The Mathematical Education of Engineers

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The Mathematical Expertise of Mechanical Engineers – The Case of Machine Element Dimensioning

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Abstract

This contribution reports on a project that tries to capture the mathematical expertise a mechanical engineer needs in his or her daily work. We study how mechanical engineering students in their final semester work on typical tasks. The task considered in this article is concerned with machine element dimensioning in a typical gearing mechanism using an industry standard dimensioning program. One major competency we identified was "keeping track" in a large algebraic model containing a considerable number of variables. Coming to an initial design based on estimations and making variations for improvement based on mathematical and non-mathematical arguments is essential for efficient and effective work.

Introduction

The mathematical education of engineers should enable students to use mathematical concepts, models and procedures for solving daily problems in their "engineering life". For a sound educational provision we therefore need a better understanding of the role mathematics plays or might play in such daily life problems. For non-academic professions, there are several investigations of the usage of mathematics in comparison with what is taught at school (for an overview see Gainsburg (2005)). Only recently has this research been extended to engineering professions where it is much harder for a non-professional to understand the work and the role of mathematical thinking. Kent and Noss (2002) and Gainsburg (2006) investigated civil engineers and Cardella and Atman (2005, 2006) observed industrial engineering students doing their capstone projects. Using an ethnographic qualitative method of research (for engineers and students, respectively), they discovered several aspects and patterns of mathematical thinking. The work described in this contribution is concerned with the mathematical expertise of mechanical engineers. Taking an approach similar to that of Cardella and Atman, we investigate how students work on typical practical tasks. The approach and results on typical static design and mechanism design tasks can be found in Alpers (2006, 2007). Here, we report our findings when students worked on a typical machine element dimensioning task. In the next section we briefly recap the method of investigation and present the task. We then describe the principal approach of the students and discuss the findings concerning the mathematical thinking processes. Finally, we draw some conclusions for the mathematical education of mechanical engineers.

Method of Investigation

Two students in their final semester of study are given a practical task which is identified by a colleague teaching machine elements, CAD and FEM who worked in the car industry for several years. They are paid for working 100 hours on the task and they are asked to document their work process. They can seek advice from the colleague who plays the role of a group leader. The author investigates the documents of the students and conducts interviews with the students and the colleague in order to detect necessary mathematical knowledge as well as situations where a more mathematical approach might have been more efficient. The interview with the colleague also serves to check whether the work done by the students resembles the work of a junior engineer in industry and thus to recognise the limitations of the approach.

The task within this project deals with machine element dimensioning. The students are to dimension the gearing mechanism sketched in Figure 1 where an input rotational velocity of 3000 rpm (revolutions per minute) has to be transformed to approximately 1200 rpm. The input power amounts to 15 KW, the distance between the shafts is approximately 200mm and the life expectancy should exceed 10000 hrs.

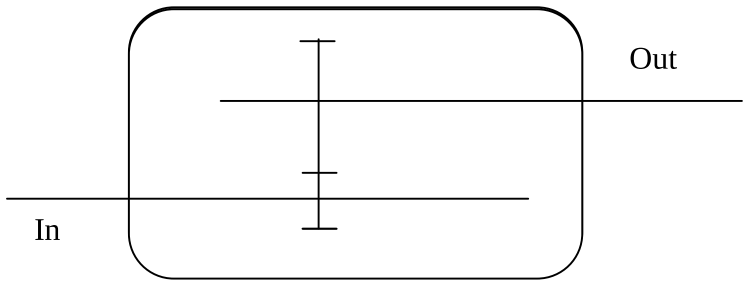

Figure 1. Sketch of Gearing Mechanism.

Approach of the Students

The essential steps for dimensioning the transformation mechanism are quite straight forward. First, the gears have to be dimensioned such that the requirements concerning transformation (3000 to 1200 rpm), distance of shafts and stiffness (see below) are met. Then, the loads for the shafts are known and the latter can be dimensioned correspondingly. Finally, the bearings for the shafts have to be sized and chosen from catalogues. Procedures and formulae for all these subtasks are written down in text books on machine elements. The students used the book by Jahnasch (2007) which is widely used at German universities and in industry as well. For performing the computations there are three modules in the machine element computation program MDesign® which is also quite widespread. The students also performed computations by hand (and pocket calculator) for the following reasons:

- for providing input data for MDesign®;
- in order to understand or repeat the computation since MDesign® just provides results;
- to check the results of MDesign® mainly for detecting errors in input values.

According to the colleague, a senior engineer would have known the computational scheme but a junior engineer would be likely to first look it up in a book. A senior engineer would perform computations by hand if the results of a program seemed to be strange.

The computational schemes can be looked up in Jahnasch (2007). In what follows we outline one major scheme that is concerned with the stiffness of the gear wheels and is typical also for other stiffness computations. There are two interesting damage situations: a tooth might break at its bottom or the tooth flank might be damaged by pressure. The computational models used for dimensioning and analysis are structurally the same. On the one hand, the allowed stress is successively computed starting with an allowed stress for the material used. Then there are five factors modelling different aspects including the geometry and the surface quality. These factors have been gained by experiments or experience and are tabulated. The product of all these quantities then is the allowed stress for the component (bottom or flank). Similarly, in order to compute the actual stress, one starts with a basic stress where the tangential force and some geometric quantities are used. Then, there are eight factors modelling influences of the geometry and of the application situation. Again, these factors have partially been gained by experiments or experience. Moreover, some of them can be computed by going into more detailed models which also contain geometric properties. The product of all these quantities is the actual stress. Finally, the quotient of both products is taken in order to compute the so called safety factor. For a non-mathematician, this is quite a complex algebraic (multiplicative) model where the quantities used are often estimated and uncertain.

Findings and Discussion

In this section we outline the essential findings concerning the use of mathematics and the necessary mathematical knowledge. We also relate the findings to the results of Gainsburg (2006) and Cardella and Atman (2005).

When working on the task and using the program the students had to have in mind the computational models for adequate dimensioning. They did not have to set up the models but the latter were given in text books and also – in a more sophisticated form – in guidelines of the German Institute for Standardisation (DIN). The models are based on many experiments and experiences and consequently the practising engineer does not know the exact reasons for all the assumptions leading to the model. Yet, he or she has to use it and work thoroughly through the sometimes lengthy computational algorithms.

The most sophisticated model outlined in the previous section is algebraic in nature. Its complexity stems mainly from the large number of variables the influence of which cannot always be seen immediately. It is essentially a multiplicative model where basic values are transformed into real values (of actual stress and bearable stress) by multiplying factors. The quotient provides the safety factor. Engineers should understand this basic algebraic structure in order to work effectively and efficiently with the model. It gives insight into the effects of potential variations. Since the model contains several different quantities representing stress, “keeping track” (Gainsburg (2006)) is a major challenge.

The values of many variables must be estimated based on the application of the mechanism which necessarily results in uncertainty. Estimation and uncertainty were identified by Cardella and Atman (2005) as well as Gainsburg (2006) as being characteristic in engineering work. Uncertainty is to be taken into account when interpreting the results of computations in the models. Safety factors must not be considered as precise quantities. When a safety factor equals 1 that does not mean that the mechanism is safe. It makes sense to compute variations for different estimations in order to check the influence on the safety factor. The requirements on safety factors cannot be justified mathematically but must be negotiated. This influence of the "social context" has already been stated by Gainsburg (2006).

When working in a model with many variables, the first problem is to get a reasonable initial design which is then transformed into an acceptable solution in several iteration cycles. Experience, rough calculations and rules of thumb play a major role in getting such an initial design. According to the colleague involved in the project there are many rough models for standard engineering tasks. When making variations of the initial design, the students often did not go back to the mathematical model but used their qualitative knowledge and the program for getting quick results. There was no optimisation in a mathematical sense but rather iterative improvement that was also detected by Gainsburg (2006) and Cardella and Atman (2005). According to the colleague, this is not always the case in real engineering life. Particularly in larger serial production, it is quite important to achieve the "final five percent" of improvement, and for this the underlying model has to be investigated thoroughly for finding additional gains. This shows that in practice the tasks and requirements differ considerably depending on the application conditions.

Beside the model for computing safety factors one also has to use a mechanical model for determining the forces in order to dimension the shafts and the bearings properly. Here, the students were insecure and made – unknowingly – several simplifications which still led to an acceptable design. Yet, in situations where the "final five percent" are important, engineers should know where they simplify and be able to change to more detailed and precise models.

The investigation of program usage showed that it is by no means possible to simply delegate the work to the machine element computation program. Often, input values have to be determined by hand in simple models. There are different program parts for the computation of gears, shafts and bearings, and to go from one part to the other requires data adaptation and computation of further data. Moreover, erroneous input is very likely to occur. Without having a good understanding of the underlying mechanical model it is hard to avoid errors.

As was emphasized by Gainsburg (2006), there were many extra-mathematical aspects that had to be taken into account. The goal of the design consists of finding a functional **and** cost-efficient solution. For the latter goal, logistic and production-related aspects play a very important role. Moreover, experience available in the company is also an essential argument for favouring certain alternatives over others. These factors can hardly be included in a mathematical optimisation model, so even if the current state of machine element computation programs will improve, one cannot expect a simple delegation of such a design task to a program.

Finally, we want to discuss the merits and limitations of our approach. The task that was suggested by the colleague seems to be quite realistic and resembles other machine element dimensioning tasks that can be seen in books on machine elements like Jahnasch (2007). By chance, the author recently supervised a diploma thesis performed by one of the students in a company producing axles and gears and similar tasks came up. Yet, there is also a limitation in that there was no “unforeseen” new application situation resulting in the necessity of model adaptation or development. The latter was recognised by Gainsburg (2006) when observing structural engineers. In our approach we cannot investigate the behaviour in such situations. There simply is no rich environment with many sources of information influencing the design process. When the students needed more information (for example, finding values for the many factors), they sometimes made (justifiable) assumptions on their own and sometimes asked the colleague acting as a mentor. Without a real application background it was also hard to decide when the work was finished, so there was no real incentive for improvement by iteration. Given the timeframe of 100 hours, the students made only very few variations. A further limitation of the method is that one can just observe behaviour similar to the one of junior engineers. We cannot capture a realistic dialogue between junior and senior engineers as was observed by Gainsburg (2006). When the students had problems we could at least ask the colleague whether the problems were specific to the students or might also be seen in real engineering life.

Beside the limitations there are also clear advantages of the method. Having the students “at hand” permits extensive interviews and gives them opportunity to show their program usage. For a mathematician or a math educator, it is not easy to understand the work of mechanical engineers. Understanding is also an iterative process. Therefore, it is very helpful when the students and the colleague are available for questions many times. This is much harder to achieve in a real company where explanations to researchers mean loss of hours billable to the client. If one does not have the opportunity to obtain a deeper understanding of the engineering task it is hardly possible to investigate whether at some stages a more mathematically oriented approach would have made the work more effective and efficient.

Conclusions for Education

One should be very careful when drawing conclusions concerning mathematical education since the observations are restricted to a few tasks. Moreover, mathematical education not only aims at providing the mathematical expertise needed in later jobs but also has to provide the mathematical concepts and procedures needed in application subjects like mechanics or control theory. Nevertheless, the results of the investigation suggest some changes or shifts in emphasis.

Modelling and working with models plays an important role for efficient work. So, setting up models and solving problems with models should be an essential part of engineering education. It is not clear, though, how the educational efforts should be distributed. Several application subjects in mechanical engineering (e.g. engineering mechanics, machine dynamics, control theory) deal mainly with developing mathematical models and solving problems. Working with such models is virtually one of the main differences between technicians/craftsmen on the one hand and engineers on the other hand. Whatever the distribution might be, it is certainly reasonable to work with models in the mathematical education and to discuss the problem of estimation and uncertainty. Methods for estimating faults resulting from insecure input data are also important. Moreover, an investigation of the influence of variables in models is important for efficient variation. Students should know the difference between qualitative improvement strategies on the one hand and mathematical optimisation models on the other hand. The merits of both approaches should be discussed in examples such that students gain a better understanding of when to use what. Mathematical application projects provide good learning opportunities in this respect. Here, estimations for application situations have to be made as is often the case in real engineering life.

The investigation has also shown the value of precise and diligent work using complex algorithms or schemes that are not fully understood. There are simply too many models and computational schemes, so for practical reasons engineers are eager to accept models set up by others particularly when they are established as guidelines or standards. So, even the often criticised traditional mathematical education solving problems using computational schemes which are only partially understood seems to have some value.

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The Work of the SEFI Mathematics Working Group

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Abstract

The paper describes the work and activities of the SEFI Mathematics Working Group (MWG). The MWG is a group of academics from European universities; it advises upon the teaching of mathematics to engineering students and the development of the curriculum. Its current focus is engineering mathematics education within Bachelors and Masters degree programmes following the Bologna Agreement, and examining the highly varied forms of assessment across institutions and countries.

Introduction

The European Society for Engineering Education, SEFI (Société Européenne pour la Formation des Ingenieurs), was established in Brussels in 1973 to monitor the formation of professional engineers within European institutions. It holds an annual conference but most of its work is undertaken by working groups dedicated to specific needs. The Mathematics Working Group (SEFI-MWG) was set up in 1982, initially drawing delegates from post-war Western Europe but from 1990 nearly all countries from Finisterre to the Urals came to be represented. Working 3-day seminars are held at approximately two-year intervals to progress the Group’s work and a smaller executive sub-committee called the Steering Committee, meeting more regularly, coordinates the efforts meantime.

The SEFI-MWG was set up with the following aims:

- To provide a forum for the exchange of views and ideas among those interested in engineering mathematics
- To promote a fuller understanding of the role of mathematics in the engineering curriculum, and its relevance to industrial needs
- To foster cooperation in the development of courses and support material
- To recognise and promote the role of mathematics in the continuing education of engineers in collaboration with industry

To this one needs to add the following tasks, achievements and subsequent roles of the SEFI-MWG as these have evolved over the Group’s 25-year existence:

The production of an advisory core curriculum in mathematics for the formation of the professional engineer within universities in the developed world (Curricula published in 1992/2002)

The provision of an international forum for the exchange and development of ideas and practice in the teaching of mathematics to engineers

The establishment of international initiatives to compare and contrast different practices in converting teaching into effective learning (Assessment Project)

The monitoring and adaptation of widely varying curricular practice into a common internationally accepted form (Bologna Agreement)

The Evolution of the Core Curriculum

The Seminars in the 1980s enabled the widely different practices in engineer formation across Western Europe to be fully aired and possibly for the first time, to be mutually understood. There is general agreement that the specialisations of engineering; for example, aeronautical, electrical, mechanical, marine, nuclear, etc have a distinct meaning to members of the public in the developed world but if one asked what a fully qualified professional engineer actually might do on a day-to-day basis there is a much less clear view than there might be over the duties of other professionals such as those in medicine or the law. Also, in the scientific community in many continental European countries some applied mathematicians and physicists are very often termed engineers. This has meant that in specifying a core curriculum of mathematical study the needs of more mathematically orientated professionals has been an issue.

