SEFI
European Society for Engineering Education
Europäische Gesellschaft für Ingenieur-Ausbildung
Société Européenne pour la Formation des Ingénieurs

PROCEEDINGS

16th SEFI MWG Seminar

Mathematical Educations of Engineers

28th - 30th June 2012
Salamanca (Spain)

VNiVERSiDAD D SALAMANCA

Junta de Castilla y León

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Introduction

SEFI's Mathematics Working Group is particularly concerned with providing useful documents and organizing fruitful discussion for those colleagues who are involved in the mathematical education of engineers. Currently, the group works on a new edition of its curriculum document which was last issued in 2002. For this purpose, a discussion document has been set up which is based on the concept of mathematical competence. Mathematical competence is defined as the ability to master the mathematical challenges in situations where mathematics could be helpful. The group's 16^{th} seminar in Salamanca, Spain, is predominantly dedicated to discussing this concept and its implications for learning and assessment arrangements. The response to the corresponding call for papers was very satisfying such that a rich programme resulted from this call which is reflected in the proceedings of the seminar.

There are three invited speakers who deal with aspects of the competence concept from different perspectives. Jaworski relates the concept to theory in mathematics education regarding inquiry-based teaching. Fant illuminates the problem of aligning competence-based teaching and assessment. Finally, Ramos reflects on the requirements regarding applied mathematics in the electricity industry.

The paper presentations are grouped in six sessions:

1. Mathematical competence
2. Special competencies
3. Competence acquisition and assessment
4. Understanding learning and learning for understanding
5. Assessment, diagnosis, and support
6. Applications and inter-disciplinarity

Moreover, there are special discussion sessions on the topics:

- How could the competence approach be helpful in guiding mathematics education?
- What learning and assessment scenarios are suitable for competence-oriented mathematics education of engineers?

The programme is completed by software demonstrations and poster presentations giving a rich overview of tendencies and developments all over Europe. Most contributions are accompanied by a paper in the proceedings such that the latter provide an excellent summary of the topics dealt with at the seminar.

Aalen, June 2012

Burkhard Alpers

Editors' Note

The 16th SEFIMWG Seminar will take place on June 28-30, 2012, in Salamanca (Spain).

The agenda includes keynote lectures, poster sessions, software demonstrations, oral presentations and discussion groups.

The proceedings are organised in three different folders as follows:

- The folder **Key_Note_Lectures** with the plenary speakers' documents.
- The folder **Documents_pdf** with the documents provided by the different authors. In this folder you will find several subfolders, corresponding to the different sessions of the Seminar.
- The folder **Draft_Curriculum_Document** with the current version of the curriculum draft document. This document will be used in the discussion groups during the Seminar.

We hope that this structure of Seminar´s documentation will be useful and convenient.

The editors wish to thank **Dr Les Mustoe**, **Ciaran O'Sullivan** and **Stephen Broughton** for their work as language editors.

Salamanca, June 2012

Burkhard Alpers, Carol Robinson, Gerardo Rodriguez, Angel Martin, Agustin de la Villa (Editors).

Programme of the 16th SEFI MWG Seminar. Salamanca (Spain)

	Thursday, June 28
9:00	**Welcome Addresses**
9:30	**Invited Lecture 1,** Chair: B. Alpers
	B. Jaworski: Mathematics for Engineering Students: Relating competency to theory in inquiry-based teaching
10:30	**Session 1:** Mathematical Competence, Chair: M. Demlova
	D. Schott: Mathematical curriculum, mathematical competencies and critical thinking
	C. O'Sullivan: Building mathematical competence – A community approach
	A. Canton-Pire et al.: Mathematics in Engineering: When, how and why?
11:30	Coffee
12:00	**Discussion Introduction,** Chair: B. Alpers
12:30	**Group Discussion 1:** How could the competence approach be helpful in guiding mathematics educators?
13:30	**Plenary Discussion**
14:00	Lunch
15:30	**Session 2:** Special Competencies, Chair: D. Velichova
	P. Hernandez-Martinez: Mathematical modeling competencies in engineering: more than facts, skills and knowing what to do with them
	H. Kinnari-Korpela, K.-M. Rinneheimo: MALog: A new way to teach and learn mathematical logic
	M. Alsina, J. Bonet: Improving symbolic language comprehension
16:30	Coffee
17:00	**Poster Session**
17:30	**Software Demonstration Maplesoft,** Chair: T. Gustaffson
	J. Friebe: Educational Techniques for the next generation of engineers
18:00 – 19:00	**Software Demonstrations,** Chair: T. Gustaffson
	M. Brown, C.D.C. Steele: The Mathexplorer System: Student Exploration with a Matlab-based system
	T. Miilumäki et al.: Mathematics remedial instruction with Math-Bridge e-learning system
	Th. Schramm: Mathematical assessment at its best: News from Maple TA R8

	Friday, June 29
9:00	**Invited Lecture 2,** Chair: B. Alpers
	C. H. Fant: Aligned Assessment
10:00	**Session 3:** Competence Acquisition and Assessment, Chair: C. Robinson
	B. Alpers, M. Demlova: Competence acquisition in different learning arrangements
	A.G. Garcia et al.: Learning and assessing competencies: New challenges for mathematics in engineering degrees in Spain
	S. Broughton et al.: Lecturers' beliefs and practices on the use of computer-aided assessment to enhance learning
11:00	Coffee
11:30	**Session 4:** Understanding learning and learning for understanding, Chair: D. Schott
	J. Matthews et al.: Engineering Students Understanding Mathematics (ESUM) – Increasing conceptual understanding and engagement
	M. E. Bigotte de Almeida et al.: Mathematics' Teaching in Undergraduate Engineering
	P. Riegler: Students' conceptions of nothingness and their implications for a competency driven approach to curriculum
12:30	**Group Discussion 2:** What learning and assessment scenarios are suitable for competence-oriented mathematics education of engineers? Chair: B. Olsson-Lehtonen
13:30	**Plenary Discussion**
14:00	Lunch
15:30	**Invited Lecture 3,** Chair: G. Rodriguez-Sanchez
	A. Ramos: Applied Mathematics in the Electricity Industry Management
16:30	Coffee
17:00	Excursion: Sightseeing tour in Salamanca
21:00	Conference Dinner

	Saturday, June 30
9:30	**Session 5:** Assessment, Diagnosis and Support, Chair: C.D.C. Steele
	D. Velichova: Multiple choice tests revisited
	K. Lehtonen, M. Pauna: Technology enhanced tutoring and automated assessment in engineering mathematics
	M. Carr et al.: Addressing continuing mathematical deficiencies with advanced mathematical diagnostic testing
	A. Csakany: Assessment of first year engineering students in mathematics at Budapest University of Technology and Economics
	K. Roegner: Cognitive levels and approaches taken by students failing written examinations in mathematics
11:30	Coffee
12:00	**Session 6:** Applications and interdisciplinarity, Chair: K. Roegner
	A. H. Encinas et al.: Interdisciplinary tasks: Mathematics to solve specific engineering problems
	S. Klymchuk et al.: Engineering students' difficulties in solving application problems in calculus
	C.M.R. Caridade: Image Processing to Motivate Linear Algebra Students
13:00	**Closure of the seminar**
	C. O'Sullivan, M. Carr: Presentation for 17th SEFI MWG Seminar in Dublin 2014
	Final addresses
13:30	Lunch
15:00	End of seminar

The teaching of Mathematics in Engineering: The ACAM – Assessment of competencies/improvement actions project

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Abstract

This paper aims to explain the entire action plan undertaken in an exploratory study, framed in the project 'ACAM – Assessment of competencies/improvement actions' with the objective of understanding better how, where and why students learn or do not learn mathematics. The final purpose is the drawing up of programmes of support to address the lack of skills considered essential for obtaining success in mathematics units. After an initial investigation of Bigotte de Almeida, E., Fidalgo, C. Rasteiro, D.M.L.D., 2011 and Bigotte de Almeida, E. et al 2011, whose intention was to perceive why students do not succeed in Differential and Integral Calculus, taught in the first year of undergraduate engineering in Coimbra Engineering Institute (ISEC), it was concluded that further work is needed in order to obtain a greater understanding of the difficulties encountered by these students. We indicate procedures which are already being adopted, and others to be implemented, whose aim is directed to the identification of issues relating to the learning process. This research uses a data qualitative approach and checks for understanding the difficulties presented by the students, through the analysis of common mistakes in performing the assessment and other proposed activities, focusing our study on determining/evaluating which mathematic skills relating to calculus are or are not present. This analysis will be an excellent tool for enlightening the students' state of knowledge, essential in the teaching/learning process and will contribute, through an interpretation and categorisation of these errors, to the selection of activities that promote the reorganisation of educational practices contributing to the development of fundamental skills necessary for undergraduate success in engineering.

Introduction

The results of students' national assessments in first and second level/access to third level are often found to be poor, a situation that is also reflected within the Program for International Student Assessment (PISA) reports. The above-mentioned concludes that students who gain access to higher education have, in general, difficulties in basic and elementary mathematics content. This lack of preparation, not being a unique situation of Portuguese education, is compounded by the heterogeneity of the formation at the entrance of undergraduate engineering degrees, as a consequence of the diversity of access exams to higher education. In recent decades it became notorious the mass participation and democratization of higher education, verified by the considerable expansion in the diversity of student backgrounds, creating different contexts in motivation and expectation, requiring adaptation, both personal and social, to a new reality. On the assumption that higher education is intended to stimulate a greater initiative and student autonomy, teaching practices used at this level of education are generally developed in an environment that is not as structured as in first and second level education, namely without supporting instruments that fit specifically the rhythm of materials and lessons. This assumption can represent some anxiety and puzzlement for many students that undoubtedly is reflected in their attitude towards school and learning. The students' difficulties in the undergraduate engineering degrees in the area of Mathematics, especially for the Differential and Integral Calculus, have contributed to raise the failure rate and therefore have led to a lack of motivation of all involved in the education process. However, the importance of mathematics and its structural role as a supporting science and tool for a logical and structured thinking, essential for engineering, has avoided the temptation of teacher agreement with that situation, not considering it to be a normal situation. Efforts are being undertaken to better understand the factors that contribute to success. In this context of concern, a research group GIDiMatE - Grupo de Investigação em Didática da Matemática na Engenharia, integrated in the Department of Physics and Mathematics (DFM) at the Coimbra Engineering Institute (ISEC), was established. The research group's intention is to contribute to a participatory reflection and thoughtful pedagogical practice in higher education, which may influence, above all, a possible modification of concepts and an improvement in teachers' professional performance.

This investigation, which is based on student mathematical errors, does not intend to evaluate the student, but to contribute to understanding how he takes ownership of certain knowledge and what difficulties he still needs to overcome before being able to work with the content in question. Thus, the analysis of errors can also be understood as a teaching methodology, if classroom activities in which the students' errors are exploited and taken as learning tools are developed. Several studies claim that the integration of students in the scientific area of mathematics curricular units is related to the degree of knowledge gained in previous school levels. Thus the importance of making a survey of errors that students usually make, which prevent them from correctly solving the questions proposed in the assessment, is high. Indeed, the observation of mistakes made by students as well as the various versions in which they arise in problem resolutions associated with the syllabus of curricular units may allow the construction of a strategic reference base frame to be included in teaching practice. In this study the analysis of errors has been understood as a teaching methodology since in its interpretation will be to highlight the absence of basic and/or elementary skills critical to the contents of Differential and Integral Calculus avoiding the lack of content follow-up which, could involve student demoralisation and lead necessarily to dropout and school failure.

The European Society for Engineering Education, SEFI, through its workgroup MWG (Mathematics Working Group) promotes a discussion forum addressed to all those interested in the Mathematics education of engineering students in Europe. In this context, and in order to define what teaching mathematics contents were appropriate for engineering, the first curricular guidance document was created in 1992. That document makes a detailed and structured list of topics, organized by levels, which correspond to specific contents essential to Mathematics learning in engineering degrees. Subsequently, in 2002 and 2011, the MWG has updated the document, releasing a report, Mathematics for the European Engineer – A Curriculum for the Twenty-First Century (SEFI, 2002) with learning outcomes rather than a simple list of topics. Regarding the minimum knowledge advised to higher education entry to an engineering course, these learning outcomes are detailed by areas and identified by topics in subsection 3.1.1 Core Zero, pp-24. From among these areas and according to the

Portuguese school education programme, GIDiMatE gave special attention to algebra, analysis and calculus, geometry and trigonometry. These areas were considered by the group as being the most relevant, because they can be assumed to be the essential content of most topics in the area of mathematics in higher education engineering.
The research question is: “How to overcome mathematical difficulties that were pointed out by the most common errors made by 1st year students of undergraduate engineering?”

Method of Investigation

Generally the methodology followed the steps of content analysis where a first reading, which coincided with the correction of assessments, helped to decide on the units that would be the object of analysis. Subsequently, categories were defined according to some criteria previously established but also adapted subsequently in harmony with the information obtained, either in the sample collected or in literature, in order to understand the situation. By comparison, systematic encoding and extraction of some regularity, the researchers wanted to obtain information that may explain the difficulties in mathematics, namely in the syllabus, regarding the second and third cycles of basic education. The description of each category will give an in-depth errors understanding since their causality is interpreted on a description basis. Starting from the research question referred above the following action strategies were developed: 1) Analyse the regularity with which certain errors in basic knowledge, essential to full integration in the Differential and Integral Calculus subjects, taught by the undergraduate Civil Engineering and Computer Science Engineering, appear: 2) Categorise those errors and identify the basic mathematical knowledge necessary to overcome them: 3) Design an errors data grid and perform a quantitative and qualitative analysis: 4) Do an in-depth study of some examples found in assessments in order to better understand the students’ difficulties: 5) Propose a reference frame that allows the selection and construction of activities to promote the reorganisation of educational practices. The realisation of those activities, particularly in a Competence Centre (to be created) open to all students enrolled in ISEC, will be a contribution to the development of fundamental knowledge that is necessary to succeed in undergraduate engineering.

The sample

Students who access the Coimbra Engineering Institute have very different characteristics, both at the origin (on average 60% of students placed on engineering degrees do not come from the district of Coimbra) and with respect to basic academic training (the majority of students placed, approximately 63%, are from science and technology courses). In the current academic year, 2011/2012, application grades of Computer Science Engineering students range from 108.3 to 168.3 and 133 students were accepted (81 have chosen ISEC as first option). In Civil Engineering the application grades varied from 110 to 180.3 and 70 students were accepted (43 have chosen ISEC as first option). This exploratory study was carried out in the 1st semester of the academic year 2011/2012.
In order to define strategies for teaching/learning that lead to meaningful learning, the error analysis in the students’ assessments was made for the courses of Mathematical Analysis I, degree in Computer Science Engineering and of Mathematics I, Civil

Engineering degree. These curriculum units are taught in the 1st year of undergraduate study and integrated into their syllabus: differential calculus (as High School revision plan), primitives and respective techniques, improper integrals and definite integral calculus, integral calculus application (areas, volumes and curve lengths) and differential equations. The evaluation methodologies proposed in these two curricular units are different: while the first integrates distributed evaluation processes (Mid-term tests – T1 and T2, with taught contents distribution, without requiring a minimum classification and with the possibility of performing each one of the assessment tests in regular times scheduled in the exams calendar – EX) the second provides only summative evaluation, by normal examination at regular times scheduled in the exams calendar – EX. The researchers' experience, and the collected literature reviewed (Cury, 2005, 2007; Gill, 2007), led them to identify a set of errors that empirically were considered as being the most common and an impediment to answering correctly certain questions.
Under these assumptions a draft of an important errors grid was designed. The option in this approach fell in 'The National Curriculum of Basic Education-Core Competencies' for Mathematics.

Errors Categorisation

<u>Error A: Powers functions product</u>
Frequently students don't know how to generalize powers product properties, included in the elementary mathematics programme, to algebraic expressions (power functions product). This appears to be a very common error. Included in this case are the errors made by the wrong application of negative powers.
<u>Error B: Powers of sums</u>
The easy generalisation of powers product properties to power functions summation is verified with pronounced regularity, assuming, in general, the form. In this case, students additionally reveal not to know how to apply common polynomials multiplication properties using wrongly the identity, which confirms that those properties weren't understood and therefore aren't correctly applied.
<u>Error C: Sum of fractions</u>
The rules for determining the sum of rational numbers (with the need to reduce to the same common denominator) is not understood by students and, therefore they generalise this error to the sum of two rational algebraic expressions. This error occurs very frequently.
From the interpretation of the various ways in which this error occurs, we can induce that it arises as a result of a non-identification of the addition operation properties, showing once again some misperception with the multiplication aspect.
<u>Error D: Distributive law from elementary algebra</u>
The wrong application of this property led to the detection of a set of errors associated with it and applied in various situations, in particular, in simplifying fractions and in solving equations, as for example:

$$\frac{a(x)+b(x)}{a(x)}=b(x) \quad \text{or} \quad \frac{a(x)}{a(x)+b(x)}=\frac{1}{b(x)} \quad \text{or} \quad a(x)b(x)+c(x)=d(x)\Leftrightarrow a(x)+c(x)=\frac{d(x)}{b(x)}$$

Once again, it becomes clear that inappropriate 'over-generalisation' of the addition properties manifests itself by rearrangement of the corresponding multiplication properties. This classification also had support on the learning outcomes related to

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knowledge and capacities proposed by the Mathematics Working Group of SEFI (European Society for Engineering Education). The four categories defined above were framed in Core Zero which contains a list of topics that covers, in addition to the High School or pre-university school syllabus lectured in European countries, requirements considered by the group as constituting a solid platform of knowledge essential for the study of mathematics in engineering degrees. The analysis performed is on the Algebra categorisation area, in particular regarding the arithmetic of real numbers and algebraic expressions and formulas.

Findings and Discussion

With regard to quantitative analysis research, students' assessment tests effectively evaluated in the undergraduate Civil Engineering and Computer Science Engineering were observed in accordance with the adopted methodology. The aim was to count the number of tests that contained at least one error in basic knowledge at the mathematics basic education curriculum level, essential to the full understanding in the Mathematical Analysis I course.
It was found that the percentage of errors is higher than 72% and if we consider the average percentage of errors in case of the Computer Science Engineering examination, there is not a significant difference between the two courses analysed.

		Computer Science Engineering		**Civil Engineering**	
		Tests with errors		**Tests with errors**	
	T1	72	74%		
1st Exam	**T1**	5	100%		
	T2	45	75%		
	EX	16	80%	60	72%
2nd Exam	**T1**	6	86%		
	T2	12	67%		
	EX	27	69%	32	76%

Figure 1 – Percentage of assessments with errors

In a more refined analysis of the various assessments performed by students the errors found were counted and, based on the first approach taken, included into the categorisation set. The preparation of the several assessments held throughout the semester did not have as a prior concern the detection of errors for later content analysis, and so it was realised that a certain type of error was associated with some particular questions. Furthermore, in this approach the questions that have not been answered by the students were not removed from the sample, which implies that the percentages observed refer to all assessments. It should be noted that the absence of a response does not infer any conclusion or allow for the same error occurring more than once in any given assessment test or question or even multiple errors occurring of in the same answer. Although the aim of this exploratory study is to define a basic reference guide enabling the selection and construction of activities, that promote the reorganisation of educational practices, helpful to knowledge development fundamental to the success of engineering undergraduate students, the researchers assumed that the ratio of errors should be determined taking into account the total number of observed errors. It has been found that the most common error made by Computer Science and Engineering students was the one included in category D 'Distributive law from elementary algebra' (35%) and for Civil Engineering's students was the one included in category B 'Powers of sums' (32%).

	Computer Science Engineering	**# type of error /# total of errors**	**Civil Engineering**	**# type of error /# total of errors**
error A	44	18%	22	25%
error B	65	27%	28	32%
error C	47	20%	17	20%
error D	83	35%	20	23%

Figure 2 – Relative frequencies distribution of errors by category and by analyzed undergraduate engineering degree

Conclusions for Education

A more specific analysis in interpretation of these errors, as well as its various representations in student's assessments indicates the absence of basic knowledge at the level of understanding and differentiation of properties which are inherent to addition and multiplication operations. It was also noted that the error frequency is directly related to the question posed, which infers the need to implement strategies in the classroom or outside classroom additional work that could meet student needs. Thus, activity preparation focused on the acquisition of basic knowledge, which may be done in parallel with the course development in order to overcome the difficulties which were detected, are suggested. One of the possible strategies to adopt in the classroom may be peer learning, since it is a form of cooperative learning that enhances the value of student-student interaction and results in various advantageous learning outcomes. To implement modular mini-courses, inside classroom and/or using an e-learning scheme, can also contribute to the full integration of students in higher education, in particular in engineering degrees. According to the Report of the Mathematics Working Group (SEFI), it was concluded that students whose solutions have errors reveal difficulties with the arithmetic of real numbers, namely: understand the rules governing the existence of powers of a number, combine powers of a number, evaluate negative powers of the number, carry out arithmetic operations on fractions, represent roots the fractional powers and algebraic expressions and formulae, such as add and subtract algebraic expressions and simplify the result, multiply two algebraic expressions, remove brackets, carry out the operations add, subtract, multiply and divide on algebraic fractions. For the proper functioning of Differential and Integral Calculus subjects it is recommended that, at its beginning lecturers acquire awareness, through a diagnostic test, of the early student difficulties in order to make a timely work routine, avoiding demotivation and the consequent school dropout, factors that lead inevitably to school failure.

References

Bigotte de Almeida, E., Fidalgo, C., Rasteiro, D.M.L.D. (2011) Compreender o insucesso da Matemática nas licenciaturas de Engenharia: Um estudo de caso. XIX Colóquio AFIRSE, 2 a 4 de fevereiro, Lisboa.

Bigotte de Almeida, E. *et al*. (2011) Projeto “ACAM - Avaliação de Competências/Ações de Melhoria”. XIX Colóquio AFIRSE, 2 a 4 de fevereiro, Lisboa.

Cury, H. *et al*. (2005) *Análise de Erros em disciplinas Matemáticas: um estudo com alunos de engenharia e ciência da computação”.* Accessed via http://www.eee2005.uerj.br/artigos/artigo04.prn.pdf (27 december 2011)

Cury,H. (2007) *Análise de erros em disciplinas matemáticas de cursos superiores”*.. 110f.relatório Técnico-Faculdade de Matemática PUCRS, Porto Alegre.

Gill O., O’Donoguhue J. (2007) *The Mathematical deficiencies of Students Entering Third Level: an item by item analysis oh student diagnostic test”*.. Proceedings of 2nd National Conference on Research in Mathematics Education-MEI 2.

Niss, M. *“Mathematical competencies and the learning of mathematics: The Danish Kom Project”.* Accessed via http://www.w3.msi.vxu.se/users/hso/aaa-niss.pdf. (27 december 2011).

Puerto, M., Minnaard, C. Seminara, S. (2006) *Análisis de los erros: una valiosa fuente de información acerca del aprendizagem de las Matemáticas”.* Revista Iberoamericana de Educación, nº38/4, ISSN:1681-5653.

A Framework for Mathematics Curricula in Engineering Education, (2011) A Report of the Mathematics Working Group Editors, , European Society for Engineering Education (SEFI).

Competence acquisition in different learning arrangements

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Abstract

The next edition of the SEFI Mathematics Working Group's curriculum document will be based on the notion of mathematical competence. This approach aims particularly at capturing higher-level learning goals which go beyond the traditional content-related and often small-scale description of learning outcomes. In this contribution, "classical" and more recent learning arrangements are investigated regarding their potential for competence acquisition. The learning arrangements include lectures, assignments, tutorials, laboratories, projects, and e-learning arrangements. We relate these to the eight competencies identified in the current discussion document (SEFI MWG 2011) and state which contribution the different learning arrangements could provide for competency acquisition.

Introduction

The next edition of the SEFI Mathematics Working Group's curriculum document will be based on the notion of mathematical competence (SEFI MWG 2011; Niss/Højgaard 2011) which is split up into eight competencies (mathematical thinking, reasoning, problem solving etc.). This approach aims particularly at enabling students to use mathematical concepts and procedures to meet the challenges in their daily engineering life. Having this goal, there still remains the task of setting up suitable learning arrangements where this goal can be achieved. In this contribution, "classical" and more recent learning arrangements are investigated regarding their potential for competence acquisition. The learning arrangements include lectures, assignments, tutorials, laboratories, projects, and e-learning arrangements which we relate to the eight competencies. We state which contribution the different learning arrangements could provide for competency acquisition.

Competence acquisition in learning arrangements

Lectures

We start with the most traditional, and probably also most widespread, form of mathematics teaching settings – the lecture. Even in problem-based learning settings as described below, lectures still play a certain role. Lectures can take many different forms: Giving a lecture can mean the classical uni-directional presentation of material but it can also include different degrees of student involvement (active learning components). The main reason for lectures is to introduce a larger audience to certain mathematical concepts and procedures. The goal consists of gaining a "first familiarity" with the material that needs subsequent individual activities by the learners to increase understanding, recognise usage scenarios and be able to apply the concepts and procedures in intra- and extra-mathematical contexts. A good lecture should motivate the material, relate it to previous concepts and provide the “overall picture” (cf. Slomson 2010). How does this correspond with the concept of mathematical competencies, i.e. how can competence acquisition be envisaged in this learning setting? In subsequent sections we present some ideas by going through the eight mathematical competencies. We restrict ourselves to classical introductory mathematics lectures for engineers. More advanced mathematical lectures (e.g. courses in discrete mathematics or mathematical logic) may contribute in a slightly different way to acquiring competencies. There might also be specific lectures on mathematical modelling or problem solving which are dedicated to address specific competencies but – given the usual curricular restrictions – there will be mostly lectures covering the classical concepts and procedures in analysis and linear algebra. Moreover, the size of a lecture plays an important role – smaller lectures (up to 50 students) allow more easily an active role of students than bigger ones (100 - 200 students).

Mathematical thinking: In order to enhance the mathematical thinking competency lecturers should emphasise in their lectures what mathematics is able to contribute to engineering work. For example, by arguing logically one can show that a certain geometrical construction in a technical drawing is fixed by certain data, or an ODE modelling a damped mass-spring system can only behave in a few ways. Moreover, by having a mathematical model one can compute reasonable or even optimal configurations in advance, thereby avoiding endless experimentation.

Mathematical reasoning: In a lecture the lecturer provides examples of correct professional mathematical reasoning when proving or justifying certain assumptions or ways of solving a problem. If the theory is laid out as a finished piece of mathematics students do not see the process of creation and thinking behind the theory (as is also often the case with mathematical articles). Therefore, the lecturer should explain the kind of reasoning behind setting up definitions and theorems. (S)he should not just present the formal definitions and arguments but should provide a lot of explanatory material (e.g. why do Laplace transforms?, to transform DE problems into equations; why compute Fourier series?, to see which frequencies have which “weight”, explain different ways of solving a system of linear equations).

Mathematical problem solving: Again, in a lecture the students do not see the real problem-solving process but merely the ‘polished’ final version (which often gives the wrong impression that everything in mathematics is straightforward once you have learned the correct procedure). Therefore, a lecture is quite restricted here. Nevertheless, the lecturer should explicitly name the problem-solving strategies that are applied, e.g. analogy (do it as in the case of …), transforming into a familiar domain; ‘divide and conquer’ (split up into special cases); try to make use of information/properties you have, i.e. relate them to things you want to know or understand.).

Mathematical modelling: As stated above regarding problem solving, the modelling process can only be shown in simple examples (not the real going back and forth in the modelling cycle). One can explain and emphasise which kind of situation or behavior can be modelled with a certain mathematical concept (e.g. vibration with sine functions, certain kinds of growth and decay with exponential functions, static behavior with equations, …). When the students carry out their own modelling activities in other learning arrangements, they then have at least ‘material’ with which to experiment. As Niss (2010) stated, if one wants to set up a model one has to anticipate what might work, and the lecture might help in the process of anticipation (real experience with many modelling activities will be of greater help, though).

Representing mathematical entities: In lectures the value of different representations can be, and should be, demonstrated (and therefore the necessity to switch between representations). There are many places in undergraduate mathematics where this can be done (different representations of lines: parameter form and equation form; graphical and algebraic representations of equations and inequalities; representations of functions; time domain and frequency domain; …). Therefore, the ‘theme’ of different representations can be explicitly emphasised at several places in a lecture, enlarging the probability that students see and retain the value for later use.

Handling mathematical symbols and formalism: The lecture gives examples for the correct use of symbols and formalism in mathematics. This need not be as formal as in lectures for mathematics students (which would be “too formal” for most engineering students) but a semi-formal presentation should also serve as example for own lines of computation and argumentation. For example, the use of set notation or short notations like S for a sum at several places in the lecture should help students to familiarise themselves with this formal notation and language.

Communicating in, with and about mathematics: In a classical lecture the passive side of this competency is emphasised. Students are to listen and follow the oral (in the lecture theatre) and written (in accompanying scripts) argument of the lecturer. Here again, the lecturer should provide good examples of mathematical presentation for a certain audience (explain your reasoning, make the structure of your argumentation clear, try to make connections to previous experience of the audience, emphasise important topics and de-emphasise technicalities, …). The students should try to relate the new concepts and procedures to their previous knowledge base and gain a ‘first’ understanding to be enhanced in their own active studies later.

Making use of aids and tools: The lecturer can provide demonstrations of the reasonable use of tools and other aids (visualisation of complicated concepts; animation of processes; choice of adequate representation; quick computation of larger examples; …). These examples can then be used by students later when working on assignments or projects.

There are several attempts to make the classroom scenario more interactive, even in larger classrooms (Mason 2002; Gavalcova 2008; Robinson 2010). In smaller groups (20 to 50) it is possible to create a “guided, directed dialogue” (Gavalcoca 2008) by asking questions and letting students give and explain answers. The answers can also be given by electronic voting systems (EVS, see Robinson 2010), which provide an overall picture of the current understanding of the audience to the lecturer. One can also include student activities by giving them small problems to discuss with each other in pairs or to make individual computations using their own technology. These active learning methods can enhance the acquisition of additional aspects of competencies compared to the classical unidirectional situation. If students are given questions that go beyond mere facts and require some sort of mathematical reasoning, the acquisition of the respective competency is being trained. If students are to exchange their arguments in pairs, the active side of the communication competency is also addressed. There are also many conceptual questions (for a question bank for use in EVS see Robinson 2010) regarding different forms of representations and their relationships. Moreover, when questions require the use of technology (normally pocket calculators) then the respective competency is also included in an active way. In summary, there are several ways of involving students actively even within a lecture scenario which help them acquire the ‘active’ side of mathematical competencies.

Assignments

By assignments we mean all kinds of ‘smaller’ tasks students have to undertake on their own, be it in groups or individually. These include standard computational tasks that serve to get more familiarity with notation, formalism and procedures but also more open and investigative assignments, with or without technology. Larger problems or projects are not included here but dealt with separately below.

Mathematical thinking: Mathematical thinking could be fostered in more open application tasks where students have to work with application models and solve questions that are of practical interest. This would demonstrate to students that having a mathematical model is helpful when working on practical tasks like machine dimensioning or choosing adequate parameters in control devices. On the other hand, a complete restriction to standard procedural computation tasks could lead the students to think that mathematics has nothing to do with real engineering work and hence is just an obstacle to be overcome during the early semesters.

Mathematical reasoning: In standard tasks very restricted forms of procedural reasoning can be exercised but in more open assignments the development of chains of logical arguments can be trained (e.g. show that a certain geometric configuration is uniquely determined by certain data; or even more open: by which data is the configuration uniquely determined). Advanced mathematical courses provide even more material for exercising mathematical reasoning, e.g. courses on discrete mathematics, mathematical logic.

Mathematical problem solving: Standard problems (e.g. how to integrate a function using one of the standard methods) can be learned using standard tasks (e.g. integrate a rational function using the partial fraction method). More open assignments (like “construct a function to move from A to B given certain restrictions”) can serve to reflect on the principal procedure to tackle such a problem. It is a question, though, whether many students are able to work on such a problem without tutorial support. So, the problems in such a learning arrangement are still likely to be rather ‘well-formulated’.

Mathematical modelling: In standard tasks only that part of mathematical modelling is practised where mathematically-formulated problems are solved using given mathematical models. In more open assignments there will still be a well-defined application situation but the ‘translation task’ (as in word problems) to be performed might be more challenging.

Representing mathematical entities: In standard tasks one can train students to switch between different representations (the computational part). In more open assignments one can also train them to choose an adequate representation for a particular problem.

Handling mathematical symbols and formalism: Standard tasks are necessary and important for students to become familiar with concepts and procedures in order to handle them without getting stuck very quickly. A certain fluency in dealing with symbols and formalism needs more or less permanent training (like fitness in sports) depending on individual abilities.

Communicating in, with and about mathematics: If students have to hand in written assignments they have to state clearly their arguments and in this way learn to actively communicate mathematically. If students work on assignments in groups, the oral component of this competency is also taken into account.

Making use of aids and tools: If the assignments include the use of aids like formulae books, pocket calculators or even mathematical programs then the competency of making adequate use of aids and tools is also addressed.

Tutorials

By tutorials we mean learning arrangements where a tutor (teaching assistant, maybe a student) works with students in order to improve their understanding related to a lecture. Such tutorials can differ a lot with regard to the way of teaching and learning. There might be tutorials where the tutor mainly performs example computations such that the situation is not much different from the lecture. But there are also forms with active involvement of students who work on standard tasks or on more open assignments with the help of tutors. Students might also give presentations of their solutions on the blackboard.

Since in tutorials similar tasks are dealt with as in the assignments, the statements made in the previous section on assignments also hold for tutorials. In addition to this, tutorials provide the opportunity to have group discussions and presentations by students such that the communication competency can be better addressed. Moreover, because tutorial support is available, tasks can be more open since students can ask the tutor for help.

Projects

By projects we mean learning arrangements where students work – mostly in groups – on problems which are more open and investigative in nature (for guidelines see Alpers 2002). Usually, students have to document and present their work at the end. In so-called problem-based learning settings (cf. Christensen 2008) this is the predominant way of learning although even there mixed forms including lectures can be found.

Mathematical thinking, reasoning, problem solving and modelling: In projects, particularly in application projects, students can extend their understanding of what mathematics can do for them as prospective engineers. Students have to think about how to proceed, which steps to take in tackling the given problem and to check how far they got in the process, and what still needs to be done. This planning, monitoring and control work is of general nature but when it comes to the mathematical kernel of a project it also addresses the mathematical problem-solving competency. Larger projects allow students to experience the full modeling cycle. Students set up and work in mathematical models reflecting an application situation which allows them to make variations and to experiment in order to get a better understanding of the situation and/or to achieve certain properties. This reflects real engineering work with programs implementing a mathematical model. Critical mathematical thinking (What are the restrictions of what mathematics can do for you?) can also be fostered when students think about the assumptions in models and parameters of models. When students do not simply play around but reason mathematically about the influence of parameters and dependency on assumptions, the respective competency is also developed. Note that whether this potential can really be put into reality depends strongly on the quality of the project tasks and the tutorial support.

Communication in, with, and about mathematics; representation: When students have to read mathematical texts on their own (including short web pages on mathematical concepts) and when they have to understand the mathematical explanations of a project group member, the passive side of the communication competency is addressed. When they explain themselves, write project documentation and make an oral presentation to other students, the active side is also taken into account. Moreover, in documentation and presentation questions of adequate representation very often turn up in the need to get a clear message across to the audience.

Making use of aids and tools: More realistic problems usually require the use of mathematical software, so students also improve their competency of using tools properly. When they create their own experimentation environment for an application situation and try to use it in a goal-directed way (make informed changes and interpret the effect), they get accustomed to the way engineers use mathematically-based software programs in their work.

Mathematics laboratories

By mathematics laboratories we mean learning scenarios where students work in a PC laboratory on tasks related to mathematical software such as numerical programs (Matlab®), CAS (Maple®, Mathematica®) or spreadsheet programs (Excel®). In such lab sessions students practise the usage of the programs and see how they can be used for standard tasks. They might also be used for experimenting with more open tasks of an investigative nature.

The same competency potential that was outlined in the section on assignments can also be claimed for laboratories. In addition, the tool usage competency is specifically addressed. Moreover, since mathematical programs require mathematical notation and formalism as input, the respective competency is also trained. Regarding the representation competency, work with mathematical programs in labs has also high potential since students can switch flexibly between different representations. This must be embedded in adequate tasks to be meaningful and not just ‘playing around’.

E-Learning scenarios

E-learning scenarios comprise quite a few arrangements. There might be presentation material potentially using multimedia which students can use to repeat or better understand certain contents. One can also find training tasks/exercises with worked-out solutions. Testing systems (like STACK or Maple TA®) allow to check, and maybe assess, the understanding of, and by, students. E-learning environments can also allow for more interactive learning scenarios: There can be environments where students can make changes (e.g. parameter variations) and recognise the effect, or work on tasks to achieve certain properties by making variations. E-learning also comprises more sophisticated intelligent tutoring systems which allow the insertion of single steps and provide tutorial help. Finally, an electronic forum might also be used as a means of collaboration and communication between students, or between students and tutors/lecturers.

E-learning scenarios offer competence acquisition opportunities similar to the other learning arrangements discussed above. The passive side of the communication competency is addressed when students have to read and understand mathematical material presented electronically. The active side is particularly taken into account when students work in discussion groups (forum) and explain mathematical material to each other. Using electronic aids and tools can also be trained in an e-learning environment. Working on larger problems or projects needs human interaction and tutoring which could in theory also be provided via electronic communication channels but personal dialogue is still stronger here.

Conclusions

In the discussion above we outlined how different learning arrangements contribute in different ways to competency acquisition. The classical ‘lecture theatre’ arrangement has still its potential, in particular when it is enhanced by components of active learning but it is certainly not sufficient. It can be considered as an example for ‘cognitive apprenticeship’ where students see mathematical competence in action as performed by the lecturer but students have to work on mathematical tasks and problems themselves to become really competent. For this, classical and more open assignments done at home or in a tutorial can be used. Projects are particularly suitable for acquiring more realistic modelling and problem-solving competencies but they are also very costly concerning tutorial resources. The tool competency can be specifically addressed in mathematics laboratories. Elaborated e-learning scenarios might offer learning opportunities similar to those of the other arrangements discussed. If they provide an adequate communication infrastructure, the communication competency can also be developed. In conclusion, one can state that a mixture of several learning arrangements seems to be appropriate where the particular offering certainly depends on circumstances like group size and available resources. Moreover, mathematical competencies are also acquired in application subjects where the setting up of and working in mathematical models play an important role as in engineering mechanics.

References

Alpers, B. (2002) Mathematical application projects for mechanical engineers – concept, guidelines and examples. In M.Borovcnik, H. Kautschitsch, eds. Technology in Mathematics Teaching. Proc. ICTMT 5 Klagenfurt 2001, Plenary Lectures and Strands. Öbv&hpt: Wien, 393-396.

Christensen, O.R. (2008) Closing the gap between formalism and application – PBL and mathematical skills in engineering. Teaching Mathematics and its Applications, 27: 131-139.

Gavalcova, T. (2008) On strategies contributing to active learning. Teaching Mathematics and its Applications, 27: 116-122.

Mason, J.H. (2002). Mathematics Teaching Practice. Guide for university and college lecturers. Horwood Publishing.

Niss, M. (2010). Modelling a Crucial Aspect of Students' Mathematical Modelling. In Lesh, R., Galbraith, P.L., Haines, C. H., Hurford, A., eds. Modeling Students' Mathematical Modeling Competencies. ICTMA 13. Springer: New York, 43-59.

Niss, M., Højgaard, T. (eds.) (2011). Competencies and Mathematical Learning. Ideas and inspiration for the development of mathematics teaching and learning in Denmark, English Edition, Roskilde University.

Robinson, C. (2010) Using Electronic Voting Systems for Active Learning. In Proc. of the 15th SEFI Math Working Group Seminar. Wismar.

SEFI Mathematics Working Group (MWG) (2011). A Framework for Mathematics Curricula in Engineering Education, Draft 7.12.2011, version 7.1

(http://sefi.htw-aalen.de/Curriculum/Competency%20based%20curriculum_discussion_document_December_2011.pdf , accessed 8 Feb. 2012).

Slomson, A. (2010). What makes a good maths lecture? MSOR Connections, 10: 17-20.

Improving Symbolic Language Comprehension

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Abstract

We focus on the goal of "Handling mathematical symbols and formalism" through the methodology of Content and Language Integrated Learning. The use of foreign language highlights, and possibly increases, the difficulties in the point of mathematical competence, but it can also be used to fix them. That is, making explicit the equivalence between formal and verbal language could improve symbolic language comprehension.

Multilingual Formulae, an on-line resource at http://mformulae.epsem.upc.edu, is designed to give support in that direction, as equivalence is not found explicitly in textbooks or research papers. It contains sets of formulas with the corresponding written and oral version in several languages. The project, conducted by professors at the UPC Engineering School at Manresa Campus, is addressed to lecturers and students as a support to ensure effective communication when both Symbolic and Foreign language are used.

1. The challenge to enhance multilingualism at EHEA

From the Bologna Declaration, the institutions involved in the European Higher Education Area (EHEA) are adapting their curricula according to the basic principles of quality, mobility, diversity and competitiveness. In that landscape, it is clear that the full command of at least one other language is a core competence, in order to be internationally competitive and culturally aware.

Among the principal recommendations given by the European Commission to enhance multilingual competence, one of the most promising alternatives is teaching curricular subjects in a foreign language (Council of Europe, 1995), in a similar way to what is called Content and Language Integrated Learning (CLIL or AICLE) at other educational levels. Even different terminology is used, as for example *Enseignement d'une Matière par l'Integration d'une Langue Étrangère* (EMILE) in French, and there are related but different systems (Immersion, Language in Content Instruction), all of them show contact between language and discipline. This contact works as a good motivation for learning, and the universities in the Vives Network considered it as one of the main ways to achieve linguistic competence. For example the Linguistic Plan of the Universitat Politècnica de Catalunya-Barcelona Tech (UPC), approved in 2010, deals with the third language competence, taking in account the good command of Catalan and Spanish of their members. Check Lasagabaster-Zarobe (2010) and Lasagabaster (2008) for a general overview of CLIL in Spain and Europe.

However, maximal quality in the achievement of other competences needs to be guaranteed, so translation of contents is not the only thing to be done. Concretely, in the framework for the Mathematics curricula in Engineering Education, it is important to deal with the implications of the use of foreign language on the development of mathematical competence. We need to make arrangements in order to keep in parallel the achievement of both competences: not allowing foreign language competence to improve at the expense of mathematical competence.

English courses for encouraging and training teachers were organised by the Institut de Ciències de l'Educació (ICE) of the UPC to promote CLIL through English, considered as *lingua franca*. In that context, the authors with a team of professors teaching different engineering subjects at the Escola Politècnica Superior d'Enginyeria de Manresa (EPSEM) were involved in projects analysing the current situation.

To counter envisaged language difficulties, our first step to ensure good communication between teachers and students was to create Class-Talk, an on-line trilingual university teaching phrasebook, in collaboration of the Language and Terminology Service of the UPC. The aim of this phrasebook, available at http://www.upc.edu/slt/classtalk/, is to help university teaching staff and students to communicate more effectively in a generic university classroom in a language that is not their mother tongue. It contains around 600 expressions classified according to the situation (starting the lecture, exams, etc.). Audio files are provided to improve listening and speaking skills.

Questionnaires were designed to collected incoming students' English language level, taking into account their certification needs. Figure 1 shows the results for a sample of 400 students enrolled in the new degrees.

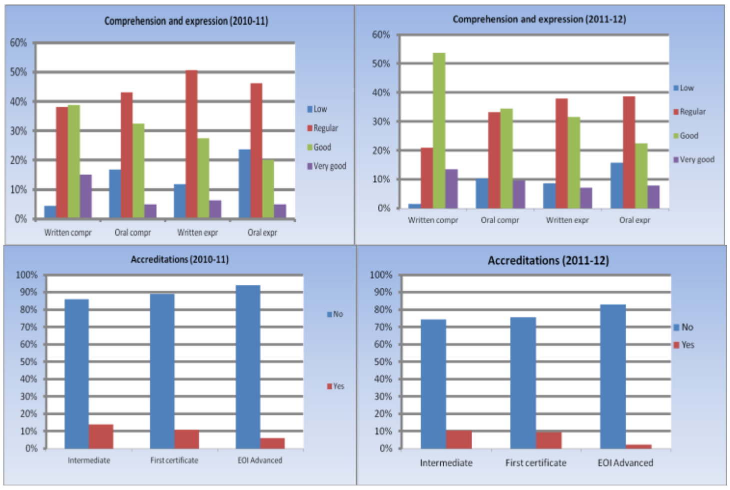

Figure 1. Data representing English level and certification of students.

The conclusion of the analysis was that scaffolding and support material was necessary for teaching content through English. With this aim the research group Linguatech-Rima (Research group on Scientific and Technological Communication) was created, with more than 20 professors from different areas involved in Engineering Education (as Mathematics, Electronics, Electricity, ICT, Chemistry, Mechanics). Members of this group are currently working on the Multilingual Formulae website, presented in this paper.

In section 2 we focus our attention on the parallelism between mathematical symbols and usual language, to stress difficulties, and why support resources are needed. In section 3, the open access resource Multilingual Formulae is presented, to deal with the verbal expression for the mathematical symbols, as a tool to give support to lecturers and students. Section 4 contains some final remarks.

2. Parallelism between languages

There is a widespread agreement that mathematics is the language of the universe, as it was stated by Galileo (Opere VI, 232): "*... questo grandissimo libro che continuament ci sta aperto innanzi a gli occhi (io dico l'universo), ma non si può intendere se prima non s'impara a intender la lingua e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezzi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro laberinto."*

Commonly this is used to note the value of mathematics as a problem-modeling tool. But note that it also uses the parallelism between mathematics and usual language, and states that without the characters no word of the language can be understood. This can be applied to both mathematics and foreign language. Thus, might mathematics or symbolic language – rather than the communication bridge – become a barrier?

From now, we focus our attention on the competence of *Handling mathematical symbols and formalism*, n.6 at *KOM project*, by Niss (2003). However, it is obviously tied in with the others: n.5 *Representing mathematical entities*, n. 7 *Communicating in, with, and about mathematics*, n.8 *making use of aids and tools*. All of them are concerned with "the ability to deal with and manage mathematical language and tools", used in problem solving and mathematical thinking in general.

The ability to understand symbolic and formal mathematical language seems to be inherent to the translation process between formal and natural language, which is included in the reflection dimension of the competence, in the Report of Mathematic Working Group. Usually, thoughts are formulated through language. This is the reason we use the natural language when reading symbolic language.

Writing and talking at the board uses this equivalence explicitly, in order to learn. But what is happening when a foreign language is used? On the one hand, verbal understanding is not so direct; difficulties can increase, so we need to be more careful to make explicit the equivalence. On the other hand, we need to use suitable expressions native to the foreign language that are not found explicitly in mathematical textbooks or research papers, and of course not studied in language courses.

In the context of engineering degrees, formulas and algebraic expressions are widely used in almost all subjects, not only Mathematics. Teaching any subject in English could be a problem if students and teachers are not fluent enough to read the mathematical language. From our point of view, the lack of language fluency may become a useful tool for improving mathematical competence. Let us remark that this equivalence is also used in benefit of handling the symbolic language. Advice on mathematical writing (Halmos (1970) or Tomforde (2007)) states that mathematical expressions are no different from the words they represent, so they should be punctuated accordingly. Also, they need to be complete sentences, thereby preventing meaningless expressions.

Questionnaires with linked audio files were designed to check the real oral comprehension of formulas read in English. They were implemented in the digital campus of our university, based on Moodle. The result was the confirmation of difficulties of teachers and students, varying according to speed, gender of the speaker, native or non-native, and its power as a self-learning tool, as stated by Alsina et al (2012b). Our next step was to elaborate a suitable resource for learning how to read symbolic language, related with engineering education. Details are outlined below.

3. Multilingual Formulae

In this section we describe main characteristics of Multilingual Formulae resource, elaborated with the collaboration of the authors in the research group Linguatech.

Multilingual Formulae is an open access on-line collaborative resource available at http://mformulae.epsem.upc.edu. The main content includes tables of symbols with English support and sets of formulas for different topics. More concretely it contains tables to support English speech of symbolic language such as binary relations, symbols, scientific notation, and so on, including examples and audio files. Additionally, more than 600 formulas from different areas of engineering have been introduced. Each formula is expressed in terms of symbolic language, and text and audio corresponding to its speech form in several languages (Catalan, Spanish, English and some in French). Examples are showed in Figure 2.

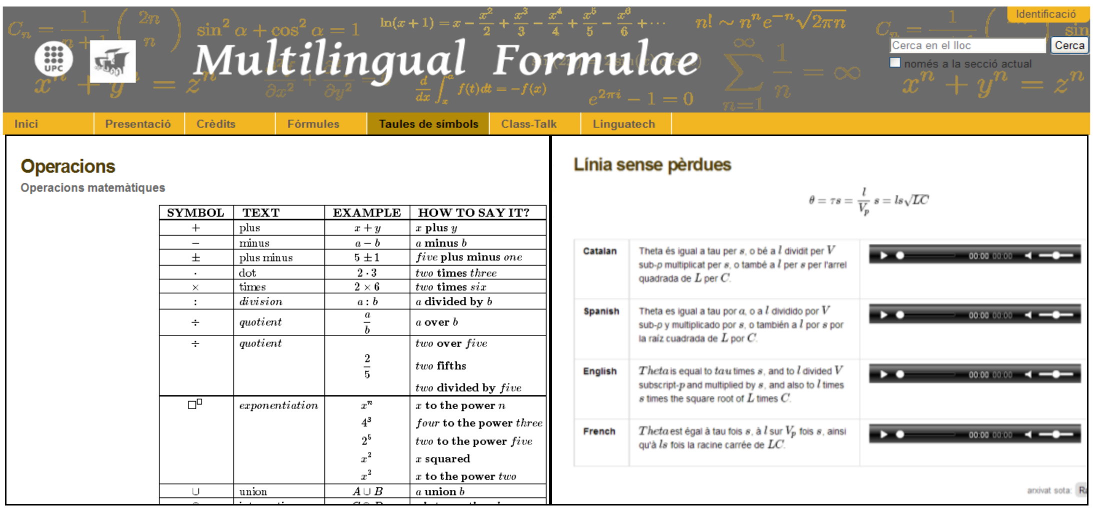

Figure 2. Examples in Multilingual Formulae at http://mformulae.epsem.upc.edu/

The project has been developed using Plone and TeX. It is the result of the teamwork of professors from different areas in EPSEM, who were in charge of designing the application, and suggesting and reviewing formulas for the different subjects. It cannot be considered finished as new formulas are being added after technical and linguistic revision.

The resource is addressed to lecturers and students as a support for the lack of fluency, to ensure effective communication when symbolic language is used. It also highlights the mathematical part of the formulas, improving content learning. Furthermore, it can also be helpful to increase self-confidence when oral presentations in a foreign language at professional or research level are involved.

4. Final remarks

The introduction of linguistic competence in addition to mathematical competence, motivated the analysis of context: the level of incoming students and difficulties reading symbolic language. But the parallelism between symbolic and natural languages becomes a learning tool when a foreign language is used, since it highlights the language equivalence. Indeed, language-aware positively supports content-aware. Furthermore, support resources are needed to avoid excessive pressure and assure quality learning. Consequently, the Multilingual Formulae resource is being developed to improve the natural reading of symbolic language in a foreign language.

Despite the focus of this paper being the handling symbolic language, we stress that it is just a tool, and we need to be aware not to trivialise mathematics, in the same way that an English curriculum would be impoverished if it focused largely on grammar issues (Schoenfeld (1992)).

Coming back to the parallelism between mathematics and language learning, let us add that besides handling language, recommendations for CLIL and mathematics have a lot in common: paraphrasing, reformulating, decrease speed of speech, etc. to make the discourse more understandable.

Finally, let us turn to attitude. It is well known that attitude significantly affects learning in general. In particular, the attitude a student has towards mathematics has a strong influence on the achievement of the mathematical competence and the mathematical behaviour of students. Moreover, the attitude of students towards mathematics is more positive when the environment provided by universities is perceived as being supportive (Shaw & Shaw (1999)). In that sense, the resources and support material built for scaffolding, with the excuse of foreign language, can have a double positive effect on mathematic learning. This is very encouraging for our Research group Linguatech.

References

Alsina, M., Soler, M. (2012) “Multilinguistic verbalization of symbolic language, a learning tool”. In N. Evnitskaya et al eds. *Proceedings of II International Round Table on Clil programes, VI Colloquium on Clil in Catalonia, TRICLIL-2012,* pp.148-152.

Alsina, M., Argelaguet, R., Martinez, I. and Vicente, J. (2012) "Material didáctico para el aprendizaje en inglés en el campo de la Electrónica”. *Proceedings of International Symposium on Innovation and Quality in Engineering Education* (to appear).

Council of Europe (1995), “Teaching and Learning, towards the learning Society”, *White Paper on Education and Training*, COM(95) 590.

Lasagabaster, D. and Ruiz de Zarobe, Y. (2010) “CLIL in Spain”: Implementation, Results and Teacher Training. Cambridge Scholar Publishing.

Lasagabaster, D. (2008) “Foreign Language Competence in Content and Language Integrated Courses”. *The Open Applied Linguistics Journal* Vol.1:31-42.

Niss, M. (2003).”Mathematical competencies and the learning of mathematics:The Danish KOM project. In A. Gagatsis and S. Papastravidis eds. *3rd Mediterranean Conference on Mathematics Education*, Hellenic Math. Soc. And Cyprus Math. Soc. pp.115-124.

Halmos, P.R. (1970) “How to write mathematics”, L’enseignement Mathématiques, Vol. 16 (No. 2):123-152.

Tomforde, M. (2007) "Mathemathical writing: a brief guide”, University of Houson. Accessed via http://www.math.uh.edu/~tomforde/MathWriting.pdf (february 2012).

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning*.MacMillan pp. 334-370.

Shaw, C.T. and Shaw, V.F. (1999) “Attitudes of engineering students to mathematics – a comparison across universities”. *International Journal of Mathematica Education in Science and Technology*, Vol. 30 (No. 1):47-63.

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The teaching of Mathematics in Engineering: a students' contribution

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Abstract

The paradigm that came as a result of the Bologna Declaration is underpinned by an assumption of change that aims to replace a teacher-centered model in the transmission of knowledge with a model of student-centered learning and knowledge construction. Assuming that higher education is intended to stimulate a greater initiative and autonomy of students, teaching practices used at this level of education are generally developed in an environment which is intentionally less structured, without supporting instruments that fit, specifically, the rhythm of materials and lessons. This process can cause many students some anxiety and puzzlement which undoubtedly is reflected in their attitude towards college and learning, as it not being particularly encouraged in primary and second level of education. This new teaching and learning environment gives rise to a greater demand on teachers of higher education, requiring greater critical capacity and greater reflection on pedagogical practice and a constant adjustment of educational strategies to address the difficulties and individual characteristics of their students. On the other hand, the need also emerges to align the implementation of strategies to the interests, motivations of students and also to adjust to the new student profiles that are currently in higher education so that the changes in attitude and behaviour towards college result in meaningful learning and therefore lead to greater success. In this context of these concerns, a research group GIDiMatE - Grupo de Investigação em Didática da Matemática na Engenharia, part of the Department of Physics and Mathematics (DFM) at the Coimbra Institute of Engineering (ISEC) was established. The intention of this research group is to contribute to participatory reflection and thoughtful pedagogical practice in higher education, which may influence, above all, a possible modification of concepts and an improvement of teachers' professional performance.

Arising from the high rates of repetition and dropout in higher education, particularly in modules on Differential and Integral Calculus during the 1st semester of this academic year a study was performed by the research group. The aim of the study was to carry out an analysis of the students' perceptions in such a way that they could be factored into the construction of a learning environment that allows a shared responsibility of the educational process by the participants and which could improve the quality of teaching and increase educational accomplishment. The conclusions reached as a result of this work will lead to the definition of a set of measures to implement in the next academic year. The monitoring of the actions proposed and subsequent evaluation to be carried out by participants in this research will enable the introduction of improvements to the quality of teaching. This exploratory study is framed within the action-research project "CAME-Understand Learning to Better Teach" which aims to deepen the knowledge of the causes of dropout in mathematics curriculum units, in order to develop strategies in the teaching process which lead to more meaningful learning for students.

Introduction

Arising from the determination to develop the quality of education in higher education the need for greater attention to be paid to student failure is increasing, making the promotion of student success a fundamental objective of institutions' action. Although in the last decade many studies have tried to understand student failure in higher education by conducting investigations that seek to know, in detail, how students learn at this level of education, some analysis on the relationship between the teaching methods and how students learn has not yet been developed. (Chaleta, E. et al., 2005).

