browsing through pre-generated content the course author is enabled to select and place these contents into the course. Typically these L-CMS are intended for use by course authors (Krämer and Han, 2009) and are too sophisticated for practical use by the common lecturer.

In this paper we assume that lecturers want to use their own manuscript for reasons of authenticity, didactic concept and domain-specific application examples and exercises etc. In order to increase usage of LMS we want to support the common lecturer in generating light-weight digital courses by providing tool support for enriching their manuscripts with reusable micro-modular LOs without learning paths (in agreement with the constructivism). This way we support a university-wide network of lecturers who exchange their materials and LOs, a topic of recent interest (Han et al., 2008).

The rest of the paper is organized as follows: In the next section we will depict the actual problems of teaching mathematics at our university. To compensate that deficency we propose our approach in the subsequent two sections. The last section gives a technical overview of the software tools that have been developed as a first solution.

Situation and Actual Problems

Mathematics courses form the basis of technical study courses at the Faculty of Engineering and Computer Science at the Hamburg University of Applied Sciences. The study success rate indicates several problems in mathematics courses whose analysis exhibits many possible influencing factors that are only slightly different for different study courses. Most relevant such factors are

- lack of interest in mathematics,
- lack of focus on the application domain,
- no compulsory exercises in some courses,
- incomplete coverage of the subject because of different lecturers,
- pure formula-oriented lectures discouraging engineering students.

Increasing the success rate in mathematics is a key contribution to enhancing the overall study success rate.

Objectives of the approach

As a result of cause analysis we propose an activating e-learning environment for mathematics courses which is described below. This approach supports students as well as lecturers in preparing and presenting the mathematical contents. It is based on micro-modular LOs which can be aggregated easily to mathematical learning modules. The activating online environment for engineering mathematics courses should contain the following components:

- interactive visualizing gadgets for supporting comprehension
- online-exercises with solution hints depending on results
- tests for verification of learned knowledge
- application examples with subject specific problems for the course of study
- the lecturer's script
- exercises matching the current lecture material with links to the above mentioned visualizing gadgets

The content should be made available for the learners on an e-learning platform. Interactive visualizing gadgets and sample applications are required to assist the mathematical understanding for subject-specific problems. Communication should be improved. The online availability enables independent learning and improving lecture materials at every time and every place. These objectives will improve the learning conditions and the learner's motivation.

Modular Conception

The compilation of lecture materials to mathematical learning modules should retain the individuality and authenticity of a lecturer. The modular conception will support the lecturer in the composition of lecture materials, which are important for his specific lecture. Interactive visualizing gadgets, exercises and tests as well as the sample applications are to be provided for the lecturer, so he can choose the necessary content for his lecture. These micro-modular LOs should be arranged thematically in a web application.

The presented concept explicitly abstains from the allocation of mathematical lecture materials in form of a learning path, like it is handled in other e-learning environments (e.g. MUMIE System (integral-learning GmbH, 2009)). In our approach the lecturer supplements the prepared interactive contents with his own manuscripts to preserve the lecturer's individuality and authenticity. Since none of the existing solutions support the here presented concept the development of the proposed solution has become necessary.

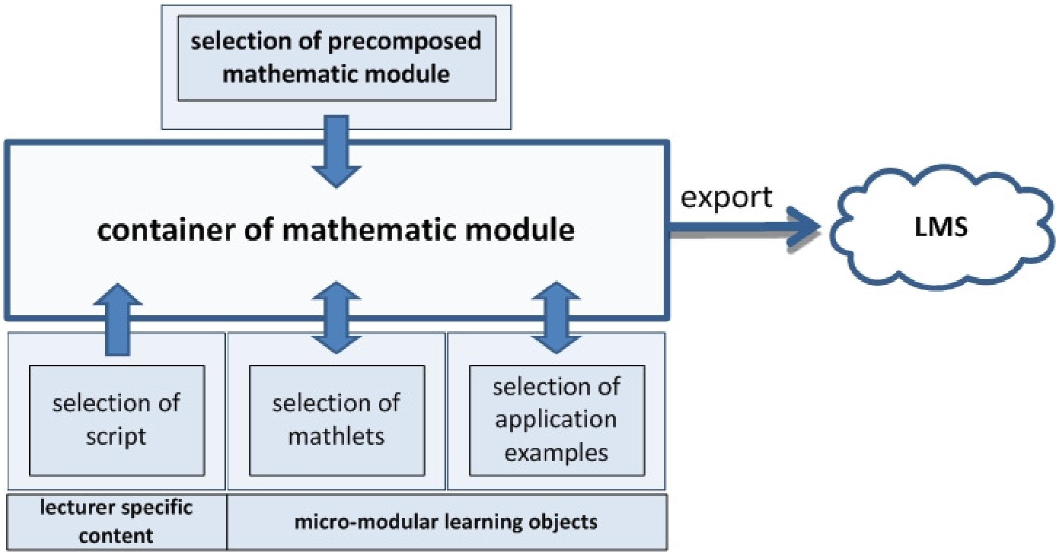

Figure 2. Concept for composition of micro modules.

Fig. 2 clarifies the above mentioned concept. From a set of pre-composed mathematics module templates, the lecturer selects one specific template according to the topic of his course. Out of this template a mathematics module is generated. In a second step the lecturer completes the module with his own script. The lecturer can rearrange the structure of the module and also add and remove LOs. The final module can be exported to an LMS.

Techniques and Tool-Support

The Hamburg University of Applied Sciences has decided to use Moodle as the central LMS. So our approach is to export the mathematical learning modules to Moodle. Therefore our goal is the development of a tool suite to assist the teachers in the creation of their Moodle courses.

One type of micro-modular content, which we want to compose are the so called Mathlets. Mathlets are small Java-Applications (Applets) to visualize and test mathematical topics. Mathlets are developed using a Java library called MathletFactory, which is a part of the MUMIE System (integral-learning GmbH, 2009).

Our first approach to reach the previously described concepts was to implement the MOO-System (Mathlet Online Organization) for storing and managing Mathlets.

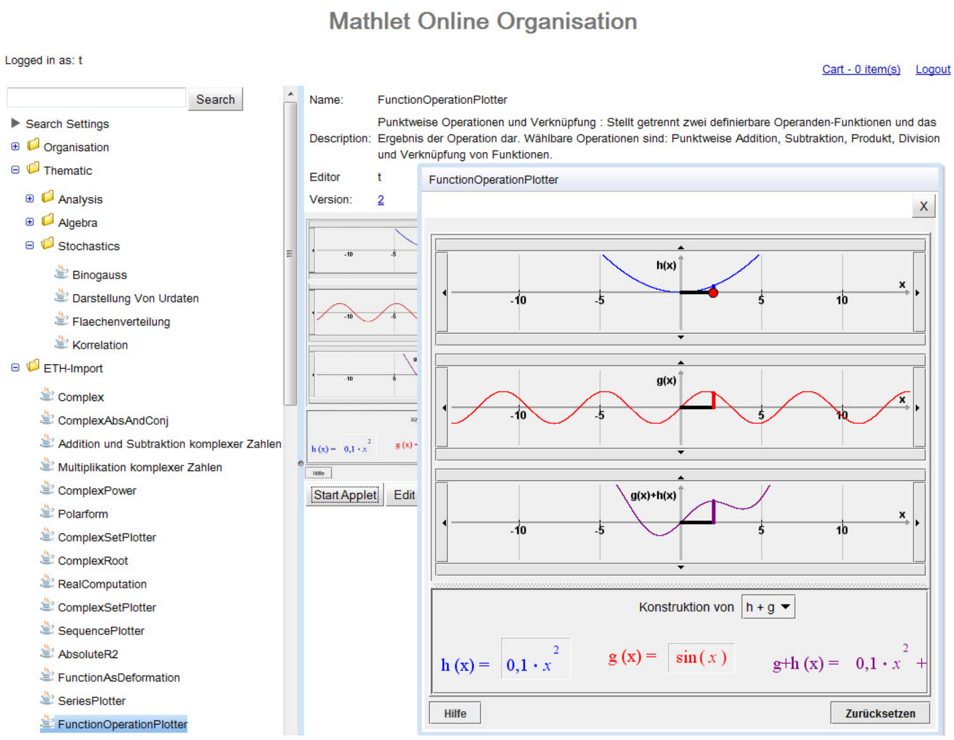

Figure 3. A screenshot of MOO with a started Mathlet.

MOO organizes Mathlets in logical categories and enriches them with metadata. Therefore the Mathlet and its metadata are combined in MathUnits. The metadata is composed of textual data (e.g. title and description), a screenshot of the Mathlet and structural information. The screenshot helps the user to get a first impression about the Mathlets appearance and improves the recognition value. There exist several points of view on a Mathlet, so one Mathlet may be assigned to more than one category. Categories themselves are structured hierarchically. This hierarchically structure offers a compact view, which consequently enables the user to navigate easily through the Mathlets. The categories and the contained Mathlets can be filtered using the search function. The search function uses all available textual metadata of a Mathlet. The metadata is not only used for the search function, but also to give the user information about the purpose of a Mathlet. To explore the whole functionality of the Mathlet it can be executed in place as shown in Figure 3.

MOO follows a wiki-like approach. All the metadata, the categories and also the assignment of Mathlets to categories can be edited by the users. As pointed out earlier there are several points of view on a Mathlet. Every user can contribute his/her personal point of view (keywords, description, categories) to the data base. This supports other users in the process of finding the appropriate Mathlet. To avoid the loss of information by intentional or unintentional data manipulation MOO keeps track of all changes by creating versions. All versions are annotated with the editing user. If necessary users can rollback to any previous version so no information is ever lost.

While navigating through the Mathlets users can collect them like goods in a shopping cart (step 1). From this cart the Mathlets can be exported to the users Moodle course(step 2) as described in section “Objective of the approach”. This simplified workflow is shown in Figure 4. To export Mathlets MOO uses web services to connect to the central Moodle platform on behalf of the user. The application then displays the user a list of his courses in Moodle. The user then selects a category within the list of courses to which the Mathlets from the cart should be added.

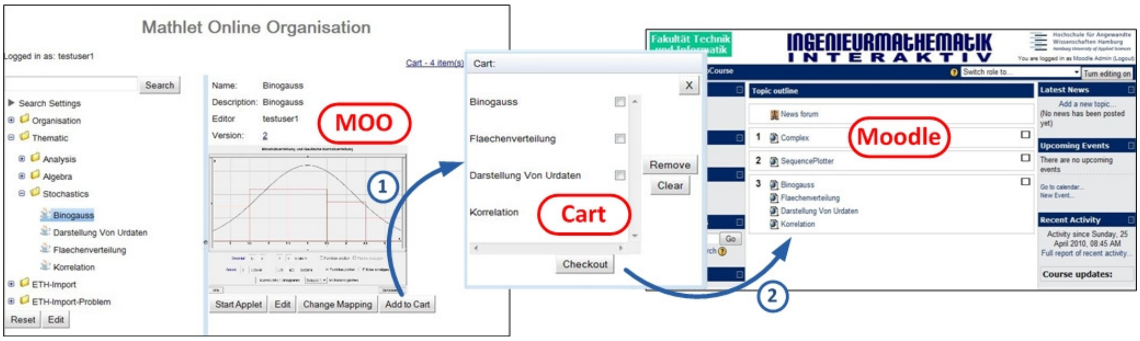

Figure 4. Mathlet Online Organisation(MOO) – simplified workflow

MOO was developed as a Rich Internet Application using Google WebToolkit (GWT), it runs on a Tomcat 6 web container and all the data is stored in a MySQL Database. All used technologies are open-source.

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Towards Mathematics Education Research – Does Physics Education Research serve as a model?

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Abstract

Mathematics and Physics are certainly among the most closely related academic fields. In terms of Education Research the Physics branch has undergone an impressive development during the past decades and has proven to be remarkably advanced and successful. This contribution gives a concise overview of the important aspects of Physics Education Research and analyses the reasons for its success. Based on this it will be investigated if and to what extent Physics Education Research can serve as a paradigm for Mathematics Education Research.

Introduction

Up until the late 1900s Mathematics and Physics had closely shared a long common path of their histories. Mathematics continues to be of paramount importance to Physics while Mathematics has created new scientific insights by separating itself from its physical and particularly its geometrical roots. Given this split, it appears quite natural that the mathematical and physical strands of Education Research are separate research fields today. Yet in terms of educational practice, both scientific fields share the top rank on the list of subjects students typically most dislike.

Physics Education Research (PER) has evolved to a consistent, coherent, and noticeable successful subfield of academic Physics over the past three decades. Major successes encompass theoretical (e.g., how do students learn physics?) and applied aspects (e.g., how does one measure the effectiveness of physics instruction?). Interestingly, despite its importance to Physics, Mathematics has played a minor role in PER so far. From this, however, it cannot be concluded that Physics cannot provide support to its twin sister Mathematics when it comes to Education Research. Quite to the contrary, given the recent success of PER and the relation between the two fields PER can serve as a model for the emergence of strongly coherent Mathematics Educational Research (MER). This is the central hypothesis of this paper which will be analysed after an introduction to PER and the factors of its success in the following sections.

Overview of PER

PER focuses on what happens as students struggle to grasp and use the concepts of physics and how students can be efficiently supported in their struggle. One of its many roots can be traced back to the Piaget-based work by Karplus (1977) on formal reasoning and other cognitive aspects of science and mathematics. Early research with a focus on physics was influenced by psychology and investigated what used to be termed misconceptions and how to overcome them. Misconceptions refer to ideas students bring with them to physics class and which are at odds with established physical concepts. For instance, a popular misconception is the belief that motion at constant velocity is caused by a constant net force.

This line of research consolidated with the formulation of phenomenological primitives (p-prims) by diSessa (1990). P-prims refer to fundamental ideas held by students that serve as a source for their reasoning. They typically are inarticulate and are considered to be self-evident by students. One of the most frequently applied p-prims is what diSessa calls Ohm's p-prim. Its logic is the qualitative reasoning that more effort implies more result or inversely that more resistance implies less result.

P-prims are mentally structured as a set of loosely connected ideas and are of limited scope and applicability. Such "knowledge in pieces" is in stark contrast to the coherent structure of ideas typically referred to as a theory. The notion of p-prims has been very influential in the development of cognitive models in PER.

Of major importance for the evolution of PER was the publication of the Force Concept Inventory (FCI) by Hestenes (1992) and coworkers. This test covers elementary concepts of Newtonian mechanics and is deceptively trivial. Yet, even after having studied Newton's laws many students perform poorly on the FCI, even if they have scored well in exams. For instance, they typically state on the FCI that a body's movement at constant velocity requires a nonzero net force. By administering the FCI, many physics instructors have been shocked by finding that their students hardly gained any physical understanding over the course of the lecture. The FCI is of ongoing importance in PER, as it documents common deficits in physics education and serves as a benchmark for the effectiveness of physics instruction, see Hake (1998). From a cognitive perspective, however, the FCI also measures how successfully students reconstructed their "knowledge in pieces" to coherent Newtonian concepts.

Like physics in general, PER has a basic branch and an applied branch. While "basic PER" investigates how student learn physics, "applied PER" uses the results of these investigations to design effective teaching materials. Redish (2003) provides a good overview of such materials. "Basic PER" and "applied PER" are interconnected via a feedback loop with the latter providing empirical data to the former and the former providing measurement instruments for evaluating the success of teaching materials. Obviously data acquired in PER will show much more variability than that in other subfields of physics. Yet, repeated patterns in such data can lead and have led to theories that explain other situations and have predictive power.

The success of PER is to a large extent based on prediction of how students will perform on specific tasks and explaining why this is the case. In addition, PER provides easy to adopt teaching materials which, when suitably applied, help to substantially and measurably improve physics instruction.

Researchers engaged in PER more often than not work in physics departments rather than departments of education. Likewise they are not necessarily responsible for teacher education. Rather, they pursue research in PER as other physicists do so in their subfields of physics. The success of PER is also manifested by the fact that Physics departments, in particular in the US, are hiring faculty to conduct PER. Typically PER inquiry is targeted at students at college or university level rather than primary or secondary school.

Factors of Success

Compared to other subfields of Educational Research PER has proven to be substantially successful during the past decades. The factors of success are certainly multiple, among them those given below.

First of all, PER has led to *precision* in the description of *reproducible* students' difficulties and deficits in Physics Education. This aspect cannot be overemphasized, in particular, as its importance is frequently underestimated. Some experienced instructors tend to be unimpressed, as PER describes phenomena which match their daily experience. The difference, however, is between knowing of a phenomenon and being able to describe it precisely or even model it. This can be likened to Galilei's scientific description of falling objects as opposed to the experienced knowledge of all human beings that objects fall.

Second, PER is coherent. The coherence of PER can most easily be tested by picking up a textbook of the field or reading a number of recently published papers. By reading such material a novice to the field will quickly be able to distil the main theories, methods, and current research trends.

Next, PER research projects in general are easy to repeat. This is essential in many respects. Repeatability facilitates that empirical findings and theories derived from them can be scrutinized and put under test by other researchers. Verification of research results increases their faithfulness which again contributes to the coherence of the field. In addition, PER provides established measurement instruments for the effectiveness of teaching and learning. Again, these measurement instruments like the FCI on Newtonian concepts or the MPEX by Redish (1998) on student attitudes are easy to apply.

Ease of repeatability is achieved by focusing on small and well-contained research topics like students' understanding of a particular concept or the investigation of students' epistemological beliefs rather than, for example, the effectiveness of large scale curricular reforms. Reforms triggered by PER again are structured in such a way that they can be easily implemented on small scale at rather low cost. For instance, "Tutorial in introductory Physics" by McDermott (1997) contains topical building blocks which can flexibly be integrated into physics classes.

Educational Research has frequently been put under substantial. Burkhardt and Schoenfeld (2003), for instance, attest a lack of attention to coherent theory building. They also call for an "engineering component" targeted at research-based development of educational tools (not curricula!) for practitioners. PER either fulfils or comes close to most of the requirements put forward by these authors.

Implications for Mathematics Education Research

Mathematics Education Research (MER) is far less developed than PER and considerably less coherent. Again, the acid test is to pick up the research literature and identifying strands of research and to identify easy to implement and effective teaching methodology. A noticeable exemption is the research on probability theory, see e.g. Fischbein (1975), which shows striking parallels to PER in terms of coherence, reproducibility and methodology.

Due to the relatedness of Physics and Mathematics, people engaged in MER will be able to read and understand the research literature of PER and use that as a starting point for the creation of a comparably successful MER. In following the spirit of PER, researchers need to focus on research projects which can be repeated by others. They also need to be willing to reproduce results gained by others. This requires an academic value system which honours such work.

Furthermore, MER needs to develop and establish tools for measuring the effectiveness of instruction, comparable to the FCI. That is, such measurement tools need to be targeted on a rather small number of related mathematical concepts and need to be administered easily. Efforts are already underway, e.g. Epstein (2007). Again, community support is needed to field test and validate such tools. Results need to be published in order to make instructors not engaged in MER aware of the availability of such instruments.

Finally, MER needs to make an early effort to develop teaching materials which can be broadly applied and easily adopted at various places. Actually, some teaching materials developed by PER, in particular those related to calculus concepts, can be readily integrated into mathematics teaching.

An Example for PER-inspired MER

In order to give an example how MER in the spirit of PER might look like, I am going to describe ongoing work along this line. In this investigation students are to model a piecewise defined function represented by a given graph like the one shown in Figure 1.

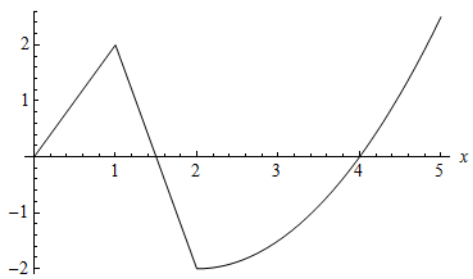

Figure 1. Graph of a function to be modelled by students. See text for details.

Typical and frequent students' answers will be $-x$ in the range $1<x<2$, x^2 in the range $2<x<5$, and somewhat less frequent x in the range $0<x<1$. When asked for a verbal justification students give statements like "It's a line and decreasing, hence it's $-x$ " or "It looks like a parabola, hence it's x^2." When prompted to compute a function value for an exemplary value like x=3/2 or x=3, students realize the inappropriateness of their models.

In order to correct their previous answers students typically come up with -4x and x^2-2, respectively. Prompted for justification results in statements of the form "It's a line and the slope is -4" and "The parabola is shifted by -2." Again, students usually need to be triggered to test their results for specific values of x. At this point students typically realize that there are more parameters to a line than its slope and likewise for a parabola. Yet, at this stage many of them are not capable of determining these parameters and by that modelling the function.

There are certainly several issues involved here. The one, I would like to point out, is that students seem to associate specific graphical representations with prototypes of functions ($\pm x$ for line segments and x^2 for parabolas) in a hierarchical fashion. Realising the inappropriateness of these prototypes students come up with a increasingly refined version (e.g. for line segments $\pm|a|\cdot x$ and subsequently $a\cdot x+b$). Hence, for novices such prototypes seem to be hierarchically structured while for experts such prototypes are much richer (like using both $a\cdot x+b$ and $a\cdot(x-x_0)$ for a line segment) and less hierarchically organized.