Beginning with the 5th Seminar held in Plymouth, UK in 1988, sub-groups of the SEFI-MWG set about defining an advisory curriculum of flexible length ranging from 220 to 320 hours aimed at appropriate institutions and countries. This was published in English, French and German by SEFI in 1992 (Barry & Steele (1992)). The content comprised:

- Analysis and Calculus
- Linear Algebra
- Discrete Mathematics
- Probability and Statistics

in the proportions of about one half for the analysis and calculus and about one sixth each for the others, together with a proviso that numerical methods be infused within the curriculum. Prerequisite mathematical study, i.e. highschool mathematics, was specified and overlying the curriculum elective study in appropriate areas of mathematics was detailed for the differing specialisations of engineer. Introducing the curriculum was a discussion and commentary which recognised that so-called ‘high technology’ is really a mathematical technology and noted that advances in computation would limit the validity of the document to a 10-year lifespan.

A revised core curriculum document was published by SEFI in 2002 (Mustoe & Lawson (2002)). In many respects this retained the main features of the earlier document but the concept of the Core as well as the above four components was extended into the underlying and overlying material. New features were the inclusion of Geometry as a specific topic and the special emphasis placed upon student learning outcomes. Also teaching practice in Central and Eastern Europe received more importance. The prerequisites are now termed Core Zero; the core itself is split into two hierarchical tiers, Core One and Core Two and the electives called Core Three. During the 1990s serious concern over the mathematical fitness and changing background of new entry engineering students emerged, firstly in the United Kingdom, but later on in many other countries. The underlying reasons appear to be cultural as well as educational but the impact on the curriculum has been that Core Zero material has encroached upon university mathematics. Also, the time allocated to teaching mathematics, i.e. the Core itself, came under pressure in many institutions and countries as engineering programmes evolved.

The Evolving Focus of the Seminars

The 6th Seminar in Balatonfured, Hungary in 1991 brought in Central and Eastern European delegates and further exchanges of the wide disposition in engineering mathematics education. Later in the 1990s focus came in special out-of-sequence seminars on areas of need, notably statistics at the Prague Seminar in 1994 and geometry at the Bratislava Seminar in 1997. By the 9th Seminar in Helsinki, Finland in 1998 the decline in entry competency was widely reported from within Europe and beyond. Emphasis too went to considering the study of the Core as the main component of lifelong learning. The widespread use of computer technology was emerging as an issue by this time, and whilst welcome in its role in the removal of drudgery in calculations delegates reported that considerable care was needed not to trivialise its use, notably with computer algebra. Interest too was taken in looking forward to a time when computational capacity would become almost optimal and whether or not it would be possible to define an irreducible core of mathematical knowledge that an engineer would need to have irrespective of computational advances. From the mid 1990s and onwards delegates came from outside Europe, notably from Argentina, Australia and the USA. The same themes were revisited at the 10th and 11th Seminars in Miskolc, Hungary in 2000 (SEFI MWG (2000)), and Gothenburg, Sweden in 2002 (SEFI MWG (2002)). With learning outcomes becoming paramount emphasis was moving to assessment, a theme that would play a major part in the subsequent and present activities of the SEFI-MWG.

Recent Activities

The SEFI-MWG has a reputation for doing rather than talking and its early seminars in the 1980s placed strong emphasis on sub-group discussions, such as the ones on analysis and calculus, discrete mathematics etc that led to the four main components of the Core Curriculum. Many of these sub-groups made considerable progress, for example by writing questionnaires, and produced valuable interim reports that were carefully distilled as input into larger reports such as the Core Curricula. In the early 1990s there was a move towards more presented papers mainly to draw in Eastern European delegates who had to present a paper in order to receive financial support to attend. From 2000 onwards a way was found to move the balance back to the earlier model, i.e. round table working. Activities within the SEFI-MWG are currently concentrated on the Bologna Agreement and the Assessment Project and meanwhile valuable debates have taken place at the most recent seminars, namely the 12th Seminar in Vienna, Austria in 2004 (SEFI MWG (2004)), and the 13th Seminar at Kongsberg, Norway in 2006 (SEFI MWG (2006)).

The Bologna Agreement

The Bologna Agreement calls for Bachelor programmes of 180 European Credits (ECTS), taken over about 3 years or 6/7 semesters, followed by 120 ECTS up to masters level, or about 2 years or 4 semesters.

The decision to opt for this was dictated by the aim of transference in academic study between European institutions. This might be good in principle but might be inapplicable and inconsistent with the programmes that some institutions offer. However other institutions are moving this way and some countries such as Belgium are more committed than others. SEFI-MWG delegates however are worried about what Bachelor qualifications might come to mean noting that there could be a wide variety of levels and that mathematics might be put under pressure and reduced. It would also risk being undermined at the lower end of the curriculum with an increasing amount of 'levelling-up' Core Zero material. Some however have commented that Bologna could work if given time to bed in but there may be a need to distinguish between those Bachelor programmes that naturally lead on to Master programmes and research and others that constitute an exit pathway, i.e. an end of formal academic study.

Delegates were asked to go away from the 13th Seminar and address the mathematical curriculum requirements of a Bachelor programme. There are two considerations to be accounted for initially. Firstly, as the start point level of mathematical study seems to get lower with every passing year, this would need to be rationalised in terms of Core Zero and Core One of the Core Curriculum. Also, the endpoint of study would come before the end of Core Two: this too would need to be rationalised in terms of the type of programme and engineering discipline. What the Group might do, is define curricula for Bachelor Type A, (i.e. proceeding to masters) and Bachelor Type B (terminating). Less academically able students might go for Type B and it might be more open as to the start level of what such a mathematics programme should be.

The Assessment Project

In 2005 the SEFI-MWG began looking into the many different assessment models that operate across the many institutions and countries in Europe. The written examination appears internationally to be a well-tried, tested, and administratively proven model for assessment in many subjects. The UK prefers 3-hour exams whilst other countries have examinations up to 5 hours (Norway) or as little as 1.5 hours, perhaps supported by an oral examination (Central Europe). In many institutions large numbers of students dictate that oral exams are unaffordable at lower undergraduate level in terms of time and effort, as are the assessment, trustworthiness and reliability of coursework or take-away assignments. There is however much respect for assessment via assignment for project and other work at higher undergraduate levels. Learning outcomes spell out precisely what the student should be able to do having covered a particular element in a unit though the final examination cannot and maybe should not necessarily test all of these. Rather more specific learning outcomes can be assessed by formative testing. Delegates in Kongsberg spoke about the use of computerised tests to reinforce learning and gave excellent examples of the assessment of concepts with multiple-choice questions. There is a strong feeling that such precision testing be used carefully and proportionately in the formative phase and that students should have full powers of expression and explanation in their final summative examination. Whatever form of assessment is adopted all assessors agree that the main aim of assessment is to measure the ability of students to communicate their mathematics most efficiently and effectively.

The next stage for the SEFI-MWG Assessment Project is to compare actual assessments from different countries and institutions across Europe. This is expected to start with a focus on first year calculus and linear algebra in programmes for engineers.

Future Investigations

In addition to discussing Assessment and the Bologna Agreement delegates at the recent seminars have also been looking at the following:

The key issues in teaching engineering mathematics for understanding.
Students need to understand mathematics as a language of scientific communication. Their understanding needs to be robust and versatile enough to cope with the unpredictable challenges of being a practising engineer.
Good teaching facilitates learning, irons out misconception, and enables students of diverse mathematical background to reach a common minimal level of understanding.
The goals of students are often short-term for a subject such as mathematics and maybe assessment orientated, i.e. they seek just enough knowledge and guidance to pass a specific examination. They can thus risk developing only a surface learning of the subject and need to deepen this by dealing with appropriately chosen mathematical challenges within an engineering context.

Innovative ways of teaching mathematics.
New undergraduates can experience a considerable culture shock when they reach university and find themselves in large classes for lectures.
All participating institutions in the SEFI-MWG use Information Technology (IT) to some extent and have moved forward from the view that its introduction is to reduce staff involvement. Rather IT, and Web-based technology enhance the quality of delivery.
A new level of support has emerged in an environment of popular culture where e/mail is the most common form of academic communication.
There is some evidence however that ill-considered computer aided teaching and

assessment can cause frustration and anxiety so care needs to be taken to measure carefully the quality of electronically available material.

The 14th Seminar in Loughborough, UK in April 2008

The current Seminar being held at Loughborough University, United Kingdom on 6th to 9th April 2008 is a joint event. This is being held with the Institute of Mathematics and its Applications (IMA) that will be holding its 6th Conference on the Mathematical Education of Engineers. The IMA conferences, held since 1994, have paralleled the SEFI-MWG seminars in many respects whilst focusing on UK related issues and in particular setting in train reports and measures to counterbalance the declining mathematical preparedness of engineering students.

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Attracting the best students of mathematics into engineering

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Abstract

In late spring 2007, Mathematics in Education and Industry (MEI) and The Institution of Engineering and Technology (IET) held a joint invitation conference entitled *Attracting the best students of mathematics into engineering*.
This paper describes that conference and explains: how it came about; what its purpose was; who the delegates were, and how they were identified for invitation. It then describes some of the key suggestions that were made and how MEI and the IET have taken these forward. Throughout, the focus will be on how the actions following the conference link to the mathematical education of engineers in school/college, university and the workplace.
Suggestions from the participants were that MEI's skills and expertise in mathematics education, including management of the Further Mathematics Network, could be used to the benefit of university students and employers alike. Specific avenues in which this could be pursued will be detailed in the paper, these include: supporting engineering departments with tailored resources, supporting employers in delivering mathematics training, and supporting the teaching and learning of the forthcoming Engineering Diploma.
The central role that the Further Mathematics Network can play in helping to support a drive to attract more of the best young mathematics students to take up engineering will also be made clear.

Introduction

In May 2007, Mathematics in Education and Industry (MEI) and The Institution of Engineering and Technology (IET) held a joint invitation conference entitled *Attracting the best students of mathematics into engineering*. The origins of this conference lie in the synergy between the two bodies involved.

MEI is a curriculum development body, promoting approaches to school mathematics that make the subject interesting in its own right and fostering skills and knowledge that are relevant to its use in the workplace.

The IET is one of the world's leading professional societies for the engineering and technology community. The Institution provides a global knowledge network to facilitate the exchange of knowledge and ideas and promotes the positive role of Science, Technology, Engineering and Mathematics in the world. Through its Education 5-19 Department, the Institution supports the teaching of STEM subjects with a wide range of resources, activities and training for teachers.

Among MEI's various programmes is the Further Mathematics Network (FMN) which it manages for the government; it provides many young people with the opportunity to take AS or A level Further Mathematics in addition to the standard Mathematics A level. The network is proving very successful with large numbers of students embracing this new opportunity; however, a review of their subsequent choice of university courses revealed that very few were progressing to read engineering. This was a surprising result; it seems that engineering is no longer attracting its share of the best students. The reasons for this, and how they can be addressed, provided major themes for the conference.

This concern is echoed in a number of recent reports.

An HM Treasury report, HM Treasury (2005), had commented on ‘the importance of a strong supply of scientists, engineers and technologists to the long-term health of the science base and the wider UK economy’.

A related issue was highlighted in the Institution of Engineering and Technology *2007 Skills Survey*, IET (2007), which found that the engineering and technology sector is facing a growing recruitment crisis, and that there is little confidence in the situation improving in the short or medium term. The annual survey of 500 companies found that, in 2007, 52% of businesses expect to face difficulties in recruiting adequate suitably qualified engineers, technicians or technologists, compared with 40% in 2006.

Similarly, a Royal Academy of Engineering report, Royal Academy of Engineering (2006), found that ‘over the next 10 years there will be a worsening shortage of high calibre UK engineering graduates going into industry. This shortage will impact the productivity and creativity of UK-based businesses unless it can be addressed’.

Finally, the ETB, ETB (2007), noted that: ‘Volumes of engineering and technology undergraduates have remained stable over the last six years at just under 100,000. However, the proportion of undergraduate and post-graduate students studying engineering and technology subjects has declined from just over 9% to just over 6% in this period.’

The conference

Identifying the delegates

Most of those invited fell into one of a number of categories: significant stakeholders from both academic and industrial branches of the engineering community, including people with an interest in educational issues; contacts in the engineering institutions and other central bodies; engineers working in industry; guidance specialists from organisations involved in careers education, both nationally and in some regions; mathematics educators with an interest in engineering; and FMN regional centre managers.

An outline of the programme

The morning's programme contained five interesting presentations which highlighted aspects of the situation. Dr Nina Skorupska from RWE npower indicated some of the issues facing industry in recruiting high quality engineers. Dawn Ohlson from Thales set out how she herself had used mathematics as an engineer at various stages of her career (it turned out that in a defence environment, tools from A Level Mathematics and Further Mathematics were the most useful). Dr Richard Pike of the Royal Society of Chemistry spoke about relative ability in problem solving of Chinese and UK students. Prof Peter Hicks discussed the advantage to engineering undergraduates of possessing a qualification in AS or A level Further Mathematics. Finally, Charlie Stripp described how the Further Mathematics Network had come into being, and why it had made such an impact on Further Mathematics examination entries.

After lunch, delegates divided into groups to discuss the following key questions. How might university engineering departments make use of the FMN and how might the FMN make use of university engineering departments? How might employers benefit from their local FMN Centres and how might FMN Centres benefit from local employers? How might careers guidance specialists encourage greater take up of Further Mathematics and engineering?

After the breakout groups, feedback was given in a plenary session and then questions were posed to a panel consisting of speakers from the morning.

Summary of principal conclusions reached in the conference discussions

It would be helpful to increase the visibility of Further Mathematics as ideal preparation for Higher Education engineering courses. Some university departments could perhaps make Further Mathematics a course requirement. It is very important for departments to be honest and transparent in setting out both entry requirements and the demands of their courses.

Engineering departments and the FMN centres should collaborate. The Network could offer online resources to support undergraduates in learning mathematics.