The policies of access to higher education in Portugal, as exemplified by the regime aimed at those over 23 years old, or students from the professional and technological courses, brought new challenges to the university and polytechnic institutions, requiring them to adjust their provision, making them more attractive and competitive and able to motivate a heterogeneous audience as a result of their diverse needs and expectations. Additionally, the paradigm stemming from the Bologna Declaration suggests the need for a change of approach: to replace a teacher-centered transmission of knowledge model by a learner-centered model and assigning greater significance to the associated construction of knowledge by students. Thus, it is accepted that in addition to knowledge acquisition, the training component must include stimulating the development of personal and professional skills that enable students to better adjust to the flexibility, complexity and adaptability in different life contexts. The high rates of students needing to repeat and students dropping out in higher education, particularly in math curriculum units, have highlighted the need for questioning which teaching styles and methodologies should be applied and what learning environments, evaluation practices should be used so as to best enable students' co-responsibility in the educational process that may be necessary for their school success and lead to significant learning. (Barbosa, 2004; Resende, 2004; Rose 2011; Bigotte 2011). Curricular changes in higher education, introduced by the implementation of the Bologna process, have imposed higher requirements on their pedagogical practices of teachers, a constant involvement in students' careers and, consequently, an adjustment of educational opportunities to students' characteristics and difficulties with the aim of establishing strategies to improve student learning. Despite all this effort students have not reached our expectations and a high level of failure or high drop-out rates (in classroom attendance school and final or intermediate evaluations) are still noticeable. This work aims to implement various actions, including the development of several educational tools and instruments, particularly making use of information and communication technologies (ICT) with the development of a platform for e-learning, contributing to the consolidation and acquisition of skills. For the promotion of the various activities that make up the project, it was felt it was necessary to carry out a preliminary study, which would analyse the degree of knowledge of the syllabus material of Mathematics students engaged in ISEC undergraduate engineering degrees, and a consequent active reflection on higher education students' mathematics skills.

Today it is also generally accepted that evaluation practices must rely on the active participation of all students and contribute positively to the improvement of learning (Fernandes 2009). However, for that to happen, it is essential to integrate evaluation into the teaching and learning processes and in addition that evaluation becomes predominantly of a formative nature. While there is much research that attempts to explain the failure of engineering students in curricular units of Differential and Integral Calculus (DIC) there is limited scientific evidence of schemes that propose an integrated intervention to articulate, describe and analyse the teaching and practice of formative assessment, implemented in the context of a real classroom and relating it to the learning and academic success of students.

It is in this context and taking into account the high rates of repeating and drop-out in math curriculum units taught at ISEC undergraduate engineering degrees, a study during the 1st semester of this academic year was carried out which aimed to analyse students perceptions and to use this information to inform the construction of a learning environment that allows students and lecturers co-responsibility in the educational process and which could improve the quality of teaching and increase educational attainment. The conclusions of this work will lead to the definition of a set of measures to be implemented in the next academic year, with the monitoring of these proposed actions and subsequent associated evaluation by the research participants leading to improvements in the quality of teaching.

Method of Investigation

Once the central problem to be investigated was established, a qualitative approach was used to investigate students' perceptions relating teaching/learning strategies applied in curricular units of Differential and Integral Calculus (DIC). Qualitative research is a methodology of study that focuses on how to interpret and give meaning to the experience being examined, allowing the variables under study to be broadened, exploring and bringing a variety of possibilities to the interpretation of the phenomena under consideration. The study focused on a set of students enrolled in curricular units associated with Differential and Integral Calculus (DIC) taught in undergraduate engineering degrees. The sample consisted of 300 students distributed as follows: Biomedical Engineering degree (45), Civil Engineering degree (81), Electro-technical Engineering degree (50) and Computer Science Engineering degree (125).

To gather information which would inform the development of environments that allow student co-responsibility in the educational process and these relate to academic success, a questionnaire designed to collect information about the students' perspective in relation to the strategies applied in DIC, was distributed. Four civil engineering students who were attending a Numerical Analysis course and had been approved on DIC answered a pre-test pilot questionnaire in order to eliminate from the final questionnaire any questions which could be misinterpreted. The questionnaire was divided into 6 questions groups: - Group I personal data (9 items, e.g. sex; age, number of scheme, enrolments, …); GROUP II -data concerning the student's organization (3 items, e.g.: "asks questions to teacher(s); prepares on basis of …) ; GROUP III -data relating to curricular unit (8 items, e.g.: quantify the number of students who attend; rate study material provided a /support;…); GROUP IV -use of ICT in the learning environment (7 items, e.g.: do you think it is important to use ICT in learning; uses of the internet in its learning process …;…); GROUP V-indicators of students performance improvement in DIC (note this question was answered only by students who attended less than 60% of lessons) (e.g.: contribution of each of the following factors for drop-out and schooling (note this question was answered only by students who were not attending DIC for the first time) (e.g.: contribution of each of the following factors for failure,…); Group VI -to improve the learning strategy developed and also asked for suggestions, in the form of an open answer, corresponding to an overall assessment .The question responses were structured using a likert scale with five points, to capture the level of student agreement to the question posed, where 1 was the minimum and 5 was the maximum value.

The questionnaire was given in the 1st semester of the academic year (2011/2012) to students who undertook assessment by examination. In order to develop further studies within the framework of the attitudes and behaviours of individual students and to allow for tracking of the progress of individual students, student identification was requested on a voluntary basis, with the confidentiality of each student being guaranteed in the processing and dissemination of data.

Findings and Discussion

For analysis of the results a simple approach was adopted using only descriptive statistics. Since the data collected is mostly a set of ordinal variables the measure of central location used was the median.

The results from personal data -Group I section show that of those who answered the questionnaire 20% were female and 80% were male students. This means that the sample is a representative of the gender proportions of ISEC students. The results from the Group I section also show that 15.3% are working students. The background student profile is that they have mostly completed Scientific-Humanistic courses (56.7%) although 18% have completed Technology courses. Following the changes introduced as part of the reform of upper secondary school education, a growing number of students have access to ISEC via Technological Specialization courses, Professional courses and as mature students i.e. older than 23 years old (evidenced by the 12.9% recorded). From the total number of valid student responses (281), 77.2% attended Mathematics A. Student age and its frequency distribution is shown in the following table (Table 1)

	Frequency	**Percentage**
<19 years	98	32,7
19-23 years	147	49,0
24-27 years	30	10,0
27-30 years	4	1,3
>30 years	18	6,0
Not answered	3	1,0
	300	100,0

Table 1-Age distribution of students answering the questionnaire

In Group II questions "relating to the organisation of Student Data" the results show that students ask questions especially during classes, neglecting completely the personal contact with the teachers on the pre-established hours or at another time agreed between the parties. Students' preparation for the UC show a strong preference in solving hand-out problem sheets, personal notes, teachers' texts and solving assessments from previous years. It should be noted that the use of outside class personal explanations is the least valued item (18% of the students scored this variable with less or equal than 3). In responses to the questions in Group III that refer to "data relating to Curricular Unit" surveyed students when compared to other UC students rank the preparation time required and the Requirement of DIC more highly,. The same is true with regard to the Articulation between theoretical and practical material which achieves an equal weighting with the existence of practical work and exercises. In particular, the valuation given by the students to the item related to the ease of understanding (median 3, with 76.7% of students that give rate not less than 3) is stressed. This result may be inconsistent with their behaviour as evidenced by drop-out and school failure rates. Students recognize that preparation time dedicated to DCI takes place essentially right before when the assessment is scheduled to take place rather than being a continuous study process. Daily study (which was assumed to be of at least 1 hour) was not identified as a preparatory practice which the students adopted (71.3% of the students assigned this a score less than 3). In relation to the adequacy of the evaluation system students showed a clear preference for distributed assessment being reinforced by the implementation of achievement of mini-tests and homework as complementary strategies to be applied. An exploratory study conducted by the authors found that ISEC students have a predominant profile of learning styles, like their counterparts in other colleges, indicating that students who attend DCI are mainly, active, sensory, visual and sequential learners. Even assuming a specific profile for students of engineering, teachers should recognize that their class groups include all types of learners, and that to meet the engineering needs of all of them an effective lecturing approach must appeal to all learning styles through balanced activities (Bigotte, E., Fidalgo, C., Rasteiro D.M.L.D., 2012). To this end and with the intention of establishing the strategies to be used by mathematics teachers in the classroom context a questionnaire was deployed with a group of questions that lists a set of activities that, according to various authors (Felder, 2002; Gomes, A., 2010), are more suitable to each type of learning style. The analysis of the questionnaire concluded that the items most valued by students are consistent with what may be found in the literature and so the activities being considered were adjusted to the learning styles advocated. However, the students' options for items that apparently would not be of a type that fit the learning styles identified in the previous study can be explained for several reasons. Included among the reasons for this might be the application of these strategies by the teacher, which may prompt students to engage in ways that run counter to habits acquired already and also the recognition of the important need for students to develop other forms of learning as a way to increase the benefit to be obtained by contact with different education methods and their teachers preferences.

As outlined in various scientific studies, information and communication technologies (ICT) have emerged as a mean that enables the construction of a set of support instruments complementing the student presence in class and allowing its users greater interaction and ease of communication, therefore bringing a greater involvement of students in the learning process. Email, the internet, virtual learning platforms and other communication tools, are considered as tools to help diversify ways of teaching in comparison to traditional teaching methods, so their use has been increased, requiring of teachers an increasingly continuous transformation in their teaching models. It was with this in mind that we intended to obtain an insight into student opinion on the use of ICT in the learning environment, through the group of questions – GROUP IV, and hence lead to a more detailed analysis on the topic. It was concluded that ISEC students have a very easy access to computers (only 2.5% have limitations in this regard) and also very easy access to the internet (only one student revealed difficulties in accessing the internet).

The importance that students give to the use of ICT in learning its relevance is well-known, and as expected the Use of the platform was an item that was rated highly. Regarding the use of the internet in the process of learning students mainly valued the platform (Virtual Laboratory of Mathematics or Moodle) where the curriculum unit is integrated, which corroborates the data analyzed previously. However, the Exchange of collateral (e.g. digitalisation of notes) is an item that was also quite appreciated by students. Taking into consideration the importance of online media in today's society interestingly the students attach little significance to the feature of sending email to teachers except when it comes to questions on the subject. Taking into consideration the diverse audience that accesses the ISEC, there has been a need to provide more flexible arrangements. Already for several years BY learning in the Department of Physics and Mathematics (DFM) has been encouraged through the adoption of a distance component of b-learning. The "LVM-Virtual Lab of Mathematics" (http://lvm.isec.pt/) which is based on an e-learning platform (Moodle) (Correia, a., Alves, J.P.) and is a supplement to face-to-face education that supports all aspects of the Scientific Area of Mathematics taught in undergraduate engineering in ISEC. This is an environment that aims above all to facilitate access to content and provide a means, in addition to the classroom, where teachers and students can interact and communicate with each other and this environment has been shown to be a solution as a strategy for teaching/learning which encourages student involvement, and hence helps if one wants more student participation in the educational process. To examine this aspect, a group of items was organized to query student's perception regarding the use of LVM/Moodle in the learning process. The analysis of the respective data suggests that students strongly use the platform for extraction of support material, and that its use is also very highly valued in Consultation of materials made available by colleagues and notices/information. It should be noted that the values obtained from the survey answers might suggest that the use of the platform as a pedagogical resource inserted into the learning environment is not a usual practice by teachers and that the students rate the option the use By teaching requirement lowly. However, the complementary data collection needed to consider this last hypothesis with a view to gaining a better understanding of the relationship between the behaviours of students and teaching practices adopted by teachers is lacking. With regard to use of software adapted to syllabus students indicated their preferences for the option, checking the solution of the problems resolved, rather than other options given which were to do with obtaining support in the problem solving process.

As mathematics teachers, teaching on various degrees in ISEC, the authors are confronted with growing issues of de-motivation, disaffection and consequent students drop-out from classes and assessments, which inevitably leads to high failure rates and subsequent concern by teachers. To get the student perspective of their co-responsibility and effectiveness in the educational process a set of questions relating to this area - Group V "indicators of improvement for performance of pupils" was included in the survey. With respect to resources in this regard, Support they consider useful to improve the learning process, student responses show that DIC functioning in sliding system was the most valued option. This mode of operation which was initiated, in the year 2002/2003, in an attempt to remedy the failure detected over the years, in mathematics courses taught in 1st year. The pedagogical approach "Sliding Disciplines", which became part of the teaching approach, after analysis and corrections were made to optimize resources and improve results, is based on some specific criteria. The most relevant one is that the students are only allowed to take the final exam if they have attended a minimum of 60% of the total lectures. By virtue of DIC using a cumulative study plan scheme, the analysis of students ' behaviour regarding this requirement of attendance, means that the number of students admitted to the final assessment is lower than the observed in the regular semester in which the curriculum unit takes place and this is a scenario that facilitates teaching strategies which are effectively closer to the needs of the students. This approach is reason enough for students to assess very positively the continuation of this initiative as it improves the teacher/students interaction, essential for achieving academic success. The students also showed preference for supplementary classroom arrangements such as Short Courses and Support Centre, to promote an improvement in their learning process, would further encourage the authors to increase efforts to foster the conditions for the creation of learning supports not yet implemented in ISEC.

It was decided it would be relevant to find out, in the opinion of the students, what the Factors of drop-out to school are so that once these were identified they could inform the development of a set of strategies to counteract high drop-out rates shown in ISEC and already examined by the authors (Bigotte, E., Fidalgo, C., Rasteiro, D.M.L.D., 2012). To this end only students who had attended less than 60% of lessons could answer group of questions associated with this aspect. The percentage of student answers in this category was 47.7%. These students clearly indicated their own lack of effort/persistence (85.2% with a score of not less than 4) as the factor that contributes to the school abandonment, and did not assign great significance to the Lack of competence of the teachers (81.3% of those who answered rated 1 or 2 to this item). The items lack of teaching strategies, the lack of follow-up of the teachers and the intellectual difficulties were indicated as less important by the students as well. Generally, students attribute disengaging with college to very personal reasons and these reasons should be the object of a more detailed study.

Given the high levels of failure, especially on summative assessments, of students who do not attend DIC for the first time, namely the students repeating the course, a set of survey items were included to try to establish a measure of the contribution of a set of factors for failure (44.7% of respondents answered questions related to these items group). From examining students responses in this part of the survey there was no strong correlation with any specific item and providing evidence that Inadequate Evaluation is a factor with little influence on the results obtained by the students. These findings reflect the need to develop further studies that examine the relationship between teaching, assessment and learning in existing educational interventions for DIC.

Conclusions for Education

The evidence gathered in this exploratory study is based mainly on student responses to the questionnaire, which was intended to identify student perceptions regarding teaching/learning strategies applied in courses taught in the 1st semester of the 1st academic year and the impact of these in Differential and Integral Calculus modules in undergraduate engineering degrees, and to use the results gathered to inform the development of environments that foster and promote student co-responsibility in their education and related academic success. In general the survey indicates that students: 1) pose their questions during lectures, and also use email for the same purpose; 2) prepare for the assessments by completing problem sheets, personal notes, other teachers texts/notes and by the completing tests from previous years; 3) consider that curricular units of Differential and Integral Calculus need greater preparation time and demand than the other units of the same year of the degree course; 4) dedicate their time learning the curriculum unit only in the time period close to the exams; 5) have a preference for distributed evaluation models with completion of mini-tests and additional work; 6) prefer the activities that happen in classrooms that most suit their learning styles; 7) prefer to use the Math/Moodle Virtual Lab to exchange materials, consult notes and look at warnings/information posted by teachers.

The survey results show that the students consider the lack of their effort/persistence as the key factor that contributes to the school abandonment. The survey results also indicate that there is no strong correlation with any specific item that can be related to failure. These findings suggest the need to develop further studies that examine the relation between teaching, assessment and learning in existing educational interventions for DIC.

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References

Barbosa, M. A. (2004) O insucesso no ensino e aprendizagem na disciplina de cálculo diferencial e integral. PUCPR. (Dissertação de Mestrado).

Bigotte de Almeida, E., Fidalgo, C. Rasteiro, D.M.L.D., (2012). “Compreender o insucesso da Matemática nas licenciaturas de Engenharia: Um estudo de caso”, XIX Colóquio AFIRSE: Revisitar os estudos Curriculares: onde estamos e para onde vamos?

Bigotte, M.E., Fidalgo, C., Rasteiro, D.M.L.D.(2012), Estilos de aprendizagem e estilos de ensino: Um contributo para o sucesso da Matemática nas licenciaturas de Engenharia Comunicação apresentada no XIX Colóquio AFIRSE: Revisitar os estudos Curriculares: onde estamos e para onde vamos?

Bigotte, M.E., Pessoa, M.T.(2011), Aprender matemática nas Licenciaturas de Engenharia:um estudo de caso.In Atas do XI Congreso Internacional Galego-Português de Psicopedagoxia (pp.3495-3503). Corunha: Universidade da Corunha (Cd-Rom)

Bigotte, M.E., et al.(2012), Projeto “ACAM - Avaliação de Competências/Ações de Melhoria”.Comunicação apresentada no XIX Colóquio AFIRSE: Revisitar os estudos Curriculares: onde estamos e para onde vamos?

Chaleta, E., Rosário, P., & Grácio, M. L. (2005). Atribuição causal do sucesso académico em Estudantes do Ensino Superior. In Actas do VIII Congresso Galaico-Português de Psicopedagogia (pp. 819-826). Braga: Universidade do Minho (Cd-Rom).

Felder, R. M., Felder, G. N., & Dietz, E. J. (2002). The effects of personality type on engineering student performance and attitudes. Journal of Engineering Education, 91 (1), 3-17.

Gomes, A. (2010) PhD Thesis, Dificuldades de aprendizagem de programação de computadores:contributos para a sua compreensão e resolução, Universidade de Coimbra

Resende, W. M.(2003) O ensino de Cálculo: Dificuldades de Natureza Epistemológica. São Paulo, Tese (Doutorado em Educação). Faculdade de Educação, Universidade de São Paulo, USP.

Rosa, O.S.,(2011) Aspectos Motivacionais do Cálculo Diferencial e Integral, Universidade de Severino Sombra,Programa de Pós-Graduação Stricto Sensu, Mestrado Profissional em Educação Matemática, http://www.uss.br/arquivos/Mestrado_Edumat/DISSERTACAO_ODILEIA_DA_SILVA_ROSA.pdf, accessed on 27th April 2012.

Lecturers' beliefs and practices on the use of computer-aided assessment to enhance learning

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Abstract

Computer-aided assessment (CAA) has been used at a university with one of the largest mathematics and engineering undergraduate cohorts in the UK for ten years. Lecturers teaching mathematics to first year students were asked about their current use of CAA in a questionnaire and in interviews.

This paper presents the issues that these lecturers face as they make use of this efficient and timesaving assessment tool. Lecturers explain how they have attempted to overcome these issues. These findings are a progression towards our overarching aims to identify best practices in the delivery of CAA to mathematics and engineering students and to evaluate the effectiveness of CAA at assessing and advancing students, from the perspectives of both the lecturer and the student.

Introduction

Computer-aided assessment (CAA) has become a popular and efficient method for assessing large cohorts of students. There are many reasons why CAA is so keenly adopted: it allows more frequent assessments covering more material, providing more feedback while reducing marking load – among other benefits suggested by Bull and McKenna (2003).

In the last twenty years, several CAA systems have been developed to capture these benefits in the teaching of mathematics in higher education – including STACK (Sangwin 2006) and Mathletics (Gill and Greenhow 2007). The HELM project (2006) was a joint development, between several UK universities, of learning materials to help engineers learn mathematics – including a CAA system based upon the QuestionMark Perception assessment management system (Harrison et al. 2007). A separate bank of questions designed to be used on the same system, but for mathematics students, was developed concurrently.

Since the start of the development of the HELM project ten years ago, there has been a call for further research on the use of CAA with students. Conole and Warburton (2005) suggest that further research on the teaching, learning and assessment outcomes of CAA would be welcome. Miller (2009) adds that research is desired to explore how students use both the formative and the summative aspects of CAA simultaneously.

In light of these calls for research, this study serves as a review of current CAA practice at one of the core institutions charged with the development of the HELM project. At this university there are currently over 200 first year mathematics students and around 600 first year engineering students, of which nearly all experience CAA in their first year.

The current situation

In spite of the initial advantages that CAA provided, the current situation is not so straightforward. Most classes are too large to be accommodated in one computer laboratory so some lecturers lack the resources to invigilate the online summative test. Some lecturers are concerned that the questions do not fully cover all aspects of a developing syllabus; and the time and learning requirements to develop new questions are prohibitive. For some, the existing questions are too focussed on testing the students' ability to carry out procedures, rather than exploring conceptual understanding.

These problems and others suggest that lecturers might not be able to teach according to their pedagogical ideals. Engeström (2000) suggests that such contradictions drive actions towards an "expansive solution" (p.966). However, while Engeström envisaged a collaborative and coordinated response from a body of practitioners, this has not happened since lecturers may respond to their own teaching needs.

This paper examines the lecturers' experiences of the use of CAA: the problems they have faced; how they have sought to mitigate the effects of these problems; current practice and how this has been shaped by these problems; and how they anticipate using CAA in the future. Thus we ask the following research questions:

- **RQ1.** How is CAA implemented in first year mathematics modules for mathematics and engineering students at this university?
- **RQ2.** Why are lecturers using CAA?
- **RQ3.** What are lecturers' perceptions of issues arising?
- **RQ4.** How are lecturers dealing with these issues?

Method of investigation

All thirteen lecturers of first year mathematics modules were approached to complete a questionnaire. Four lecturers reported they did not use CAA; the remaining nine returned completed questionnaires.

The first half of the questionnaire was aimed at addressing RQ1. Each lecturer was asked how CAA was implemented and delivered to students, covering the availability of practice tests, the format of the coursework tests, the type of feedback provided, whether the test is online or paper-based and the policy on collaboration between students. The second half of the questionnaire focussed on RQ2, asking lecturers what CAA assesses, the reasons for using CAA and the authoring of new CAA questions.

Six of the nine lecturers that completed the questionnaire indicated they would be willing to take part in follow-up interviews. The first author conducted semi-structured interviews with these six lecturers, which lasted between 27 minutes and 54 minutes. The interviews elaborated on the responses lecturers gave in the questionnaires, and explored lecturers' perceptions of the issues arising (RQ3) and how they are dealing with those issues (RQ4).

We present the findings of both the questionnaires and the interviews by considering each research question in turn.

Lecturers' implementation of CAA

All nine lecturers use CAA practice tests with their students. Seven lecturers use CAA for the online summative test: of the remaining two, one tests students using paper-based tests and, for the other, the CAA practice tests are not followed by an analogous coursework test.

Three of the seven lecturers that use the CAA coursework test invigilate the test in a computer lab; and the paper test is invigilated in a lecture theatre. The remaining four lecturers allow the students to conduct the coursework tests at the location and time of their choosing within a specified time period (usually two or three days).

The availability of practice tests before and after coursework tests differed substantially between lecturers. Of the seven groups of students that perform coursework tests online, three groups were permitted to revisit the tests after the coursework test was complete and four could not. How long practice tests are available to students before the online coursework test also varied between lecturers.

Why lecturers use CAA

In the questionnaire, the lecturers were presented with nine reasons for using CAA with which they could agree or disagree on a 5-part Likert scale. There was strong agreement that CAA frees up time, is convenient, provides opportunities and motivation for students to practice and provides immediate feedback. However, it is also evident that lecturers did not universally believe that the feedback CAA offers is reason enough to use it (one lecturer strongly agreed that students receive good quality feedback from CAA is a reason for using it; three agreed; three neither agreed nor disagreed; and two disagreed).

The reasons for using CAA were discussed further in the interviews and four prominent themes emerged.

All six of the interviewed lecturers say they use CAA because previous lecturers of the modules have used it, thus it has become *established practice*. Inheritance was a common theme in the interviews: "that's what I inherited" (Participant 3); "I've inherited it that way" (P4); "I've inherited it with the Calculus module that was taught previously" (P6).

The participants explained they have had informal discussions with colleagues on the topic of CAA; however, the effect of these discussions on the use of CAA differs between lecturers. While some lecturers believe that the *departmental influence* was an intrinsic reason for using CAA – "I use it because I've been told to use it" (P5); "I suppose [these discussions] have been a strong influence, because I hadn't used it before" (P1) – one lecturer replied, "Not much" (P4), when asked how much influence discussions with other lecturers have had on the use of CAA.

Lecturers are keen not so spend an inordinate amount of time on assessment, particularly given it is easy to do so with groups as large as 230 students: “If I’m going to consider a written piece of assessment… there’s too much marking involved” (P5). CAA not only marks and provides feedback instantly – it also handles the distribution of the assessment, *saving time*: “it certainly frees up your time; it’s convenient” (P3).

CAA can also be used as a tool to foster *student responsibility* and maturity for learning at university level. Lecturers said: “[With CAA] they have to take a strong degree of responsibility for their own learning” (P1); “with the computer tests, I think you encourage them to go and do some work” (P2). Another lecturer reported that students seem to accept this responsibility readily upon viewing the access data: “it turns out they do this [the practice tests] quite a bit… For me it was a surprisingly high average of how many times students do these tests” (P4).

Lecturers’ perceptions of issues arising

While lecturers are strongly encouraged to use CAA when presented with the four themes, they acknowledge that there are issues.

The lecturers disagreed when asked whether the CAA questions provide sufficient challenge to students (two believe they do; five believe they do not; two neither agree nor disagree). Most lecturers feel that CAA is most effective at testing procedural ability (“it’s quite effective at making sure that they can carry out the procedures” (P6)), but there is less scope for testing conceptual knowledge and recall (“I don’t think it tests their recall, because they can have all their materials in front of them” (P6)).

In order to test recall, it would be desirable to invigilate students as they attempt the summative test. However, while CAA is effective for assessing large groups of students, accommodating large cohorts in a single computer laboratory is not possible. To overcome this, four of the lecturers give students much more freedom over their environment and timing when performing the practice tests and summative test. For these students, lecturers cannot be sure how the test was completed: “how do you know who’s done it? How do you know that they’ve done it on their own? How do you know if they’ve copied from somebody else or from the book?” (P1).

New questions would be needed to assess conceptual understanding. However, only three of the lecturers have attempted to do this; and only two felt it would be worthwhile to learn how to do so. Although some lecturers would be confident in developing questions that suit the system, learning how to develop such questions for the system is time-consuming and, therefore, not worth such effort. One lecturer explained: “There is a system where you can write your own questions, but that is a lot of work. I think it’s five hours for one question, and you really have to learn the system” (P5).

The issues that lecturers have encountered are often attributed to the age of the system: “this is such an antiquated system” (P4), for example. While some lecturers are keen to move onto another system, the existing bank of questions remains a treasured resource (“if we use something else, then that means we’ve got to leave behind the question bank that we’re using” (P6)). Instead, lecturers have adjusted their practices in order to mitigate the effect of these issues.

How lecturers deal with CAA issues

The issues that lecturers have identified while using CAA cause potential conflicts. For example, a lecturer may want to assess students' deeper understanding, though it seems that CAA does not test students' conceptual knowledge so well. Such conflicts give scope for change (Engeström 2000).

In this case, some of the lecturers in our study have accommodated other assessment techniques in their modules that better test conceptual knowledge. Lecturers use coursework and projects to explore the grasp of underlying concepts, while exams cover aspects of recall, procedural ability and conceptual understanding. CAA remains a useful tool for testing procedural knowledge: "[CAA] is quite effective at making sure that they can carry out the procedures… I just test the procedures through computer courseworks, and I test the conceptual things through written courseworks" (P6).

A consequence of these adjustments is that the weighting of existing assessments had to be changed in order to accommodate new assessments. Indeed, the allocation of module scores given to CAA decreased in some modules: "we actually restructured the whole assessment and reduced the number of CAAs and also the weighting, because the coursework then took that weighting away" (P5).

The low weighting of the CAA component across these modules also mitigates the problem of not being able to invigilate students' attempts at the summative test. The impact of cheating is small ("For 2.5%, I just don't think it's worth putting up a major police operation to find out what students really do" (P4)) and lecturers are more concerned that students are motivated to practise for and complete these assessments ("I'm not sufficiently worried about it to really make my own life and theirs [the students'] much more difficult by starting to run it as an invigilated test" (P3)).

Lecturers still have misgivings about the CAA system. Some describe it as "awful" (P4), "a nightmare" (P6) and "poor quality" (P3); though they acknowledge that, for some things, CAA is "good" (P5). Further change of the assessment structure is not anticipated: rather, any development in CAA would be of the system ("I guess it's only a matter of time before we get some other kind of system, which will undoubtedly do something things better" (P3)).

Discussion and conclusions

This study of the use of computer-aided assessment by lecturers has identified a number of dilemmas that lecturers face. Furthermore, it has highlighted the means by which these lecturers have addressed these issues and why they continue to use CAA with their students.

- CAA saves time in compiling, distributing and marking assessments. This is particularly advantageous with large student groups. Other aspects of CAA are less efficient, however: developing new questions remains a challenge and is not considered worthwhile.

- The large question bank is considered a valuable resource and ensures that compiling assessments remains efficient. However, lecturers feel that there are aspects of students' understanding that are untested by CAA and students are not sufficiently challenged. Lecturers have made use of other assessment techniques in order to acquire a more rounded understanding of their students' progress.

- By giving students a routine of practising before a test, it allows them to develop a mature and independent approach to their learning. In large groups, where summative tests are not invigilated, there is a threat that students could abuse this freedom. In response, the CAA summative tests matter less towards the overall module work – and despite this, students remain motivated to complete the tests.

The lecturers in this study believe that the advantages that CAA gives them are greater than the disadvantages. The disadvantages are such that the lecturers have had to make compromises: there remain contradictions between lecturers' pedagogical ideals and the practice they are able to employ.

Of great disadvantage to the lecturer is that the CAA system does not test all aspects of a student's understanding of these mathematical topics. The result is that lecturers have had to adopt other, more time-consuming assessment techniques.

A clear message the lecturers give in the data is that they acknowledge the shortcomings of the system and have tried to address them. They would welcome a CAA system that maintains the benefits of the existing system while also addressing the shortcomings.

References

Bull, J. and McKenna, C. (2003) *Blueprint for computer-assisted assessment*. London: RoutledgeFlamer.

Conole, G. and Warburton, B. (2005) A review of computer-assisted assessment. *Research in Learning Technology*, 13(1):17-31.

Engeström, Y. (2000) Activity theory as a framework for analyzing and redesigning work. *Ergonomics*, 43(7): 960-974.

Harrison, M.C. et al. (2007) HELM educational transfer. *MSOR Connections*, 7(1): 20-22.

HELM (2006) *HELM - Helping Engineers Learn Mathematics*. Available at http://helm.lboro.ac.uk [Accessed 30 April 2012].

Miller, T. (2009) Formative computer-based assessment in higher education: the effectiveness of feedback in supporting student learning. *Assessment and Evaluation in Higher Education*, 34(2): 181-192.

Sangwin, C. (2006) Assessing elementary algebra with STACK. *International Journal of Mathematical Education in Science and Technology*, 38(8): 987-1002.

The Mathexplorer System : Student Exploration with a Matlab-based system

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Abstract

This article describes an approach currently being carried out at the University of Manchester where students are encouraged to explore and appreciate the mathematics as a complement to the traditional-style lectures. 1st year students in Electrical Engineering use various 'notebooks' making us of Mupad in order to explore some mathematical topics and gain an appreciation of the wider concepts surrounding the topics and how they fit into the appropriate topics within Electrical Engineering. Students submit some printouts from their explorations each week and these are discussed in classes and further feedback is given. There are currently plans for parts of the scheme to be extended to second-year topics.

Motivation

The school of Electrical and Electronic Engineering (EEE) exists within the Faculty of Engineering and Physical Sciences at the University of Manchester and admits about 180 students to the first year of Undergraduate study. Of the 120 credits of study in year 1, 20 credits (10 in each semester) of mathematics courses are taken along with 100 credits of Electrical Engineering courses, some of which e.g. Circuit Theory, Electromagnetic Fields, make extensive use of the mathematics.

The material taught in the mathematics courses to EEE students has been of high-quality; it has enabled students to carry out routine (and sometimes less routine) calculations of the type encountered in Electrical Engineering. However, there are some aspects where it was felt that an additional tool or emphasis would be appreciated.

- Students could often carry out a calculation involving specific parameters e.g. analysing a circuit with particular values of resistance, capacitance etc. but did not see a more general picture of how the solution changed when the parameters were varied

- Students often struggled to use the mathematics in an Electrical Engineering situation despite the context often being shown. It was almost as if they struggled to comprehend that it was indeed the 'same' mathematics.

- Students often had difficulty visualising some geometrical situations, particularly in three dimensions, or properties of functions.

- Students were in danger of taking certain principles and properties for granted rather than exploring them properly.

In response to those points, it was decided to look for/create a tool which would allow students to experiment with functions, find solutions for various parameters, visualise situations etc. Initially two tools were considered i.e. Geogebra and Mupad. Mupad was chosen, partly due to its relationship with Matlab which the students were due to meet later in the course. Clearly, merely allowing the students to carry out unstructured exploration would not give meaningful results other than for the very brightest of students. Instead, it would be necessary to produce a structured series of materials or worksheets each giving the students the option to explore particular mathematical topics within particular parameter and function ranges.

Construction of Notebooks

In Summer 2011, the school of EEE awarded £ 4 000 to the first author to produce a series of workbooks for use in academic year 2011-12 and beyond. The plan was for two students to carry out authoring of a series of notebooks covering various topics from year 1 of the mathematics course for EEE students. However, the preliminary stages involved extensive interviews with members of staff in both the school of EEE and the school of mathematics, in the former case to ensure that any applications were indeed relevant and in the latter case to ensure that the pattern of the notebooks followed the syllabus and that matters of notation etc. were consistent; this was indeed a joint project between the two schools. The intention was always that the notebooks would complement existing courses and materials rather than replace them.

For two months in Summer 2011, Mustafa Ali and Jahangir Saif were employed as part of the project. Following a period of consultation and planning, they wrote 3-4 notebooks per week. Generally each notebook would undergo a time of authoring, followed a few days later by a time of checking and testing. Each student was generally working on several notebooks at a given time, with the relevant notebooks being at different stages.

A typical Notebook

Generally, the notebooks would all follow the same structure with there being differences in the detail given the subject matter. An introduction is short as the main description forms part of the relevant lecture course rather than the notebooks as other materials are available e.g. Harrison et. al (2004). The introduction section also allows students to define any functions that they wish to use and to define horizontal and vertical extents for graphs.

Figure 1 shows the relevant part of a typical notebook (on Maxima and Minima). The three central lines (in green on the screen) allow the student to control the functions and parameters. The left part contains the functions and parameters actually used with the central and right parts (effectively there as comments) giving the description and the default values. Students are not required to do any Matlab coding but are required to be able to enter simple functions.

Notebook's global variables (*editable, remember "Notebook - Evaluate All"*)

```
f := x^3-6*x^2+9*x+3:  // Function to evaluate (f := 2*x^3-9*x^2+12*x-3:)
xRange := 0..4:          // x-Range for plot     (xRange := 0..3:)
yRange := -6..9:         // y-Range for plot     (yRange := -2..6:)
```

2 Minima and Maxima

Figure 1 : The part of a notebook where functions and parameters are defined.

The middle parts of a notebook cover various sub-topics and reflect the choices that the student has made in the introductory section. For example, staying with maxima and minima, students can see the effect of their choice of function. The system calculates the derivative of the function and the position of any critical points as well as plotting the function and the derivative (see Figure 2) drawing attention to the fact that the derivative is zero at critical points.

$f(x) = x^3 - 6x^2 + 9x + 3, \quad f'(x) = 3x^2 - 12x + 9$

x	f(x)	f'(x)
1.0	7.0	0.0
3.0	3.0	0.0

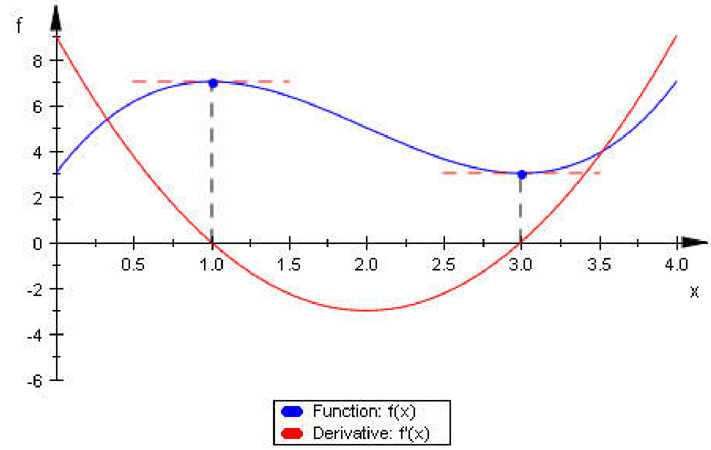

Figure 2 : Students choose a function and this is plotted along with its derivative and information about critical points.

The notebook on maxima and minima, has a third section which distinguishes between the two extrema. Once more, it uses the function provided in section 1 and plots the function and the second derivative along with the local quadratic approximation (second order Taylor series, although this terminology has not been developed by this point of the course) at each of the critical points, thus demonstrating the role of the second derivative on the nature of the critical point.

$f(x) = x^3 - 6x^2 + 9x + 3, \quad f'(x) = 3x^2 - 12x + 9, \quad f''(x) = 6x - 12$

x	f''(x)	t(x)
1.0	-6.0	7.0 - 3.0*(x - 1)^2
3.0	6.0	3.0*(x - 3)^2 + 3.0

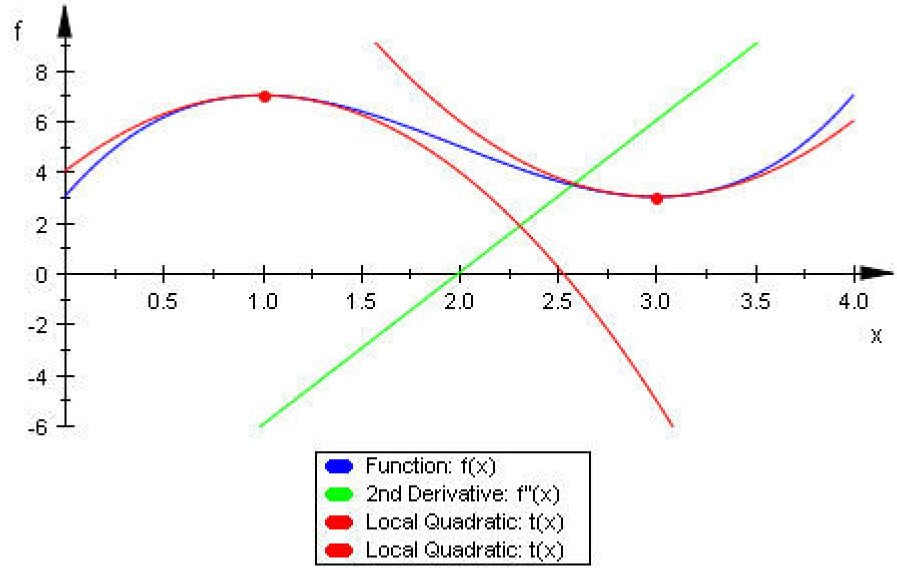

Figure 3 : The second derivative and the local quadratic approximation at each critical point.

The final section of each notebook is a series of exercises that students should carry out (see Figure 4). Of course, students are encouraged to explore further and find out other interesting facts for themselves.

Q1) For the default function, describe the difference between the two points of inflection at x = 1 and 2.

Q2) For the following functions:

```
a) f := 2*x^2 - 8*x + 8:     xRange := 0..4:    yRange := -4..8:
b) f := x^3-6*x^2+9*x+10:    xRange := 0..4:    yRange := -10..20:
c) f := x^2 + cos(2*x):      xRange := -2..2:   yRange := -2..4:
```

* Write down the expression for the function's derivative (example (a)).

* Plot the function and list the critical points (example (a)).

* For each critical point, state whether the point is a maximum or minimum, by calculating the value of the 2nd derivative at that point (example (b)).

Figure 4 : Exercises on Maxima and Minima

Use of Notebooks

The notebooks form an integral part of the 1st year for EEE students and are a vital link between the mathematics course and the core EEE activities. At the beginning of the academic year, students are introduced to the notebooks and invited to download them using a wordpress site (Brown (2011)). The introduction takes place in a supervised session and students can then start work on the first notebook, on exponentials. This is a topic that the great majority will have met previously (or which others may experience on a diagnostic followup (Steele, 2009)) so that the emphasis can be on getting used to the notebooks. Each week subsequently, students are informed of a new notebook relevant to the studies at that point. They are expected to carry out several exercises, make relevant printouts and hand these in, in advance of a small group tutorial session each week.

This small group tutorial session deals with up to six different courses per week but will treat one course as the 'main' topic each week so, other than at the extreme end of the semester, students will not know which subject will be the main one. On the occasions when mathematics is the main topic, there is a discussion based on the various submissions. On other occasions, there may be a very brief discussion on these topics.

Semester 1 notebooks include exponentials, vectors, cross products, complex numbers, complex arithmetic, differentiation, Newton Raphson, Maxima and Minima and Impedence. Semester 2 notebooks include indefinite integrals, definite integrals, Taylor series, multivariable functions & partial differentiation, multivariable Taylor series, multivariable stationary points, introduction to ordinary differential equations, unforced ODEs & complementary functions, Step-response ODEs & Particular integrals, Sinusoidal response ODEs & Particular integrals and applications to LRC circuits.

$$0.5\,\frac{\partial}{\partial t}\,y(t) + y(t) = \mathrm{heaviside}(t), \qquad R = 100, \qquad C = 0.005$$

$$y(t) = -\,\mathrm{heaviside}(t)\left(\frac{1}{e^{2.0\,t}} - 1.0\right), \qquad \tau = 0.5$$

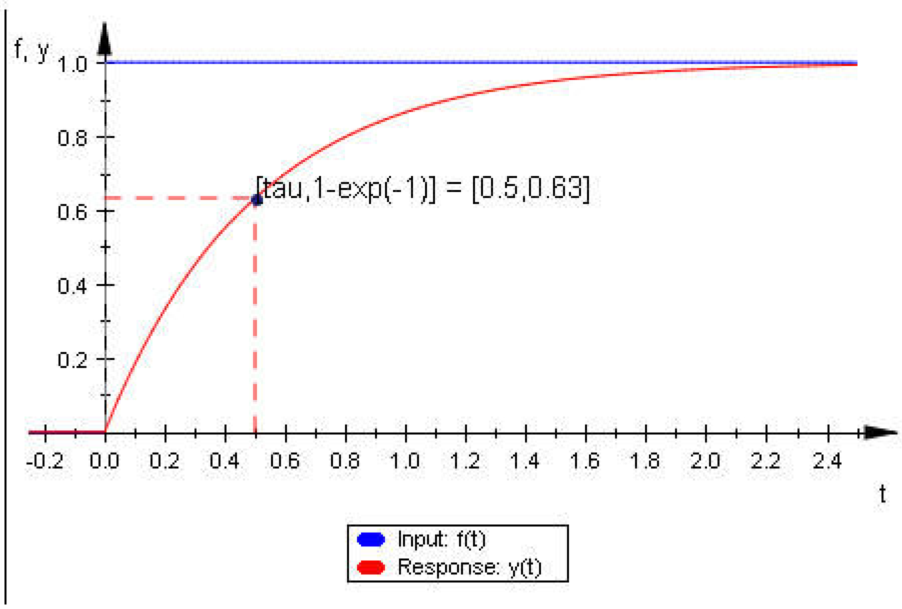

Figure 5 : Voltage across a capacitor subject to a unit-step voltage input.

Several of the notebooks e.g. Impedence, LRC Circuits, tend towards applications in Electrical Engineering. For example, when RC circuits are being considered, students are invited to enter their own values for R and C and are presented with the relevant differential equation and solution for the voltage across the capacitor (Figure 5)

Extension to second year

During the period March to July 2012, notebooks are being written for the second year courses delivered by the school of mathematics to students in the school of EEE concentrating on such topics as Laplace transforms, Vector Calculus, Linear Algebra (matrices, eigenvalues, Gaussian Elimination). This work is funded (£ 7 000) by the TESS fund within the Faculty of Engineering and Physical Sciences and is being used to employ the PhD student Ebtihal Gismalla to create the appropriate notebooks. It is planned to use the resultant notebooks with year 2 students during the first semester of academic year 2012-13.

Conclusions

Notebooks have been produced for the majority of topics in year 1 and are in construction for topics in year 2. The notebooks have been an integral part of the 1st year for EEE students. The majority of students have engaged with the notebooks and a number have carried out their own explorations. There do remain a few for whom the use of the notebooks has turned into an exercise in following a few instructions and attention will be given to getting them to engage more.

References

M Brown (2011) “Mathexplorer” Accessed via http://mathexplorer.wordpress.com/ (3rd May 2012)

Harrison, M.C., Attenborough, K.A., Stirling, D., Farrow, M. & Steele, C., (2004) 18 Months at the HELM, MSOR Connections , Vol 4 (No 2), 31-34.

Steele, C.D.C. (2009) Diagnostic Testing : Only the beginning. Proceedings of the Conference on Mathematical Education of Engineering students, Berlin.

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Mathematics in Engineering: why, when and how?

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Abstract

Education for engineers should be able to show the students why, when, and how mathematics can be useful to solve problems in engineering. This work provides some examples of learning tasks: mathematical challenges (problems set-up and solving) in applied contextsrelated to engineering professional activity. The proposed illustrations are: *interesting* due to their real applications, *attractive* due to their intrinsic beauty and *accessible* because they do not require a very sophisticated mathematical language. We analyse them following the mathematical competence approach, describing how they can contribute to acquire mathematical competencies and, at the same time, how they can motivate a positive students' attitude towards mathematics.

1. Introduction

Arguably, finding *good* examples is one of the key issues when teaching mathematics toengineers. "Good" means here: interesting, eye-catching, motivating, useful, beautiful, and able to awake creativity. In this work, we provide some of these examples. Using the "case study" format, the proposed tasks pretend to cover some of the basic mathematical skills: interpreting the data, representing the available information in a mathematical way, building models, solving them (both with mathematical language – theorems and mathematical reasoning – and with the aid of the computer), testing the results, extending the problem, looking for connections with other fields, etc. We are convinced that all these examples should be interesting to the students because of their *real applications*, their intrinsic *mathematical beauty* and their *accessibility*, as they do not require, in general, sophisticated mathematical language.

These tasks have been tested by the authors in different courses, and they seem to work. (See Cantón*et. al.*, 2008.) They are ready-to-use, so anyone can adapt them to his/her own teaching context and to different educational levels, from Core Zero to Level 3 (see SEFI report, 2011).

In each example, mathematical modelling is organised in several steps similar to those in Gainsburg (2006):

1. IDENTIFY the real-world PROBLEM one wants to solve.
2. SIMPLIFY: Select the relevant information and make some assumptions in order to make the problem easier/tractable.
3. Get a MODEL: Represent the idealised problem in a mathematical way.
4. SOLVE THE MODEL: Apply the appropriate mathematical or computational techniques to solve the problem.

5. INTERPRET the mathematical solution and test it with respect to reality or expected results.

6. EXTEND and CONNECT the problem.

Since it is not possible to explain in detail all the examples and their development in this short communication, we will describe briefly two of them, andgive a list ofthe other illustrations.

2. Rankings and web search engines.

Context: A (basic) Linear Algebra course (Core level 1). Or an advanced Linear Algebra course, for more technical developments (Level 2).

Requirements: Matrix algebra (matrices, systems of equations). Eigenvalues and eigenvectors.Any software able to perform matrix calculations.

1. IDENTIFYtwo PROBLEMS: one for fun; the other one, real stuff.

1) The regular season of some sports competition (football league, NBA, etc.) has finished. We need to build a final *ranking* (to get the winner, or to classify for playoffs, or…). We may just add the number of wins of each team. But somebody argues that not all the victories should have the same *significance*.

2) A search engine (such as Google).Let us suppose that, after a certain query, we have determined that one hundred webpages enclose information that might, in some sense, be relevant to the user. Now, in which order should they be displayed? What we need, once more, is a *ranking* of all these webpages that reflects the *significance* of each of them (Fernández, 2007).

2. SIMPLIFY. To understand the ingredients, we first consider the *available* information. For problem 1), we have the names of the teams, and the complete list of results of the games between them. In problem 2), there is a lot of information: the list of webpages, their contents, the links between them, and so on. We then select the *relevant* information for our purposes. In problem 1), we just need the number of wins of each team against the rest. In problem 2), after some consideration, we decide that we just need to register whether there is (or not) a link between each pair of webpages.

What should be our ordering criterion? The number of wins seems to be a plausible choice in Problem 1);and the number of links that each page receives should be an indicator of its relevance in Problem 2). It would be like an enormous voting system: each web-maker “votes” with links to other pages. There are two reasonable assumptions:

- The significance is proportional to the number of wins/links. Or:
- The significance is proportional to the number of wins/links, but weighted with the significance of the opposing teams/pages linking to it.

The first one is just brute force: number of wins/links, a commonly used method in sport leagues rankings. But for the webpages ranking there is some concern about its efficiency: for instance, it would be easy to get a spurious significance, just by creating many pages with the only purpose of adding links to our webpage. The second ranking criterion seems to be better: one needs to get many links, but from pages that get many links themselves.

3. THE MODEL. In problem 1), if there are n teams, we have an$n\times n$ matrix, whose entries are whole numbers. In problem 2) we have, again, an $n\times n$ matrix M, now with entries 0 or 1 – or, if the students know this language, a directed graph. Notice that the matrices are not symmetric, in general, and they could be huge.

Now considering the mathematicswe focus on the second problem, on which the significance of a webpage will be proportional to the number of links weighted by the significance of the pages linking to it. Let $P_1,...,P_n$ be the webpages on the Internet (or in the search engine databases). Let us denote by $\boldsymbol{x}=(x_1,\dots,x_n)$ the significances we want to assign them. For example, if P_1 receives links only from pages P_3, P_{10} and P_{66}, x_1 should be proportional to $x_3+x_{10}+x_{66}$, and so on for the rest of the webpages. Thus the desired vector $\boldsymbol{x}$ is a solution of a system of equations, that can be written in matrix form as$\boldsymbol{M}\boldsymbol{x}=\lambda\boldsymbol{x}$,where M is the matrix of the web. So we are done, the model is posed: ordering the webpages of Internet by significance is a Linear Algebra problem.

4. SOLVE THE MODEL**.** But this system of equations is quite special. The input is the matrix M, and the output, a vector $\boldsymbol{x}$ and a numberλ. At this point, there are acouple of possible detours, depending on the level of the course you are teaching. If the language of eigenvalues and eigenvectors is available, you may use it. Otherwise, use computational techniques to solve the problem.

The problem $M\boldsymbol{x}= \lambda\boldsymbol{x}$ has, of course, many different solutions, but we need just one: the one we can use as an ordering. The entries of the vector $\boldsymbol{x}$that we are looking for should be all positive or all negative. Any mathematical software (Maple, Matlab, Sage, R, even a spreadsheet such as Excel) can be used if $M$is not too large. So we deal with some “small” problems – the sport league ranking, a small Internet web – and see what happens.

5. INTERPRET. It seems that, if the matrix M is “reasonable”, there is always a unique solution with the required characteristics. But, whichare the conditions (on matrix M) that guarantee the solution we are looking for to be *unique*?

6. EXTEND and CONNECT.

- Using the computer to solve our problem is quite straightforward: it requires writing a small piece of code. But knowing *a priori*that the problem has a solution – thatis, establishing the conditions that M should satisfy – requires mathematical knowledge. We could search for information about the theorems (Perron-Frobenius) that guarantee the existence of a unique solution. Some notions will arise: irreducible matrices, connected digraphs, and so on.
- Computational issues. The matrix M could be huge. It is not easy to solve an eigenvalue problem if the matrix is, say, 1000×1000. Some specific computational techniques (power method, for instance) may be needed.

- Rewrite the model in a probabilistic language (Markov chain models).
- Find different contexts in which these ideas/techniques could be applied: economic and financial models (Leontieff input-output models, ratings in Finance), Biology (Leslie matrices, migrations), etc.

3. Transition curves in roads and railways design.

Context: Any first year Mathematics course (Core level 1) or an Interdisciplinary course (Level 2), in a Civil Engineering Degree, for example.

Requirements: Vector calculus, geometry of the circumference, trigonometry, derivatives and integrals.

1. IDENTIFY THE PROBLEM. As highway engineers, we need to design a transition curve between a straight line and a circular curve in order to give comfort and safety to road users (see Kobryń, 2011).

The design should ensure that a driver could leave a straight section of the road without being suddenly affected by the centrifugal forces that will be developed as the vehicle enters the curve. In other words, we want to build a curve whose tangent direction changes softly.

2. SIMPLIFY. We need not simplify the problem too much in this case. We are merely interested in tangent directions. A straight-line segment has a constant tangent direction – given by the line itself. A circular arc has a tangent direction that changes at a constant rate as onemoves along the circle: always perpendicular to the radius.

3. THE MODEL. The *curvature* of a curve measures the rate of change of the tangent direction as one moves along the curve: that is, we obtain a derivative. Straight lines have zero curvature and circles have constant curvature. Recall that we want the trajectory to be described by a vehicle moving at a constant speed and rotating also at a constant speed. Thus, we need a curve with slowly increasing curvature.A natural choice is taking curvature proportional to arc length. This leads to a differential equation involving the derivatives of $x(s)$ and $y(s)$ – the coordinates of the position on the curve parameterised by arc length. Depending on the students' level, one can explore with more or less depth the geometrical concepts arising here.

4. SOLVE THE MODEL. One finds that the solutions for x and y of the differential equations are Fresnel Integrals. These are definite integrals between 0 and s (arc length parameter) of elementary functions,such as $\cos(t^2)$ and $\sin(t^2)$, but that cannot be expressed in terms of elementary functions.

How is the real position on the curve for a given arc length found? We can use mathematical software to evaluate the corresponding Fresnel integrals. (Some years ago, tables with their values were provided, in the same way that there were tables with the values of logarithms).

5. INTERPRET. What should the shape of the curve in the plane we have obtained be? After some guessing, using mathematical software, we find out that the transition curve

is the so-called *clothoid*. The suggestive name of this curve is due to its double spiral shape that reminds of a thread wound around a spindle (as Clothos' thread in the Greek mythology).

6. EXTEND and CONNECT.

- Again, using the computer to solve our problem is straightforward, but the result is just an estimation of the integral, for which Taylor polynomials are used. Applying appropriate mathematical knowledge allows us to control the error.
- Look for information aboutTaylor's theorem and the convergence of the infinite (Taylor) series to the original function.
- Read about the curvature of planar curves and 3D curves.
- Find different contexts in which these methods could apply: diffraction phenomena, for example.
- Find other curves with special geometric properties that make them suitable for some concrete applications.

4. Other examples

Other examples also developed (and used) by the authors deal with:

- Fitting curves to data sets;
- Simulation/Monte Carlo methods;
- The exponential function and exponential dynamics;
- Ship stability.

We will be happy to provide the details about these issues at the request of any interested person.

5. Final remarks

We have exhibited two examples of learning tasks that would help Engineering students to develop complex mathematical competencies such as posing and solving problems, and modelling. Computational issues are also covered.These learning tasks can be used in basic terms, or in a more complicated form, so they can be adapted to any educational level.

They act to show the conspicuous role that Mathematics plays in the solution of concrete real problems in Engineering. The students will discover, through these illustrations, how Mathematics provides a flexible language to describe and focus the problem and how strong its solving techniques are. Moreover, they will realise how theoretical considerations are usually needed to find out the solution to slippery problems.

The abstract structure of mathematical language provides the engineers with a wealth of resources that can be useful in unexpected contexts. Matrices and Markov chains did exist long before Brin and Page used them to develop their Pagerank algorithm that eventually led Google to its current status. Differential Geometry was there much before the problem of abrupt changes of curvature in roads would cause discomfort to the passengers of a vehicle. Showing this “unreasonable effectiveness of Mathematics” (following Wigner) should be a “must” in the mathematical education of engineers.

References

Cantón, A., Fernández, P., Gil, O., Pérez, S. and Vázquez, M. (2008): “Aprendizajemultidisciplinar en Ingeniería y Arquitectura”. In *Proc. of the INECE08. Universidad Politécnica de Madrid.*

Gainsburg, J. (2006): The mathematical modelling of structural engineers. *Mathematical Thinking and Learning,* 8: 3-36.

Kobryń, A. (2011): “Polynomial solutions of transition curves”, *J. Surveying Eng.* 137, 71. American Society of Civil Engineers.

Fernández, P. (2007): “Google’s secret and linear algebra”.*EMS Newsletter,* 63.

Mathematics Working Group, European Society for Engineering Education (2011): “A framework for Mathematics curricula in engineering education”.

Image processing to Motivate Linear Algebra Students

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Abstract

The increasing development of computer technologies has given rise to educational alternatives, facilitating the creation of new forms of learning in an attractive and motivating way. Teachers develop efforts to create learning techniques using methods that can facilitate the learning in different areas of knowledge. The purpose of this experiment is to describe a case study that the author made with first year students of Electromechanical and Mechanics Engineering in a Linear Algebra course at the Coimbra Institute of Engineering. The study involved the application of Digital Image Processing to teaching and learning the basic concepts of Linear Algebra. The experience was very enriching for the author as a teacher, as well as for the students, because it let them awaken the motivation to learn, the interest and taste for Linear Algebra.