The relevance of such findings, if consolidated, is that they can form the baseline for the creation of teaching materials to make students more successful modellers. The layout for such a research endeavour has been given in PER by McDermott (1992).

Conclusion

The endeavour of PER had been early and aptly summarized by Aarons (1972):

"I am deeply convinced that a statistically significant improvement would occur if more of us learned to listen to our students [...] By listening to what they say in answer to carefully phrased, leading questions, we can begin to understand what does and does not happen in their minds, anticipate the hurdles they encounter, and provide the kind of help needed to master a concept or line of reasoning without simply "telling them the answer." [...] Were more of us willing to relearn our physics by the dialog and listening process I have described, we would see a discontinuous upward shift in the quality of physics teaching. I am satisfied that this is fully within the competence of our colleagues; the question is one of humility and desire."

This dialog and listening process has flourished in PER. It has rather quickly led to the development of teaching materials which, when suitably applied, realize the proclaimed quality of physics teaching. Similar successes can be reasonably expected in MER when it adopts the paradigm laid out by PER. Such an adaption is a question of scientific methodology, but also, as Aarons has pointed out, one of humility and desire.

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SAGE, the open source CAS to end all CASs?

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Abstract

SAGE, the *'Software for Algebra and Geometry Experimentation'*, is an open source computer algebra system (CAS). It comprises and allows access to well known specialized open source CASs like GAP, MAXIMA, PARI/GP, R, SINGULAR etc. and other optional ones. SAGE is highly practical and supports education in mathematics, in many situations and at all levels; like many CASs, it is powerful enough to handle a wide range of mathematical problem — in calculus, algebra, numerics, number theory etc. Because it is open source and easily installed or accessed via its splendid web interface, it is available to anyone, any time, anywhere. In this way, it supports instructors to teach and learners to learn and explore mathematics.

Introduction

Especially in engineering oriented degree courses, computer algebra systems, CASs, are, or should, be part of the education in mathematics because of the following interrelated reasons:

- *CASs are problem solving tools:* students learn how to judiciously use a CAS to solve problems in mathematics, engineering etc; they learn he user interface as well as the potential and limits of a CAS;
- *CASs allow the investigation of algorithms:* students learn how to implement algorithms using the CAS's programming interface; then they can study algorithms, their complexity, the performance of an implementation and how the performance depends on the input data;
- *Deployment of CASs helps understanding in mathematics:* instructors and students can easily visualize or animate examples and procedures, thereby they illustrate concepts; in doing so students become active, they engage and accept challenges more readily. In the best case, a CAS boosts the process of learning mathematics.
- *Analysis of the CAS itself reveals the underlying mathematics:* advanced students investigate the CAS itself, Wester (1995); as is necessary for any tool. They learn how features are implemented and extended and can compare the performance of different CASs.

These reasons lead to the following requirements that determine the role of CASs in education in mathematics.

- *Power:* CASs have to be powerful enough for learning, teaching and research.

- *Usability:* CASs should be intuitively usable in order to minimize the time and effort needed to learn how to use them; users should focus on the problem at hand; they are not to be burdened with handling the CAS, Wester (1995).
- *Availability:* To be suited to education in mathematics, CASs should be easy to install and immediately ready to use.

It is from these perspectives that we consider the open source CAS SAGE (2010) and the proprietary CAS MATLAB. SAGE (2010) is a collection of CASs accessible via a common user interface or user web interface. MATLAB is a proprietary product of The Mathworks (2010a) with its *Symbolic Math Toolbox* for symbolic computations. There is an open source equivalent, Octave (2010), Eaton (2002), with its symbolic extension GiNaC (2010), Bauer et al (2002).

In the following, we will not emphasize the fact that many CASs may hide implementational detail thereby preventing scrutiny of the code for bugs or unintended behaviour. Hence, like cryptographic algorithms, CASs should be open source software as a prerequisite, Joyner (2007).

SAGE vs MATLAB

SAGE (2010) is a free open-source mathematics software system licensed under the GPL. It combines the power of many existing open-source packages into a common Python-based interface. The mission of the community of its developers is to create a viable free open source alternative to Magma, Maple, Mathematica and MATLAB. To assess SAGE, we will compare it to MATLAB ─ SAGEbenchmark (2010) takes a similar approach in comparing the performance of SAGE to the performance of Mathematica and MATLAB.

In the following examples we show the SAGE-code on the left and the MATLAB-code on the right.

Function Graphs: We only compare the simplest 2D-graphics features, namely graphs of functions of one variable

plot(sin(x)) # some comment	ezplot('sin(x)') % some comment

or in a slightly more elaborate example

p=plot(sin(x),-3,3,color='red');
q=plot(cos(x),-3,3,color='blue');
show(p+q);
	hdl=ezplot('sin(x)',-3,3);
	set(hdl,'color','r'); hold on;
	hdl=ezplot('cos(x)',-3,3);
	set(hdl,'color','b'); hold off;

SAGE's `plot` can detect poles and show the two separate parts of the graph, as MATLAB's `ezplot` does by default. Like MATLAB, SAGE offers many of other plot options for 2D, 3D, explicit and implicit, parametric curves and surfaces, etc.

Differentiation and Integration (Quadrature): Symbolic differentiation and integration is provided by SAGE the same way as by MATLABs *Symbolic Math Toolbox*

diff(sin(x)/x,x)	diff(sin(x)/x,x)

and

integral(x*sin(x),x)	int(x*sin(x),x)

Of course, proper integrals are computed by SAGE and MATLAB, together with the usual numerical evaluation of symbolic expressions by SAGE's `N()` or MATLAB's `double` respectively. Multiple integration works analogously.

Solving Equations: SAGE's `solve` and MATLAB's `solve`, both solve linear and nonlinear equations

vars = var('a b c d e f x y'); solve([a*x+b*y==c,d*x+e*y==f],x,y)	syms a b c d e f x y; [x y] = solve('a*x+b*y-c','d*x+e*y-f',x,y)

Solving Ordinary Differential Equations: SAGE's desolve and MATLAB's dsolve, both solve (systems of) simple ordinary differential equations symbolically

t = var('t'); x = function('x',t); desolve(diff(x,t)+x==1, [x,t])	% independent variable is t by default dsolve('Dx+x=1')

Vectors and Matrices: Both SAGE and MATLAB support vectors, matrices and sparse vectors and matrices. In the example below, we generate at random and render a polygon in the plane, rotate it around the origin and render it again in a different colour.

pnts=[(random(),random()) \\
for _ in range(5)]; p = line(pnts);
c = cos(pi/4); s = sin(pi/4);
R = N(matrix($[[$c,s$]$,$[$-s,c$]]$));
pnts = matrix(pnts)*R;
q = line([(pnts[i,0],pnts[i,1]) \\
for i in range(5)],color='red');
(p+q).show()
	plgn = rand(2,5);
	plot(plgn(1,:),plgn(2,:));
	hold on;
	c = cos(pi/4); s = sin(pi/4);
	R = [$c s;-s c]; plgn = R*plgn;
	plot(plgn(1,:),plgn(2,:),'r');
	hold off;

In SAGE, there might be a better solution than to convert between the different data types: `pnts` is a list of points, because the function `line` expects input arguments of this type. To do matrix multiplication, we convert to a matrix by `matrix(pnts)`. Multiplication by the rotation matrix `R` produces a matrix which we have to convert back to a list of points so that the function `line` can render the polygon.

Solving Linear Equations: Both SAGE and MATLAB offer the backslash-operator to solve linear equations symbolically

vars = var('a,b,c,d,e,f'); A = matrix([[a,b],[c,d]]); b = vector([e,f]); A\b	syms a b c d e f; A = [a b;c d]; b =[e;f]; A\b

Here SAGE detects the types of vectors and matrices involved and caters for transposition where necessary.

Eigenvalues and Eigenvectors: Of course, SAGE and MATLAB symbolically compute eigenvalues and eigenvectors

A = Matrix([[1,2,3],[3,2,1],[1,1,1]]); A.eigenvalues(); A.eigenvectors_right();	A = sym([1,2,3;3,2,1;1,1,1]); eig(A) [V,D] = eig(A);

Here SAGE returns triples of eigenvalue, eigenvector and multiplicity of the eigenvalue, whereas MATLAB produces the diagonal matrix D of eigenvalues and the matrix V whose columns are the corresponding eigenvectors such that AV = VD.

Numerics: From the standard set of numerical problems, Risse (2010), we here present only a glimpse of the features of SAGE and MATLAB. SAGE and MATLAB support sparse vectors and matrices. Unlike MATLAB, SAGE natively supports matrices over several rings and fields.

Solving linear Equations: SAGE, here SciPy (2010), solves linear equations by providing all well-known decomposition methods based on optimized BLAS and LAPACK routines. SAGE solves linear least squares problems either by solving the (system of) normal equations or by computing the pseudo-inverse of the coefficient matrix. MATLAB solves linear equations and linear least squares problems numerically. It does so with partial pivoting in the case of a linear equation and by QR-factorization in the case of a least squares problem, based on BLAS and LAPACK routines.

Solving Non-linear Equations: For scalar equations we use

x = var('x'); # solution in (0,pi/2) find_root(cos(x)==sin(x),0,pi/2)}	% x0 is start guess or intervall fzero(@(x)(cos(x)-sin(x)), x0)

Quadrature: SAGE, here SciPy, offers well known methods like trapezoidal, Simpson, Romberg and Gaussian rules for numerical quadrature. MATLAB's `quad` implements adaptive Simpson quadrature, `quadl` implements adaptive Lobatto quadrature.

Solving Differential Equations: We take the well-known Volterra-Lottka model of predator and prey populations using a system of differential equations $y_1' = y_1(a - by_2)$, $y_2' = y_2(-c + dy_1)$ with constants a, b, c, d as well as initial population sizes $y_1(0)$ and $y_2(0)$.

SAGE	MATLAB

`# initialize a,b,c,d,y0`
`def odefun(y,t):`
`   return [y[0]*(a-b*y[1]), \`
`            y[1]*(-c+d*y[0])];`
`t = srange(0,25, 0.01);`
`y = odeint(odefun, y0, t)`
	`% initialize a,b,c,d,y0`
	`function fval = odefun(t,y)`
	`   fval = [y(1)*(a-b*y(2));`
	`           y(2)*(-c+d*y(1))];`
	`end`
	`tspan = [0, 25]; [t, y_ode45] = …`
	`ode45(@odefun,tspan,y0);`

Both SAGE and MATLAB provide various solvers for stiff and non-stiff systems of ordinary differential equations.

Programming

SAGE builds on scripts in Python; MATLAB on scripts in m-files — both are interpreted or (pre-) compiled. We focus on animation and interaction only.

Animation: SAGE’s `animate` allows for very easy generation of animations consisting of just a couple of frames which are shown one after another in an infinite loop. As a drawback, every single frame is generated before the whole animation is shown. In contrast, MATLAB only offers the option `animate` for the set of `plot` commands, so that a dot is moved along a curve in the plane or in space.

If one is interested in proper animations, one has to specify graphical objects whose attributes like position, size or color are changed by a program in time. Then, the `drawnow` command forces the system to immediately render the graphical objects. This is shown below for “the swinging dot” — `h` is the handle of the graphical object 'dot' whose position is changed in the while loop.

SAGE	MATLAB

`anmt = animate( \`
`[point([2*sin(pi/8*cos(t)), \`
`2*(1-cos(pi/8*cos(t)))]) \`
`for t in srange(0,11,0.1)],\`
`xmin=-1,xmax=1,ymin=0,ymax=2);`
	`plot([-2,2,0,0],[0,0,0,2],'k');% axes`
	`hold on; h=plot(0,0,'.k');    % pendulum`
	`while true`
	`   set(h,'xData',2*sin(pi/8*cos(t)));`
	`   set(h,'yData',2-2*cos(pi/8*cos(t)));`

anmt.show()	drawnow; end; hold off;

Interaction: SAGEs `interact` provides very easy generation of interaction with the input parameters of some function, here `NN`. The specification of each parameter determines the type of interaction control out of the set of usual controls. In contrast, MATLAB offers the capability to build ─ supported by the tool `guide` ─ more general graphical user interfaces, however, at a higher programming cost.

In our example we plot parabolas whose coefficients we want to control interactively. We use a slider for the coefficient a of x^2, an input box for the coefficient b of x and a radio button group for the three values of the constant c. The MATLAB code on the right hand side does not show the specification of the interaction components with their call back functions, it only shows a section of the function which renders the graph of the parabola.

@interact
def NN(a=(0,2),b=1,c=[0,2,4])
   p=plot(a*x^2+b*x+c,-2,2);
   p.show();
	a = get(ha,'Value');
	b = double(get(hb,'String'));
	c = get(hc,'Value');
	f = @(x) a*x.^2+b*x+c; ezplot(f);

Pros and Cons

SAGE provides many data types, for example, the data structure returned by `eigenvectors_right()` whereas MATLAB supports only a few data types, for example, the data returned by `eig` which is either a one-dimensional array of eigenvalues or the matrix `V` of eigenvectors with the diagonal matrix `D` of the eigenvalues.

SAGE, here SciPy, provides explicit vectorization by the command `vectorize` whereas MATLAB supports vectorization in a very generic way as all built in functions, such as the elementary functions, vectorize by default. SAGE's strong point is discrete mathematics: polynomial rings, groups, elliptic curves, and much of algebra. Hence, SAGE supports applications in combinatorics, cryptography, coding, and graph theory where MATLAB natively does not offer equivalents. Similar features are provided in toolboxes, for example, arithmetic in some finite fields is provided in the *communication toolbox*.

SAGE's symbolic computation engine seems to be more powerful, for example, in the simplification of algebraic expressions, than the Maple/MuPad kernel which MATLABs *Symbolic Math Toolbox* uses.

To summarize our comparison of SAGE and MATLAB we turn to Wester's comparison: in 1994, Michael Wester (1995) put forward 123 problems that a reasonable computer algebra system should be able to solve and tested the CASs Axiom, Derive, Macsyma, Maple, Mathematica, MuPAD and Reduce. Because MATLABs *Symbolic Math Toolbox* nowadays is based on the MuPAD engine by default it is fair to assess both CASs by Wester's problems. Even though Wester's investigation of course is no longer up to date, it still provides valid criteria for how to compare CASs. As SAGEreference (2010) shows, the current version of SAGE can do most of Wester's problems natively now.

Conclusion

Even though we have sketched only a small part of SAGE's features, it should have become apparent that SAGE has considerable capacity to support education in mathematics. In particular, SAGE's web interface makes SAGE most valuable in teaching. We presented symbolical and numerical examples. The packages NumPy (2010) and SciPy (2010) contain more numerical algorithms for integration of differential equations, optimization, linear programming, etc. On top of this, SAGE offers support for discrete mathematics by, for example supporting arithmetic in finite fields of both prime and powers of prime order. This feature is not part of MATLAB or any of its toolboxes (except for the *Communication Toolbox* which offers to work with Galois arrays over GF(2^m) for m=1,2,...,16). Arithmetic over other finite fields has to be programmed and provided by additional classes, TheMathworks (2010b).

Specifically, SAGE's web interface allows for all sorts of mathematical experiments. SAGE is available any time, anywhere by way of its web interface – either on the general SAGE server or, for example, on a local copy. To sum up, SAGE allows the user

- to tackle problems in many areas of mathematics
- to natively solve problems numerically and symbolically
- to access specialized computer algebra systems in a unified way.

Users can tap SAGE's huge potential in mathematics

- via user interfaces familiar to GAP, MAXIMA, PARI/GP, R, SINGULAR users,
- with web interface, without the need to install any software (SAGE runs on servers, e.g. http://www.sagenb.org or http://SAGE.informatik.hs-bremen.de),
- without the need to worry about licences and licence fees.

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Using Electronic Voting Systems for Active Learning

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Abstract

The focus of this paper is the use of Electronic Voting Systems to engage engineering students in the learning of mathematics. Some academic staff members from the Mathematics Education Centre at Loughborough University have been using Electronic Voting Systems since 2007/2008 year to teach mathematics to undergraduate students from Mechanical, Automotive and Aeronautical Engineering departments. A study was designed to investigate the views of staff and affected students about the use of Electronic Voting Systems in mathematics classes. At the end of the first year of use, staff generally perceived them as an effective teaching tool. However there remained many unanswered questions for staff. In particular, how do we ascertain the types of questions which better engage students and which facilitate deeper learning?

This paper discusses findings from the literature. It then describes the setting up of a University-wide staff interest group in Electronic Voting Systems at Loughborough University and some of the issues addressed and different pedagogic approaches which were adopted by staff when using Electronic Voting Systems. Finally it highlights resources which have been developed for using Electronic Voting Systems to teach mathematics. These have been developed following mini-project funding and include a website with over 300 questions, designed in PowerPoint and ready for use, and links to question banks developed elsewhere.

Introduction and Literature Review

Throughout the UK, there has been concern with the level of engagement of students in the teaching that takes place in universities. Sometimes this manifests itself in poor attendance. At other times students attend but play a passive role in the process. In 2007, Loughborough University, via its Teaching Support Unit, purchased an Electronic Voting System to, amongst other things, facilitate student interaction and engagement in lectures.

Some of the earliest reports of the use of Electronic Voting Systems (EVS) in classrooms include those of Cue (1998), and Hake (1998). Caldwell's (2007) review of existing literature on handset use is a comprehensive and detailed work that covers every aspect of handset use including description of the technology, use of questions, effect on student performance and association of handsets with 'peer learning'. The study also includes guidelines for writing good questions and best practice tips. A very helpful book is that edited by Banks (2006). It provides some historical context, followed by practical cases in a variety of subjects, with associated discussions of the pedagogy associated with them, and finally outlines some of the directions that EVS may take in the future. Further electronic resources with comprehensive information on the use of EVS include the repository created by Steve Draper of Glasgow University (http://www.psy.gla.ac.uk/~steve/ilig/).

Papers with specific focus on the use of EVS in Mathematics include McCabe, Heal & White (2001), Lomen and Robinson (2004), and Cline, Zullo and Parker (2007). A more comprehensive overview of 10 publications on the use of EVS in Mathematics and Statistics can be found in Retkute (2009).

The single, most important benefit of EVS use, identified from literature review, is its capacity to enhance, catalyse or increase student engagement during lectures. This was reinforced by the finding from surveys of students at Loughborough University who were introduced to EVS in the study of engineering mathematics. 145 students completed a questionnaire on the use of EVS in class. The results showed that the majority of students were extremely positive about the usefulness and overall advantageousness of EVS use in classes. Results also showed that EVS use did increase the likelihood of students participating and engaging in class. Students identified the main benefits of EVS use and their two most important benefits related to feedback. These were, 'Checks whether I'm understanding course material as I thought I was' and 'Allows learners to identify problem areas'. Further details are available in King and Robinson (2009).

Following a description of the technology involved, this paper now presents results from a staff survey at Loughborough University, which led to the setting up of a very successful staff interest group in EVS. The work of the staff interest group is then briefly described. Finally the paper describes the setting up of a website with resources to be used with EVS in the teaching of mathematics. Example questions are provided to demonstrate questions with different pedagogic benefits.

The Technology

EVS is a technology that affords a lecturer the means to give students, especially in a large class, the chance to engage with course material by having them answer questions in class - with immediate feedback provided. The EVS system being used by Loughborough University is TurningPoint (www.turningtechnologies.co.uk). Its enabling software is embedded in Microsoft PowerPoint. So a lecturer can prepare multiple choice questions (MCQs) as a series of PowerPoint slides for, for example, formative assessment purposes. The students respond by clicking the corresponding alphanumeric answer choice on their EVS handsets (Figure 1).

Figure 1 - Students using TurningPoint (EVS) handsets to register their responses to a question in class. Used with permission of Turning Technologies

Student responses are then displayed on the PowerPoint slide in the form of a suitable chart (see Figure 3). The lecturer may then decide to elaborate on any relevant issues arising out of the question and answer display session.

Results of Staff Survey and Setting up of Staff Interest Group in EVS

The views of 8 staff that used EVS at Loughborough University were ascertained via a questionnaire, observations, a 'blog' and follow-up interviews. The results showed that EVS is generally seen as an effective teaching tool, as its use can enhance student engagement by increasing their participation in class, give lecturers valuable feedback on student understanding, make the classroom more 'fun', and enable lecturers to change teaching practice and curriculum in response to student feedback. However, there are technical and pedagogical issues to be overcome in realising the full potential of EVS. Further details are available in King and Robinson (2009b).

In discussions with staff, it became clear however, that after the initial training session, there was little additional professional development for staff with regards to the best pedagogic practice in introducing EVS into lectures. The author thus successfully applied for an internal Academic Practice Award for the 2008-9 academic year to, amongst other things, set up a staff interest group in EVS.