We must involve the engineering industry in the process of awakening interest in engineering and raising expectations of what engineers can contribute to society. There is a need for good presentations directed at school students.

Participants also suggested ways of improving the mathematical competence of new engineers. MEI and the FMN could support employers in delivering mathematics training, whether this was for apprentices and technicians, for those on degree courses or for graduates who would benefit from mathematical development.

There were many expressions of disquiet about the careers advice and information given to young people about STEM related careers. In particular, careers advice in schools is often far from adequate in providing detailed information about opportunities in engineering.

Careers guidance specialists should be aware of entrance requirements for careers in engineering, including the vital importance of qualifications in Mathematics.

Conference report

Following the conference, a report was published in November 2007. The report may be found online at http://www.mei.org.uk/files/pdf/MEI_IET_Report.pdf or on the IET website www.theiet.org

The final section of the report contains an action plan for MEI and the IET. The majority of the proposed actions have been taken forward, and the final part of this paper discusses progress to date.

Taking the conference Action Plan forward

The Action Plan in the report was set out in four sections, each with a number of suggestions for action. This paper focuses principally on the first two sections, headed 'Higher Education', and 'Industry', and discusses progress to date.

Higher Education

In a paper written for the Department for Children, Schools and Families (DCSF) Maths Board in December 2007, Stripp (2007), Charlie Stripp observed that university teachers in engineering and the sciences could provide the encouragement for Further Mathematics that would ensure significant growth in its take up in schools and colleges. He pointed out that encouragement can be given without 'raising the bar' for entry, quoting the example from the university of Derby given below:

'*Further Mathematics AS/A2 is not a requirement for entry onto our programmes, but if you have the opportunity to take further maths at AS or A level, we strongly recommend it. We find that students who have taken these extra qualifications handle the transition from school or college to university very well. The University is prepared to be more flexible with students who have studied Further Mathematics but not met the standard offer.'*

He also emphasised that AS Further Mathematics, which can be studied in Year 12 or Year 13, is an excellent option for any student wishing to study engineering at university.

The IET and MEI/FMN have made significant progress in encouraging HE institutions to improve the clarity of their course requirements and offers. The FMN website now contains quotes from 16 universities indicating that they value Further Mathematics for particular courses. Unfortunately, by January 2008, only 6 of these included reference to engineering courses.

The FMN website may also be seen as one way of informing HE departments about the increases in numbers of potential students with qualifications in Further Mathematics, thus enabling more universities and departments to consider asking for these qualifications. FMN centres' management committees usually include university teachers as members, and these are well-placed to pass on positive messages to their colleagues. Charlie Stripp's presentation to a conference of Professors and Heads of Electrical Engineering in January 2008, emphasised these points.

MEI/FMN is producing a poster and associated exemplars that detail the applications of school mathematics in university degrees, including engineering. Using contextualised examples from undergraduate examinations enables students to see how the mathematics they study in their A levels can be applied to more advanced problems, some of which are in 'industrial' contexts.

MEI has begun to work with the Royal Academy of Engineering to support mathematical aspects of the Engineering Diploma. The diploma will include an Additional and Specialist Learning unit designed to provide sufficient mathematics content to permit students who take it to apply for engineering courses at university.

The FMN intends to support activities carried out by the Professors and Heads of Electrical Engineering to offer enrichment for students of A level Mathematics and Further Mathematics and inform teachers about how mathematics is used in electrical engineering. Similar support can be extended to other strands of HE engineering.

Industry

Working with STEMNET and local Setpoints, several FMN centres have made use of Science and Engineering Ambassadors to provide further mathematics students with direct information from young engineers about their career and their choice of degree courses. In many instances this has been part of a day designed to provide enrichment for a mathematics course, but in at least one case, the session was part of a day whose main purpose was to provide a revision session shortly before an A level examination.

MEI has sought to develop courses for mathematics in the workplace by seeking an opportunity to work with an industrial partner. Several possible ways for this collaboration to develop have been discussed. These include provision of suitable online learning resources for reference and study, support for mathematical aspects of in-house technical courses, and support for learning of more elementary mathematics where employees demand this.

The IET and MEI/FMN will consider holding a small-scale conference in 2008/9, to discuss how best we can influence careers guidance specialists.

Conclusion

The MEI/IET conference was held to find ways in which the IET, MEI and the FMN can collaborate to try to improve take up of university engineering courses and careers in engineering amongst the best students of mathematics in schools and colleges. The conference and its follow-up activities have begun to develop, on a small scale, a model of collaborative practice involving engineers in industry and HE, those working in central bodies, and mathematics educators principally in the FMN.

This approach is clearly comparable to that suggested by the ETB, ETB (2007): 'The findings of *Engineering UK 2007* clearly show the need to improve the supply of suitably qualified engineers and technicians in order to match the on-going and future anticipated demand for skills. To make this happen will need the involvement of, not only public sector interventions, but also employers, training providers, educational establishments – from primary school to FE and HE – professional bodies and associations, campaigning groups and other stakeholders with an interest in the success of engineering and technology, and in the part it must play in sustaining competitiveness and growth in the wider UK economy.'

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Which Mathematics Should We Teach Engineering Students? An Empirically-Grounded Case for Mathematical Thinking

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Abstract

While many engineering educators have proposed changes to the way that mathematics is taught to engineers, the focus has often been on mathematical content knowledge. Work from the mathematics education community suggests that it may be beneficial to consider a broader notion of mathematics: mathematical thinking. Schoenfeld identifies five aspects of mathematical thinking: the mathematics content knowledge we want engineering students to learn as well as problem solving strategies, use of resources, attitudes and practices. If we further consider the social and material resources available to students and the mathematical practices students engage in, we have a more complete understanding of the breadth of mathematics and mathematical thinking necessary for engineering practice. This paper further discusses each of these aspects of mathematical thinking and offers examples of mathematical thinking practices based in the authors' previous empirical studies of engineering students' and practitioners' uses of mathematics. The paper also offers insights to inform the teaching of mathematics to engineering students.

Introduction

Many engineering educators have proposed changes to the way that mathematics is taught to engineers: by having engineering professors teach mathematics rather than mathematics professors, by better integrating mathematics and engineering or by making large changes to the engineering curriculum. For example, Booth suggests uniting mathematics and engineering courses, or having mathematics and engineering educators collaborate in providing mathematics instruction to engineers. Her recommendations are based in a phenomenographic study of first year students' experiences learning mathematics (Booth (2004)). Similar to Booth's suggestion, Central Queensland University recognized the need for more dialogue between mathematicians and engineers, and established a group of three engineering academics and three mathematicians in order to generate greater awareness of the changes taking place in engineering education and the way mathematics can make a positive contribution to a changing engineering education (Fuller (2004)). Others suggest changing the mathematical education of engineers by coupling innovative mathematics courses with mathematical modeling courses in order to improve students' mathematical abilities (Gong et al. (2007)). These proposed changes vary in terms of their bases (e.g. based in teaching experiences vs. empirical studies), but the proposed changes generally share a commonality of focusing on mathematical content knowledge.

What should we teach? Mathematical Thinking

Work from the mathematics education community suggests that it may be beneficial to consider a broader notion of mathematics: mathematical thinking. Schoenfeld (1992) describes mathematical thinking as not only involving the mathematics content knowledge we want engineering students to learn, but also problem solving strategies, metacognitive processess, beliefs and affects, and practices. Schoenfeld refers to the mathematical content knowledge that is often the focus of mathematics courses, as well as efforts to improve the mathematical education of engineers, as the mathematical Knowledge Base. The second aspect that Schoenfeld discusses is Problem Solving Strategies, which he relates to the heuristics identified by Pólya (1946). Examples of heuristics or problem solving strategies include “decomposing and recombining” (Pólya (1946)) and “guess and verify” (Schoenfeld (1992)). Schoenfeld’s third aspect of mathematical thinking consists of metacognitive actitivities such as self- monitoring and planning that allow a problem solver to effectively use limited cognitive resources (such as the limited working memory that can hold approximately seven pieces of information at a time). In his fourth aspect of mathematical thinking, Schoenfeld recognizes that the beliefs that an individual holds or affects around mathematics and mathematical thinking will impact their use of their mathematical Knowledge Base or Problem Solving Strategies. Finally, Schoenfeld recognizes that the environment in which someone learns mathematics plays a large role in the extent to which they learn to engage in mathematical thinking. Educators might teach students Practices associated with “school-mathematics” or alternatively the Practices that are associated with expert mathematicians. An individual who learns the Practices of “school-mathematics” will develop a different perspective of what mathematics is compared with a student who learns the Practices of an expert mathematician. Schoenfelds’s approach to mathematics education also resonates with the Knowledge, Attitudes and Behavior (KAB) Approach that is common in medical education. The KAB approach suggests that a learner’s knowledge, attitudes and behaviors are dynamically related, such that each of these three aspects reflects and affects the other two (Schrader & Lawless (2004)).

Aspect	Definition/Description	Main Theoretical Source
Knowledge Base	Cognitive Resources: Mathematical Content Knowledge	Schoenfeld (1992)
Problem Solving Strategies	Global or local strategies learned from mathematics courses	Pólya (1946), Schoenfeld (1992)
Use of Resources	Social Resource: Peers, Experts Material Resources: textbooks, time, computers Use of Resources: metacognitive processes such as planning and monitoring	McGinn and Boote (2003) Schoenfeld (1992)
Beliefs and Affects	Beliefs about mathematics and one’s mathematical ability, Feelings towards mathematics, Emotions or feelings experienced	Schoenfeld (1992)
Mathematical Practices	Activities or actions that engineers or mathematicians engage in, or activities that involve mathematics.	Schoenfeld (1992); Cardella (2006)

Table 1: Aspects of Mathematical Thinking that could be included in the education of engineers

Schoenfeld’s discussion of mathematical thinking illuminates many aspects of learning mathematics that are not always captured in conversations on how we might teach mathematics to engineering students. However, his discussion is largely based on a cognitive perspective. For an even fuller view of how engineers use and learn to use mathematics, we should also consider the larger environment. This work extends the perspective Booth considered in her phenomenographic studies of students' experiences learning mathematics, considering "what the individual is doing, with respect to the object of knowledge, and even with respect to the pedagogical context in which she is situated and the socio-cognitive and collaborative context of fellow learners and teachers" (Booth (2004, p. 13)). Thus we consider the aspects of mathematical thinking described by Schoenfeld but also consider the social and material resources available to students (McGinn & Boote (2003)) and the mathematical practices students engage in, for a more complete understanding of the breadth of mathematics and mathematical thinking necessary for engineering practice. In summary, this paper proposes that we consider each of the aspects of mathematical thinking summarized in Table 1, as we study and design the mathematical education of engineers.

Why teach mathematical thinking? Empirical Basis

The recommendation that we consider a broad notion of mathematical thinking, as we are considering what we should teach engineering students, is based not only in the mathematics education literature but also in previous studies of engineering students' and practitioners' uses of mathematics in engineering practice (Cardella (2006, 2007), Cardella & Lande (2007)). In a series of studies, three groups—engineering students completing their undergraduate education, engineering students beginning their graduate studies (Masters degrees) and practicing engineers—were interviewed about and observed[1] in order to learn what the engineers (both students and practitioners) believed they learned from their mathematics courses and to characterize the ways in which the engineers used mathematics in their practices. The undergraduate students were collaborating with a local industry partner on a five-month long "capstone" project, the graduate students were collaborating with an industry sponsor on a nine-month long intensive design project and the practitioners were engaged in a brainstorming session as part of their normal work practices.

Undergraduates: Capstone Design Projects

Nine students, representing five engineering disciplines, answered interview questions on what they had learned from their mathematics courses, and how they used what they had learned from their mathematics courses during their capstone design project. The students' responses to the interview questions "What did you learn from your mathematics courses?" and "Is there anything else you gained?"covered all five aspects of mathematical thinking in Table 1 (Cardella (2006)). Additionally, the interview responses served as the primary filter for analyzing the observation data. In some cases, the observed activity could have been considered "mathematical thinking" just as easily as "engineering thinking" or "critical thinking." Therefore, if the particular activity had

[1] The two groups of students were observed first-hand while the practicing engineers were indirectly observed through video-recordings collected by another research team.

been previously mentioned as something that a student had specifically learned from a mathematics course, it was counted as mathematical thinking.

The undergraduate students who participated in the observation portion of the study worked on a five-month-long capstone design project designing a satellite center for a shipping company. The following excerpt is an example of how this team engaged in mathematical thinking early in their project (one month into the project):

> During the first observed meeting, the team reviewed the problem brief that they had received from their Industry Partner and discussed the quantitative variables they needed to consider. They reviewed some data that they had received from an Industry Partner. John was explaining the data to his teammates, and mentioned that he knew how to input the data into Excel, but he needed someone to show him how to take averages in Excel. Two of his teammates, Diego and Mei, quickly offered multiple suggestions on how he could go about finding averages in Excel. In the middle of his explanation, Diego stopped himself and said "oh wait… I think this is right… that's fine."
>
> During this episode, the mathematical knowledge base the team drew on included content knowledge of how to take averages and how to use Excel. They used a resource of data from their Industry Partner, as well as Excel, and John used his teammates as social resources. Diego engaged in monitoring behavior as he paused to consider whether or not he and his teammates were proceeding correctly. The team took a mathematical perspective to the problem as they "mathematized" the problem by focusing on the quantitative aspect of the problem. Also, in talking about taking averages, they used a mathematical vocabulary (Cardella (2006)).

Graduate students: Year-long Design Project

Over the course of one academic year, a 9-month period, a team of four mechanical engineering graduate students designed a compressed air and vacuum delivery system for use in small dental offices. The team worked with an industry client—because the client was in a different state, most of the team's interactions with the client were by telephone and email. The main criteria that the team considered in their design decisions were weight, cost and the system's ability to deliver the required air flow. The team created both mathematical "simulation" models and physical "experimental" models (by purchasing and then testing different motors, compressors and other parts) of their system.

During the February 27th meeting, the students review their latest acquisitions: a new Gast compressor, a new vacuum (pump) and a new capacitor. After reviewing and plotting the data that they already have (gathered from parts they had previously purchased), they collect data for the new compressor and the new vacuum using Labview. Although the manufacturer has provided the information that they need—air flow and air pressure—they collect data themselves to make sure that the manufacturer's specifications are accurate. Throughout the process of reviewing the data they collected earlier, plotting the data using Engineering Equation Solver, collecting new data and comparing their "experimental" results to the manufacturer specifications, the students seek out instructors (professors, consultants and teaching assistants) for advice and feedback as well as help in communicating with their client (the instructor emphasizes that it is a *student* project). They also spend time planning for their next official meeting with their instructors that will take place two days later; during this meeting, they will present their work to date for their instructors to evaluate. Overall, the meeting is an iterative cycle of these events.