Introduction

Arousing curiosity and motivation in students is not always an easy task. To motivate students is necessary as the content or activities enrich experiences that encourage student interest and curiosity. Because not all students learn in the same way, the teacher makes content more engaging and motivates the students to perform the various activities with interest. Thus, the use of different methodologies can contribute to motivating students to find more meaning in what is expected of them. Through this motivation, students find reasons to learn and improve all their skills. This explains the concern of teachers, specifically teachers of mathematics, in seeking to motivate their students and get positive results, reducing the failure of mathematics.

Investigations on methodologies for teaching strategies focus on identifying different types of learning and motivation of students associated with this learning [Bulut (2011), Habash (2010), Meece (2006)]. The use of technology and examples of application of Linear Algebra help enrich the traditional methodology in order to facilitate and encourage student learning (Berriochoa (2009), Caridade (2011)). The combination of learning with more explorative activities enables students to acquire concepts, and terminology while developing self-confidence in using mathematics (Silverman(2010)). Some experiments have been made: the use of the Web to improve student learning (Waldock(2002)), the development of multimedia applications laboratory activities to develop mathematical concepts of trigonometry and pre-calculus (Rosen(2005)), the ATLAS project to encourage and facilitate the use of software in teaching Linear Algebra (Roberts(1996)) and the programme of research on teaching and learning of Linear Algebra in the first year of science courses in French universities (Dorier(2000)).

The teaching of Linear Algebra and Digital Image Processing

Linear Algebra is taught to a large and diverse number of students. Who are the students of the Linear Algebra course? How to motivate these students? What is the best way for students to acquire knowledge and skills in this area?

MATLAB (Mathworks (2002)), is a computer algebra system, designed for professional use in solving problems that require mathematical methods. It is one of the packages for the most natural application of Linear Algebra, since it is specially developed for matrix operations. Digital Image Processing is a highly relevant field, extremely rich in mathematical ideas that allow learning of Linear Algebra in a way completely different from the standard one.

In this context, I intend to present my experience to interest and motivate students to develop skills using a methodology that allows interdisciplinary between Linear Algebra (LA) and Digital Image Processing (DIP). This experience was developed in October 2011, at the first LA lectures with 40 students of Electromechanical Engineering and 73 students of Mechanical Engineering at Coimbra Institute of Engineering. The theoretical concepts of LA were presented by the DIP applied to images. This study is based on the following objectives: enable the acquisition of LA knowledge; develop reasoning and critical thinking of the students; encourage self-learning; increase student motivation in LA.

Development activities

One of the first concepts that student get exposed to in LA course is matrix operations. Let A and B be two matrices (images) with the same dimension $m \times n$. The matrix obtained by adding (or subtraction) the previous matrices A and B, called $A+B$ (or $A-B$) is shown in Figure 1 by the addition/subtraction some parts of the smiley face.

$$A \pm B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \pm \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11} \pm b_{11} & a_{12} \pm b_{12} & \cdots & a_{1n} \pm b_{1n} \\ a_{21} \pm b_{21} & a_{22} \pm b_{22} & \cdots & a_{2n} \pm b_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} \pm b_{m1} & a_{m2} \pm b_{m2} & \cdots & a_{mn} \pm b_{mn} \end{bmatrix}$$

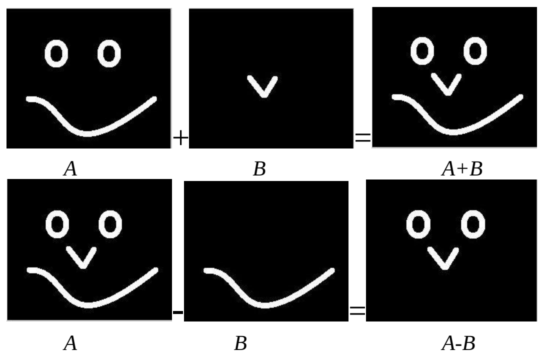
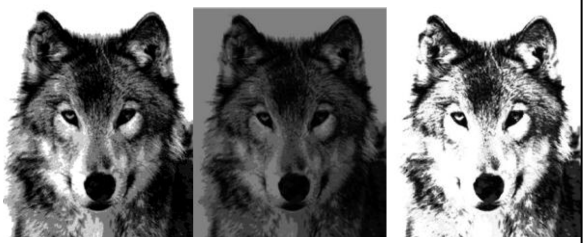


Figure 1. Addition and subtraction of two matrices.

The scalar multiplication of $k \in \Re$ by a matrix A, represented in Figure 2 can be teaching by DIP, using the increase or decrease light image. In the case represented in Figure 2, the image of the wolf in the centre ($2A$) was darker and the image on the right ($0.5A$) was lighter than the original one, on left.

$$kA = k\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{bmatrix}$$



A *2A* *0.5A*

Figure 2. Scalar multiplication of one matrix.

The transpose of a matrix A is another matrix A^T which is formed by turning all the rows of matrix A into columns and vice versa. An example of this operation is presented in Figure 3. The image of a flower is transpose. The students can see the difference between a transpose image and a rotate image by 90 degrees in positive or negative angle.

$$A^T = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix}^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \cdots & \vdots \\ a_{1p} & a_{2p} & \cdots & a_{mp} \end{bmatrix}$$

A A^T

Figure 3. Transpose a matrix.

The multiplication of two matrices A and B, are represented in Figure 4. In the top right, the image represents the multiplication of a starfish by matrix I' on the left, and the bottom image represent the same multiplication but on the right. In the first case the starfish is a horizontal reflection of the original image and in the second a vertical reflection.

$$A \times B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pn} \end{bmatrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}, \quad com$$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ip}b_{pj}, i = 1, \ldots, m; j = 1, \ldots, n$$

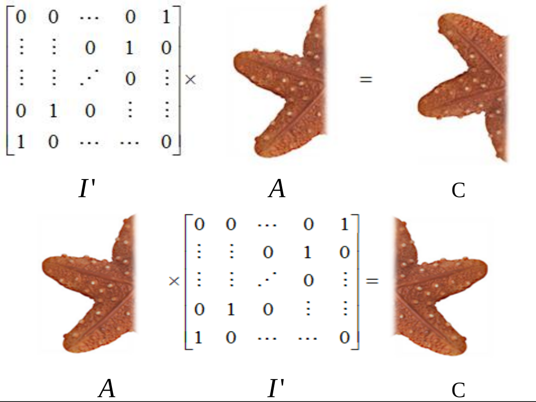


Figure 4. Multiplication of two matrices.

The properties of the matrix operations can also be operated by the application of DIP images. For example in Figure 4 it is possible to illustrate that the multiplication of matrices are not commutative ($A \times B \neq B \times A$). Another example of the proprieties is the transpose of the sum is the sum of the transposed matrices ($(A+B)^T = A^T + B^T$) that are represented in Figure 5. The transpose of the image $A+B$ is equal to the sum of the image A^T with B^T.

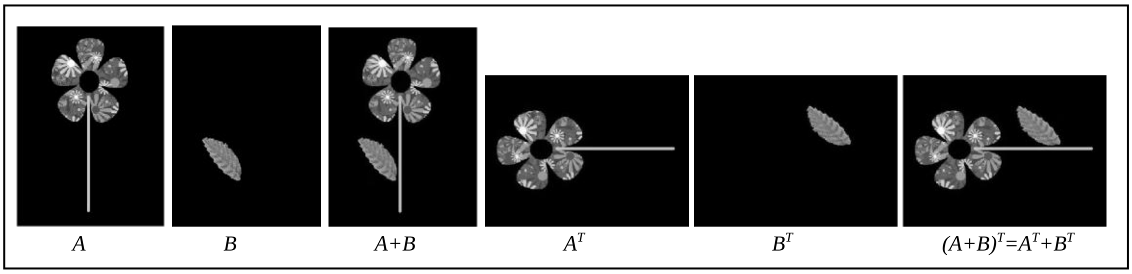


Figure 5. The transpose of a sum of two images.

Another context that the student can explore in LA classes are geometric transformations applied to image. Let A be a matrix of dimension 3×3 and u a column matrix of dimension 3×1. A transformation matrix of the function $f:\Re^3 \to \Re^3$ is defined by $f(u) = Au$, where u represents a pixel and A represents a matrix of scaling, rotation or translation. In Figure 6 is represented some geometric transforms applied to different images.

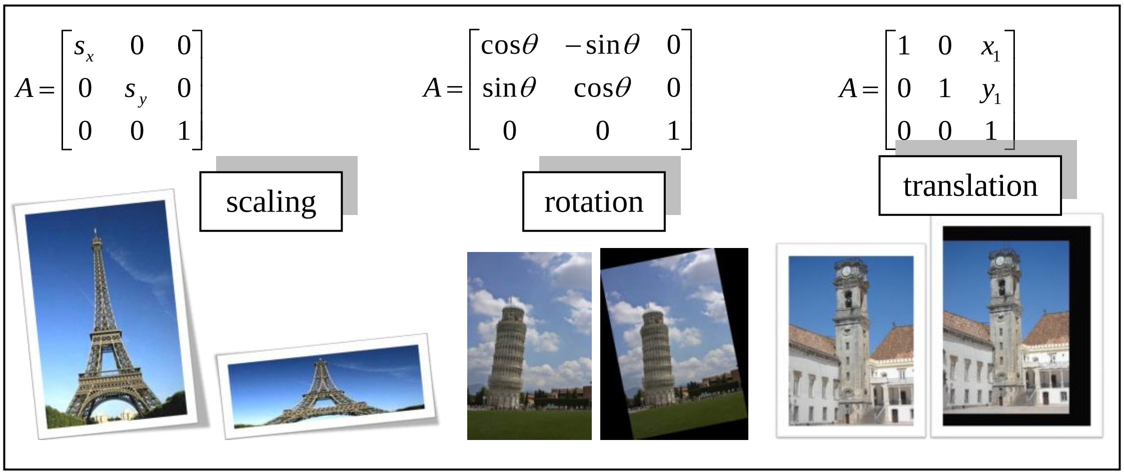


Figure 6. Geometric transformations applied to images.

The left image represent a scaling of the Eiffel Tower, the centre image a rotation of the Pizza Tower and the right image a translation of Coimbra Tower. In these examples the matrix A is applied at all the pixels on the image.

Results and Discussion

All forms of manipulating images (to improve contrast, the increase or decrease light, etc.) may be performed by applying operations to mathematical matrices associated with each image. A natural link between LA and DIP, supported by computational tools and contemporary technologies can be explored in the LA course.

My teaching experience has shown that the traditional methodologies are no longer responding to the new educational paradigm. The students eager to learn, and especially to know how to do, require dynamic new and practical activities of learning where they are the centre of the teaching-learning process. The teacher was the transmitter of knowledge and has become the guide of learning, whose objective focuses on the student and in the construction of their learning. Thus, education should be motivating and exciting for student learning, and have meaning. In this sense, use of MATLAB software and applications of DIP in LA courses allow a considerable improvement in student learning.

The experience described is performed for the purpose of determining whether the use of this new methodology in the classroom increases student learning. It always has, as its main objective, to captivate students' interest throughout the lesson, providing an environment of teaching and learning enjoyable and motivating. Students cooperated and answered the questionnaire that I proposed, showing great interest and curiosity, mainly by reference to DIP. The programme content learned in most of the first students was facilitated by using the relationship between matrices and images. It was evident that the experience was very enriching for the students; it led them to think more consciously about matrices and their properties; and was enriching for me as a teacher.

At the end of the lesson, students completed a survey about their interest in the new methodology. Some of the answers are presented below:

1. The activity performed with the Image Processing was interesting?

"Yes, it was a funny and interesting way to understanding the operations between matrices"; "I found it very enlightening and appealing"; "It was interesting, motivating and easier to understand".

2. What is your assessment of the activity performed with the Image Processing?

"This type of practical activity has the advantage to make the students more interested in Linear Algebra classes"; "An activity that arouses a great interest in the subject and in matter itself"; "Positive. Interesting until the end"; "I think it was not possible to improve".

Conclusions

This experience was very positive, in the sense that the introduction of this methodology helped to awaken the motivation to learn, interest and taste in LA. We all benefited by developing this kind of mathematical activity. Students were able to learn mathematics in an engaging way, as a tool that facilitated their learning. The teacher can see their students motivated and interesting with the entire context presented in lessons.

Although the experience has been limited to some content, I believe the importance of this type of methodology justifies its presentation. By the way, it is evident from the reference of students that learning is more exciting when using a different methodology that can motivate the interest of the students. This can be seen by the feedback from students. Most of the students enjoy the experience, join it and they were very active. In the next school year I can extend the project to other contents of LA, such as the calculation of eigenvalues and eigenvectors.

References

Berriochoa, E. et al. (2009) An approach for teaching the linear algebra for students of engineering. *Proc. of International Conference of Education, Research and Innovation*, 3756–3763.

Bulut, M. and Bulut, N. (2011) Pre Service Teachers' usage of dinamic Mathematics Software. *TOJET: The Turkish Online Journal of Educational Technology*, Vol. 10(No. 4): 294-299.

Caridade, C.M.R. (2011) Applying Image Processing Techniques to motivate students in Linear Algebra Classes, *Proc. of the 1st World Engineering Education Flash Week*, Lisbon, 114-121.

Dorier, J.L. et al. (2000) On a research programme concerning the teaching and learning of linear algebra in the first-year of a French science university. *International Journal of Mathematical Education in Science and Technology*, Vol. 31(No. 1):27–35.

Habash, R.W.Y. and Suurtamm, C. (2010) Engaging high school and engineering students: A multifaceted outreach program based on a mechatronics platform. *IEEE Transaction on Education*, Vol. 53(No. 1):136–143.

Mathworks (2002) *Using Matlab, Version 6.5*. MathWorkss Inc. Natick MA.

Meece, J.L. et al. (2006) Classroom goal structure, student motivation, and academic achievement. *Annual Review of Psychology*, No. 57:487–503.

Roberts, L.F. (1996) Software activities for linear algebra: concepts and caveats. *Problems, Resources and Issues in Mathematics Undergraduate Studies (PRIMUS)*, Vol. 6(No. 2):140–150.

Rosen, G. et al. (2005) Relating high school mathematics through sound and images. *ASEE Southeast Section Conference*.

Silverman, J. and Rosen, G. (2010) Supporting students interest in mathematics through applications from digital image processing. *Journal of the Research Center for Education Technology*, Vol. 6(No. 2):63–77.

Waldock, J. et al. (2002) Using the web to enhance student learning. *Proc. of the Second International Conference on the Teaching of Mathematics in Hersonissos*, Crete, Greece.

Mathematical basic knowledge of study-beginners: evaluation of 23 years preparatory course.

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Abstract

Since long was observed, that the students of the above-mentioned bachelor degree course had difficulties in visiting the mathematical lectures with success, because they had problems with their mathematical basic knowledge. So, a preparatory course was installed, which now exists since 1988 until today. It lasts two weeks and takes place just before the beginning of the first semester. The participation for the students is recommended, but voluntarily.

To valuate the mathematical abilities of the beginners, at the first day of preparatory course an entry test, respectively diagnostic test is written by the students. For being able to compare the results of the test about all this years, the form and tasks of the test were never been changed. The – often poor – results on the test are a good motivation for the students to visit the course indeed completely.

The focal point of interest shall be the evaluation of the diagnostic test about a timeframe of more than twenty years. In which way has the basic knowledge in elementary mathematics changed during this time, respectively which variation took place?

In addition there will be a report about the kind of performance and the contents of the preparatory course. The course is really helpful and there is also the sight of the students. For them it`s not only the mathematical aspect but also there is a social effect. This also should not be underestimated for a successful start of studies.

Assessment of Engineering Students in Mathematics at Budapest University of Technology and Economics

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Abstract

Freshmen admitted by the Budapest University of Technology and Economics (BME) are required to take a new first test, called 'test zero' in mathematics since 2010. There are more than 3000 students who are tested every year. Among the participants there are hundreds of students majoring in a variety of fields such as engineering (civil, mechanical, chemical, environmental, transportation or electrical), computer sciences, or economics and management. The test covers the topics of competency-based high school mathematics curriculum that are assumed to be of higher importance. The need for the test as part of the course requirements in first semester mathematics was generated by the current inconsistencies of the admission system of Hungarian higher education.

It is essential for higher education institutions to implement strategies or steps to improve the teaching methods used in mathematics sessions, so as to subsequently reduce the problem of non-completion. This is why BME, among several other Hungarian universities, introduced new catch-up courses for students with a weaker mathematics background.

This paper presents the results and summary of the mathematics assessment project of BME. It highlights the aspects of the assemblage of the proposed mathematics test, and summarises the conclusions of the project. The paper also outlines the results of the actions taken by professors and instructors of BME to help students with a weaker background to reach their mathematical potential.

Introduction

In the 2002 SEFI (European Society for Engineering Education) Mathematics Working Group (MWG) report "Mathematics for the European Engineer, a Curriculum for the Twenty-first Century" the authors noted that **"***The symbiosis between mathematics and engineering is a long-standing one. Almost all branches of engineering rely on mathematics as a language of description and analysis. … However, in increasingly more countries there is concern over the deterioration in the mathematical ability of new entrants to engineering degree programs*." (SEFI, 2002, p. 3)

Hungary is no exception to this experience. There are increasing concerns in Hungary about the skills and competencies of students entering higher education. In 2009 a nationwide survey was conducted among university students. This study concluded that incoming undergraduates were deficient in basic mathematics. A lack of mathematical competencies and abilities has been identified as a factor resulting in non-completion of courses in Hungarian higher education institutions.

In the Hungarian enrolment system admission points are based only on high school achievements. Points are calculated on the results of the final exams (graduation exams) in two high school subjects, one of which is typically mathematics for applicants for engineering programs. The maximum of total admission points is 500 including extra points. 400 points are based on the grades in the last two years of secondary school as well as / or the results of the graduation exams, while applicants may earn a maximum of 100 extra points for extra achievements. High school students are given options before they sign up for the exams: they may take the graduation exams at advanced or intermediate level in each subject. The advanced level exam is more difficult, covers more topics and has a written and an oral part. The intermediate mathematics exam only has one written part. It is more likely that one can earn better test scores at the intermediate level. Even though students are encouraged to choose advanced level exams by the possibility that they may earn 50 extra points for each one, very few of them make this choice.

In 2011 SEFI Mathematics Working Group produced the first revision to their own report on a mathematics curriculum for engineers. In this 2011 revision report they say: "*Mathematical competence is the ability to recognize, use and apply mathematical concepts in relevant contexts and situations which certainly is the predominant goal of the mathematical education for engineers."* (SEFI, 2011, p. 3)

Pilot programmes for measuring the mathematical competence of freshmen when they start their studies were introduced at different institutions of higher education in different countries in Europe, including Hungary, during the recent years. Based on the results of these surveys universities and colleges may select the students who are very much in the need of extra help or a group of the outstanding and gifted students who can take extra mathematics courses of higher level and who can study at a faster pace.

Budapest University of Technology and Economics (BME) with its 23 000 students is a leading institution in engineering in Hungary. Lecturers and professors in mathematics of BME also have the impression that preparedness of freshmen is becoming worse and worse every year. The failure rate is increasing together with the proportion of dropouts or withdrawals. To maintain the necessary standards in elementary subjects has become a challenge. This has created the need to identify why so many students struggle with important basic parts of their curricula including mathematics. The idea of adding 'test zero' to the items of the course requirements of first semester mathematics courses was motivated by the fact that students are not required to take an admission exam and the university has no other information about the knowledge and the educational background of the student than his / her total number of admission points. As one element of these steps freshmen admitted by the BME have been required to take the test called 'test zero' in mathematics since 2010.

Objectives and Methods

The goals of 'test zero' are

- to define clearly the prerequisites of first semester mathematics,

- to enforce students to refresh their former knowledge in mathematics,
- to test certain mathematical competencies,
- to give feedback to students at the very beginning of their studies,
- to give the opportunity to students who have failed to register for catch-up courses to improve their skills,
- to obtain data about the mathematics background of freshmen,
- to get a list of topics in which the students are less successful,
- to give feedback to high school teachers and educators,
- to point out that testing (and the following additional support) can reduce the number of failures and withdrawals.

Students take the test in the second week of the fall semester. The time for the test is 50 minutes; the paper-based test includes 15 multiple-choice questions with 5 possible answers of which exactly one is correct. Students have to write the code (A, B, C, D or E) of the correct answer into the answer box. Since the number of participants in the fall semesters was over 3000, 6 different versions of the problem sheet were used with all similar problems. Students are not allowed to use pocket calculators, formula books or formula sheets. The topics for the problems are intentionally chosen according to the needs of higher education and not exactly according to the proportions of the chapters of high school mathematics. One correct answer is worth 4 points, one wrong answer is worth -1 point and the student gets no score (zero point) if he / she had left the answer box blank. The maximum test score was 60, the minimum test score is -15 points. The test result is regarded as successful if its score is above 24 points (40%).

The participants are first year students of BME, among them larger groups of students of the following faculties: Faculty of Civil Engineering, Faculty of Mechanical Engineering, Faculty of Chemical and Bioengineering, Faculty of Transportation Engineering and Vehicle Engineering, Faculty of Electrical Engineering and Informatics, Faculty of Economic and Social Sciences, Faculty of Natural Sciences. In September 2011 the total number of the participants in mathematics ‘test zero’ was 3344.

Results

It is particularly interesting to compare the admission points and test results. On Fig. 1 every point represents one student, identical points may appear. A major part of the box is almost uniformly spread by points. That is, having a student with excellent admission points we can hardly suggest what we can expect from him / her in mathematics and knowing that the student has a bad test score we cannot conclude that he / she belongs to the lower group with respect to admission points. But if the student has a good test score it is more likely that he / she will have good admission points. These highlight the fact that admission points and test scores correlate weakly. The bad news is that, knowing the admission points of the student we can hardly predict his / her performance on mathematics ‘test zero’. Admission points give us insufficient information about the knowledge of the student.

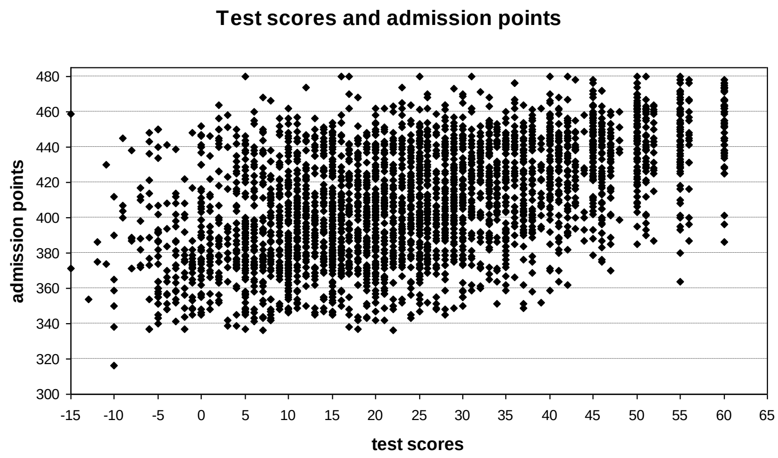

Fig. 1. Admission points and test scores for all participants

Even though many students have excellent admission points, students with a non-traditional mathematics background are at a risk of struggling with problems of 'test zero' and consequently with the other requirements of college-level mathematics courses, due to possible gaps in their mathematical knowledge.

The levels of high school graduation examination reults in mathematics and test results are obviously correlated. Knowing that high school students may make a choice on the level of their graduation exam in every subject it is not surprising that very few of them choose the advanced level in mathematics, which is assumed to be difficult. In 2011 only 32.7% of the students of BME involved in 'test zero' arrived with an advanced level graduation exam pass in mathematics.

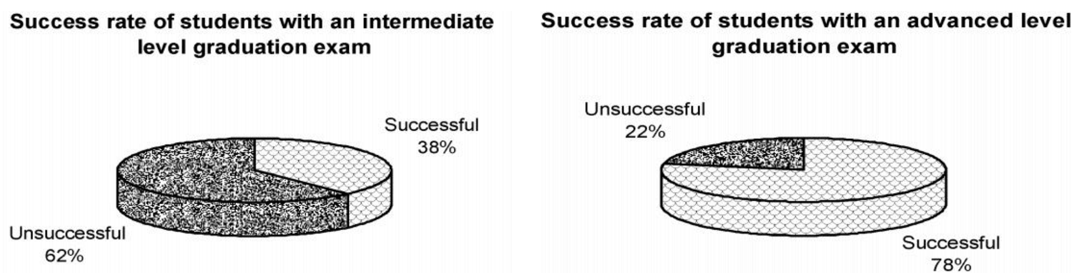

Fig. 2. Result of students with different levels of graduation exam

It is important to emphasize that it is much more likely that a student with an advanced level graduation exam pass meets the requirements in 'test zero' of BME. *Fig 2.* demonstrates this fact.

There were problems in which the success rate was lower than in others. Among these problem areas we found the ones on algebraic skills, like recognizing and being able to use identities of exponents or logarithms, trigonometry, word problems, inequalities, geometry, and vector algebra.

A general conclusion of the test results is that in certain parts of secondary school mathematics students fail to show an acceptable success rate. A distressingly large proportion of students were not taught, or failed to learn, some of the most fundamental parts of the high school mathematics curriculum. This could be the number one reason why a large number of university students are struggling in traditional lectures, where a certain level of knowledge is assumed.

Seeing these results, BME decided to open extra elementary bridging mathematics courses that offer the additional benefit of aiding the students in their transition into higher education. In the fall semester of 2011 1074 students registered for such an extra course.

Conclusions

The study presents informative findings and results from a recently-conducted survey of mathematics knowledge of BME students when they enter higher education. Conclusions that were obtained from the analysis of the test results in 2011 are the following:

1. Admission points and test scores are scarcely related. This means that admission points cannot be used to predict the performance of the student.
2. The rate of success in the test is closely related to the level of the high school graduation exam pass in mathematics. Students who arrive with an advanced level exam pass are more likely to meet the required standard in ‘test zero’.
3. Several students enter higher education in the engineering programmes of BME with insufficient knowledge of mathematics. Students can be informed about their test result at the very beginning of their studies. In the form of extra elementary mathematics courses the university provides support to students with non-traditional backgrounds.
4. The topics that seemed to be more difficult for students, or in which the success rate is lower, are identified: algebraic skills, including ability to use identities for logarithms, trigonometry, inequalities, vector algebra.
5. Failure rate increases together with number of answer boxes a student has left blank.

It is essential for higher education institutions to implement strategies or steps to improve the teaching methods used in mathematics sessions, so as to subsequently reduce the problem of non-completion. This is why several Hungarian universities introduced new catch-up courses for students with a weaker mathematics background. The topics of these courses are typically the important chapters of high school mathematics curriculum. 1074 freshmen of BME decided to register for a mathematics make up course in the fall semester of 2011. 857 (79,8%) of them completed the course successfully. Having completed the catch-up course the total number of credits they earned together with their success rate in their compulsory mathematics subjects was significantly higher than that of those who did not complete the catch-up course even though they had been advised to do so.

The provision of mathematics support through extra elementary courses seems to work successfully at BME.

References

European Society for Engineering Education (SEFI) Mathematics Working Group (MWG) (2002). Mustoe, L. and Lawson. D. (Eds): Mathematics for the European Engineer: A Curriculum for the Twenty-First Century. Accessed via http://sefi.htw-aalen.de/ (10 March 2012)

European Society for Engineering Education (SEFI) (2011). A Framework for Mathematics Curricula in Engineering Education First Revision of Report by the SEFI Mathematics Working Group "Mathematics for the European Engineer: A Curriculum for the Twenty-First Century". Accessed via http://sefi.htw-aalen.de/ (10 March 2012)

Csakany, A. and Pipek, J. (2010) "On the results of the survey on preparedness of freshmen entering higher education in engineering or natural sciences programs". *Matematikai Lapok*, Vol. 2010/1, pp. 1-15 (in Hungarian)

LTSN MathsTEAM (2003). "Diagnostic Testing for Mathematics". Accessed via http://www.mathstore.ac.uk/mathsteam/packs/diagnostic_test.pdf (10 March 2012)

Russell, M. (2005) "Academic Success, Failure and Withdrawal among First Year Engineering Students: was poor mathematical knowledge a critical factor?" Level 3, (3). Accessed via http://level3.dit.ie/html/issue3_list.html (10 March 2012)

Csakany, A. (2011) Results of Mathematics 'test zero' at Budapest University of Technology and Economics in 2010, *Mathematics in Architecture and Civil Engineering Design and Education, conference proceedings*, ISBN 978-963-7298-44-8

Csakany, A. (2011) "Gap-Bridging Mathematics Courses at the Budapest University of Technology and Economics", *Economica*, vol. IV, special issue, ISSN 1585-6216 (in Hungarian)

Institute of Mathematics at Budapest University of Technology and Economics (2012). Csakany, A. and Prok, I. (Eds): "Information about Mathematics Test Zero" Accessed via http://www.ttk.bme.hu/altalanos/nyilt/NulladikZH/ (10 March 2012) (in Hungarian)

Aligned Assessment

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Abstract

This contribution is based on the author's experience of teaching and assessing students' learning. We discuss the alignment of assessment and expected learning outcome including the alignment of the expected learning outcome and the requirements for passing. Examples on aligned assessment are presented.

Introduction

Since John Biggs and Catherine Tang wrote the book "Teaching for Quality Learning at University" in 1999, the idea behind the book, constructive alignment, has had an increasing impact on the teaching – learning – assessing cycle at many universities. There is now a fourth edition of the book available, Biggs, J and Tang, C (2011) and many short stories can be found on internet.

One guideline in constructive alignment, there is of course a lot more included in the concept, is that the planning of a course or a module must give answers to three questions:

What shall the students learn, what is the expected learning outcome?

What shall the students do to learn, what is the best way to organize the teaching and learning?

How can the students' knowledge be evaluated, what forms of assessment are most suitable?

In this paper we discuss the third question, that of assessment, and begin with an overview of the different forms of assessment that are in use around Europe and was identified in a SEFI MWG project, the assessment project, reported at the SEFI MWG seminar in Vienna 2004 Lawson (2004).

Forms of assessment

The most common assessment method is a written examination, with closed books, at the end of the course. Less common is a written examination with open books or computer facilities to support the problem-solving. There is an ongoing debate on the use of calculators in university mathematics. In secondary school this debate is since long ended; the graphics calculators are seen to be essential tools for mathematical modeling and experiments. At university level computers are used for these purposes. When it comes to written examination with closed books calculators are in general not allowed. If they are there is, or should be, restrictions on the types or models. No graphics, no CAS. Modern advanced calculators are more or less equivalent to open books and personal notes being allowed.

Another difference worth mentioning is the written exam duration. In some countries four or five hours are standard, in other only one or two. Every written exam is a spot check but the shorter duration the less of the contents can be covered and the more it opens for gambling strategies. The shorter duration can be compensated, like in many central European institutions, by a follow-up oral examination. Either for all students or for those that scored well enough to get a higher grade. Teachers comment on oral examination that it is highly staff intensive but give the best opportunity to test in-depth understanding.

Take away assignments are used at several institutions, but always as one amongst a number of methods of assessment and never as the only or primary method. They give students an opportunity to explore more realistic problems than they can in an ordinary written examination and for this reason often require the use of computer software to complete the assessment task. Some teachers have reservations about this method of assessment because it is impossible to be certain that the student submitting the work actually did it by him/her-self. When the take away assignment is followed up with an oral presentation of the work the legal certainty is stronger.

Only a few institutions use multiple choice tests and those that do use them do so only occasionally. Such tests can be cheap to administer as they can be computer delivered and marked. They can be useful in giving formative feedback during the course. There are reasons to believe that the use of this kind of assessment is increasing. We will discuss this in detail later in this paper. Again it is impossible to be certain that the student submitting the work actually did it by him/her-self, unless the test is implemented under invigilation and on computers that are not connected to any net. Furthermore, as all that is marked is the student's final answer, they have limitations when being used for summative assessment.

Other methods of assessment such as project work, group work and oral presentations are not widely used. However, when it comes to examination of mathematical competencies, these methods can be more interesting. It is difficult to give individual grading of group work, but individual time-logs, progress-logs and contribution-reports together with the project report can help.

Aligned assessment and the requirements for passing

In the previous chapter we recalled the findings of the SEFI MWG assessment survey; the major part of the assessment is based on a traditional final written exam with closed books. Also, as shown by examples provided in the survey, the construction of these exams is similar across Europe, possibly around the world. They consist of a number of problems more or less similar to the problems in the textbooks and questions related to the theory, each given a certain maximum score and together covering most or some of the intended learning outcomes. When marking an exam the examiner gives the student a score for each problem depending on how successful the student’s attempt to solve the problem, or to give an answer to the question, turned out to be. The examiner then decides whether the student should pass or fail or get a better grade. Traditionally this decision is entirely depending on the student’s total score. The limit between fail and pass is often set to a percentage of the maximum score. This percentage varies from 40 to 50 or 60. The requirements for passing were not included in the SEFI MWG assessment survey but discussions with colleges from different universities around Europe lead to this conclusion. The grading systems vary from country to country, sometimes between universities in the same country. When the ETCS grading system is adopted across Europe it will be of interest to investigate the equality of the grading. But even then the differences between the course modules at different universities will make the comparison very problematic. The aim of this chapter is to discuss the requirements for passing in relation to the expected learning outcome.

Expected learning outcomes (ELOs) specify depth and what students should be able to do at the end of the module. Typical ELO statements begin `On successful completion of this modules students will be able to' followed by a verb like calculate, solve, explain or prove. The information in the list of ELO statements for a course module is twofold. Firstly it tells the student: ‘if you can do all this on the day of the exam you will pass’. Secondly it tells everybody else; ‘a student who passed this course module was able to do all this on the day of the assessment’. Or does it really?

Here is the examiners dilemma; if the ELO is expressed in a vague manner it is of little help to the student but one can easily claim equivalence between ‘passing the exam’ and ‘being able to do all this’. For instance an ELO statement of the type ‘after this module the student will be able to solve standard problems in this field of mathematics’ gives no or very little information to the students what to learn. Neither will it be of help for teachers in other disciplines what knowledge they can assume. But anyone who has passed the exam has certainly managed to do something related to the statement. On the other hand, if the ELO statements are expressed in a precise manner, they are of great help for the students but the equivalence between ‘passing the exam’ and ‘being able to do all this’ becomes hard to establish.

To improve quality in teaching and learning and in the entire education of engineers we are, according to the constructive alignment principle, supposed to state the ELO in such a way that it supplies the students with a proper guidance for their learning. When we select ELO statements for a course module we have to think both as mathematicians – “what is most essential in this field of mathematics, at this level?” and as teachers taking part in the engineering education, we have to reason interdisciplinary and in long term. These thoughts have been fundamental in the SEFI MWG core curriculum project since it started around 1980. However, an improved quality in the education is not achieved automatically just by giving an excellent list of ELO statements. The quality is of course heavily dependent on what the students actually learn. Thus, there are strong arguments for aligning the expectations not only with the assessment design, but also with the requirements for passing.

The prevailing principle: ‘a student who is given a certain percentage of all possible points on an exam will pass’ implies that a high score in some parts of the course module can compensate for a complete failure in other parts, unless the percentage is close to 100. If we accept the principle: ‘the requirement for passing should guarantee that the student has demonstrated an acceptable level of knowledge, skill and understanding of every part of the expected learning outcome’, then we have to alter the requirements for passing. Furthermore, if the assessment consists only of a final written exam, that exam should be designed in a way that is consistent with the principle and also give the students opportunity to show both ability to perform routine tasks and a deeper understanding. Quite a challenge, not only if but in particular when the exam duration is only one or two hours!

What to do instead? I have practiced two different designs of final exams with four hours duration. In the first design the exam is split into two parts. The first part covers methods and procedures, some very easy to apply and some rather complicated but standardized. It also includes theoretical questions which require a limited understanding. The second part covers problem solving and a higher level of understanding. To pass the student must score a large portion of the first part, basically only minor mistakes in the calculations and minor lack of knowledge is accepted. The second part is used only for grades above ‘passed’. The students and the programme managers are very pleased with this design. The ELO statements tells the students what knowledge or level of understanding that is required to pass and what deeper knowledge and understanding to strive for if they aim for a better grade. I, and some of my colleagues, have used this design for five years in two courses: Linear algebra in the second semester of the first year for several programmes and Multivariable calculus in the first semester of the second year for two programmes. When we introduced this exam design there was an increase in the number of students that passed Linear algebra, but a drop in Multivariable calculus. One explanation of this difference lays in the nature of the courses. Linear algebra starts, in a sense, from nothing. All they need to know is elementary mathematics, if they didn’t learn it in school they have learnt during the first semester. Multivariable calculus on the other hand builds upon calculus in one dimension. That course is assessed by a traditional exam and you pass if you score 40% of the maximum score. Thus, as pointed out before in this chapter, the student may have severe failures in some parts of the course. Those failures can be crucial when the ideas and methods are extended to higher dimensions. Students have complained, they are disappointed that the traditional design opens for bad choices. They soon find out that you can neglect to study some chapters in the book if you learn other well enough.

The second design consists of a number of items where the student can show both low and high level of understanding or ability to apply either standard techniques or genuine problem solving in the same field. The criterion for pass is then to score reasonably good on all items. We have tried this design for two years in one course in mathematics in the education of marine engineers with very good result. The students on that programme are in general not very interested in mathematics; their future work will be in the engine room in a ship. To motivate them to work hard enough we introduced very short mid-term-exams, one single item on each. If they were successful, they didn't have to work with the corresponding item on the final exam. Together with very clear statements on what they had to be able to do to pass this lead to a pass rate above 90%. The first design had not been a good choice for this course and this group of students.

It is a well-known fact that from a student's point of view the assessment is the curriculum. Most students use old exams to find out what to learn. No matter how we state our expectations on their learning, if those statements are not aligned with the assessment, the actual learning outcome will differ from the expected. The assessment design and the requirements for passing will have the same impact on the students' learning.

Assessing competencies

The SEFI MWG core curriculum project has entered a new phase; we are now adding the higher-level learning goals based on the concept mathematical competencies described in the Danish KOM-project presented in Niss, M. (2003).

The competencies provide a framework for our discussions and thoughts about what we expect our students to be able to do with the mathematics they have learnt, not directly related to a specific field of mathematics. The competencies are developed when the student study different courses, not necessarily mathematics. For instance the student's competence in mathematical modelling can be improved in any subject where mathematical models are in use. Therefor the competencies are also to be seen as expected learning outcomes of the programme, the entire education. It would be a benefit for the education if at least some mathematical competencies were included among the general competencies that are, or ought to be, included in the description of the expected learning outcome of the engineering programme. We could then discuss with our engineering colleagues how each competence best is developed, what the student shall do to obtain the competence and how we shall assess it. Some of the competencies are best developed in project work like bachelor or master thesis projects. Others are mainly developed in studies of mathematics. Thus, in the near future we must broaden this discussion and include engineering colleagues and program managers. There is a lot to be done in this field.

There are eight competencies described in the KOM-project. The following declarative list uses the text in the latest version of the SEFI MWG "core curriculum" [to appear on the SEFI MWG website] in which the ideas are discussed in detail and references are given.

Thinking mathematically which comprises knowledge of the kind of questions that are dealt with in mathematics and the types of answers mathematics can and cannot provide, and the ability to pose such questions. It includes the recognition of mathematical concepts and an understanding of their scope and limitations as well as extending the scope by abstraction and generalization of results. This also includes an understanding of the certainty mathematical considerations can provide.

Reasoning mathematically which includes on the one hand the ability to understand and assess an already existing mathematical argumentation (chain of logical arguments), in particular to understand the notion of proof and to recognize the central ideas in proofs. It also includes the knowledge and ability to distinguish between different kinds of mathematical statements (definition, if-then-statement, iff-statement etc.). On the other hand, it includes the construction of own chains of logical arguments and hence of transforming heuristic reasoning into own proofs (reasoning logically).

Posing and solving mathematical problems which comprises on the one hand the ability to identify and specify mathematical problems (be they pure or applied, open-ended or closed) and on the other hand the ability to solve mathematical problems (including knowledge of the adequate algorithms). What really constitutes a problem is not well-defined and it depends on personal capabilities whether or not a question is considered as a problem. This has to be kept in mind, for example when identifying problems for a certain group of students.

Modelling mathematically which also has essentially two components: The ability to analyze and work in existing models (find properties, investigate range and validity, relate to modeled reality) and the ability to "perform active modelling" (structure the part of reality that is of interest, set up a mathematical model and transform the questions of interest into mathematical questions, answer the questions mathematically, interpret the results in reality and investigate the validity of the model, monitor and control the whole modelling process). This competency has been investigated in more detail in (Blomhoj & Jensen 2003, 2007).

Representing mathematical entities which includes the ability to understand and use mathematical representations (be they symbolic, numeric, graphical and visual, verbal, material objects etc.) and to know their relations, advantages and limitations. It also includes the ability to choose and switch between representations based on this knowledge.

Handling mathematical symbols and formalism which includes the ability to understand symbolic and formal mathematical language and its relation to natural language as well as the translation between both. It also includes the rules of formal mathematical systems and the ability to use and manipulate symbolic statements and expressions according to the rules.

Communicating in, with, and about mathematics which includes on the one hand the ability to understand mathematical statements (oral, written or other) made by others and on the other hand the ability to express oneself mathematically in different ways.

Making use of aids and tools which includes knowledge about the aids and tools that are available as well as their potential and limitations. Additionally it includes the ability to use them thoughtfully and efficiently.

The competencies are not isolated from each other, on the contrary they are very closely related and you cannot have one without another. For instance, you cannot communicate mathematics without being able to represent mathematical identities, handle mathematical symbols or think and reason mathematically. When we plan to include this type of competence in or description of the expected learning outcome we have to think in a new way, new compared to traditional end-of-course assessment. Using Biggs words: The learner shall in a sense be 'trapped', and find it difficult to escape without learning what is intended should be learned. This can be done by observing the learning process and letting the final result of that process also be what we assess.

In this way the competencies Posing and solving mathematical problems or Modelling mathematically together with all the other, in particular Communicating in, with, and about mathematics and Making use of aids and tools can be practiced and assessed by letting students or groups of 2 – 4 students solve genuine mathematical problems or implement mathematical models and then present their solution orally and/or in a written report to a teacher-student audience. The problem solving or modelling can include numerical calculations or experiments using software, the presentation can include graphical representations of the result. All this can be done as a minor part of a single course module or as a larger project. At least in the latter case it is important that we observe the process as well. This can be done, as mentioned in the first chapter, by individual time-logs, progress-logs and contribution-reports. If a group work is graded, not only pass-fail, then we have to decide whether all students in a group shall have the same grade. Quite often that is the case but we have to know if it is fair. Individual time-logs in one single document handed in by the group give some information to the teacher. But it is also good for the climate in the group. A student not contributing to the joint work will realize that for him/herself. This also holds for the progress-log. In the final contribution-report we can identify strengths and weaknesses. Quite often the students write in the report that they all have contributed equally in discussions and problem solving but are first writer of different parts of the project report. The students make it quite clear that they want to have the same grade. But I have also experienced groups in which one of the students makes it clear that he/she is satisfied with a lower grade than another and points out the other student's strengths. In case the students are only supposed to work in a given mathematical model and improve their ability to use software in the implementation of the model, it can be sufficient to observe that they work and at the end check the implementation. The curriculum report will eventually include examples for this kind of assignments; all of us can contribute to that.

The competence in communication can also be practiced and assessed in class, working with the ordinary exercises from the text book. One method is often called 'ticking'. The idea is to select a number of exercises or theoretical questions like true-false statements which the students can solve at home or somewhere else, even together with other students, but out of scheduled time. They also prepare to give a presentation of their solutions to the other students in the class, using for instance the black board. When it is time for presentation they tick-mark which exercises or questions they are prepared to talk about. This activity is not only good for the communication skill; it also activates the learners which many students comment on in course evaluations. I have practiced this in several different settings and in different courses, as a mandatory part of the assessment and as an optional part giving bonus points to the final exam. Neither the student's performance nor the quality of the explanations has been graded only the willingness to try to explain. I have motivated the students by mentioning their future work as 'teachers' and the general communication skill that is an expected learning outcome of the education. To motivate the bonus points for better grades I have pointed at the ability to explain as a result of a deeper understanding. To be honest; all students have not demonstrated that deeper understanding in their presentations, but they have tried to reach it. Assuming a clear learning outcome statement on communication in the course I would at least think of making the ticking compulsory to some extent and perhaps also grade the performances.

The competence in mathematical reasoning can be developed and tested in many ways, both in the final exam and as mentioned above, in 'ticking' activities. True-false statements are in general quite good for this purpose. Sometimes the students feel cheated when they give incorrect answers to such questions. The statement reminds them of a true statement but some part is altered to make it false. It takes quite good understanding of both the concepts and the logics to give a correct answer and to prove it. More than we can expect from an average student. Thus, be careful when selecting statements for a written exam. In a ticking activity an incorrect answer from a student can benefit the entire group.

Technology-supported assessment

The use of technology-support in mathematics has several reasons. One, which is not within the scope of this chapter, is to simplify or improve the students' work, like in modelling projects or problem solving when a mathematical model is developed, implemented, tested or simulated with support of computer algebra systems or computations. Most universities offer the students commercial software like Mathematica or Maple (CAS) and Matlab or the freeware Octave for computations. There are also many commercial or free programs designed for specific types of problems. The students have to spend considerable time to learn how to use the software and there is a strong need for support from teachers when they do so. In most engineering programmes the proficiency to use computer software in problem solving is an expected learning outcome of the programme.

Next we will discuss how technology may support formative assessment during the course and summative assessment after the course or course module.

In its most primitive setting a test suitable for a computer-supported assessment system consists of a number of multiple choice questions consisting of a question together with one correct answer and a number of incorrect answers, distractors. The distractors must be close enough to the correct answer. The student has to select the correct answer to all or most questions in order to pass or to get a positive feedback. The student's work is not simplified or improved; she could do the same with paper and pen, as long as the questions are similar to those in the textbook. The advantage for the teacher is that once the system is there and a suitable set of exercises or questions are imported to the system, the system will do the work. The advantage for the student is that she can often do the test anywhere and anytime. If the test is created by a randomized selection of questions out of a large question bank the student can do the test many times. She will get immediate feedback and the teacher will get immediate information about the student's progress. The need of multiple choice question decreases if the test system is supported by a computer algebra system. But instead there can be specific demands on how the answer is given or formulated. A correct answer in wrong format is considered to be an incorrect answer, confusing the student of course. One challenge for the teacher is to find questions that assess a deeper understanding of the subject and still has one correct answer. Another challenge is to rethink the assessment and find questions that could not be asked when only paper and pen were available.

The feedback to the student in a simple system consists only of a mark correct/incorrect; the student has to find out what to do to improve. A more advanced system includes also learning support for the students. There are many reasons to strive towards complete, computer supported systems, so called Intelligent Tutoring Systems (ITS), where the student gets information not only about his/her errors or mistakes but also about the underlying misconceptions or lack of knowledge together with support to fill the gaps. At the moment no such system exists, the student needs help from a human teacher to figure out the nature of the misconceptions and what to do to improve. Some of this can be e-support linked to the test; some may be personal given by the teacher or a support center on request from the student. The nature of the support can also vary from "read this example" or "view this explanation" to "read again chapter X in your textbook".

There are specific problems with the legal certainty when computers are used in summative assessment or when students get some kind of credit (bonus points) for the performance in a formative assessment. In general the computers at universities are connected to a network and to internet. To prevent cheating the network connections must be closed, perhaps some other programs must be blocked and the students' work must be invigilated. If the number of students is greater than the number of available computers, the need of randomized tests is obvious. The tests cannot be too similar otherwise the last students can have some unlawful help from the first. Still all tests must be of the same difficulty. If the test is done out of campus or out of office hours then the examiner does not know who actually took the test. For these reasons at most a minor part of the entire assessment should be computer-supported and not invigilated.

Conole and Warburton gives a survey of both the use of technology for assessment and of the research on this use in the paper "A review of computer-assisted assessment", Conole, G. and Warburton, B. (2005).

There is an obvious need for a thorough survey of today's use of technology for assessment in mathematics and a deep discussion concerning the consequences of that use.

Final comments

We all know that the students are assessment driven. Whatever changes we make to the curriculum and the statements about what we want the students to learn nothing will really change until the assessment is aligned with our wishes. Constructive alignment is a system organizing the entire chain expected learning outcome – student activities –assessment. Whatever ingredient we put in the expectation pot we must ensure that we assess the students' knowledge of that ingredient in a proper way. And nothing else should be assessed.

References

Biggs, John and Tang, Catherine (2011) *Teaching for Quality Learning at University* 4:th ed. The Open University Press

Lawson, Duncan (2004) Assessment in Engineering Mathematics – in the 12th SEFI Maths Working Group Seminar Proceedings.

Niss, Mogens. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagatsis, S. Papastravidis (Eds.), *3rd Mediterranean Conference on Mathematics Education*, Athens, Greece: Hellenic Mathematical Society and Cyprus Mathematical Society, pp. 115-124

Conole, Gráinne and Warburton, Bill (2005) A review of computer-assisted assessment. ALT Journal, *Research in Learning Technology Vol. 13, No. 1, March 2005, pp. 17–31*

The teaching of Mathematics in Engineering: contributions of learning in b-learning

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Abstract

The paradigm of putting students at the centre of the teaching/learning that came as a result of Bologna's Declaration motivates the need to bring together the implementation of information strategies and communication models, with the motivations, interests, and learning styles of students so that the changes in attitude and behaviour towards education and learning result in meaningful higher educational attainment. Additionally, the new avenues of access to higher education, also pose new challenges for polytechnics and universities, requiring teachers to acquire greater skills in the motivation of this heterogeneous audience, particularly in the development of a diversification of strategies in their pedagogical practices. It is universally accepted and proven by scientific studies, that information and communication technologies (ICT) have allowed the construction of a set of support mechanisms that complement the student attendance in class and allows its users greater interaction and ease of communication, therefore leading to a greater student involvement of in the learning process. E-mail, the internet, virtual learning platforms such as Moodle and other communication tools, are considered as important organizational tools with regards to traditional teaching methods, so that their usage has been increased, demanding of teachers an increasingly and continuous transformation in their teaching models.

With this in mind, and given the high rates of student repeats and dropout in higher education, particularly in mathematics curriculum units taught in undergraduate engineering, an exploratory study has been carried out. The exploratory study aims to contribute to the construction of a learning environment that enables students to feel responsible in the educational process and their commitment to academic success. This work is framed within the action-research project "CAME – Understand Learning to Better Teach" developed in the framework of the plan of action of the GIDiMatE – Grupo de Investigação em Didática da Matemática na Engenharia, within the Department of Physics and Mathematics (DFM) at the Coimbra Institute of Engineering (ISEC). One of the objectives of this research group is to deepen the knowledge of college dropout causes in mathematics modules so as to develop strategies in the teaching process which lead to improved student learning methods.

Introduction

The quality of learning in higher education emerged as a consideration for the scientific community in the form of a problem arising from a further deepening of democratic systems around the 90's. A growth in the number of students attending this level of education showing a variety of personal and motivational characteristics is being observed. The policies of higher education access in Portugal, for example the regime aimed at those students over 23 years old, or students from the professional and technological courses, brought new challenges to the university and polytechnic institutions, requiring them to be able to adjust their institutions, making them more attractive and competitive and able to motivate a heterogeneous audience based on their needs and expectations. Additionally the paradigm stemming from the Bologna Declaration requires a change in emphasis: to replace a teacher-centered transmission of knowledge model by a learner-centered model and requiring the construction of knowledge by assigning significance to it. Thus, it is accepted that in addition to the acquisition of knowledge, stimulating the development of personal and professional skills that enable students to better adjust to the flexibility, complexity and adaptability in different life contexts must become part of the training. In addition to these challenges it is widely accepted that the information and communication technologies play a fundamental role in this reorganization, as a complement to student learning, both in terms of access to and ease of dissemination of information and in the development of new contexts for student achievement. It is recognized and corroborated by scientific studies, that information and communication technologies have allowed the construction of a set of support tools that complement classroom teaching and allows its users greater interaction and ease of communication, leading to a greater involvement of students in the learning process - "*The introduction of computers in education is seen as particularly suitable to enrich teacher's pedagogical strategies and stimulate various educational contexts, methodologies that encourage activity, participation, collaboration, initiative, creativity ...*" (Ponte, J., 1994). E-mail, the internet, virtual platforms such as Moodle and other communication tools are considered to be forms of learning organization tools with respect to traditional teaching methods, so their usage has been increasingly employed, requiring teachers to be engaged in a continuously changing teaching model. In this sense the use of ICT adds extra value to the teaching/learning process, particularly with regard to flexibility and access, making the physical attendance requirement unnecessary for access to materials and enabling the improvement of communication channels and an increase in collaborative work, which are critical engines of a more and more frequent interaction in the current job market. No less important, as a result the on-going emphasis on developing the quality of higher education training, increased attention is paid to college failure, making the promotion of success a fundamental objective of the institutions' action. In the last decade, although many studies have sought to understand the reality of academic failure in higher education, prompting investigations that seek to understand, in depth, how student learning takes place, analysis of the relationship between teaching methods and how students learn (Grácio et al, 2005) needs to be conducted further. It is also important to contextualize these concerns particularly in the case of teaching and learning mathematics. The authors, mathematics teachers in the various degree courses taught at the Coimbra Institute of Engineering (ISEC) are confronted with an increasing lack of motivation, disinterest and the consequent neglect of students in relation to their mathematics courses. This situation, compounded by the difficulty shown by students in elementary and basic concepts which are essential to successfully engaging with the syllabus, inevitably leads to high failure rates and subsequent concern of the teachers.

In this new environment of teaching/learning a greater priority arises for teachers with regard to higher education teaching, higher criticism, a further reflection on their teaching practice and an on-going adjustment of the educational courses' difficulties to particular characteristics of their students with regards to the meaningful construction of knowledge. By focusing the teaching/learning process on the student, there emerges the need to implement these student centred strategies and for dissemination of information and communication models to be aligned to the interests, motivations and learning styles of students so as to have changes in attitude and behaviour of students towards college that makes the process of learning more meaningful and therefore lead to greater academic success. However, any learning environment can be considered as a unique

and unrepeatable space built by the teacher, based on their views of the educational process and mastery of its knowledge, in particular in relation to how ICT is used to continuously adjust the interaction between its actors (Marin, 2009).
The present work arises from these assumptions and discusses an exploratory study that aims to carry out the analysis of students' perceptions of the introduction of ICT as a teaching strategy that contributes to support the construction of a virtual learning environment, allowing a shared responsibility of students in the educational process and one that may be related to increased educational success.

Method of Investigation

Students who attend the Coimbra Institute of Engineering have very different characteristics, both in terms of where they come from (on average 60% of students on engineering degrees do not come from the district of Coimbra) and also in terms of their basic academic training. For example, it appears that the majority of students attending (approximately 63%) are from science and technology courses, but this number is decreasing over the years, giving rise to a greater variety of basic courses taken by the students. Considering this diverse audience and the need to offer more flexible ways of learning, ISEC and in particular the Physics and Mathematics Department has encouraged the adoption of a distance component of b-learning (-Blended learning platform which has been in use for the last eight years). The "*LVM*-Virtual Laboratory of Mathematics" (http://lvm.isec.pt/) is based on an e-learning platform (Moodle) and it is a complement to classroom learning that takes place in all mathematic courses taught in ISEC undergraduate engineering degrees. In an approach that aims primarily to enable the access to the content and provide a way, beyond the classroom, for teachers and students to interact and communicate with each other, this environment has proven to be a good solution as a teaching/learning strategy, also providing an opportunity for a more active involvement in the educational process.
In order to enhance the functionality of this platform, with a view to stimulating interaction and experimentation through the technological resources, it is important to reflect on the integration of ICT as a promoter of students' meaningful learning that may result in academic success. Since 2009 a survey has been developed that includes several case studies which are aimed to investigate, in more detail, the implications of the use of ICT in educational practices applied in some mathematic courses taught in undergraduate engineering at ISEC. By analyzing the behaviour of students on the implementation of this teaching support strategy, the purpose is to reflect on teaching practices and investigate what learning environments allow students a joint responsibility in the educational process, and which may prompt better learning. The research follows a qualitative research methodology, considering the observation of several specific cases, which allows us to understand and explain holistically and in a constructive perspective, as suggested by Stake (Stake, R., 1998), the virtual learning environment phenomena, using the Moodle platform through "*LVM*". The intention is to get explanations to the questions:

Q1: What are the behaviours that students using "*LVM*" show?

Q2: What is the students' profile of student interaction in the activities undertaken via *LVM*?

Q3: What virtual environments for teaching/learning enable a shared responsibility of students in the educational process?