From the very start it was emphasised that the aim of the meetings was to enable staff who use, or are interested in using, EVS to get together and learn more, from each other and the literature, about how to use these to best effect. The first meeting involved small group discussion and feedback to the main group on participants' experiences and issues arising following the first year of EVS. Subsequent meetings have seen some very experienced teaching staff demonstrate different types of pedagogic uses of EVS. These included ice-breaker questions, fact-finding questions, questions checking prior knowledge, questions checking understanding of lecture material, questions testing understanding of a diagram or video, question seeking opinions, questions which initiate discussion, questions involving calculations, questions testing application of knowledge in a new situation and ConcepTesting. The latter has been used with great success in mechanics (see, for example, Mazur (1997) and is often combined with peer instruction. The staff interest group also included a report of a pilot by two participants with more advanced text-entry voting systems and the opportunity for participants to try these out. There was also a report from the author of a visit, funded by the award, to the University of Arizona, where staff have gained much experience in the use of EVS. Attention has been drawn to key literature on the use of EVS. In addition, results of surveys, of staff and student experiences of the system at Loughborough University have been reported. At the end of each meeting participants suggest what they would like to discuss at the following meeting. This involvement of all participants in planning and contributing to the sessions was one of the main factors in the continuing success of these meetings.

Resources for Using EVS in the Teaching of Mathematics

There are many people starting to use EVS and one of the first things they need to do is to start to write questions for the topics they teach. Many are unaware that there have been projects, particularly in the USA, which have resulted in question banks being developed. The author thus successfully applied for sigma-cetl (http://www.sigma-cetl.ac.uk/) mini-project funding to collate information about existing resources for using Electronic Voting Systems (EVS) **for mathematics** and also to develop questions. A project website was developed (http://mec.lboro.ac.uk/evs) which provides information and links to relevant material. The website also provides over 300 mathematics questions which have been developed in the Mathematics Education Centre at Loughborough University. Also, the visitor to this website will find links to some key papers, and websites which review papers, on the use of EVS in general and EVS in mathematics in particular. The website was launched on the 30th of March 2010, with a one-day conference. Key-note speakers at this conference were Steve Draper, Glasgow University, and Mark Russell, the University of Hertfordshire, UK. Their presentations are also available.

Questions developed at Loughborough University cover topics typically found in a first year university course in engineering mathematics. These include differentiation, integration, differential equations, complex numbers, matrices and vectors. Figures 2 and 3 contain examples of questions developed for the author's course on mathematics for Sport Technology. The two questions are related to the vectors part of the course. The first question (Figure 2) is an example of a straightforward calculation and requires students to use text-entry handsets to input a numerical answer. The second question (Figure 3) is a multiple-choice question and the correct answer can be selected using a standard handset. (Note that this latter question is an adaptation of a question developed by the Mathquest project in America (http://mathquest.carroll.edu/). The second question requires much more thought by the student. In this type of question one could ask the students to respond initially and then, given the responses indicated, could ask them to discuss their answers with a fellow-student and then vote again. This peer-instruction can be a very valuable pedagogical tool to encourage deep learning in class.

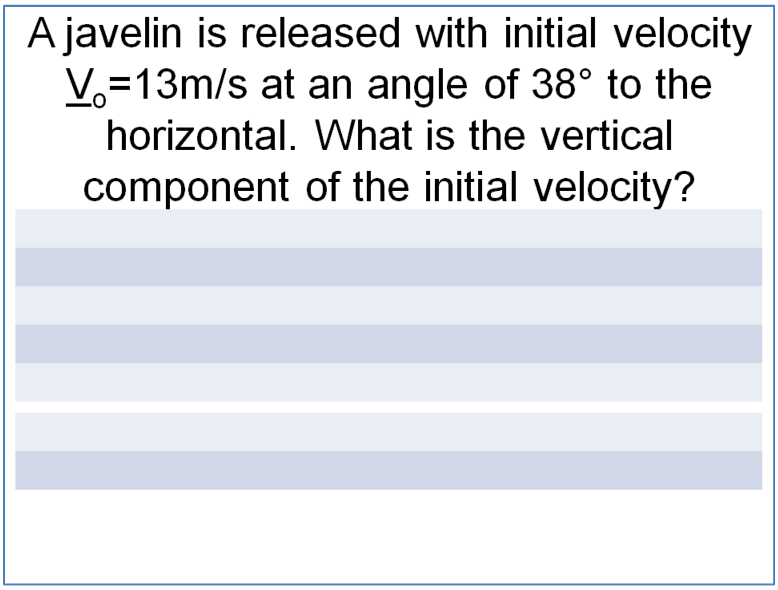

Figure 2: Straightforward question checking that students can calculate components of vectors. It requires the use of text-entry handsets.

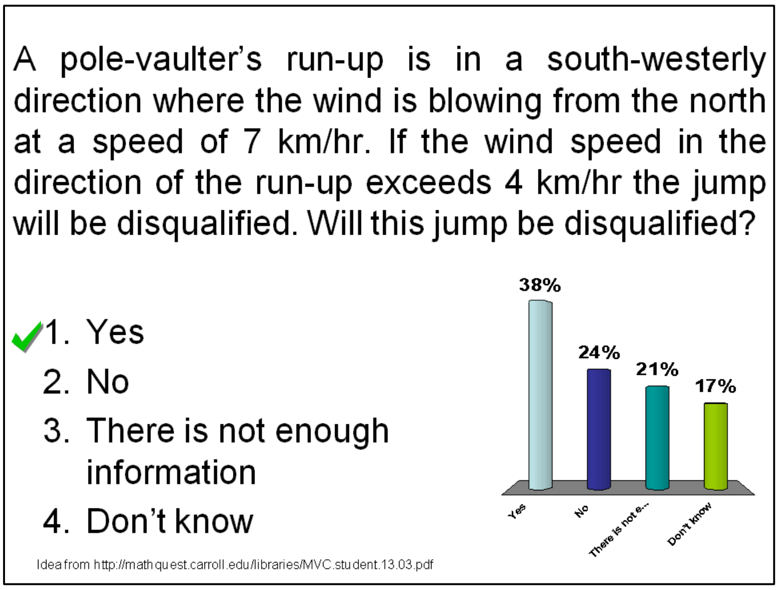

Figure 3: A more demanding question which requires the student to apply knowledge of vectors to a sporting situation.

Within the EVS project website (http://mec.lboro.ac.uk/evs) there are inventories of the questions contained. Thus if you are looking for a question on differential equations, you could look up the inventory and then browse the questions which have been developed for that topic. For example, Cornell University has developed many questions for calculus, as part of its 'Good questions project' (http://www.math.cornell.edu/~GoodQuestions/materials.htm). The questions have been designed to inspire a deeper level of thinking and understanding.

Finally the project website (http://mec.lboro.ac.uk/evs) is an evolving resource which I hope will develop as more people start to develop questions for EVS use. If you have questions to share with the academic community, please do get in touch with the author.

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Longitudinal Analysis of a Key Skills in Mathematics Initiative

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Abstract

The drive towards mass education, and the year-on-year fall in popularity of technically based subjects in Ireland (and other western countries), has created cohorts of students in many technical courses who are ill-equipped to succeed on those courses. Compounding this problem is the fact that most students in Ireland are now on semesterized courses, which seems to encourage students to learn enough for the regular examinations, without necessarily taking time to reflect on what they have learnt.

At IT Tallaght we have implemented an initiative called Key Skills Testing in Mathematics as a simple and efficient means to encourage knowledge persistence and reflection. Using the Moodle platform, Key Skills consists of;

1. creating many categories of multiple-choice and numerical input questions which we believe our cohorts of students MUST be able to do. Each question comes with feedback and reference to a book chapter and an electronic resource.
2. creating tests which draw randomly from particular categories of questions. These tests may be repeated several times over a semester and only a high mark is rewarded with credit.
3. different tests run for different groups and in different semesters, reflecting the Key Skills of previously taught material required for that semester.

The aim is for students to expect Key Skills in each semester, re-inforcing and repeating their learning. We view our project as a simple, structured attempt to embed reflective learning into each mathematics module.

This project entered its third year in September 2009. We would like to present an analysis of our results so far, looking at performance on Key Skills tests over 3 and 4 consecutive semesters for different groups of students. We will present evidence that students improve over a semester and that, semester on semester, students' first attempts in a new semester are better than in previous ones (hinting at long term skills retention). We will also see evidence that a greater proportion of students gain a "high" mark as the student group proceeds through each semester.

Introduction

The Institute of Technology Tallaght Dublin (ITTD) is located in South County Dublin and was established in 1992. The Institute caters for a student population of approximately 2,300 full-time and 1,200 part-time students and offers a wide range of programmes from Higher Certificate, Ordinary and Honours Degree to Masters Degree and Doctoral level.

This decade has seen a gradual trend of falling numbers applying for Engineering and Technology based courses in Ireland. Compounding this fall has been a fall in the preparedness of students for their course, in terms of ability in mathematics and physics. The proportion of engineering students at ITTD with reasonably good maths (B or better in Ordinary Leaving Certificate maths, roughly the equivalent of English O Level and combining B1, B2 and B3) and reasonably poor maths (C or less at OLC, combining C1 to

C3 and D1 to D3) has also been roughly 30%: 70% for several years prior to 2007.See *Robinson et al* (2007) for a detailed discussion of these trends.
Low uptake of Engineering and a high proportion of mathematically weak students has continued into 2008/09, though there has been a slight increase in engineering students in the 2009 academic year. These trends are well documented elsewhere – see for example *Bamforth et al* (2007).

Several initiatives to improve basic mathematics ability have been tried at ITTD, *Robinson et al* (2007) and *Marjoram et al* (2004). These initiatives have targeted first year students with the aim of improving first year retention. Unfortunately, while these initiatives appear to have been moderately successful in the fight to retain students, there is still a worrying lack of mathematical ability and knowledge amongst most second and third year students. The semesterized examination system does not help, leaving students with little time to reflect on their learning. Typically, the weak student does not bring key mathematical knowledge with him/her from one semester to the next. How can we help them to do that, and continuously refresh key skills?
In October 2006 we applied for an internal seed fund grant of €3, 000 to implement a project which we call Key Skills. The idea is to continuously test key mathematical skills over a semester until a high mark is achieved (a high threshold competency based test). Such tests need to be randomized so that adjacent students do not see the same questions. They need to be repeatable, automatically marked, and provide immediate feedback on learning resources that students might go to in order to do better next time. The tests must be difficult to cheat on and marks gained must go towards the continual assessment for their maths module – a currency all students understand. Such frequent testing has been shown to be effective in motivating students to learn, especially weaker students. See Tuckman (2000) for example.

The Moodle Quiz Platform was chosen to implement the Key Skills initiative. This platform has proved to be extremely stable, with no performance issues or corrupted data to date. It possesses all of the features we require, as well as keeping excellent student record data that can be further processed in Excel, for instance. It is also open source and has a very active Quiz component user group.

Implementation of Key Skills

The pilot phase of implementation is discussed in detail in *Marjoram et al* (2008). The roll out to all students in years 1, 2 and 3 in the School of Engineering began in September 2007 and we will look at data, in this paper, for the 6 semesters Sep 2007/Jan 2008, Sep 2008/Jan 2009 and Sep 2009/Jan 2010.

Our Key Skills initiative has been set up as *one* Moodle course that all students enrol on. This cuts down on explanatory material for students and also gives lecturing staff a single location and log on process to describe. The idea is that students will do Key Skills tests in all semesters, so that this single location is valuable for continuity of student access and login details. All student records are also kept in the same location, which will allow us to do a longitudinal analysis of student performance over several semesters.

Currently Key Skills consists of over 1000 questions across 50 categories with at least 20 questions in each. These are a mix of multiple-choice and numerical input questions. We have also managed to incorporate a downloadable version of CALMAT Version 5.0 as a revision resource that students can install on their own machine at home, as it has recently become available under a Creative Commons License (See FETLAR project or JORUM Library). Each question feedback has a reference to CALMAT material and a well known introductory mathematics text for Engineers.

Key Skills tests last one hour and have 15 questions each, most of which are multiple-choice. The topics included in the tests have all been studied in earlier semesters. Usually a topic is included because it is considered to be essential for the current semester's mathematics and other modules. However, sometimes a topic is included in the test because it is not covered in current semester course work but is considered to be essential in future, such as basic calculus. The questions are generally straightforward, such as identifying the equation of a straight line from its graph, solving a linear equation, adding two algebraic fractions, or performing a differentiation or integration. As the tests are based on material from earlier semesters, the first Key Skills test is given in Semester 2. A new initiative in 2009 in semesters 5 and 6 for Electronic Engineering students is for those students to fill in "Reflection Sheets" between test attempts, forcing them to reflect. This will be discussed in detail later.

The best mark in the semester's Key Skills test is half of the continuous assessment for that semester (typically 15% of the module mark). At the moment, for a 15 question test, a student gets 6% for 10 correct, 7% for 11 correct, 9% for 12 correct, 10% for 13 correct, 15% for 14 or 15 correct and 0% for 9 or less correct. Our entire Moodle course is available from the National Digital Learning Repository. It is publicly available for New Users and can be searched for under KeySkills.

Test uptake and the testing process

Table 1 below shows the uptake of tests in the six semesters from Sep 2007 to May 2010.

Module	Total Tests	Total Students	>=5 tests	Max tests	Tests/Stu	Stan. Dev
Elec 2	209	85	20	12	2.46	1.87
Elec 3	32	23	0	3	1.39	0.5
Elec 5	293	74	27	12	3.96	2.82
Elec 6	168	64	10	7	2.63	1.63
Tech 2	595	214	37	12	2.78	1.54
Tech 3	149	41	8	6	3.63	1.07
Mech 3	73	34	3	12	2.15	2.26
Tech 4	147	40	15	7	3.68	1.53
MechElec 4	122	42	16	7	2.90	1.74
Mech 5	263	85	22	11	3.09	2.24
Mech 6	488	126	44	12	3.87	2.39
Total	2539					

Table 1: *Numbers of students and their test frequency*

In Sep 2008 the mechanical engineering Sem 2, 3 and 4 modules were changed. This required a slight change in the questions to reflect prior learning and Mech 3 and MechElec 4 were replaced with Tech 3 and Tech 4 respectively. We have taken them to be essentially the same in considering student results from semesters 3 and 4 going into 5 and 6. The Mech 4 and Elec 4 tests are the same and there is no Mech 2 data as it was overwritten by Tech 2 in terms of results. From this table we can see that *students are persistent*, with up to 12 tests taken in some semesters. For modules with substantial numbers of students/tests we see that about a quarter of students do 5 or more tests in the semester. It is clear that the process is well understood by students, and that email and text messaging of test opportunities was effective. Some modules have very low test uptake e.g. Elec 3. In all cases, this was due to the lecturer on that course not being one of the 4 named authors. The logistics of arranging labs, supervision and prior warning to students of tests was considerable and most results in Tech 3 and Tech 4 are for the 2009/10 academic year. Some lecturers were also unwilling for their group to do KeySkills , regardless of whether it occurred outside class time. More detail on the make up of Key Skills testing and advice on running such a regular testing regime is in *Marjoram et al* (2008).

Main Findings for Mechanical Engineering Students
Semester on semester progress

Our data is a bit thin in Semesters 3 and 4, which gives us low student counts involving those semesters. Tables 2 below is for the 12 students who did at least 3 tests in semesters 4, 5 and 6 and did not get 14, 15 in their first test. Table 3 is for the 26 similar students in Sem 2, 3, 4.

Semester	4	5	6
Average Grade	6.98	9.78	10.08
No with 14,15	1	5	6
No with >=12	2	9	10
Mark in Test 1	4.67	9.17	9.00

Table 2: 12 students Sem 4, 5 and 6 Mech

Semester	2	3	4
Average Grade	8.50	8.67	7.53
No with 14,15	1	2	1
No with >=12	16	14	6
Mark in Test 1	7.47	7.65	7.00

Table 3: 26 students Sem 2, 3 and 4 Mech

Table 2 is very encouraging, Table 3 less so! In Table 2 we see the numbers getting high marks (12 or more) climbing steadily. We also see improvements in Test 1 in each semester, and improvements in average test mark through semesters 4, 5 and 6, suggesting knowledge retention. Table 3 shows none of these things. In mitigation, Semester 2 tests mostly algebra which is covered in Semester 1. Test 4 is then based mostly on Semester 1 and 2 material, which students have not seen or used for some time, as the semester 3 course is all statistics. The Semester 3 and 4 testing regime was also somewhat haphazard for these students as none of the authors had these groups when they did their tests.

Semester	3	4	5	6
Average Grade	9.69	8.41	10.19	10.21
No with 14,15	3	4	8	4
No with >=12	3	4	9	7
Mark in Test 1	9.33	6.22	10.78	10.56

Table 4: 9 students Sem 3, 4, 5 and 6 Mech

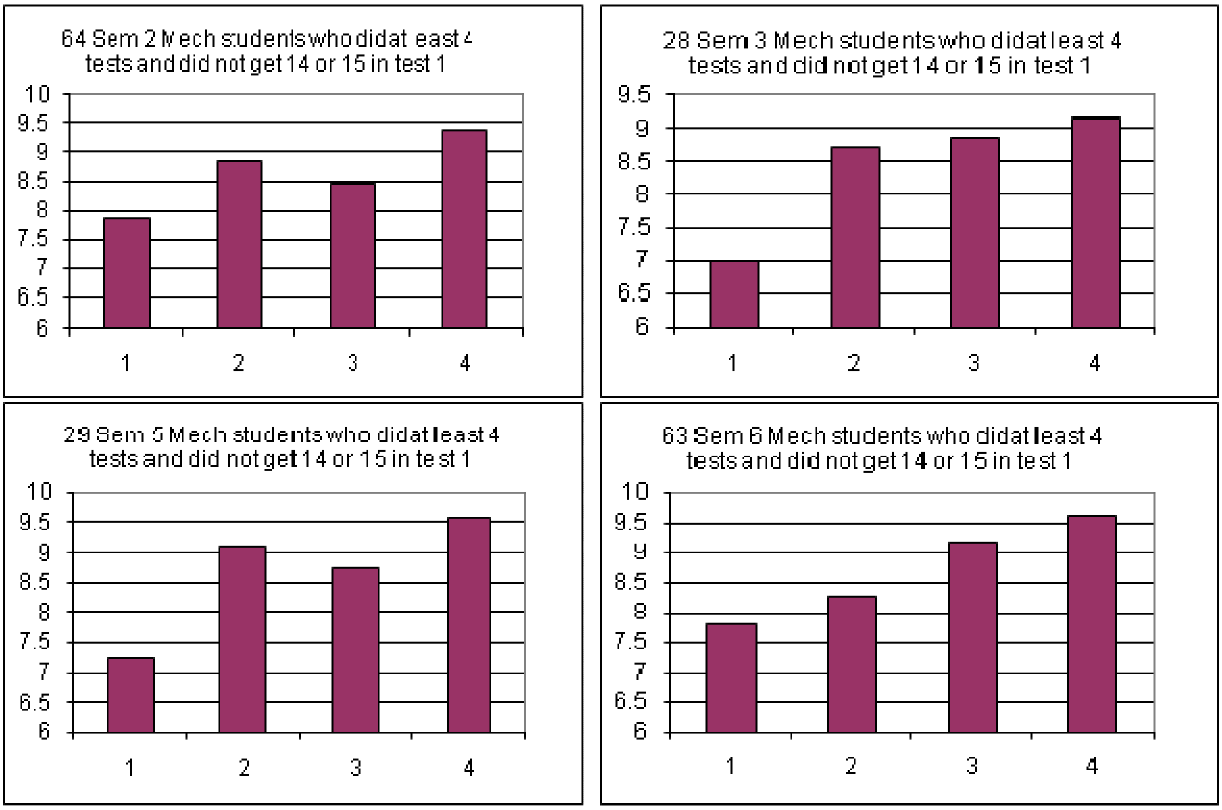
Semester	2	3	5	6
Average Grade	8.43	8.97	9.94	8.75
No with 14,15	0	3	7	7
No with >=12	4	3	10	11
Mark in Test 1	7.46	8.46	8.92	8.15

Table 5: 13 students Sem 2, 3, 5 and 6 Mech

The drop in Semester 6 may be due to a much greater proportion of numerical input questions this semester (and higher number of tests) than in the previous 3 years, which is born out by looking at the semester 6 results in 2007, 8, 9 and 10. Space does not allow us to present this data. The climb in good results (12 or more) generally is encouraging.

Progress Within a Semester

Progress within a Semester is generally positive. The graphs below show average results within a sample of Semesters for students who have done *at least* 4 tests and did not get 14 or 15 in their first test. All other Groups in Table 1 show a similar trend. Only the first 4 test results are shown



Tables 6: Student improvement in their first 4 consecutive tests

Students do, on the whole, get better as they take more tests. Improvements are not particularly marked. In the next section we will make a more detailed statistical analysis on the semester 5 Electronic Department students and examine the effect of a reflection strategy.

Main Findings for electronic Engineering Students

Given the marking structure, students are offered several (usually 6 or more) opportunities each semester to take a test. The authors were concerned that some students exhibited no evidence of having worked on the test topics between tests. When asked about preparation for the next test, some students admitted having no record of the question categories where they gave an incorrect answer, although their performance in each individual test was readily accessible through Moodle. To address this problem, a “Key Skills Reflection Sheet” has been introduced this year for third year Electronic Engineering. The sheet identifies the question categories, both with a short description and a sample question drawn from a practice test. Students must then mark on the sheet those question categories in which they gave an incorrect answer. This is best done directly after each test. The sheet must be returned to the lecturer prior to the next test with actions filled in against some or all of the incorrect answers. Recommended actions include attendance at mathematics support sessions or use of online resources targeted at the problem category.