In this episode, the mechanical engineering graduate students drew on a mathematical knowledge base that included knowledge of exponential functions and graphing dependent and independent variables as well as knowledge of how to use LabView to collect data and how to use Engineering Equation Solver (EES) to conduct mathematical simulations. They used a social resource of their instructors for expert advice, and used several material resources: LabView, EES and the numerical information provided as the manufacturer specifications (although they did not trust this resource, it provided them with baseline values to evaluate their data against). They engaged in planning behavior (Use of Resources) as they prepared for their next meeting their instructors, and engaged in several mathematical practices: using a mathematical vocabulary, interpreting numbers, being precise, creating and using mathematical models, calculating, checking if results seem reasonable and checking to see if theoretical results match experimental data (see Cardella (2006) or Cardella (2007) for a more detailed description of these practices).

Practitioners: One Brainstorm Session

Video data was collected during two brainstorming meetings, each two hours in length. This analysis focused on the first meeting. The team of practitioners was tasked with generating ideas for a thermal pen—the company had a particular technology in mind and was generating new uses for this technology. The expertise represented by the meeting participants included electronics, mechanical engineering, ergonomics, industrial design and business. The practitioners took turns suggesting different forms for the potential thermal pen, referring frequently to existing objects. In the course of considering different forms for the pen, the practitioners also considered how children might use the pen, and the implications that the uses might have for their design. At times the practitioners were tempted to determine specific details of potential solutions, with the facilitator, at times, reminding the group that their goal was to brainstorm ideas rather than leave with a detailed solution.

While the practitioners did not perform many calculations or use any software to simulate data or construct mathematical models, the practitioners did engage in several mathematical thinking activities. In this context, the mathematical Knowledge Base that they exhibited covered mainly arithmetic and geometry. The resources that they drew upon were social—they talked about who had knowledge about the numerical and mathematical information relevant to the design of the pen. They expressed a concern for precision, suggesting that they held a belief that precision was important, but in this context they dealt with estimates and approximations. They used a mathematical vocabulary as they talked about the potential forms (including weights and other dimensions) of the pen, and used numbers as they communicated their ideas. As they approximated dimensions of the pen, they reflected on the approximations to check that they seemed reasonable, checked if the results from the data that did exist seemed reasonable, and used the approximations and existing information to check if their proposed ideas were feasible (e.g. if the proposed pen would provide a "reasonable force"). Additionally, they engaged in proportional reasoning in a qualitative manner:

> Charles described a proportional relationship between pressure and amount of "ink" produced by the pen as: "if the child doesn't press hard then you activate every other one so you get a faint line" (E1, 586-587) and Tom also suggests that there is a relationship between speed and amount of ink: "if you go fast it will be a paler colour if you go slowly it will be darker, won't it" (E1, 921-922). (Cardella & Lande (2007))

How should we teach mathematical thinking?

Clearly we still need to ensure that engineering students have a solid, foundational understanding of mathematical content knowledge. However, beyond the mathematical content knowledge that is necessary for engineering practice, we should consider Problem Solving Strategies, Resources and Use of Resources, Beliefs and Affects, Mathematical Practices, and the Environment in which mathematics is taught. At this time, there is still much work to be done in order to understand how we can address these other aspects of mathematical thinking as we consider the mathematical education of engineers. Nevertheless, it is important to consider the full space of what we would like engineering students to learn and how they might engage in learning related to all five aspects of mathematical thinking.

As we consider this set of questions, there are many pedagogical approaches that we might adopt in order to address the full space of mathematical thinking. Within this conference, some presenters focus on Problem Based Learning, active learning , using games, using technology, a modeling approach and a learning centre environment. Another innovative approach that has gained increasing interest globally, in teaching mathematics not only to engineering students but to students learning mathematics all throughout their academic experiences (from primary school to university) is the use of Model Eliciting Activities (MEAs). MEAs are open-ended, real-world, client-driven problems based on the models and modeling perspective. MEAs require students to interpret and mathematize (e.g. quantify, fit to a mathematical formula) a scenario that contains some amount of data provided by a fictional client (Diefes-Dux et al. (2004)).

Finally, learning scientists within the United States have recently adopted a Challenge-Based Instructional approach. In the Challenge-Based approach, problems are posed as a series of interesting challenges that require the students to search for and acquire knowledge and expertise, as needed, to solve the challenge. When coupled with a team approach, the challenge-based approach to learning stimulates the students to develop a deep understanding of the discipline while at the same time building problem-solving, collaboration, and communication skills (Barr et al. (2007)).

Conclusion

Regardless of instructional strategy, it is important that we consider the full space of mathematical thinking—the mathematical Knowledge Base as well as Problem Solving Strategies, Resources, Use of Resources, Beliefs and Affects, and Mathematical Practices—as we contemplate the mathematical education of engineers. In the empirical data, it was evident that engineering students' mathematical education encompassed all of these aspects. Each of these aspects of mathematical thinking will impact the way in which an engineer is able to use mathematics in practice, and each deserves attention as we further study how to educate engineers and further design course, curricula, interventions and tools aimed at educating engineers in mathematics.

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Reducing Choice = Increasing Learning or Decreasing Marks?

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Abstract

In early-year engineering mathematics programmes in Dublin Institute of Technology (DIT), students were commonly required to attempt five out of eight questions in their end-of-year examination. As these questions were based on well-defined areas, it allowed students to omit certain topics and still perform impressively. This observation was re-iterated by the fact that the most common problem for which engineers sought help in the Maths Learning Centre last year was basic integration, with 56% coming from second or third year, at which stage they should be very familiar with integration. One way to address the problem is by reducing (or even eliminating) choice questions on mathematics papers in earlier years: if the material covered is necessary groundwork for later years, it should not be possible for students to omit it entirely. In this study, we build upon the results of an anonymous survey to determine students' opinions of reduced choice in early years. Within DIT, last year's changeover from year-long to semester-long modules afforded a natural move from a three-hour end-of-year exam to two two-hour end-of-semester exams. Two new approaches to choice were introduced by the authors. The first, with mechanical engineers, completely eliminated choice: students were required to answer all questions on the end-of-semester papers. The second approach, with building services engineers, replaced the final exam (a choice of five out of eight questions with question one compulsory) by end-of-semester exams featuring a choice of three out of four questions with question one compulsory. This greatly decreased choice without eliminating it. We compare and contrast both approaches, and address the question of whether reduced choice encourages students to learn difficult topics or adversely affects their marks.

Introduction

Within Dublin Institute of Technology (DIT), a wide range of engineering programmes are offered, at both Ordinary and Honours degree levels. The former programmes take three years to complete, and have a prerequisite of a D3 (the lowest pass grade) in Ordinary Level Leaving Certificate Mathematics, the final mathematics exam at the end of secondary school in Ireland. The latter programmes take four years to complete and have a prerequisite of a C3 in Higher Level Leaving Certificate Mathematics. If a student achieves sufficiently high grades in their final examinations in an Ordinary degree, they may enter into the third year of the Honours programme, and so leave with an Honours qualification within five years of commencing their studies.

In first-year engineering programmes, both at Ordinary and Honours levels, the standard format of many end-of-year mathematics exams to date has been to require students to answer five questions out of eight. Due to the fact that these questions were based on well-defined areas, it was possible for students to entirely ignore certain subject areas and still receive a good grade in the exam. This led to serious difficulties for some students in later years of their programmes, as they lacked the basic mathematical knowledge required to learn more advanced topics.

This approach caused particular problems in calculus, with many students unable to cope with more advanced differential equations due to problems with basic integration. This was noted both anecdotally in the classroom and also in the Students' Maths Learning Centre, in which the most common topic for which engineering students sought help was basic integration, with 56% of the queries coming from second or third year students. It had been observed that students who had successfully completed an Ordinary degree and subsequently entered directly into the third year of an Honours degree programme experienced considerable difficulties with the mathematics component, although they tended to cope well with the more practical elements of the programme. Given that more than half the Ordinary degree students now progress to an Honours programme, this is an area of particular concern.

Many students expressed regret at having omitted integration in earlier years, leading us to conduct an anonymous, online survey to determine engineering students' attitudes towards choice in mathematics exams in the early years of their programmes (Carr (2007)). Students were asked if they had struggled as a result of omitting specific subject areas and if they felt that certain topics should always be compulsory. From the 276 responses received, 64% had avoided integration at some point, and 47% had struggled as a result; 39% had avoided differentiation at some point, and 32% had struggled as a result. A quarter of respondents felt that choice should be removed from at least some maths papers, while a massive 60% felt there should be compulsory questions on certain topics.

Concurrent with this research, DIT was in the process of changing over from year-long to semester-long modules. This afforded a natural move from a single three-hour end-of-year exam to two two-hour end-of-semester exams. Two new approaches were introduced by the authors: in the first of these, choice was completely eliminated, and students were required to answer all questions on both end-of-semester papers; the second approach replaced the final exam (a choice of five out of eight questions with question one compulsory) by end-of-semester exams featuring a choice of three out of four questions with question one compulsory. This greatly decreased choice without eliminating it. We will now look at each of these approaches in greater detail, contrasting the exam results on a question-by-question basis to determine the effects, if any, of these new layouts.

Elimination of Choice: 3rd Year Ordinary Degree Mechanical Engineering

As shown in Table 1 below, third-year Ordinary degree students in mechanical engineering traditionally had to answer five out of eight questions, with large numbers avoiding the integration or differential equations questions (as can be seen from the "Attempts" column) and still achieving good marks. With the changeover to two semesterised maths exams in this programme, the decision was made to make all questions compulsory, in an attempt to ensure students had a good grounding of all relevant areas before obtaining their degree. This would be of particular use to students continuing into the Honours programme.

Questions	**2005** (Pass: 38/51)			**2006** (Pass: 39/55)		
	Mean	***Sigma***	**Attempts**	**Mean**	***Sigma***	**Attempts**
1: Differential Eqns	54	*24*	30	43	*26*	29
2: Integration	37	*20*	23	67	*29*	25
3: Laplace Trans.	45	*26*	39	44	*29*	51
4: Radius of Curvature	44	*25*	6	62	*29*	9
5: Regression	65	*28*	39	65	*23*	53
6: Hypothesis Tests, Control Charts	65	*28*	29	60	*24*	35
7: Gauss-Seidel	67	*28*	40	58	*31*	30
8: Runge-Kutta	75	*25*	38	67	*22*	48
Total	**57**	***20***	**51**	**58**	***22***	**55**

Table 1: Mean mark and standard deviation (sigma) for each exam question in 2005 and 2006. Students had to answer five questions out of eight. The number of students who attempted each question is shown, along with the number who passed the exam.

Table 2 below shows the division of topics and subsequent results for this newly modularised and semesterised version of the course. Obviously, only three sets of results are available at this point in time, but this is sufficient for a preliminary study.

Questions	**Jan 2007** (Pass: 39/53)			**Jan 2008** (Pass: 40/51)		
	Mean	***Sigma***	**Attempts**	**Mean**	***Sigma***	**Attempts**
1: Runge-Kutta	70	*26*	51	80	*24*	50
2: Laplace Trans.	50	*24*	53	61	*29*	51
3: Diff Eqns	52	*32*	49	52	*29*	50
4: Diff Eqns	66	*32*	53	58	*27*	46
Total	**56**	***22***	**53**	**59**	***23***	**51**
	May 2007 (Pass: 47/54)			**May 2008**		
1: Integration	64	*32*	52	Data not yet available		
2: Control Charts	87	*24*	53			
3: Regression	79	*24*	54			
4: Gauss Seidel	72	*31*	51			
5: Hypothesis Tests	78	*30*	50			
6: Confidence Intervals	82	*26*	49			
Total	**70**	***22***	**54**			

Table 2: Mean mark and standard deviation for each exam question in each semester of 2007 and 2008. Students had to answer all questions. The number of students who attempted each question is shown, along with the number who passed the exam.

Reduction of Choice: 1st Year Ordinary Degree Building Services Engineering

Before modularisation, first-year Ordinary degree building services students were required to answer five out of eight questions on the end of year exam. Question 1 was compulsory and consisted of ten short questions covering algebra, trigonometry, differentiation and integration. The remaining questions each covered a specific topic including a question each on differentiation and integration. Due to the format of the paper it was possible for a student to completely avoid questions considered difficult and concentrate instead on those considered the easier options. This usually resulted in the students avoiding the question on integration with the number of attempts being consistently low (again, as evident in the "Attempts" column of Table 3 below). On average only 45% would attempt the question on integration against 90% for differentiation. As has been stated this led to difficulties in later years with integration.

	2005 (Pass: 29/40)			**2006** (Pass: 28/35)		
Questions	**Mean**	***Sigma***	**Attempts**	**Mean**	***Sigma***	**Attempts**
1: Compulsory	73	*19*	40	69	*18*	35
2: Binomial	34	*31*	17	40	*23*	20
3: Graphs	28	*36*	11	5	*0*	2
4: Differentiation	48	*26*	39	42	*23*	30
5: Trigonometry	36	*21*	35	56	*22*	26
6: Integration	36	*19*	23	42	*16*	11
7: Complex Numbers	45	*25*	28	50	*16*	26
8: Stats/ Probability	39	*22*	27	48	*22*	24
Total	**47**	***18***	**40**	**51**	***15***	**35**

Table 3: Mean mark and standard deviation (sigma) for each exam question in 2005 and 2006. Students had to answer five questions out of eight, with question 1 compulsory. The number of students who attempted each question is shown, along with the number who passed the exam.

After modularisation, it was decided to still allow an element of choice but, due to the importance of differentiation and integration, these were incorporated into the same question and made compulsory. The student was required to answer three out of four questions with question 1 compulsory. It is hoped that this approach will lead to students taking integration more seriously and assist them in succeeding years of the course. The division of topics and subsequent marks is shown in Table 4 below.