The sample

The sample consisted of 50 students who satisfied conditions for admission to the Mathematical Analysis course assessment of the undergraduate Computer Science Engineering degree, which takes place in the 2nd semester of the 1st year, in a sliding mode, in the academic year 2010/2011. This sliding mode regime arises from an attempt to overcome the failure detected over the years in mathematic courses taught in the 1st year. In fact, the Mathematic Scientific Committee of ISEC implemented, in 2010/2011, the pedagogical structure known as "Sliding Courses" which, after analysis and corrections made in order to optimize resources and improve results, was included in the teaching service on the following assumptions: 1) slider courses work in alternative semesters, in addition to the undergraduate curriculum; 2) the students cohort for the course is all students that have not been approved in the previous semester; 3) only students who have attended a minimum of 60% of all classes are allowed to take the exam. These restrictions are not applied to students who have passed Elementary Mathematic course in the corresponding academic year or to students for whom Law 105/2009 of 14 September 2009 is applicable (employed students). Students engaged in the undergraduate Computer Science Engineering degree have a great diversity in their basic training, with mathematics access grades ranging from 12 to 18 (out of 20), and yet still some of them have difficulties with the content taught in higher education. They often exhibit very high levels of non-attendance at lectures, demonstrating inconsistent effort in the classroom or in doing learning activities set by the teacher. In order to establish the students' profile, other data was collected, by these same 1st year students fulfilling a questionnaire regarding their ICT behaviour.

Description of the learning environment

The teacher responsible for the course that was the focus of this exploratory study was a person who was a dedicated researcher in terms of her own teaching and who has, over the years, reflected on her teaching practice and questioned herself about what teaching and learning strategies best fit students needs in an attempt to make their learning a more meaningful process, and so result in effective academic success for these students. In the classroom, given the nature of the syllabus to be covered, the teacher used lectures or introductory explanation of subjects, with exemplification through problem solving for the acquisition of basic knowledge, while in the remaining classes (theoretical-practical and practical) other teaching methods were applied and shared solution of exercises led to the understanding and application of materials by the students. Specific activities were proposed to create the spirit of synthesis and analysis in the students which is necessary to obtain the desired learning outcomes. In an attempt to complement the classroom with a virtual learning environment, the strategy of ICT integration was considered as means to support the teaching, both pedagogical (content publishing, dissemination of materials and miscellaneous items) and administrative (dissemination of warnings and other information, display of patterns of results, booking of additional classes). The use of this complementary structure was promoted throughout the semester to students. All necessary materials for each chapter (hand-outs and exercise sheets) were placed on the platform in advance. The work published by the students in LVM and discussed among peers was monitored by the teacher, thus allowing further monitoring of issues and it also contributed to improved communication channels and encouraged cooperation. Other activities were also set up for sending, in advance, files to correct and hence integrate the process of individual formative assessment. The setting adopted in Moodle was the topic format. Five topics were established, corresponding to each syllabus chapter (revisions, primitives, integral calculus, improper integrals and differential equations). In order to promote the application of constructive teaching methodologies and enhance ICT in a laboratory learning environment the use of computer algebra software was also made available via a LVM link to *GeoGebra*. The notes to support lectures, basic texts written by another teacher (a non-lecturer on the course) and worksheets that were set by the teacher, were posted in the general Moodle page, and five thematic forums were also established (revisions, integrals, integral calculus, improper integrals and differential equations) that aimed to achieve peer to peer interaction and also between students and teachers, through the exchange of messages, questions, the presentation of various forms of exercises proposed, solutions and also sharing of notes for each syllabus unit. Four activities were also set which resulted in individual work being submitted for correction. These shared spaces were launched simultaneously with the introduction of subject in the theoretical classes.

Instruments for data collection

In order to answer the questions Q1 and Q2 in this investigation the data was collected from the Moodle platform, regarding the activity reports of each participant included in the sample. For this purpose, a grid was built based on the information relevant to the study and the number of hits per item was recorded. The following categories were considered: CAT 1- Material resources: integrating the set of tests/exams from previous years, the hand-outs designed by another teacher and a link to external software application (*GeoGebra*); CAT 2- Revisions, integrals, integral calculus, improper integrals and differential equations which were divided into two subcategories: SUBCAT 1-Material resources: which included the set of worksheets developed by the teacher, by desired level of learning outcome and supporting texts provided by teachers not involved in this course; SUBCAT2 - Thematic forums: including comments, posted activities and self-study material.

In order to examine students' perceptions regarding the use of ICT, with regards to allowing a deep reflection on the construction of virtual environments for teaching/learning and permitting a joint responsibility for students in the educational process (Q3 above), a questionnaire designed to collect information on four groups of questions outlined below was distributed to the students:

Question Group A: Students' characterization regarding ICT access (5 items, e.g.: Do you have access to a computer? Do you have internet access?);

Question Group B: Students' perception about the importance of ICT use for learning (4 items, e.g.: in the explanation of the contents, when using software adapted to the syllabus content);

Question Group C: How students use the internet in their learning process (5 items, e.g.: talk about problem-solving in forums, exchange of support materials);

Question Group D: The use of LVM in the learning process (6 items, e.g.: to consult notices/information, to consult proposed discussion forums);

Question Group E: the use of software custom-made to the syllabus (4 items, e.g.: problem solving, troubleshooting check);

A *Likert* type scale format has been used, with 4 levels, depending on the degree of students' agreement and including an extra option - not applicable.

Findings and Discussion

The software used in data analysis was SPSS. Regarding behaviours evidenced by students' use of "LVM" (Q1), it was found that the highest average number of accesses to the platform per item was for integrals. The download of resource materials provided by the course teacher, complemented with other materials given by teachers outside the course, and the explanation of the thematic unit tests on the syllabus from previous years was also a teaching tool widely used by students. It should be noted that there was a decrease in access to the platform during the semester, demonstrating a drop in study methods and engagement with the subject, and that this feature was confirmed by the course teacher, by virtue of her experience in teaching Computer Science Engineering students. This trend is confirmed by the discrepancy between the number of accesses to the exercise worksheets set by the teacher, which leads to an autonomous work and to the material resources that allow contents taught in the lectures to be read.

With regard to how students followed the thematic forums which were launched, it was found that 18 students participated in posting comments and only five posted activities. This contribution tended to decrease drastically to no action in the forums associated with improper integrals and differential equations; indeed, in the forum dedicated to the integral calculus only 7 students posted comments, and only one student posted study resources. This behaviour indicates an absence of a culture of sharing with respect to knowledge construction and no collaboration among peers.

Observing the pattern of accesses to the LVM platform it is clear that, in general, there is not a positive influence on grades, i.e., the students who accessed the LVM more (as indicated by counting of resource materials consultations/downloads) are not the ones achieving higher grades. Although, if the students that really showed interaction between teachers and colleagues are considered then there may be evidence to sustain the hypothesis that virtual learning and discussion improves success and perhaps contribute to higher grades.

To gain a better understanding of which virtual teaching/learning environments allow a higher students' enrolment analysis was performed on the data collected via the questionnaire mentioned above. Only 38 out of 50 students answered the questionnaire. Arising from this data analysis one relevant conclusion is that almost all students have access to computers and that most students use internet at home. Smaller importance is given to internet access at public Wi-Fi free places. Students confer similar levels of importance on the following aspects when considering ICT as a learning resource: contextualising of the explanation area, use of software tailor-made to the syllabus and the exemplification of practical cases of application. In considering the use of the internet in the personal learning, students indicate that the Moodle course page is noted as the most important feature and some importance is also given to discussion of problem solving in chat rooms. The LVM-platform Moodle is used in the individual learning process as an important place to consult information and warnings. The discussion forums and the teachers' requirements were not valued. Students use the software tailor-made program content primarily for checking the problems' completed solution, assigning little relevance to its application in the solving of the problem itself.

Conclusions for Education

The poor adherence to the proposed publication of material developed in class was clear, confirming the results from the academic year 2009/2010 (Bigotte, E., 2011), which shows no culture of sharing to build knowledge, or collaboration between colleagues. It seems reasonable to assume that students' participation in discussion forums encouraged interaction between the teacher and students and contents lectured.

The observed decreasing interaction as the semester progressed suggests the need to introduce diversified strategies in order to create internal and/or external stimulation to keep discussion forums active. There was no evidence that the student who uses the LVM platform more frequently achieves better final grades. This conclusion indicates the need for further studies to be conducted to better allow cross information, filtering it to the level of individual student trajectories in order to be able to establish correlations and draw a profile of students' activities and interactions in the "LVM-Virtual Laboratory Mathematics". With respect to virtual environments for teaching/learning, students consider the Moodle platform as an important source of information/material, as a repository of resources necessary for sharing, but do not appreciate it as a strategy for autonomous learning, do not make use of it as a environment to provide for knowledge construction through interaction between peers or with the teaching staff and also do not consider the publication of their own reviews or studying materials.

It is worth mentioning that, since there was not any type of pressure by the course teacher that would induce a "requirement" to use of the LVM platform in student's learning process, the suggestion is that this type of student participation in the LVM platform may eventually be included as component in the formative evaluation parameters. The use of educational software tailored to the syllabus was considered of medium relevance in its application to problem solving, solution's verification and acquisition of relevant technical suggestions for its implementation. The internet was not regarded as an important place for interaction and research of complementary expertise. For the point of view of social background, either by the characteristics of the student background, or by the ones associated to their prior knowledge, it can concluded that all students accessed the material available regularly, although not all students participated in the forums.

In terms of the ICT strategy applied there was a significant interaction by the students throughout the learning process, reflecting primarily the simplification of access to the content to support the classes. Specifically in terms of the course, the study showed the need to create a more flexible intervention concerning the learning rate of the individual student that would allow more individualized monitoring, the requirement for building dynamic learning objects, that can easily be updated and adjusted to each context, which would create an environment that promotes group cohesion and would help to ensure the social presence of all participants.

This contribution is an essential addition to a participatory and careful reflection of the pedagogical practice in higher education. It is intended that its conclusions indicate that the dynamic of integration of ICT into course instruction, allowing, overall an eventual modification of the ideas and an improvement of teacher's professional performance.

It is intended that changes of attitude and behaviour towards college of students result in more significant learning and therefore lead to higher student success. In addition it is necessary to align the application of information strategies and models of communication to the interests, motivation and student's learning styles.

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References

Bigotte, E., 2011, As TIC e o Ensino da Matemática na Engenharia: um contributo, I Encontro Internacional TIC e Educação "Inovação Curricular com TIC", Instituto de Educação da Universidade de Lisboa, Lisboa, Potugal.

Chaleta, E., Rosário, P., & Grácio, M. L., 2005, Atribuição causal do sucesso académico em Estudantes do Ensino Superior. In Actas do VIII Congresso Galaico-Português de Psicopedagogia, Braga: Universidade do Minho (Cd-Rom), Portugal, pp. 819-826.

Jaramillo, P., Castañeda, P., Pimienta., 2009, M. Qué hacer con la tecnología en el aula: inventario de usos de las TIC para aprender y enseñar,Educación y Educadores, Vol. 12, Núm.2, 2009, pp. 159-179,Universidad de La Sabana, Colombia.

Marin, J., Elena, P., Quiroga, R., 2009, Un caso de integración de TIC que no agrega valor al aprendizage, Revista Latino Americana de Ciências Sociales, vol.7, n.1, pp 267-287.

Morais, N., Cabrita, I., 2010, Ambientes virtuais de aprendizagem: comunicação (as)síncrona e interacção no ensino superior, Prisma.Com n.º 6; pp 158-178 [ISBN: 1646-3153].

Pais, S., Cabrita, I., Anjo, A., B., 2011, A Aprendizagem da Matemática e o Projecto Matemática Ensino – Um estudo de caso no Ensino Superior, VII Conferência Internacional de TIC na Educação, Lisboa, Portugal, pp 1043-1058.

Ponte, J., 1994, "O Projecto MINERVA Itroduzindo as NTI na Educação em Portugal". Relatório de avaliação do projecto MINERVA. Lisboa: Ministério da Educação.

Relatório de Actividades-Serviços Académicos (RAC-SA), 2011, Instituto Superior de Engenharia de Coimbra.

Stake, R. E., 1998, Investigación com estúdio de casos. Accessed in http://www.google.pt/books.

Learning and Assessing Competencies: New challenges for Mathematics in Engineering Degrees in Spain

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Abstract

The introduction of new degrees adapted to the European Area of Higher Education (EAHE) has involved a radically different approach to the curriculum. The new programs are structured around competencies that should be acquired. Considering the competencies, teachers must define and develop learning objectives, design teaching methods and establish appropriate evaluation systems. While most Spanish universities have incorporated methodological innovations and evaluation systems different from traditional exams, there is enough confusion about how to teach and assess competencies and learning outcomes, as traditionally the teaching and assessment have focused on knowledge. In this paper we analyze the state-of-the-art in the mathematical courses of the new engineering degrees in some Spanish universities.

Introduction

The Bologna process encourages the transition of higher education from knowledge possession to understanding performances and from a teaching-centered to a student-centered approach via learning outcomes. The *European Credit Transfer and AccumulationSystem* (ECTS) is a system based on learning outcomes and competencies (European Commission, 2009).

All degrees are defined in terms of the competencies that students should have acquired with a view to entering the job market. Such competencies are divided in generic and specific. All academic subjects, including mathematics, must define their learning outcomes in such a way that the acquisition of such competencies will be facilitated.

The concept of competency can be defined as *the ability of carry out tasks or to deal with situations effectively using knowledge; skills and attitudes* (see Weinert, 2001). Learning outcomes are statements of what a learner is expected to know, understand and/or be able to demonstrateafter completion of a process of learning.

The Tuning-AHELO conceptual Framework (OECD, 2011) defines Engineering as *the profession that deals with the application of technical, scientific, and mathematical knowledge in order to use natural laws and physical resources to help design and implement materials, structures, machines, devices, systems and processes that safely accomplish a desired objective*. This framework offers a summary of some of the most influential learning outcomes in the Engineering field. Graduates should possess generic skills needed to practice Engineering. Among these are: *The capacity to analyze and synthesize, apply knowledge to practice, adapt to new situations, ensure quality, manage information, and generate new ideas.* More particularly, graduates are expected to have achieved the following learning outcomes*:the ability to function effectively as* *an individual and as a member of a team;the ability to communicate effectively with the engineering community and with society at large;the ability to recognize the need for and engage in independent life-long learning; andthe ability to demonstrate awareness of the wider multidisciplinary context of engineering.*

Other international references for competencies and learning outcomes of Engineering are ABET (Felder and Brent, 2003) or CDIO (Crawley et al., 2011), with similar learning goals even though different words are often used for the same idea.

Concerning the Spanish case, regulation RD1393/2007 is a detailed procedure to implement the new grades adapted to the EHEA. No catalogue of degrees has been drafted; instead we have a system for the verification and accreditation of university degrees. This is run through a Quality Agency and a register of universities and degrees (RUCT,2008). The degrees are grouped into five areas of knowledge, one of which is *Engineering and Architecture.* According to the data available from this source, in Spain there are 50 public universities and 31 private ones. Only four of these universities are defined as polytechnic, but nearly all of them include *Bachelor Degrees* in the fieldof *Engineering and Architecture* (EABD) in their offer, there being (in April 2012) a total offer of 606 EABD.

All Spanish EABD have 240 ECTS credits, 60 of which correspond to basic subjects concentrated during the first three academic semesters. The generic competencies, described in Table 1, are collected in the definitions of most of these EABD.

Competencies	Description
GC1:Self Learning	The ability to engage in independent life-long learning
GC2:Critical Thinking	The ability to select, analyze, synthesize and apply relevant information, knowledge, methods and logical and well- motivated argument
GC3:Use of ICT	The ability to use modern ICT technology for communication and engineering practice
GC4:Problem solving	The ability to apply knowledge of mathematics, science, and engineering for formulating and solving engineering problems
GC5:Technical Communication	The ability to communicate effectively, by oral o written form, with the engineering community and with society at large
GC6:Team work	The ability to function effectively as a member of a multi-disciplinary team

Table 1: Basic Generic Competencies for Engineering

The purpose of this paper is to analyze the treatment afforded to thesegeneric competencies in the mathematics subjects of the Spanish EABD.

Competencies Associated with MathematicalSubjects

All students of Engineering and Architecture must follow different mathematics subjects (calculus, linear algebra, numerical methods, differential equations, statistics, etc.). In some EABD mathematical contents are limited to two 6-ECTS subjects, followed during the first two semesters. In many universities, in order to economize resources, the same subject, calculus or algebra for example, is offered to students following different EABD whose basic mathematics requirements are similar.

For each subject, teachers must prepare and publish a learning guide (LG) in which they outline: competencies to be acquired, learning outcomes, programs, methodology, assessment, planning, etc. To explore the treatment afforded to competencies in mathematics subjects we have analyzed a set of different LGs. We have chosen a varied and sufficiently representative sample of 60 subjects, imparted by 13 universities.

Table 2 shows the universities chosen for the study, with the number of EABD offered, by each of them, during the 2011-2012 academic year, and the number of LGs chosen for our research.

University		EABD	LG
USAL: Universidad de Salamanca www.usal.es	Public	16	5
UPM: Universidad Politécnica de Madrid www.upm.es	Public	38	12
UPCOMILLAS: Universidad Pontificia de Comillaswww.upcomillas.es	Private	4	2
UAL: Universidad de Almería www.ual.es	Public	5	2
UCLM: Universidad de Castilla la Mancha www.uclm.es	Public	14	4
UEM: Universidad Europea de Madrid www.uem.es	Private	11	4
ULPGC: Universidad de las Palmas de G. Canaria www.ulpgc.es	Public	9	3
UNED: Universidad Nacional de Ed. a Distancia www.uned.es	Public	6	2
UNIOVI: Universidad de Oviedo www.uniovi.es	Public	16	3
UNIZAR: Universidad de Zaragozawww.unizar.es	Public	14	3
UPV: Universidad Politécnica de Valencia www.upv.es	Public	19	10
US: Universidad de Sevilla www.us.es	Public	24	7
UVIGO: Universidad de Vigo www.uvigo.es	Public	12	3
Total	13	188	60

Table 2: Learning Guides Analyzed

For each LG, we have analyzed the competencies sought, the learning activities foreseen and the proposed methods of evaluation. All the analyzed LGs include as a specific competency: *The ability of students to demonstrate knowledge and understanding of the mathematical principles underlying their branch of engineering*. Also, all LGs aim at developing one or several generic competencies that coincide with, or are related to, the six generic competencies of our research. Table 3 shows the frequencies where the analyzed competencies appear in the LGs.

University	LG	GC1	GC2	GC3	GC4	GC5	GC6
USAL	5	5	5	4	5	3	2
UPM	12	10	7	7	10	4	4
UPCOMILLAS	2	0	2	2	2	2	2
UAL	2	2	2	1	2	0	0
UCLM	4	3	4	1	4	3	2
UEM	4	4	2	0	2	3	2
ULPG	3	2	0	3	1	0	1
UNIOVI	3	2	2	3	3	1	2

UNED	3	2	2	2	0	2	1
UNIZAR	3	3	2	2	3	3	3
UPV	10	7	5	9	10	3	6
US	7	5	4	2	7	1	2
UVIGO	3	0	3	3	3	0	0
Total/Percentage	60	45/75%	40/67%	39/65%	52/86%	25/41%	27/45%

Table 3: Frequency Table of Generic Competencies included in the LG analyzed

Although it is not possible to determine whether the students really do acquire the competencies, there is broad consensus with regard to ensuring that the activities carried out by students in mathematics subjects promotes the acquisition of competencies GC1 and GC4. Additionally, competencies from GC1 to GC5 are tightly linked to the mathematics competencies defined in the KOM Project (Niss and Højgaard, 2011).

Methodological changes

The student-centered programs, based on the development of competencies, require other methodologies and strategies than the traditional programs.

The CDIO Standard8 states*: Active learning methods engage students directly in thinking and problem solving activities. There is less emphasis on passive transmission of information, and more on engaging students in manipulating, applying, analyzing, and evaluating ideas. Active learning in lecture-based courses can include such methods as partner and small-group discussions, demonstrations, debates, concept questions, and feedback from students about what they are learning* (Crawley et al.,2011).

Regarding the LGs it may be deduced that many teachers have attempted to incorporate methodological changes aimed at adapting to the new scenario. These changes are mainly relatedto two aspects: the way to teach, increasing the use of the powerful technological support available, and the aims sought in the teaching activities, directed towards the acquisition of the different competencies mentioned above.

From the LGs studied:

- 70%propose solving problems with mathematical software. This activity allows the development of GC1 to GC5 competencies (Díaz, García and Villa, 2011).
- 55% incorporate teaching materials, managed through educational platforms such as MOODLE. This activity develops GC3 and promotes GC1.
- 38% include some method of active learning, which permits the development of the GC1, GC2 and GC3.
- 25% propose some collaborative learning activities activity for the development of GC6.

However the teaching based on the transmission of information persists in many mathematical subjects. That is, some teachers have tried to adapt their situation to the EHEA with as few changes as possible.

Assessment of Competencies

The change to competency-based learning implies differences in the assessment methods used to adequately determine the acquisition of those competencies. Baartman et al. (2006) state that *one single assessment method seems not to be sufficient*. They propose some quality criteria for a Competency Assessment Program.

In Spain no procedures have been defined for the separate evaluation of generic competencies. These competencies are evaluated together with the specific competencies in the subjects. There are universities that offer advice onhow to develop and assess competencies (VOAPE-UPM, 2011). But 23.3% of the LGs analyzed propose an assessment model based exclusively on traditional written exams.

For assessing each competency a set of measurable learning outcomes can be defined. For example, the learning outcomes for GC4 (Problem Solving) could be: gather and organize relevant information; translate the problem, expressed in usual language, to technical language in order to separate data from aims and choose a model; choose an effective strategy; use mathematical knowledge for solving the problem and interpret the result; and express the reasonableness of the solution. Also Niss and Højgaard(2011) proposea varietyof learning activitiesfor assessing mathematical competencies, which can be used for assessing generic competencies.

Other models for the assessment of generic competencies, based on indicators and rubrics (see Villa and Poblete, 2008) or using Miller's pyramid, can be used.

Student Performance

From a general point of view, academic results have improved in the new system. Nevertheless, the feeling amongmany students and instructors is that the new learning methods require more work time from both sides. In some cases, students continue to demand traditional expositive techniques and look unkindly upon attempts to match teaching and evaluation practices with what is demanded by the design of the degree. Despite this, and little by little, resistance is being worn down.

Fenoll, Vizcarro and Vieira (2012) made a study about the opinions of leaders of Spanish universities, teachers and students with respect to the Bologna Process. They conclude that leaders perceive the process as a driver for a positive change. Teachers' perceptions are diverse. The spectrum varies from the enthusiast innovators to the immobile teachers. Students are skeptics, but anti-Bologna sentiment has weakened.

Proposals of Learning Activities

Among the active learning activities that develop generic competencies, the following can be mentioned: solving problems using mathematical software (see Díaz et al., 2011); small projects for team work (García, García, Rodríguez and de la Villa, 2011); multidisciplinary projects (García, Bollain and Corral, 2011) and students' competitions (García, García, Rodríguez, Vila andde la Villa, 2011).

Conclusions

Mathematics teachers in EABD are making important efforts to change towards a competency-based teaching style. However, there is still considerable confusion regarding which teaching practices are best and the optimum way of assessing such competencies.

There is an interesting process of diversification of teaching scenarios, with the incorporation of Mathematics laboratories and the use of on-line methods with Learning Management Systems such as MOODLE.

Nevertheless, it should also be noted that the students' poor initial mathematical knowledge hinders opportunities for them to produce autonomous work – resources that could spectacularly increase the developmentof competencies.

It is indeed possible to appreciate an improvement in the results for the students following the courses with certain regularity and doingthe tasks set by their instructors, but we still need to design specific assessment tests that will allow the evaluation of competencies.

References

Baartman, L.K.J.; Bastiaens, T.J.; Kirschner, P.A. and Van der Vleuten, C.P.M. (2006)The Wheel of Competency Assessment: Presenting Quality Criteria for Competency Assessment Programs.*Studies in Educational Evaluation,* 32:153-170.

Crawley, E.F.; Malmqvist, J.; Lucas, W.A. and Brodeur, D.R. (2011)"The CDIO Syllabus v2.0. An Updated Statement of Goals for Engineering Education".In *Proc. of the 7th International CDIO Conference*, Technical University of Denmark, Copenhagen. Accessed via: http://www.cdio.org/files/project/file/cdio_syllabus_v2.pdf (23 April 2012).

European Commission, Directorate-General for Education and Cultures (2009) "ECTS users' guide". Accessed via: http://ec.europa.eu/dgs/education_culture/publ/(23 April 2012).

Felder, R.M. and Brent, R. (2003)Designing and Teaching Courses to Satisfy the ABET Engineering Criteria. *Journal of Engineering Education*, 92(No. 1):7-25.

Fenoll, C; Vizcarro, C. and Vieira, M.J. (2012)The Bologna process in Spain, as seen by university leaders, teachers and students: Perceptions and issues. *Preprint.*

Díaz, A.; García, A. and de la Villa, A. (2011) An example of learning based on competences: Use of Maxima in Linear Algebra for Engineers.*The International Journal for Technology in Mathematics Education (IJTME),*18 (No. 4):177-181.

García, A.; García, F.; Rodríguez, G. and de la Villa, A. (2011) "Small Projects: A Method for improving Learning."In *Proc. of the 3rd International Research Symposium on PBL.*Coventry University, 460-470.

García, A.; García, F.; Rodríguez, G.; Vila, J. and de la Villa, A. (2011) "Orthogonal Transformations with DERIVE". *The 17th International Conference on Applications of Computer Algebra*. Houston.

García, J.; Bollaín, M. and Corral, A. (2011) "Applying problem-based project-organized learning in a traditional system."In *Proc. of the 3rd International Research Symposium on PBL*.Coventry University, 684-676.

RUCT (2008). Gobierno de España. Registro de Universidades, Centros y Títulos. Accessed via: http://www.educacion.gob.es/educacion/que-estudiar-y-donde/donde-estudiar-universitarias.html(23 April 2012).

Niss, M. and Højgaard, T. (eds) (2011) *Competencies and Mathematical Learning. Ideas and inspiration for the development of mathematics teaching and learning in Denmark* .IMFUFA tekst nr. 485-2011, Roskilde University.

OECD (2011)A Tuning-AHELO Conceptual Framework of Expected Desired/Learning Outcomes in Engineering.*OECD Education Working Papers*, No. 60.Accessed via:http://www.oecd-ilibrary.org/education/a-tuning-ahelo-conceptual-framework-of-expected-desired-learning-outcomes-in-engineering_5kghtchn8mbn-en (23 April 2012).

VOAPE-UPM (2011)"VOAPE. Formación y evaluación de Competencias Genéricas."http://innovacioneducativa.upm.es/competencias-genericas/formacionyevaluacion
(23 April 2012).

Villa, Aurelio and Poblete, Manuel (2008)*Aprendizaje basado en competencias.* Universidad de Deusto.

Weinert, F.E. (2001)"Concept of competence: A conceptual clarification." In Rychen, D.S. and Salganik, L.H., eds.*Defining and Selecting Key Competencies*. Hogrefe& Huber Pub.,pp 45-65.

Interdisciplinary Tasks: Mathematics to Solve Specific Engineering Problems

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Abstract

The authors have been working for some years in the application of Mathematics to different engineering problems. The main reason is to improve student motivation when studying the authors' subjects and to make possible for students to connect and apply what they learn in these classes with other engineering subjects. In this study some examples of our work is shown and the authors request expressions of interest from other educators at this conference for a future collaboration with this work.

Introduction

To engineering students, Mathematics subjects usually become a difficult task. These students find difficulties trying to understand the usefulness of the knowledge acquired during their university courses and also the usefulness of the tools used in Math classes to solve numerical problems. Students often do not see the connection with the rest of the subjects.

In every new course the authors search for problems and applications that can be found in other disciplines, like Chemistry, Economics, Mechanics, Computer Science, and so on. In some optional subjects that are taught at the College of Industrial Engineering at Béjar, students are asked to develop teamwork or an individual work that relates different subjects, such as Circuits, Automation or Electromagnetism, with those imparted by the authors. The students had to analyse and give details about how they used some of the tools that were explained in the mathematical subjects to solve problems in other disciplines, and the authors were able to check, for example, how Electronic Engineers use Laplace transforms or Z transforms to solve differential equations, but this is not common in other specialties.

As it is well known, the new university emphasis (after Bologna accord, available at the official Bologna Process website) has lead to subjects being changed so as to be more practical. This is the case for subjects such as Linear Algebra, Calculus or Numerical Methods, that form a part of the curriculum in all careers related to science, particularly engineering grades. This led the authors to redesign those subjects.

On the one hand, the challenge for lecturers is to make these subjects more attractive to students not attending Mathematics degrees, who generally do not like maths. On the other hand, the use of computers as part of daily classes, with specific software, like Matlab, Mathematica, simulation programs, or web tools, changes the old idea of learning to a new teaching-learning concept (for an overview see Díaz et al. (2009)).

Gradually, the authors have noticed that the needs of the students and our concerns were common; hence several years ago the authors of this paper began to work together.

Currently, this group is working with other teachers from the same department who teach in different Colleges and also with teachers from other departments at the University of Salamanca, and with professors from other universities. In fact, the Department of Applied Mathematics obtained a teaching innovation project from the University of Salamanca, whose main objective is to look for materials and problems on the applications of Mathematics to Science in general and Engineering in particular. More recently, in 2010 a paper on the work of this project was presented at the international conference entitled "5th European Workshop on Mathematical and Scientific e-Contents", Díaz et al. (2010). During this conference the authors contacted professors from other universities who had similar difficulties to those encountered in our courses. These professors had spent considerable time collecting industrial engineering problems that require the knowledge of the mathematical tools used by our students. These are real problems that future engineers need to solve throughout their careers and later in their professional works.

In the next section some of the possibilities that the Internet offers us to achieve our goals are discussed and in the section following that some examples of real problems whose solution requires knowledge of basic tools from Linear Algebra are presented. Finally, our conclusions are described.

New methods of learning

One methodology useful for students who are doing degrees in Science and Engineering, is that this type of education is based on new technologies with new methods of learning, where software-based tools and other tools available on the Internet play an important role, as mentioned in Kraaijenbrink (2007).

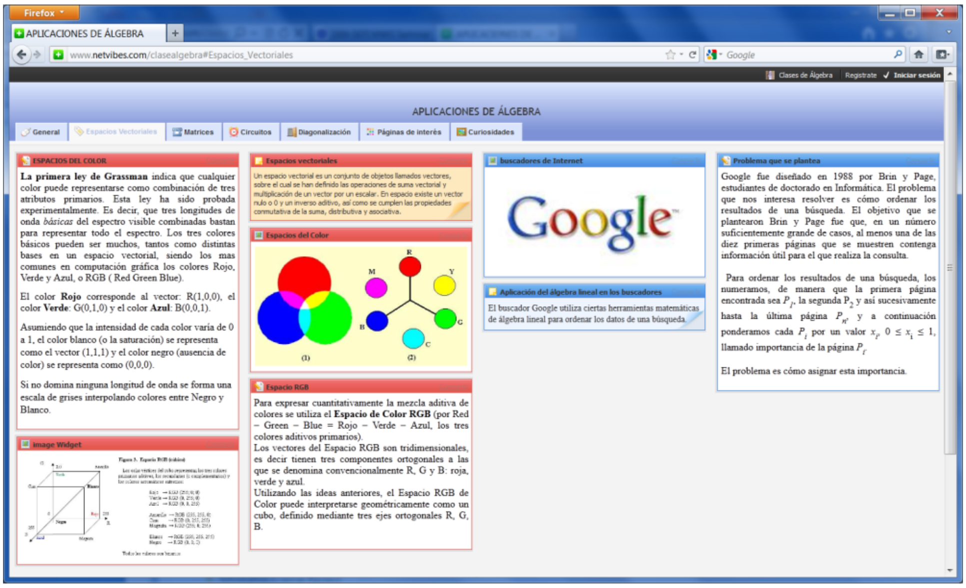

Figure 1. Web site (http://www.netvibes.com) with vector space applications.

Virtual teaching environments, the Internet or different computer tools are useful and represent an important part of the engineers' education. With the help of the new media, the transfer of knowledge can be much more illustrative and instructive than printed media. The authors set out to create a common framework for engineering students who may be studying subjects of mathematics in the early years of their career (Figure 1).

The subjects of these early math content courses are common and what makes them different is the application and suitability to the different degrees. Topics such as vector spaces, matrices or solving systems of linear equations are repeated in most science and engineering degrees. This fact led the authors to consider the idea of directing more theoretical foundations to practical aspects of the concepts being studied.

Interdisciplinary problems

In this section some examples for different specialities are shown:

1. Electrical Engineering

   An electrical network is an interconnection of electrical elements such as resistors, inductors, capacitors, transmission lines, voltage sources, current sources and switches (see Figure 2).

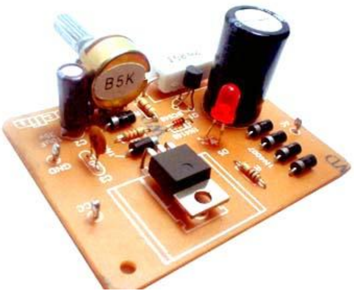

Figure 2. Electrical circuit used in classes.

   An electrical circuit is a special type of network, one that has a closed loop giving a return path for the current.
   The mathematical problem would be to estimate the current intensity in the electrical circuit shown in Figure 3 (see De la Villa (2010)).

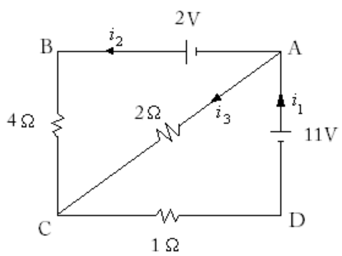

Figure 3. Example with an electrical circuit.

2. Chemical Engineering

The sulphuric acid (H_2SO_4), which has the historical name 'oil of vitriol', has many applications in the chemical industry. When it is with potassium bromate ($KBrO_3$) it is transformed by oxidation to potassium bromide (K Br) to obtain bromine (Br_2), potassium sulphate (K_2SO_4, also called sulphate of potash, arcanite or archaically known as patash of sulphur), and water (H_2O).

Potassium sulfate is a non-flammable white crystalline salt which is soluble in water. These chemical compounds are commonly used in fertilizers, providing both potassium and sulfur.

Figure 4. Chemical application.

Our mathematical problem based on this would be: Adjust the reaction to obtain potassium sulfate, bromine and water from sulphuric acid, bromate potassium and bromide potassium.

3. Textile Engineering

The Kubelka-Munk theory is generally used to connect the spectra reflectance properties of a sample with its constitution. It provides a correlation between reflectance and concentration. The concentration of an absorbing species can be determined using the Kubelka-Munk formula:

$$F\,(R) = (1\text{–}R)^2/2R = k/s = Ac/k$$

where, R is the reflectance, k the absorption coefficient, s the scattering coefficient, c the concentration of the absorbing species, and A the absorbance.

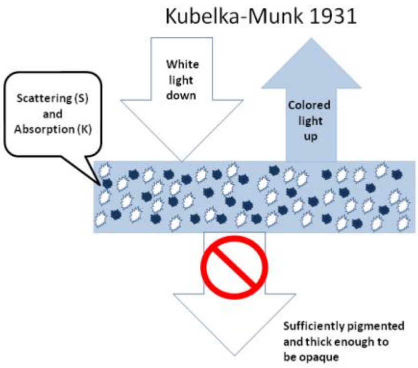

Figure 5. Textile application.

Another way to write this formula is:

$$\Delta F\,(R_{\lambda 1}) = A_1c_1 + B_1c_2 = (1-R_{\lambda 1})^2/2R_{\lambda 1} - (1-R_{t\lambda 1})^2/2R_{t\lambda 1}$$

$$\Delta F\,(R_{\lambda 2}) = A_2c_1 + B_2c_2 = (1-R_{\lambda 2})^2/2R_{\lambda 2} - (1-R_{t\lambda 2})^2/2R_{t\lambda 2}$$

Where A_i and B_i are the corresponding coefficients, c_1 and c_2 are the concentrations of dyes 1 and 2 respectively.

From this formula some equations could be obtained to calculate the tincture concentration of each dye employed in the mix, as a function of reflectance measured at different wavelengths and the corresponding coefficient. Calculate the concentrations when the following data is known:

Coefficients: $A_1 = 3$, $A_2 = 2.25$, $A_3 = 1.5$; $B_1 = 0.4$, $B_2 = 1.3$, $B_3 = 2.8$

Reflectances: $R_{\lambda 1} = 40\%$, $R_{\lambda 2} = 33\%$, $R_{\lambda 3} = 28\%$, $(R_t)_{\lambda 1} = 70\%$, $(R_t)_{\lambda 2} = 75\%$, $(R_t)_{\lambda 3} = 80\%$.

Conclusions

As a result of the work undertaken several examples of various problems using algebra concepts and procedures to solve very different problems have been uploaded to net vibes (http://www.netvibes.com). With these examples it has been shown that rather abstract concepts, such as vector spaces, base changes or diagonalization of matrices, may be of interest in modern control in Computer Science or something as common as the intensity of an electrical circuit, the vibrations that common structures have or the search routine used by the Google searching machine.

Although some of the examples are quite simple, there are other more complex ones used by students on their final courses. The different Math's applications allow the lecturers to relate parts of the mathematics subjects with different aspects of other subject areas. In this way the students understand that the knowledge gained should not be stored in separate compartments of the memory but should enable them to integrate into a total view of their course. Thus we believe that the engineering students find more reasons to be active in classes and they are motivated to find new applications of the mathematical concepts and tools. Hence the students understand other disciplines more easily and this makes them capable of being able to apply them to formulate mathematically any real-life situation.

Until now, the authors have focused on applications that certain tools of Linear Algebra and Numerical Methods provide to solve several engineering problems. However, the authors are interested in making a large and attractive collection of problems for students of other degrees. Thus, any cooperation with this project would be welcomed.

References

The official Bologna Process website, available at http://www.ond.vlaanderen.be/ hogeronderwijs/bologna/.

De la Villa Cuenca, A. (2010) Problemas de Álgebra, 4ª Ed. CLAGSA.

Díaz Len, R., Hernández Encinas, A., Martín Vaquero, J., Queiruga Dios, A., and Ruiz Visus, I. (2009) A Collaborative Learning case study: Mathematics and the use of specific software. *V International Conference on Multimedia and Information Communication Technologies in Education (m-ICTE 2009).*

Kraaijenbrink, J. (2007) Engineers and the Web: An analysis of real life gaps in information usage. *Information Processing & Management*, 43(5): 1368-1382.

Flores Mesa, V., García García, B., Hernández Encinas, A., Martín del Rey, A., Martín Vaquero, J., Queiruga Dios, A. (2010) Three technological tools for learning Coding Theory. *5th European Workshop Mathematical & Scientific e-Contents (MSEC-2010).*

Vallperas Morell, J; Sánchez Martín J. R (2005). *Problemas de Tintorería*. Ed. Golden.

Mathematical modelling competencies in engineering: more than facts, skills and knowing what to do with them

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Abstract

In the United Kingdom, the use of modelling in the teaching and learning of mathematics became prominent around 30 years ago when coursework "projects" were included in the assessment schemes at school level and some universities, mainly the former Polytechnics and the Open University, incorporated it in their instruction. Since then, mathematical modelling has disappeared almost entirely from pre-university education and it was never really embraced by the more "academic", research-led universities.

Alongside this decline in the use of mathematical modelling in classrooms and lecture theatres in the UK (and other countries with westernized educational systems) there has been an increased interest in research in this area (e.g. ICTMA, SEFI). The abundant literature shows in general that mathematical modelling (or the acquisition of modelling competencies) can motivate the study of mathematics and offer learners the opportunity to develop a conceptual understanding of the subject, given the appropriate setting (Hernandez-Martinez et al, 2011; Blomhoj & Jensen, 2003).

However, in my point of view, there are some issues that need consideration:

The majority of the research papers and professional reports present mathematical modelling in an overly optimistic light (almost as a panacea), and although I firmly agree with some of the research results that point out its benefits, this literature fails to show many of the problems (practical and pedagogical) that are associated with the design, implementation and assessment of mathematical modelling tasks in engineering and science courses.

Despite the benefits of mathematical modelling reported in the literature in terms of increased motivation and attainment, few teachers and lecturers are willing or able to implement a modelling approach in their courses.

In this paper/presentation I report on my experience of creating and introducing mathematical modelling in a second year Materials Engineering mathematics module, focusing on the practical difficulties and obstacles I encountered (e.g. employing a student to help me design the tasks, communicating with the engineering department/lecturers, etc.), while at the same time trying to make sense of the learning benefits that such modelling tasks could bring to my particular students. In my attempt to organize, understand and communicate this process to the reader/audience, I frame my thoughts in Activity Theory. This theoretical framework allowed me to view mathematical modelling in a "new light" where learning, conceived as a dynamic social enterprise, not only means acquiring knowledge and skills or even knowing when/how to use them but it means that learning is an experience of identity development where learners become someone or something new. I explore the implications of viewing mathematical modelling in this way, and how the theory can help in resolving some of the issues and dilemmas that I encountered. It is my intention that this experience will help other practitioners that are thinking of introducing mathematical modelling in their practices and to consider the consequences that this might have for policy in the context of Higher Education.

References

Blomhoj, M. and Jensen, T.H. (2003) “Developing mathematical modelling competence: conceptual clarification and educational planning.” *Teaching mathematics and its applications, 22(3), 123-139.*

Hernandez-Martinez, P., Williams, J., Black, L., Davis, P., Pampaka, M. and Wake, G. (2011) “Mathematics coursework as facilitator of formative assessment, student-centred activity and understanding.” *Research in Mathematics Education ,13(2), 201 – 216.*

Competency Development Aspects in Learning Engineering Mathematics

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Abstract

One of the main goals of higher educational institutions is to establish the conditions under which students are induced not only to develop their own competencies but also to improve constantly, via lifelong learning. The search for new teaching models and learning strategies, ensuring clear development of students' competencies, is necessary. In this study, an overview of the aspects that should form part of the contemporary study programme on engineering mathematics is presented. First and foremost, the notion of mathematical competency, with the most common reference points to concentrate on, is discussed. With reference to research analysis of the scientific literature, we reconsider the whole spectrum of competencies (attributable to university graduates) which meet the expectations of the community. Furthermore, the engineering mathematics learning goals and objectives, as well as necessary changes in the learning process itself, are briefly discussed. Finally, some ideas concerning the preparation of study programmes on engineering mathematics and competency (instrumental, operational, holistic) development trends, with associated abilities, are highlighted.

Introduction

To keep abreast with the times, a young person should be prepared to live and work in the continually changing world. That is why one of the main goals of higher educational institutions (according to Barnett (1990) and Bowden and Marton (1998)) is to establish the conditions under which students are induced not only to develop their own competencies but also to improve constantly, via lifelong learning. In pursuing this goal, educational institutions realise that traditional teaching, based solely on delivering class lectures to students-listeners, does not serve the purpose anymore. The search for new teaching (learning) models, facilitating development of students' competencies, is necessary. We have to understand what the competency actually means. What abilities are the leading ones? What abilities can be exercised in lecturing mathematics? Under what circumstances those abilities would increase? In this study, an attempt is made to answer these questions and to highlight the aspects that should form part of a contemporary study programme on engineering mathematics.

What Is Mathematical Competency?

In the scientific literature, the term 'competency' is used to describe abilities associated with an individual's experience (skills, qualification, intelligence, etc.) in tackling problems (Krogh and Roos (1996)). In order to illustrate the definition of 'competency', L. Spencer and S. Spencer (1993) introduced an 'iceberg' model comprising both the visible and the hidden parts. The visible part represents human knowledge, skills, as well as the ability to apply them in problem solving, i.e. it covers everything under the name 'qualification'. One can evaluate the latter using tests and/or professional grading by points. The hidden part of the competency model comprises human viewpoints, values, encouraging an individual to act in one way or the other, and some personal qualities that may either be inborn (physical attractiveness, charisma) or may have developed over time (will, self-sufficiency, responsibility, courtesy, etc.). The latter components most often determine the specialist's career success.

What is meant by 'competency' also depends on the expectations of the social environment. We may be accused of non-competence if we do not demonstrate philosophy of the field experts. Since the outside expectations vary, everything that refers to competency varies also.

In management science, the concept of competency is closely related to individual's abilities in science and learning, culture and interaction with other people, and an individual's ability to create. An employee is considered to be competent if his/her experience allows him/her to embrace the values and operating requirements of the organisation ensuring that it does not fail in competition. The competency consists of an individual's behaviour, knowledge, abilities and skills, as well as individual qualities, viewpoints and values, allowing the performance of any task in line with community expectations.

Returning to the study programmes designed for the development of competency, we have to establish the whole spectrum of competencies (attributable to university graduates) that meet the expectations of the community. R. Barnet, an ideologist of higher education, points out four types of competency, namely: subject competency, activity competency, intellectual competency and personal competency. These competencies should be established and developed at university education level. According to Longworth (1999), the educational institution should establish conditions under which the man was able: to learn democracy (to express one‘s thoughts reasonably, accept difference in opinion, understand the subtlety of thinking, respect the law and other individual, obey the established rules); to develop intellectual capabilities (to contemplate, analyse, generalize, apply, model and assess critically) and to act responsibly (to master team-work, obtain and use proper literature, be able to collect, organise and present data, learn self-help and self-education). According to Reich (cited in Stoll and Fink (1998)), success is guaranteed by the following skills and abilities: generalisation (ability to envisage the essence and regularities, ability to generalise); systematic thinking (ability to relate phenomena); experimentation (ability to progress in lifelong learning); social skills (communication, etc).

Similar aspects are discussed by many researchers (Bowden and Marton (1998)), with the addition of technological literacy, personal qualities (positive attitude, responsibility, self-control, curiosity, adaptability, ability to present and defend ideas), promotion of university values (democracy, critical thinking, parity relations, striving for lifelong learning). Many authors agree the above abilities to be general competencies which should be developed in all programmes, irrespective of the level of education (Longworth (1999)). General competencies (communication and social interaction, problem solving and intellectual abilities, access to information resources and control capabilities, self-education) are the key since they enable a person to learn anything, i.e. acquire any professional abilities, pursue personal goals and grant successful career. In the Recommendation of the European Parliament and of the Council on key competencies for lifelong learning (December 18, 2006), mathematical competency is considered to be equally as important as the other seven key competencies.

Changes in Organising Mathematical Learning

The goals of mathematics, as a subject in engineering studies, reflect two aspects: an individual's training for *common activity* and training for *specialized activity*.

For common activity, development of all general competencies is a priority: first and foremost is the development of intellectual competencies, that are developed in pursuing the objectives of Bloom's cognitive taxonomy; secondly, social interaction competencies, that can be developed through the active learning methods along with the appropriately planned student activity; competencies concerning the ability to use sources of information can be developed through a properly-organised study process where the participants/students are forced to form and analyse basic questions themselves, take notes, etc.; competencies associated with development of the right sort of personal qualities and values are promoted and developed through artificially stage-managed or casual academic situations, personal experiences, through appropriately chosen learning models and tasks that require personal self-control, responsibility, tolerance and other features.

In specialised activity, subject questions, in all their aspects, are essential if they represent an integral part of prospective engineering studies. Thus, subject to study process organisation, some competencies may be either developed or not. The traditional teaching of mathematics (applied science presentation model) focuses on science, i.e. an abstract theory which, being isolated from reality, often turns out to be inaccessible, mysterious and dry. Mathematical science cannot be identified with mathematics as a study subject. Academic knowledge should be acquired through research, i.e. knowledge cannot be presented as the absolute (Barnett, 1990). Therefore, mathematics, being the study subject, should encourage all students to take part in mathematical activities: to use scientific literature, analyse, compare, generalise, encourage imagination, memory and attention, master mathematical symbols and notions, form logical statements, prove statements, seek causalities, discover thought differences, find and tackle errors. Mathematical activity calls students to show attention, diligence, willingness and firm resolution to overcome difficulties, and develops self-control, as well as the ability to estimate learning outcomes and take responsibility for them (i.e. mathematical activity promotes such personal qualities that are highly important in the world of science, business and living). Hence, mathematical activity implies competency development. In other words, the lecturer should model real-world situations (instead of giving absolute mathematical facts), enabling students to notice regularities, form and prove their own hypotheses and theories. Maybe, such theories would not qualify for the theory of mathematical science, but they would definitely meet the engineering requirements. An engineer must have an up-and-doing theory, and have acquired abilities and skills that could be applied in specific problem solving in a particular area of research.

To conclude, previously the lecturer would class lectures focusing on the contents of a subject and his own activities; now the lecturer must keep in mind students' activities, learning approaches and highly active learning methods in line with the available methodical means that are used to stimulate experience and research work.

The study programme on mathematics: preparation aspects

With reference to her own experience, D. Lepaitė (2003) singles out three types of university study programmes:

- *subject type study programmes*. The objective is to pass on some knowledge. It is assumed that the subject knowledge exists independently of the student, i.e. represents an academic wisdom which must be absorbed by students;
- *operational type study programmes*. These programmes test students' abilities to perform a task. Often, the range of particular operations and procedures, within the subject framework, is indicated;
- *competency development study programmes*. These programmes test not only students' abilities in the subject area but also on mastering work, seeking high quality and perfection, i.e. they focus on the whole spectrum of competencies (attributable to university graduates) that meet community expectations.

In the competency development programme, the most important thing is how an individual can achieve (through subject contents) human developmental goals, such as: intellectual goals (analysis, systematisation, evaluation, prediction, problem solving, optimization, efficiency, etc.); policy goals (the use of scientific literature and technical facilities, communication, argumentation, democratic lifestyle, teamwork, etc.); personal qualities promotion goals (willingness, self-support, attentiveness, rationality, leadership, common cultural values, personal hygiene, etc.). In this instance teaching (learning) methods, learning atmosphere and individual features of a lecturer play an important role.

In summary, we can distinguish three types of learning outcomes (Figure 1), namely: *mathematical knowledge*, leading to intellectual abilities; *operational learning outcomes*, leading to social interaction, communication and information control abilities; *character features and values* (attributable to university graduates) that satisfy the expectations of the community.

So, the subject contents should be revised with attempt to select content elements that turn out to be essential not only in the sense of future engineering studies but also with respect to the development and analysis of intellectual abilities at different scales of Bloom's cognitive taxonomy goals. The above three types of learning outcomes are closely interrelated. Actually, the proper choice of the study programme ensures the development of all types of abilities. It means that it is possible to plan different competency development levels:

- *instrumental competency*, which is developed by analysing mathematical textbooks, taking notes, applying educational software programmes, etc. The above activity is bound with values trend, learning motivation, will endeavour, as well as with knowledge acquisition, perception and combination with existing knowledge (first circle in the diagram; Figure 1);

- *operational competency*, which is developed by performing practical projects, communication, presentations, by submitting questions, preparing problem-solving algorithms, performing self-control tests. These activities are bound up with the usage of knowledge, require self-control, attention, communication, teamwork skills, tolerance towards other viewpoints, ability to hear, phrase, and develop both the mathematical language and mathematical logic (second circle in the diagram; Figure 1);
- *holistic competency*, which is developed by asking to prove, give arguments pros or cons (when solving problems in a few different ways and searching for the best solution), in attempts to reveal essential features, stating regularities, tackling creative tasks, organising seminars and discussions. This activity requires supreme (according to Bloom's taxonomy) intellectual abilities to analyse, generalise, lift knowledge into a higher abstraction level, qualify and model a situation. Profound knowledge allows an individual to feel safe, feel responsible and be able to assume command. All this liberates and floods with self-satisfaction.

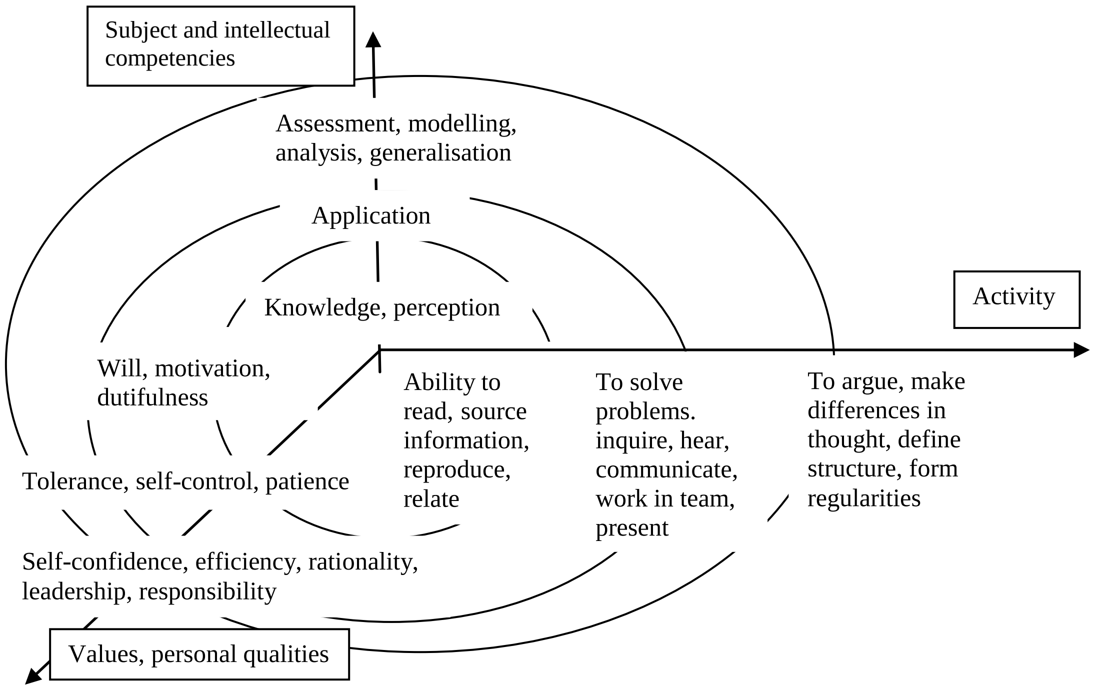

Figure 1. Competency development model.

L. M. Spencer classifies competency levels based on labour intensity, complexity, abstraction rate, perfection, importance and influence extent. Consequently, the study programmes designed for students of different levels (courses) should differ in the amount of generalisable information, abstraction level and the complexity of solvable problems. On the other hand, all study programmes should encourage the development of the same common abilities and promotion of educational values. The lower level competency programmes should lay the foundations for the acquisition of higher competency.

In the competency development model (Figure 1), objective abilities turn out to be more complicated along each coordinate. For instance, consider the activity ‘to argue’. In the vertical direction, there are three distinct argumentation planes: argumentation at the knowledge level means ability to explain (recall) what is written down; argumentation at the application level means ability to explain and substantiate solving schemes; argumentation at the intellectual level means ability to explain theoretical aspects, to prove theorems, to derive formulae, to make conclusions. The level achieved is a matter of great relevance to the assessment results of the students’ knowledge (Sanchez (2008)).

Conclusions

In summary, three essential dimensions with regard to mathematical development of a future engineer have been distinguished: subject dimension, where an individual can seek for knowledge, understanding and application, ability to contemplate, generalise and model (in accordance with Bloom’s goals taxonomy); activity dimension, where an individual is encouraged to strive for mathematical competencies (ability to read, write, calculate, communicate); value dimension, representing both the academic and human values. In compiling competency development study programmes, one must plan one’s own activities, with focus on the students. The competency development model presented outlines possible evolution trends, as well as links between capability aspirations.

References

Barnet, R. (1990) *The Idea of Higher Education*. SHRE / Open University Press.

Bowden, J. and Marton, F. (1998) *The University of Learning.* Routledge & Falmer Press.

Kline, P. and Saunders, B. (1993) *Ten Steps to a Learning Organization.* Great Ocean Publishers.

Krogh, G. and Roos, I. (1996) *Managing Knowledge: Perspectives on Cooperation and Competition.* Cambridge University Press.

Lepaitė, D. (2003) *Kompetenciją plėtojančių studijų programų lygio nustatymo metodologija.* Kaunas : Technologija. (in Lithuanian)

Longworth, N. (1999) *Making lifelong work: Learning cities for a learning century*. Kogan Page.

Sanchez A.V. and Ruiz, M. P. (eds) (2008) *Competence-based learning. A proposal for the assessment of generic competences.* University of Deusto.

Spencer, L. M. and Spencer, S. M. (1993) *Competence at Work. Models for Superrior Performance.* John Wiley & Sons, Inc.

Stoll, L. and Fink, D. (1998) *Keičiame mokyklą.* – Vilnius: Margi raštai. (in Lithuanian)

Mathematics for Engineering Students: Relating competency to theory in inquiry-based teaching

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Abstract

In this talk I relate a number of themes. Principally, I am talking about teaching mathematics to engineering students. In doing so I address the role of *competencies* for teaching mathematics, drawing on the Danish "KOM" project. I address *theory* in considerations of teaching, drawing particularly on theory of constructivism and on sociocultural theory. I also address theory in *inquiry-based practice* in learning-teaching mathematics. I exemplify these areas of theory with reference to a research project in the UK, ESUM, *Engineering students understanding mathematics*. I show how mathematical competencies and different theoretical perspectives may be seen to provide analytical frameworks for the conceptualisation of teaching and relate them to ideas of inquiry-based practice and its role in fostering conceptual learning of mathematics. I consider further how developmental research enables the development of teaching through teachers' and researchers' inquiries into the teaching process and concomitant growth of knowledge about teaching.

Mathematics Remedial Instruction with Math-Bridge e-learning system

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Abstract

Mathematics is important in science, technology and economics. Unfortunately, in recent decades students' mathematical skills seem to have deteriorated in western countries.