The number of students in third year electronic engineering who took *multiple* tests in semester 5 prior to the introduction of reflection sheets (2007 and 2008) was 44. This year (2009), there are 19 such students all of whom needed to fill in and return a reflection sheet in order to be allowed to resit. The quality of information recorded on the reflection sheets varies. Some students are careless even about recording which questions they got wrong and give minimal or overly general entries as "Actions", e.g. "studied" or "revised". Others fill in the forms very conscientiously and give detailed "Actions", e.g. specific online resources used.

Results:
[Statistical significance will be taken at the 5% level, so that any number in the analysis below, less than 0.05, is considered significant.]

There appears to be a statistically significant improvement in test performance since the reflection sheets were introduced, particularly for those students who failed to pass the threshold of 10 right answers on their first attempt (whom we will call "Low Threshold" or LT students for short) A complication is that the 2007 students were offered the opportunity to compensate for a low Key Skills mark through their performance in other elements of the mathematics module. This may have acted as a disincentive to engage fully in the process. For this reason, this year's students were analysed against the 2007 and 2008 classes combined, as well as against the 2008 class only, where the comparison is more clearly an investigation of the effect of the reflection sheets. This gives us 4 groupings to test against the 2009 students (19 Students, 14 of whom were LT)

2007/2008 Combined (44 Students)	*2008 Only* (24 Students)
2007/2008 Combined LT (28 Students)	*2008 Only LT* (16 Students)

An analysis of test performance for each of the groupings reveals no statistically significant difference in starting level (Test 1), either in terms of the mean score on the first test or the proportion of LT students. Test performance is analysed in four ways:

The first analysis examines the best score achieved, i.e. the maximum number of correct answers achieved by each student. There is *ambiguous evidence to support the hypothesis that reflection sheets lead to better scores.* **Evidence:** there has been an increase in the mean best score since the reflection sheets were introduced. Two-sample t-tests were used to investigate whether the increase is statistically significant. The Welch t-test was preferred because of the significantly different variances between the samples; F-tests support the hypothesis that the reflection sheet evened out performance. Indeed, better performance leads to greater concentrations of marks near 15 and hence decreased variance. The p-values were 0.0315 for *2007/2008 Combined* and 0.1141 for *2008 Only*. For LT students, the p-values were 0.0108 and 0.0761 respectively. However, the nature of the data, including the fact that there is a maximum score on the test, may lead to enough asymmetry to undermine the normal assumption required for a t-test. The Mann-Whitney test for LT students gives p-values of 0.0674 against *2007/2008 Combined* and 0.1368 against *2008 Only*, giving weak support to the hypothesis that there was a real increase in best score. The p-values for LT students are both above 0.2.

The second analysis is to calculate the increase from first test to best test for our groups. There is *strong support* for the hypothesis that the mean increase has gone up in the case of the comparison with *2007/2008 Combined, 2008 Only* and *2007/2008 Combined LT,* with *weak support* for the hypothesis in the case of *2008 Only LT.* **Evidence:** this increase is higher on average for the group with reflection sheets (2009 group) and there is less reason to doubt that the distributions are normal, being samples of differences. The analysis shows no evidence of different variances between samples (*2007/2008 Combined* and *2008 Only*) and the two-sample *t*-test gives p-values of 0.0080 against *2007/2008 Combined* and 0.0461 against *2008 Only*. For LT students, the hypothesis of different variances is *supported* by the *F*-test p-values of 0.0312 for *2007/2008 Combined LT* and 0.0495 for *2008 Only LT*. The Welch two-sample *t*-test then gives p-values of 0.0087 and 0.0748 respectively. To address concerns that the underlying distribution is not normal, Mann-Whitney tests have been carried out on this data. The p-values for students are 0.0095 against *2007/2008 Combined* and 0.0485 against *2008 Only*. For LT students, the p-values are 0.0381 against *2007/2008 Combined LT* and 0.1261 against *2008 Only LT*. The normal assumption certainly looks reasonable apart from the case *2008 Only LT*.

The third analysis is to count the number of students reaching the threshold of 10 right answers. There is *strong support in all four cases for the hypothesis that the number of students reaching 10 or more has increased by using reflection sheets.* **Evidence:** Considering *2007/2008 Combined* first, this proportion has moved for all repeating students from 30 out of 44 (68%) with no reflection sheet to 18 out of 19 (95%) with reflection sheets. Fisher's exact test gives a p-value of 0.0198. For *2007/2008 Combined LT* students, the move is from 14 out of 28 (50%) to 13 out of 14 (93%) with a p-value of 0.0061. For *2008 Only*, the proportion has moved from 17 out of 24 (71%) with a p-value of 0.0504, and for *2008 Only LT* students the proportion has moved from 9 out of 16 (56%) with a p-value of 0.0296.

The fourth analysis is to count, for each student, the number of test results which were worse than the previous test, the number which were the same as the previous test, and the number which were better than the previous test. Since reflection sheets were introduced, the proportions in all three categories have moved in the desirable direction, including for LT student comparison. Splitting the test data more simply into "no improvement" and "improvement" categories, Fisher's exact test gives p-values of about 10% for *2007/2008 Combined* (and about 20% against *2008 Only*), so there is *only weak evidence of real test on test improvement.*

Concluding comments

Though our analysis includes less students than we would like, we believe that there is good evidence that a Key Skills testing regime, made possible by Moodle's stable Quiz environment and data storage/retrieval facility, can lead to improvements in students mathematical skills over both one semester and several semesters. There is also good evidence that forcing engagement and reflection through a reflection sheet regime can produce significant improvements in performance.

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IMPETUS for the Engineering Sciences

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Abstract

Since the introduction of the Bachelor degree in Germany, engineering students have even less time for learning mathematics. Required courses in mathematics provide basics, but much reduction in terms of content has occurred. Offering non-required mathematics courses can only partially solve this problem. Engineering students in Germany have precious few credit points for which they can freely choose from elective courses, and here, they are more apt to choose such a course in their own field of studies. The aim of this contribution is to present the IMPETUS model for a non-required course that has mathematics as its core, yet is nonetheless appealing for engineering students. Within this model, students are motivated to take an active role in their own learning process. The knowledge gained in the course (currently statistics and MATLAB) can be immediately applied in a different setting, whereby statistical models are developed and assessed for actual research projects.

Introduction

The signing of the Bologna Declaration in 1999 by Ministers of Education from 29 European countries and the subsequent Bologna Process have led to major reforms in higher education. The principle aims of the declaration were to establish standards with respect to both academic degrees and quality assurance that are compatible throughout Europe. The conditions of the new bachelor degree programs unfortunately impose severe time constraints on students, especially those in engineering programs at German universities. There is a great deal of pressure on students to complete the bachelor degree within three years. As the average engineering student at the Technical University (TU) Berlin has only nine credit hours electives over a three-year period, they will, for the most part, only complete those mathematics courses that are required for their degree. For example, Numerical Mathematics I for Engineers is required by five of the fourteen bachelor degree programs in engineering at the TU Berlin. During the Winter Semester 2009/2010, only 15 of the 162 students attending the course elected to do so.

Not only the lack of free electives contributes to the poor attendence in higher-level mathematics courses for engineers. Nearly all engineering students are required to complete courses in analysis (one-variable and multi-variable) and linear algebra during their first year. Time constraints imposed on lecturers due to the volume of material covered means that little time is available for applications. Examples relating the mathematics to real problems are important to students, as a survey of 135 students attending the course Analysis I for Engineers in Winter Semester 2006/2007 confirms. The questions, “On a scale of 1 (very important) to 5 (not at all important) how important are engineering examples to you?”

“On a scale of 1 (very important) to 5 (not at all important) how important are examples having practical relevance to you?”

received an average response of 2.6 and 1.9, respectively. Yet first-year students are only beginning to acquire the background knowledge necessary for their field, so that the inclusion of meaningful examples necessarily shifts from the actual mathematics to the topic of the application. For this reason, many instructors omit engineering applications during the lecture, leaving these for self-study, which are largely ignored by students because of the heavy workload. The outcome is that many students regard higher mathematics as being too theoretical and become demotivated during their first-year courses.

Even before the reforms, colleagues in both mathematics and engineering departments have deemed that engineering students do not possess sufficient skills in mathematics; see e.g. Grünwald, Kossow and Schott (2000) or Berger and Schwenk (2001). Deficiencies of incoming freshman and time constraints have contributed to further alterations concerning the content of first-year courses at the TU Berlin. For example, although complex vector spaces have not completely disappeared from the linear algebra course, the topic is only covered perfunctorily and not included in examinations. On the other hand, requiring more mathematics would mean even less electives, culminating in an inflexible system that offers no freedom to students. One of the main activities of the IMPETUS project is to experiment with a model for elective mathematics courses that are suitable for and appealing to upper-level engineering students.

Background of IMPETUS for the Engineering Sciences

The IMPETUS project began in 2007 as part of the TU Berlin’s “Knowledge Through Learning Offensive” as a strategic project for improving the situation of women enrolled in the engineering sciences. It recently merged together with the project “Zielgerade” (Aim Straight Ahead) to provide women with programs dedicated to all different phases of their bachelor studies: orientation in university life, orientation in the professional culture of engineering disciplines, support during the senior thesis phase. IMPETUS is concerned with the last two phases and currently offers workshops in writing for scientific and technical fields, student seminars, excursions to companies or research institutions employing engineers, and a summer program, which is the focal issue of this contribution. It should be mentioned that these activities would be beneficial for all students. However, the need to develop them to appeal to women is extremely important due to the acute underrepresentation of women in engineering fields.

The intrinsic goals of the IMPETUS project are various, yet connected through the aim to reduce high attrition rates. The choice of an engineering field of study is often based on the desire of a hands-on career. The first semesters at the university are often deemed by engineering students as too theoretical, especially when it comes to mathematics, and frequently a main reason for dropping out (see e.g. Gollub (2009)). The need arises to confirm students' choice of an engineering degree and to motivate them to complete their studies. Additionally, internal studies over the past decades reveal that the percentage of women in engineering disciplines decreases dramatically at each educational stage. With the introduction of the bachelor degree in Germany, the fear is that the additional academic stage will result in even less women, percentagewise, in the upper stages. The IMPETUS project therefore strives to increase the percentage of women in engineering programs by stimulating their interest in research.

The IMPETUS Summer School

A comprehensive summer school was developed by extending upon the Research Experience for Undergraduates (REU) programs popular in the USA. These summer programs are quite frequently hosted by universities to provide talented undergraduate students with research opportunities to encourage them to continue in a graduate program. The National Science Foundation alone funds about 145 REUs in the engineering sciences (see for example the REU search engine)[1] allowing universities to supply participants with a stipend (approximately 3500 Euros plus housing) over a typical 8 week program. The budget of IMPETUS is rather modest, so that compensation for participation is not possible. For this reason, the acquisition of students and participating research leaders was expected to be and is more difficult than in the USA. Nevertheless the resonance has been good and the evaluation excellent, so that a continuation was recently granted.

The IMPETUS project offers a four-week intensive summer school for women engineering students who have successfully completed at least two semesters at the TU Berlin. Just as in REUs, students work within an assigned research group three days a week. But the IMPETUS concept goes futher than most REUs in several ways. One of the most important differences is that participants attend an interdisciplinary educational component that combines engineering, mathematics and computer programming (three hours twice per week). This course ideally serves as a connection between the various research projects. In addition, two workshops are offered for developing technical writing skills. One focuses on the content and structuring of a scientific paper; the other gives a quick introduction to typesetting in LaTeX. Other workshops focus on soft skills, such as presentation techniques or networking. The summer school is rounded off by visits to research institutions or companies, providing students with an orientation as to possible career opportunities. In order to earn credits, students are required to deliver an oral presentation and to submit a written report concerning their research project.

The course that is currently being offered is "Quantitative Methods in Data Analysis". The motivation for the choice of this topic was brought about by previous requests from engineering disciplines at the TU Berlin for a one-semester statistics course suitable for engineers. With the user disciplines in mind, this course was developed based in part upon the classical treatise by Mandel of the National Institute of Standards and Technology. It begins with linear and nonlinear regression, so that statistical models are immediately introduced. The validation of the models obtained through examples and student research projects becomes the focal point, motivating students to learn the basic definitions and theory in statistics. Distribution functions, confidence intervals, hypothesis testing, and the planning of experiments are all included. A treatment of accuracy and precision is currently under construction. The course material is distributed in the form of packets, including practical, interdisciplinary exercises that are developed for solution by hand as well as with MATLAB.

There are a host of statistical software packages, so one might wonder as to the choice of MATLAB for a course in data analysis. Only about 25 % of the female students (45 % of the male students) surveyed at the TU Berlin (2005, 2009) have programming experience when entering the university. Not all are required to take a programming course yet most are expected to have acquired this skill at some point in their academic career. As the goal was for participants to develop these needed programming skills, not just to utilize statistical software, the students in the IMPETUS summer school are required to write and assess their own basic statistics routines. The choice of MATLAB was based upon the preferences of colleagues in the engineering departments.

The course itself is held in a computer laboratory in a guided independent study approach. The role of the instructor is to specify the plan for the day and to help students explore solutions to their questions. Small groups are highly encouraged for discussing ideas, but actual write-ups and implementations of routines are completed individually. Due to the motivation students receive from immediately applying the new material to their own research project, students willingly take an active role in their own learning process. Two sample projects are listed below to provide a glimpse into suitable projects for students in a relatively early stage of their university career.

Calibration of a five-hole sensor

(Research Leader : Dr. Ing. Andreas Becker, Institute for Aviation and Astronautics, TU Berlin)

From the student's abstract: Five-hole sensors are used in flight measuring techniques to determine flight attitude (or orientation) parameters experimentally. They serve for calculating the norm and direction of the acceleration vector of the flow, which is dependent upon the angle of approach (horizontal) and angle of sideslip (vertical). In order to use the sensor for flight measurements, it needs to be calibrated. The calibration for this project was achieved by simulating various angle combinations in a wind tunnel. The resulting pressures were recorded and then used to generate Eich curves. A MATLAB-program was written to evaluate the data and to represent it graphically.

T-V-Model: Parameter-based prediction of IPTV Quality

(Research Leader: Alexander Raake, Telekom Labs, TU Berlin)

From the student's abstract: With the increasing propagation of real-time video services in the Internet, for example the numerous IPTV offers of large providers, issues concerning quality become more and more relevant. The question of how well audio content is received by the average viewer was answered a few years ago after intensive research in the area of speech communications led to a suitable model.

The objective assessment of video streams on the other hand is a young discipline. The research strives for a standard in which a model describes the objective quality of video streams. The T-V-model project develops such a model that allows for estimations concerning the perceived quality and which can be used for the monitoring and planning of communication networks.

In this project, the PSNR (peak signal to noise ratio) algorithm was implemented in MATLAB and applied to the test sequences. The results were then compared with those of the SSIM (structural similarity) algorithm using correlation principles.

Evaluation

The summer school is evaluated annually by the participants and their research leaders. The goals of the project were successfully met. All participants confirmed that their motivation to complete their degree was increased due to their participation in the IMPETUS summer school. They were integrated into their research groups and were able to take part in interesting, interdisciplinary research topics. Several of the participants remained active in the research group even after the end of the summer school. The diverse activities, aside from the course and research project, provided a glimpse of different career opportunities in the engineering sciences as well as to what practising engineers actually do.

The practical concept of the IMPETUS summer school was welcomed by research leaders: “The lower-level students at the TU Berlin become familiar with the basics of their discipline during their first-year courses. What these are good for and how they can be used can be demonstrated very well in this type of module. The concept of the module, to simultaneously convey methodology and computer-supported solution strategies is exactly the right way to prepare these students for the practical competencies expected of them.” The idea to expand this concept to first-year courses would for this reason be desirable. Unfortunately, participants who had completed only two semesters prior to the summer school were somewhat overwhelmed, as they needed to work harder to obtain missing basics in their own fields.

One of the most difficult problems in the summer school is that faculty are not necessarily present. Some of the student lab workers supervising the IMPETUS participants have admitted having little to no knowledge of statistics, so that connecting the research project to the course was entirely left up to the participants. This is reflected by the fact that most of the reports contained very little discussion on statistical topics other than counting statistics. More coordination and support for the student lab workers appear to be necessary.

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Endnotes

[1] see the REU website of the NSF http://www.nsf.gov/crssprgm/reu/reu_search.cfm (accessed 29 April 2010)

2 see the IMPETUS website for more details http://www.impetus.tu-berlin.de (accessed 29 April 2010)

Computer Tomography as a Project for Students

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Abstract

Computer Tomography is a very popular subject which many people encounter during the process of medical diagnoses. Many people are surprised to learn that there is a great deal of mathematics behind the corresponding image reconstruction process. A teaching project with students about this subject is very efficient because many important teaching aims can be supported including: activation of learners, project work, inter-disciplinarity, subject adaptability to students' abilities, training in modelling, problem solving and use of technology. Discussing the tasks and problems within the project leads to the following conclusion: The education of engineers needs many mathematical basics and appropriate use of computer software.

1. Introduction

The mathematical education of engineers should give students insights in the process of mathematical modelling and simulation as well as in important mathematical concepts and methods used by engineers to solve practical problems. Typically problems are interdisciplinary, with teams of scientists and collaborators working together. Computer Tomography (CT) is such a problem. It is an important diagnostic method of radiology where the human body or parts of it (e.g. cross-sections) are screened by X-rays from different directions. The attenuation of rays depends on the density and thickness of material. From the measured attenuation of radiation, a computer calculates the structures (density distribution) of the part under consideration. An image shows these structures by using colours or grey values. CT is an instructive example to illustrate the interplay of different disciplines and methods to solve problems. Image reconstructions are established in clinical tomographs on the basis of up-to-date higher mathematics. However, for teaching aims it can be also realized using relatively simple modelling allied with physical and mathematical means. The original integral equation for the unknown density function of the material can be approximated by a finite system of linear equations with unknown density values of the planar object (cross-section) is covered by a square pixel grid with assumed constant densities. Starting with only a few pixels simple linear equations arise which can be solved easily. Straight lines in the plane representing X-rays must be established and intersections of straight lines must be calculated to find the coefficients in the linear equations. In practice high dimensional systems occur. They need matrix and vector concepts of linear algebra, knowledge of the solution structure of such systems as well as iterative solution methods. The approximation procedure becomes clearer if integration and its numerical realization are known (e.g. Riemann sums instead of integration).

By use of computer software students can carry out experiments during the project and come to important conclusions. They can work with software packages or develop their own small routines. Important numerical concepts and effects can be discussed.

Organizing the project in parallel competing groups increases its attraction. In the next sections the basics of the project are described to illustrate the character of the CT problem and to encourage colleagues to start this or similar projects at their universities (additional material is given in the references).

2. Physical Modelling

Wilhelm Conrad Röntgen discovered X-rays in 1895. For applications in medicine, the attenuation during transition through material is relevant since parts of rays are absorbed. There are different effects causing attenuation, e.g. photoelectric effect, Compton Effect (scattering) and pair production. These attenuation parts combine approximately linearly. Furthermore, attenuation depends on the material (atomic charge) and on the energy (frequency) of the X-rays. Experiments show that rays (such as light, X-rays, γ-rays, electron rays) in accordance with to the law of Lambert-Beer. This states that there is an exponential decrease of ray intensity during transition through material. For simplicity the following is assumed:

- Rays are mono-energetic (point spectre) and travel along straight lines.
- The structure of the organism is described by its density distribution.
- The attenuation of rays is proportional to the density.

Now we consider a ray transmitting through a homogeneous object. If I_0 is the intensity entering the object (at the ray source), f is its constant density, s is the actual and s_E is the whole path length of the ray in the object, then the intensity within and after leaving the object (at the detector) is

$$I(s) = I_0 \exp(-f \cdot s), \quad I_1 = I_0 \exp(-f \cdot s_E).$$

Hence the attenuation exponent is both proportional to the density and to the path length. For an inhomogeneous bounded object with variable density f we have to sum up small attenuation parts $f\,ds$ within the object along the ray L. This means

$$I_1 = I_0 \exp\left(-\int_L f(\vec{r})\, ds \right).$$

Here $\vec{r}$ is the three-dimensional position vector with coordinates x, y, z. Rearranging the integral representing the global attenuation along L gives the integral equation

$$\int_L f(\vec{r})\, ds = \ln \frac{I_0}{I_1} =: g(L). \tag{1}$$

If the right-hand side is known, we must try to determine the structure function f (density distribution) of the object.

We know that the density *f* is positive within the object. If it is relatively isolated, then *f* can be supposed to be zero outside the object. This means

$$f(\vec{r}) \geq 0 \quad \text{for all} \quad \vec{r}. \tag{2}$$

It is the job of mathematicians to clarify under which assumptions unique solutions exist. The problem can be simplified by using cross-sections. Hence the following is assumed:

- The body can be decomposed into a sequence of thin planar sections.

We consider one section Ω. Then equation (1) has the same form as before, but $\vec{r}$ has only the coordinates *x*, *y*. This section is a region with density $f(x, y)$.

Generalisations: It is possible to modify or to cancel modelling assumptions. Rays can be modelled in a plane also by thin strips or small sectors or in space by thin cylinders or small cones. Attenuation can be described as a function of space and intensity (energy). The object can be reconstructed in 3D without using cross-sections.