	Jan 2007 (Pass:32/49)				**Jan 2008** (Pass: 22/38)		
Questions	**Mean**	***Sigma***	**Atmpt**	**Questions**	**Mean**	***Sigma***	**Atmpt**
1: Compuls.	28	*24*	49	1: Compuls.	52	*21*	38
2: Complex Numbers	33	*32*	47	2: Binomial	43	*31*	36
3: Graphs	42	*42*	5	3: Trig	58	*43*	37
4: Trig	56	*56*	46	4: Trig	46	*42*	27
Total	**41**	***40***	**49**	**Total**	**37**	***22***	**38**
	May 2007 (Pass:30/39)				**May 2008**		
1: Diff / Int (Compuls.)	48	27	39				
2: Max/ Min	57	30	17	Data not yet available			
3: Simpson	70	33	31				
4: Stats/Prob	63	30	30				
Total	**58**	**23**	**39**				

Table 4: Mean mark and standard deviation (sigma) for each exam question in 2007 and 2008. Students had to answer question 1, and two of the other three questions.

The number of students who attempted each question is shown, along with the number who passed the exam.

More Detailed Analysis of Calculus Questions

There is a huge amount of information contained within the four preceding tables. However, we are particularly interested in some of the calculus questions, and therefore we now analyse these figures to see what effect, if any, this has had on the numbers answering these questions and the mean mark. Table 5 shows the results for the third-year cohorts' questions on integration and differential equations. The number of students attempting these questions has almost doubled, but this has not had any adverse effect on the marks for the questions, which is an encouraging result.

<table>
<tr><th rowspan="2">Year</th><th colspan="3">Integration</th><th colspan="6">Differential Eqns</th></tr>
<tr><th>Mean</th><th>Sigma</th><th>% Attempts</th><th colspan="2">Mean</th><th colspan="2">Sigma</th><th colspan="2">% Attempts</th></tr>
<tr><td>2005</td><td>37</td><td>20</td><td>45%</td><td colspan="2">54</td><td colspan="2">24</td><td colspan="2">59%</td></tr>
<tr><td>2006</td><td>67</td><td>29</td><td>46%</td><td colspan="2">43</td><td colspan="2">26</td><td colspan="2">53%</td></tr>
<tr><td>2007</td><td>64</td><td>32</td><td>96%</td><td>52</td><td>66</td><td>32</td><td>32</td><td>93%</td><td>100%</td></tr>
<tr><td>2008</td><td colspan="3">n/a</td><td>52</td><td>58</td><td>29</td><td>27</td><td>98%</td><td>100%</td></tr>
</table>

Table 5: Mean and standard deviation for integration and differential equations questions for third-year mechanical engineers from 2005-2008. In 2007-2008, there were two questions on differential equations, which are shown in split columns.

Table 6 shows the results for the first-year cohorts' questions on integration and differentiation. These are now merged into one compulsory question; again, we see a dramatic increase in the numbers answering integration but no subsequent drop in marks (although the marks for 2007 are merged for differentiation and integration).

Year	Integration			Differentiation		
	Mean	***Sigma***	**% Attempts**	**Mean**	***Sigma***	**% Attempts**
2005	36	*19*	58%	48	*26*	98%
2006	42	*16*	32%	42	*23*	86%
2007	48*	*27**	100%*	48*	*27**	100%*

Table 6: Mean and standard deviation for integration and differentiation questions for first-year building services from 2005-2007. * In 2007, integration and differentiation was combined in one compulsory question, meaning the results are identical.

Mean Marks and Pass Rates

As well as looking at particular questions, which were known to be avoided previously, we must also consider the overall mean marks and pass rates for the exams, to determine if reducing or eliminating choice has had any impact on these. Table 7 below shows the mean marks, standard deviations and percentage pass rates for exams from 2005-2008 for both programmes.

In the case of the third-year mechanical engineering class, the average mark has not been adversely affected by the decision to make all questions compulsory, or by covering most of the difficult questions in the first semester, nor is there any evidence of a reduction in pass rate; if anything, the pass rate in the second semester exam has increased. This may be as a result of increasing the bar in the first semester, thus encouraging students to perform better in the second semester.

Year	1st Yr Building Services Eng			3rd Yr Mechanical Eng		
	Mean	***Sigma***	**Pass Rate**	**Mean**	***Sigma***	**% Pass Rate**
2005	47	*18*	73%	57	*20*	75%
2006	51	*15*	80%	58	*22*	71%
Jan 2007	41	*40*	65%	56	*22*	74%
May 2007	58	*23*	77%	70	*22*	87%
Jan 2008	37	*22*	56%	59	*23*	78%

Table 7: Mean mark, standard deviations and pass rate (%) for exams from 2005-2008

However, looking at both the marks and pass rates for the first-year building services engineering class, an interesting result emerges: the mean mark and pass rate for the first semester has dropped significantly, but the second semester is consistent with previous years. The compulsory question on integration/differentiation was on the exam at the end of the second semester. This suggests that this drop may in fact have considerably more to do with students' transition to semesterisation, rather than the change in the format of the exam, with first-year students less prepared to sit exams a mere twelve weeks after the start of their college career.

Conclusions and Future Work

Third-year mechanical engineering students are now compelled to answer all questions on their maths papers, including both integration and differential equations, which were commonly avoided previously. Almost all students are now attempting these questions. The major concern about this approach was that it would significantly reduce the mean mark or the pass rate. Neither of these has deteriorated; in fact, the students' marks actually increased in the second semester. It must be remembered that we only possess preliminary data at this point, as this is the second year in which this approach has been implemented, but, as yet, there is no evidence to suggest that by eliminating choice on the paper, we have reduced the mean mark or the pass rate for this cohort.

A decrease in both mean marks and pass rates is evident for the first-year building services students – but only in the first semester exam, with the second semester exam showing no significant deviation from previous years. As mentioned above, this increased early failure rate is likely to be due to other factors, such as difficulties adapting to third-level. However, another important factor is that ten fewer students sat the second semester exam than the first semester one; such students would not have been included in the compilation of pass rates for end-of-year exams in previous years, but did affect the pass rate in first semester, possibly making it artificially low. Further analysis of this trend will be conducted in the future, when more data is available. We plan to conduct a follow-up of the maths grades of the first-year students in later years in order to further ascertain potential benefits of this approach.

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Computer-aided formative assessment of ordinary differential equations: a new approach to feedback

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Abstract

There are a large number of systems which deal with different levels and areas of mathematics and the use of computer technology to enhance student learning, understanding or assessment. Historically, these systems have often been used as an electronic repository for past questions, tests and exams, with their contents simply being copied across to the new medium with little or no amendment beyond that technically necessary.
The work discussed looks specifically at computer-aided formative assessment, and a new way of trying to enhance the feedback which can be given to students at the different stages of their working. The aim of the research is to design and implement these types of feedback in a computer environment and investigate their effectiveness on students' perceptions, understandings and achievement with undergraduate ordinary differential equation problems. Results will be presented from the design and trial of a new system, built within a computer assessment environment which allows students to enter their attempt at the solution to a question line-by-line and receive various different types of feedback. The development and use of the system, by both the researcher and the student-users, will be reflected upon.

Introduction

Computer-aided assessment (CAA) of mathematical topics has been around for a long time, e.g. O'Shea (1982). As the sophistication of computer technology, as well as the use and accessibility of the internet, has increased, more evolved forms of CAA have arisen. These systems generally make use of a computer algebra system (CAS) or specialist functions with much the same outcome.

There are many different ways in which CAA has been used, for example to provide multiple-choice or multiple-response questions. The addition of a CAS to CAA, often credited to Klai et al (2000), allowed it to move forward and be able to check a student's response in a more mathematically rigorous sense.

The single-line user response entry often used in CAA allows not only simple checking of the response against a 'model' one, but also against certain properties a correct response should have. STACK (Sangwin and Grove (2006)) is an example of one such CAA, which uses the CAS Maxima, where students can be presented with mathematical questions to which they must provide a syntactically linear response – it is this system which is focused upon in this article.

The work to be discussed looks at linear ordinary differential equations (ODEs) of first and second order. This topic was chosen for two main reasons:

1) breadth of applicability – ODEs are studied by students of mathematics, engineering, sports science and more;

2) depth of mathematical content – To be able to work confidently and competently with ODEs students need strong understanding of arithmetic, algebra, substitution, differentiation and integration, to say nothing of the specific methods of the topic itself.

Having provided a brief background to the research, the design and its rationale are discussed before providing some results from initial trials of the system.

Design

Even within the narrow area under consideration, i.e. that of ODEs, computers have previously been used to allow students to explore and learn (see for example Tall (1986) and James et al (1997)), but these programs have not approached matters from an assessment point of view. It is the author's belief that, whilst learning is certainly important, the role of *practice* in consolidation of understanding in mathematics should not be neglected; and where one practises, assessment automatically follows, even if only informally.

Until recently, STACK made provision only for single-part questions, and though it now allows multi-part questions, each part still accepts only a single line of CAS text as a response. Within the context of summative assessment, in the sense of Busuttil-Reynaud and Winkley (2006), this is perhaps all that one might consider necessary. However, there is one particular consideration which must be addressed concerning this position.

For the majority of students entering an undergraduate mathematics-related course, their most recent summative assessment experience will have been that of A-level exams. In these exams, for mathematics at least, students are able to gain partial credit for incorrect solutions if their attempts have shown certain levels of understanding. Where CAA is used with first year undergraduates it may be that this expectation of 'working' or 'follow-through' marking is carried forward, yet it cannot be met. See Ashton *et al* (2006) for an example of incorporating partial credit in multi-part mixed-entry mathematics questions.

Find the general solution to the differential equation

$$(x^2-1)\frac{dy}{dx} = -(x^2+1)y,$$

given x>1.

Your last answer was interpreted as:

$$A\cdot(x^2-1)^{-[1/x]}\cdot e^{-x}$$

Incorrect answer.

Your solution is incorrect. Substituting your solution into the left-hand side of the differential equation gives

$$(x^2-1)\frac{dy}{dx} = (x^2-1)\cdot\left(\frac{e^{-x}\cdot\left(\frac{\ln(x^2-1)}{x^2}-\frac{2}{x^2-1}\right)\cdot A}{(x^2-1)^{[1/x]}} - \frac{e^{-x}\cdot A}{(x^2-1)^{[1/x]}}\right).$$

Whereas substituting your solution into the right-hand side of the differential equation gives

$$-(x^2)+1)*y = \frac{(-(x^2)-1)\cdot e^{-x}\cdot A}{(x^2-1)^{[1/x]}}.$$

Answer: A*(x^2-1)^-(1/x) * e^-x

Figure 1: Sample STACK question with attempt and feedback

When one considers formative assessment, the issue of marking or feeding back based on working becomes rather more significant. Figure 1 shows a sample ODE question from STACK along with an incorrect attempt and the associated feedback. Whilst the CAS here is able to provide derivatives of the left- and right-hand sides of the attempt, to show that the ODE has not been satisfied, it cannot comment on where the error might have arisen. Indeed, whilst in some cases it may be possible to predict all (or at least most) of the mathematically relevant errors and pre-define feedback for each, this is not a practical nor time-efficient solution in general.

One of the fundamental concepts behind the design of the computer-aided formative assessment system within STACK was that understanding and proficiency can be improved via formative assessment, more specifically line-by-line feedback, based on students entering their working line-by-line.

The system is intended to bridge a perceived gap between 'traditional' and usual computer-centred marking by providing a facility through which students can attempt multiple ODE questions showing as much working as they wish, and receive feedback at around the same level of detail as if they had asked their lecturer to mark it.

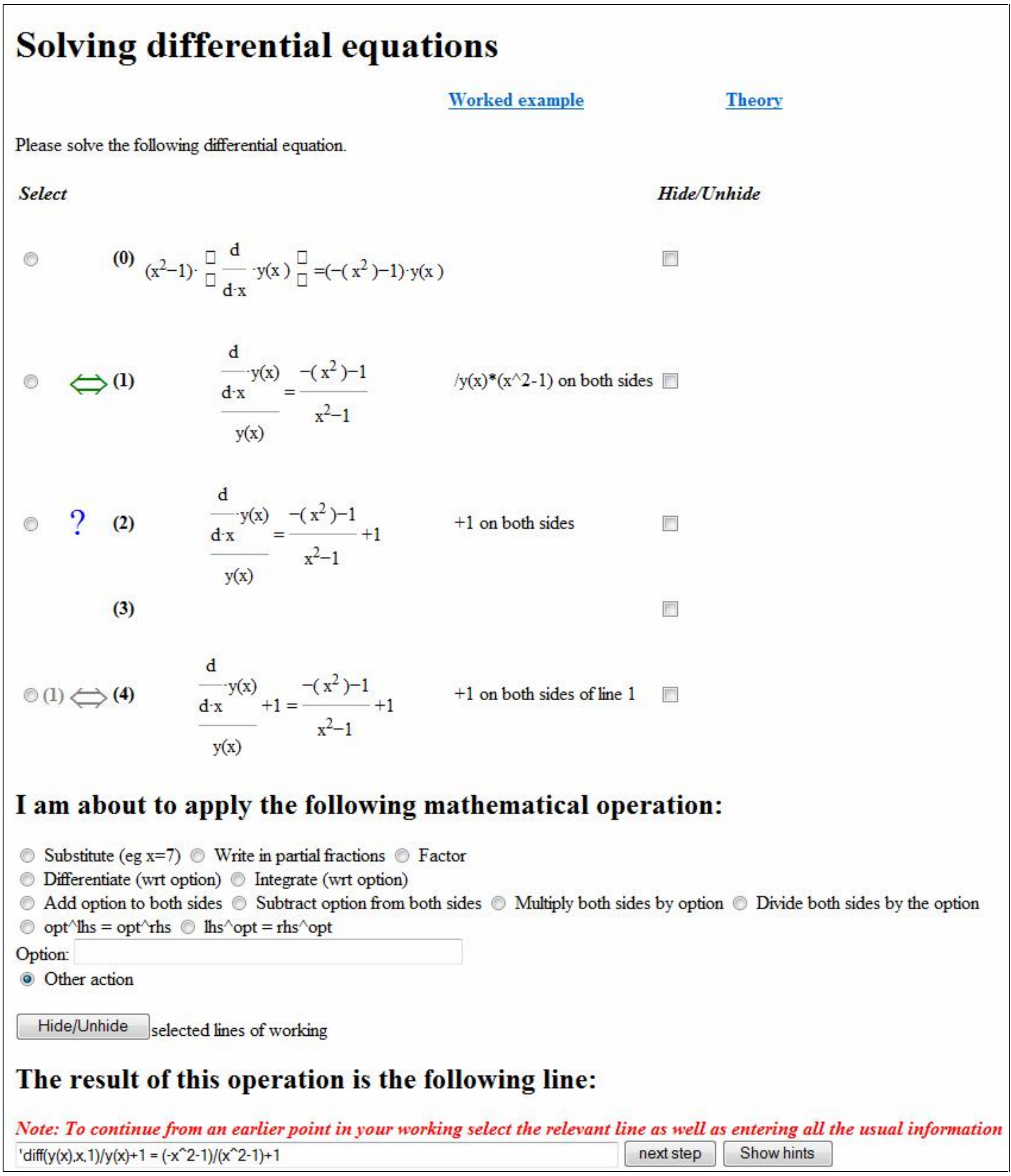

Figure 2: A question attempt in the current system

Figure 2 shows the present layout of the system, whose main features include:

- Multi-line response entry;
    - Includes capacity to refer back to, or hide, any previous lines of working
- Line by line mathematical equivalence feedback;

- Teacher's hints based on student's progress;
- Worked Example and Theory pages;
- Commenting of working;

Furthermore, at the time of writing, a facility to automatically give feedback based on the comments the student makes is being added, i.e. the system will be able to check whether the student has correctly done what they stated they would in their comments.