Tampere University of Technology (TUT) has an extensive set of measures to support and help freshmen with their mathematics skills. At the start of their studies, all students have to take the Basic Skills Test (BST) in Mathematics. Students who do not pass the BST must participate in the Mathematics Remedial Instruction (MRI). MRI is carried out with a new e-learning system, Math-Bridge.

Keywords: Mathematics, learning support, remedial studying, e-learning, Math-Bridge

Introduction

Good competency in mathematics is important in science, technology and economy as mathematics can be considered not only as a language of nature and technology but also an important methodology in economics and social sciences. A study by Hanushek and Wößsman (2007) shows that the quality of education has a strong positive influence on economic growth. In their research, students' skills were measured using 13 international tests that included mathematics, science, and reading.

Despite the fact that the value of mathematics in society and economics is understood, unfortunately in recent decades students' mathematics skills have deteriorated in western countries. The report "Mathematics for the European Engineer" (2002) by SEFI (The European Society for Engineering Education) states that this phenomenon prevails in Europe.

The results of the PISA surveys were very flattering to mathematics education in Finnish comprehensive schools. In the survey Finnish 15-year-olds were successful in solving real-life problems, which means that the survey mainly measured students' procedural fluency and adaptive reasoning in mathematics. However, conceptual understanding and strategic competence, which are very important features in university mathematics, were not satisfactorily developed in upper-secondary school mathematics (Joutsenlahti 2008).

As mathematical competency is a prerequisite for studying technical sciences, weak mathematical competence slows down studies. For example, 57% of MSc. students starting their studies in Finland at Tampere University of Technology (TUT) in 2005 had completed all mandatory first year mathematics courses by May 2009 – in four and a half years. Students who had progressed fastest in their studies had typically completed first year mathematics courses according to the recommended schedule. Students who faced problems in studying mathematics more often progressed slowly with their studies in general (Pajarre, Lukkari, Lahtinen, 2010).

According to the above-mentioned SEFI report, universities in the western world have observed a decline in mathematical proficiency among new university students and have taken action to remedy the situation. The most common measures are (1) reducing syllabus content – replacing some of the harder material with more revisions of lower level work; (2) developing additional units of study; (3) establishing mathematics support centres; (4) doing nothing.

Math-Bridge was a joint project of nine European universities from seven countries started in May 2009. The aim of the project was to build up a bridge between school mathematics and university mathematics by building up an e-learning platform for online courses of mathematics including learning material in seven languages: German, English, Finnish, French, Dutch, Spanish and Hungarian. This learning material can be used in two different ways of learning: in self-directed learning of individuals and as a "bridging course" that can be found at most European universities (Math-Bridge, 2009, 2011).

Tampere University of Technology has set up an extensive set of measures to support and help freshmen with their mathematics skills. These include Basic Skills Test, Mathematics Remedial Instruction, and Mathematics Clinic (Pohjolainen et.al, 2010).

Basic Skills Test

Since 2002 every TUT freshman has participated the Basic Skills Test (BST) to identify the students lacking mathematical skills. BST is a computer-aided test with 16 upper-secondary school mathematics problems to be solved within 45 minutes. The test uses STACK system (System for Teaching and Assessment using a Computer Algebra Kernel) (Sangwin, 2010) making it possible to generate slightly different problems for each student. Moreover, STACK automatically assesses students' inputs and gives immediate feedback. Thus, students get their test results right after completing the test.

Mathematics Remedial Instruction with Math-Bridge

Those who do not pass the BST must participate in the Mathematics Remedial Instruction (MRI) and pass it in the following four weeks. In Fall 2011, MRI was carried out with e-learning system Math-Bridge. In this and following sections the pedagogical remedial scenario of Mathematics Remedial Instruction with Math-Bridge e-learning system is described together with results of the pilot run of MRI with Math-Bridge

Mathematics Remedial Instruction is a computer-aided brush-up program that includes 71 upper-secondary school level mathematics problems to be solved. The remedial instruction is based on a pure e-learning scenario where a student independently solves given problems within 4 weeks. MRI is realised using a STACK system (Sangwin (2010), which generalises randomly parameterised problems, checks the correctness of students' answers and saves the results in a database.

In MRI the STACK system is integrated to Math-Bridge so that all 71 STACK exercises used in TUT Mathematics Remedial Instruction are executed in the Math-Bridge system. Moreover, a specially designed content from school mathematics is available in Math-Bridge for a student to support his/her studies.

The system built up during the project, is also called Math-Bridge. The users of the Math-Bridge system can be grouped as follows: administrators, authors, tutors and learners/students. They all have a different role in using the Math-Bridge system. The Math-Bridge system serves many pre-defined courses, the option to build own courses from thousands of mathematical learning objects or to use the adaptive course generation tool. The learning objects include theorems, proofs and definitions as well as instructional examples and interactive exercises. The Math-Bridge system pays attention to a student's individual needs by making it easy to find mathematical learning objects necessary for him/her to study. Because of the multilingual learning material, mathematical knowledge in other languages is increased, while knowledge of languages is increased on average (Math-Bridge, 2011).

The Mathematics Remedial Instruction was brought into the Math-Bridge system. There was a special book for MRI, which consisted of theory material, examples and interactive STACK exercises. Hence students were offered self-learning material and not only the exercises, which students had to solve to pass the MRI. Furthermore, students could also make use of many the other learning material served in several languages.

Students who failed to pass BST were directed to participate in Mathematics Remedial Instruction, and other students were also permitted to participate. Thus, instead of 172 students that were directed to MRI, there were 226 students participating in MRI altogether. Subsequently, 182 students accomplished MRI successfully.

Mathematics Remedial Instruction was started on September 19th 2011 with an opening lecture that consisted of information about MRI study procedures and a demonstration of the Math-Bridge e-learning system. On September 20th 2011 a rerun of the opening lecture was arranged so that as many students as possible could get the required information and see the demonstration.

MRI is based on a distance-learning scenario. There are only a few face-to-face teaching sessions: the opening and closing lectures and two-hour tutorials arranged twice a week during the four weeks of MRI. The tutorials were set up for the students as opportunities to ask for help in difficult or problematic parts of MRI. It was not mandatory to participate in the tutorials and this might be the reason why students did not participate in them at all. There was only one student that actually came in a tutorial session during the whole MRI.

All the mandatory tasks of MRI were done in the e-learning system Math-Bridge that included study materials and interactive STACK exercises of MRI. Moreover, after successfully solving the interactive exercises students were directed to rerun the Basic Skills Test to see if their results have been improved. Thus, for those students that finished MRI, there were BST results for before and after the course, as well as pre- and post-test results, to give the student information about his/her progress and possible knowledge gain.

Data and learning results

The learning gain obtained from the MRI with Math-Bridge was studied. It was examined whether MRI with Math-Bridge was helping students to learn topics of mathematics that are pre-requisites in university level mathematics courses and also tested in Basic Skills Test at TUT.

The data used in the analysis consisted of the BST scores before and after MRI. BST scores of all the students were added to data so that possible improvement in BST scores of the students who finished MRI could also be compared to them. Data was analysed by using MathWorks Matlab.

The statistics of Basic Skills Test before and after MRI with Math-Bridge are in the Table 1. There were 148 students that took both BSTs in fall 2011. Thus, there were 34 students that successfully passed MRI, but had not participated in BST at the beginning of their studies. To make sure that the populations are the same in the comparison of BSTs, the results of these 34 students are omitted from the results of BST after MRI.

TABLE 1. Basic Skills Test results before and after Mathematics Remedial Instruction with Math-Bridge. Basic Skills Test consists of 16 school mathematics problems. Each problem is assessed with grade 0 if the student could not answer correctly and with grade 1 if the student answered correctly.

Statistics	Before MRI	After MRI
n	148	148
Average	4,86	11,49
Standard Deviation	1,74	3,01
Min	0	1
Max	9	16
Median	5	12
Average Difference	6,63	
Standard Deviation of Difference	2,86	

It can be seen in the results given in TABLE 1 that both average and median scores of BST were remarkably better in the second round. Since the BST scores of the students who were required to participate in MRI were not normally distributed, non-parametric tests were needed in analysis. Difference in medians was confirmed significant using one-way sign-test (p-value $2.72*10^{-31}$). A boxplot of students' BST scores (Figure 1) supports this.

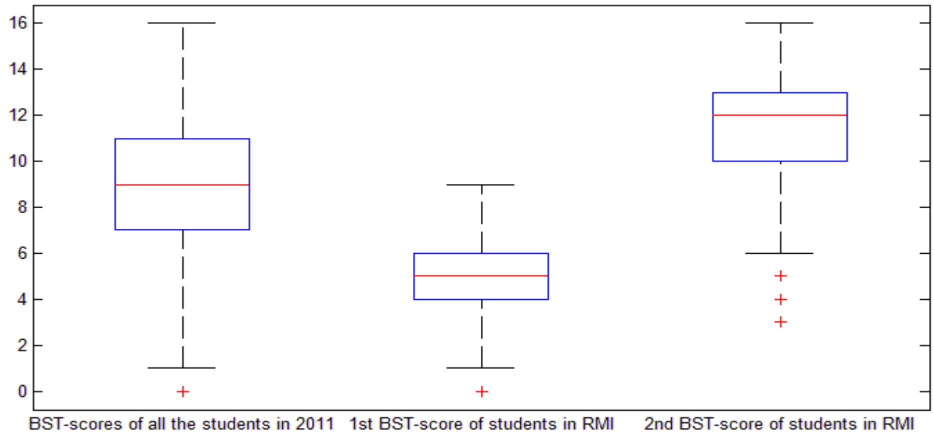

Figure 1. Boxplot of BST-scores.

Figure 1 also shows that the scores of BST taken after MRI seem to be higher than the BST scores of all the students. Median of all the BST scores was compared to the median of the scores of BST taken after MRI using Wilcoxon rank sum test and it was found to be significant (p-value $1.23*10^{-16}$).

So not only did students achieve better on the second BST than on the first but their scores were clearly better with regard to the median even when compared to all the BST scores. The reason for this improvement could be explained with the 71 interactive STACK exercises executed by each student in MRI. These exercises are rehearsing exactly the topics of mathematics that are tested in BST and they are executed in the same way to that the BST problems. Furthermore, after MRI, each student took the Basic Skills Test on his/her own but had access to other materials, such as those offered in Math-Bridge; however, students were told not to use supporting materials during BST after MRI and that test was for them to test their knowledge. Moreover, there was no pressure of passing the BST like it was in the first BST.

Conclusions

Every year students with inadequate mathematical skills begin their studies at TUT. Supportive actions are needed so that these students can complete mandatory mathematics courses and get the tools they need not only to complete their engineering studies but also for other challenges they face after graduation.

Mathematics Remedial Instruction and the Math-Bridge e-learning system are examples of supportive actions carried out at TUT.

It can be said that the overall outcomes of Mathematics Remedial Instruction with Math-Bridge system seem very promising. Although it must be remembered that conditions under which students took the second BST were somewhat different of conditions of the first BST, students' achievement on second BST implies that MRI improved students' basic skills in mathematics.

References

Hanushek E.A., Wößmann L., (2007). The Role of Education Quality in Economic Growth. World Bank Policy Research Working Paper No. 4122, February 2007. (http:/info.worldbank.org/etools/docs/library/242798/day1hanushekgrowth.pdf)

Joutsenlahti J., (2008). The features of mathematical thinking among Finnish students in upper-secondary school. In Carl Winslow (eds.) Nordic Research in Mathematics Education: Proceedings from NORMA 08 in Copenhagen. Rotterdam: Sense Publishers, pp. 315–320.

Math-Bridge, "Annex 1, Description of Work," ECP 2008 EDU 428046 Math Bridge, Version 12/02/2009.

Math-Bridge. Math-Bridge project website. [WWW]. [Cited 16/9/2011]. Available at: http://www.math-bridge.org/.

Mustoe L., and Lawson D., (eds.) (2002). Mathematics for the European engineer. A curriculum for the twenty-first century. SEFI Mathematics Working Group. (http://sefi.htw-aalen.de/Curriculum/sefimarch2002.pdf).

Pajarre E., Lukkari H., & Lahtinen P. (2010). Poissaolojen ja opintojen suoritusjärjestyksen vaikutus opintojen etenemiseen Tampereen teknillisellä yliopistolla. Tampereen teknillinen yliopisto, Yliopistopalvelut, Tampere, Finland.

Pohjolainen, S. & Silius, K. (2011). Teaching and learning mathematics at the university level - with and without technology. In H. Silfverberg & J. Joutsenlahti (Eds.) Integrating Research into Mathematics and Science Education in the 2010's. Annual symposium of the Finnish Mathematics and Science Education Research Association 14.-15.10.2010 in Tampere. University of Tampere Press, pp. 25–45.

Sangwin C.J,, "Who uses STACK? A report on the use of the STACK CAA system," 2010. http://web.mat.bham.ac.uk/C.J.Sangwin/Publications/2010-3-1-STACK.pdf

University Students' Difficulties in Solving Application Problems in Calculus: Students' Perspectives

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Abstract

Common application problems from a typical first-year university calculus course are normally mathematised to a large extent, so there is no need to collect or analyse the data, make assumptions, and so on. But still in many application problems students have to go through the formulation step of the mathematical modelling process that often requires choosing/constructing a formula or setting up a function for further investigation. This paper deals with engineering students' difficulties in the formulation step of solving a typical application problem from a test in a first-year engineering mathematics course. The research question was to find out why most of the students could not use their knowledge to construct a simple function in a familiar context. It was neither the lack of mathematics knowledge nor an issue with the context. The students' difficulties are analysed and presented in the paper along with their suggestions on what can be done from their perspectives to improve their skills in solving application problems.

Introduction

We support the view expressed by Niss, Blum, and Galbraith (2007) that solving application problems can be considered as a subset of the mathematical modelling process which can be described as "consisting of structuring, generating real world facts and data, mathematising, working mathematically and interpreting/validating (perhaps several times round the loop)" (pp.9-10). Craig (2002) described a number of classifications of the word (application) problems. Most common classes are algorithmic and interpretive. Galbraith & Haines (2000) used a similar approach in their study with undergraduate students dividing the problems into mechanical, interpretive and constructive. Another common classification is the division the problems into ill-structured (real-life) and well-structured (school word problems). In our study we use the framework of Galbraith & Haines (2000) and examine the students' difficulties with the constructive type of a problem that requires carrying out some calculations and draw a conclusion from some given information and the existing knowledge of mathematics.
Normally the information given in an application problem includes some numbers, expressions, and stories. This is the stage where many students have difficulties in translating the word problem into the mathematical formula and then deciding which mathematics they should use. As Clement et al (1981) pointed out "rather than being an

immediate aid in learning mathematics, the process of “translation” between a practical situation and mathematical notation presents the student with a fresh difficulty that must be overcome if the application (or the mathematics) is to make any sense to the student in the long run” (p. 287). Talking about the nature of that “translation” they continue: “What makes teaching (and learning) of these translation skills so difficult is that behind them there are many unarticulated mental processes that guide one in construction a new equation on paper. These processes are not identical with the symbols; in fact, the symbols themselves, as they appear on the blackboard or in the book, communicate to the student very little about the processes used to produce them. There seems to be no way to explain such translation processes to students quickly” (p. 289).

In this paper we report university students’ difficulties in solving a common application problem from a typical first-year calculus course. The research question is why most of the participated students could not use their knowledge to construct a simple function in an application problem in a very familiar context. Many researchers and practitioners in different ways ask a similar question (e.g. Crouch & Haines, 2004): “Why is it that students of engineering science, technology and allied subjects find it difficult to move freely between the real world and the mathematical world, when by their own choice of applied discipline one might have expected strong engagement in modelling or pseudo-modelling tasks?” (p. 199).

The Study

We, the lecturers from a New Zealand university and a German university, were surprised with some results from a routine mid-semester test given to our engineering students. Although many students performed well in the procedural test questions that required mathematical techniques and manipulations the vast majority of the students failed to solve the application problem below:

Problem 6. The cost of running a heavy truck at a constant velocity of v km/h is estimated to be $4+\frac{v^2}{200}$ dollars per hour. Show that to minimize the total cost of a journey of 100 km in the truck at constant velocity the truck should run approximately 28 km/h.

The above is the English version and the students from the German university received it in the German language. From our (lecturers’) point of view the students had sufficient knowledge, skills, common sense and practice to solve the problem easily. The vast majority of them studied calculus at school where similar problems would have been common. In our tutorials and lectures about a quarter of the suggested problems were application problems. However, the result was very surprising: within about 6 minutes (an average time a student had for Problem 6 on the test) only 4 out of 92 students in the New Zealand university and 6 out of 105 students from the German university were able to set up the total cost function correctly recognising that:

TOTAL COST = COST PER HOUR × TIME IN HOURS

that is

$$\text{TOTAL COST, } F(v) = \left(4 + \frac{v^2}{200}\right) \times \frac{100}{v}.$$

As usual we give our students the model answers to the test questions and discuss them in a class in the following week. Before discussing the solution to Problem 6 we asked the students the following question: *If someone works at the rate of 10 dollars per hour for 6 hours, what is his/her total earnings?* All students gave the correct answer immediately: 10 dollars per hour × 6 hours = 60 dollars. Then we asked them the next question: *If someone works at the rate of R dollars per hour for T hours, what is his/her total earnings?* Again all the students replied: $R \times T$. That is, they used the formula:

TOTAL EARNINGS = RATE PER HOUR × TIME IN HOURS.

But this formula for Total Earnings and the formula for Total Cost in Problem 6 have the same nature and structure. The students did numerous basic algebra problems in the past at school and university when they used quantities expressed as letters rather than numbers. They did applications problems in calculus at school and in our tutorials. The context of the problem did not require any special knowledge – just common sense. The students knew the formula relating the distance, constant velocity and time. The vast majority of the students were well motivated and hard working and they definitely tried to perform well on the test. The students were majoring in engineering and yet the vast majority of them failed to set up the function in Problem 6 correctly. Why? To find this out we gave our students the following short questionnaire after the discussion of this problem in class in the following week after the test:

> **Question 1.** What difficulties did you have while trying to set up the total cost function in Problem 6?
>
> **Question 2.** What can be done to improve your skills in doing this step of an application problem?

Answering the (anonymous) questionnaire was voluntary so it was a self-selected sampling. We received 104 responses, 54 from the New Zealand group and 50 from the German group, so the response rate was 57% in the New Zealand group and 48% in the German group.

Students' Responses

Question 1. What difficulties did you have while trying to set up the total cost function in Problem 6?

The main students' difficulties fell in the following two categories.

1. Difficulties related to understanding of the problem (language, use of the given information, identifying the variables): 48% in the New Zealand group and 36% in the German group. Some typical comments were as follows:
   "The wording was ambiguous";
   "I did not understand the question"
   "It was confusing";

"Hard to understand";
"I thought it was too complicated";
"I had trouble deciding how to use the information";
"I did not know how to convert the real life problem into one to solve mathematically";
"I was confused because the result was given".

In the subsequent informal interviews with the selected students it was revealed that the last comment was quite common. The fact that the answer was given hindered many students identifying the unknown variable. The students were used to a different way of formulation of the question in min-max problems. They reported that it would be easier for them if the question had been formulated like this "find the velocity that minimizes the total cost of a journey of 100 km" instead of "show that to minimize the total cost of a journey of 100 km in the truck at constant velocity the truck should run approximately 28 km/h". Although they met mathematics questions like "show that" before but it was less common for them to see it in application problems. The observed sensitivity to the wording of the question in the problem is consistent with another interesting fact. In New Zealand every three years there is a notable drop followed by a two-year increase in the students' performance at the final school year mathematics exam. It was reported by a chief school mathematics examiner that the reason for the regular drops was the change of a chief school examiner every three years. A new chief examiner used their own language style of setting up exam questions that was different from the wording used by their predecessor.

2. Difficulties related to the identification and usage of the formula: 35% in the New Zealand group and 42% in the German group. Some typical comments were as follows:

"Couldn't figure out time as $\frac{100}{v}$";
"I had trouble with the units, because in the given formula I only knew the unit of *v*";
"Could not see the connection between costs per hour and time";
"Did not know where to use $(4+\frac{v^2}{200})$".

Those students apparently understood the problem but could not use their existing mathematical knowledge of familiar formulae to set up the required function correctly.
The above two sorts of difficulties are in agreement with the classification from the study by Anaya et al (2007) on novice engineering students' difficulties in mental processes doing a more general modelling task:

- Difficulties related to the relational understanding of the situation to be modelled, including difficulties in the identification of variables and unknowns.
- Difficulties related to creativity in establishing associations and relationships between pieces of knowledge that eventually might not have been related up to that moment.
- Difficulties related to the choice of the available knowledge and the use of the given information (p.428).

Question 2. What can be done to improve your skills in doing this step of an application problem?
The vast majority of the students (87% in the New Zealand Group and 78% in the German group) thought that they needed more practice in solving application problems similar to problem 6 in class to improve their problem solving skills. Some of them asked to be taught detailed steps in solving such application problems and give more "demonstration". Only a small number of the students (6% in the New Zealand group and none in the German group) suggested making wording of a problem easier to understand in spite of the fact that almost half of the students in the New Zealand group stated that they did not understand the problem.

Discussion and Conclusions

The two major students' difficulties reported in this study dealt with understanding of the wording of the question in the problem and with using their existing knowledge of familiar formulae for setting up the required function in a familiar context. Both those difficulties are typical characteristics of novices that were observed in a number of studies (e.g. Galbraith & Haines (2000), Crouch & Haines (2004), Anaya (2007)). Quite surprising for us was the observed sensitivity of many students to the wording of the question where they were asked to show that the required velocity was equal to the given value instead of finding the required velocity. The vast majority of the students participated in the study thought that they needed more practice to improve their skills in solving application problems. It is consistent with another study (Klymchuk & Zverkova, 2001) with more than 500 university students from 9 countries where the students also indicated that they felt it difficult to move from the real world to the mathematical world because of the lack of practice in application tasks. Practice is certainly one of the ways that helps students to progress from novices to experts. We can increase time spent on application problems and show their importance by including more such problems in the assessment. At the moment in our (authors') calculus courses about 25% of all test and assignment questions are application problems. This can be increased to 50%. It might encourage students to practise more with application problems preparing for their tests and exams.

The comments of the students that participated in the study make us think that our assumptions about reasonable application problem solving skills of our students were too optimistic. It demonstrates again that assumptions lecturers make about students regarding their knowledge base and successful completion of earlier modules and/or examinations cannot be relied upon as it was shown in Anderson et al. (1998). In our diagnostic test at the beginning of the course we check only students' basic mathematical techniques but not their skills in solving application problems. This study shows that there is a need to teach the students basic skills in solving application problems from the beginning of a calculus course. We should encourage the students to write all steps of the modelling process in detail, even for simple application problems. This can prepare them to deal with real-life problems that require advanced mathematical modelling skills in their other courses and also at work. We agree with Kadijevich (1999) who pointed out an important aspect of doing even simple mathematical modelling activity by first-year undergraduate students: “Although through solving such … [simple modelling] … tasks students will not realise the examined nature of modelling, it is certain that mathematical knowledge will become alive for them and that they will begin to perceive mathematics as a human enterprise, which improves our lives” (p. 36).

References

Anaya, M., Cavallaro, M.I. & Domingiez, M.C. (2007). Modelling: Difficulties for novice engineering students. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), Mathematical Modelling (ICTMA 12): Education, Engineering and Economics (pp.324-332). Chichester, UK: Horwood.

Anderson, J., Austin, K., Bernard, T. & Jagger, J. (1998). Do third year mathematics undergraduates know what they are supposed to know? International Journal of Mathematical Education in Science and Technology, 29(3), 401-420.

Clement, J., Lochhead, J. & Monk, G. (1981) Translation Difficulties in Learning Mathematics The American Mathematical Monthly, 88(4), 286-290.

Craig, T. (2002) Factors affecting students’ perceptions of difficulty in calculus word problems. In I.Vakalis, D.H.Hallet, C.Kourouonoitis, D.Quinney, C.Tzanakis (Eds.) Proceedings of the Second International Conference on the Teaching of Mathematics at the Undergraduate Level – ICTM-2. Crete, Greece. Wiley, CD format, #411.

Crouch, R. & Haines, C. (2004). Mathematical modelling: Transitions between the real world and the mathematical world. International Journal on Mathematics Education in Science and Technology, 35(2), 197-206.

Galbraith, P. & Haines, C. (2000) Conceptual mis(understandings) of beginning undergraduates. International Journal of Mathematical Education in Science and Technology 31(5), 651-678.

Kadijevich, D. (1999). What may be neglected by an application-centred approach to mathematics education? Nordisk Matematikkdidatikk, 1, 29-39.

Klymchuk, S. S. & Zverkova T. S. (2001). Role of mathematical modelling and applications in university service courses: An across countries study. In J. F. Matos, W. Blum, S. K. Houston, & S. P. Carreira (Eds.) *Modelling, applications and Mathematics Education – Trends and Issues* (pp. 227-235). Chichester, UK: Ellis Horwood.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. Galbraith, M. Niss, & H-W.Henn (Eds.), Modelling and Applications in Mathematics Education: The 14th ICMI Study (New ICMI Series), (Vol. 10, pp. 3-33). New York: Springer.

Technology enhanced tutoring and automated assessment in engineering mathematics

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Abstract

Engineering programmes in Helsinki Metropolia University of Applied Sciences have decided to give special emphasis to the start of the studies by placing the students in small groups appropriate to their school background and their mathematical abilities. Students' background is very variable and many students have a low motivation. Students are encouraged to take a special mathematics course 'Expressions and equations' in the beginning of their studies. The placement tests and the course assignments are based on the STACK computer aided assignment system. In this paper we present an overview and discussion of the principles of the system and the pedagogical principles applied.

Together with the University of Helsinki, Metropolia, and some other European universities, including the University of Salamanca, has also started developments towards more sophisticated tutoring and learning environment. Its idea is to provide suggestions about course of content and activities, called learning paths, to students. Based on machine learning techniques and automatic student profiling the system is able to predict suitable learning modules for a student depending on his or her previous learning history.

The pedagogical guideline is to encourage the students into independent study and problem solving. The automatic problem and quiz feedback is immediate and available any time. The learning environment includes also audiovisual minilectures on core topics. It is also possible to arrange diagnostic and formative tests that apply the same assignment system. As the practising of the basic techniques is mainly managed through the online system, we have also more time for motivating applied projects. We have automatic problems also for other first-year mathematics courses and audiovisual minilectures are under development.

Assessment is an integral part of the learning process. Teachers and digital classrooms will embed continuous assessment in instruction in a fundamentally new way. Instructors will be able to monitor their students' advancement continuously. If a student is in danger of dropping out, immediate action can be taken. Instructors also get meaningful feedback on how effective their teaching is, and hence will be able to adjust to the needs of the class as a whole and to students as individuals. The present form of assessment of learning outcomes has been replaced by assessment for learning instead of assessment of learning. Also the automatic tutoring based on machine learning techniques from a large amount of student activity data helps to give feedback at the appropriate point before a student has fallen too far behind of the curriculum.

We have found that this kind of a system will motivate the students to work harder and solve more problems that is possible in an entirely teacher-driven pedagogical framework. The system gives immediate feedback that is extremely important in steering the learning process and encouraging the student. The results show that activity in STACK problems correlates strongly

with achievements in traditional tests. In addition to discussing the pedagogical and technical principles we will also give a few practical demonstrations of the system in our talk.

Structured Teaching Approaches for Students with Under-Developed Mathematical Skills

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Abstract

Ireland has one of the highest third-level participation rates in the world, with more than 60% of Irish students going on to further education. The state-funded third-level education sector consists mainly of 7 universities and 14 institutes of technology. The majority of high achievers in the Irish Leaving Certificate Examination (the terminal examination of post-primary education) attend universities, so the institutes of technology must meet the challenge that a large proportion of their students have average or below-average achievement in school.

It is found that many of the basic habits and thought processes of good learners are absent or under-developed and must be actively encouraged. A small selection of specific teaching approaches are presented, aimed at addressing this problem in particular areas of difficulty in Mathematics, including expression parsing, rearranging formulas and inverse functions, integration by parts, the chain rule of differentiation and matrix multiplication.

A theme running through the various methods is to provide a structure within which the students can operate, as it is found that serious difficulties regularly arise from disorganized work based on poor parsing of the original problem. At IT Tallaght, we have also found that we have a high proportion of so called "visual learners" on our programmes who should find it easier to work with a highly structured tabular layout.

Expression Parsing

Much of our teaching of basic algebra (if it is taught explicitly at all) consists of taking an expression and showing the student how it can be changed into another expression. This pre-supposes that the original expression held any meaning at all to the student. By this, we do not mean what it may represent as an engineering quantity, but simply what it might mean to the student as a piece of grammar. We believe that many of our students make no sense of most algebraic expressions in terms of how it is put together through arithmetic operations. This leads many students to fail to transform this essentially meaningless collection of symbols correctly into some other meaningless collection.

As Merlin (2008) says in his thesis abstract

Evidence shows that transforming expressions is a major stumbling block for many algebra students. Using Sfard's (1991) theory of reification, I highlight the important roles that the process of parsing and the notions of subexpression and structural template play in competent expression transformation. Based on these observations, I argue that one reason students struggle with expression transformation is the inattentiveness of traditional curricula to parsing, subexpressions, and structural templates.

Merlin goes on to discuss instructional strategies and emphasizes that

Algebra curricula need to give explicit attention to parsing and to structural notions in ways that will make structure a strong competitor for perceptual salience among the many impulses competing for student attention.

In his Thesis Merlin introduces expression trees as a strong visual structural template to help students learn how to parse an expression. Such trees are common in computer algebra systems. In IT Tallaght we use a tabular approach to parsing, incorporating as it does the very comforting Step1, Step2, .. type hierarchy, as an easier route to expression parsing than trees for weak students.

To introduce expression parsing, operations on numbers are discussed and a basic table of operations built up. This table will be added to as we need to do more operations. The basic table simply contains +, -, × and ÷. We must then introduce *terms* of expressions and *factors* of terms. The parsing strategy is then to create one term containing the starting variable and then to add the other terms. This tabular approach is particularly suited to the many "visual learners" doing Engineering at IT Tallaght Cranley et al (2005)

Example 1: Parse the expression $3(x+2y)+z$ starting from x.
Solution:

Operation starting at x	Result
+2y	$x+2y$
×3	$3(x+2y)$
+z	$3(x+2y)+z$

Since we add or subtract *terms*, we must start by adding the term 2y, not the number 2. Now we have the factor in the first term which contains x. We multiply by any factors to finish creating the first term. Now add any other terms.

Example 2: Parse the expression $3(x+2y)+z$ starting from y
Solution:

Operation starting at y	Result
×2	$2y$
$+x$	$x+2y$
×3	$3(x+2y)$
$+z$	$3(x+2y)+z$

We go on to introduce the operations $(\ )^2$, √, $(\ )^n$ and $(\ )^{-1}$.

Example 3: Parse the expression $\dfrac{2T}{3x^2-1}$ starting from x.

Solution:

Operation starting at x	Result
^{2}	x^2
$\times 3$	$3x^2$
-1	$3x^2 - 1$
$^{-1}$	$\dfrac{1}{3x^2 - 1}$
$\times 2T$	$\dfrac{2T}{3x^2 - 1}$

Operations exp( ), ln( ), cos( ), sin( ) are further introduced, and so on. Clearly, we can go the other way with this process and give the students a parsing table to be converted into an expression. As a class activity we use a variation on the game of "Chinese whispers". The first student in a row is handed an expression to parse. The parsing table is then passed to the next student, who must turn it back into an expression. This process goes on until we reach the end of the row and the final expression is revealed… This works best in groups of 3 (expression, table, expression or table, expression, table) as any discrepancy between the start and finish can be resolved within the group rather than by the lecturer.

Transposition of formulae and Inverse functions

Transposition is a common task for all engineering students in many of their modules, not just mathematics. For rearranging formulas, the original formula must be parsed and the list of operations applied to the variable of interest listed in order. Once this is done, the correct sequence of inverse operations is easily deduced. The sequence of inverse operations is performed to both sides of the equation in turn, eventually producing the required result. The production of a plan is found to be of particular value to students who would not usually take such an organized approach to solving a problem.

Example 4: Rearrange the formula $L = \dfrac{2T}{3x^2 - 1}$ to make x the subject.

Solution: By parsing the original equation, formulate a plan (read upwards on the right).

Original Operations applied to x	Sequence of inverse operations (to be performed in reverse order)
^{2}	$\pm\sqrt{\ }$
$\times 3$	$\div 3$
-1	$+1$
$\dfrac{1}{\ }$	$\dfrac{1}{\ }$
$\times 2T$	$\div 2T$
$= L$	L

Next implement the plan by applying the reverse sequence of inverse operations to both sides of the equation:

$$
\begin{aligned}
L = \frac{2T}{3x^2 - 1} \quad &\Leftrightarrow \quad \frac{L}{2T} = \frac{1}{3x^2 - 1} && \text{dividing both sides by } 2T \\
&\Leftrightarrow \quad \frac{2T}{L} = 3x^2 - 1 && \text{inverting both sides} \\
&\Leftrightarrow \quad \frac{2T}{L} + 1 = 3x^2 && \text{adding 1 to both sides} \\
&\Leftrightarrow \quad \frac{1}{3}\left(\frac{2T}{L} + 1\right) = x^2 && \text{dividing both sides by 3} \\
&\Leftrightarrow \quad \pm\sqrt{\frac{1}{3}\left(\frac{2T}{L} + 1\right)} = x && \text{taking square roots of both sides}
\end{aligned}
$$

This solution method is particularly effective when solving equations with exp( ) and ln( ) functions. Students successfully see exp( ) and ln( ) take their place in the table as operations with their inverse, and treat them exactly like they would + and – .

Example 5: Find the time t when $3e^{-4t} = 2$

Solution:

t	= t
$\times\ -4$	$\div\ -4 \quad = 0.1$
e	$ln \quad = -0.4005$
$\times 3$	$\div 3 \quad = 0.67$
= 2	2

So that t = 0.1sec.

This tabular layout also gives the student some insight into the buttons on their calculator and that each operation has an inverse or "2nd function". Finally, exactly the same layout can be used to introduce inverse functions in general and the relationship $fog^{-1} = g^{-1} o f^{-1}$ has a clear meaning as a table of operations and their inverse.

To further re-inforce the function/inverse function relationship lecturers at IT Tallaght have employed the following technique. Students are given a non- permanent over head projector (OHP) pen and an OHP transparency sheet on which the positive X axis and Y axis are marked distinctly along with a background grid. Students are asked to sketch a function onto the sheet using the pen. Figure 1 below shows *y=ln(x)* sketched on such a transparency sheet. The students then rotate the transparency sheet clockwise 90° (see Figure 2) Finally the students flip the transparency sheet back to front so that now the

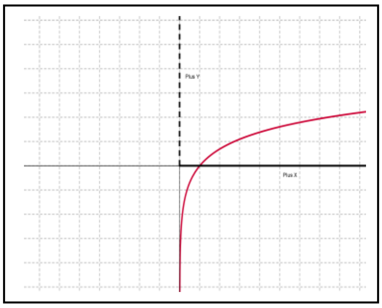

Figure 1
Original Sketch $y=ln(x)$

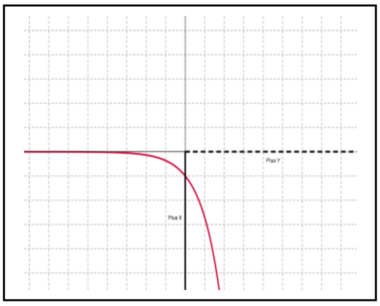

Figure 2
Original Sketch rotated clockwise 90°

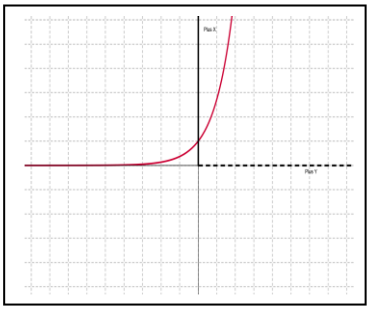

Figure 3
Original Sketch rotated clockwise 90° and flipped

positive X and Y axis have swapped position and the sketch is now that of the inverse function of the original sketch (see Figure 3 where the inverse function is $y = e^x$). Students re- use the transparency sheet to investigate other function/inverse function pairs. The technique is particularly beneficial in emphasising the importance of domain and range for inverse functions for functions such as y = sin(x).

Further Structured methods

Matrix Tables

Matrix multiplication is aided by placing the two matrices to be multiplied as shown; now each row by column calculation has a natural position for the answer in the new matrix product.

Example 6: Multiply the matrices $A = \begin{pmatrix} 3 & 4 \\ 2 & 1 \\ -5 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 1 & -7 \\ 6 & -3 & 2 \end{pmatrix}$.

$$B = \begin{pmatrix} 5 & 1 & -7 \\ 6 & -3 & 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 4 \\ 2 & 1 \\ -5 & 7 \end{pmatrix} \quad AB = \begin{pmatrix} 3\times5 + 4\times6 & 3\times1 + 4\times-3 & 3\times-7 + 4\times2 \\ 2\times5 + 1\times6 & 2\times1 + 1\times-3 & 2\times-7 + 1\times2 \\ -5\times5 + 7\times6 & -5\times1 + 7\times-3 & -5\times-7 + 7\times2 \end{pmatrix} = \begin{pmatrix} 39 & -9 & -13 \\ 16 & -1 & -12 \\ 17 & -26 & 49 \end{pmatrix}$$

Tableau method of integration

The tableau method of integration by parts is not new, see Horowitz (1990), but is not well-known in Ireland. The functions u and $\frac{dv}{dx}$ are identified and included in the differentiation and integration columns of the table, which also has a sign column. The process can be stopped correctly at any point so long as it is understood that multiplication takes place along the indicated arrows, while the final integral of products (which is across – across instead of across – down) can be done without further iteration of the table. The method is clearly of particular value where several repetitions of the integration by parts formula must be applied.

Example 7: Tableau method of Integration by parts to integrate $\int 5t^3 \cos 6t \; dt$.

Sign	**Diff**	**Int**
$+$	$5t^3$	$\cos 6t$
$-$	$15t^2$	$\frac{1}{6}\sin 6t$
$+$	$30t$	$-\frac{1}{36}\cos 6t$
$-$	30	$-\frac{1}{216}\sin 6t$
$+$	0	$\frac{1}{1296}\cos 6t$

$$\int 5t^3 \cos 6t \; dt = \frac{5}{6}t^3 \sin 6t + \frac{15}{36}t^2 \cos 6t - \frac{30}{216}t \sin 6t - \frac{30}{1296}\cos 6t + C .$$

We see that the remaining integration (along the dashed arrows) is trivial to do.

The method can also be applied to examples producing an iteration formula.

Example 8: $I = \int e^x \cos(x) dx$

Sign	**Diff**	**Int**
$+$	e^x	$\cos x$
$-$	e^x	$\sin x$
$+$	e^x	$-\cos x$

We see that $I = e^x \sin(x) - e^x(-\cos(x)) - I$ from which the integral I can be found.

The Chain Rule

For the Chain Rule of Differentiation, one tabular approach is to identify the functions involved in the composition and to write them in the correct order independently of input variables. Differentiation of each individual function takes place only at the top of each of the columns in the resulting table; otherwise the functions are simply reproduced for inclusion in the appropriate compositions of functions for the final answer. This is best illustrated by an example.

Example 9: The Chain Rule to differentiate $\frac{d}{dx}\ \sin\ \sqrt[5]{x}\ ^{4}$.

Input	1st	2nd	3rd	Output
x	$^{\frac{1}{5}}$	$\sin$	^{4}	$\left(\sin\left(x^{\frac{1}{5}}\right)\right)^{4}$
x	$\frac{1}{5}\ ^{-\frac{4}{5}}$			$\frac{1}{5}x^{-\frac{4}{5}}$
x	$^{\frac{1}{5}}$	$\cos$		$\cos\left(x^{\frac{1}{5}}\right)$
x	$^{\frac{1}{5}}$	$\sin$	$4\ ^{3}$	$4\left[\sin\left(x^{\frac{1}{5}}\right)\right]^{3}$

Hence $\frac{d}{dx}\ \sin\ \sqrt[5]{x}\ ^{4} = \frac{1}{5}x^{-\frac{4}{5}}.\cos\left(x^{\frac{1}{5}}\right).4\left[\sin\left(x^{\frac{1}{5}}\right)\right]^{3}$.

Conclusion

A variety of structured methods have been introduced which, we believe, can help weak students in particular to complete some of their regular mathematical tasks accurately.

References

Merlin, Ethan Michael (2008) "Beyond PEMDAS: Teaching students to perceive algebraic structure", Master of Arts Thesis, University of Maryland. Accessed via http://drum.lib.umd.edu/bitstream/1903/8553/1/umi-umd-5639.pdf

Horowitz, David (1990) "Tabular Integration by Parts", *College Mathematics Journal* 21: 307–311.

F. Cranley, CO'Sullivan (2005) "Analysis of Learning Styles in a First Year Engineering Course." In Proceedings of 22nd International Manufacturing Conference, pp 327-331.

Engineering Students Understanding Mathematics (ESUM) – Increasing conceptual understanding and engagement

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Abstract

ESUM is a developmental research project, aiming to increase the engagement and conceptual understanding of mathematics of first year engineering students in a UK university. The research involved the design, introduction and study of a teaching innovation which included inquiry-based questioning and tasks, small group work, a group project and the use of GeoGebra software to increase student participation, understanding and mathematical competence developing the ability to recognise and apply mathematical concepts in engineering contexts.

The rationale for this research is a recognition that engineering students' engagement with mathematics is often of an instrumental nature and that traditional university mathematics teaching may foster instrumental learning rather than a desired conceptual understanding. An inquiry-based approach to learning mathematics develops collaborative exploratory activity leading to increased conceptual understanding within a community of practice of university mathematics teaching.

Both the teaching process and outcomes were studied. Feedback led to concomitant modification to teaching practice. Evidenced by the observation of activities, analysis of student surveys, focus groups and interviews, improved test scores and examination results compared to previous cohorts, the innovation was found to have increased student engagement and understanding. In spite of successful outcomes, however, the study revealed tensions between the lecturer's aims in designing a teaching innovation and the students' strategic approach to learning within the context and culture of university teaching and learning. Activity Theory analysis was used to reconcile tensions between the teaching aims and students' perceptions.

Introduction

ESUM (Engineering Students Understanding Mathematics) was a developmental research project, which had two aims: firstly to increase the conceptual understanding of mathematics and engagement of a cohort of engineering students and secondly, to study how such a teaching innovation may be designed and implemented. The mathematics module was taught to 48 first year engineering students at a UK university (2010-11). Students were enrolled on several *materials* engineering programmes with different mathematics entry requirements and had a wide range of prior mathematics qualifications. The research took place during the first semester of the module with a focus on pre-calculus mathematics and was presented over 15 weeks with two 50 minute lectures and one tutorial per week. The lecturer, a member of the research team, had previously taught this module for three years and was seeking to stimulate and challenge students and to encourage their involvement during lectures. Modifications to teaching this module in previous years had had limited success. (Jaworski 2008, 2010).

Rationale for the project is the recognition that engineering students' engagement with mathematics is often of an instrumental nature and that traditional university mathematics teaching may foster instrumental learning rather than a desired conceptual understanding (Skemp, 1976; Hiebert 1986; Hawkes and Savage, 2000). An inquiry-based approach to learning mathematics develops collaborative exploratory activity intended to lead to increased conceptual understanding (Jaworski 2004; Wells 1991) within a community of practice of university mathematics teaching (Wenger, 1998).

Both the design of teaching and its implementation were researched with data being collected at all phases of the project. The research team, which collaborated closely with colleagues from engineering, consisted of three lecturers from the Mathematics Education Centre (MEC) at Loughborough University with considerable experience in the teaching of mathematics to engineers, and a research officer. Expertise of others was drawn on as appropriate for question design, tutorial assistance and a literature review conducted by a post-doctoral researcher. Working within the research team, the research officer collected data through audio recordings and observations of lectures and tutorials and obtained feedback from students through two surveys, focus groups and interviews. The lecturer wrote a weekly reflective log considering issues that had arisen and modifications that could be made immediately or be considered in the longer term. The team met regularly during the course of the project.

ESUM was supported by funding from the HE STEM programme and the Royal Academy of Engineering. (http://www.hestem.ac.uk) The project has been reported in detail in two case studies. (Jaworski & Matthews, 2011a; Jaworski, Matthews, Robinson & Croft, 2012)

The Teaching Innovation

The teaching innovation consisted of a coherent approach to promote mathematical conceptual understanding and engagement. This included the integral use in all activities of GeoGebra (http://www.geogebra.org/), free multi-platform software, which combines geometry, algebra, tables, graphing, statistics and calculus and was used as a tool to promote mathematical conceptualisation. To encourage collaboration, students were organised in groups of 3 or 4 based on the programme for which they were registered to facilitate meeting in their own study time. They worked in the same groups on exploratory activities in tutorials and on an assessed group project. Tasks were designed to stimulate questioning and inquiry, and to challenge students' mathematical thinking. During tutorials, the lecturer and a graduate assistant encouraged both discussion and conceptual engagement.

Assessment of the module was modified from prior years. An examination (60%), taking place at the end of the second semester on the work in both, was left unchanged. Eight Computer Aided Assessment (CAA) tests were reduced to four (20%), two tests across each semester. The remaining marks were allocated to the purpose-designed inquiry-based project, assessed on a group report (15%) and an optional project poster (5%). A mathematics graduate worked with two of the research team to design three versions of the group project. Completed projects were circulated to another group for comment providing scope for students to learn from each other's work.

Two research team members worked with a PhD student to explore sources of inquiry-based questions, which were modified and incorporated in lectures and tutorial materials. The lecturer also made use of other modes of questioning (open, closed) to gain an insight into levels of understanding and encourage discussion. Various teaching materials such a series of HELM (Helping Engineers Learn Mathematics http://helm.lboro.ac.uk) workbooks and a VLE page with links to lecture materials and past examination papers were made available. During lectures PowerPoint presentations and OHP slides were used together with a mode of teaching that opened up mathematic topics for conceptual consideration.

Findings and Analysis

Data analysis took place at two levels – ongoing reflective analysis as part of the design and teaching phases and analysis of data after the end of teaching. Qualitative analysis focussed on aspects of the innovation such as the students' use and reflections on GeoGebra in relation to the teaching intentions for its use and the ways in which inquiry-based questions promoted mathematical conceptual understanding and engagement. The literature review showed emerging trends in approaches to mathematics instruction for STEM subjects in higher education motivated by a desire to achieve a more conceptual or in-depth understanding of mathematics by students. (Abdulwahed et al, 2011)

Computer-based test results were about the same as previous cohorts of students even though each CAA test had twice the number of questions for the same amount of time. Final examination scores showed higher levels of achievement (on average +17.5%) than previous cohorts indicating evidence of some level of understanding. More students responded to the questions on functions than in previous years and the scores were higher. Although these results may indicate a higher quality intake (which we are unable to check since data from previous years is not available), they also reflect positively on the teaching approach. Of the thirteen groups, eleven submitted a project report and the average score was 61.3% (excluding zeros). Twelve of the groups submitted the optional poster. The group who did not submit a poster had received 75% for their project report. Three students failed this module. One of these was admitted to the module with a GCSE grade C and found the work very difficult. Another received low marks throughout and had low attendance.

The attendance in the first semester increased compared with previous years, which is evidence of increased engagement. (2006/07 – 48%; 2007/08 – 54%; 2008/09 – 47%; 2009/10 – 58%; 2010/11 – 66%)

Analyses of observations and reflective data suggest that the style of lecturing, inquiry-based questions and tasks in tutorials resulted in an increased level of engagement/interaction. In tutorials, although there was some discussion off task or some use of social media, most students focused on mathematical inquiry working through the tutorial or group project questions. The second semester lecturer observed that these students seemed more ready to communicate and work interactively than previous cohorts.

GeoGebra was used both in demonstration mode by the lecturer and by the students individually and in groups for tutorials and for the group project. Comments in project reports, focus groups and interviews indicated students' perceptions of the contribution of GeoGebra to their learning and understanding.

"This project has been very useful to all members of the group in understanding functions; one of the main observations was how much the line of a function could be changed by changing [parameters]. To in fact produce a line that is completely different from the original, all functions have a basic shape but the gradient, size and even direction can be changed." [Group L – project report]

"We think that people, including us, may think that they understand functions but it is always very useful to see a graphical form of the function and this does give a better understanding of what happens as (x,y) values increase." [Group A – project report]

"As a group we looked at many different functions using GeoGebra and found that having a visual representation of graphs in front of us gave a better understanding of the functions and how they worked. In the project the ability to see the graphs that were talked about helped us to spot patterns and trends that would have been impossible to spot without the use of GeoGebra." [Group F – project report]

However comments from the focus groups were less positive. It was suggested that GeoGebra was just plugging numbers into a computer and that too much time was spent on its use and they felt that it did not help their ability to do well in the exam.

"Understanding maths - that was the point of GeoGebra wasn't it? Just because I understand maths better doesn't mean I'll do better in the exam. I have done less past paper practice."[S3 - interview]

"I found GG almost detrimental because it is akin to getting the question and then looking at the answer in the back of the book. I find I can understand the graph better if I take some values for x and some values for y, plot it, work it out then I understand it….then change the equation. If you just type in some numbers and get a graph then you don't really see where it came from."[S1 - interview]

"GeoGebra is flawed as the biggest method of assessment is exams and you are not going to have any[thing] graphical apart from what you can draw in there with you" [S2 - interview]

It would seem that although students recognise that GeoGebra plays a role in promoting understanding, their main focus is passing the examination and developing strategies that will assist this. Several students claimed that they did not do any work outside formal sessions other than preparation for CAA tests or the group project, both assessed components. At the time of the focus groups and interviews, the examination had not yet taken place and students reported they would access the additional HELM and VLE materials in preparation for these.

Inquiry-based questions were used in lectures to stimulate interest and engagement and in tutorials to encourage group discussion. Whilst aspects of engagement were observed, aspects of understanding were harder to discern and it was difficult to measure an increase in conceptual understanding. Work has begun on research for the further development of inquiry-based questions and an instrument to measure conceptual understanding.

Students reported that the work on functions in the early part of the module was a repetition of previous work and perhaps gave a mistaken impression that their mathematical understanding was at an appropriate level. A recommendation was made to begin the module in the next academic year with matrices which most students had not met previously.

Conclusions

Analysis revealed a complexity of interlinked issues including not only the activities of the teachers, researchers and students and other stakeholders but also constraints in which activities took place such as university regulations, social culture, and individual goals. Activity Theory analysis (AT) (Engeström, 1999; Leont'ev 1979) was employed to seek to understand the relationships between these as outlined in the areas 1 and 2 below, and a and b.

1) a) teaching intentions and approaches *and*
   b) students' engagement, responses and performance

2) a) the purposes of the intervention and associated findings *and*
   b) the context in which the innovation is embedded

Reported in Jaworski, et al. 2011b, AT helped to make sense of the tensions that were observed including:

1) Students' perceptions and use of GeoGebra and inquiry based questions and ways in which these differ from teaching design and expectations.
2) Tutorials: students engage with differing degrees of intensity often without the depth of engagement the teacher is hoping for.
3) Students' strategic approach acts potentially against teaching objectives

The research furthered practical experience in a number of areas - the design and use of inquiry-based tasks; the use of GeoGebra in institutional settings and its perception by students; student engagement and epistemology; the importance of assessment; the difficulties in discerning degrees of conceptual understanding.

Further development will include modification to the teaching innovation in response to feedback; reflection on practice to develop knowledge of students, their perceptions and expectations; consideration of how to address institutional and cultural constraints; further research into the nature of assessment and its relation to student activity and epistemology; and further research into the nature of mathematical understanding.

References

Abdulwahed, M., Jaworski, B. and Crawford, A. (2011) Constructivist methods for mathematics instruction in STEM higher education; Review and current trends. *SEFI Annual Conference 2011*, Lisbon, Portugal,28-30 September 2011, Lisbon, SEFI.

Engeström, Y. (1999). "Activity theory and individual and social transformation." In Y. Engeström, R. Miettinen & R-L Punamäki, eds. *Perspectives on activity theory* Cambridge: Cambridge University Press, pp. 19-38.

Hawkes, T., & Savage, M. (eds) (2000). *Measuring the Mathematics Problem*. London, Engineering Council.

Hiebert, J. (1986) *Conceptual and procedural knowledge: The case of mathematics*. Hillslade, NJ: Erlbaum

Jaworski, B. (2004) "Grappling with complexity: Co-learning in inquiry communities in mathematics teaching development." In M. J. Høines & A. B. Fuglestad, eds. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* Vol. x, Bergen: Bergen University College, pp. 17-32.

Jaworski, B. (2008) Helping engineers learn mathematics: a developmental research approach. *Teach. Math. Appl.*, 27:160-166

Jaworski, B. (2010) Challenge and support in undergraduate mathematics for engineers in a Geogebra medium. *MSOR Connections*, 10:10-14

Jaworski, B. and Matthews, J. (2011a) Engineering Students Understanding Mathematics (ESUM): an innovative teaching approach, [online] http://www.hestem.ac.uk/resources/case-studies/engineering-students-understanding-mathematics-1

Jaworski, B., Robinson, C., Matthews, J. & Croft A.C. (2011b) Issues in teaching mathematics to engineering students to promote conceptual understanding: a study of the use of GeoGebra and inquiry-based tasks. Paper presented at the symposium devoted to "*Activity theoretic approaches to technology enhanced mathematics learning orchestration*", Paris November.

Jaworski, B. and Matthews, (2012) J. Engineering Students Understanding Mathematics (ESUM): research rigour and dissemination, [online] < http://www.hestem.ac.uk/resources/esum>

Leont'ev, A. N. (1979) "The problem of activity in psychology." In J. V. Wertsch, ed., *The concept of activity in Soviet psychology*. New York: M. E. Sharpe, pp. 37-71.

Skemp, R. (1976) Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.

Wells, G. (1999) *Dialogic Inquiry*. Cambridge: Cambridge University Press.

Wenger, E. (1998) *Communities of practice*. Cambridge: Cambridge University Press.

Addressing Continuing Mathematical Deficiencies with Advanced Mathematical Diagnostic Testing.

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Abstract

Dublin Institute of Technology offers students a number of different routes into engineering, allowing many non-standard entrants the opportunity to study the discipline provided they fulfil certain criteria. The final aim of many of these students is to achieve an Honours Degree in Engineering, which takes a minimum of four years. Apart from the first year of the course, the other main entry point is at the start of the third year, at which stage students who have performed well in a three-year Ordinary Degree can begin. However, these students have a wide range of mathematical abilities and prior knowledge, and many are missing the basic skills required for completion of a mathematics module at this level.

It is common practice for students to be diagnostically tested upon entry to third level; however, anecdotally, it appeared that many of the mathematical issues uncovered at that point had not been rectified during the students' subsequent studies. In an attempt to quantify the problem, it was decided to pilot an Advanced Maths Diagnostic Test which covered many of the key concepts from the early years of Engineering Mathematics. A pass-mark of 90% was set in this assessment. 167 third-year students studying for an Honours Engineering degree were tested during the pilot study, only two of whom achieved the pass mark on the first sitting. To encourage the other students to revise this crucial material, multiple re-sit opportunities were provided, and a weighting of 10% of the continuous assessment mark for the mathematics module was given to the diagnostic test. Online resources and special classes covering the relevant material were also provided, with the result that 131 of the 167 students reached the necessary threshold by the end of the semester.

KEYWORDS: Mathematical Competencies, Engineering Mathematics, Transition from Ordinary to Honours degree

1. Introduction

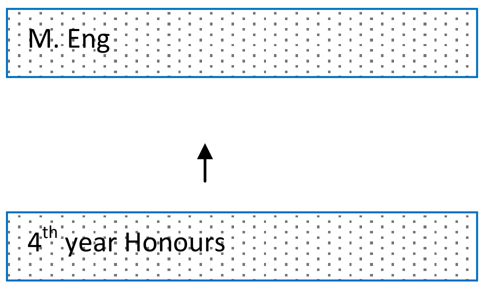
Many students who enter third level engineering programmes have problems with core mathematical skills. This has been borne out in the results of diagnostic tests carried out in many third level institutions, both in Ireland ([1], [2]) and in the U.K. ([3]). These problems with core concepts can lead to comprehension difficulties in numerous modules, both in mathematics itself and in related subjects. In addition some students enter the later years of Engineering courses in Ireland having come through several different routes, both within a particular college and from other colleges. It can be difficult to quantify exactly what knowledge of Engineering maths they have from their earlier years. In an attempt to quantify this problem we are devising an Advanced Maths Diagnostic exercise similar in concept to the Diagnostic exercises already given to many students on entry to third level, but with a higher level of content. Students who fail to reach a satisfactory result in the Advanced Maths Diagnostic exercise will be required to sit online tests until such time as they reach the required standard in Engineering Maths to complete the 3rd and 4th year of an Honours degree in Engineering.