3. Mathematical Modelling

In 1917, the Austrian mathematician Radon solved the problem of reconstructing a function from its line integrals by presenting an analytical solution formula. This problem is nothing other than an integral equation (1). It is an inverse problem which needs some care to get reasonable solutions. The inventors of CT did not know these results. They developed a simple numerical solution which fits in our project and which we will describe with some modifications. The standard solution in modern medical applications is based on the analytical solution.

Radon assumed that the function $g(L)$ on the right-hand side of (1) is given for all *L*. In practice we can choose only a finite number of rays L_i ($i=1,...,m$). Thus we have only a finite set of values $g(L_i)$. Hence, we do not expect that we can find the unique solution $f(x, y)$ in Ω. Instead we get only approximate values f_j at certain points in Ω ($j=1,...,n$). A simple idea is to discretize the original equation (1). We assume that Ω is contained in the square domain *Q*. As before $f(x, y)$ is taken to be 0 outside Ω (see Fig. 1a). Now *f* can be approximated within a finite-dimensional function class by a linear combination of basis functions:

$$f(x, y) \approx f_a(x, y) = \sum_{j=1}^{n} f_j \cdot b_j(x, y). \tag{3}$$

Besides, the right-hand side in (1) can be calculated from measured intensities. We have

$$g_i \approx g(L_i) = \int_{L_i} f(x,y)\,\mathrm{d}s \approx \int_{L_i} f_a(x,y)\,\mathrm{d}s = \sum_{j=1}^{n} a_{ij} f_j, \quad a_{ij} = \int_{L_i} b_j(x,y)\,\mathrm{d}s, \quad (i = 1,\ldots,m).$$

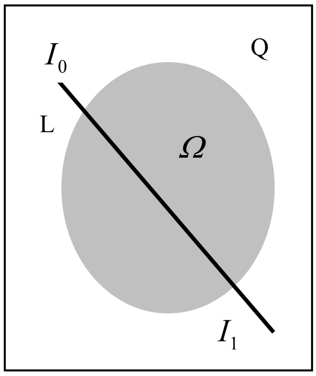

Figure 1a: Cross-section and X-ray

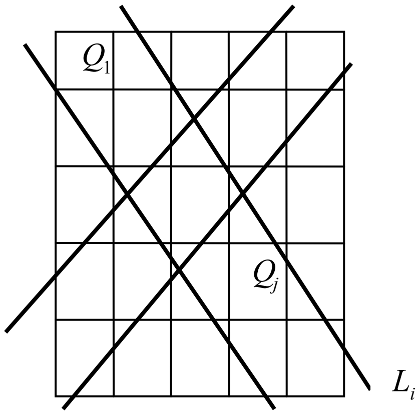

Figure 1b: Rays in the pixel grid

Consequently, we get m linear equations with n unknowns f_j as approximated attenuation equations, which can written compactly as

$$A \cdot \vec{f} = \vec{g}. \tag{4}$$

The m single equations of (4) represent hyper-planes H_i in n-dimensional space (for $n=2$ straight lines, for $n=3$ planes, and so on). The solution set of (4) corresponds to the intersection set of these hyper-planes. The solutions f_j of (4) supply the approximate density function which can be smoothed by interpolation methods. For instance we decompose Q into $n = l^2$ uniform small squares (pixels) Q_j ($j=1,\ldots,n$) and assume that f has the constant value f_j in Q_j, which is approximately fulfilled at least for small Q_j (see Fig. 1b). This corresponds to (3) choosing the basis functions

$$b_j(x,y) = \begin{cases} 1: & (x,y) \in Q_j \\ 0: & (x,y) \notin Q_j \end{cases}. \tag{5}$$

Then the approximate function $f_a(x,y)$ is piecewise constant. The corresponding line integrals a_{ij} are just the *cut lengths* of straight lines L_i in pixels Q_j. If L_i does not meet Q_j the cut length is set to 0. The attenuation area along L_i is then the sum of small rectangles with lengths a_{ij} and heights f_j. The additional condition (2) turns into

$$\vec{f} \geq 0 \quad (f_j \geq 0: \quad j = 1,\ldots,n). \tag{6}$$

Thus we have a system of linear equations with inequality constraints.

Generalizations: a) Use of other basis functions, b) Replacement of (4) by a linear inequality system, c) Use of interpolation and smoothing to improve the quality**.**

4. Systems of linear equations and quadratic optimization

Matrix A in (4) has the size (*m,n*), where *m* is the number of rays (measures) and *n* is the number of pixels (small quadratic regions). The following situations can occur:

- The system is under-determined (more pixel than rays). Then there are often infinitely many solutions.
- The system is over-determined (more rays then pixels). Then there is often no solution.
- The system is balanced (number of rays and pixels is equal). Then there can be a unique solution.

In practice the matrix A is *sparse*, because a lot of pixels Q_j are not met by a ray L_i. Since often there is no solution, a generalized concept is necessary: least-squares minimization of the residual $A\vec{f} - \vec{g}$. Observing that *m* single linear equations might be scaled by factors $c_i \neq 0$ without changing the solution set, we wish to minimize

$$\| C(A\vec{f} - \vec{g}) \|^2 = \sum_{i=1}^{m} c_i^2 \cdot \sum_{j=1}^{n} \left(a_{ij} f_j - g_j \right)^2, \quad C = \operatorname{diag}(c_1, \ldots, c_m). \tag{7}$$

Here ||.|| denotes the Euclidian norm (vector magnitude). In addition to the standard case $C = E$ (identity), the Hesse form is interesting. Here the norm of the *m* row vectors of CA is 1. The corresponding least-squares solution minimizes just the sum of the squared distances to the hyper-planes H_i belonging to (4). There is an equivalent system of linear equations (Gaussian normal equations). But it is not recommended to solve these because the numerical difficulties increase. If the columns of A are linearly independent, there is a unique least-squares solution of (7). But this solution depends on scaling, and may have no solution (the minimum is positive). Otherwise, usually a mixed quadratic optimization problem

$$\text{Minimize} \quad \| C(A\vec{f} - \vec{g}) \|^2 + \alpha^2 \| \vec{f} \|^2 \quad \text{with respect to } \vec{f}, \tag{8}$$

is solved with a small positive *regularization parameter* α.

5. Solution

In practice the dimension of matrix A is high. Hence an iterative solution method for (4), (7) or (8) should be used. The pioneers of CT developed a simple method which was extended by Herman (1980) and others (ART: algebraic reconstruction technique). A first idea is to project in each step *k* orthogonally onto one of the hyper-planes H_i. If they are chosen cyclically and (4) has solutions, then iterates f_k of ART converge to a solution. For accelerating convergence, the relaxation parameters λ_k are introduced, shortening or prolonging the projection step. Also, negative coordinates of f_k are put to zero to meet constraint (6). By appropriate choice of parameters we get approximate solutions of (7), (6) or (8), (6). We have realized the method with cyclic choice of hyper-planes and with relaxation parameters tending moderately and cyclically to zero.

Alternatives: a) Gaussian elimination, QR decomposition for small systems, b) Generalizations of ART using blocks of hyper-planes (BART), Method of conjugate gradients (CGLS, Hochbruck and Sautter 2002).

6. CT in the Computer Laboratory

Specifications: For simplicity we assume that *Q* is the unit square and Ω is the unit circle. Now we have to decide about the arrangement of rays. Naturally there are *p* ray combs in different directions (angles), each of them containing a group of *q* rays (which can be parallel or in a fan). So we have a net of $m = p \cdot q$ rays covering *Q*, normally uniformly distributed. The next step is to decide about the representation of rays (straight lines) L. We can use $y = ax + b$ (usual form), $\cos\varphi \cdot x + \sin\varphi \cdot y = s$ (Hesse form) or $x = x(t),\ y = y(t)$ (parameter form).

Matrix generation: Matrix *A* in (4) contains the cut lengths a_{ij} of rays L_i in pixels Q_j. Row *i* refers to ray L_i and its cut lengths in all pixels. Column *j* refers to pixel Q_j and its cut lengths with all rays. The cut lengths can be calculated analytically (using formulas) or numerically (using algorithms). The calculation itself is very elementary. But there are a lot of different algorithms. At first we can start with one ray and one pixel. The whole procedure can use this part $m \cdot n$-times (for each ray and each pixel). But this algorithm is not efficient. In the next step we can consider a ray in the whole pixel field *Q*. If a certain pixel is met by the ray we can look in the neighbourhood to find the next met pixels (ray tracing). Or we calculate the points where the line enters *Q* and leaves *Q*. All other positive cut lengths result from cut points of ray with grid lines of *Q*. Furthermore, we can select special cases, where the cut lengths are equal to the pixel lengths. Considering the whole ray family in the pixel field *Q* we can exploit symmetries if the rays are chosen suitable.

Data generation: The first possibility is to investigate natural objects (as parts of human body or some material). Then we have to use Computer tomographs or we have to obtain available data. The disadvantage is the effort and complexity of the object. A second possibility is to choose arbitrary or random measure data. The disadvantage is that we get no information whether the reconstruction is good or not. A third possibility is to create or to use artificial objects (*phantoms*). The advantage is that we can produce more or less complex objects and that we can compare originals and reconstructions. So reconstruction methods can be adjusted by training with known objects. If the quality of reconstruction is satisfactory the methods can also be used for unknown objects.

We have chosen planar phantom objects. They can be given by pictures. A scan supplies digital information about the colour or grey level distribution. Or they are given by mathematical objects $z = f(x, y)$, where the heights z represent the densities. These objects (surfaces in space) can also be transformed into planar pictures (2D contour plots, checkerboard plots). If the objects are simple enough, we get the exact data $g_i = g(L_i)$ by analytical calculation using (1). Otherwise we get approximate data $g_i \approx g(L_i)$ by numerical integration.

Object zoo: A very simple object f is a circular or an ellipsoidal cylinder (homogeneous object). We can choose also rises over ellipsoidal regions (inhomogeneous objects with densities increasing to the centre). These objects can be composed to construct suitable phantoms with certain properties (e.g. models of human body cross-sections).

Software: During the project MATLAB was used. The following programming routines were developed: a) matrix generator, b) data generator, c) ART solver, d) image representation of objects, e) graphical user interface for the CT package (GUI).

Experiments: A simple example in the project was a *double peak* consisting of two cut elliptic paraboloids. We chose $m = 15 \cdot 25 = 375$ rays and $n = 15 \cdot 15 = 225$ pixels. If $\vec{f}_a$ is the reconstruction (obtained by ART) and $\vec{f}_n$ is a grid embedding of original f, then we used

$$d = d(\vec{f}_a) = \frac{1}{n} \cdot \| \vec{f}_a - \vec{f}_n \|$$

as quality number characterizing the error within the grid. We got $d = 0.06$ for our example. In Figure 2 $\vec{f}_n$ is shown in the first row and $\vec{f}_a$ in the second.

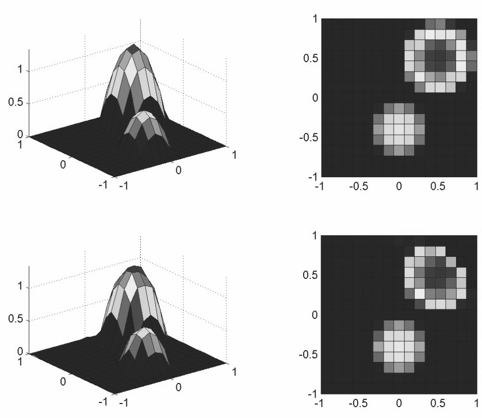

Figure 2: Double peak original and reconstruction in 3D and 2D

Many experiments changing *m*, *n* and parameters in the solver are possible.

7. Findings and Conclusions for Education

Modelling and working with models plays an important role for efficient work. So, setting up models and solving problems with models should be an essential part of engineering education. The described project realizes this aim in the framework of mathematical teaching. The work was organized in groups. The features were:

- Activation of learners (subject of practical relevance, team work, competition between groups, initiatives and creativity of students, experiments and evaluation, presentation and evaluation of results),
- Teaching modelling competencies (discussion of modelling assumptions),
- Use of Technology (software MATLAB, own routines, own user interface),
- Project based teaching (arranging contents according to the project),
- Interdisciplinary character (physics, computer science, modelling, simulation, methodology, economy can be included),
- Flexibility of the subject (adaptation to students level and time budget, distribution of tasks according to talents),
- Training of problem solving (including aspects of efficiency and costs).

Working in the project led to the following conclusions:

- Mathematics is needed everywhere.
- Complex problems often contain a lot of stumbling blocks.
- Without good modelling and without good mathematics many practical problems cannot be solved.
- Practical problems are solved by mathematical methods using computers.
- Education of engineers without solid mathematical basics is dangerous.

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The Impact of Cooperative Learning Scenarios on the Applied Mathematics Education of Geomatics Students

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Abstract

Geomatics students need a profound knowledge in mathematics to master their studies and to be prepared for their life as an engineer. The best way to realise this necessity is to apply the mathematical methods and techniques in scientifically sound projects. We offer summer camps in Spain, to work in international student teams on surveying projects in archaeological sites in Spain. The students apply laser-scanning, photogrammetry and other high tech surveying methods. The resulting data must be assessed, evaluated and visualized using the methods learned before. Because of the scientific relevance of the results, the students are highly motivated.

Cooperative Learning (CL) as a learning or teaching strategy is a part of Civic Education and successfully applied at schools and universities. The idea is that students of different levels of ability learn and act together in a group to achieve a common goal and improve thereby their understanding of a subject.

We show that the categories of CL can be used to describe our projects and help to improve the learning and teaching processes.

Introduction

The challenge of our bachelor and master degree programs is to achieve a professional qualification and employability of graduates at all levels. They have to face a global world with complex tasks and should be prepared for lifelong learning. In addition to having profound knowledge in their chosen field they must be able to apply this to new and unknown fields often in collaboration with scientists with a different cultural and scientific background.

It is practically impossible to teach every fact that could possibly be used in the future life of an engineer. The best way to overcome this problem is to provide a thorough education in maths and natural sciences such as physics.

From this point of view it is a lucky coincidence that there is a strong demand in the field of Archaeology for the skills our students are taught. In collaboration with colleagues from Spain we can define projects at archaeological sites or excavations, where Spanish and German students from Geomatics and Archaeology can share and exchange their knowledge and achieve scientifically important results.

The aim of these projects is to impart knowledge in new technologies, practical skills and the consolidation of the theory and the mathematical background of treating subjects like Practical Geodesy, Photogrammetry, Geodetical Networks and Engineering Geodesy.

In the following we briefly discuss the mathematical implications of terrestrial laser-scanning. The learning theoretical background of our projects can be described by the concept of the "Cooperative Learning Scenario". We introduce this concept in a dedicated section and then show how it applies to our projects.

The Maths behind Terrestrial Laser-Scanning

The main task for a Geomatics engineer (called surveyor in former times) is to determine the coordinates of points and objects in space. They use a (laser-) optical device called "total station" to measure angles and distances to and between single distinct points of the objects. Using this technique objects are represented by a (small) set of typical points e.g. of corners or the rooftops of buildings. In terrestrial laser-scanners (TLS) this idea is enhanced. This device sends out laser beams in a certain direction which are then reflected by the object. Measuring e.g. the light travel-time, the distance can be determined. Together with the direction, this gives the coordinates of the reflection point in the system of the scanner. Rotating the device and using mirrors the direction can change automatically so that it is possible to measure thousands of points per second. Figure 1. shows schematically the result.

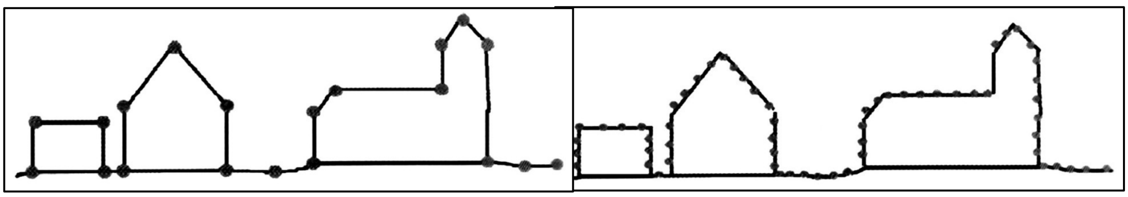

Figure 1: (a) Typical points of buildings as measured by a total station.
Figure 1: (b) Grid of points of buildings as measured by a TLS.

To look into any corner of an object and to have a full 3d-view it must be scanned from different positions. For this purpose marks or targets are laid out and scanned and must be recognized in any scan to join the data to a common coordinate system. Additionally, the marks are measured by total stations together with known points of the surroundings to embed the data into a global coordinate system. The result is a 3d-point cloud. Normally, it must be thinned out and smoothed. Connecting the points to triangles yields a surface which can be coloured or textured to get a realistic model of the object.

The first step of this measurement is the preparation and the planning as, for example, shown in Figure 2. Besides the technical skills to set up the equipment the student must have a genuine understanding of the geometrical concepts. One problem is to distribute the marks so that they do not lie in a line or on a plane or that the triangles built by the marks and the scanners do not have too small angles. The achieved accuracy of the scans would be bad if these conditions are not fulfilled and the student would realize this at home. Thus, a student must know everything about *points*, *lines*, *planes*, their mutual intersections and *triangles* in the sense of pure and *analytical geometry*. To enhance the accuracy there are normally more points and marks used than absolutely necessary. Mathematically, this leads *to overdetermined linear systems*. The student must know how to solve linear systems in the determined and overdetermined case using the *Gaussian elimination* and the *least squares* method with a background in *multivariate calculus*.

The next step is performing the measurements which are normally no problem if the preparation was done well. First data analysis is done in the field so that large errors can be avoided or measurements can be repeated.

The data must be evaluated at home. In this phase, which can take up to ten times more time than the measurements, software packages are used to make a model from the data. The student needs a lot of experience and cannot use the software as a black-box. The marks must be identified in each scan to combine the data. The raw data sets can be quite big and one needs a good understanding of the *statistics* to smooth and thin them out. Making surfaces from the point clouds is not a standard task. The software offers some possibilities but sometimes it must be helped manually. For special data sets new methods must be tried out. For this purpose students must know about graphical algorithms as *Delaunay-triangulation* or its *Voronoi* representation. Sometimes even spline- or *NURBS*-surfaces do the trick or software must be developed to detect and fit certain geometrical shapes. Mathematically, this leads to a *linearization of systems of multivariate equations* which leads in this case to an *eigenvalue* problem.

To summarize: preparing, performing and evaluating laser-scanning is a task which needs a lot of engineering mathematics as background. Besides geometrical concepts of surveying, linear algebra, multivariate calculus and statistics is needed in some depth to achieve reliable results.

Aspects of learning theory

Cooperative Learning (CL) as a learning or teaching strategy is a part of Civic Education and successfully applied at schools and universities [3]. The idea is that students of different levels of ability learn and act together in a group to achieve a common goal and thereby improve their understanding of a subject.

Usually, this is a strategy for a teacher who carefully designs the lessons and activities for teams in a classroom to teach a dedicated subject. However, our well-prepared projects, applying geodetical concepts to Archaeology in small international student teams, seem to fit the categories of CL quite well.

The students learn, practise and apply complex scientific and technical skills. Besides the pure fact-oriented view, we achieve benefits including: higher self-esteem and academic achievement, an increase in level of reasoning and development of social skills and skills in oral communication and some more [2].

To insure that CL, and therefore its benefits really occur, five conditions must be fulfilled [1]:

1. **Positive interdependence** (sink or swim together)
   In our projects small student teams receive clearly defined tasks in surveying or scanning parts of archaeological sites. Since the groups are working on real and not on toy-science, the contribution of each member becomes important. Their work must be done very carefully, because its quality is mostly assessed later, with no chance for repetition.
2. **Face-to-Face interaction** (promote each other's success)
   Although instructors are available, the teams are mostly working on their own. Advanced students must teach the other (often foreign) team members in a chosen common language. Subjects, learned in the past, must be recapitulated, discussed and successfully applied.
3. **Individual and group accountability** (no hitchhiking! no social loafing)
   As stated in condition one, the quality of the results of the work is important per se and often also important for the future work (thesis) of some group members. The group tasks are connected, so that the results are important for other groups. These constraints lead to self-control of the group with a positive effect on the engagement of the members.
4. **Interpersonal and small group skills**
   The students learn-by-doing to act inside a team. Since there are different levels of ability, we find an interchanging structure of teaching and learning. This leads to a respected (changing) hierarchy inside the group, founded by knowledge, ability or choice. The international composition of the groups helps to overcome prejudices against foreign nations or races.
5. **Group processing**
   The data are often pre-processed after work and the quality becomes obvious. Discussing the results and planning the next activities leads to a reflection and improvement of the roles of each group member.

CL summary

Obviously, CL is a suitable framework in which our student projects can be discussed from an educational point of view. Since the learning theoretical background is quite evolved, we can use the categories to reflect the success (or even sometimes the failure) of the projects. CL makes clear that interdisciplinary science cannot only be learned from books but also has to do with the interaction of human beings.

Description of the projects

Primarily, the HafenCity University Hamburg and the Polytechnic University Madrid exchanged lecturers through the European Education Program ERASMUS. After two years of cooperation, they started working in 2006 with the first common project (BARRIOS et al, 2008), two diploma thesis about the data acquisition and 3D Modelling of a castle in Villavellid, North Spain (Figure 3).