This feedback, in addition to the equivalence and teacher-hint types, is intended to provide detailed information to the student not only on whether they have done what they said they would and if they have done something permissible, but also if they are going in the right direction and what to try if they are not sure what to do. In short, the system aims to support the student through their working as much as possible without taking ownership of the attempt from them.

Implementation

In November and December of 2007 three trials were run at Birmingham, Coventry and Loughborough universities, in which students were presented with the current version of the system and, after some explanation, left to explore it and attempt the questions. Following up to an hour and a half of use the students were then drawn into a group discussion of their reflections and experiences with the system.

In general students were receptive to the ideas behind the system and its implementation, giving positive comments about the equivalence feedback and the Theory/Worked Example files. Though some students struggled initially with the necessary syntax for entering their working into the system most felt this was worthwhile; one such student even asked if/when it would be possible to purchase the final release of the software.

At the time of writing, the main studies are being reviewed and re-evaluated, including the role the system will play. Work will also be done to ensure the system is ready for use, and results of this study will appear once they have been collected and analysed.

Conclusion

This article has sought to briefly present and justify the above system as a new approach to computer-aided *formative* assessment, or even potentially CAA in general. The argument for multi-line response entry has been given and the author's proposed solution to the problem presented. Though the final results of the implementation of the system have yet to be seen, the trial data seems positive and it is hoped that the study will provide useful information about the value of different types of feedback in a computer environment.

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Using the SONG approach to teaching mathematics

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Abstract

In 2003, as part of the UK MathsTeam project, we presented a case study illustrating our way of encouraging engineering students to learn mathematics. This is based on SONG, a mix of Symbolic, Oral, Numerical and Graphical approaches. Students are thus exposed to multiple representations of the mathematical concepts they need to understand, exploiting in a variety of ways the technology-rich context in which they find themselves. Since that project the world, and our university, have inevitably moved on. We must now teach CEng, IEng and a smattering of Foundation degree students together in the same group, and this increased diversity exacerbates some of the difficulties, such as the range of mathematical backgrounds, and the varying levels of confidence, skill and motivation, which we and our colleagues in the UK mathematical community have been facing.

Given the increasingly diverse backgrounds, where in many cases students have struggled with mathematics; can we give the chance of a fresh approach rather than just revisiting their previous difficulties?

In a technology-rich environment, with multiple representations and multiple independent ways of getting answers, arguably no student should ever get any sums wrong; but what is the best way to use this technological power to motivate students whatever their approach to learning?

Introduction - can things only get better?

When we were one team amongst many in the UK MathsTeam project (LTSN (2003)), we presented our ideas and practice in teaching mathematics to engineering students, one aspect of which was to exploit mathematical technology to allow multiple representations of mathematical ideas and concepts. Our acronym was SONG, a reminder that we use a mix of Symbolic, Oral, Numerical and Graphical approaches, rather than over-emphasising say an algebraic approach.

We might ask in what ways things have moved on since then. It is useful to consider the wider context within which we live and work, and from which we draw our students. The recent Leitch Report (HM Treasury (2006)) states that despite having made good progress over the last decade, aspects of the UK skills base remain weaker than those in other developed economies, for example:

- 5 million adults in the UK lack functional literacy;
- 17 million adults in the UK have difficulty with numbers.

In terms of factors which might affect these figures in future, more than one in six young people leave school unable to read, write or add up properly. The report notes that such low skills levels, if not addressed, will inevitably not only damage business, but will also result in increasing inequality. The figures describe part of the context from which we draw our engineering students, and betray something of the mathematical tone in the country. We do of course have some control over those we admit to our courses, but economic pressures on universities, and indeed the need to provide a qualified engineering workforce, are significant factors. We select those who are more likely to be competent, but we do teach an increasingly broad range of levels and abilities. Thus those teaching elite courses may be able to rely upon a certain level of competence, although even in that case the difficulties with identifying exactly what can be expected upon entry have been well explored over the last few years. The situation with lower level courses, such as preparatory years or Foundation degree (FdSc) entry years is even less predictable, and we do find ourselves addressing issues of what we would like to treat as the basic numerical awareness we would hope all citizens might achieve.

In our own context, we must now teach CEng, IEng and a smattering of Foundation degree students together in the same group, and this increased diversity exacerbates the difficulties, such as the range of mathematical backgrounds, and the varying levels of confidence, skill and motivation.

While we do operate the standard “sticking plaster” measures, such as extra classes for selected students, and a drop-in Maths Help, we find we need to constantly examine what wider educational factors we take into account when designing learning activities.

Skills and Learning as well as Mathematics

What skills? What learners?

We are not teaching mathematics for mathematics sake, even though in our enthusiastic hearts we believe that is an intrinsically valuable thing to do! We are teaching mathematical ideas so that our students can become competent engineers. Mathematics must seem like an integral part of an engineering student’s course, not a parallel activity. It therefore behoves us to know about the generalities of engineering education. This means not only possessing an awareness of the range of applicability of the mathematics we teach, but also those more general skills that the engineering profession itself says that it values and wishes to develop in its members. We have written about this previously in a context of both engineering and mathematics (e.g. Challis et al. (2002)).

A glance at the HEA Engineering Subject Network (HEA Engineering Subject Network (2008)) reveals a familiar story. Still the most commonly valued generic key skills are communication, literacy, numeracy, team work, problem solving, information technology (IT) and self-management. Integrating the development of these skills into mathematical activity can help both in providing freshness of approach to mathematics, and also in integrating mathematics into the rest of a student’s engineering course.

Moving on to learners, we once gave a talk with unofficial subtitle “Mathematicians are people too”. The same probably applies to engineers. As mathematicians we are very good at designing coherent curricula, with clear pre-requisite structure and mathematical inter-relationships and appropriate content. But do we design our range of learning activities to take account of the range of learners that we encounter?

What follows in the next few paragraphs contributes to the debate about the relationship between mathematicians and mathematical educators, the currency of which is indicated by activity within the HEA Mathematics, Statistics and Operational Research Subject Network on precisely this topic. With some noble exceptions, this relationship seems at best to achieve mutual toleration, and yet if each community could free itself from its jargon, we have much to learn from each other.

One area in which this is the case is in achieving awareness that different students have different ways of learning, and that it is useful to recognise that in designing learning activities. There are various sources which can help in developing that awareness (e.g. Support4Learning (2008) or Felder & Solomon (2008)). Categorisations are many, so one must not be too dogmatic or definitive about this. Also it is worth noting that no one person always behaves as one type of learner, and perhaps the best learners adopt different styles as the occasion demands. But nevertheless the ideas can be thought-provoking.

For instance, Felder and Solomon (2008) describe active learners as those who “tend to retain and understand information best by *doing* something active”. They say one of their phrases is “Let’s try it out and see how it works". Reflective learners, on the other hand, “prefer to think about it quietly first”. Significantly for much of our practice, they propose that “sitting through lectures without getting to do anything physical but take notes is hard for both learning types!

They draw a different contrast between visual learners, who (obviously) like pictures; and verbal learners, who prefer descriptions and words, but they propose that “all learn more when information is presented both visually and verbally”. Is there a parallel here with the historical development of our own subject, as developments have swung between algebraic and geometric approaches?

As a third and final example, Felder and Solomon distinguish between sequential learners who progress in small linear steps (the “inchworm”), and global learners, who tend to progress in large jumps, and suddenly “get it” (the “grasshopper”).

We should aim to switch on to mathematics, students exhibiting all these traits. We can use any means which comes to hand. One possibility lies in integrating basic skills development into our mathematical developments, for instance by getting students to write reports which are fully integrated into the course. Another is to exploit technology in all its aspects, for instance both to allow powerful visualisation, and to provide motivation.

We would point out that it is difficult to motivate properly unless it comes from within the student anyway; but it is easier to de-motivate! What we have to do is provide a positive environment, but given that the current generation of students is that which has been tested at every stage of their education as part of an unwise and destructive UK government strategy, the extrinsic motivation of an assessment which counts can sometimes be necessary to provoke serious mathematical activity!

The role of technology and other factors

Mathematics has four main ingredients (Challis and Gretton (2007)): symbols, numbers, words and images. Each contributes to effective understanding and communication. Representing the same mathematical concept in different forms helps in the process of making connections within and between topics, provides a mix which takes account of differing learning styles, and makes it more likely that the symbolic mathematics will be correct.

The strength of computers and graphic calculators is that they can be used to provide multiple representations of mathematical concepts quickly, correctly and easily. There does arise the question of which tools to use, but it is important first to consider what mathematical idea is being addressed, and the range of approaches to it, and then one can use whatever tools come to hand. Technology also works when used as a motivational aid. Using a technological aid, alternative what-ifs can be tested, incorrect working is easily discarded, feedback is virtually immediate and the student is enabled by knowing they are always correct!

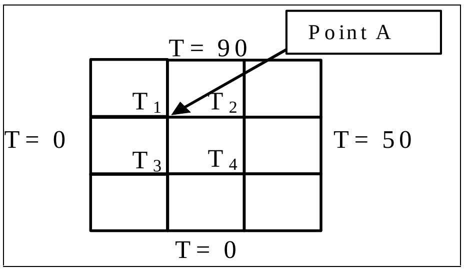
Indeed, the use of technology such as CAS, spreadsheets, and other visual technology does mean we all are enabled to get correct answers, validated by multiple means, but means we must develop wider skills. What do the results mean? How can we best communicate them to various different audiences? How do we convince others that we are right? These are essential meta-skills for engineers as for all of us.

We find we must develop a “rich” set of mathematical tasks. Ahmed (1987) describes these as ones which are accessible to all, allow challenges, get students to make decisions, get students to explain, reflect, interpret, and so on. Finally the task should be enjoyable!

Example

In Figure 1, we give an illustrative example of one part of a rich task, drawn from Challis and Gretton (1997): *"Well, here's another nice mes(h) you've gotten me into.”* (Apologies to Stan and Ollie.). It is one we use with first year engineering students as part of their learning to handle linear simultaneous equations. There are some points worth mentioning here. The task exploits commonly available technology, a spreadsheet. This enables students eventually to solve realistically large sets of equations. The problem involves a realistic context. The traditional order of the curriculum is disrupted once we allow use of technology. The problem requires some Symbolic manipulation; students can be required to give an Oral report at the end; the method is Numerical; and the technology allows easy Graphical presentation of results via the Excel plotting tools (Figure 2). Students can thus deploy a variety of learning styles in understanding and solving this problem. Some imagination must be applied to make the marking efficient: if a report section is to be included, then the purely instrumental parts of the marking must be streamlined as far as they can, and some indication of that is seen in Figure 1.

A flat, square plate as shown below has the temperatures on the edges held at the values shown:



You can use the discrete form of the energy conservation law, which says that the temperature at any point is the weighted average of the temperatures at the four surrounding points, to find the four equations necessary to give the approximate values of temperatures T_1 to T_4 at the points shown.

Now answer the questions below in the grid provided

	Your answer	
6. Write down in matrix form AT= B the equations satisfied by T_1 to T_4.		/4
7. Solve these equations using whatever technology you need, writing down here the value of T_1.	T_1 =	/4
8. Implement a strategy of mesh refinement as demonstrated in Excel in the lecture, to find the value of the temperature at the point A (which is the point where T_1 was originally specified) correct to 1 d.p., and write that value down here.	Temperature at point A to 1 d.p. =	/7

Figure 1

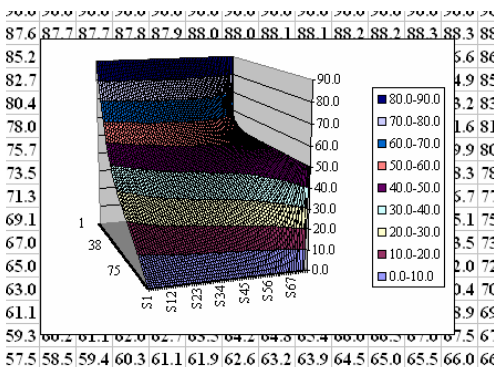


Figure 2

Conclusion

In a technology-rich environment, with multiple-representations and multiple independent ways of getting answers, arguably no student should ever get any sums wrong, in the narrow sense. But there is more than this, as mathematical learning is not only about getting right answers. Technological power can be used to motivate students, to allow the tackling of more realistic problems, and to vary the approach to learning, so that visual and verbal aspects, and active and reflective aspects, are all deployed in solving a rich problem.

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Closing the gap between formalism and application – PBL and mathematical skills in engineering

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Abstract

A common problem in learning mathematics concerns the gap between, on the one hand, doing the formalisms and calculations of abstract mathematics and, on the other hand, applying these in a specific contextualised setting in for example the engineering world. The skills acquired through problem based learning (PBL), in the special model used at Aalborg University, Denmark, may give us some idea of how to bridge this gap. Through an investigation of a series of examples of student projects concerning the application of mathematical subjects – such as matrices, differential equations, cluster analysis, graph theory etc. – the skills attained by participating students will be mapped out and discussed.

(The full paper will be published elsewhere.)

Trends in the Choice of A-Level Mechanics Options by Students Entering Engineering Degree Courses

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Introduction

In recent years the A-level Mathematics syllabus has steadily broadened from its traditional base of 'pure' mathematics and classical mechanics, introducing material on probability and statistics and decision mathematics. At the same time the examination of the subject has been increasingly modularised and the choice of optional modules has been greatly increased. As a result the level of knowledge of classical mechanics demonstrated by students who have a mathematics A-level qualification has declined markedly.