1.1 Entry to an Honours Degree in Engineering.

There are two distinct routes to achieving a Honours degree (Level 8) in Engineering in the Dublin Institute of Technology. Students who have achieved a C (55%) or better in higher level Mathematics in the Irish Leaving Certificate (final secondary school exam in Ireland) are eligible to enter directly onto a 4 year Honours degree in Engineering. Students who do not have this level of mathematics but have a pass in Ordinary level Maths may enter onto a 3 year Ordinary Degree(Level 7) in Engineering (see Figure 1 below). Upon successful completion of this award students may progress to the 3rd year of the Honours degree. These students tend to struggle with the Mathematical level of the Honours degree [4].

1.2 Choice on Papers

As early as 1992, Ramsden noted that “From our students’ point of view, assessment always defines the actual curriculum”[5]. Brown et. al went even further, observing “Assessment defines what students regard as important, how they spend their time and how they come to see themselves as students and then as graduates. If you want to change student learning then change the methods of assessment.’ [6]. Despite this, many mathematics papers in Ireland give students a choice between questions on different topics. This allows students the opportunity to omit or avoid more difficult topics. This was strongly borne out by a survey carried out in the DIT [7,8] in which engineering students were asked if they had struggled as a result of omitting specific subject areas and if they felt that certain topics should always be compulsory. From the 276 responses received, 64% had avoided integration at some point, and 47% felt they had struggled as a result. A quarter of respondents felt that choice should be removed from at least some mathematics papers, while a massive 60% felt there should be compulsory questions on certain topics.



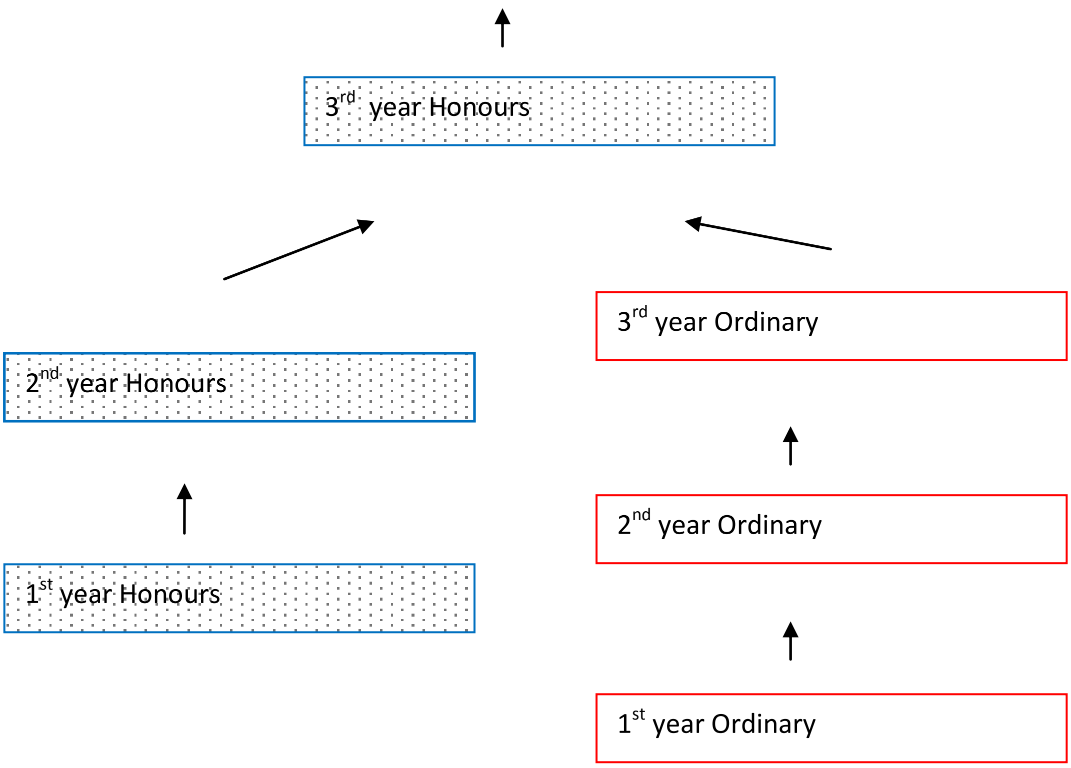


Figure 1: Routes to an Honours degree in Engineering.

Since undertaking this survey the level of choice has been reduced on most mathematics papers in Honours degree programmes in the DIT but not on the Ordinary degree programmes. Thus many students particularly those from an Ordinary degree background, have completely avoided differentiation and integration in their early years.

2. Core Skills Assessment

A large number of engineering undergraduates begin their third-level education with significant deficiencies in their core mathematical skills. Clearly our aim must be to produce engineers who "able to activate mathematical knowledge, insights, and skills in a variety of situations and contexts"[14] therefore every year, in the Dublin Institute of Technology (DIT), a diagnostic test is given to incoming first-year students, consistently revealing problems in basic mathematics. As noted by Niss ""prerequisites for mathematical competence are lots of factual knowledge and technical skills , in the same way as vocabulary, orthography, and grammar are necessary but not sufficient pre-requisites for literacy"[9]. It is difficult to motivate many students to seek help in the Maths Learning Centre to address these problems. As a result, they struggle through several years of engineering, carrying a serious handicap of poor core mathematical skills, as confirmed by exploratory testing of final year students[7]. In order to improve these skills in engineering students, a "module" in core mathematics was developed. The course material was basic, but a grade of 90% or higher was required to pass the module. Students were allowed to repeat the module as often as they liked until they passed. An automated examination for this module was developed on WebCT, and a bank of questions created for it. There has been a systematic improvement in the core mathematical abilities of the students participating in the core skills initiative [4] .

2.1 Advanced Core Skills Assessment

It was decided to set up an advanced core skills assessment in mathematics, similar to the core skills initiative already developed for first year students in the DIT described above [10]. Similar tests are in existence in many institutes including the Institute of Technology Tallaght, Dublin [11]. This consists of a multiple-choice quiz on WebCT, based on a randomised question bank. The material covered by the test consists of the more important aspects of undergraduate engineering mathematics covered in the first two years of the Honours degree programme and/or the three years of the Ordinary degree programme . The pass mark is set at 90 %. Students will be allowed to re-sit the assessment every two weeks until they pass.

2.2 Pilot Project Overview

Beginning September 2011 we piloted the advanced core skills assessment. As in incentive to ensure students participate it is worth 10% of the mathematics module. In the first instance, the students sit the Advanced Mathematics Diagnostic. Those who score 90% receive nine marks out of ten, whilst those who scored less than 90% will receive no marks and have to take the advanced core skills assessment at a later date. These students continued to sit the assessment on a monthly basis until they achieved the required pass mark.

The Advanced Core Skills assessment will mirror the Advanced Mathematics Diagnostic test with 5 versions of each type of question. Each time a student sits the test they will be randomly assigned different versions of each question. After their first attempt, students will be given access to a WebCT site with resources tailored for each question and will be encouraged to attend the Student Maths Learning Centre (SMLC). After their second and subsequent attempts, special classes two hour classes on problem topics were provided and there was an average attendance of 20-25 students at these weekly classes.

2.3 Advanced Diagnostic test

The test consists of 10 paired questions on each of the subtopics shown below. For logistical reasons the test is restricted to 1 hour. In this time it is not possible to cover the wide range of topics that we would like our students to know from the previous years. Lee & Robinson [12] found that “to get full advantage of the paired question approach it is essential that the pair test exactly the same skill and have the same number of steps involved. In addition, we would recommend that in order to minimize the potential for making a slip it would be wise to keep the number of steps involved in a question to a minimum”[13]. We have endeavoured where possible to keep the level of difficulty of the paired questions the same and an analysis of this will form part of our pilot. After several iterations between the lecturers in the Faculty, we settled on the following 10 subtopics.

Topic	Sub-Topic	Number of Questions
Differentiation	Basic	2
	Product Rule	2
	Quotient Rule	2
	Chain Rule	2
Integration	Basic	2
	Substitution	2
Differential Equations	1st Order ODE's	2
	2nd Order ODE's	2
Matrices	Multiplication of Matrices	2
Complex Numbers	Multiplication	2

Table 1. List of topics covered in the advanced maths diagnostic test.

For all of our questions we attempted to create incorrect answers based on common errors we routinely see on our exam papers.

2.4. Results

A total of 167 students attempted the Advanced Mathematics Diagnostic Test. On the first attempt only 2 out of 167 students achieved a the pass mark of 90%. Six more attempts were given at the test, on average every 2 weeks and by the end of the semester 131 students achieved the pass mark of 90% (see table 2 below).

Attempt	Pass	Fail
First Attempt	2	165
End of Semester	131	36

Table 2: Number of students who passed the advanced maths diagnostic test on their first attempt and before the end of semester.

When we break the students into two groups, those who have passed the advanced diagnostic test/ core skills and those who didn't we see a marked difference in the average mark. The 131 students who passed the diagnostic test scored on average 54.6% in the Maths module and 83% (108/131) of them passed the module. In comparison those who failed the advanced diagnostic test scored an average of 30% in the Maths module and only 39% of them passed the Maths module (see table 3 below).

	Pass (Advanced Diagnostic)	Fail (Advanced diagnostic)
Number	131	36
Average mark(maths module)	54.6	30.3
Pass(maths module)	108	14
Fail(maths module)	23	22

Table 3: Average mark and numbers of passes and fails on the Maths module for those who passed/failed the advanced diagnostic test.

Finally when we divide the students according to their background we see that the students who come from an Ordinary degree background are more than twice as likely to fail their third year Maths module and almost twice as likely to fail the Advanced Diagnostic test at the end of the semester (see table 3 below). In conclusion students who enter third level with weaker Maths ability, are still struggling with Maths with respect to their peers , even after an extra years tuition. This is hardly surprising given that even though these students have an extra year in College they only receive 24 hours extra tuition over this period(see table 5 below).

Background	**Direct Entry to Honours Degree**	**Ordinary Degree**
Number of Students	78	89
Maths Module Average Mark	41.6%	55.7%
Pass in Maths Module	47	73
Fail in Maths Module	31 (40%)	16 (18%)
Fail in Advanced Diagnostic	22 (28%)	14 (16%)

Table 4: Comparison of performance of Students with direct entry onto honours degree course versus entry via the ordinary degree.

Background	**Maths Hours**	**Total**
3 years Ordinary degree	8 hours x 24 weeks	192
2 years Honours degree	7 hours x 24 weeks	168

Table 5: Total number of hours of Maths tuition prior to entering 3rd year of an Honours degree.

3. Future Work

To complete this work all of these students will be diagnostically tested in the first week of Fourth year Maths and this grade will be compared with their first attempt at the advanced diagnostic test and their final grade in this test at the end of third year. This grade will also be compared with the results of the test which were given to this year’s fourth year class. In addition a series of practice tests will be created for each topic on the test. At the beginning of fourth year an anonymous online survey of each of the students will be carried out along with a series of focus groups and interviews of selected students.

This work will allow us to identify areas in the earlier years of the undergraduate mathematics that need improvement. This test will also be useful in assessing the suitability of graduates from other colleges who apply to enter the 3rd year of our Honours degree and in the placement of Erasmus students within our courses.

4. References

[1] Cleary, J.. “Diagnostic testing – An evaluation 1998-2007.” In S. Close, D. Corcoran & T. Dooley (Eds.), Proceedings of Second National Conference on Research in Mathematics Education (MEI2) (pp. 215-227), St. Patrick’s College, Dublin.2007

[2] Ni Fhloinn, E. “Maths Diagnostic Report”, Internal report, Dublin Institute of Technology 2006

[3] LTSN MathsTEAM (2003). “Diagnostic Testing for Mathematics”. Accessed via http://www.mathstore.ac.uk/mathsteam/packs/diagnostic_test.pdf (Accessed 6 November 2009)

[4] Carr,M., Bowe, G., & Ni Fhloinn, E. Improving core mathematical skills in engineering undergraduates. 15th SEFI MWG, Wismar 2010

[5] Ramsden,P., Learning to teach in higher education. New York: Routledge, 1992

[6] Brown, G., Bull, J. And Pendelbury, M Assessing student learning in higher education. London: Routledge (1997)

[7] Carr,M., Shiels,D., & Ní Fhloinn,E. Reducing Choice = Increasing Learning or Decreasing Marks?, Mathematical Education of Engineers, Loughbourough, 2008

[8] Carr, M, & Ni Fhloinn E. Increasing Learning by decreasing Choice: What do students think ? , , Adults Learning Maths 14, Limerick, 2007

[9] Niss, N., Mathematical Competencies and the learning of Mathematics: The Danish KOM project http://www7.nationalacademies.org/mseb/mathematical_competencies_and_the_learning_of_mathematics.pdf (accessed May 2012)

[10] Carr,M., & Ni Fhloinn, E.,, Assessment and development of core skills in Engineering mathematics. “Opening Windows on Mathematics and Statistics”, Open University, Milton Keynes, September 2009

[11] Marjoram, M., Moore, D., O’Sullivan, C. and Robinson, P Implementing a Key skills in Mathematics Initiative, Proceedings of Mathematical Education of Engineers, Loughborough 2008

[12] Lee, S. and Robinson C.L, Diagnostic testing in mathematics: paired questions, Teaching Mathematics and its applications, Vol 24, 2005

[13] Fritz, C.O., Morris,P.E., Bjork,R.A,Gelman,R., and Wickens,T.D., “When further learning fails: stability and change following repeated presentation of text”, British Journal of Psychology,91,pp.493-511, 2000

[14] Gill, O. & O'Donoghue, J. . "The mathematical deficiencies of students entering third level: An item by item analysis of student diagnostic tests". In S. Close, D. Corcoran & T. Dooley (Eds.), Proceedings of Second National Conference on Research in Mathematics Education (MEI2) (pp. 228-239), St. Patrick's College, Dublin. 2007

[15]What do they really need to know? Mathematics requirements for incoming Engineering undergraduates. E. Ni Fhloinn & M.Carr 15th SEFI MWG, Wismar 2010

[16] Mark Russell "Academic Success, Failure and Withdrawal Among First Year Engineering Students: was poor mathematical knowledge a critical factor?" Level 3 , (3). Accessed via http://level3.dit.ie/html/issue3_list.html 2005(accessed May 2012)

[17] Savage, M., Kitchen, A., Sutherland, R. & Porkess, R. In Hawkes, T. & Savage, S. (Eds.), Measuring the Mathematics Problem. London: Engineering Council (2000)

[18] OECD: measuring Student knowledge and Skills-A new framework for Assessment, OECD, Programme for International Student Assessment(PISA),1-104, Paris, France,1999

5.APPENDIX: PROVISIONAL ADVANCED DIAGNOSTIC TEST

1. Find $\frac{dy}{dx}$ $where\ y = x^4$

a)$4x^3$ b) $4x^4$ c) $5x^5$ d) x^3

2. Find $\frac{dy}{dx}$ $where\ y = x^7$

a) $7x^6$ b) x^6 c) $7x^8$ d) $\frac{x^8}{7}$

3. Find the solution of the following first order Differential Equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$$

a)$Ae^{-3x} + Be^{x}$ b) $Ae^{-3x} + Be^{-x}$ c) $Ae^{3x} + Be^{x}$ d) $Ae^{x} + Be^{3x}$

4.Find the solution of the following second order Differential Equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 10y = 0$$

a)$e^{-3x}Acosx + Bsin$ x

b)$e^{1x}Acos3x + Bsin3x$

c) $Acos3x + Bsinx$

d) $e^{1x}Acos3x + Bsin3x$

5.Find $\int x^3 dx$

a)$\frac{x^4}{4}$ b)$4x^4$ c)$3x^3$ d) x^3

6.Find $\int x^{-4} dx$

a)$-4x^{-3}$ b)$-\frac{x^{-3}}{3}$ c)x^{-5} d) $\frac{x^{-5}}{5}$

7. $\begin{pmatrix} -1 & -2 & 1 \\ 2 & -3 & 2 \\ 1 & 2 & -3 \end{pmatrix}\begin{pmatrix} -2 & 7 \\ 0 & 3 \\ 5 & -1 \end{pmatrix} =$

a)$\begin{pmatrix} 7 & -14 & 6 \\ 3 & 3 & 4 \\ 2 & 1 & 7 \end{pmatrix}$ **b**) $\begin{pmatrix} 7 & -14 \\ 6 & 3 \\ -17 & 16 \end{pmatrix}$

c)$\begin{pmatrix} 3 & -14 \\ 6 & 7 \\ -13 & 16 \end{pmatrix}$ d)$\begin{pmatrix} 3 & -14 \\ 6 & 7 \\ -13 & 10 \end{pmatrix}$

8. $\begin{pmatrix} 6 & 3 & 0 \\ 2 & 5 & 1 \\ 9 & 8 & 6 \end{pmatrix}\begin{pmatrix} 7 & 4 \\ 6 & 7 \\ 5 & 0 \end{pmatrix} =$

$a)$ $\begin{pmatrix} 65 & 45 \\ 49 & 42 \\ 141 & 92 \end{pmatrix}$ **b**) $\begin{pmatrix} 60 & 45 \\ 49 & 43 \\ 141 & 92 \end{pmatrix}$

c)$\begin{pmatrix} 65 & 45 \\ 49 & 43 \\ 141 & 98 \end{pmatrix}$ d) $\begin{pmatrix} 65 & 42 \\ 49 & 43 \\ 140 & 98 \end{pmatrix}$

9. Solve $\frac{dy}{dx} = \frac{2x}{y+1}$

a) $\frac{y^2}{2} + y = x^2 + c$ b) $\frac{y^2}{2} + y = 2x^2 + c$

c) $\frac{y^2}{2} + y = \frac{x^2}{2} + c$ d) $y^2 + y = x^2 + c$

10 . Solve $\frac{dy}{dx} = (1+x)(1+y)$

a)$1 + \mathrm{y} = x + \frac{x^2}{2} + c$ b)$1 + \mathrm{y}^2 = x + \frac{x^2}{2} + c$ **c**)$\mathrm{l\,n}(1+y) = x + x^2 + c$ d) $\ln(1+y) = x + \frac{x^2}{2} + c$

11. $\frac{d}{dx}\sin(2x+1) =$

a) $2\cos(2x+1)$ b) $2\sin(2x+1)$ c) $\cos(2x+1)$ d) $-\cos(2x+1)$

12. Solve $\frac{d}{dx}\cos(5x-4) =$

a) sin(5x-4) **b**) -5sin(5x-4)

c)-5cos(5x-4) d)-sin(5x-4) +5

13. Solve $\frac{d}{dx}x^3 cosx =$

a) $3x^2cosx - x^3sinx$ b) $3x^2 + cosx$ **c)** $3x^2 + x^3cosx$ d)$-x^3sinx$

14. Find $\frac{dy}{dx}$ where $y = x^2sinx$

a)$2xsinx + 2cosx$ b) $2xsinx - x^2cosx$ c) $2sinx + x^2cosx$ **d**) $2xsinx + x^2cosx$

15.Find $\frac{dy}{dx}$ where $y = \frac{\cos x}{x^2}$

a) $\frac{(xsinx+2cosx)}{x^2}$ **b)**$-\frac{(xsinx+2cosx)}{x^3}$ c) $\frac{-(sinx+2cosx)}{x^4}$ d) $\frac{-(xsinx+cosx)}{x^3}$

16. Find $\frac{dy}{dx}$ where $y = \frac{x^2+5}{2x-4}$

a) $\frac{(20-8x)}{(2x-4)^2}$ **b)** $\frac{(2x^2-8x-10)}{(2x-4)^2}$ c) $\frac{(6x^2-8x-10)}{(2x-4)^2}$ d) $\frac{(2x^2-8x-5)}{(2x-4)^2}$

17. If $z_1 = 9 - 2j$ and $z_2 = 2 - 4j$ find $z_1 z_2$

a) 18-32 j b) 18+40j c)18-40j **d**) 10-40j

18. If $z_1 = 4 + 2j$ and $z_2 = 1 - 8\,j$ find $z_1 z_2$

a) 4+46j b) 4-46j c)20-34j **d)** 20-30j

19. Evaluate the following integral $\int \cos(x + 2)\, dx$

a) –sin(x+2) b)sin(x+2) c) 2cos(x+2) d)-2cos(x)

20. Evaluate the following $\int \mathrm{x}(4\mathrm{x}^2 - 7)^3 dx$

a) $\frac{(4x^2-7)^3}{8}$ b) $\frac{(4x^2-7)^3}{32}$ c) $\frac{(4x^2-7)^4}{8}$ **d**) $\frac{(4x^2-7)^4}{32}$

Teaching applied mathematics for naval architecture – an example

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Abstract

A course in applied mathematics is part of the curriculum of the master program in naval architecture. The first part of the course consists of five lectures where the main topics in vector calculus, line and surface integrals, integral theorems are presented. Attendance of these lectures is obliged. After each lecture home-work assignments are given to the students. The second part of the course is devoted to different individual projects. In most cases a computer program (e.g. maple) for simulating a phenomenon beside common physics or the life sciences has to be developed. At the end a written report of the project has to be handed in and an oral presentation must be held.

Pre-Knowledge of Basic Mathematics Topics in Engineering Students in Spain

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Abstract

Successful engineering studies assume the existence of a pre-requisite knowledge in mathematics, at least at basic level. This knowledge seems to be guaranteed in Spain by the government educational institutions, since before entering the University level, students have to pass a national exam to access at the University (called PAU): nevertheless, this situation does not seem to correspond to the "real" knowledge of the basic mathematical topics. In addition to that, nowadays in Spain it is possible to enter the University level without having taken a course in mathematics at the secondary level, and it is possible to pass the national exam without considering any mathematical item. This situation has been analysed by developing a very simple test of basic skills in mathematics, which has been distributed to a group of students entering at the first year of Engineering Graduate Studies at the University of Salamanca. Those students present different ways to enter at the University level (from secondary school to pre-professional studies) and different levels of pre-university mathematical knowledge. They have taken this test twice, the former at the beginning of the course and the latter when having the final exam. The results of both performances of the test are analysed and related to final qualifications of the course. As a conclusion of our work, we make a strong recommendation to mathematics teachers about being as conscious as possible about the way their students are accessing at university level.

Introduction

It is well known that a basic knowledge of mathematical topics is a pre-requisite for success in engineering studies at University level. But unfortunately, we also know that these basic skills are not standard among our students, as can be seen at Kent and Noos (2002), Mustoe (2002), Mustoe and Lawson (2002), Bowen et al (2007), and is also supported by studies of several institutions as the Institute of Physics (2011), the Engineering Council (2000) or the International Commission on Mathematical Instruction (1997). Different causes have been proposed to explain this lack of knowledge, and one that seems to arise is the method used by the national educational organisations to assess the mathematical performance of undergraduate students, as shown recently by Ni Fhloinn and Carr (2010) in Ireland or Kurz (2010) in Germany.

In this work, we claim to not only assess the level of mathematical performance of our students, but also to gain a major understanding of their strategies when entering the University, analysing their degree of improvement and the relationship between the entrance level on basic mathematics and their development on the standard examinations. The part of the curriculum chosen to accomplish this study has been

Calculus skills, because they are the first mathematics content seen by undergraduate students when entering the current Engineering studies.

Data of Investigation

To carry on this work we have designed a very simple proof that has two main parts. In the first one, there is a heading whose aim is to obtain some data on students that are relevant to the analysis, especially those related to the type of access to the University. These data are relevant from 2008, because the new legislation regarding access into University in Spain allows students to pass a national examination (called PAU) without performing any mathematical proof, even if they are going to enter a technical, scientific or engineering degree. The percentage of students of Vocational Training Modules, not requiring any specific mathematical proof, also influences the results.

In the second part, it is proposed the resolution of some simple Calculus exercises that are included in the curriculum of secondary and high school mathematics, and that, therefore, students should know. They are operations in routine use in the classroom and that they must be known at these levels. The test has the following questions:

- Question 1. Simplify the following: $(x^2-1)(x-2)/(x+1)(x^2-4)$
- Question 2. Simplify the following: $1/(x+2)+(2-x)/(x^2-4)+1/(x-2)$
- Question 3. Approximately draw the graph of $f(x) = \sin x$
- Question 4. Approximately draw the graph of $g(x) = e^x$
- Question 5. Approximately draw the graph of $h(x) = \log x$
- Question 6. Calculate the derivative of $f(x) = 1/x$
- Question 7. Calculate the derivative of $g(x) = x\, e^x$
- Question 8. Calculate the derivative of $h(x) = x/(x^2+1)$
- Question 9. Calculate the primitive of $f(x) = 1/x$
- Question 10. Calculate the primitive of $g(x) = x\, e^x$
- Question 11. Calculate the primitive of $h(x) = x/(x^2+1)$

This test has been answered twice by a group of 73 students of the Mechanical Engineering Degree at the Polytechnic School in Zamora from the University of Salamanca (Spain): the former at the beginning of the course, and the latter at the end of the course. All of them have been also qualified in a standard exam of Calculus skills at the end of the course. So we have three qualifications for each student (pre-test, post-test and examination) and we try to relate these results with the type of access to the University.

For each item there were three possible answers: correct, erroneous, or empty. For our purposes, the erroneous and empty questions have been grouped as "incorrect".

Findings and Discussion

First at all, the results for the whole group of 73 students in pre-test and post-test are presented at Table 1 for the eleven items. As seen below, the results have improved in all items, but not too much on the integration skills (10^{th} and 11^{th} items). Both of them have been failed for the majority of the students, even after having received a Calculus course at the University.

	PRE-TEST		POST-TEST	
Item	correct	incorrect	correct	incorrect
1	49 (67.1%)	24 (32.9%)	51 (69.9%)	22 (30.1%)
2	32 (43.8%)	41 (56.2%)	40 (54.8%)	33 (45.2%)
3	50 (68.5%)	23 (31.5%)	62 (84.9%)	11 (15.1%)
4	49 (67.1%)	24 (32.9%)	54 (74.0%)	19 (26.0%)
5	37 (50.7%)	36 (49.3%)	47 (64.4%)	26 (35.6%)
6	39 (53.4%)	34 (46.6%)	45 (61.6%)	28 (38.4%)
7	31 (42.5%)	42 (57.5%)	46 (63.0%)	27 (37.0%)
8	30 (41.1%)	43 (58.9%)	47 (64.4%)	26 (35.6%)
9	29 (39.7%)	44 (60.3%)	48 (65.8%)	25 (34.2%)
10	5 (6.8%)	68 (93.2%)	24 (32.9%)	49 (67.1%)
11	1 (1.4%)	72 (98.6%)	17 (23.3%)	56 (76.7%)

Table 1: results of pre-test and post-test for the whole group.

As a first approximation to the problem, we can look at whether having success in the pre or post-test has some relationship with the results at the final exam of the course for the whole group of students. The results of this analysis are shown in Table 2, where success in the test has been considered as having six or more correct answers.

	Pre-test		Post-test		Final exam	
	pass	fail	pass	fail	pass	fail
Number of students (73)	31 (42.5%)	42 (57.5%)	50 (68.5%)	23 (31.5%)	27 (37.0%)	46 (63.0%)

Table 2: relationship between pre-test, post-test and final exam

As it is seen, for the whole group an improvement for the simple test has no direct relationship with passing the final exam, because there are students that have improved their results on the test, but they have not been capable of passing the final exam.

This analysis becomes more accurate if we divide the group attending to the way of access to the University. For our purposes, we have considered three groups:

- Group A: students from post-secondary studies (called bachelor in Spain) and with exam of Mathematics at the PAU. There are 49 students in this group.

- Group B: students from post-secondary studies but without exam of Mathematics at the PAU. There are 15 students in this group.

- Group C: students from Vocational Training Modules. There are 9 students in this group.

Results of the pre-test and post-test having into account this division are shown in Table 3 (group A), Table 4 (group B) and Table 5 (group C).

GROUP A (49)	PRE-TEST		POST-TEST	
Item	correct	incorrect	correct	incorrect
1	35 (71.4%)	14 (28.6%)	39 (79.6%)	10 (20.4%)
2	26 (53.1%)	23 (46.9%)	31 (63.3%)	18 (36.7%)
3	35 (71.4%)	14 (28.6%)	45 (91.9%)	4 (8.1%)
4	39 (79.6%)	10 (20.4%)	40 (81.6%)	9 (18.4%)
5	28 (57.1%)	21 (42.9%)	32 (65.3%)	17 (34.7%)
6	30 (61.2%)	19 (38.8%)	34 (69.4%)	15 (30.6%)
7	24 (49.0%)	25 (51.0%)	34 (69.4%)	15 (30.6%)
8	25 (51.0%)	24 (49.0%)	33 (67.4%)	16 (32.6%)
9	26 (53.1%)	23 (46.9%)	35 (71.4%)	14 (28.6%)
10	5 (10.2%)	44 (89.8%)	20 (40.8%)	29 (59.2%)
11	1 (2.1%)	48 (97.9%)	12 (24.5%)	37 (75.5%)

Table 3: results of pre-test and post-test for group A

For group A, the results of the pre-test were good even at the beginning of the course, except for the last two questions concerning the integration skills. All of the items have better results in the post-test, but also have good results in the pre-test. Although the two items related to integration (10^{th} and 11^{th}) have the worst results, the improvement at the end of the course is also significant for these questions.

GROUP B (15)	PRE-TEST		POST-TEST	
Item	correct	incorrect	correct	incorrect
1	10 (66.7%)	5 (33.3%)	9 (60.0%)	6 (40.0%)
2	5 (33.3%)	10 (66.7%)	8 (53.3%)	7 (46.7%)
3	10 (66.7%)	5 (33.3%)	12 (80.0%)	3 (20.0%)
4	9 (60.0%)	6 (40.0%)	10 (66.7%)	5 (33.3%)
5	8 (53.3%)	7 (46.7%)	13 (86.7%)	2 (13.3%)
6	9 (60.0%)	6 (40.0%)	9 (60.0%)	6 (40.0%)
7	6 (40.0%)	9 (60.0%)	10 (66.7%)	5 (33.3%)
8	5 (33.3%)	10 (66.7%)	11 (73.3%)	4 (26.7%)
9	3 (20.0%)	12 (80.0%)	10 (66.7%)	5 (33.3%)
10	0 (0.0%)	15 (100.0 %)	4 (26.7%)	11 (73.3%)
11	0 (0.0%)	15 (100.0 %)	3 (20.0%)	12 (80.0%)

Table 4: results of pre-test and post-test for the group B

For group B, the results are worse than for group A. In the pre-test, there are more questions that are not well answered (for example, items 2 and 8), and none of the 15 students were capable to answer the questions related with integration. In the post-test, we can see some degree of improvement, but it is less important than in group A. Globally, this group has worse performance than group A, even having studied mathematics in the post-secondary studies (the same as the students in group A).

GROUP C (9)	PRE-TEST		POST-TEST	
Item	correct	incorrect	correct	incorrect
1	4 (44.4%)	5 (55.6%)	3 (33.3%)	6 (66.7%)
2	1 (11.1%)	8 (88.9%)	1 (11.1%)	8 (88.9%)
3	5 (55.5%)	4 (44.5%)	5 (55.5%)	4 (44.5%)
4	1 (11.1%)	8 (88.9%)	4 (44.4%)	5 (55.6%)
5	1 (11.1%)	8 (88.9%)	2 (22.2%)	7 (77.8%)
6	0 (0.0%)	9 (100.0 %)	2 (22.2%)	7 (77.8%)
7	1 (11.1%)	8 (88.9%)	2 (22.2%)	7 (77.8%)
8	0 (0.0%)	9 (100.0 %)	3 (33.3%)	6 (66.7%)
9	0 (0.0%)	9 (100.0 %)	3 (33.3%)	6 (66.7%)
10	0 (0.0%)	9 (100.0 %)	0 (0.0%)	9 (100.0 %)
11	0 (0.0%)	9 (100.0 %)	2 (22.2%)	7 (77.8%)

Table 5: results of pre-test and post-test for group C

Students in group C are clearly different from those in groups A and B. With the exception of question number 3 (graph of the exponential function), more students answered the questions incorrectly. They show no substantial improvement along the course, due to their poor initial knowledge of mathematics. This bad performance is going to be a very important obstacle to pass the final exam.

Now we are going to analyse the relationship between results in the pre-test, the post-test and the final exam, but taking into account the groups of students determined by the circumstances of their entry to University. The results are shown in Table 6.

	PRE-TEST		POST-TEST		EXAM	
	pass	fail	pass	fail	pass	fail
GROUP A (49)	25 (51.0%)	24 (49.0%)	38 (77.5%)	11 (22.5%)	17 (34.7%)	32 (65.3%)
GROUP B (15)	6 (40.0%)	9 (60.0%)	10 (66.7%)	5 (33.3%)	7 (46.7%)	8 (53.3%)
GROUP C (9)	0 (0.0%)	9 (100.0%)	2 (22.2%)	7 (77.8%)	3 (33.3%)	6 (66.7%)
ALL (73)	31	42	50	23	27	46

Table 6: pre-test, post-test and final exam for the groups considered

All three groups have similar increases in the percentage of correct answers of the post-test compared to the pre-test. This increase was an average of 25.113%. Of the three groups, group B obtained better results in terms of passing the exam. This is because although groups A and B had received similar pre-university mathematics training, the fact that group B did not prepare for the mathematics test for the PAU makes the results in the pre-test worse than in group A. Nevertheless, after the course group B is capable of reaching a similar level to group A. It should be noted, however, that the scores of group A were higher than those of group B. The students of group C had the worst qualifications in both tests, but require a special mention because, despite its initial low level, a third of them have passed the exam. It is obvious that this group needs special attention.

Conclusions

Engineering students' University entrance information gives teachers a natural division of the class in order to determine the possible degree of improvement for these students. The ones coming from Vocational Training Modules seem to be less capable of reaching the desired level to pass the final examination of the course. Nonetheless, all of the students have an important lack of basic knowledge, especially in questions related to integration skills.

It is interesting to note the fact that 20% of students had poorer scores on the final test than in the initial one. This is because the concepts have not been understood properly. To overcome these shortcomings and try to match the initial level of all students, it would be interesting to consider the development of a bridging course or some similar initiative (Crisman, 2012), as is being done in fact in some universities.

References

Bowen, E., Prior, J., Lloyd, S., Thomas S., and Newman-Ford, L. (2007) Engineering more engineers -bridging the mathematics and careers advice gap *Engineering Education* Vol. 2 (No. 1): 23-31.

Crisman, K.D. (2012) Using Notebooks to Supplement Bridge Courses, *PRIMUS: Problems, Resources and Issues in Mathematics Undergraduate Studies*. Vol. 22 (No. 1): 1-9.

Engineering Council (2000) *Measuring the mathematics problem.* The Engineering Council, London. Accessed via http://www.engc.org.uk

Institute of Physics (2011) *Mind the gap. Mathematics and the transition to A-levels to physics and engineering degrees*. The Institute of Physics. London. Accessed via http://www.iop.org

International Commission on Mathematical Instruction (1997) On the teaching and learning of mathematics at university level. *ICMI Bulletin No.43.*

Kent, P. and Noss, R. (2003) Mathematics in the university education of engineers. *Ove Arup Foundation Report*, Ove Arup Foundation, London

Kurz, G. (2010) A Never-ending story: mathematics skills & deficiencies of engineering students at the beginning of their studies. *Proc. 15th SEFI-MWG European Seminar on Mathematics in Engineering Education.* Wismar, Germany

Mustoe, L and Lawson, D. (2002) *Mathematics for the European engineer. A curriculum for the twenty-first century*. Accessed via http://sefi.htw-aalen.de

Mustoe, L, (2002) The mathematics background of the undergraduate engineers. *International Journal of Electrical Engineering Education*, 39 (3):192-200.

Ni Fhloinn, E. and Carr, M. (2010) What do they really need to know? Mathematics requirements for incoming engineering undergraduates, in *Proc. 15th SEFI-MWG European Seminar on Mathematics in Engineering Education*. Wismar, Germany

Building Mathematical Competence – A Community Approach

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Abstract

This paper describes an innovative and novel approach to sowing the seeds of the recognition of the importance of mathematics competence in real life situations. Mathematical competence *is the ability to recognize, use and apply mathematical concepts in relevant contexts and situations which certainly is the predominant goal of the mathematical education for engineers,* (SEFI Mathematics Working Group 2011). Traditional methods of mathematics teaching and learning have resulted in a maturing population who do not appreciate the mathematics they use in their everyday lives. These 'everyday' mathematics skills often involve the use of complicated mathematical ideas and techniques. However, many people often consider the mathematics they can do as 'common sense' and the tasks they can't do as 'mathematics'.

The paper describes a successful initiative 'Looking at Tallaght with Maths Eyes' that took place in June 2011 to coincide with the 18th International Conference Adult Learning mathematics (Mathematical Eyes: A Bridge between Adults, the World and Mathematics), hosted by the Institute of Technology Tallaght. Dublin, Ireland.

The initiative aimed to:

- Develop the maths eyes of the Tallaght community: (Every member of the community has maths eyes – they just need to be opened).
- Help the Tallaght community to make the link between mathematics and the real world. (A key focus was to encourage the community to use Maths Eyes when they think about their water usage and water conservation).
- Build people's confidence in their use of maths in their life.
- Empower people and build their confidence in their own maths knowledge and skills (empowered parents are more confident in supporting their children's learning; more confident citizens can make more informed evaluations of the information that bombards them every day and have a better understanding of the impact of their actions and decisions in their life, work and leisure).
- Build a positive image of maths.

The paper outlines the different approaches that were used to encourage participation from a range of stakeholders. These included a community wide 'curiosity' campaign; the development and piloting of a resource pack for educators called 'Developing Maths Eyes; An Innovative Approach to Building a Positive image of Mathematics' (2011); primary schools showcase; adult learners showcase; a maths poster exhibition and a curated photographic exhibition of instances of mathematics seen in the local area; the development of maths trails for the local parks and an audio maths 'I-walk' for Tallaght. In addition it describes how the initiative has since been adapted as a model for use both nationally and internationally.

Mathematical Competence

In the normal course of life and work, people encounter and manage mathematics-containing situations. The mathematics information may be presented in a variety of ways and elicit a range of responses, (Gal et al., 2009) that are underpinned by mathematics knowledge, skills and competences.

More than ever, problem solving, spatial awareness, estimation, interpretation and communication skills are essential for active citizenship and highly valued in the modern worker, to support change, reaction and response (Expert group on Future Skills Needs, 2009; O'Donoghue, 2000), especially given the pervasiveness of ICT and 'black boxes'. However, the mathematical knowledge, skill and competence that underpin work, may be dismissed as 'just part of the job' (Coben & Thumpston, 1995). Skills deployed from a 'common sense' perspective may tend to conceal mathematical ability rather than expose it for development (Coben, 2009). As school mathematics is the primary source of quantitative literacy for most adults (Steen, 1997) many individuals are left blind to the mathematics that they use in their everyday lives. A key concern is that these may be denied, or dismissed as common sense, (Coben & Thumpston, 1995) indeed anything but mathematics.

Mathematics invisibility poses significant challenges for education and training programmes, not least for want of a starting point, i.e. the so-called 'bootstrap problem' (Klinger, 2009). The self-perception of 'not being a maths person', confirmed by mathematics invisibility, may be transmitted across generations and their communities, restrict education choice and constrain employment mobility. 'Developing Maths Eyes' offers a grounded approach to building confidence in a community in their own mathematics. In the longer term an individual with maths eyes will appreciate the mathematics that they do in their everyday life and work and challenge their self concept of 'not being a maths person'. This new confidence in their own mathematics and in the usefulness of mathematics will be a new inheritance of future generations.

'Looking at Tallaght with Maths Eyes' – A Community Initiative

'Looking at Tallaght with Maths Eyes' was an innovative and novel approach to sowing the seeds of the importance of mathematics competence in real life situations in a local community in Dublin 24, Ireland, in June 2011. The initiative was timely as it was supported by both national and international drivers.

Firstly the Institute of Technology Tallaght (IT Tallaght) was to host the 18th International Conference of Adult Learning Mathematics in June 2011. Adults Learning Mathematics is an international research forum which brings together practitioners and researchers from around the world who are involved in the mathematics education of adult learners at all levels. The organisation through its' research and scholarly activity informs international and national policy and practices to support all aspects of adult mathematics education (see http://www.alm-online.net). The need to celebrate and welcome the conference to Tallaght provided, for the first time, an event to discuss and mathematics at a community level.

Secondly the largest curriculum reform in second level mathematics in Ireland, *Project Maths*, is in the process of being implemented on a phased basis (see www.projectmaths.ie). The emphasis of this reformed curriculum is to link the mathematics in the classroom to real world applications: *'Project Maths aims to provide for an enhanced student learning experience and greater levels of achievement for all. Much greater emphasis will be placed on student understanding of mathematical concepts, with increased use of contexts and applications that will enable students to relate mathematics to everyday experience'*, Project Maths Development Team *(2012).*

Finally at parental/community level the individual concept of maths was strongly linked to the individuals own school experience of school maths and was considered as abstract, and not relevant to their everyday lives. There was a sense that *'Project Maths'* was new and unknown (and as a consequence feared).

When these drivers were considered together, the Institute took the decision to help the community to realise that maths is an integral part of their everyday life and that the new curriculum is doing is using real life applications as resources for teaching. Central to the thinking and planning from the outset was the notion that the target audience was the community of Tallaght (not just schools). As a result a number of different approaches were incorporated into the initiative to introduce and reinforce the concept of having maths eyes and the link between mathematics and the real world. The key aims of the initiative were to:

- Develop the maths eyes of the Tallaght community.
- Help the Tallaght community to make the link between mathematics and the real world. (A key focus was to encourage the community to use Maths Eyes when they think about their water usage and water conservation).
- Build people's confidence in their use of maths in their life.
- Empower people and build their confidence in own maths knowledge and skills (empowered parents are more confident in supporting their children's learning, more confident citizens can make more informed evaluations of the information that bombards them every day and have a better understanding of the impact of their actions and decisions in their life, work and leisure).
- Build a positive image of maths.
- As a community celebrate the hosting of an International Conference at the Institute.

The theme of the annual ALM conference was selected by the local organising committee. To provide synergy between the conference and the community initiative the theme selected for the conference *'Mathematical Eyes: A Bridge between Adults, the World and Mathematics'* mirrored to a large extent the theme of the local initiative 'Looking at Tallaght with Maths Eyes'.

Engagement with community

In order to keep the community focus and reach as many stakeholders as possible a range of different events were hosted. These included: Curiosity Campaign, family events, library events, and exhibitions (See Figure 1)

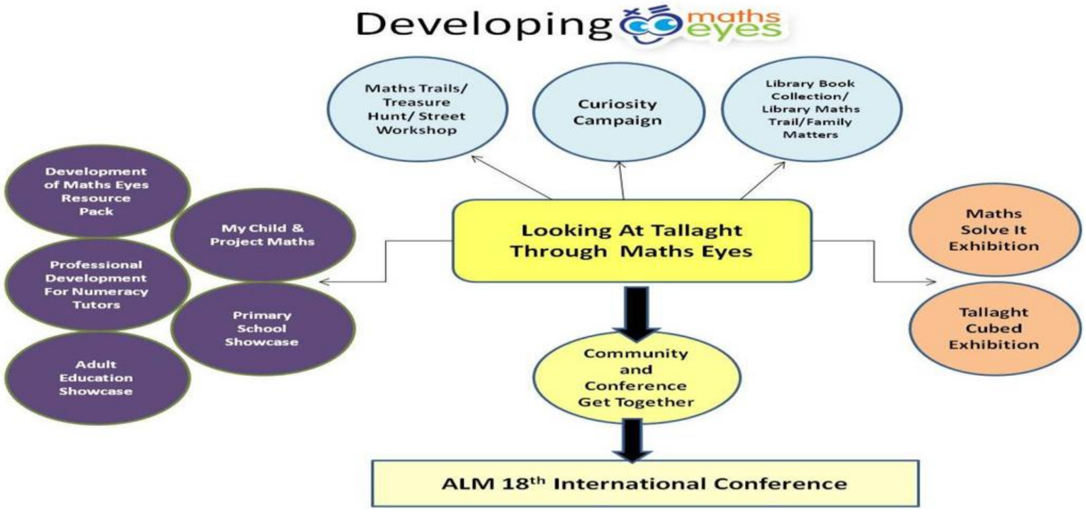

Figure 1: Overview of 'Looking at Tallaght with 'Maths Eyes' community engagement

A curiosity campaign is a recognised marketing approach which aims to create interest by generating curiosity about a particular theme or brand. The curiosity campaign carried out as part of 'Looking at Tallaght Through Maths Eyes' took familiar places/pictures/activities and linked them, in novel ways to real world mathematics through challenges/statements. In all a series of 10 election-style A0 size lamppost posters were developed (see Figure 2). Six of the posters were general in theme and four were specifically targeted to prompt awareness of water use and conservation (this was to resonate with the current priority of South Dublin County Council one of the funding partners).

Figure 2: Curiosity Campaign Posters

Over 500 of the colourful posters were erected on lampposts around the Tallaght area in early June 2011. The campaign was supported by our media partner 'Tallaght Echo' who ran the posters as advertisements without comment in their newspaper over the weeks of the campaign. The posters were then formally uncovered through a dedicated supplement in the local newspaper. The posters along with an explanation of the mathematics that underpinned the statement or challenge were included. In addition an information leaflet similar to the supplement was distributed and made available through several outlets in the local area. The uncovering of the posters launched a week of Maths Eyes activities in the area.

Among the other strategies to engage the community was the collaboration of the Mathematics and Computing lecturers at IT Tallaght to develop of a Maths ‘EyeWalk’ podcast which was designed to guide people through Tallaght on a guided 40 minute walk. This was available to download via the South Dublin County Library website (2011). A maths trail for a local park was also developed through collaboration between mathematics lecturers in IT Tallaght and the parks department of SDCC**.** This was used for an event where families were invited to exercise their minds and their bodies by following the trail of clues through a park located in the heart of Tallaght. Finally a street workshop was designed and delivered on a busy square to engage passers-by with the theme of the proportion in the context of the amount of water used in various domestic tasks and how to conserve it.

In addition to these events South Dublin Libraries were key partners in hosting a programme of events tailored to involve all age groups including; pre-school children and their carers, primary school children and their teachers, teenagers, adult learners and their educators, families, everyone. As part of this the library assembled collections of picture books for young children featuring maths themes and vocabulary, hosted a Maths trail within the library, facilitated a workshop about developing maths skills in pre-school children with numeracy sacks and hosted a family maths day

Using maths problem pictures or posters is an excellent way to help individuals of all ages to develop their Maths Eyes”. The best pictures for developing maths eyes are snapshots of familiar things that capture some aspect of real life mathematics. It was decided to use the concept of engaging the community with mathematics through these problem posters in two ways. Firstly a set of photographs of everyday items and locations in the Tallaght area was assembled. The mathematics group at IT Tallaght added statements to these images to prompt the viewer of the photo to explore the image for some mathematical aspect. In doing this a range of images and questions were chosen so that not just shape but other mathematical aspects such as number, pattern etc. would be suggested for exploration in the set of problem pictures. This collection of problem pictures were assembled as an exhibition entitled ‘Solve-It’ in the Institute foyer. An interaction sheet for those looking at the exhibition was constructed. In keeping with the emphasis on whole community engagement, a second photography exhibition was assembled in collaboration with the local Tallaght Photographic Society. The amateur photographers in the society were asked to explore the Tallaght area with their maths eyes open. From the images taken an exhibition of the most suitable photographs was selected jointly by an external photographer and Institute mathematics staff An interaction sheet for those looking at the exhibition was constructed for this exhibition also. The resulting exhibition was entitled ‘Tallaght Cubed’ and the public were invited to view and comment on the photographs which were exhibited in the County Library. This exhibition was a fine example of what can be achieved by maximizing the human capital in the locality.

Linking with those in Education

A number of activities were developed to encourage participation from those involved in all levels of education. In partnership with the Schools Liaison officer from Dublin City Libraries all primary schools in the local area were encouraged to 'Show Their Maths' Eyes' and attend a showcase event in the Institute. To encourage participation all schools were invited to send a representative to an information session on 'Developing Maths Eyes' that was facilitated by the Institute. Subsequently one of the staff from the Maths Eyes Project visited schools to speak to all staff and to explain the initiative to them to encourage their participation. These visits were requested by the schools and facilitated through the South Dublin Library. In all 14 of 28 schools in the area attended the showcase. In total 600 children (senior infants – sixth class) and 100 adults showed their 'maths eyes'. In some cases schools took a whole school approach while others limited participation to an individual class. The National Centre for Excellence in Teaching Science and Mathematics (http://www.nce-mstl.ie/) facilitated 'Fun with Maths' workshops for all children attending the event. The event was supported by national media figure John Murray, from RTE1.

From a secondary school perspective the timing of 'Looking at Tallaght Through Maths Eyes' was determined by the dates of the International Conference and was not ideal. Hence the decision was made to concentrate on those who would be entering secondary school in September 2011. The Project Maths team gave a presentation 'My Child and Project Maths' to parents of children in the final year of primary school. The presentation gave parents the opportunity to find out more about Project Maths and how it will impact on the mathematics education of their child.

To engage with those involved in who are involved in numeracy teaching and learning in adult basic education the Institute of Technology Tallaght in partnership with the National Adult Literacy Agency (http://www.nala.ie/) hosted the event ' *Sharing innovative numeracy teaching and learning: Adult, Youth and community showcase'*. Over 100 numeracy tutors from the Adult Education sector and their students showcased how they linked maths to real life. Exhibitions covered: the development and use of numeracy sacks; numeracy and art, photography, architecture, money and exchange, fossils and botany. All those at the showcase attended a number of workshops on innovative teaching of mathematics in adult education.

Strength in Partnership

A key to success of the 'Looking at Tallaght through Mathematical Eyes' as a community initiative was the strong partnership approach with key stakeholders in the Tallaght Community. Partners included: Institute of Technology Tallaght (many departments), South Dublin County Council, South Dublin County Libraries, Dublin West Education Centre, Tallaght Photographic Society and South Dublin Vocational Education Committee. The key media partner was the Tallaght Echo (the local newspaper). There was also support from the National Centre for Excellence in Mathematics and Science Teaching and Learning, National Adult Literacy Agency, Irish Mathematics Society and the Learning Innovation Network.

To strengthen the links between community and the International conference a reception was held in the Institute of Technology Tallaght. The event was a unique and successful opportunity for engagement between the local and international participants.

Local, National and International Dissemination

As part of the overall initiative a resource pack was compiled. The pack 'Developing Maths Eyes; An Innovative Approach to Building a Positive Image of Mathematics' (2011) was designed to help all those in education to develop the maths eyes of their students. The pack which was funded by Dublin West Education Centre was developed by a team including Institute staff with expertise in the area of Numeracy in conjunction with educators from primary and adult education. The pack was written in language which enabled the resources developed to be used in a variety of situations with both children and adults.

The resources developed spanned the five strands in the Irish mathematics curriculum: Number, Algebra, Statistics/Probability, Geometry/Trigonometry and Functions.

Following on from the initiative further work in disseminating the ideas and approach have taken place. As part of a national event 'Maths Week' which is held in October each year (www.mathsweek.ie) key elements of the Maths Eyes initiative were repurposed. For example a selection of the recycled *Curiosity* campaign posters and accompanying explanations of the mathematics on which they were based were distributed to all of the schools and adult education centres in the South Dublin County Council area for displaying in their yards/ halls/ classrooms to help celebrate Maths Week 2011. Further, the *Solve-It* and *Tallaght Cubed* exhibitions were made available to be exhibited in a selection schools in the area.

Building the success of the resource pack for 'Developing Maths Eyes' and the other associated ideas it was decided to proceed with making the resources and learning approaches more widely available. To this end the images and exhibitions associated with Maths Eyes are now digitised and available online (for free) at www.haveyougotmathseyes.com. A mechanism has been put in place for Maths Eyes posters to be assembled by learners themselves with a competition using this facility being piloted in the Dublin area in June 2012. The Maths Eyes development team is also working with NALA, and the Department of Education and Science Home School, Community Liaison Officers. In addition work is ongoing with organisers of the national Maths Week to investigate how the 'Maths Eyes' infra-structure and model of community engagement can be used to best effect nationally.

The strong synergy between the community initiative and the Adults Learning Mathematics International Conference meant that participants at the conference developed a good understanding of the approach that was used and were well positioned to replicate this approach in their own countries. To date the model has been adapted for use in America, Norway, Holland and the United Kingdom.

References:

SEFI Mathematics Working Group (2011). *A Framework for Mathematics Curricula in Engineering Education -A Report of the Mathematics Working Group*. Accessed via http://sefi.htw-aalen.de/Curriculum/Competency%20based%20curriculum_discussion_document_December_2011.pdf (02- 05-2012)

Gal, I., Alatorre, S., Close, S., Evans, J., Johansen, L., Maguire, T. et al. (2009). *PIAAC Numeracy: A Conceptual Framework* (Rep. No. No. 35). OECD Directorate for Education.

Maths Eyes Development Team (2011), *Developing Maths Eyes: An Innovative Approach to Building a Positive image of Mathematics* . Accessed via http://www.haveyougotmathseyes.com/wp-content/uploads/resources/maths_eyes_2012_resource_pack.pdf .

(02- 05-2012)

Expert group on Future Skills Needs (2009). National Skills Bulletin. Accessed via http://www.skillsireland.ie/media/egfsn090703_national_skills_bulletin.pdf (02-05-2012).

O'Donoghue, J. (2000). Assessing Numeracy. In D.Coben, J. O'Donoghue, & G. E. Fitzsimons (Eds.), Perspectives on Adults Learning Mathematics (pp. 271-287). Dordrecht, Netherlands: Kluwer Academic Publishers.

Coben, D. & Thumpston, G. (1995). Common Sense, Good Sense and Invisible Mathematics. In T.Kjaergard & A. L. N. Kvame (Eds.), The Third International Conference on the Political Dimensions of Mathematics Education - PDME III (pp. 284-297). Norway: Caspar F A/Sorlag.

Coben, D. (2009). Numeracy for Nursing:The Scope for International Collaboration. In K. Safford-Ramus, G. E. Fitzsimons, L. Ginsburg, & J. Kanter (Eds.), A Declaration of Numeracy:Empowering Adults through mathematics Education (pp. 3-16). Philadelphia, Pennsylvania.: Chestnut Hill College.

Steen, L. (1997), Preface : The New Literacy . In Why Numbers Count, Quantitative Literacy for tomorrow's America. Eds. Steen, L.. College Entrance Examination Board.

Klinger, C. (2009). Passing it on: Linking Adult innumeracy to Mathematics Attitudes, Low Self-Efficacy Beliefs, and Math-Anxiety in Student Primary Teachers. In K. Safford-Ramus (Ed.), 15th International Conference of Adults Learning Mathematics A Research Forum (pp. 123-132). Philadelphia: Adults learning mathematics (ALM) - A Research Forum.

Project Maths Development Team (2012). "*Overview of Project Maths*". Accessed via http://www.projectmaths.ie/overview/ (02-05-2012)

South Dublin County Library, (2011). *'EyeWalk'* podcast. Accessed via *http://www.southdublinlibraries.ie/podcasts*. (17-10-2011)

Applied Mathematics in the Electricity Industry Management

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Abstract

This paper shows that optimization models are a part of the curriculum of the School of Engineering at different levels from undergraduate to graduate students. Also it illustrates how optimization models can contribute to support decisions in the electricity industry. Mock-up models are used as learning tools for the students while high-end models are fully integrated in the real decision making of the electric companies.

Introduction

Operational Research (OR) can be defined as the application of advanced scientific analytical methods in improving the effectiveness in operations, decisions and management of a company. Other names also used for naming it are *Management Science*, *Business Analytics* or *Decision Science*. In short, it is also defined as the science of better (motto of the Operational Research Societies

(http://www.scienceofbetter.org/))

or as decision support models or advanced analytical methods. We can consider OR as a science in the frontier between primarily economics and engineering.

Life itself is a matter of OR is the slogan of the EURO (European Operational Research Societies). Effectively, there are many fields where OR methods can be applied to improve decision making. The electricity industry and the OR have always been good companions for many years, see Garver (1962). Decision support tools allow managing the operation and to evaluate long-term capital-intensive decisions in the electricity industry, see Delson (1992).

In this paper we present the practical experience of teaching applied mathematical methods at undergraduate, master and doctorate levels in a School of Engineering and some real decision support tools we have developed to help in taking operation and investment decisions in the electricity industry.

The paper is organised as follows. First, we introduce an overview of the different courses that constitute the curriculum of mathematical optimisation methods of the engineering students and the learning material they utilize. Then, we enumerate the competencies that the students attain within these courses. Later, we show the importance of developing mock-up models to consolidate their learning process. Finally, we establish the transition between the small models and the high-end ones used in the electricity industry and extract some conclusions.

Structure of the optimisation courses

In the paper, we are going to focus on optimization, as the most frequently used technique, although other classical OR techniques such as simulation are also adopted. Optimization selects the best decisions among the innumerable feasible options available. It is a prescriptive technique. Simulation evaluates the performance of a system under different conditions including mainly stochastic parameters or events. It is a descriptive approach.

At undergraduate level in the *Engineering Degree* we present just and introduction/overview of the many techniques that are under the name of *Mathematical Methods* (see the hourly content of the course in

http://www.iit.upcomillas.es/aramos/MME.htm)

see figure 1.