Figure 2: (left) *Preparation of a laser-scanning of an archaeological excavation in Mara, Spain.*
Figure 3: (right) *Villavellid castle, Spain*

Each student, one from each university, works first together in Madrid and later in Hamburg on the calculation, processing and creation of the 3D model of the castle. After its completion, each student elaborates his own work in different theses with different priorities.

After the good experiences and results of the initial project, both universities agreed to organize a Summer Camp to be carried out every year. The first summer camp in 2007 took place in Atapuerca, Burgos and Hellín (MAS et al. 2008a,b). Sixteen students together with four lecturers from Hamburg and Madrid attended this project. The second summer camp (2008) took place in Atapuerca, Burgos and Mara (Calatayud). Twenty students together with seven lecturers from Hamburg, Madrid and the United States attended this project. The third summer camp (2009) took place in Mara (Calatayud), where twelve students attended the project together with four lecturers from Hamburg and Madrid. The next camp starts in July in Logroño.

Each project starts with a meeting of all attendees and traditionally with a conference about “The Physical Geography of Spain”, specially designed for the foreign students, and also with a presentation of the projects. Upon arrival on the site, the students begin with a tour, conducted by the local researchers, through the excavations and a presentation of the work to be undertaken.

After the visit, the students have to organize themselves and to form working groups or teams to perform the various tasks (Figure 4a). The teams develop a working strategy to dispatch the tasks which they present to all participants. The lecturers and scientists serve as consultants in this part of the project in order to remove possible ambiguities.

Once the strategy has been set up for the project, the students begin with the data acquisition (Figure 4b). Here, they are instructed in the use of the instruments by students with these skills (Figure 5a).

Figure 4: (a)*Tour through the excavation, Mara, Spain. (b) Data acquisition with three teams*

In the evening, the backup of the day´s work has to be done through specially prepared computers.

Figure 5: (a)*Instruction by students in laser- scanning. (b)Evening presentation*

After partial completion of work (usually after two days) new teams can be formed. These groups will begin to document, evaluate and update the results. Occasionally detected errors or gaps are immediately communicated to the measuring groups, so that the work can be complemented and fully completed. Additionally, evening presentations of a high scientific standard are presented by the lecturers and scientists of the participating universities and guests from research institutes or industries (Figure 5b). At the end of the project the teams present their results (as they are) to the plenum of all other teams and instructors and open a panel discussion about the projects and the positive and negative experiences.

Conclusions

We introduced the theoretical framework of Cooperative Learning and showed that it applies to our concept of geomatic archaeological projects which we presented in some detail. Of course, we did not consider CL as an end in itself but propose to use the concept with some care to increase the success of the projects.

We point out that not only the geodesic and mathematical subjects should be practised and dealt with here. We seek the transfer of technology to other sciences, as in our particular case, archaeology. It is important to the whole process of learning and experimentation.

Other features of this cooperation are:

- Provides the possibility for the students to work in a field of science which, until now, was not in direct contact with Geomatics.
- Increases the collaboration between students from several countries in Europe and opens the understanding and perception of these.
- Opens the possibility to carry out attractive and practice oriented Bachelor and Master Theses.
- Solves problems using new technologies such as laser-scanning, GPS, GIS, fringe projection.
- Allows one to delve deeper into the history and culture of other countries and opens the horizon for free thinking.
- These kinds of projects opened the cooperation with researchers from different countries, such as Spain, USA, Germany and other countries.

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Does CAS at school help freshmen in engineering sciences?

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Abstract

We have known for a long time that students have many difficulties concerning basic skills in mathematics. Moreover during recent years students display new kinds of mathematical mistakes: they lack the recognition of patterns and structures within mathematical expressions. Examples are given. Furthermore, during recent years there has also been a change of paradigm in teaching mathematics at German schools, due to bad PISA results. The focus is now on modelling and solving problems in the "real world". CAS is often used. Even the final maths exams of secondary schools are written with the help of CAS calculators. As a consequence practice in applying rules (for differentiation, integration etc.) is missing.

The theses are:
The lack in basic skills is caused by using calculators too early.
The lack in applying rules is caused by using CAS too early and too much.

Symptoms

We have known for a long time that students have many difficulties concerning basic skills in mathematics. Moreover during recent years students display new kinds of mathematical mistakes: they lack the recognition of patterns and structures within mathematical expressions. Here are some examples: well known are the problems students have in extending an expression of the form x^2+3x to a complete a square. They have difficulties recognising the pattern and comparing with $a^2+2ab+b^2$ to find $(3/2)^2$ as the missing term. We have also known for a long time that the recognition of structures e.g. in a double fraction, is very weak.

In addition during recent years further problems concerning structure have arisen: for example, log, sin, cos are not identified as functions, they are treated as "normal" symbols. Calling the function $\sin(x)$ is understood as a multiplication like $\sin * x$. This becomes obvious in the following mistakes:

- Applying a kind of a commutative law for simplifying the expression: $\sin(x)\cos(x) + \cos(x)\sin(x) = 2\,x \sin \cos$,
- applying the product rule for differentiation: $d/dx\,(\sin(x)) = \cos(x) + \sin$.
- These newer kinds of mistakes give a different look to an old mistake: $\sin(x+y) = \sin(x) + \sin(y)$ can be interpreted as consequence of the distributive law.

Often in bridging courses at university students are not able to identify rules that are the basis when they transform terms. For examples, even when all rules for dealing with powers are projected on a screen, many students are not able to identify the rule behind $(-x)^2=x^2$. If they are forced to do this, a typical answer is: "I do not know, I am following always my gut feeling." But, they do not understand this observation. The students know in principle how to deal with mathematical terms. For example they know how to simplify *uv*+*vu*, but if the situations gets a little bit more difficult, then they get completely confused.

Reasons

If we look for the reasons behind these mistakes, we have to take into account that during recent years there has been also a change of paradigm in teaching mathematics at German schools, due to bad PISA results. The focus is now on modelling and solving problems in the “real world”. CAS is often used and CAS is a necessary tool for it. Even the final maths exams at many secondary schools in Berlin are written with the help of CAS calculators. It is the aim that all secondary schools in Berlin will do so.

As a consequence the computer does the differentiation and integration and not the student. Thus an opportunity to practise applying rules and therefore an opportunity to practise recognizing structures has gone. The reduction of practice at school has also to do with the fact that the focus of the education is on subjects that the majority of students at school will need in their future jobs. Arnold a Campo (2007), the head of the MNU (Deutscher Verein zur Förderung des mathematischen und naturwissen-schaftlichen Unterrichts e.V., Society to promote mathematics and science education), states that only a small percentage of the students at school will become an engineer or a scientist.

For a long time it has been common that normal calculators are used at school. The usage of calculators starts at least after the first six years of school, just when fractions are introduced. The consequence is a lack of basic skills like dealing with fractions and later with symbolic fractions. Today the increased use of CAS will play a similar role concerning recognition of structures.

The change of paradigm of teaching mathematics at German schools had started earlier, at the end to the eighties, in the Netherlands. Krieg, Verhulst and Walcher reported about a protest of students in the Netherlands against the low level of mathematics teaching at school. They expressed their alarm in a public letter to the minister of education, Maria van der Hoeven, signed by 10 000 students. This letter has become popular under the motto “Lieve Maria”.

We also must take into account that student’ life, and our daily life in general, has changed. There is a flood of information. Looking for solutions is now a quick and impatient online search. New devices like digital cameras, mobile phones etc. must be self-explanatory without long instructions. The young generation can be regarded as natives of our digital world, while the older generation, born in the non-digital world, are immigrants. The natives of the digital world are experts in the trial-and-error-method and in looking for answers just by one click. But they are not trained for a time-consuming, systematic and deductive acquisition of knowledge. They are not well trained to follow formal rules of either a natural language or a mathematical language. The digital world promotes more intuitive than rational competencies.

Role of practice

Elsbeth Stern is an expert in cognitive psychology at the ETH Zürich, Switzerland. Stern (2006) emphasises the role of practice. “A person, who is not experienced in reading, has to transform every letter into a phoneme and has arduously to construct a word out of it. RAM capacity is occupied that is lost for understanding the content.” [Stern]. Our engineering freshmen have comparable difficulties. They are not experienced in reading formulas, they spell the mathematical expressions. Thus RAM capacity is bound that goes lost to capture the meaning and the structure of the expression. Like reading, doing the basics in mathematics has to be automated. Automating releases RAM capacity, this is needed for the creative process of understanding and problem solving and saving new information.

“Discussions with staff led to the identification of some specific mathematics content viewed as exceptionally difficult for learning. These include spatial visualisation, algebraic manipulations, abstract notion, complex numbers and mechanics. It is noted that elements of such content depict more mathematical skills and application than knowledge.” [Mansons, Alpay, 2010]

What to do?

Primarily the students have to be forced to train their mathematical skills. As a consequence the usage of computers in the exercise lessons has to be reduced. Also a part of the written assessments should be done without the help of calculators.

Twenty years ago the Beuth University of Applied Sciences established a bridging course. It takes place in a compact form just in the last eight days before the freshmen will start their lectures. The participants form groups of about no more than forty. So a kind of individual help is possible. The problem is that eight days of exercising are not enough to substitute for fewer practices that were not developed during twelve years in school. Two further remedial actions at Beuth University have been established: an online bridging course and “L+”. The online bridging course is available for all students at all times, it is available before the beginning of, as well as during, their studies. “L +” means “Learning plus”, it is a one-to-one help system. Members of the teaching staff (retired and temporarily teachers) guide students to help themselves. L+ is offered not only for mathematics but also for other subjects like mechanics.

There are also problems other than the lack of basic skills within the mathematical education of engineers: the motivation of students and the ability of linking and recognising mathematics in the subjects of the engineers. To overcome these problems a careful setup of the curriculum and permanent time-consuming discussions among the concerned teaching stuff is needed. Also integrating project works taken from the engineering subjects into the mathematics education helps. Alpers (2006) and Diercksen (2008) gave convincing examples for mechanics and for electrical and electronic engineering, respectively.

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Qualitative methods for nonlinear differential equations – the Solow model of long time economic growth

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Abstract

Many basic mathematical subjects and procedures of great relevance for engineering practice are not included in mathematics curriculum of universities. Utilizing computers actually requires more "higher mathematics" knowledge than was necessary in the past. This fact will be demonstrated for nonlinear differential equations. Simple qualitative methods to determine the essential solution behavior of nonlinear equations are given in the paper. This will be applied to the Solow model.

Introduction

Over recent years the mathematical starting knowledge of students has decreased to a very poor standard which persists at the moment (Berger, M. and Schwenk, A. 2006, Brüning, H. 2004). It is the most important problem to overcome to ensure high mathematical standards in teaching process. (Schott, D. 2007). Future engineers need computers for calculation but there is no time to develop maths skills and the sensible use of computers. Indeed one cannot trust numerical computer results and there is a need for advanced mathematical methods for verifying the results of computing. In the following some qualitative methods for linear and nonlinear autonomous differential equations are explained briefly and applied to the Solow model. Further the stability in the sense of Lyapunov of equilibrium solutions is considered. This paper is a proposal to adapt the methods under consideration in math lectures for engineers. Until now they are not included in maths for European engineers (Mustoe, Lawson 2002).

Autonomous Differential Equations

An *autonomous system* of ordinary differential equations has the form:

$$\frac{d\boldsymbol{x}}{d\boldsymbol{t}} = \boldsymbol{F}(\boldsymbol{x}). \qquad (1)$$

For every $\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{n}}$ a velocity $\boldsymbol{F}(\boldsymbol{x}) \in \mathbb{R}^{\boldsymbol{n}}$ is given. From the vector $\boldsymbol{F}(\boldsymbol{x})$ we know how $\boldsymbol{x}$ changes. *Existence and Uniqueness* of the solution $\boldsymbol{x_1(t)}, \boldsymbol{x_2(t)}, \ldots, \boldsymbol{x_n(t)}$ can be proved by requiring the function $\boldsymbol{F}$ to be Lipschitz continuous. By uniform boundedness of the *Jacobian*

$$\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}}$$

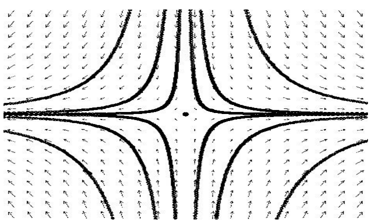
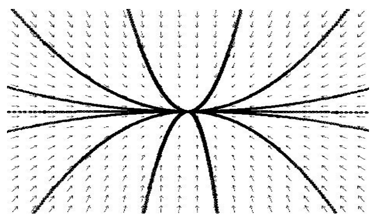
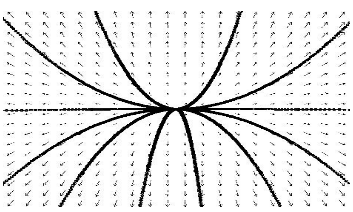
Lipschitz continuity of $\boldsymbol{F}$ is ensured. There is a maximal interval *I* where the solution is defined. We restrict ourselves to planar systems of equations having two state variables $(\boldsymbol{n = 2})$. Each system state corresponds to a point of the $(\boldsymbol{x_1}, \boldsymbol{x_2}) -$ plane called *phase plane*. If *t* varies over the maximal interval *I* the curve $(\boldsymbol{x_1(t)}, \boldsymbol{x_2(t)})$ is called a *trajectory*, *path or orbit*. The set of all trajectories is called *phase portrait*. Every trajectory is uniquely defined by an initial value $\boldsymbol{x_0} = \boldsymbol{x}(\boldsymbol{t_0}) = (\boldsymbol{x_1}(\boldsymbol{t_0}), \boldsymbol{x_2}(\boldsymbol{t_0}))$. Different trajectories will not intersect at any time. An *equilibrium or fixed point* $\boldsymbol{x_e}$ of an autonomous system is a state with zero velocity. Equivalently, every zero of *F* is a fixed point, i.e. $\boldsymbol{F}(\boldsymbol{x_e}) = \boldsymbol{0}$. The last equation is the stationary condition for the dynamics of the system. Equilibrium points may be stable or unstable. More generally, a solution $\boldsymbol{x}(\boldsymbol{t})$ to the differential equation is called *stable in the sense of Ljapunov* if for all *ε>0,* there exists a *δ>0* such that for every solution *y(t)* the implication holds

$$||\boldsymbol{x}(\boldsymbol{t_0}) - \boldsymbol{y}(\boldsymbol{t_0})|| < \delta \rightarrow ||\boldsymbol{x}(\boldsymbol{t}) - \boldsymbol{y}(\boldsymbol{t})|| < \varepsilon$$

for all t>0. It is called unstable otherwise. If *x(t)*= $\boldsymbol{x_e}$ is an equilibrium point one says that the fixed point is stable or unstable. A solution is *asymptotically stable* if it is stable and furthermore *y(t) → x(t)* as *t→∞*. An asymptotically stable solution represents the long time behavior of nearby solutions of the dynamical system. In this way stability is a local property. If all solutions converge to *x(t)* as *t→∞* independently of the initial values than *x(t)* is a *globally stable* solution. An asymptotically stable fixed point $\boldsymbol{x_e}$ is called a sink. An unstable equilibrium point is a source if all trajectories run away from the equilibrium point or a saddle if some trajectories move to $\boldsymbol{x_e}$ and some lead away from it. If a fixed point is stable but not asymptotically stable than it is called a *center.* We can classify fixed points for linear systems given by the equation

$$\frac{\boldsymbol{dx}}{\boldsymbol{dt}} = \boldsymbol{Ax}.$$

Here $\boldsymbol{A}$ is a *(2x2)*-matrix with trace τ and determinant Δ. For every *A* the solution *x(t)=0* is a fixed point. There may exist more than one equilibrium point (Δ=0). Linear systems are fully characterized by the eigenvalues and eigenvectors of *A*. From *Δ≠0* it follows that *0* is a unique equilibrium point. Suppose that *A* has two real eigenvalues $\boldsymbol{\lambda_1} < \boldsymbol{\lambda_2}$. Let *Δ≠0,* there are three cases to consider

- Saddle:  $\boldsymbol{\lambda_1} < \boldsymbol{0} < \lambda_2$;
- Sink:  $\boldsymbol{\lambda_1} < \lambda_2 < 0$;
- Source:  $\boldsymbol{0} < \boldsymbol{\lambda_1} < \lambda_2$.

The saddle and the source are unstable, the sink is globally stable. For the saddle the eigendirection of $\boldsymbol{\lambda_1}$ ($\boldsymbol{\lambda_2}$) is a so-called stable (unstable) manifold. The remaining cases (complex, multiple, zero eigenvalues) can be found in detail in Hirsch, Smale and Devaney (2004). Obviously a fixed point $\boldsymbol{x_e} = \mathbf{0}$ is asymptotically stable if all eigenvalues have negative real part. Contrary the fixed point is unstable if one eigenvalue has positive real part.

For our purposes the Hartman-Grobman theorem for the so called hyperbolic case where all eigenvalues of the Jacobian have nonzero real part suffices for the nonlinear case too (Olver 2006).

Theorem Let $\boldsymbol{x_e}$ be an equilibrium point for the first order ordinary differential equation $\frac{d\boldsymbol{x}}{dt} = \boldsymbol{F}(\boldsymbol{x})$. If all eigenvalues of the Jacobian matrix $\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}}(\boldsymbol{x_e})$ have negative real part then $\boldsymbol{x_e}$ is asymptotically stable. On the other hand, if the Jacobian $\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}}(\boldsymbol{x_e})$ has one (or more) eigenvalues with positive real part, then $\boldsymbol{x_e}$ is an unstable equilibrium point.

If the Jacobian at a fixed point has eigenvalues with zero real part the theorem provides no insight for the stability. In fact, in this case the fixed point can be stable or unstable. An answer can be found by Lyapunov's methods, in particular a *Lyapunov function V(x)*: $\mathbb{R}^n \to \mathbb{R}$ is required.

Definition A Lyapunov function for the first order autonomous system (1) is a continuous function *V(x)* that is non-increasing on all solutions *x(t)*, meaning that $V(x(t)) \leq V(x(\boldsymbol{t_0}))$ for all $t > \boldsymbol{t_0}$. A strict Lyapunov function satisfies the strict inequality $V(x(t)) < V(x(\boldsymbol{t_0}))$ for all $t > \boldsymbol{t_0}$, whenever *x(t)* is an non-equilibrium solution. If *V* is continuously differentiable it is a (strict) Lyapunov function if and only if it satisfies the inequality

$$\frac{d}{dt} \boldsymbol{V}(\boldsymbol{x}(\boldsymbol{t})) = \boldsymbol{\nabla V}(\boldsymbol{x}) \cdot \boldsymbol{F}(\boldsymbol{x})(<) \leq 0. \qquad (2)$$

Theorem Let *V(x)* be a (strict) Ljapunov function for the system (1). If $\boldsymbol{x_e}$ is a strict local minimum of *V,* then $\boldsymbol{x_e}$ is a (asymptotically stable) stable equilibrium point. On the contrary, any critical point of a strict Lyapunov function which is not a local minimum is an unstable equilibrium point.

For general nonlinear systems it can be difficult to find a Lyapunov function. The Krasovski-Lasalle theorem gives less restrictive conditions, sufficient for an equilibrium point being asymptotically stable.

In systems theory stationary points and periodic solutions are the most important characteristics of a dynamic system. In the case of planar autonomous systems the theorem of Pioncaré-Bendixson gives sufficient conditions for the existence of closed orbits (Hirsch, M.W., Smale, S. and Devaney, R. L. 2004). The theorem of Bendixson-Dulac gives a condition implying that no closed orbit exists. In the next section the theory is applied to an economical application.

The Solow-Model

We consider the *production function* F for one final good *Y=F(K,AL)*. Here *Y, A, K, L* denote output, labour productivity, capital, and labour. All these variables are functions of time. The function *F* satisfies some properties:

- *F* ∈ $\boldsymbol{C^1}$, *F* is strictly increasing and strictly concave
- *F* is an *CRS function*

The latter means if the inputs of *F* are changed by a certain proportion, then the output also changes by the same proportion (e.g. *F* is homogeneous of degree one) . A standard example is the *Cobb-Douglas* function

$$F(K,AL)=\boldsymbol{K}^{\alpha}(\boldsymbol{AL})^{1-\alpha},\ 0<\boldsymbol{\alpha}<1.$$

Solow assumed that $A(t)=A(0)\boldsymbol{e}^{\gamma t}$ and $L=L(0)\ \boldsymbol{e}^{nt}$ with exogenous rates of growth $\boldsymbol{\gamma}$ and *n*. For given *K(0)* the accumulation of capital is

$$\dot{\boldsymbol{K}} = \boldsymbol{I} - \boldsymbol{\delta K}$$

The investment *I* is assumed to be equal to the savings *sY* a constant fraction of output *Y*. Accordingly, the capital accumulation equation reads

$$\dot{\boldsymbol{K}} = \boldsymbol{sY} - \boldsymbol{\delta K} = sF(K,AL) - \boldsymbol{\delta K} = AL\ sF(K/(AL),1) - \boldsymbol{\delta K}.$$

Defining $\boldsymbol{k} = \boldsymbol{K}/(\boldsymbol{AL})$ and the intensive production function *f(k)=F(K/(AL),1)* one arrives at

$$\frac{\dot{\boldsymbol{K}}}{\boldsymbol{K}}\boldsymbol{k} = \frac{\dot{\boldsymbol{K}}}{\boldsymbol{AL}} = \boldsymbol{sf}(\boldsymbol{k}) - \ \boldsymbol{\delta K}.$$

By logarithmic differentiation we find

$$\frac{\dot{\boldsymbol{k}}}{\boldsymbol{k}} = \frac{\dot{\boldsymbol{K}}}{\boldsymbol{K}} - (\boldsymbol{n} + \ \boldsymbol{\gamma}).$$

Finally the *fundamental equation of growth* is

$$\dot{\boldsymbol{k}} = \boldsymbol{sf}(\boldsymbol{k}) - (\boldsymbol{n} + \ \boldsymbol{\gamma} + \boldsymbol{\delta})\boldsymbol{k}.$$

A fixed point $\boldsymbol{k_e}$ is given as solution of

$$\boldsymbol{sf}(\boldsymbol{k_e}\,) = (\boldsymbol{n} + \ \boldsymbol{\gamma} + \boldsymbol{\delta})\boldsymbol{k_e}.$$

For the Cobb-Douglas production function $\boldsymbol{f}(\boldsymbol{k}) = \boldsymbol{k}^{\alpha}$ one computes $\boldsymbol{k_e} = (\frac{\boldsymbol{s}}{\boldsymbol{n}+\boldsymbol{\gamma}+\boldsymbol{\delta}})^{\frac{1}{1-\alpha}}$.