Some University disciplines, notably Physics and most types of Engineering, have traditionally relied on a students arriving at university with a good level of knowledge of classical mechanics. Lecturers in these areas have noticed a steadily declining familiarity with the standard models and methods of classical mechanics. This trend is set to continue with the introduction of the 2004 changes to the structure of A-level mathematics under which students offering single subject mathematics will study no more than two modules of mechanics. Further, there is no requirement for students to study any mechanics at all; a single or double subject A-level in mathematics can be obtained by following application modules in statistics and decision mathematics only.

Included in Robinson et al. (2005) is a report of the findings of a survey of first year engineering students at the Universities of Loughborough, Nottingham and Leicester undertaken during the academic year 2004-05. This survey discovered that, of the students who had taken A-level mathematics, 9% had studied no mechanics at all and 23% had studied only one mechanics module. Robinson et al commented that a further decline might well be anticipated after the effects of the 2004 syllabus revision had worked through to undergraduate level.

In a paper (Clements 2006) presented at the previous conference in the MEE series in 2006 the author reported the results of a survey of students entering the first year of engineering degree courses at Bristol University in 2005. The survey investigated the A-level option modules taken by those students. The cohort included a percentage who had taken a double mathematics A-level although this is relatively uncommon compared with the situation pertaining in the 1980s and even the 1990s.

Overall 89% of students (those who had taken A-level as their 18+ qualification) had taken module M1 with the percentage in individual Departments varying from 80% to 96%. This does not suggest any strong bias by discipline. 72% of students overall had taken module M2 with a variation between Departments from 63% to 79%, again not suggesting a strong bias by discipline. Module M3, M4 and M5 were taken by only

23%, 20% and 3% of students overall. These findings obviously show considerable consistency with the findings reported by Robinson et al.

A further survey, similar to that undertaken in 2005, was carried out in October 2006. The significance of this survey lies in the changes to the A-level mathematics syllabus introduced in 2004. Prior to those changes students studying a single subject A-level in mathematics were required to take three core modules and three additional modules chosen from a wide selection which included at least five modules of mechanics as well as modules in statistics and decision mathematics. Thus students could, in theory, take three modules in mechanics although, in practice, most students included a broader mixture of subjects amongst their options. Students commencing mathematics A-level from September 2004 onwards studied a modified syllabus which comprised four compulsory core modules and only two option modules chosen from a similarly broad collection as previously. So following the 2004 syllabus the students taking a single A-level in mathematics could study a maximum of two modules of mechanics and, in practice, were more likely to study a single mechanics module (M1) and an introductory module in statistics or decision mathematics (S1 or D1). Both before and after the 2004 changes it was possible to study A-level mathematics without taking any mechanics modules at all.

The students surveyed in October 2005 had commenced their A-level studies in 2003 (or earlier if they had not proceeded directly from school to university) whilst most of those surveyed in October 2006 had commenced their A-level studies in September 2004. Thus a comparison of the results of the two surveys would potentially reveal any trends and changes in the pattern of options choices under the pre-2004 syllabus and the syllabus for 2004 onwards.

Findings of the 2006 Survey

In the 2005 survey 12% of the students were found to have taken a secondary education leaving qualification other than A-levels; in 2006 the proportion was 14%. These students were ignored in the subsequent processing reported here. The grouping of students into subject areas was changed slightly between the 2005 and the 2006 surveys. In particular, in 2006 the computer science students were not surveyed since they had no requirement to study mechanics during their degree course. The smaller degree courses, Engineering Mathematics, Engineering Design and Computer Systems Engineering (CSE) had, by force of circumstance, been grouped in the 2005 survey but were separated in the 2006 survey. These groups, however, make up less than 20% of the cohort surveyed.

Tables 1 and 2 show the numbers and proportions of students taking the various mechanics modules broken down by degree course.

2005	n	None	M1	M2	M3	M4	M5+
Aeronautical	54	11 (20%)	43 (80%)	34 (63%)	9 (17%)	3 (6%)	1 (2%)
Civil	55	2 (4%)	53 (96%)	36 (65%)	9 (16%)	5 (9%)	1 (2%)
Electrical	47	7 (15%)	40 (85%)	32 (68%)	10 (21%)	5 (11%)	2 (4%)
Mechanical	83	6 (7%)	77 (93%)	62 (75%)	23 (28%)	7 (8%)	3 (4%)
Eng Maths & Design	46	4 (9%)	42 (91%)	34 (74%)	14 (30%)	10 (22%)	3 (7%)
Comp Science & CSE	47	7 (15%)	40 (85%)	37 (79%)	12 (26%)	4 (9%)	1 (2%)
Overall	**332**	**37 (11%)**	**295 (89%)**	**235 (71%)**	**77 (23%)**	**34 (10%)**	**11 (3%)**

Table 1. Proportions of students taking the available mechanics modules, 2005

2006	n	None	M1	M2	M3	M4	M5+
Aeronautical	62	7 (11%)	55 (89%)	41 (66%)	13 (21%)	5 (8%)	0 (0%)
Civil	56	7 (13%)	49 (88%)	27 (48%)	9 (16%)	1 (2%)	0 (0%)
Electrical	54	8 (15%)	46 (85%)	30 (56%)	12 (22%)	2 (4%)	0 (0%)
Mechanical	75	5 (7%)	70 (93%)	45 (60%)	19 (25%)	4 (5%)	1 (1%)
Computer Systems	15	6 (40%)	9 (60%)	2 (13%)	1 (7%)	0 (0%)	0 (0%)
Engineering Maths	17	0 (0%)	17 (100%)	12 (71%)	2 (12%)	1 (6%)	0 (0%)
Engineering Design	20	0 (0%)	20 (100%)	17 (85%)	6 (30%)	1 (5%)	0 (0%)
Overall	**302**	**34 (11%)**	**268 (89%)**	**176 (58%)**	**63 (21%)**	**14 (5%)**	**1 (0%)**

Table 2. Proportions of students taking the available mechanics modules, 2006

It is interesting to note that the overall proportion taking no mechanics at all and taking only one mechanics module is unchanged. The significant change seems to be the reduction of the numbers studying a second mechanics module from 71% to 58%.

Although Clements (2006) did not report these results, the 2005 survey also collected data on the uptake of statistics modules and decision mathematics modules. The 2006 survey also collected this data. Hence it is possible to compare the proportions taking various numbers of modules in these areas in a similar manner to the comparison undertaken in mechanics. Tables 3 and 4 show the comparative results for statistics. Here a similar pattern prevails with the proportion taking no statistics modules and taking only one statistics module almost unchanged whilst there is a slight increase in the percentage taking S2.

2005	n	None	S1	S2	S3	S4
Aeronautical	54	18 (33%)	36 (67%)	16 (30%)	2 (4%)	0 (0%)
Civil	55	13 (24%)	42 (76%)	14 (25%)	3 (5%)	0 (0%)
Electrical	47	12 (26%)	35 (74%)	16 (34%)	4 (9%)	3 (6%)
Mechanical	83	17 (20%)	66 (80%)	25 (30%)	8 (10%)	2 (2%)
Eng Maths & Design	46	7 (15%)	39 (85%)	18 (39%)	3 (7%)	2 (4%)
Comp Science & CSE	47	15 (32%)	32 (68%)	17 (36%)	5 (11%)	1 (2%)
Overall	**332**	**82 (25%)**	**250 (75%)**	**106 (32%)**	**25 (8%)**	**8 (2%)**

Table 3. Proportions of students taking the available statistics modules, 2005

2006		None	S1	S2	S3	S4
Aeronautical	62	22 (35%)	40 (65%)	12 (19%)	3 (5%)	2 (3%)
Civil	56	13 (23%)	43 (77%)	12 (21%)	1 (2%)	0 (0%)
Electrical	54	19 (35%)	35 (65%)	11 (20%)	1 (2%)	0 (0%)
Mechanical	75	18 (24%)	57 (76%)	45 (60%)	19 (25%)	4 (5%)
Computer Systems	15	3 (20%)	12 (80%)	5 (33%)	0 (0%)	0 (9%)
Engineering Maths	17	1 (6%)	16 (94%)	12 (71%)	2 (12%)	1 (6%)
Engineering Design	20	3 (15%)	17 (85%)	17 (85%)	6 (30%)	1 (5%)
Overall	**302**	**80 (26%)**	**222 (74%)**	**116 (38%)**	**33 (11%)**	**8 (3%)**

Table 4. Proportions of students taking the available statistics modules, 2006

Tables 5 and 6 show a similar comparison for decision mathematics. In that case there is negligible change.

2005	n	None	D1	D2
Aeronautical	54	37 (69%)	17 (31%)	4 (7%)
Civil	55	48 (87%)	7 (13%)	2 (4%)
Electrical	47	30 (64%)	17 (36%)	1 (2%)
Mechanical	83	66 (80%)	17 (20%)	4 (5%)
Eng Maths & Design	46	25 (54%)	21 (46%)	4 (9%)
Comp Science & CSE	47	31 (66%)	16 (34%)	5 (11%)
Overall	**332**	**237 (71%)**	**95 (29%)**	**20 (6%)**

Table 5. Proportions of students taking the available decision maths modules, 2005

2006		None	D1	D2
Aeronautical	62	47 (76%)	15 (24%)	2 (3%)
Civil	56	41 (73%)	15 (27%)	1 (2%)
Electrical	54	36 (67%)	18 (33%)	4 (7%)
Mechanical	75	58 (77%)	17 (23%)	3 (4%)
Computer Systems	15	13 (87%)	2 (13%)	0 (0%)
Engineering Maths	17	4 (24%)	13 (76%)	5 (29%)
Engineering Design	20	15 (75%)	5 (25%)	0 (0%)
Overall	**302**	**216 (72%)**	**86 (28%)**	**15 (5%)**

Table 6. Proportions of students taking the available decision maths modules, 2006

Conclusions

It seems that the most obvious and significant effect of reducing the number of option modules which are required in a single subject A-level in mathematics has been a further reduction in the number of students studying mechanics beyond the level of the first mechanics module M1. The effect on the uptake of statistics modules and decision mathematics modules has been, by comparison, small or negligible.

This change further reinforces the conclusions of Robinson et al. (2005) and Clements (2006) concerning the declining level of preparedness of students entering degree courses in engineering in a subject, mechanics, which is an important enabler for a proper understanding of engineering science and design analysis. Continuing change in the syllabus of university degree courses will be required to compensate for the lower entry skills of the students studying these degree courses.

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Redesigning the Calculus Sequence for Engineering Students

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Abstract

Modeling, programming and computing are core activities of modern day engineering and require a certain fluency with mathematics. In striking contrast, many engineering students lack familiarity with elementary mathematical notions and their difficulties only accumulate in mathematical disciplines. This article presents some efforts to deal with this situation at the Technische Universität Berlin (TU Berlin), where the project "Innovations in Mathematics Education for the Engineering Sciences" is currently in the initial phase of developing and testing material for the calculus sequence. This project is embedded in the Matheon Research Center and funded by the German Research Foundation, with the overall aim to completely redesign the first-year calculus education for engineering students at the TU Berlin. A first emphasis of the project is the development of applets containing background information on and visualizations of key notions, as well as electronically corrected exercises.

Background

All engineering students at the TU Berlin (approximately 4000 first-year students from over 20 different fields) attend a two-semester calculus course and a one-semester linear algebra course. Both courses are taught by members of the mathematics department and the syllabuses, as well as the learning goals and grading issues, rely on a consensus reached with the engineering departments. This curriculum acknowledges the need for modern day engineers to possess a sound understanding of mathematical ideas and techniques, much in accordance with different studies (see Nguyen (1998) and Mustoe (2002)).

On the other hand, feedback from lectures and tutorials confirms that our incoming students have very heterogeneous mathematics ability and background. Since there are no entrance examinations, it remains difficult to obtain precise data on this important point. The rapid pace imposed upon the lecturer (due to the amount of material to be taught) and the impossibility of individual follow-up in tutorials are clearly a disadvantage for students with a weaker background in mathematics, for whom difficulties often keep accumulating. Even when performing well in engineering disciplines, they may fail in mathematics and thus seriously compromise their chance of obtaining a degree.

Aim of the Project

A primary goal of the project is to develop new and efficient material for the first-year calculus sequence so that students reach a deeper understanding of mathematical concepts. This should happen without lowering standards nor changing the emphasis of existing courses.

Attending lectures and solving tutorial problems are presently the main possibilities for our students to become familiar with the notions of calculus. It is therefore appropriate to introduce complementary material the students can refer to at any time and without supervision. At this stage, applets are the ideal medium, since they embody the possibility of visualization, coupled with interactivity and immediate feedback. To intensify student engagement and promote reflective learning, we shall further introduce our students to computer software used by practicing engineers. This should give students the possibility of generating their own visualizations of calculus concepts and of seeing these concepts in a computational context.

This is of course not the first effort in this direction, and we refer the reader to Edwards (2003), Schott (2005), Tiedt (2001), Wlodkowski (2006), where interesting accounts of implementing graphical applets or computing software can be found.

Graphical Applets

Each applet contains a theory part explaining in a concise and self-contained way the concept under consideration, using the notations introduced in the lecture and providing an aide to memorizing for the student. A demonstration part contains a visualization of the concept, requiring the student to type in a limited amount of data. The emphasis is on the graphical illustration of the concept to make it meaningful and lively. In the training part, the student is expected to solve a problem centered on the mathematical point and enter the solution data. The solution is automatically corrected and immediate feedback is provided. If the student solution is found to be incorrect, a step by step check of the solution process is proposed.

A typical example would be the graphical applet devoted to the Taylor formula. The Taylor approximation is not only at the core of the calculus sequence but the very justification of discretization procedures widely used in engineering sciences. The theory part introduces the constituents of the Taylor formula: the function we want to approximate at a given point, the Taylor polynomial formed with the successive derivatives of the function at this point, and the remainder, which assesses the approximation.