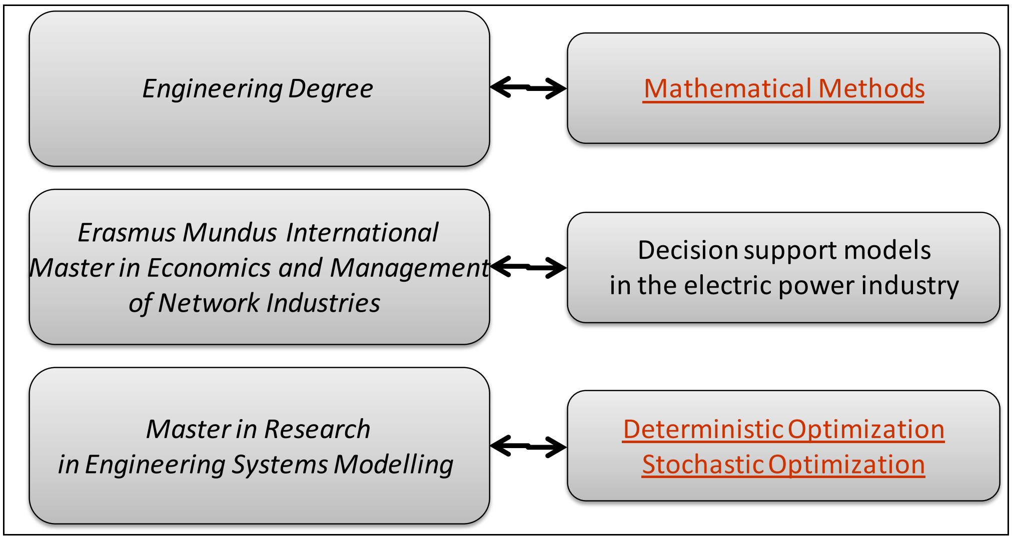

Figure 1. Summary of optimization methods in the engineering curriculum.

At graduate level we need to discriminate between professional-oriented master programs and research-oriented doctorate programs. As an example of the first one we can observe in the *Erasmus Mundus International Master in Economics and Management of Network Industries* whose general program is in this link (http://www.upcomillas.es/emin/Program.aspx).

In this master the spotlight is on decision support applications presented in the context of the electricity industry operation functions with a course entitled *Decision support models in the electric power industry*. As an example of the research-oriented doctorate program we can take the *Master in Research in Engineering Systems Modelling* following this link to the syllabus (http://www.upcomillas.es/eng/estudios/estu_mast_inve_mode_cont.aspx?idIdioma=ENG).

In this research-oriented doctorate program the optimization theory is explained but still with an engineering point of view. Deterministic Optimization

(http://www.iit.upcomillas.es/aramos/O.htm)

and Stochastic Optimization

(http://www.iit.upcomillas.es/aramos/OE.htm)

are the two courses in the OR field.

Learning material

We follow an OpenCourseWare approach to provide all the material fully accessible via the web. For example, in the link

http://www.doi.icai.upcomillas.es/intro_simio.htm

you can find the lecture notes and slides while in the link

http://www.iit.upcomillas.es/aramos/Ramos_CV.htm#ModelosAyudaDecision

are the more specific applications to the electricity industry and the mock-up models we provide as a basic step.

Competencies

In these courses we pay emphasis in two main objectives: learn how to build models for a certain decision problem and to understand the technique used to solve them. The student has to be able to develop optimization models using high-level languages. We encompass theory, numerical examples and computer examples along the course.

The specific contents of optimisation that are reviewed with different prominence are: linear programming (LP), mixed integer programming (MIP), nonlinear programming (NLP), mixed complementarity problem (MCP) and stochastic programming (SP). Also we review some specialised algorithms used to solve large-scale optimization problems such as Benders' decomposition or Lagrangian relaxation.

The general competencies of the courses can be summarised in the following list:

- Recognise the diverse fields where optimisation techniques can be applied
- Understand and apply the techniques used for decision making

- Model and solve prototype optimisation problems of diverse nature using an algebraic modelling language
- Analyse, synthesise and interpret the solutions obtained
- Present the model in a written report and orally
- Learn how to work in a group for doing the practice

Professional-oriented master students need to achieve a general understanding of the mathematical models. They must understand their input and output and their use in an industry context. A specific competency of this master is to understand how different functions of the company are done by means of mathematical models.

However, research-oriented master students must develop from scratch their own mathematical optimization model and, therefore. They must been able to develop a model following the different steps and to present orally and to write a research paper about it. Other specific competencies include:

- Understanding the mathematical principles that support the algorithms and their potential application
- Achieve mathematical rigorousness
- Learning how to model efficiently

Model development

To achieve these competencies the students learn by doing. They have to build mock-up optimization models by using an algebraic modelling language (in particular, we use General Algebraic Modelling System GAMS (http://www.gams.de/)) at both undergraduate and master levels. Relevant characteristics of these algebraic languages are:

- High-level computer programming languages for the formulation of complex mathematical optimization problems
- Notation similar to algebraic notation. They provide a concise and readable definition of problems in the domain of optimization
- Do not solve problems directly, but ready-for-use links to state-of-the-art algorithms. Therefore, allow the modeller to concentrate in the modelling process

These languages are used for rapid prototyping given that allow flexibility for continuous refinement of the model and therefore generate a huge decrease in maintenance time. Their main advantages are:

- Independency of the mathematical model and data, solution method (solver), operating system or user interface

- At the same time, models can benefit from advances in hardware, solution methods or interfaces to other systems

These advantages are also important for developing high-end models. Since more than ten years we have been using the GAMS language for teaching and for professional model development at the School of Engineering.

For practical purposes, Microsoft Excel is the preferred user interface (inputting the data and output of the results including their graphical representation).

High-end models

Students develop mock-up optimization models to learn their use and to attain the competencies. However, commercial-grade models are needed to support decisions for large-scale electric systems. Scaling-up models is a major task, not only from a computational point of view but also from a mathematical point of view. A very careful computer implementation has to be followed and probably a specific optimization algorithm must be used (e.g., Benders' decomposition). People from the electric companies are aware of the usefulness of mathematical models. However, in these models it is crucial to balance the mathematics and algorithms involved and the practical solutions provided by them.

In the next section we present some of the paradigmatic models used in the electricity industry and how we have stated and solved it.

Short-term daily unit commitment model

ROM model (http://www.iit.upcomillas.es/aramos/ROM.htm) objective is to determine the technical and economic impact of intermittent generation (IG) and other types of emerging technologies (active demand response, electric vehicles, concentrated solar power, solar photovoltaic) into the medium-term system operation including reliability assessment, see Dietrich (2012). Results consist of generation output including IG surplus, pumped storage hydro and storage hydro usage, and adequacy reliability measures. The benefits of improving IG predictions can also be determined by changing forecast error distributions and re-running the model.

Next there is a list of the main characteristics of the model:

- A daily stochastic optimization model formulated as a mixed integer programming (MIP) problem followed by a sequential hourly simulation

  This system modelling in two phases reproduces the usual decision mechanism of the system operator. Detailed operation constraints such as minimum load, ramp-rate, minimum up-time and downtime of thermal units and power reserve provision are included into the daily stochastic unit commitment model. The hourly simulation is run for the same day to account for IG production errors, demand forecast errors and unit failure and therefore revising the previous schedule.

- A chronological approach to sequentially evaluate every day of a year

  Decisions above this scope as the weekly scheduling of pumped storage hydro plants are done internally in the model by heuristic criteria. Yearly hydro scheduling of storage hydro plants is done by higher hierarchy models, as for example, a hydrothermal scheduling model (see next section).

- Monte Carlo simulation of many yearly scenarios that deal with IG or hydro inflows stochasticity

The model scheme based on a daily sequence of planning and simulation is similar to an open-loop feedback control used in control theory.

A mock-up Stochastic Daily Unit Commitment Model for the master students can be found at (http://www.iit.upcomillas.es/aramos/StarNetLite_SDUC.zip).

Hydrothermal scheduling model

Hydrothermal scheduling models (HTCM) manage the integrated operation planning of both hydro and thermal power plants, see Ramos (2011).

By nature, these models are high-dimensional, dynamic, nonlinear, stochastic and multiobjective. Solving these models is still a challenging task for large-scale systems. One key question for them is to obtain a feasible operation for each hydro plant, which is very difficult because the models require a huge amount of data, by the complexity of hydro subsystems, and by the need to evaluate multiple hydrological scenarios. For these models no aggregation or disaggregation process for hydro power input and output is established. Besides, thermal power units are considered individually.

A HTCM determines the optimal yearly operation of all the thermal and hydro power plants taking into account multiple cascaded reservoirs in multiple basins. The objective function is based on cost minimization because the main goal is the medium term hydro operation.

This model is connected with other models within a hierarchical structure. At an upper level, a stochastic market equilibrium model (see next section) with monthly periods is run to determine the hydro basin production. At a lower level, a stochastic simulation model with daily periods details hydro plant power output, see Latorre (2007). This later model analyzes for several scenarios the optimal operational policies proposed by the HTCM. Adjustment feedbacks are allowed to assure the coherence among the output results.

This model has the following main characteristics:

- Specially suited for large-scale hydroelectric systems
- Deals with multireservoir, multiple cascaded hydro plants
- Consider nonlinear water head effects
- Takes into account stochastic hydro inflows
- Formulated as a multi-stage stochastic optimization solved by a state-of-the-art solution method, stochastic dual dynamic programming, see Cerisola (2012)

A mock-up Medium Term Stochastic Hydrothermal Coordination Model can be found at (http://www.iit.upcomillas.es/aramos/StarGenLite_SHTCM.zip).

Market equilibrium model

The market equilibrium model is stated as the profit maximization problem of each generation company (GENCO) subject to the constraint that determines the electricity price as a function of the demand, which is the sum of all the power produced by the companies. Each company profit maximization problem includes all the operational constraints that the generating units must satisfy.

When considering the Cournot's approach the decision variable for each company is its total output while the output from competitors is considered constant. In the conjectural variation approach the reaction from competitors is included into the model by a function that defines the sensitivity of the electricity price with respect to the output of the company. This function may be different for each company.

Operating constraints include fuel scheduling of the power plants, hydro reservoir management for storage and pumped-storage hydro plants, run-of-the-river hydro plants and operation limits of all the generating units.

The model incorporates several sources of uncertainty that are relevant in the long term, such as water inflows, fuel prices, demand, electricity prices and output of each company sold to the market. This is done by classifying historical data into a multivariate scenario tree. The introduction of uncertainty extends the model to a stochastic equilibrium problem and gives the company the possibility of finding a hedging strategy to manage its market risk. With this intention, we force currently future prices to coincide with the expected value of future spot prices that the equilibrium returns for each node of the scenario tree. Future's revenues are calculated as gain and losses of future contracts that are cancelled at the difference between future and spot price at maturity. Transition costs are associated to contracts and computed when signed.

The risk measure used is the *Conditional Value at Risk* (CVaR), which computes the expected value of losses for all the scenarios in which the loss exceeds the *Value at Risk* (VaR) with a certain probability.

All these components set up the mathematical programming problem for each company, which maximizes the expected revenues from the spot and the futures market minus the expected thermal variable costs and minus the expected contract transaction costs. The operating constraints deal with fuel scheduling, hydro reservoir management, operating limits of the units for each scenario, while the financial constraints compute the CVaR for the company for the set of scenarios. Linking constraints for the optimization problems of the companies are the spot price equation and the relation of future price as the expectation of future spot prices.

The KKT optimality conditions of the profit maximization problem of each company together with the linear function for the price define a *mixed linear complementarity problem*. Thus the market equilibrium problem is created with the set of KKT conditions of each GENCO plus the price equation of the system, see Rivier (2001). The problem is linear if the terms of the original profit maximization problem are quadratic and, therefore, the derivatives of the KKT conditions become linear.

The results of this model are the output of each production technology for each period and each scenario, the market share of each company and the resulting electricity spot price for each load level in each period and each scenario. Monthly hydro system and thermal plant production are the magnitudes passed to the medium-term hydrothermal coordination model, explained below.

A mock-up Cournot Model can be found at

(http://www.iit.upcomillas.es/aramos/StarMrkLite_CournotEn.gms)

Transmission expansion planning model

Transmission expansion planning determines the investment plans of new facilities (lines and other network equipment) for supplying the forecasted demand at minimum cost. Tactical planning is concerned with time horizons of 15-30 years. Its objective is to evaluate the future network needs. The main results are the guidelines for future structure of the transmission network.

Here we present a decision support system for defining the transmission expansion plan of a large-scale electric system at a tactical level. A transmission expansion plan will be defined as a set of network investment decisions for future years. The candidate lines are pre-defined by the user so the model determines the optimal decisions among those specified by the user.

TEPES transmission expansion planning model

(http://www.iit.upcomillas.es/aramos/TEPES.htm)

will determine automatically optimal expansion plans that satisfy simultaneously several main attributes. Their main characteristics are:

- Dynamic: the scope of the model will be several years at a long-term horizon, 2020 or 2030 for example.

- Stochastic: several stochastic parameters that can influence the optimal transmission expansion decisions will be considered. Besides, the model must consider stochasticity scenarios associated to: renewable energy sources, electricity demand, hydro inflows, and fuel costs. These yearly scenarios are grouped in: operation scenarios (hydroelectricity, etc.) and reliability scenarios (N-1 generation and transmission contingencies)

- Multicriteria: some of the main quantifiable objectives will be incorporated in the objective function, the model considers: transmission investment and variable operation costs (including generation emission cost), reliability cost associated to N-1 generation and transmission contingencies.

The optimization method used is based on a functional decomposition between an automatic plan generator (based on optimization) and an evaluator of the transmission plans from different points of view (operation costs for several operating conditions, reliability assessment for N-1 generation and transmission contingencies, etc.). The model is formulated as a two-stage stochastic optimization solved by *Benders' decomposition* where the master problem proposes line investment decisions and the operation subproblem determines the operation cost for this investment decisions and the reliability subproblems determine the not served power for the generation contingencies given that investment decisions.

The operation model (evaluator) is based on a DC load flow. By nature the transmission investment decisions will be binary. The current network topology will be considered as starting point.

A mock-up Long Term Transmission Expansion Model can be found at

(http://www.iit.upcomillas.es/aramos/StarNetLite_TEPM.zip)

Conclusions

Mathematical model development and the use of models for taking decisions is a part of the curriculum of the School of Engineering at undergraduate and graduate level. The main competencies associated to the courses are focused on understanding the process to develop models and their potential application. There is a natural continuation between mock-up models that are explained to the students and high-end models that are developed as part of funded research.

Mathematical formulation of models allows the students to advance in their logical thinking, writing them in an algebraic modelling language, familiarize them with reality and how the models can be employed for decision support.

References

Cerisola, S., Latorre, J.M., Ramos, A. (2012) "Stochastic Dual Dynamic Programming Applied to Nonconvex Hydrothermal Models" *European Journal of Operational Research*, 218, 687–697 10.1016/j.ejor.2011.11.040

Delson, J.K. and Shahidehpour, S.M. (1992) "Linear Programming Applications to Power System Economics, Planning and Operations" *IEEE Transactions on Power Systems* (7) 3: 1155-1163

Dietrich, K., Latorre, J.M., Olmos, L., Ramos, A. (2012) "Demand Response in an Isolated System with high Wind Integration" *IEEE Transactions on Power Systems* 27 (1): 20-29 10.1109/TPWRS.2011.2159252

Garver, L.L. (1962) "Power generation scheduling by integer Programming-Development of theory," *Power Apparatus and Systems, Part III. Transactions of the American Institute of Electrical Engineers*, 81 (3), 730–734.

Latorre, J.M. Cerisola, S. Ramos, A. Bellido, R. Perea A. (2007) "Creation of Hydroelectric System Scheduling by Simulation" in the book H. Qudrat-Ullah, J.M. Spector and P. Davidsen (eds.) *Complex Decision Making: Theory and Practice* pp. 83-96 Springer ISBN 9783540736646 10.1007/978-3-540-73665-3_5

Ramos, A., Cerisola, S., Latorre, J.M. (2010) "A Decision Support System for Generation Planning and Operation in Electricity Markets" in the book P.M. Pardalos, S. Rebennack, M.V.F. Pereira and N.A. Iliadis (eds.) *Handbook of Power Systems II* pp. 797-817 Springer ISBN 9783642126857

Ramos, A. Cerisola, S. Latorre, J.M. Bellido, R. Perea, A. and Lopez E. (2011) "A Decision Support Model for Weekly Operation of Hydrothermal Systems by Stochastic Nonlinear Optimization" in the book G. Consigli, M. Bertocchi and M.A.H. Dempster (eds.) *Stochastic Optimization Methods in Finance and Energy*. Springer ISBN 9781441995858 10.1007/978-1-4419-9586-5_7

Rivier, M., Ventosa, M., Ramos, A., Martínez-Córcoles, F., Chiarri, A. (2001) "A Generation Operation Planning Model in Deregulated Electricity Markets based on the Complementarity Problem" in the book M.C. Ferris, O.L. Mangasarian and J-S. Pang (eds.) *Complementarity: Applications, Algorithms and Extensions* pp. 273-295 Kluwer Academic Publishers ISBN 0792368169

Students' Conceptions of Nothingness and their Implications for a Competency driven Approach to Curriculum

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Abstract

Competency has been the buzzword of higher education for at least a decade. The reasonable approach to describe what students should be able to do after completing coursework, however, falls short of the fact that mathematics, like any subject matter, contains inherent difficulties for students. Students usually need assistance in overcoming such difficulties. Competency driven approaches to curriculum tend to ignore this issue. Here, this problematic issue will be exemplified by investigating students' difficulties with the concept of empty set on one hand and SEFI's framework for a mathematics curriculum on the other hand.

Introduction

Students do not enter the learning process as empty vessels. What they bring with them in terms of preconceptions and prior beliefs significantly influences their learning. Competency driven approaches to teaching tend to underemphasise this issue by following a deductive approach: define a rather abstract concept to start with (competency, in this case) and deduce what that means for student learning.

Research needs to complement this arguably reasonable method with an inductive approach: observe students' difficulties with subject matter and induce what that means for teaching. To support this argument I will present findings of an on-going research project focusing on students' understanding of the concepts of empty set and empty word (the latter one being of paramount importance in automata theory). While both concepts in a way are an "incarnation of nothingness" they are unrelated to a large extent. Yet the data collected so far in exams, formative assessments, and student interviews strongly suggest that many students enter (and leave) their learning process with an incorrect understanding of the empty set. Furthermore they use this understanding of the empty set to construct their own understanding of the empty word.

Empty word and empty set are but two concepts in the vast "concept space" of mathematics. The SEFI framework for a mathematics curriculum (SEFI MWG, 2010) views such concepts as a key in defining mathematical competence as "the ability to recognize, use and apply mathematical concepts in relevant contexts and situations" (p. 3). In fact, the document specifically mentions the ability to "understand the concepts of a set, a subset and the empty set" (p. 28) as one of many very fundamental content-related mathematical competencies. But what does it mean to *understand* the concept of the empty set?

Basically, competency in general and understanding in particular are ill-defined concepts in the sense that scientific concepts need to be defined operationally. But what are the operations that we need to carry out in order to decide whether someone understands something or whether someone has a certain competency?

I argue that an important (but by no means sufficient) step towards an operationalization of notions like competency and understanding consists in collecting evidence about typical difficulties students encounter with subject matter. In the case of empty set that would mean that we need to know about students' difficulties with a particular concept in order to decide whether students understand this concept. If students show certain characteristic difficulties with this concept we can be sure that they do not understand it sufficiently. Unfortunately, it does not tell us that much if students do not show such characteristic difficulties.

Students' difficulties and competencies are two quite opposite lenses through which students' learning can be viewed. They are rather complementary to each other. For this reason I argue that we need to complement competency based approaches to curricula by descriptions of students' difficulties or misconceptions related to relevant topics. Such descriptions are also of paramount importance for effective teaching. Hestenes (1996) suggests the teacher be "equipped with a taxonomy of typical student misconceptions to be addressed as students are induced to articulate, analyse and justify their personal beliefs". The next section is intended to provide seminal information for such a list related to the concept of empty set.

Students' conceptions of the empty set

In automata theory there are two important concepts of "nothingness": the empty set and the empty word. The empty word ε is the neutral element of string concatenation (denoted by $\bullet$), i.e. $\varepsilon \bullet w = w \bullet \varepsilon = w$ for any word w. Another important construct of the theory is the set of words, called language. For any language L, as for any set, the empty set $\emptyset$ is the neutral element with respect to the union operation, i.e. $\emptyset \cup L = L \cup \emptyset = L$.

Note that for any symbol a the operation $w \bullet a$ "adds something" to the word w in that it appends the symbol a to w. Likewise the operation $L \cup \{w\}$ "adds" something to the language L. The analogy, however, is weak: the union operation requires its arguments to be sets. Hence, for adding a word w to a language, w has first to be made an element of a set ($w \rightarrow \{w\}$).

Like any set a language is an unordered collection of words. Words, on the contrary, can be viewed as ordered collections of symbols. For instance, the word *foo* can be represented as the ordered collection ("f","o","o") and

$$(\text{"f"},\text{"o"},\text{"o"}) = (\text{"f"},\varepsilon,\text{"o"},\text{"o"}) \qquad (1)$$

is a correct statement. Hence, in a way ε is a neutral member of the ordered collection of symbols comprising a word. For sets, however, there is nothing like a neutral member (call it ν) such that, for instance,

$$\{1,2,3\} = \{1,\nu,2,3\} \qquad (2)$$

holds. In fact such a concept would lead to inconsistencies with the concept of cardinality of a set.

Many students have severe and persistent difficulties to apply the concepts related to ε and ∅ and also to tell them apart. Many instructors of automata theory are actually aware of this. However, it is *a priori* not obvious what causes these difficulties and consequently how to address them effectively in teaching. In order to learn more about this issue I devised the following problem to be used in the very first online formative assessment of an automata theory class for computer science undergraduates (see Kortemeyer (2010) for technical details about the online formative assessments used here).

Problem (N): Which of the following statements about sets are true or false, respectively?

(N1) {8,3,5} = {3,5,8}
(N2) {∅,8,3,5} = {3,5,8}
(N3) {∅} = ∅
(N4) {3,{8,3},5} = {3,5,8}
(N5) {{5,1},{1,3}} = {{1,3},{1,5}}
(N6) {3,{8,3},8} = {3,8,{3,8}}

In designing this problem I had been guided by two previous observations: first, that it is unclear to some students that sets are orderless collections (items N1, N5, N6 address this), second, that many students have difficulties to generalise the concept of sets with "atomic" elements to sets of sets (N2-N6 address this). I did not expect students' problems related to a confusion of ∅ and ε at that point, as the empty word was to be introduced later. Also (somewhat naively) I did not expect students' problems to be related to the empty set alone, as students had to take a course covering set theory as a prerequisite. I felt that this course would have provided sufficient experience related to ∅. The results of the online formative assessment, however, told me that indeed there are students' difficulties related to ∅.

Item	**N1**	**N2**	**N3**	**N4**	**N5**	**N6**
Initial test						
1st try (N = 53, c = 25)	0.96	0.58	0.75	0.83	0.81	0.79
2nd try (N = 23, c = 9)	0.96	0.52	0.70	0.87	0.83	0.56
3rd try (N = 14, c = 9)	0.86	0.64	1.0	0.93	0.93	0.79
Recapitulation						
1st try (N = 41, c = 28)	1.0	0.66	1.0	0.93	0.93	0.90
2nd try (N = 13, c = 8)	1.0	0.62	0.70	1.0	0.77	0.70
3rd try (N = 5, c = 5)	1.0	1.0	1.0	1.0	1.0	1.0
Final exam (N = 43)	-[a]	0.79	0.86	0.93	0.95	-[a]
Final exam of subsequent class (N = 62)	-[a]	0.56	0.67	0.72	0.75	-[a]

[a] Item has not been given in exam.

Table 1. Statistics of the relative number of correct answers for Problem (N). *N* denotes the number of students having worked on this problem. If there has been more than one possible try *c* counts the number of students having answered the problem fully correctly in the respective try.

Table 1 lists the results of this online formative assessment within the rubric "Initial test". Note that items N1-N6 had been encapsulated into one problem such that the online grading engine graded the problem as correct only if *all* items N1-N6 had been answered correctly. Students whose answers had not been fully correct were granted up to two more trials without being given any hint which of the items N1-N6 had actually been answered correctly or incorrectly. A more detailed analysis of the data given in Table 1 indicates that students were particularly reluctant to change their opinion about item N2. Hence, there might be a misconception related to the underlying mathematics.

Of course my students' answers on Problem (N) required me to address the observed difficulties with respect to sets in the class session following the online formative assessment. I decided to do this via a peer instruction question cycle (see Mazur (1996) for details on this pedagogy) focusing on items N2 and N3, for peer instruction would allow me to observe what type of arguments my students use in order to justify their answers to this problem. While eavesdropping on my students' discussion I heard two students justifying their claimed correctness of N2 using an argument along the lines of "∅ isn't really there, so if you add ∅ as an element it doesn't change anything". Note that these students perceived ∅ to have the property of ν described by Equation (2). Obviously, they considered ∅ to be the neutral element of a set (in the sense of a neutral *member* of a set), rather than the neutral element with respect to the union operation.

Having been alarmed about the issue I used some further occasions to address this issue in class. I also gave Problem (N) in another online formative assessment about 8 weeks later and included items N2-N5 in the final exam. The data related to these reassessments (see Table 1 under the rubrics "Recapitulation" and "Final exam", respectively) suggest that a number of students replaced their preconception related to (2) over the duration of the course. Yet, a considerable number of participants still adhered to (2) in the final exam. A detailed analysis shows that most of these nine students consistently answered N2 incorrectly in all assessments.

In order to rule out that these findings are an artefact of my class and to find out more about this issue I convinced the colleague who taught automata theory in the subsequent semester to include items N2-N5 in his final exam (this colleague was neither using formative assessments nor peer instruction). The corresponding results listed in Table 1 show a remarkable resemblance to the performance of my students on their first try during their very first encounter with these items. These data make it hard to deny that there is some inherent difficulty to this subject matter. This claim is consistent with the findings reported by Fischbein (1994) about other difficulties students typically show in the context of sets.

In order to investigate the issue further, I have started to conduct interviews with students on this issue. These interviews (to be reported elsewhere) strongly confirm that students view the empty set as the neutral member of any set as expressed in Equation (2). For instance, one student clearly uttered: "The empty set is part of every set. […] Therefore within each set one can write the empty set in front of any element." Another student explicitly tried to justify this by arguing that the empty set is visualised as an empty Venn diagram. According to this student, the emptiness of the Venn diagram representing ∅ can be found in the unoccupied space between the elements of any nonempty Venn diagram. Hence, the empty set is an element of any set.

In summary, it appears that many students have a misconception about the properties of the empty set ∅ in a way symbolically expressed in (2) which is analogous to the correct property (1) of the empty word ε. This might attribute to many students' difficulties to tell ∅ and ε apart.

Implications for Teaching and Research

The data presented in the previous section foremost serves the purpose of describing students' difficulties with the empty set. By following the time line of events that have led to these results the format of the previous section mimics of storytelling. This format has been chosen on purpose in order to show that uncovering students' difficulties is completely within the scope of a single instructor in any course. What is not within the scope of a single instructor, however, is to investigate and collect such difficulties on a scale encompassing all mathematical subject matter taught in higher education. This needs to be a collaborative research effort. It is necessary, though, in order for us to gain a better description of what we actually mean by certain content related competencies. In my eyes such an effort is at least as important as efforts to deductively outline competencies.

Taking into account how students' prior knowledge and beliefs influence learning sheds light on another rather important dimension of competency, namely the competencies of instructors. As emphasised by Schoenfeld (2010), instructors do not only need to have content knowledge and pedagogical knowledge, but also pedagogical content knowledge. That is, they need to know how student learning can be fostered based on knowledge about their students' difficulties with subject matter and what makes this subject matter difficult. Given how little is known today about why certain subject matter is difficult to learn, instructors need to be able to elicit students' preconceptions and difficulties. Appropriate tools for acquiring pedagogical content knowledge "on the fly" are conveniently accessible today. Formative assessments are an example for such tools. Formative assessments also had been the starting point for the deeper investigation of students' conceptions of empty set and empty word as described in the previous section. On a slightly more abstract level it relates to what Aarons (1974) already described decades ago: "I am deeply convinced that a statistically significant improvement would occur if more of us learned to listen to our students. […] By listening to what they say in answer to carefully phrased, leading questions, we can begin to understand what does and does not happen in their minds, anticipate the hurdles they encounter, and provide the kind of help needed to master a concept or line of reasoning without simply 'telling them the answer.'"

References

Aarons, A.B. (1974). Addendum to “Toward Wider Public Understanding of Science”. American Journal of Physics, Vol. 42: 157-158.

Fischbein, E. (1994). “The interaction between the formal, the algorithmic, and the intuitive components in a mathematical activity”. In R. Biehler, R.W. Scholz, R. Sträßer, and B. Winkelmann, eds. Didactics of Mathematics as a Scientific Discipline. Kluwer, pp 231-245.

Hestenes, D., (1996). Modelling Methodology for Physics Teachers. In: E. Redish and J. Rigden, eds. Proceedings of the International Conference on Undergraduate Physics Education, American Institute of Physics, pp 935-957.

Kortemeyer, G. and Riegler, P. (2010). Large-Scale E-Assessments. Zeitschrift für E-Learning, Vol. 5: 8-22.

Mazur, E. (1997). Peer Instruction: A User’s Manual, Prentice-Hall.

Schoenfeld, A.H. (2011). How We Think – A Theory of Goal-Oriented Decision Making and its Educational Applications, Routledge.

SEFI MWG (2011). “A Framework for Mathematics Curricula in Engineering Education”. Accessed via http://sefi.htw-aalen.de/Curriculum/Competency%20based%20curriculum_discussion_document_December_2011.pdf (1 May 2012).

MALog: A new way to teach and learn mathematical logic

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Abstract

An international project entitled Mathematical and Applied Logic – MALog led by Tampere University of Applied Sciences (TAMK) aims to provide pedagogically high-quality learning materials, which are created, presented and distributed by innovative use of Information and Communication Technologies (ICT) based solutions. Learning materials will be produced in a manner which creates an individual adaptive learning path for each learner. Various studies indicate that students in high schools and universities, and company employees need tools to help them learn mathematical logic effectively.

At present in TAMK the curriculum is revised to be competence-based. MALog had the objective of developing learning materials to support the development of mathematical and applied logic knowledge and competencies. In order to provide robust pedagogical support for the materials produced in MALog, an ontology of mathematical logic was created.

Introduction

Recent studies have highlighted concerns about engineering students' mathematics skills ([2], [3], [4]) and have identified students' mathematical skills at entry to their course of study as being a particular issue. Rising student numbers, decreasing hours of contact time and increasing heterogeneity of student cohorts are all challenges to university teachers. These issues, and concerns raised by industrial partners, laid the foundation for the international project Mathematical and Applied Logic – MALog – that aims to develop mathematical logic teaching and learning resources to build competencies in key mathematical logic skills. The other key factor which motivated the project was a survey [1] conducted in 2007 in Tampere University of Applied Sciences. The results of the survey on engineering students' mathematical logic skills showed that these skills were weak and that mathematical logic should be taught more extensively.

At the moment at Tampere University of Applied Sciences is carrying out reform of the curricula. The process aims to establish curricula and curricula work practices that support the best possible competence-based education, flexible learning, cross-sectoral cooperation and the use of different learning environments in teaching and advising. One aim of the curricula reform at TAMK is to include diverse learning environments to curricula inter alia by integrating RDI activities into learning.

One of TAMK's RDI projects is this three-year MALog project. This project has been partly funded from EU's Lifelong Learning Programme and started in December 2009. The project consists of universities and schools in Finland (Tampere University of Applied Sciences, Hervanta Upper Secondary School), United Kingdom (University of Warwick) and Romania (Technical University of Civil Engineering Bucharest, George Cosbuc National College). The project also involves close collaboration with several industrial partners who help to gather information on the use of mathematical logic in industry.

MALog had the objective of developing learning materials to support the development of mathematical and applied logic knowledge and competencies. The material would support learners in schools, universities and companies using the information collected from a needs analysis. Through engagement with industrial partners, real-life problems would be collected to enhance the theoretical material and connect with practical applications. All the materials produced would be translated into English, Finnish, Romanian, French and German.

This paper presents outcomes, observations and results achieved during the MALog project and reflect on ongoing curricula work. In the paper the results of the project have been considered from the point of view of the framework for mathematics curricula in engineering education.

Aims and Objectives

The main objective of MALog is to provide resources necessary for students and company employees to develop their mathematical logic skills. The resources take the form of learning materials that include theoretical and practical tutorial materials on a variety of mathematical logic and applied logic topics including practice assignments, example problems and visualisations. These learning materials are designed to meet the needs of schools, universities and business throughout Europe.

These resources are structured using a novel semantic architecture developed using an ontology of mathematical logic. Domain-specific information held in the ontology is used to create individual learning paths through the available resources for learners. Using an on-line deployment of learning materials, students can be guided through resources most appropriate for their needs and required competencies.

Mathematical Logic Competencies

At the beginning of the project an electronic questionnaire was used to ascertain how much was known about mathematical logic. The questionnaire was composed by the five partners and the respondents were 360 students from three universities and two high schools. In addition to this, the three university partners interviewed enterprises in their respective countries on the need for mathematical logic in working life. The surveys and interviews were conducted to discover what is already taught, how the material is delivered, and the types of problems encountered. An analysis of the results helped establish what kinds of material should be produced in the project and how effective they will be.

Mathematical logic is a broad description for a field of mathematics concerned with the application of formal logic and deductive reasoning. Different learning environments will require the use of different aspects of mathematical logic and also present the material in different forms.

The project has identified competencies are required in several areas of mathematical logic including set theory, Boolean algebra, propositional logic, predicate logic, and proof. Information about these competencies is gathered in an ontology of mathematical logic.

Competency structure

In order to provide robust pedagogical support for the materials produced in MALog, the ontology of mathematical logic was created and released in April 2010. With the help of the ontology, MALog aims to provide an individual adaptive learning path (IALP) for each learner and to deliver learning materials as flexibly as possible. Links between the learning materials further enhance their quality and usefulness by allowing learners to discover related material and relevant real-life problems.

Developing resources

Using the information gathered during the needs analysis and from interviews with industrial partners, the project identified specific learning resources that should be developed to support key mathematical logic competencies. These were divided into sixty units of learning material and individual plans were developed for each item.

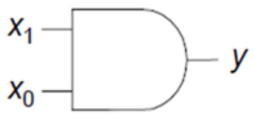
The project has made contacts with companies, representing several engineering fields, to secure the sources of learning materials, for which real-life problems from the companies were collected. When producing the learning material, each partner created material reflecting the local cultural context of real working life. These learning materials are designed to meet the needs of schools, universities and business.

In the discussion with the company representatives it emerged that more and more products entail logical functions. In software design especially good mathematical thinking and reasoning ability make it possible to comprehend large entities and dependencies between items; programming work presupposes the ability to capture the big picture in software design right from the beginning. Poor mathematical skills make it necessary to construct the program bit by bit, bottom up, and the overall conception is not achieved. So a good knowledge of a mathematics and logical thinking underlies the mastery of programming.

In the discussions with the industrial representatives and professional teachers the importance of embedded systems and how their importance will grow significantly in the future were highlighted. In the discussions it was mentioned that one of the key factors in the development of embedded systems is digital technology. Mathematical logic is needed in order to master digital technology and it has been predicted that in the near future the field of digital technology will significantly increase. For example, future multimedia mobile phones and their network systems require even more digital technology. It is possible to utilise digital technology, which is based on mathematical logic - Boolean algebra - in practically all aspects of technology. Thus, mathematical logic is a fundamental element in the development of digital technology, programming languages and all engineering fields.

Figures one and two presents mathematical logic application exercises that emerged from the interviews. The figures present the competencies of mathematical logic issues in the engineering professional studies and in real life.

x_1	x_0	y
0	0	0
0	1	0
1	0	0
1	1	1



$$y = x_1 x_0$$
$$y = x_1 \cdot x_0$$

Figure 1. Truth table, symbol of AND gate and AND function.

X	Y	Z	$F = X + Y'Z$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

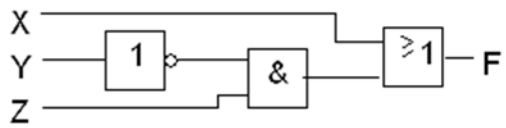


Figure 2. Truth table, logical function and circuit diagram.

Suppose that a student wants to build a truth table for the NAND logical operator. With the help of the ontology the learning path builds up in a way that it provides the materials related to this issue and suggests additionally that material on NOT and AND logical operators is related to the understanding of NAND, and the student will be offered activities to ensure he has understood NOT and AND as well. When the learning materials have been constructed using an on-line deployment of learning materials it provides a possibility of following an individual adaptive learning path so that it guides the learning of the student. The student has been guided through those resources most appropriate for his needs and required competencies.

All the materials produced in the project will be licensed under a Creative Commons license. They are free to use and modify but are not allowed for use commercially. The learning materials being developed will be freely available on-line and will enable school children, technology students and technology professionals to further develop their mathematical skills.

Evaluation of learning resources

Evaluation of the learning resources developed was undertaken by all five partner institutions using a variety of course structures and delivery formats.

In the project the pilot courses were organised so that the impact of the materials produced was tested. Feedback (in the form of questionnaires and interviews) from the courses by the students, teachers and the professionals in companies using the materials has been collected in order to receive valuable end-user feedback and student viewpoints. Based on the feedbacks and findings, the learning materials have been updated and finalised in order to best meet the needs of the target groups. Results of the project are expected by the end of the year 2012.

Conclusions for Education

In today's world people encounter technological applications everywhere from cars to coffee machines. Good mathematical skills help develop the logical and critical thinking skills necessary for good design and implementation of software and technology products.

Mathematical logic is fundamental to computer science and to all engineering fields which apply computer science. Mathematical logic skills not only advance pure logical thinking, but also enhance a learner's understanding, hence simplifying key engineering activities such as the mastering of programming skills and the development of digital devices and embedded systems.

At the moment in TAMK the curriculum is revised to be competence-based and within this process the learning materials of the project have been planned to be used in several ways. For example, in the field of computer sciences the materials have been planned to be used as a part of professional study courses. The idea is that the course will be run by a professional teacher together with a mathematics teacher. For the students, this way shows up more clearly the use of mathematics in the field of the professional studies of engineering. Also the materials will be used in a way that there will be an own course based on the materials produced in the project. With the help of learning materials, applied learning materials related to real-life problems, on-line deployment of learning materials produced in the project means that students can be guided through the issues related on mathematical logic most appropriate for their needs and required competencies.

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References

[1] Kinnari, H. & Rinneheimo, K-M. (2007). Matemaattisen logiikan opetustarve Tampereen ammattikorkeakoulun insinööriopinnoissa. Tampereen ammatillisen opettajakorkeakoulun opinnäytetyö.

[2] Kinnari, H. (2010). A study of mathematics proficiency. In Ion Mierlus-Mazilu, ed., 1st International Workshop on Mathematics and ICT: Education, Research and Applications, Bucharest, Romania.

[3] Näätänen, M. Osataanko matematiikkaa kyllin hyvin? Matematiikkalehti Solmu. Erikoisnumero 1/2005-2006. http://solmu.math.helsinki.fi/2005/erik1/naatanen.pdf

[4] Tuohi, R. (2009). Matematiikan lähtötasotestaus Turun ammattikorkeakoulussa tekniikan ja liikenteen alalla vuosina 1999–2004 ja 2008. Matematiikkalehti Solmu 3/2009. http://solmu.math.helsinki.fi/2009/3/alkutestaus.pdf

Cognitive Levels and Approaches Taken by Students Failing Written Examinations in Mathematics

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Abstract

A study was conducted at the Technical University Berlin involving students who failed the written examination in the first semester course Linear Algebra for Engineers twice in order to better understand the reasons behind their failure. The study considered student understanding in terms of Bloom's taxonomy and the ways in which students approached problem solving. The results indicate that students rely on lower-order thinking processes and these processes are linked to solution approaches. Thus, by investigating solution strategies in homework sets and in class work, an instructor can easily identify students at risk of not understanding at the appropriate level. In this contribution, the study is related to the framework set forth by the SEFI Mathematical Working Group.

Introduction

One of the goals of the European Commission (Europe 2020) is for at least 40% of the population between the ages of 30 and 34 to complete a college education. To achieve this goal, recruiting must also be combined with developing measures to improve retention rates. The crucial phase in a student's university experience is generally the transition from secondary to tertiary education. Especially in educational facilities offering courses with large lectures, the anonymity and lack of accountability can lead to undesirable results (see for example O'Shea (2005)).

The course Linear Algebra for Engineers at the Technical University (TU) Berlin serves over 3500 incoming freshmen each year. Eight lecturers (5 in winter and 3 in summer semesters) cover the same material to provide students with the theoretical foundations. The lecturers are in general professors or post-docs, the latter of who often have little teaching experience. The weekly 2-hour lecture is supplemented by a weekly 2-hour tutorial. Many of the tutors are also inexperienced pedagogically and mathematically. Students can apply to become a tutor in their third semester at the university, and the first-semester engineering mathematics courses are typically assigned to new tutors. Thus, the challenge is not only to support students but also to support instructors at different levels. Several measures, including a reorganization of the tutorials to promote more active involvement of the students as well as tutor training, took place in 2006 (see for example Roegner (2008)), coinciding with the introduction of bachelor degree programs at the TU Berlin. Although the situation has improved since then, with success rates of 42 – 47% instead of 35 – 40%, it is far from ideal.

One of the key issues to investigate how to further improve the course lies in the area of assessment. In interviews with students who failed the written examination twice, many claimed that they invested a lot of time in studying for the tests. In fact, most of these students faithfully attended weekly office hours in preparation for the oral examination. The students are thus industrious. The question is where do things go wrong? One contributing factor to the low success rates lies in the fact that there are no lower-level courses offered, save for a non-credit introductory course focusing on secondary school mathematics, in which students can be placed according to their abilities. Not being able to change this structural condition, other factors need to be investigated to improve the overall situation further.

The results of a study (Roegner (2011), (2012)) with students in an oral examination suggest that the depth at which these students learn is insufficient at the tertiary level. Their fixation on memorization and step-by-step procedures hinders them from succeeding in university-level mathematics. In terms of the MWG's Framework for Mathematics Curricula in Engineering Education (2011), which will subsequently be referred to as the Framework, the students are trapped in the Reproduction mode. The results also suggest that students' approaches to solving problems can serve as an indicator for inappropriate depths of learning, thereby supplying tutors with methods for identifying students at risk of failing.

Method of Investigation

Over the course of two years, examination questions were formulated and tested during oral examinations to ensure that various approaches were not only possible but also taken by students. The actual study, which took place during 2009, considers the answers of 43 participants to the following two problems.

Problem 1.

Given is the matrix $A := \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 3 \\ 0 & -2 & 3 \end{bmatrix} \in \mathbb{R}^{3,3}$.

(a) Determine the eigenvalues and associated eigenspaces of the matrix A.

(b) Is the matrix A diagonalizable? If so find a diagonal matrix D and an invertible matrix S so that $A = SDS^{-1}$. (In the course, the columns of S are appropriate eigenvectors of A.)

(c) Is the matrix mapping $A : \mathbb{R}^3 \to \mathbb{R}^3; \vec{x} \mapsto A\vec{x}$ injective?

Problem 2.

The (ordered) basis $\mathcal{B} := \left\{ B_1 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B_2 := \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, B_3 := \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, B_4 := \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$ of $\mathbb{R}^{2,2}$ and the linear mapping

$$L : \mathbb{R}^{2,2} \to \mathbb{R}^{2,2}; \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mapsto \begin{bmatrix} a+b & 0 \\ c+d & 0 \end{bmatrix}$$

are given.

(a) Determine the representing matrix $L_{\mathcal{B}}$ for the linear mapping L with respect to the basis $\mathcal{B}$.

(b) Is the linear mapping L injective?

(c) Can you find an eigenvalue and associated eigenvector of L?

Each part of Problem 1 has been practiced using algorithms in the course but can also be at least partially solved using other methods or chains of reasoning. In Problem 2, the first part is more or less procedural, but is somewhat tricky for students due to the underlying vector space. The last two parts of the second problem are not practiced in the course, although clearly the concepts are related to those in Problem 1.

The students were asked the problems and their sub-problems in the same order in most cases. Variations occurred when students anticipated follow-up questions or offered information relating to properties that they happened to notice along the way. In computational portions of the examination, the correct answers were agreed upon before the student went on to the next step.

The scoring chosen reflects the categories in Bloom's hierarchical taxonomy adapted to the case of mathematics (see e.g. Riegeluth and Moore (1999)): Knowledge (K), Comprehension (C), Application of Knowledge (AK), Analysis (A), Synthesis, and Evaluation.[1] A point was awarded in the positive (respectively negative) column of a given category or subcategory for each correct (respectively incorrect) written or oral statement. Furthermore, the ways in which the students approached the problems or steps therein were taken into account. Two contrasting pairs were used for this purpose: global versus local and conceptual versus procedural. A global approach was noted when a student viewed the problem holistically, at least to some degree. A local approach was recorded when the student was so focused on the current step that they failed to draw upon other information, such as previous steps. A conceptual approach was noted when a student used a theorem instead of applying the expected algorithm. Procedural was noted when the student applied the algorithm instead of a theoretical argument that a first-semester engineering student could reasonably be expected to make and which would have made the problem easier to solve. To some extent, the recording of the strategies thus provides some measure as to the efficiency of the solution, which, as Schoenfeld (2000) has suggested, students reflect too little upon in examination situations.

In terms of the Framework, the competencies Thinking Mathematically (TM), Reasoning Mathematically (RM), Problem Solving (PS), Communication (Com), Representing Mathematical Entities (RME), and Handling Mathematical Symbols and Formalism (HMSF) are addressed. It was reasonable for students to demonstrate the levels Reproduction and Connections within each of these categories and Reflection in the categories RM, PS, and RME.

The students in the study were not selected rather they specifically came to the author to be examined. Although the sample is therefore not random, much can be learned by the performance of these students.

Student Performance and Findings

Before beginning with a summary of the results presented in Roegner (2012), it should be mentioned that AK is used a bit differently than in Bloom's original sense. The students may be able to recite or explain an algorithm, but that does not mean that they can carry out the details, even though they may have seen similar problems. Carrying

[1] As the problems were constructed for the purpose of exploring the first four categories, Synthesis and Evaluation will be henceforth ignored.

out the details is for most students easier than understanding how or why the algorithm works, which is part of the category C. For this reason, the alteration in the meaning of AK switches the ordering in the hierarchy, so that AK comes before instead of after C.

The students were overall successful in demonstrating their competency in the categories K and AK. On the average, 84% of their total positive points in Problem 1 (72% in Problem 2) were earned in these two categories, corresponding well with the Reproduction levels of TM, PS, and Com in the Framework. Only 10%, respectively 1%, of the positive points earned in Problem 1 (8%, respectively 7%, in Problem 2) were in the category C, respectively A. The positive scoring in these last two categories corresponds fairly well to the Connections level of TM, RM, PS, Com, and RME.

The main portion of students' errors occurred in AK (47%) for Problem 1 and in C (40%) for Problem 2. Typical errors within each category are given for Problem 1 followed by Problem 2 in parentheses. In K, 57% (87%) of the errors were due to misstating definitions or theorems, i.e. the Reproduction level of Com, and 30% (0%) for improper forms, i.e. the Reproduction level of RME. In AK, the main errors were made in applying the techniques of linear algebra (41% (35%)), corresponding to the Reproduction level of PS. In Problem 1, 26% of the errors were basic arithmetic errors, corresponding to the Reproduction level of PS and HMSF and 21% errors in applying mathematical forms, corresponding to the Reproduction level of RME. In Problem 2, 53% of the errors in AK arose from students applying definitions incorrectly in concrete situations, most closely related to the Reproduction levels of RM and Com. Turning to Bloom's category C, 66% (21%) of the errors were due to student difficulties in discriminating between object types (for example, a number and a matrix), related to the Connections level of RME and HMSF. Another 16% (56%) of the errors made were due to misinterpretations of definitions and theorems, perhaps most closely tied to the Connections levels of RM and Com. In Problem 2, 26% of the errors involved students' inability to reformulate the eigenvalue equation to reflect the general situation of a linear mapping, thus demonstrating problems at the level of Connections in RME and HMSF. Note that not all students were asked to do this. Answers dealing with category A were for the most part avoided by students.

Students considered problems locally 1.6 times as often as globally and procedurally 10 times as often as conceptually in Problem 1, with the corresponding numbers in Problem 2 being 2 and 1.6. Students were categorized according to whether they approached their problems globally if their ratio of global to local points was at least as high as the average student and similarly for conceptually in terms of their conceptual to procedural point ratios. By considering the ratio of average positive scoring to average negative scoring within a category, then comparing these ratios with respect to the solution approaches of global (gl) versus local (l) and conceptual (c) versus procedural (p), the following table ensues.

Favoured Approach / Category	K	AK	C	A
Problem 1 gl/l [N = 26/N = 17]	1.5	0.9	5.6	0.9

	c/p [N = 8/N = 35]	1.4	0.9	0.8	16.7
	(gl and c)/(l and p) [N = 4/N = 13]	5.6	1.3	6.8	20.0
Problem 2	gl/l [N = 15/N = 23]	1.5	2.1	3.9	7.5
	c/p [N = 12/N = 26]	1.2	2.1	3.2	13.5
	(gl and c)/(l and p) [N = 11/N = 22]	1.3	2.4	4.6	15.6

The best performance in category C is by students favouring global approaches, whereas in category A, students favouring a conceptual approach had a much higher ratio of right to wrong answers. Students favouring both global and conceptual approaches substantially outperformed the students favouring local and procedural approaches in nearly all categories in each problem.

The data was conditioned to avoid division by zero and to differentiate between students with a few or no correct responses as opposed to many incorrect responses. The correlation coefficients comparing the ratios of positive to negative scoring in each category with the ratio of global to local approaches as well as the ratio of conceptual to procedural approaches were then calculated. For gl/l in the category C, the correlation coefficient is .55 (p-value .0001) for Problem 1, .60 (p-value .0001) for Problem 2, and .34 (p-value .04) combined. The corresponding results for c/p in the category A are .54 (p-value .0002) for Problem 1, .64 (p-value $< .0001$) for Problem 2, and .53 (p-value .0007) when combined. The correlation between gl/l and C is thus somewhat weak although significant, whereas that between c/p and A are rather strong.

Discussion and Conclusions for Education

The students in the study were most likely unsuccessful in the written examination twice because they had not learned to learn mathematics at a deeper level. They tended to be rule learners and shied away from attempting new solution strategies unless they were heavily prompted to do so. Their main focus during the oral examination was on Reproduction skills in nearly all competencies set forth in the Framework. Most students who attempted to demonstrate competencies at the level of Connections, again generally because they were prompted, failed to do so in a satisfactory manner. Helping shallow learners transform to deep learners is, thus, a key issue for students in making the transition from school to the university. The challenges are first of all how to identify and then how to help these students.

The findings provide at least a partial answer to the first challenge. The approaches students took in solving problems were classified into two different contrasting pairs. Students taking a global or holistic approach to the problem demonstrated many more instances of positive scoring as compared to negative scoring in Bloom’s category Comprehension. The students taking a conceptual approach instead of the usual procedural approach were able to demonstrate more connections in Bloom’s category Analysis. The correlation values provided in the previous section support these findings. Thus, even for inexperienced tutors, who incidentally have the most contact with the first-semester students in the course, analysing solution approaches during class discussions or in homework can help them to identify which students may be learning at too low of a level. Of course, the pilot study presented here should be extended to a more heterogeneous group of students, and different phases in the learning process need to be taken into account. Nevertheless, the results are promising. The second challenge is an entirely different problem altogether that still requires novel ideas and much attention.

References

European Commission (n.d.) “Europe 2020 targets.” Retrieved March 20, 2012, from http://ec.europa.eu/europe2020/reaching-the-goals/targets/index_en.htm.

O’Shea, J. (2005) “Helping students make the transition from school to university.” In *Proc. of the HELM Conference, Loughborough University, 2005*. Retrieved May 8, 2011 from http://www.engsc.ac.uk/downloads/helm/proceedings/session7a_oshea.doc.

Riegeluth, C. M. and Moore, J. (1999) “Cognitive education and the cognitive domain.” In C. M. Riegeluth, ed. *Instructional-design Theories and Models. A New Paradigm of Instructional Theory,* Mahwah, NJ: Lawrence Erlbaum Associates, 51-68.

Roegner, K. (2008) “Linear Algebra as a bridge course for first-year students.” In *Proc. MEE2008*. Retrieved October 7, 2010, from http://www.fbm.fh-aahlen.de/profumit/

alpers/sefimwg/Seminars/Loughborough2006mee2008/pages/proceedings.html.

Roegner, K. (2011) “TUMULT: A comprehensive blended learning model utilizing the MUMIE platform.“ Habilitation Thesis, TU Berlin.

Roegner, K. (submitted 2012) “Problem solving approaches and levels of understanding.”

Schoenfeld, A. (2000) “Purposes and methods of research in mathematics education.” *Notices of the American Mathematical Society*, 47(6): 2-10.

SEFI MWG (2011) “Framework for Mathematics Curricula in Engineering Education.” Retrieved April 22, 2012 from http://sefi.htw-aalen.de/Curriculum/Competency%20 based%20curriculum_discussion_document_December_2011.pdf.

Mathematical Curriculum, Mathematical Competencies and Critical Thinking

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Abstract

The new trend towards stressing the acquisition of mathematical competencies gives the opportunity to see the mathematical curriculum in a different light. Critical thinking is a very general competency which is also connected with mathematical thinking and other mathematical competencies. Mathematical education should train critical thinking not only in the mathematical field. This kind of thinking is also important for modelling and solving practical problems. Some formulas used in everyday life are critically analysed. The value of paradoxes and counterexamples in mathematical courses is pointed out.

Introduction

In Germany, and also in other countries, the relation between mathematical knowledge and mathematical competencies within the educational process is controversially discussed. Indeed, there is a close connection between both concepts. The new trend of stressing competencies at least gives the opportunity to investigate this relation deeply, to make it conscious for the public and to draw fruitful consequences for mathematical education. The mathematical curriculum not only considers competencies but also content which is necessary in order to understand other important parts of mathematics. At the engineering faculty of Hochschule Wismar it is based on Linear Algebra (vector and matrix concepts), Analysis (calculus and differential equations), Numerical Math (approximate methods, algorithmic thinking, error estimation) and Stochastics (probability distributions, conclusion from random samples). Critical thinking is developed based on logical thinking, on precision, on error estimation and on completeness of arguments. In addition, paradoxes and counterexamples in mathematics are especially suitable to train mathematical competences as critical thinking.

Often, certain formulas measuring quantities of interest are given in the public. Some of these formulas can be analysed by using basic mathematical means, whether they are of more or less practical value, which modelling assumptions are hidden behind and which features are neglected by the model. Sometimes some specialists try to award a discipline more importance by creating useless mathematical formulas. This is called pseudo-mathematisation. Further, some interesting news containing mathematically looking data are presented in the media. Mathematically educated people as engineers should be able to decide if these data are seriously used or if they serve more or less ideological interests.

Curriculum and Competence

In earlier times the curriculum was the most important educational base. The contents and its volume were fixed for different study courses and different levels. Later a catalogue of learning aims was added. Nowadays the competence model is the most favourite one. Competence has become a vogue word. It describes abilities and skills which are needed for a certain qualification, e.g. as Bachelor or Master in a certain engineering subject. There is more freedom for lecturers to fix contents and methods for acquiring of competencies. Besides, the ability for further study and the solution of everyday problems are included. Hence, the courses are also application oriented. Also, there are methods to measure the quality development in learning processes. On the other hand the definition of competencies is often rather vague and open to different interpretations. Finally, the international discussion is rather broad. So the concept is used in most cases intuitively. Essential general mathematical competencies are:

- Use of *knowledge*: understanding mathematical theory, knowing important facts, linking between disciplines;
- Correct use of *technical elements*: mathematical language, logical reasoning, rearranging or transforming of terms, use of means as tables, formulas, computer software, media;
- *Problem solving*: applying and transferring solution methods, applying heuristic methods, generalising, creating new connections and concepts;
- Use of *Methodology*: algorithmic, numerical, analytical and stochastic thinking, geometrical imagination;
- *Mathematical modelling*: creating suitable models, interpreting and validating results;
- *Critical thinking*: checking correctness and completeness of results;
- *Communication* skills: team-working, networking.

Modelling and Reasoning

Often simple models are used, for instance formulas. If the applications are successful some people do not think about mathematics at all and others believe that mathematics regulates the world. In any case, they take success for granted. But what happens, if the result turns out to be wrong. Then people often blame the formula or even mathematics for the disaster. They do not realise the true causes for misleading results:

- The formula (the model) is too simple for the given facts.
- The formula is given a wrong meaning.
- The range of application is not observed. The wrong formula is used or prerequisites of the formula are ignored.
- The formula is used in a new context (without proving the legitimacy).