We set $G(k)= \boldsymbol{sk}^{\alpha} - (\boldsymbol{n} + \ \boldsymbol{\gamma} + \boldsymbol{\delta})\boldsymbol{k}$. Assuming 0<α<1 we find $G'(\boldsymbol{k_e}) < 0$ and deduce that $\boldsymbol{k_e}$ is an asymptotically stable equilibrium point. Obviously the nontrivial steady state $\boldsymbol{k_e}$ is unique. Furthermore it can be shown using the *Inada conditions (f(0)=0, f'(0)=∞, f'(∞)=0),* for more general *f(k),* that there is also an unique nontrivial fixed point which is globally stable.

Moreover in the Cobb-Douglas case $\boldsymbol{f}(\boldsymbol{k}) = \boldsymbol{k}^{\alpha}$ the fundamental equation of growth is a Bernoulli equation

$$\dot{\boldsymbol{k}} = \boldsymbol{sk}^{\alpha} - (\boldsymbol{n} + \ \boldsymbol{\gamma} + \boldsymbol{\delta})\boldsymbol{k}.$$

It has a closed solution easily computable by hand. Formally a strict Lyapunov function for this one-dimensional equation is

$$\boldsymbol{V}(\boldsymbol{k}) = -\frac{\boldsymbol{s}}{\boldsymbol{1}+\boldsymbol{\alpha}}\boldsymbol{k}^{\alpha+1} + \frac{\boldsymbol{n}+\boldsymbol{\gamma}+\boldsymbol{\delta}}{\boldsymbol{2}}\boldsymbol{k}^{2}.$$

This can be deduced from (2) by

$$V'(k)\dot{k} = -(sk^{\alpha} - (n + \gamma + \delta)k)^2 < 0.$$

Thereby we conclude that the fixed point is asymptotically stable and also globally stable.

The Solow-Model with logistic manpower

The exponential growth $L=L(0)e^{nt}$ of Labour is unrealistic. It is replaced by logistical growth. Now for $L>0$ one has

$$\frac{\dot{L}}{L} = a - bL, \quad a>0,\ b>0.$$

This modification seems to be marginal but it makes the model much more uncomfortable. If we use the Cobb-Douglas production function as before we find a (decoupled) system of differential equations:

$$\dot{k} = sk^{\alpha} - (a - bL + \gamma + \delta)k \qquad (3)$$
$$\dot{L} = (a - bL)L\,. \qquad (4)$$

If $k(0)$ and $L(0)$ are known there exists an unique solution of the system. Furthermore the fixed point is

$$(k_e, L_e) = \left(\left(\frac{s}{\gamma+\delta}\right)^{\frac{1}{1-\alpha}}, \frac{a}{b}\right)$$

The Jacobian at the fixed point reads

$$J(k_e, L_e) = \begin{pmatrix} -(1-\alpha)(\gamma+\delta) & bk_e \\ 0 & -a \end{pmatrix}.$$

Because the eigenvalues $-(1-\alpha)(\gamma+\delta)$ and $-a$ are both negative the fixed point is an asymptotically stable sink. In this model there exists no closed orbit. Proving this we multiply $\dot{k}$ and $\dot{L}$ by $\frac{1}{kL}$ and differentiate

$$\nabla \cdot \begin{pmatrix} \frac{\dot{k}}{kL} \\ \frac{\dot{L}}{kL} \end{pmatrix} = \nabla \cdot \begin{pmatrix} \frac{sk^{\alpha-1}-(a-bL+\gamma+\delta)}{L} \\ \frac{(a-bL)}{k} \end{pmatrix} = -s(1-\alpha)\frac{k^{\alpha-1}}{L} - \frac{b}{k} < 0.$$

From the theorem of Bendixson-Dulac it now follows that there doesn't exist periodic solutions.

The solution of the logistic equation $\dot{L} = (a - bL)L,\ L(0) = L_0$ is given as $(t) = \frac{aL_0a^{at}}{a+bL_0(e^{at}-1)}$. The growth rate $n(t)$ of L is $n(t) = \frac{\dot{L}}{L} = \frac{a(a-bL)L_0}{a+bL_0(e^{at}-1)}$.

It can be inserted instead of $(a-bL)$ into the first equation of the system (3), (4). The second equation is solved and the system reduces to an equation which is not autonomous

$$\dot{k} = sk^{\alpha} - (n(t) + \gamma + \delta)k.$$

The solution of this equation can be given using Hypergeometric functions (Brida 2005).

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Creating Cognitive Connections in Mathematics

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Abstract

This paper presents a few ideas on how the dynamic multiplatform mathematical software GeoGebra can be used for creating cognitive connections in learning mathematics. GeoGebra enables production of self-standing dynamic worksheets as interactive Java applets for presentation of dynamic constructions and interactive calculations on Internet.

Introduction

Research in cognitive psychology indicates that our brains store knowledge using both words and images. Teaching that targets and engages both of these systems of representation has been shown to significantly increase students' comprehension and retention. Explicitly engaging students in the creation and usage of non-linguistic representations has even been shown to stimulate and increase activity in the brain. This leads to creating cognitive connections that reinforce the knowledge and understanding of basic concepts. Manipulatives are concrete or symbolic artefacts that students interact with while learning new topics. They are powerful instructional aids because they enable active, hands-on exploration of abstract concepts. Research has shown that computer-based manipulatives are even more effective than ones involving physical objects, in part because they can dynamically link together multiple representations of the concepts beibng investigated. At the beginning of the 3rd millennium we are facing a dramatic change in the basic nature of teaching and learning strategies caused by the massive usage of new technology. We can benefit from this development in general, and in mathematics especially, as currently available dynamic and visual learning environments could affect our perspectives in terms of the content and comprehension of mathematics education. Extensive usage of new ICT has enlarged the pool of cognitive tools and possibilities of their application in teaching and learning processes.

Cognitive tools have their irreplaceable position in the didactics and educational theory and there exist many ideas on how we can understand their role in education. These allow users to explore mathematical concepts dynamically and have been increasingly discussed recently. Historical overview of symbolization in mathematics presented in [1] can be considered as the evolution of cognitive tools. Five stages of the evolution of symbolization are identified *"from static, inert inscriptions to dynamic objects or diagrams that are constructible, manipulable and interactive"*. The "static, inert stage" of symbolization in mathematics consists of a wide spectrum of textbooks and handouts prepared by teachers for ages and still widely used. The second stage denoted as "static kinaesthetic / aesthetic stage", relates to the use of chalk and marker pens on black-green-white boards and slides with overhead projectors and allows users to erase and change the inscription. Besides the flexibility to make changes, these tools also provide opportunities to use several colours to emphasize particular notations. In the third "static computational stage" users get a static presentation of the input they provide to certain devices such as calculators and graphing tools. This type of interaction with cognitive tools gives users more opportunities to deal with high order thinking tasks if they leave low level cognitive activities to the tools, [2]. The last two stages of this evolution framework focus on the dynamic perspectives of symbolization. In the fourth, “discrete dynamic stage” discrete co-action between user and environment is described. Users have more control over the process and the output obtained from the media, they can change and manipulate input dynamically through the first interactive tools that have appeared recently in CAS. In the fifth “continuous dynamic stage” users can interact with the real objects through dynamic interactive devices and so they can get instantaneous and continuous feedback.

The dynamic worksheets prepared in GeoGebra enable users to create dynamic mathematical objects such as graphs of functions and to interact with these objects. Assuming that these mathematical objects are real objects of this platform, although the platform itself is virtual in nature, then the continuous interaction between these objects and users could be considered as a continuous dynamic interaction. Similarly, GeoGebra can be considered as a haptic device detecting the movements of the slider and adjustable objects described in this platform.

GeoGebra Features and Usage

GeoGebra is an intuitively controllable application suitable for all users without any extra mastery in computer literacy. It can serve for development of instructional materials in mathematics in many different forms, types and styles, and for all levels of mathematical education. It is downloadable from the website www.geogebra.org, while nothing but a Java 6 platform is necessary for its full operation. GeoGebra is particularly easy and intuitive to learn. Files can be saved in “ggb” format, or as dynamic web pages. GeoGebra can output files as pictures (png) or as encapsulated postscript for publication of quality illustrations. GeoGebra user interface offers a rich graphics menu for drawing various objects, while the complete construction protocol is saved and it appears in language chosen from the 45 available, therefore no translation is necessary and free sharing of developed instructional materials is genuinely supported all over the world. Users are encouraged to visit GeoGebra webpage and GeoGebra user’s forum GeoGebraWiki, a free pool of teaching materials for this dynamic mathematics software, where everyone can contribute and upload materials, see [3].

GeoGebra has a built-in Cartesian coordinate system, and accepts both geometric commands (drawing a line through 2 given points, a conic section determined in different ways, in Fig 1. by 5 points) and algebraic ones (drawing curves with given parametric equations). Among its more interesting features is the ability to draw tangent lines to algebraic and even transcendental curves at given points, while equations of these tangent lines are available immediately too. Dual representation of objects, geometric–synthetic and algebraic–analytic, is the great advantage of GeoGebra software that mostly suits the didactic aims of full comprehension of basic mathematical concepts. One can enter the objects either as geometric objects (via drop-down menus) or as algebraic objects – pairs of coordinates, functions – via the entry line. Moving the objects in the Geometry window (see Fig. 1) changes the expressions in the Algebra window accordingly (Fig. 1, on the left), and vice-versa; editing the expressions in the Algebra Window results in the respective change in the Geometry Window. This main feature of GeoGebra is meeting demands of many didactics and educators to provide for their students as many representation forms as possible.

Teachers can guide students to understand how these two representations of mathematical objects are connected to each other. Changes in the superposition of the conic section determining points in Fig. 1 are reflected in the form of its equation. Following these changes in the algebraic window students can realise the role of the coefficients of the bi-quadratic form. Existence of the tangent line to the conic section passing through the chosen point can be investigated by changing its position, while equation of the tangent line appears immediately in the updated form. In this way, students are engaged to study relations of mathematical objects in more complex form, in both ways, geometric–synthetic and algebraic–analytic, thus receiving comprehensive knowledge. Having developed an understanding of the relation between a particular type of determined conic section and its equation, and tangent line to a conic section and its equation, students have opportunities to create cognitive connections between their previous knowledge and the outcomes of the current exploration. They can be engaged in asking what-if questions, as: What if two of the conic section determining points coincide? What do I have to change in the superposition of 5 determining points in order to obtain ellipse, parabola, or hyperbola? What if I move point R to be the point on the conic section or to be its interior point? Many various questions that can be created by teachers and students are leading to the heuristic learning approach, hands-on exploration of abstract concepts and development of cognitive connections. In this sense GeoGebra represents example of a cognitive tool of the fifth, continuous dynamic stage, as users are really interacting with real objects through dynamic interactive devices and they receive instantaneous and continuous feedback in two simultaneous forms.

Students can work directly with suitable applications for demonstration of particular mathematical concepts prepared by teachers as supplementary materials. They can also be encouraged to create their own applications, developing thus a deeper insight and comprehensive cognitive connections, and cultivating their creativity and imagination. They can contribute in this way to development of variegated instructional materials and worksheets for overall usage and share in the classroom. Consequences of this approach are extremely important and significant, as usage of dynamic cognitive tools influences the former passive role of students in the process of education. Now they become actively involved in the process, acting not only as self-learners, but also in the role of demonstrators for all other students in the classroom.

Geometric functions and tools as functions

GeoGebra separates mathematical objects into free objects and dependent objects. Where the dependent objects are defined by an explicit construction (algebraic or geometric) the construction steps can be encapsulated into a tool. Once the tool has been defined a new button appears on the tool bar and a corresponding function name is available to the user. Tools are essentially geometrical functions, and may operate with geometric objects such as circles, lines and points. Using these tools it is possible to extend the software in natural way, just as mathematical domains are extended during educational process. The idea of a tool as a function appears in [4], and it represents key mathematical processes of compression and extension. Compression or encapsulation is the process by which mathematicians take a complex procedure or construction and represent it by a single step. An example of compression is arithmetic multiplication: repeated addition is compressed to a single step. Geometrical example of compression is a theorem of Euclidean geometry that given three points which are not-collinear a unique circle can be determined through them. This multi-step construction, which requires drawing perpendicular bisectors to line segments, can be encapsulated into a single operation, while GeoGebra provides a button to do so. For mathematicians it is interesting to notice that this compression construction process results in a well-defined geometrical function: take any three points and return a circle or a straight line. Users are allowed to create their own tools as macro-constructions, while buttons for calling these geometric functions are immediately available in the basic GeoGebra menu. This is one of the features that are convenient in the view of customers, who are free in customisation of the software interface menus according to their needs.

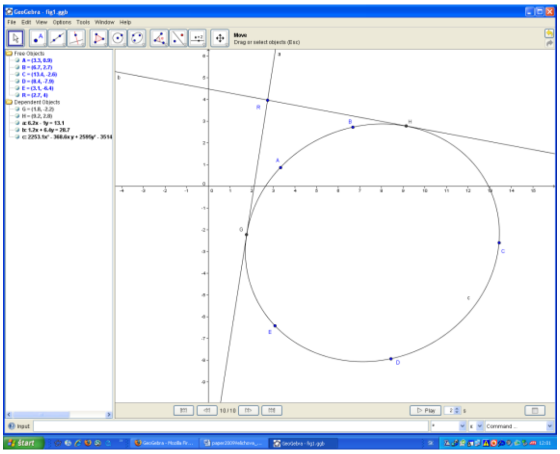

Fig. 1 GeoGebra user interface

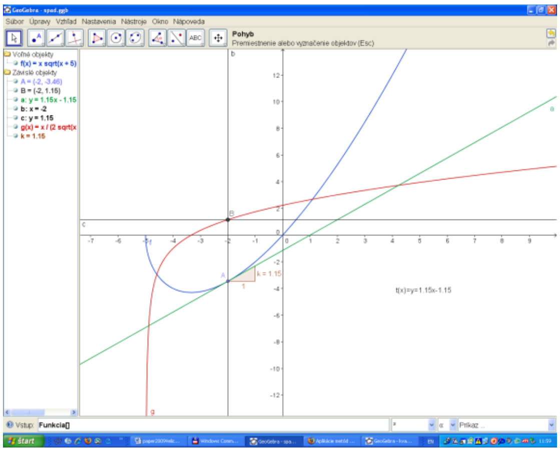

Fig. 2 Function derivative illustration

Dynamic constructions are one of the most powerful features of GeoGebra. Users are able to create dynamic worksheets demonstrating certain properties and features generally applicable to specific basic objects and dynamically change the form of these objects. Derivative of a function and its geometric meaning is a basic concept from the mathematical analysis, which is very often assued to be rather easy to understand and visualise. Dynamic illustration (Fig. 2) of the slope of the tangent line to the graph of a function of one real variable, changing according to the value of the function first derivative in the respective point, is rather crucial for the overall understanding of the function behaviour. It is easy to illustrate intervals on which function is defined and to estimate intervals, where it is increasing (positive slope) or decreasing (negative slope), and to find points, in which the first derivative equals to zero, therefore function is reaching the extremes there, its maximum or minimum. Consequently, a tangent line at an extreme point appears as parallel to the coordinate axis *x*. Also points in which the function's first derivative is not defined can be illustrated, as the slope of the tangent line to the function graph at such a point goes to infinity and the tangent line is perpendicular to axis *x*. Concepts of the global extremes and maximal and minimal values of a function of one variable on the given interval can be investigated clearly and dynamically as well.

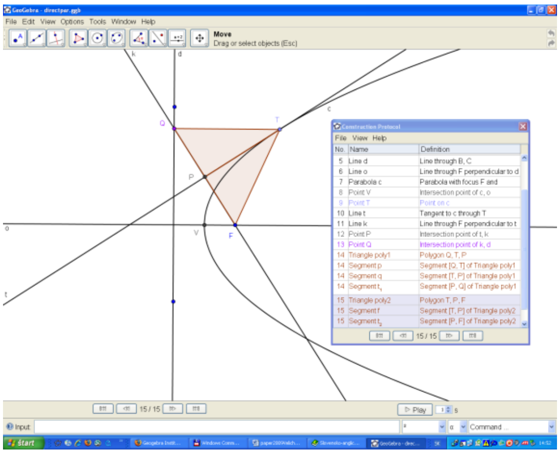

Fig. 3 GeoGebra dynamic construction

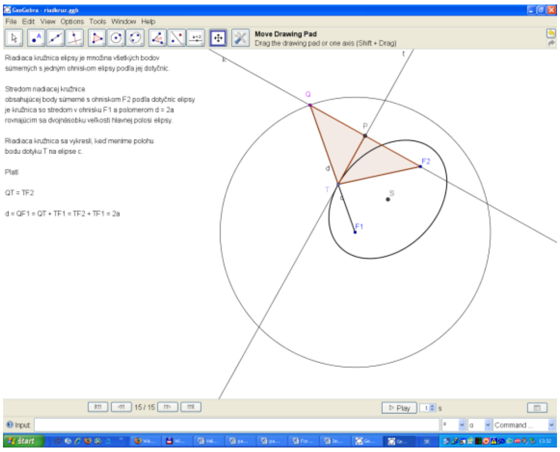

Fig. 4 GeoGebra dynamic applet

Properties of tangent lines to parabola and position of points symmetric to the parabola's focus with respect to all parabola tangent lines are illustrated in Fig. 3. Construction is stored in a construction protocol, available on request from the view menu, and can be followed step by step, or re-played via the navigation bar for construction steps in step-by-step mode, while also automatic play is available with the customisable timing.

Applications for on-line usage

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Apart from the standalone application, GeoGebra also allows the creation of interactive web pages with embedded Java applets. These targeted learning and demonstration environments are freely shared by mathematics educators on collaborative online platforms like the GeoGebraWiki. The number of GeoGebra website visitors coming from over 180 countries has reached over 300,000 per month. The International GeoGebra institute has been established, coordinating the work of thousands of volunteers all over the world in the structure of accredited national GeoGebra institutes in different countries. GeoGebra dynamic spreadsheets enable the production of interactive presentations directly on the web serving as instructional resources for e-learning solutions in the form of dynamic cognitive tools. These html pages can be used directly from Internet and are viewable in all common web-browsers, while there is no need to install GeoGebra software in the user's computer.

Fig. 4 is from the database of e-learning materials at the GeoGebra Institute of Slovakia, webpage geogebra.ssgg.sk. A collection of similar dynamic instructional materials is available also in the European Virtual Laboratory of Mathematics, from the EVLM Central Portal at the address www.evlm.stuba.sk. Many of applets in this database enable users to act interactively and to directly manipulate received results using sliders.

The scope and position of the sliders are determined by applet authors but might be kept free; therefore users can move them freely in the drawing worksheet. These enable a smooth change of shaping parameters and can be set to automatic change, creating thus a real animation available on web. Manipulated by users themselves, sliders serve for exploring relations between different parameters of an investigated object and finding their role in determination of the shape of its representation and visualisation as geometric figure, analytic expression, equation, formula, or any logic relation and statistical table.

Conclusions

Concerning the development of software GeoGebra, authors are working on a highly customizable new interface where users can easily change topics, e. g. from "geometry" to "statistics", and re-arrange different parts of the screen using drag and drop. A team of people is also working on introducing 3D geometry into the GeoGebra, in the similar dynamic mode as for the case of 2D. In the future we can expect to have a universal tool for maths education combining Dynamic Geometry, Computer Algebra, Statistics, semantics of mathematical formulas, dynamic 2D drawing worksheet and 3D Dynamic Geometry that will hopefully keep its simplicity and user friendliness.

Finally, it should be mentioned that simple drawing of mathematical objects and figures is not enough for the building of a comprehensive understanding of basic mathematical concepts. On the other hand, creative dynamic activities are essential also to the development of one's technological content knowledge. This is consistent with the general notion of mathematical understanding as one's growing competence to navigate through various representations of mathematical concepts illustrated dynamically. GeoGebra seems to be a didactic tool supporting these efforts to a high level, and in an easy, natural and user friendly way. These characteristics do predetermine this free-ware application to be used in teaching, learning and exploring mathematics into such a large extent as possible, for the benefits and satisfaction of both teachers and students.

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VLE: Virtual Learning Environment for Statistics

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Abstract

The VLE developed at the Department of Statistics and Modelling Science of the University of Strathclyde, UK, and at Chalmers University, Sweden, provides students with a unified teaching environment containing randomly generated questions (quizzes) on every subject of the course syllabus, extensive system of hints and answers, links to the class text and other support materials: statistical tables, demonstrations, help files. Since it is web-based, the VLE is accessible from anywhere anytime, it does not depend on the operating system used, it has infinite number of variations of the study questions, so it provides students with valuable resource to practise beyond the assisted lab times. The VLE also provides class organises with extensive tools for monitoring each student's activity on the VLE allowing for immediate intervention with individualised help should there be a need. It is also used for tests and examination: the students by the end of the course become very familiar with its environment and this helps to ease the examination stress. Since marking is done automatically, VLE saves a huge amount of staff time leaving them to scrutinise only boundary case submissions and appeals.

Introduction

Chalmers University of Technology is reputed for the level of its graduates in Engineering. However, modern technological challenges and ubiquitous use of computers call for changes in the way Probability and Statistics (and in fact most subjects in engineering educations) are taught at the university. Scientists working in Cognitive Psychology recently noted that modern students learn much differently than it used to be a few decades ago, see, e.g. Sinclair *et al.* (2006). A term `Nintendo Syndrome' has been dubbed to describe the approach typical for video-games generation: do not read manuals, but go and try. If ”killed”, try something else until you get to the next level. This attitude now clearly shows in studying: it becomes increasingly hard to make students read books. At the same time providing them with ways to try before reading proves educationally rewarding: after being ”killed” a few times most of the students unavoidably come to a book and actually learn deeper.

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Cornerstones in the reformed mathematics education are, see also Enelund and Larsson (2006),

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• Emphasis on the full view of problem solving, i.e., set up the mathematical model, formulate the equations, solve the equations and visualize the solution to asses the correctness of the model and the solution

• Computer assignments and exercises from engineering and physics

• Joint computer assignments between math courses and mechanics courses taught in parallel

The Virtual Learning Environment (VLE) for Probability and Statistics perfectly fits this paradigm providing students with a unified web-based framework for self-teaching. It has being developed since 2005 at the Statistics and Modelling Science department of the University of Strathclyde, UK and since 2009 also at Chalmers University of Technology. At Chalmers, the entire course of Probability and Statistics for 3rd year Mechanical Engineer students was ported to use VLE and the performance results of 208 students as well as the students evaluations are very promising. The students strongly believe that it is natural to use the computer to solve probability and statistics problem.   Moreover, they sincerely appreciate to work in the VLE and think that it facilitates their learning process.

VLE Approach

The novel approach consists in shifting the weight from formal lectures and towards self-practise and self-studying with the help of the VLE also aided with statistical routines of Matlab computer package (or of Excel or Minitab). The VLE teaching environment contains randomly generated questions (quizzes) on every subject of the course curriculum. Currently, over 700 quizzes to be used with randomly generated data are programmed and their number is growing. They cover a wide range of themes including elementary probability: events and operations on these, Full Probability and Bayes formulae, random variables and their characteristics, standard distributions and limit theorems. Basic statistics covers graphical analysis and main numeric characteristics of data, sampling, estimates: point and interval, confidence intervals, hypotheses testing (Z-, t-, F- for means and variance), two-sample tests, regression for bi-variate and multivariate data.

The VLE provides an extensive system of hints and answers to each study question, direct links to the course Study Guide and other support materials: statistical tables, demonstrations, help files. A snapshot of a web-browser running VLE is shown in Figure 1 below.

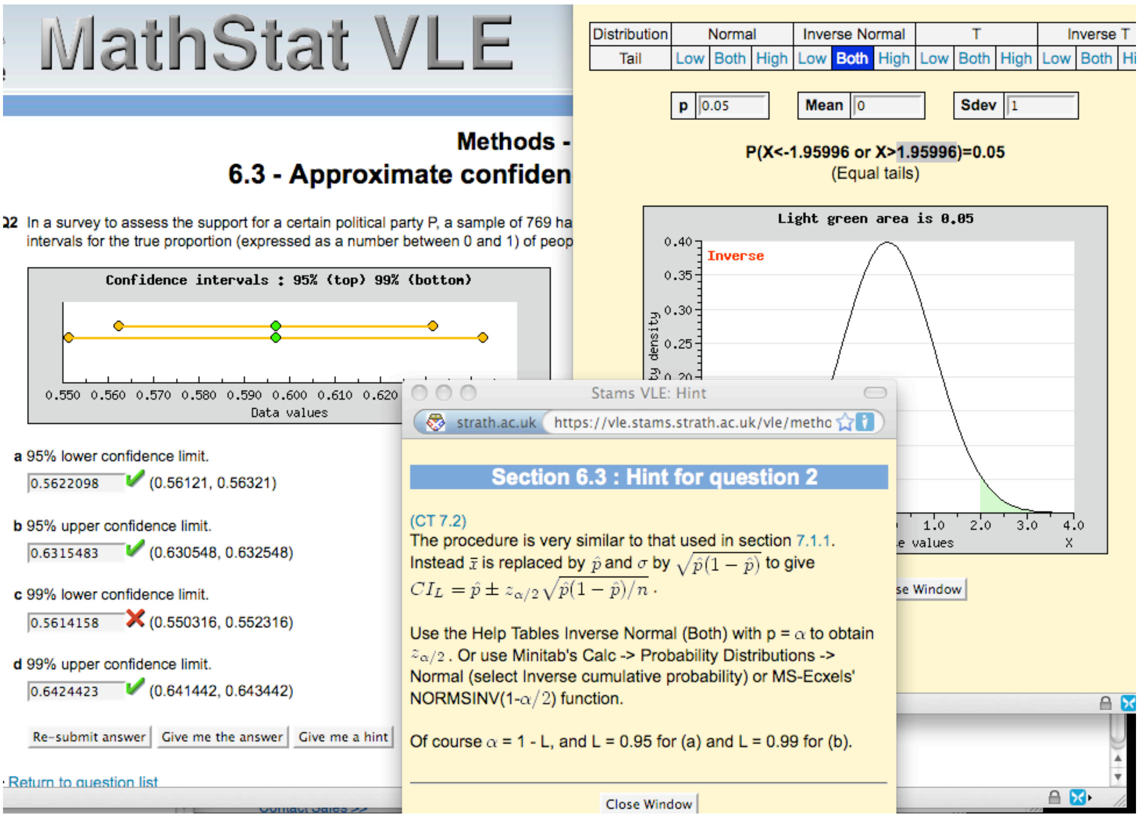

Figure 1. Student study session.

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Structure of a Typical VLE-based Course

Currently the VLE provides easy ways to set up a new course with the following components.

Methods is the main study session where students find questions categorised by themes and corresponding weeks. Each call to a question generates a new instance with new data, so that a student can try a question as many times as necessary to get consistently correct answers. Each question contains a hint with a direct link to the corresponding chapter of the Study Guide covering the theory and also a possibility to see the answers as the last resort. To provide students with assistance, Study sessions are usually organized with the teaching staff available to answer all possible problems. By our experience, a class of 20-30 students is usually well covered by just one demonstrator.

Projects is a specific task for students to work in a small group on a real world engineering problem, e.g., prediction of probability for crack growth and failure of a railway structure. It usually assumes writing a report which is then assessed by teaching staff. The VLE provides means to assign students to a group automatically as well as re-shuffling and moving students between the groups later.

Test sessions are usually organised during the teaching period and represents the same type of questions but carrying a weight in the total test mark and deprived of hints and other help systems. The marking is done by the system with the answers' accuracy pre-defined. No staff intervention is necessary apart from the overall surveyance on the test environment: no noise, no talking, etc. The system can be set in such a way that only users of the computers installed in a certain location in a certain time can access the test.

Examination is basically the same as a test session above with additional control on the timing: a test can be opened to the students for the whole week, for example, while examination session has much stricted time frame.

Help system provides students with various support materials: the Study Guide available as a whole or by chapters, electronic statistical tables, statistical demos, links to external data sets, computer software help.

Administrative tools. A great feature of the VLE is that it also provides the class organises with extensive tools for monitoring each student's activity on the VLE. Each question tried can be examined for an error which allows for immediate intervention with individualised help should there be a need, see an activity log on Figure 2.

Figure 2. VLE administrator view.

Apart from monitoring tools including mass email posting, various diagnostic views and building of surveys, the VLE contains tools for setting a new course: extensive timetabling capabilities, flexible structure and, most important, a new question builder. A new question can be added by copying and updating similar existing ones (context and keyword searches are provided for selecting questions from the bank) or by programming a completely new question from scratch using a standard template. Basically, when building a new question one has to figure out the range of plausible values for its data so that the question is meaningful whatever realisation of random data is generated.

VLE Internals

The VLE and WWW systems are hosted at Chalmers by a virtual machine running Enterprise Linux system on the university mainframe. It provides the Apache web server, the PHP server-side scripting language, MySQL database and Statistical Computing System R, see R Development Core Team (2010). In addition, various database administration tasks are accomplished using the phpMyAdmin utility. Minimal requirements are not high: 2.4Ghz processor, 1GB RAM and 12 Gb of disk space (RAID-1 preferably for production system).

The VLE is written in php which provides the web-interface and interconnection with the class data organised as a MySQL database. Many study questions are also realised in php for which an extensive library of basic statistics functions and graphics was written. Lately, the VLE has an integral support of R Statistical package so now R-routines can be directly included in the question code including graphics produced by R. This opens widest possibilities for developing even advanced statistical courses: almost any numeric procedure that R is capable of can now be put in a form of a VLE question for students to practise. The latest example is multiple regression questions that the students at Chalmers learn with the help of Matlab. These questions are actually programmed with R which mimicks the corresponding Matlab's procedures: regress and stepwise. Rserve package is used for interaction between php and R.

Learning with the VLE

Six years of experience of using VLE at Strathclyde and lately at Chalmers University of Technology proves great potential of the VLE for teaching Statistics. First and foremost, it meets the challenges with changing learning attitudes outlined above by providing the students with a great tool to learn by practise. It gives them ways to practise as much as they feel like to become confident with the topic. As results show, a typical student at Chalmers during 7 weeks of the classes answered on average 150-200 questions! It is hardly possible to achieve in a Statistics class run in a traditional way. No wonder the results shown by the students are much higher than expected from previous years.

Since the VLE uses web-interface, it is available to the registered students 24 hours from anywhere. This is not only a great helper to special needs students who now have all the time to quietly learn at their own pace, but also for ordinary students when they cannot attend assisted classes due to illness or other reasons.

The students also enjoyed the timely help and attention when the class organiser emailed some students about their alarming performance: this is certainly not what they usually expect in a big class of over 200 students.

Here is just a few quotes from emails of students who took Statistics VLE course at Chalmers this year:

```
 On 22 Apr 2010, at 18:31, a male student wrote:
First of all I'd like to say that I really like the VLE-system, I
am retaking this year's course since I failed it last year, and I
must say that this year's course is multiple times better than last
year's in every single way, partially due to the VLE-system but not
only, I also think that the study guide is very good!

On 22 Apr 2010, at 14:05, a male student wrote:
We students love your humble approach and everyone I've spoken with
is more than positive to the course as a whole in your regime. I am
engaged in education questions in the local student union at
mechanical engineering and I don't think that I've ever heard so
many positive reviews when introducing a new concept. Keep up the
good work!

On 22 Apr 2010, at 10:22, a male student wrote:
First and foremost, thank you for a good course-layout! I really
like VLE and how it eases the studying of this subject which was
previously known to be pretty hard.


On 20 May 2010, at 12:09, a female student wrote:
Even if we are test pilots for this course I must say it's one of
the most well-organized courses I've attended at Chalmers! Well
done Sergei!


On 30 May 2010, at 12:11, a male student wrote:
Thank you for a great course! This has been a great way of learning
statistics.
```

Future Prospects

Although the VLE can already be considered a solid mature system, there is still a numerous ways to develop it further. First of all, making it more modular will ease its installation at other universities and adaptation to their specific environment. Re-shaping the VLE development as an open source project will also necessitate modular approach and would provide a new huge step. Rather challenging task is internationalisation and the next year or two we should be able to see its first results.

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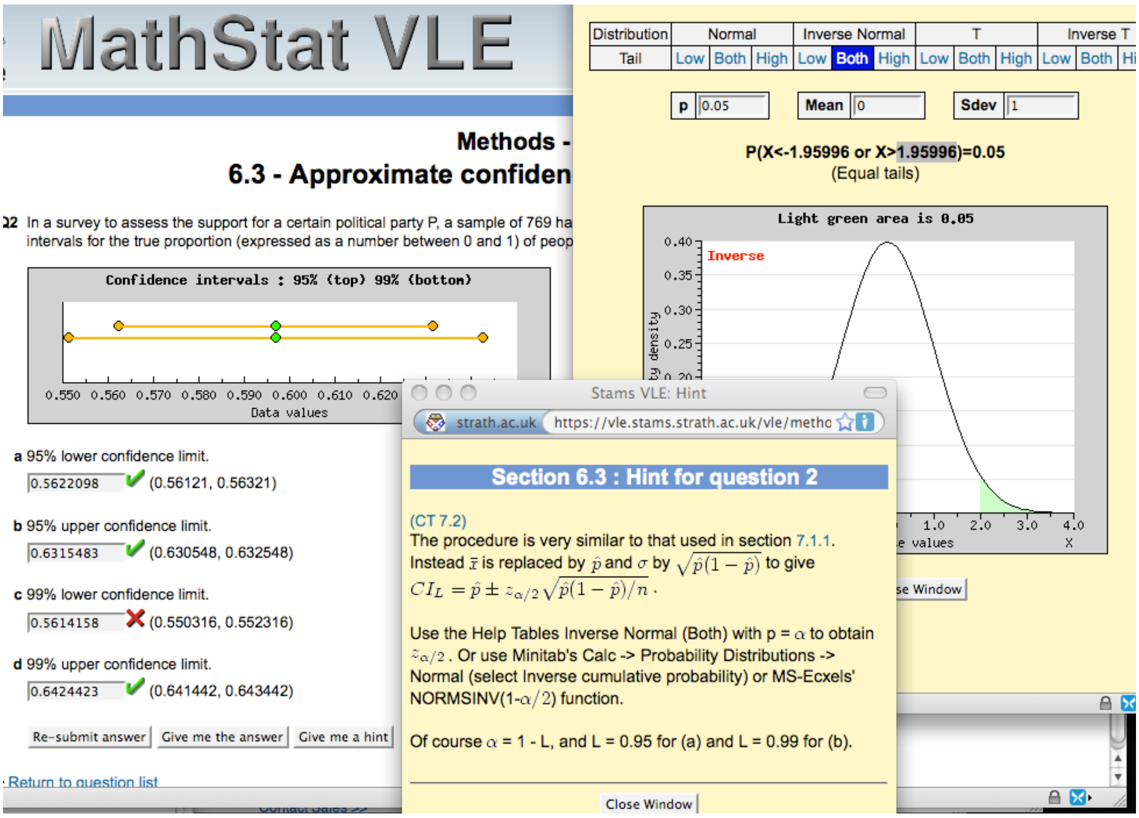

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Student		S1	S2	S3	S4	S5	S6	S7	T1	T2	T3
		29/32	32/33	31/33	19/23	29/30	5/10		79	58	40
		36/42	38/45	13/18	16/28	12/16	24/27	14/14	94	40	82
		42/42	41/42	29/29	14/18	8/10	38/38	26/28	100	49	79
		33/33	47/47	41/42	38/39	25/25	16/17	26/29	88	55	61
		30/30	39/40	40/41	39/40	24/24	11/24	14/16	77	85	95
		37/38	49/49	39/39	14/15	12/12	34/34	28/28	88	73	92
		26/27	21/23	15/16	13/14	8/12	17/20	13/16	64	63	70
		41/41	55/56	72/72	28/30	22/22	35/35	17/18	80	100	83
		37/37	46/46	44/44	43/43	29/29			88	70	
		31/31	23/27	14/16	40/41	26/26	40/41	31/31	89	67	57
		109/109	99/99	55/55	42/43	29/29	19/20	6/10	100	67	55
		91/91	30/31	24/24	29/30	12/12	29/30	20/22	80	68	87
		50/53	35/38	26/28	27/28	20/21	17/22	14/19	100	63	69
		31/31	35/35	30/30	30/30	24/24	20/20	16/16	80	81	95
		41/41	50/50	48/48	66/67	37/37	37/37	40/41	100	100	99
		71/72	92/93	52/52	34/34	21/23	22/30	15/21	82	53	32
		31/41	16/32	20/45	31/34	24/24	1/3		80	69	
		26/26	51/54	30/30	36/39	27/27	40/42	33/35	80	57	75
		37/37	30/33	36/40	28/31	34/37	21/23	13/14	88	60	82

Figure 2. VLE administrator view.

Apart from monitoring tools including mass email posting, various diagnostic views and building of surveys, the VLE contains tools for setting a new course: extensive timetabling capabilities, flexible structure and, most important, a new question builder. A new question can be added by copying and updating similar existing ones (context and keyword searches are provided for selecting questions from the bank) or by programming a completely new question from scratch using a standard template. Basically, when building a new question one has to figure out the range of plausible values for its data so that the question is meaningful whatever realisation of random data is generated.

VLE Internals

The VLE and WWW systems are hosted at Chalmers by a virtual machine running Enterprise Linux system on the university mainframe. It provides the Apache web server, the PHP server-side scripting language, MySQL database and Statistical Computing System R, see R Development Core Team (2010). In addition, various database administration tasks are accomplished using the phpMyAdmin utility. Minimal requirements are not high: 2.4Ghz processor, 1GB RAM and 12 Gb of disk space (RAID-1 preferably for production system).

The VLE is written in php which provides the web-interface and interconnection with the class data organised as a MySQL database. Many study questions are also realised in php for which an extensive library of basic statistics functions and graphics was written. Lately, the VLE has an integral support of R Statistical package so now R-routines can be directly included in the question code including graphics produced by R. This opens widest possibilities for developing even advanced statistical courses: almost any numeric procedure that R is capable of can now be put in a form of a VLE question for students to practise. The latest example is multiple regression questions that the students at Chalmers learn with the help of Matlab. These questions are actually programmed with R which mimicks the corresponding Matlab's procedures: regress and stepwise. Rserve package is used for interaction between php and R.

Learning with the VLE

Six years of experience of using VLE at Strathclyde and lately at Chalmers University of Technology proves great potential of the VLE for teaching Statistics. First and foremost, it meets the challenges with changing learning attitudes outlined above by providing the students with a great tool to learn by practise. It gives them ways to practise as much as they feel like to become confident with the topic. As results show, a typical student at Chalmers during 7 weeks of the classes answered on average 150-200 questions! It is hardly possible to achieve in a Statistics class run in a traditional way. No wonder the results shown by the students are much higher than expected from previous years.

Since the VLE uses web-interface, it is available to the registered students 24 hours from anywhere. This is not only a great helper to special needs students who now have all the time to quietly learn at their own pace, but also for ordinary students when they cannot attend assisted classes due to illness or other reasons.

The students also enjoyed the timely help and attention when the class organiser emailed some students about their alarming performance: this is certainly not what they usually expect in a big class of over 200 students.

Here is just a few quotes from emails of students who took Statistics VLE course at Chalmers this year:

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 On 22 Apr 2010, at 18:31, Mattias Mannegren wrote:
First of all I'd like to say that I really like the VLE-system, I
am retaking this year's course since I failed it last year, and I
must say that this year's course is multiple times better than last
year's in every single way, partially due to the VLE-system but not
only, I also think that the study guide is very good!

On 22 Apr 2010, at 14:05, Martin Jonsson wrote:
We students love your humble approach and everyone I've spoken with
is more than positive to the course as a whole in your regime. I am
engaged in education questions in the local student union at
mechanical engineering and I don't think that I've ever heard so
many positive reviews when introducing a new concept. Keep up the
good work!

On 22 Apr 2010, at 10:22, August von Sydow wrote:
First and foremost, thank you for a good course-layout! I really
like VLE and how it eases the studying of this subject which was
previously known to be pretty hard.


On 20 May 2010, at 12:09, Ellinor Svensson wrote:
Even if we are test pilots for this course I must say it's one of
the most well-organized courses I've attended at Chalmers! Well
done Sergei!


On 30 May 2010, at 12:11, Karl Lindqvist wrote:
Thank you for a great course! This has been a great way of learning
statistics.
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Future Prospects

Although the VLE can already be considered a solid mature system, there is still a numerous ways to develop it further. First of all, making it more modular will ease its installation at other universities and adaptation to their specific environment. Re-shaping the VLE development as an open source project will also necessitate modular approach and would provide a new huge step. Rather challenging task is internationalisation and the next year or two we should be able to see its first results.

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