In everyday life the problems are not given uniquely. It is necessary to see problems and to formulate them adequately in a mathematical model. There are many possibilities to create such models. The art is to find a sufficiently simple one which is good enough for successful applications.

Correct reasoning is very important. Often opinion polls ask for alternatives. For example: Would you prefer plan A or plan B? Assume the result is 60% for A and 40% for B. Politicians and media sometimes sell this result as: The majority of people are against B. This conclusion is not correct and is unfair. First, I do not know how many people were asked. The conclusion from a random sample to the whole community can be wrong with a positive probability. Second, the contrary of preference is not rejection. It can be useful to support both plans in a certain proportion.

The Value of Man and the Performance of Business

The Russian author Leo Tolstoy proposed a fraction rule to measure the *value V* of man, where the power (mind, true reputation and other personal properties) determines the nominator and the meaning about itself the denominator of the fraction:

$$V = f(P,S) = \frac{P}{S} \quad \left[\frac{Power}{Self - Confidence} \right].$$

According to this formula the value increases if the power increases at constant self-confidence or if the self-confidence decreases at constant power. Perhaps people believe that these statements are true or at least reasonable. But these consequences can be expressed also by more complex functions $f(P,S)$. How can we decide which of these functions is the most realistic one? Also, P and S are vaguely determined. Perhaps other variables should be considered, too. Perhaps it is useless to give a formula at all. But, if people can handle mathematical fractions, the formula suggests a social message, namely that modest people with great power are of excellent moral value. A similar slogan is from Marion Wolf, a German journalist, born in 1950. It reads: “The greatness of a person is reckoned by his ability in proportion to his modesty”.

In economics the *OEE* index is used, standing for *Overall Equipment Effectiveness*:

$$OEE = g(A,P,Q) = A \cdot P \cdot Q \quad Availability \times Performance \times Quality \ .$$

The three variables A, P and Q are given values between 0 and 1, where values near 1 are optimal. The same is true for *OEE*. Assume $A = 0.9$, $P = 0.6$ and $Q = 0.95$, then $OEE = 0.513$. *OEE* numbers greater than 0.85 are said to be top. The real background is that all three factors influence *OEE* in a positive way. Increasing one factor and holding the others constant means a better *OEE*. The formula can be criticised as above.

It is dangerous to press simple moral relations into mathematical formulas. It can be misused for manipulation of people, because they think the message has the same quality as mathematical theorems, namely to be absolutely and eternally true.

Health and Body Mass Index (BMI)

The book of Ziegler (2011) illustrates that many people believe on numbers. For example, they know whether they live healthily or not by considering their *BMI* (*Body Mass Index*). Roughly speaking, a person is thin if the *BMI* is less than 20, is of normal weight between 20 and 25, and is thick between 25 and 30. The person is fat if the *BMI* is more than 30. What can be said from a mathematical point of view about *BMI* and its practical value? The defining formula reads:

$$BMI = h(m,l) = \frac{m}{l^2} \quad \left[\frac{weight\ (mass)}{height^2} \quad \frac{kg}{m^2} \right].$$

More precisely, a function *h* is given, which calculates for each person *x* with mass *m(x)* and height *l(x)* the number *BMI*(*x*). Assuming a weight of 66 kg and a height of 1.7 m the *BMI* is 22.84. That is fine; this person is of normal weight. Where is the formula from? The Belgian mathematician and statistician Adolphe Quetelet (1796 – 1874) proposed it 1870. It can be interpreted as *average mass load per surface unit*, if the body surface is assumed to be nearly proportional to the square of the body height.

The model is quite simple, really too simple. The differences in age, sex, build, distribution of muscles, fat tissue and so on are not considered. Nevertheless the industries of beauty care and nutrition like this formula and offer *BMI* calculators on their websites. Quetelet introduced *BMI* as a statistical measure considering 5738 Scottish soldiers by recording their geometrical measures and their build. The formula corresponded quite well to his visual impression. But these soldiers were not representative. All the subjects were male, trained and of nearly the same height. Besides, they did not live in our time.

In a homogeneous body mass is proportional to volume: $m = \rho \cdot V$ (ρ density). Then *BMI* is a geometric magnitude. First let us assume that the human body is a homogeneous cuboid with width $a = \alpha \cdot l$, thickness $b = \beta \cdot l$ and height $h = l$. Then

$$BMI = \frac{\rho \cdot V}{l^2} = \frac{\rho \cdot a \cdot b \cdot h}{l^2} = \frac{\rho \cdot \alpha \cdot \beta \cdot l^3}{l^2} = \gamma \cdot l \qquad \gamma = \rho \cdot \alpha \cdot \beta \ .$$

Observe that both γ and *l* depend on the person *x*. Normally similar cuboid bodies (with constant factors α and β) should have the same *BMI*. But this is not the case. Considering people with ideal body proportions the taller ones have a greater *BMI* value. By the way, the surface of the cuboid is

$$O = 2 \cdot (ab + ah + bh) = 2 \cdot (\alpha\beta + \alpha + \beta) \cdot l^2 \ .$$

Hence, *O* is only proportional to l^2 if $\alpha\beta + \alpha + \beta$ is constant, especially if α and β are constant (fixed body proportions, see interpretation of *BMI* above). Naturally, the cuboid body model does not match reality. If the height is reduced omitting head and legs it is more realistic. But this version can be criticised by the same arguments.

Another simple body model is an ellipsoidal cylinder with parameters as above denoting the great and small halve axes of the ellipsoidal cross section and the height. Then

$$BMI = \frac{\rho \cdot V}{l^2} = \frac{\rho \cdot \pi \cdot \alpha \cdot \beta \cdot l^3}{l^2} = \delta \cdot l \qquad \delta = \rho \cdot \pi \cdot \alpha \cdot \beta \quad ,$$

with the same consequences as in the cuboid case. Certainly, the model can be improved by incorporating magnitudes which are not considered yet. Nevertheless, the value of *BMI* is questionable. It is especially useful if people in a uniform group (with many coincidences) are compared. Therefore, serious medical statements consider differences in age and sex. So the normal weight for woman should be between 19 and 24 instead of 20 and 25 for men. The scale is also changed for children and for people with amputated body parts.

It is a curiosity that nearly the same measure is used by some Chinese scientists to measure the *sex appeal* of woman. They divide the body volume by the squared body height, taken from the sole of the foot up to the chin. This measure needs no comment.

Paradoxes and Mathematical Modelling in Society

Paradoxes and counterexamples often occur in science as challenges of thinking. Paradoxes can play a very useful role in education producing fruitful discussions, provoking deeper thinking about the subject, clarifying new concepts and giving better insight to the theory. So they help removing potential conflicts between intuition, theory and reality. Besides they help forming new and correct intuitions (see e.g. Stoyanov (1997), Székely (1990), Wise and Hall (1993)).

The *arithmetic average* is a statistical measure. It estimates the expectation value of a probability distribution from a random sample. Often people think that average characterises the typical. So people look at average income in their country and compare it between different countries. The country is rich if the average income is high and poor if it is low. But the average income is not at all typical, if only some very rich people and a lot of very poor people live in the country or if middle incomes are very rare. The previously-mentioned scientist Adolphe Quetelet even made a study about the *average human*, which caused vehement debate. This concept is problematic, among other things, because the averages of single characteristics (as height and weight) do not correspond. Other people even claimed the average is the source of beauty.

Nevertheless the book of Quetelet (1869) is considered as a starting point and a milestone for the quantitative analysis of human and social qualities. Although the legitimacy of mathematics in natural sciences is recognised, the applications of mathematics in social sciences and economy are often discussed controversially. On the one hand, the complexity in society is very high and of another quality; the models are often too simple or are uncritically transferred from other fields. On the other hand, people believe the omnipotence of mathematical formulas and models and feel helpless because mathematical power seems to dictate the world. It is claimed in the March edition 2009 of the American magazine “Wired” that a mathematical formula caused the Wall Street to fall and led to the deep finance crisis in the whole world. The formula was derived by David X. Li to estimate the risk of finance institutions for investments in correlated securities (see Szpiro (2009)). The formula gives the probability that several enterprises go bust simultaneously. Li used parallels to methods in life insurance in calculating the probability that married couples die in the same year. It was simple to apply the formula, and finance managers used it widely. But the formula contains a correlation coefficient. Li assumed this coefficient to be constant and estimated it on the base of historical data. But at some stage this coefficient started to increase rapidly. The classical model failed under the new conditions. In this example people used mathematics without understanding.

Conclusions for Education

The essential findings concerning curriculum, competencies and critical thinking in mathematical education of engineering students are:

- It is especially important to know the basics (basic facts as well as basic techniques) because they are used in all mathematical disciplines and in practical applications again and again. Besides, these basics do hardly depend on the historical development. Considering the curriculum the basics relate to algebra, analysis, numerical mathematics and stochastics.
- In some sense it is more important to learn the kind of thinking in mathematics (the methodology) than to learn the solution of certain problem classes (apart from the basics).
- Modelling should be an essential part of the curriculum, since it is necessary to understand the part of mathematics in engineering and in practice.

A reasonable curriculum should supply not only problem-solving competencies for certain disciplines but also for everyday life to make general processes more transparent. Hence mathematical competencies are important to contribute essentially to the welfare of our society.

References

Stoyanov, J. (1997) *Counterexamples in probability*. Wiley.

Székely, G. (1986) Paradoxes in probability theory and in mathematical statistics. Reidel.

Szpiro, G. (2009) Eine falsch angewendete Formel und ihre Folgen. In *Mitteilungen der DMV*, Band 17, Heft 1, 44-45.

Quételet, A. (1869) Essai de Physique sociale (1835); L'homme moyen. *Physique Sociale, vol. 2*, Bruxelles.

Wise, J. and Hall, E. (1993) *Counterexamples in probability and real analysis*. Oxford University Press.

Ziegler, G. (2011) *Darf ich Zahlen? Geschichten aus der Mathematik.* Piper.

Mathematical eAssessment at its Best: News from Maple T.A. R8

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Abstract

Bologna-compliant studies enforce a strict framework and timetable for students and do not leave much room for practicing mathematical skills. eAssessment systems like Maple TA help to overcome this problem and are discussed by many authors. However, the learning curve for using these systems is often quite steep so that teachers are shying away from using them. To show that there is no need to be afraid, the focus is put on the workflow from designing mathematics exercises, combining them into tests, running the tests, grading and giving feedback to the students to keep up the records. From this point of view some general remarks on using eAssessment systems at schools and universities are made and the resulting demands on such systems are presented. Maple TA meets at least some of these demands. In the latest release some interesting new features like 'adaptive questions' or 'secure testing environment' improve the system and are discussed in some detail.

Introduction

We need more and better-qualified scientists and engineers but receive less and less well-educated freshmen in engineering and natural science courses of studies (Risse, 2008). There are many reasons but with regard to mathematics (core zero! comp. Alpers, 2011) the shrinking time for practice at schools and then at universities is easily identified as one major problem. Crash courses (often called bridging courses) do not really help and it is reasonable to use the short time available for face-to-face teaching in tutorials. Therefore, the classical teacher-centered unidirectional lecture can be replaced by a self-learning scenario if the progress of the students is assessed continuously. (Otherwise they won't do it.) This formative assessment can be done automatically using an eAssessment system, which of course also applies to final examinations i.e. summative assessments. The use of eAssessment systems in general found its way into the core curricula (Alpers, 2011) and is discussed by many authors, e.g. Lethonen (2008), Enelund (2011), Schramm (2008). Several solutions have now existed for more than 20 years and have evolved well. However, some aspects have proved quite helpful and should be considered.

- ·Separate content from administration: avoid monolithic software solutions with burned-in (hardwired) content. A good practice would be the use of a learning platform such as Moodle, OLAT or ILIAS, which have built in eAssessment properties, even for mathematics to some extent - at least for asking questions. For grading answers the situation is more subtle.

- If questions have purely numerical, calculable or otherwise fixed or foreseeable answers, there is no problem with standard systems. If the questions are dynamically generated (including diagrams) and have formula-based answers, computer algebra systems (CAS) should be used to generate the questions from templates and to check the answers for correctness.

- For the long term use it is reasonable to separate the questions or their templates from the assessments (tests, exercises, examinations) they are used in. If the questions are stored in a dedicated data-base (question banks), they can be used in different contexts and can be easily shared.

- Define and separate the roles of the administrator, the authors (instructor, teachers) and the candidates (students). Other roles to be foreseen could be tutors, scorers or proctors. While the tutor coordinates the use of the system, the scorer helps grading the answers (possibly replaced by a CAS), the proctor delivers the assessment, for example the proctor could log in the students after checking their identity cards.

Those demands on an eAssessment system can in parts also be found in the IMS QTI 2.1 (2006) specification, using somewhat different designations published by the IMS Global Learning consortium. The QTI (Question and Test Interoperability) standard defines also a lot of question types (multiple choice, fill in the blank, or true/false choice etc.) but is still under development, especially concerning mathematics. The main purpose for the standard is the exchangeability of the question banks between different systems but at the price of lacking mathematical flexibility.

A solution that meets the design pattern mentioned is Maple T.A. (2012). This server-based mathematical eAssessment platform goes far beyond the QTI standard with the transparent use of the CAS Maple 15 engine in the background. It is used and described by several authors (for example, as already mentioned, by Lethonen (2008), Enelund (2011), Schramm (2008)) and also sometimes criticized, for example by Bolton (2008) who preferred the STACK (System for Teaching and Assessment using a Computer-Algebra Kernel) system because of the adaptive and flexible response to wrong answers. STACK is an open source package under development by Sangwin (2012) in the School of Mathematics at the University of Birmingham. However, Bolton described an old release (2.5 to 3) and some progress was made particularly concerning the adaptive response in the current release 8.

Maple T.A. Release 8

As a user of Maple T.A. a teacher gets an account to the system typically as an instructor with predefined classes. Registered Students can choose their class or be enrolled by the instructor perhaps by an uploaded list. For choosing a class the typical web-interface (Internet Explorer) looks like Fig. 1. The pre-defined assignments could be chosen and altered via the Content Manager in the menu. However, the first task is to construct questions. For this purpose the Question Repository is chosen - shown in Fig. 2. It shows the question bank Math 1 containing two groups. The first one (Demo Class Questions) is inherited and contains many examples that could be used as templates.

Fig. 1.Class Homepage -- Instructor's view.

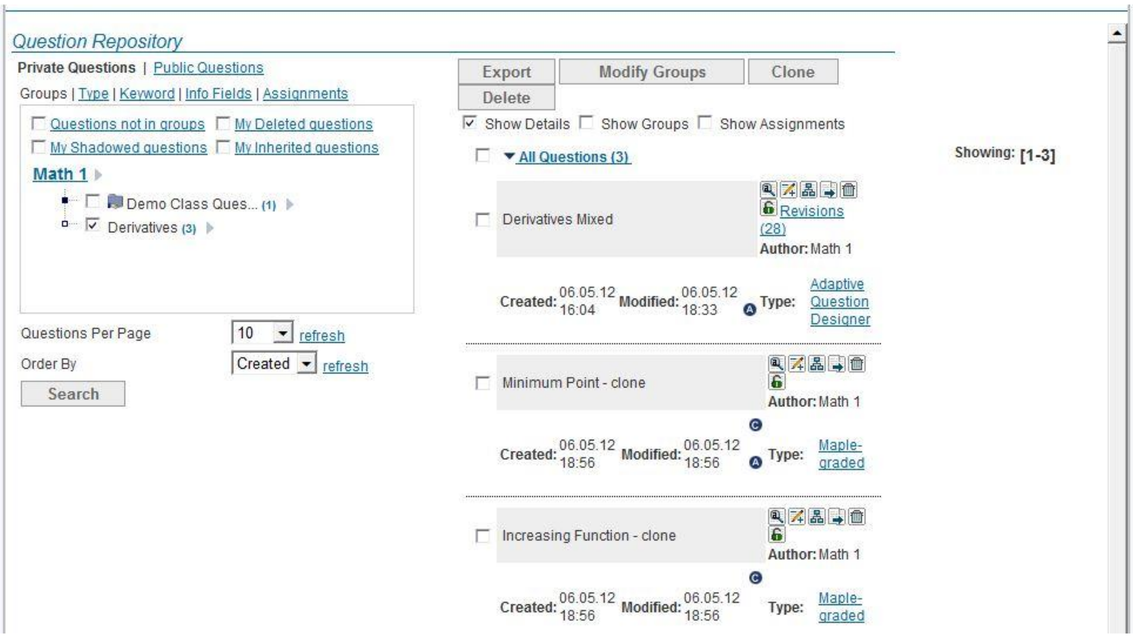

Fig. 2. Question Repository, groups and questions.

The second one (Derivatives) was introduced by the instructor, containing three different questions as an example. The last two come from the demo-group but the first one, designed by the instructor, will be used as an example for adaptive questions. From here it can be looked at, edited or cloned, to work on as a new template or to supply to other instructors. This "Derivatives Mixed" question was designed using the Adaptive Question Designer, which is shown in Fig. 4. A maple-graded question type was chosen to use the help of the CAS Maple 15. The $-preceded variables are defined in the Algorithm-Editor shown in Fig. 3 and invoke Maple commands by maple("..”). The first line defines two random but not equal whole numbers used as index to specify an element from the lists for the outer and inner part of a composed function to be differentiated.

```
$CI=range(1,4);$CO=range(1,4); condition:not(eq($CO,$CI));

$OUTER=maple("[sin,cos,tan,cot][$CO]");
$INNER=maple("[1+arcsin,1-arccos,1-arctan,2*arccot][$CI]");
$PROBLEM=maple("$OUTER($INNER(x))");
```

Fig. 3 Algorithm-Editor

As shown in Fig.4 these variables can simply be used in the text field of the Question-Designer to describe the question text.

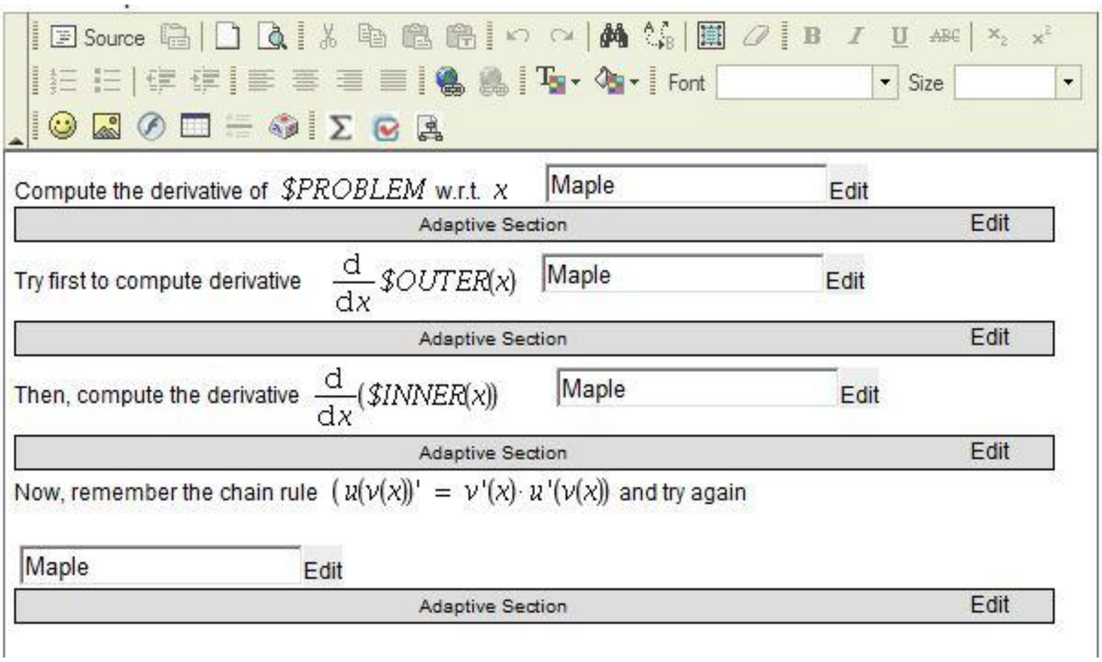

Fig. 4. Adaptive Question Designer

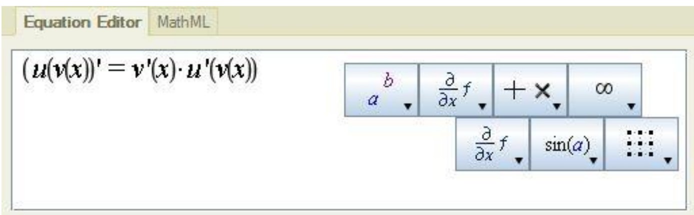

Fig. 5. Equation-Editor

For the construction of formulas an Equation-Editor can be invoked clicking on the ∑-button of the menu containing palettes with all necessary mathematical elements. This editor can later also be used by the students to enter their answers to the questions. The response area is prepared by the field  that starts the Response-Editor (Fig. 6).

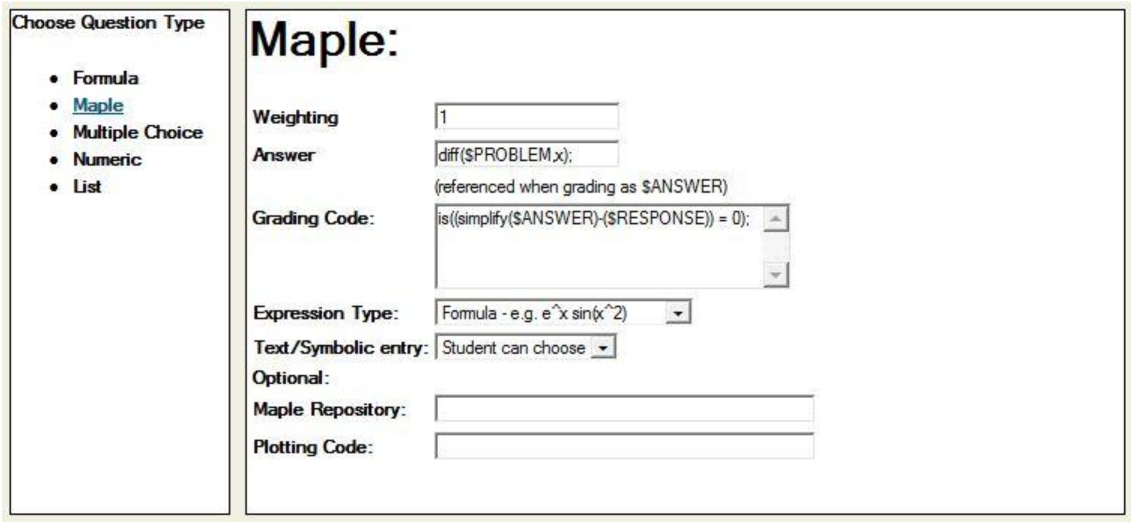

Fig. 6.Response-Editor

In this Maple-type question the correct answer is simply computed by the CAS and stored in a variable $ANSWER. The student's answer is stored in the variable $RESPONSE. The instructor must supply a Grading Code that must evaluate to a Boolean true for an answer to be correct. In this case the Maple simplify command is used for the difference $ANSWER-$RESPONSE that must be zero. Without the simplify command the check for correctness could fail because of the complex structure of all possible true answers. The simplify command invokes the knowledge of Maple about, for example, trigonometric identities or algebra. Because it is sometimes not easy to determine whether two expressions are mathematically identical it is no wonder that it is sometimes hard to find the appropriate Grading Code especially if expressions are randomly chosen.

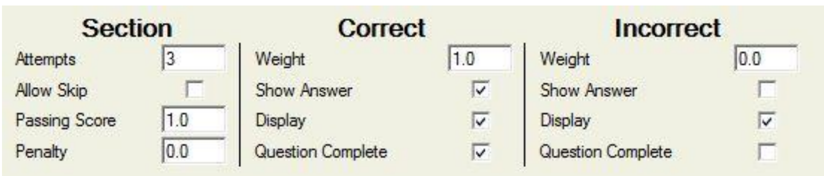

Fig. 7. Adaptive-Section-Editor

The most important point in the adaptive question types are the Adaptive-Section-fields. Using these fields it is possible to react to a possibly wrong answer. An incorrect answer can then lead to the next field where a subpart of the original question can be asked and automatically answered if the response is still wrong or skipped. In the end, the original question can be asked again to see whether the student understood all the hints and can then get at least a part of the marks available. We show an example (Fig.8) where the student fails to give the correct answer for the derivative of a composite function. Then the derivatives for the inner and outer part are separately questioned and finally the chain rule is given and the first question is asked again. Of course the student gets only a part of the points.

In this example the most complex question type of Maple T.A. was introduced. Other more simple types containing purely numerical, simple formula, multiple-choice or ‘fill in the blanks’ questions etc. are also easily built using the appropriate editors. Once having the questions stored in a question bank the assignment can be designed using this and possibly other banks with a lot of options. All the banks and assignments relating to one class can be stored in a course-module, saved, exported or imported by other instructors.

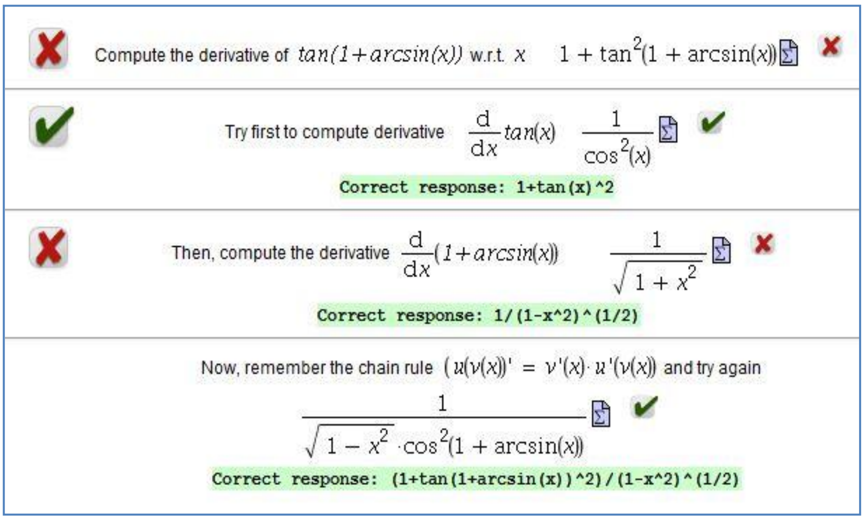

Fig. 8. Direct adaptive response

The students use the assignments perhaps for practice or for final examinations and a full record of the results is kept (if wished by the instructors) and a number of statistical reports are possible. New in the last release is the proctored browser, which offers a secure environment that hinders the student from using other applications than the browser used for the examination or to switch to other sites than specifically allowed. If the student tries, the instructor gets a cheating-message.

Conclusion

The general demands on an eAssessment system are presented and it was shown how these demands are fulfilled at least in part by a real application such as Maple T.A. To show the simplicity, the workflow for designing and using an adaptive question was shown step by step.

However, it should be noted that starting with a set of existing possible entangled questions is probably not a good idea. The instructor should first learn about the possibilities of the system and then rethink what he is going to test or what he wants to know from the students to avoid disappointment.

References

Alpers, B. (2011) “A Framework for Mathematics Curricula in Engineering Education.” Editor, Report of the Mathematics Working Group, SEFI, Draft, Version 7.1. Accessed via http://sefi.htw-aalen.de/Curriculum/Competency_based_curriculum_discussion_document_December_2011.pdf (3 May 2012)

Bolton, P. (2008) “Computer assisted assessment of mathematics.” MSOR Connections Vol. 8 (3) . Accessed via http://mathstore.ac.uk/headocs/8349_bolton_p_caamaths.pdf (3 May 2012)

Enelund, M. (2011) “Integration of a computational mathematics education in the mechanical engineering curriculum.” Proceedings of the 7th International CDIO Conference, Technical University of Denmark, Copenhagen, June 20 – 23. Accessed via http://publications.lib.chalmers.se/cpl/record/index.xsql?pubid=143187 (3 May 2012)

IMS QTI Version 2.1 (2008) Public Draft Version 2.Accessed via http://www.imsglobal.org/question (3 May 2012)

Lehotonen, K. (2008) “Testing and Teaching Mathematics with Interactive Online Technology.” In Proc. 14th SEFI Conference: Mathematical Education of Engineers, Loughborough. Accessed via http://sefi.htw-aalen.de/Seminars/Loughborough2008/mee2008/proceedings/mee2008F_Lehtonen.pdf (3 May 2012)

Maple T.A. (2012), Maplesoft, Canada Accessed via http://www.maplesoft.com/products/mapleta (3. May 2012)

Risse, T. et al. (2008) “Positions to Mathematical Education of Engineers.” In Proc. 14th SEFI Conference: Mathematical Education of Engineers, Loughborough. Accessed via http://sefi.htw-aalen.de/Seminars/Loughborough2008/mee2008/proceedings/mee2008F_risse_etal.pdf (3 May 2012)

Sangwin, C. (2012) STACK 3.0. Accessed via http://www.stack.bham.ac.uk/ (3 May 2012)

Schramm, T. (2009) “E-Assessments and E-Exams for Geomatics Studies.” In König, G. and Lehmann, H., eds., Proceedings of the ISPRS working group VI/1 - VI/2, volume XXXVIII-6/W7, 2009. Accessed via http://www.isprs.org/proceedings/XXXVIII/6-W7/Paper/12_Schramm.pdf (3 May 2012)

Diversity equality – Classifying groups of freshmen in engineering sciences

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Abstract

The results of a long-term study of the elementary mathematical skills of freshmen in engineering sciences are: The mathematical abilities of the freshmen depend on the type of the previous school education. Those groups of freshmen who have been identified in the previous studies as the weakest have become still weaker. The positive effect of the bridging course was shown more clearly than in the earlier studies. Suggestions for improvement of equal opportunities are: The university has to bear in mind that the students differ in their learning attitude and learning ability according to their educational / personal background. These groups are identified with this long-term study to give consideration to the diversity of the students. The support of the students should individually be adapted to the identified special groups. The observed results reflect the situation in Germany and especially in Berlin.

Introduction

At the beginning of the Winter Term in 2010/2011 elementary mathematical skills of freshmen in engineering sciences have been tested. The long-term study started in 1995 and has been repeated every five years with the same questionnaire. The results are evaluated with respect to four aspects: type of previous school, gender, participation at the offered previous bridging course and, in the actual study, also migration background. The university has to bear in mind that the students differ in their learning attitude and learning ability according to their educational / personal background. These groups are to be identified to give consideration to the diversity of the students and to gain enough information to install student tandems across the special groups.

Test Setup

The questionnaire consists of seven groups of over all 27 exercises. All tasks had to be carried out without the use of a pocket calculator. If required, appropriate approximate values were given. The students have 90 minutes to work with the test. Each single exercise is marked discretely by 0 point or 1 point, so the maximum of points which can be achieved is 27. The students remain anonymous. The test requires only mathematical knowledge that has been taught up until the 10th year of school. The test was performed in the first week of study for all freshmen of the Beuth University. The number of participants in the test was 1138 on the last occasion.

General overview

Figure 1 shows the results of the last three tests in 2000, 2005, and 2010. It shows the percentage of the participants over the points achieved. In 2010 nearly 70% of the participants failed at a level of 40% of the maximum points, in 2000 only 60% failed. The 40% level of points (11 points) is marked by the vertical line. There is a dramatic change in the proportion of all participants who failed totally (zero or one points out of 27 points); this has increased from 4% (year 2000) over 7% (2005) to 11% in 2010.

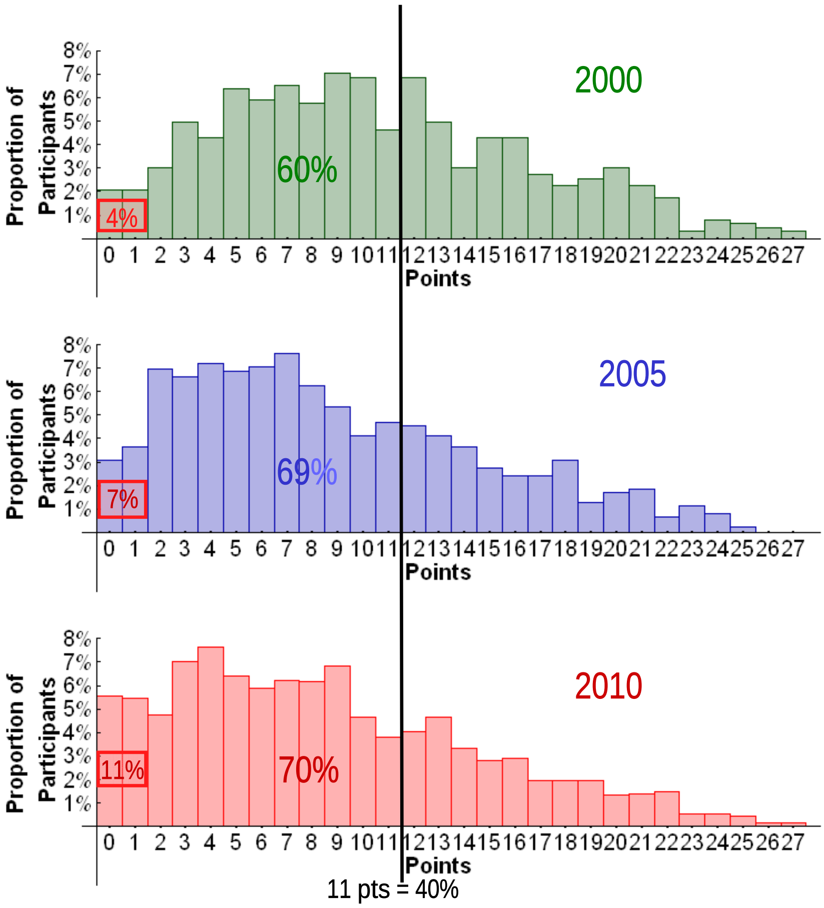

Figure 1 Percentage of the participants over the points achieved

Influence of the previous school education

In Germany there are different ways to meet the entrance requirements of the Universities of Applied Sciences: Students after 12 years of school (the so-called "Fachoberschule", below marked by "F") and after 13 years of school (the so-called "Gymnasium", below marked by "G"). The students who have 12 years of school are again separated in to two subgroups: the first subgroup consists of students who have done an apprenticeship of two or three years (below marked by "F1"), the second subgroup has been continuously in school for 12 years without an apprenticeship in between (below marked by "F2"). In Germany there is also an alternative entry pathway into university without a higher school education. That is an aim of politicians and society. What is required is a basic school education followed by an apprenticeship and several years with work experience in an appropriate job (below marked by "job"). The study proves that the mathematical abilities of the freshmen depend on the type of the previous school education.

First we take a closer look at the groups “G” and “F” who have a higher school education (Figure 2).

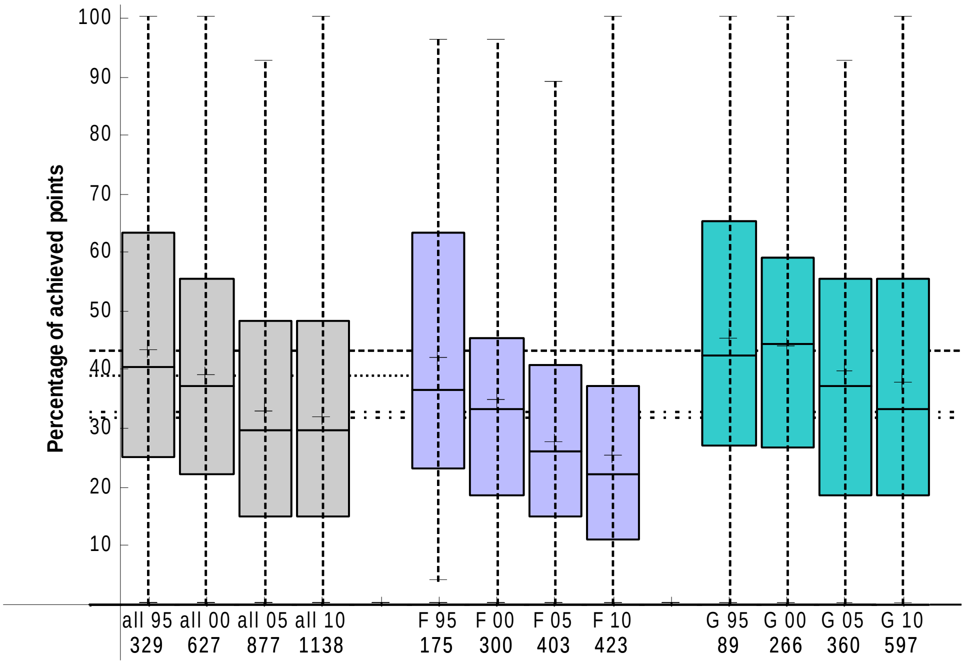

Figure 2 University entrants of the years 1995, 2000, 2005 and 2010, comparison of ‘Fachoberschule’ (F) and ‘Gymnasium’ (G)

In Figure 2 the results of the university entrants of the years 1995, 2000, 2005 and 2010 are compared by box plots. It shows the proportion of the achieved points in percentages for different years and different groups. The size of each population is given in the last line. The four boxes at the left show the results of all participants: the results of 2010 are worse than 15 years earlier. The mean value, indicated by a cross, dropped from 43% to 32%, and there is also a significant decline in the 75% mark by 15 percentage points. However, the dramatic decline seems to have stopped, considering the values in the years 2005 and 2010. The group ‘Gymnasium’ (G, the four boxes at the right) has a similar performance but on a slightly higher level. However the group ‘Fachoberschule’ (F, four boxes in the middle) is still decreasing over the whole period. Furthermore the level of the group ‘Fachoberschule’ is significantly lower than the level of the group ‘Gymnasium’.

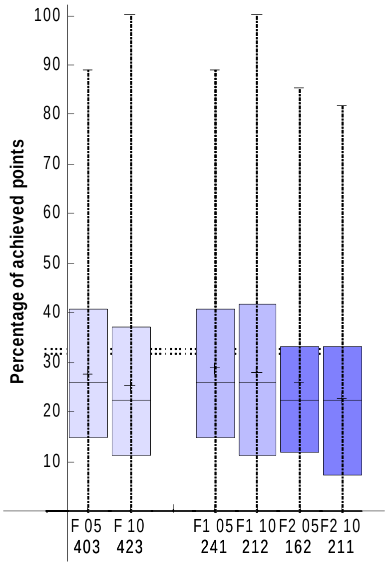

Figure 3 Comparison of the subgroups F1 and F2

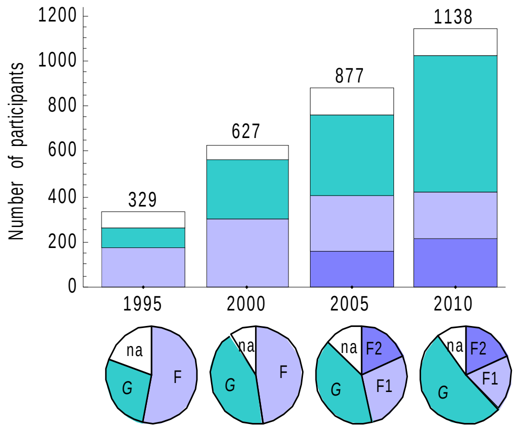

Figure 4 Population of the groups and subgroups (na = no answer)

Figure 3 takes a closer look at the group ‘Fachoberschule’ by considering also the subgroups F1 (with apprenticeship) and F2 (without apprenticeship) in the years 2005 and 2010. The results of the subgroup F2 (two boxes at most right) are significantly worse then those of subgroup F1 (two boxes in the middle). One realises that the subgroup F2 is responsible for the decline of the whole group F (two boxes on the left).

Also, the sizes of the populations of the groups and subgroups show a noticeable change over the time (see Figure 4). The number of participants in the test has clearly increased. The Beuth Hochschule specialises in engineering study programmes. In 1995 the number of engineering students in Germany was at a minimum level and is increasing since that time. Figure 4 mirrors this global development. The pie diagrams below show the most noticeable fact that the proportion of the group ‘Gymnasium’ has increased at the expense of the subgroup F1 (‘Fachoberschule’ with apprenticeship).

Those university entrants without higher school education have worked several years on the job before entering the university. Therefore two special problems are combined: the low mathematical background and the lack of practice in studying theoretical subjects. However this group is known for its high self-motivation. At the moment this group still is very small. This group unsurprisingly showed the lowest scores (see Figure 5 below). The bridging course has nearly no effect for the group because their mathematics level is so low. Student tandems can be useful which means that a student without higher school education individually gets his personal student assistant, a buddy, who is from a gymnasium.

Influence of personal distinguishing marks

The data of the test have been evaluated with respect to migration background, gender, and participation at the bridging course.

Due to political and social discussions the feature of migration background came into the focus. So only in the study 2010 data with respect to this feature have been collected by the question “Do you have a (perhaps second) not German mother tongue?”. In Berlin there is high proportion of people, especially young people, with migration background. The majority of those have (grand) parents who came from Turkey and the Middle East, so they have Turkish and Arabic backgrounds. Other groups from Russia and the Far East do not dominate.

The median of the achieved points is 22% for this migration group compared to 33% for those without migration background (i.e. only German mother tongue) (see Figure 5 below, MiY = Migration Yes, MiN = Migration No). This significant difference is a challenge for the universities in the near future.

In the study 2010 male students have slightly better scores than females (see Figure 5 below). This difference between male and female has also been observed in the previous studies. The mean differs by 5 % points - a score of 34% for the men and 29% for the women.

The Beuth University offers a mathematical bridging course in a compact form eight days before the beginning of the lectures. The participation in the bridging course reflects the individual attitude to exert oneself for learning. Low scores in the test correspond with a low participation proportion in the bridging course of that (sub)group. The lowest participation of 19% in the bridging course is shown by male migrants from F2 (Fachoberschule without apprenticeship). However for this group the effect of the bridging course has been the best. Within this group the mean score in the test differs by 19 % points, i.e. there is a score of 20% without bridging course and a score of 39% with bridging course. The highest participation of 54% in the bridging course is shown by female Germans from a gymnasium (G).

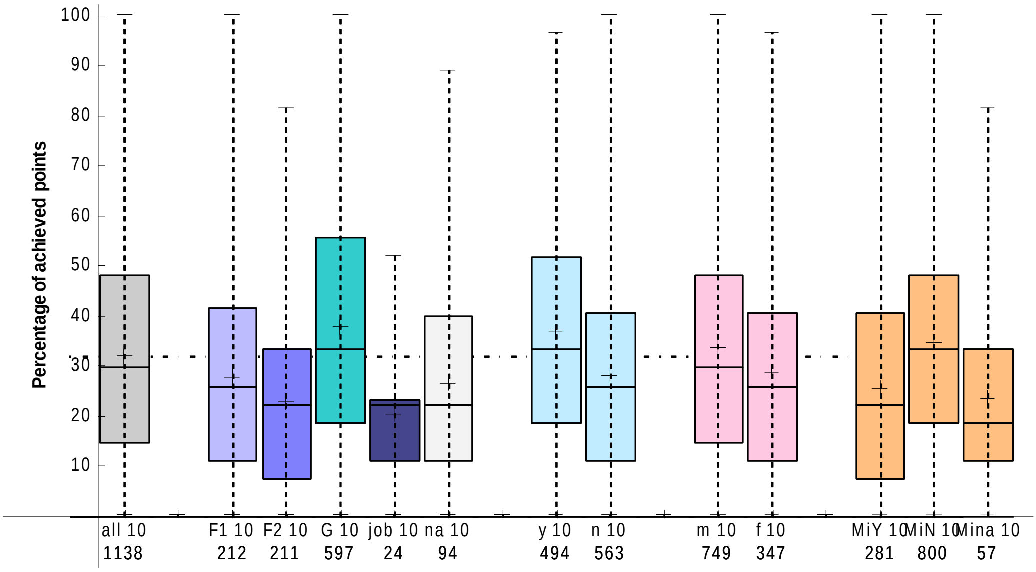

Figure 5 Overview of the scores of different groups (na = no answer)

In general the effect of bridging course has become clearer than in the previous studies. The mean score in the test differs by 8% points, i.e. there is a score of 29% without bridging course and 37% with bridging course (see Figure 5). Over the years the bridging course has been accepted more by the students; in 1995 only 36% of all participants in the test have shown up in the bridging course while in 2010 this percentage has increased to 43%.

Conclusion

The observed findings reflect the situation in Germany and especially in Berlin. In general freshmen from gymnasium (G) perform better than those from Fachoberschule (F), men perform better than women, and freshmen with German as their only mother tongue perform better than migrants. The average of the points achieved stopped decreasing at a low level. This stopping is due to the fact that the group F has become smaller in size and extremely weaker and that the best group G has become bigger in size. A closer look shows that the group identified in the previous studies as the weakest has become still weaker.

Suggestions for improvements of equal opportunities are: universities have to bear in mind that the students can be distinguished in different types according to their educational / personal background. These types have to be identified to give consideration to the diversity of the students. The support of the students has to be individually adapted to the identified special groups.

The best effect of support programmes is expected within the small group without higher school education because they have the lowest mathematical knowledge but the highest motivation. The biggest challenge is the group of migrants because some of them seem to be less motivated and seem not to realise their problems.

Support programmes like bridging course are useful. In addition student tandems across the special groups should be installed.

Multiple Choice Tests Revisited

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Abstract

This paper brings some ideas about possible methods, and forms of testing and assessment of conceptual understanding in basic mathematical courses for engineering students. Multiple-choice tests represent a rather useful tool and a verified way of testing, an efficient and effective way to assess a wide range of knowledge, skills and abilities. Discussion on how this instrument can be developed and utilised in a modified way for testing basic mathematical understanding and knowledge necessary for a successful path through engineering study programmes is included in the paper, together with practical examples of multiple-choice tests used for testing mathematical abilities and conceptual understanding of engineering students at the Faculty of Mechanical Engineering at the Slovak University of Technology in Bratislava, Slovakia.

Introduction

Multiple-choice testing still remains one of the most commonly-used assessment formats allowing broad and deep coverage of content in a relatively efficient way. They are utilised mostly by students for self-assessment during semester in the preparatory phase for the examination period. Usually a collection of multiple-choice tests prepared by mathematics lecturers is presented to students in order to check their understanding of basic concepts and probable performance during examination. The main idea is to offer students a collection of easy, straightforward questions with a choice of 3-4 answers among which usually one is correct and one is completely not relevant. This instrument developed in this way serves students as an easy and focused training on how to recognise a correct response to questions concerning fundamental mathematical concepts and relations. This activity is more or less connected to a certain degree of formalisation and memorisation of basic mathematical knowledge. It could hardly lead seriously to building a sort of better conceptual understanding, and this assessment method is far from testing it. Students should learn to think and apply knowledge. Facts and procedures are necessary for thinking, but study should not be driven by multiple-choice testing into minimising or eliminating thinking and problem-solving.

How to Design Multiple-Choice Tests

Despite the rather negative aspects mentioned above, the idea of testing by choosing a correct answer from a given set of several possibilities is not bad at all. In real-life situations, both professional and personal, we are quite often forced to make decisions, which are important and designative, while almost always there are available several quite different solutions. Undoubtedly, it is our personal responsibility to analyse the logical consequences of our choice and expected outcomes of our decision, and to choose the most appropriate solution. Developing multiple-choice tests for mathematics one can consider to a certain degree general rules and guidelines available for the design of such instruments.

Multiple-choice tests should be designed through adopting the strategy of raising students´ awareness and concentration on the choice of correct answers. Many instructions are available on how to develop a quality tests. Most standardised tests, including state exams and most commercial achievement tests, are made up primarily of multiple-choice items. A few state tests have a quarter, a half or even more ‘open-ended’ or ‘constructed-response’ items, usually short answer questions. These ask students to write and also explain, not merely select an answer. Many short-answer questions are not much more than multiple-choice items without the answer options, and they share many of the limitations and problems of multiple-choice items. To develop useful and reliable multiple choice test, the author must first of all:

- Outline the core content that has to be covered
- Identify and prioritise key points and tasks
- Write out a series of stems - incomplete statements or questions
- Write keyed responses in a clear sentence that follows the format of the stems
- Develop 2 to 4 alternatives or distractors that follow the grammatical style and are consistent with the stem
- Mark the choice

For ideal test items and stems one should consider the following rules:

- Use simple, direct language to present core information for analysis, comparison or evaluation, etc., and avoid cleverness, trickery, or verbal complexity
- Include as much of the item as possible in the stem, but avoid repeated information and briefer alternatives
- Present unique content not built upon other questions and do not supply answers to other questions
- Avoid negative stems, if they are not necessary
- Qualify significant information at the beginning of the stem
- Do not introduce unfamiliar vocabulary and concepts in the test
- Avoid generalisations that are open to interpretation
- Use the number of alternatives appropriate to a test item throughout the test, generally three to five (not necessarily a consistent number throughout the test)
- Sequence alternatives in logical or numerical order.

  Should there be no order, randomly assign correct answers in the sequence
- List alternatives on separate lines, indent, separate by a blank line, use letters or numbers for alternative answers
- Pay attention to grammatical consistency of all alternatives

Several ideas are presented about new forms and structure of this testing instrument, with its possible innovative usage in assessment of conceptual understanding.

New Design Approach

A multiple-choice test usually has dozens of questions. For each question, the test-taker is supposed to select the best choice among a set of two to five options. These types of tests are called selected-response tests. Test-makers often promote multiple-choice tests as objective, as there is no human judgement in the scoring, and this can be done mechanically, even by a machine. However, humans decide what questions to ask, how to phrase questions, and what distractors to use. All these are subjective decisions that can be biased; therefore, multiple-choice tests cannot be claimed as really objective.

It is possible to get multiple-choice items correct without knowing much or doing any real thinking, and because the answers are in front of the student, some people call these tests 'multiple-guess'. In this respect multiple-choice items can be easier than open-ended questions asking the same thing. The reason is very simple - it is harder to recall an answer than to recognise it. Test-wise students know that it is sometimes easier to work backwards from the answer options, looking for the one that best fits. It is also possible to choose the correct answer for the wrong reason or to simply make a lucky guess. One strategic change can be applied to the general evaluation criteria - receiving positive points for correct answers. For example, all incorrect hits can be rewarded by negative points. In this way, any risk and unreasonable guess might be eliminated, as minus points influence the total test score rather negatively. The total balance of points achieved then represents a more objective view, and gives a better insight to the real understanding of the core mathematical concepts in the broad sense.

Multiple-choice items are best used for checking whether students have learned facts and routine procedures with clearly correct answers. In some subjects, and mathematics in particular, carefully written multiple-choice items with good distractors can fairly accurately distinguish students who grasp a basic concept from those who do not. In this context, another idea for test design can be adopted, which is to include questions with the choice of answers that might be all incorrect, or a set of more than one correct answer available for one question, while the total number of correct answers in the whole test is given. Such a structure demands more responsibility while making the choice. It is naturally forcing students to analyse carefully in/correctness of all presented distracters in order to make a truly correct choice, not to take a risk and lose points.

Questions must be well structured, focused on basic mathematical concepts, their properties and relations, which can lead to more straightforward way of reasoning while finding a correct answer. Generally, it is rather questionable whether multiple-choice tests can be useful for measuring how deeply students can analyse material. Additional questions might arise and can be posed to and by students in connection to post-analysis, with demands of possible re-formulation of incorrect answers appearing in the test. This feedback and natural discussion during the final evaluation of the test results might lead to further study and consequent deeper knowledge and conceptual understanding of the basic concepts.

Most researchers agree that multiple-choice items are poor tools for measuring the ability to synthesise and evaluate information or apply knowledge to complex problems. In mathematics, for example, they can measure knowledge of basic facts and the ability to apply standard procedures and rules. Carefully-written multiple-choice questions also can measure somewhat more complex mathematical knowledge such as integrating information or deciding which mathematical procedures have to be used to solve problems. Multiple-choice items are efficient in checking on factual declarative knowledge and routine procedures. However, as students move toward solving non-routine problems, analysing, interpreting, and making mathematical arguments, multiple-choice questions are not useful. This testing instrument is not appropriate for assessing critical or higher order thinking in mathematical subjects, the ability to apply knowledge or to solve individually mathematical or applied problems.

Examples, Findings and Discussion

Several examples of multiple-choice tests are presented here which are used (in addition to other practised forms of assessments) for testing conceptual understanding in basic courses Mathematics I and II for bachelor students at the Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava. Students have to pass 2 exams, both in one academic year, one per semester. These consist of a written practical test with 4 problems to solve (60% of total score), a multiple-choice test with 10 questions covering the whole scope of respective subject (20% of total score), and an oral examination. Performance of students during semesters is also evaluated: 20% of total points can be achieved by solving individual projects delivered during semesters, writing two short practical tests, and by active participation at practical exercises.

The mathematics multiple-choice test is a collection of 10 questions from different parts of mathematics covered by basic courses Mathematics I and Mathematics II for the bachelor study programmes in Mechanical Engineering. Each question is presented with 4 answers to choose from. Test-takers have to consider correctness of all 40 posed distractors, as there are exactly 20 correct ones, distributed randomly among the 10 questions they have to answer. Therefore, in some stems, no correct answer is available while in some other ones there can be more than one correct answer to mark. Each correctly-chosen answer is awarded one point, while each answer marked ‘incorrect’ is awarded minus one point. The total maximal score in the test is 20 points. Under the pressure of these rules, students tend to avoid any risk and adopt the strategy of marking only those answers, about which they are absolutely sure and do not hesitate. This strategy reflects the basic attitude of test-takers to pass the assessment with the maximal possible score, on average without any guess, which might have a derogatory effect on the score. Students tend to mark only those answers about which they have no doubts and are sure about the correctness of their choice. It seems that rather than make a guess and mark a possible incorrect distractor, students decide not to mark any, and to receive no minus points in their overall score. Those test-takers, who have serious problems with their study discipline and can see a fundamental insufficiency in their knowledge of basic mathematics, are more likely adopt the multiple-guess strategy, but their test scores are with higher probability convergent to zero or even negative.

Example 1. Question from linear algebra (1 correct answer)

Homogeneous linear system of equations
a) has always an infinite number of solutions
b) has always a unique solution
c) can have no solution
d) has always at least one solution

score	+	0	-
1st exam students	25	4	15
2nd exam students	13	27	3
3rd exam students	15	25	3

Example 2. Question from functions of one real variable (no correct answer)

A real function of one real variable is a one-to-one (injective) function, if
a) it is a monotone function
b) it is a bounded function
c) any line in direction of *y*-axis intersects the function graph in a single point
d) any line in direction of *x*-axis intersects the function graph in a single point

score	+ no answer	0	-
1st exam students	21		23
2nd exam students	27		16
3rd exam students	34		9

Example 3. Question from differential calculus (2 correct answers)

If $f''(x) < 0$ for all x in an interval of real numbers, function $f(x)$ is on this interval
a) differentiable
b) decreasing
c) convex
d) continuous

score	+2 / +1	0	-
1st exam students	14	7	23
2nd exam students	17	9	17
3rd exam students	21	14	8

Example 4. Question from integral calculus (3 correct answers)

The area of an elementary region $R = \{[x, y] \in R^2 : a \le x \le b, g(x) \le y \le f(x)\}$ can be calculated using the formula

a) $A(R) = \int_a^b (g(x) - f(x))dx$

b) $A(R)=\int_a^b (f(x)-g(x))dx$

c) $A(R)=\int_a^b f(x)dx-\int_a^b g(x)dx$

d) $A(x)=\int_a^b |g(x)-f(x)|dx$

score	+3/+2/+1	0	-
1st exam students	3/25/9	5	2
2nd exam students	0/22/3	12	6
3rd exam students	2/27/11	2	1

Conclusions

Multiple-choice tests for testing basic mathematical factual and procedural knowledge have been chosen in order to obtain overview of what students really know from the subject of basic mathematics. The aim was to test how they recognise and understand basic mathematical concepts and relations, and how skilful they are in applying this knowledge finding correct responses to direct questions related to essential fundamentals of mathematics necessary to prospective engineering experts. Test results are important from the pedagogical point of view in terms of feedback, which both students and teachers receive when analysing them. There is no intention to concentrate mathematics teaching on instructions reduced to ‘drill and kill’, but to train students to be able to apply mathematics meaningfully in their specialisations. Steady knowledge of fundamental principles and concepts is necessary in order to ‘know about possible procedures and instruments available in mathematics’ that might help to solve applied problems. Students deserve their chance to ask own questions, to have doubts and lead discussions about them to clarify things, to read and challenge instructional texts, to conduct experiments, to write extended papers, to explore and invent new ideas. But all these activities depend on firm fundamental knowledge, which is irreplaceable if one really wants to learn a subject. Without prior knowledge and awareness of available mathematical tools, there is no application possible.

References

Multiple-choice tests. Study Guides and Strategies. Available online at the address: http://www.studygs.net/tsttak3.htm

Parkes, J. Multiple-choice tests. Available online at the address: http://www.flaguide.org/cat/mutiplechoicetest/multiple_choice_test1.php

Sadler, P. M. (1998) ‘Psychometric Models of Student Conceptions in Science’, *Journal of Research in Science Teaching*. V. 35, N. 3, pp. 265-296.

Stiggins, R. J., Rubel, E., Quellmalz, E. (1988). *Measuring Thinking Skills in the Classroom.* NEA Professional Library in Washington, D.C.