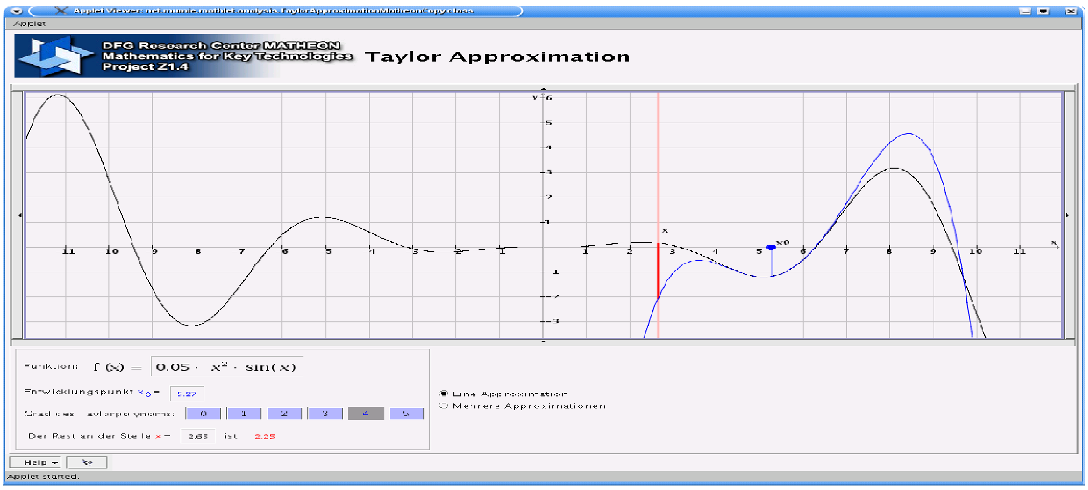

Figure 1: A Java applet on the Taylor expansion (demonstration part).

The demonstration part (Figure 1) is designed in such a way that the student enters a function (using the syntax familiar from pocket calculators), the expansion point and the degree of the Taylor polynomial. The data is processed and the applet displays a visualization of the function in the vicinity of the expansion point, as well as of the required Taylor polynomial. It is moreover possible to compute the remainder term at any given point or to display successive Taylor expansions at the same point.

The main point of the applet is to show that the Taylor expansion only holds locally and that the approximation at a given point improves with the degree of the Taylor polynomial. In the tutorial part, the student is given a function and an expansion point and is prompted to compute the corresponding first Taylor coefficients. The student's solution is then displayed and graphically compared with the actual Taylor polynomial.

Programming Exercises

Two reasons dictated our choice Scilab as the computer software to be used. Firstly, it is designed much like Matlab, making it easy for the students to switch later on to the software used by most practicing engineers; secondly, Scilab is free, making it possible for the students to download the software on their own computer and to work on the programming exercises at home rather than in a crowded university computer facility.

We do not expect programming knowledge from our students but rather introduce a small set of carefully chosen programming concepts during the first weeks of the lecture term. Bearing in mind that most of the students will be introduced to numerical methods in subsequent courses, we keep the knowledge of numerical processes to a very elementary level.

An important point is that computational software makes it possible to provide numerical evidence for many results in the calculus course. We believe this to be a satisfying approach for engineering students when many calculus results explained in the lecture are motivated but left without mathematical proof. Of course we should warn the students that numerical evidence merely provides an insight and does not constitute a proof in the mathematical sense and can even be misleading.

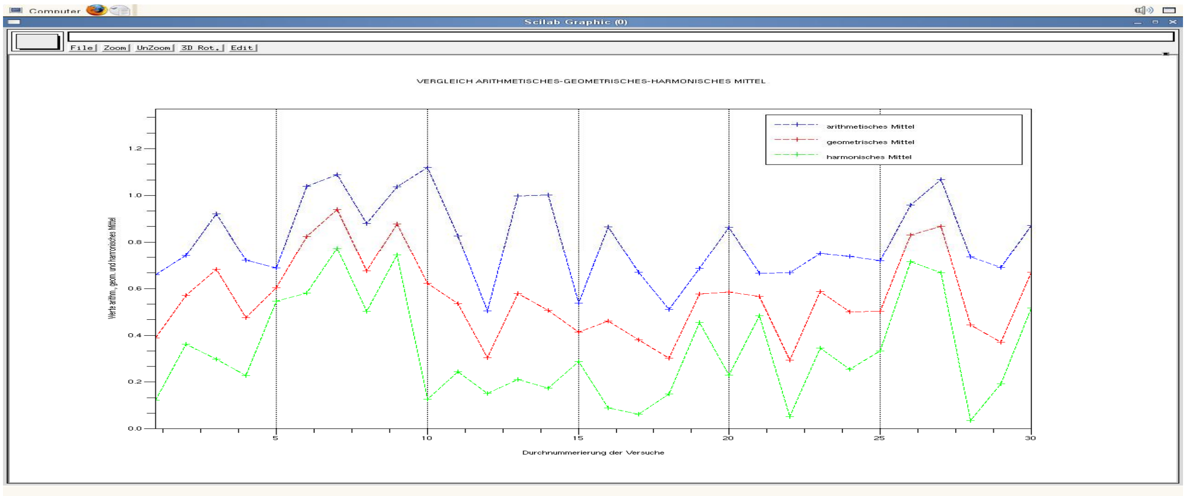

Figure 2: Visualizing the arithmetic, geometric and harmonic means of n=10 random positive numbers (30 runs).

This point of view may be illustrated by the following exercise aimed at introducing the arithmetic-geometric-harmonic inequality. The student is given the mathematical expression for the arithmetic, the geometric and the harmonic mean of positive numbers and asked to write a Scilab-function computing the means of n given positive numbers. The student is asked to test the function to ensure that it works and does generate the desired quantities. The function is then run using n-tuples of randomly generated positive numbers and their means are visualized (Figure 2).

This is done in a such a way that the student cannot fail to notice that the arithmetic, geometric and harmonic means occur in a certain order and that this situation persists when modifying the number n of positive numbers and the number of runs of the function. The student is then asked to infer a mathematical result from the visualization and to state it using sound mathematical notations.

Implementation considerations

As a conclusion we shall give a brief outline of the state of implementation of our material. For the design of the graphical applets, we rely in part on expertise and existing Java code by project MUMIE (Multimediale Mathematikausbildung für Ingenieure, teaching engineers mathematics using multimedia) at the TU Berlin. This project has been implementing applets for the linear-algebra course in the past years to much praise and positive feedback. In addition, we now have Scilab-exercises addressing all the concepts taught in single variable calculus and many ideas from multivariate calculus. We plan to test this material in an experimental tutorial during the summer term of 2008.

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Teaching Reliability Theory with the Computer Algebra System MAXIMA

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Abstract

The use of the Computer Algebra System MAXIMA as a teaching aid in an MSc lecture course in Reliability Theory is described here. Extracts from student handouts are used to show how the ideas in Reliability Theory are developed and how they are intertwined with their applications implemented in MAXIMA. Three themes from the lectures will be used in the demonstration to illustrate this: (1) Approximations, (2) Markov Modelling, (3) Laplace Transform Techniques. It will be demonstrated in the presentation that MAXIMA is a good tool for the task, since it is fairly easy to learn & use; it is well documented; it has extensive facilities; it is mature and won't undergo sudden (and perhaps undesirable) 'upgrades' causing compatibility problems; it is available for any operating system; and, finally, it can be freely downloaded from the Web. MAXIMA is a useful tool also in Reliability *research* for certain tasks. This latter feature provides a seamless link from teaching to research – an important feature in postgraduate education.

Introduction

The author has taught over successive years a one–semester lecture course entitled Reliability Modelling and Analysis (RMA) to MSc students. Students come from various academic background but all have a computing-related first degree. The RMA module, which is optional, is designed to prepare students for solving reliability-related problems in their own fields of specialisation. It is assumed that students have some prior knowledge of Probability and Statistics. A common initial standard in the basics is achieved by helping the weaker student on a one–to–one basis with reference to any standard textbook in Probability and Statistics.

It was clear to the author at the outset that computing activities, in some form, should be part of this lecture course for the following reasons:

(a) Students have a common interest and background in computers.

(b) The syllabus permits the coursework component for computer usage to be specified.

(c) In the author's experience, computer generated course material enlivens the presentation and makes the 'dry', i.e. formal, mathematics-based material more acceptable to non–mathematicians.

(d) In an applied course, the real life relevance should be emphasized by pertinent examples and exercises. There is room here for the traditional 'paper–and–pencil' work, but, to avoid boredom, they have to be accompanied by computer-based activities.

(e) There are many reported cases in the literature of successful computer usage in engineering education. Some textbooks use Computer Algebra for teaching cognate subjects such as Probability and Statistic. In all these cases commercial software is used.

Initially, the use of MATLAB was envisaged but lack of licensing for a full version of MATLAB (the *Symbolic Math Toolbox* in particular) was not available for class use. After some research, it has been decided to build up a portfolio of teaching material written in MAXIMA, a non–commercial Computer Algebra System. MAXIMA has been identified, after some experimentation, as a viable alternative to MATLAB's *Symbolic Math Toolbox*. MAXIMA is well documented and is comprehensive. Also the author had prior experience with MAXIMA in the reliability context Csenki (2007a), Csenki (2007b).

A large amount of MAXIMA based teaching material, in the form of lecture notes, exercises and coursework sheets, has been accumulated. The module descriptor allows setting a maximum of 20% programming based coursework and envisages a maximum 80% paper based examination.

Handouts are made available to show how material in Reliability Theory was combined with and illustrated by results produced in MAXIMA running under LINUX. We shall highlight three themes: (a) Approximations, (b) Markov Modelling, and, (c) Laplace Transforms.

Discussion

Details of the overall module structure, module contents, assessment and recommended textbooks are provided in the paper of Csenki (2007c).

Approximations

The ***Normal Approximation*** will be introduced in the presentation with a view to discussing later questions in Quality Control. We will start by stating an appropriate form of the Central Limit Theorem (CLT), followed by a series of examples.

The first three examples are designed to illustrate the CLT for exponentially distributed summands. A MAXIMA implementation of the convolution operation will be shown. Noteworthy is the use of recursion. Lists are an important data structure in MAXIMA as well as in LISP, the programming language which MAXIMA was written in. Our students do not have direct knowledge of LISP, the first functional programming language, but those who did our first degree in Computer Science will have been taught Haskell, a modern descendant of LISP. Therefore, MAXIMA code should be readily accessible for a great number of students on our course. MAXIMA will be used to create and plot convolution densities. Thereby students are given a 'template' for plotting graphs rather than a complete, reasoned set of instructions for doing so. It is hoped that this and subsequent examples will enable them to use the literature on MAXIMA independently e.g. Heller (1991).

After standardization, a visual impression will be given of the quality of the normal approximation afforded by the CLT. In an example we shall illustrate the CLT for a wildly oscillating density, which is interesting to students precisely because of its unusual shape and also because it allows certain numerical features of MAXIMA to be demonstrated.

Extreme Value Distributions (of the maxima type) will also be considered in the presentation with a view to preparing the ground for Structural Reliability the underlying theory for analyzing and designing structures to withstand maxima of loading variables e.g. wave loading, wind loading. The theory, embedded in engineering practice, is discussed in the established textbook Ang & Tang (1984). The recent review article Foschi (2004) may serve as a starting in the subject area. The overall framework is defined initially, followed by the student being asked to define and plot in MAXIMA the approximating densities and the limit given a specific initial density. Symbolic differentiation in MAXIMA is also an important feature here.

Markov Modelling

For the Markov material reviewed in the presentation, the students are recommended a textbook like Ramakumar (1993).

To solve Markov models, the student is expected first to consider solving the corresponding ***Komogorov Equations***. We start with the smallest, two–state system to model a process with exponential holding times. It serves to introduce the terminology and defines the Kolmogorov equations which then are solved symbolically in MAXIMA. The example is concluded with a short digression on the numerical use of SCILAB, another free piece of software alluded to in the lectures.

Next, we consider an $(n+1)$–state absorbing Markov model of the n–stage Erlang distribution. Noteworthy is here the implementation of the Kolmogorov equations by stages. Several MAXIMA features will be exemplified here: list manipulation, symbolic differentiation, recursive definition, the conditional. The example is concluded with a practical application calling for MAXIMA's numerical capabilities.

In the last item discussed in this context, the learner will be asked to apply the techniques learnt to a more complex, 6–state repair model. Students are asked to apply a dedicated MAXIMA function for obtaining the long term solution of the system *symbolically*. Two alternatives for defining an auxiliary function are shown in the model solution, one by iteration and one by recursion. It is good educational practice to consider both approaches.

Matrix Diagonalization is an alternative to solving the Kolmogorov differential equations for systems whose rate matrix can be diagonalized. MAXIMA's matrix diagonalization facilities will be exemplified here and the insight thus gained will be used to arrive at general, user defined MAXIMA functions to accomplish the task.

Laplace Transforms

There are limitations for analyzing Markov models by the previously mentioned techniques. As an alternative, Laplace transforms are also introduced in the lectures and the exponential density is taken to illustrate some of the corresponding MAXIMA functions. The illustrative example in the presentation will be a Markov model from before, now modified to be absorbing and the Laplace transform technique will be used for obtaining moments of the time to system failure.

Conclusions

Aspects of an MSc lecture course in Reliability Theory is supported by the Computer Algebra system MAXIMA. There is no attempt in the module to teach MAXIMA to the students in a systematic fashion. Instead, a collection of pertinent examples is given to them with the expectation that competence will be attained by studying these examples in conjunction with the literature on MAXIMA. Implementation in MAXIMA of additional material for this module is an ongoing task.

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On Strategies Contributing to Active Learning

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Abstract

There are well documented reasons for University staff to become even more effective teachers to meet the challenges arising from
- dropping of learning skills in mathematics during recent decades,
- changes in life conditions,
- preference for visual forms of information,
- university audiences not fully oriented to the study.

As a starting point in our considerations we take the idea that teaching means helping students to learn. In this paper we introduce and discuss ideas and strategies for active learning from Krantz (1999), together with those of Bressaud (1999). We also cite the thoughts of Zucker (1996) and a summary of strategies formulated by Zweck (2006). The inceasing web frequency of the theme of active learning shows its significance and special web pages within universities are now reserved for instructions on how to engage students in active learning.
Apart from the technical aspects there are other, inner, aspects associated with active learning: to promote positive feelings, to cultivate mastery and to provide a joy of discovery for a learner. Research of human brain implies that it is impossible to separate emotion from logical thought and learning. Some strategies that have been proved as effective include:
- questions posing,
- problem formulating as a story to be continued,
- communication with learners,
- creating a conviction that learning means self-work, mainly outside of a class.

We present our experience with some of these, and discuss also Zucker's (1996) proposal to provide a "survival guide" to students new to university study.

Introduction

We start with a short story acting as "a case study". Some years ago, I taught a student - let us call her Ann - in a two semester mathematics course. Ann was an average student, with an average knowledge of secondary school mathematics. She showed no special interests concerning mathematics, and was probably oriented more on her major study of financial management, but was able to pass all her exams at an average level. Sometimes, she came to me for advice / consultation on how to solve some more complicated problems, and I tried to show her "the whole picture", not only the single solution steps, and not only in mathematics. I also directed her to relevant chapters in the recommended reading.

Later on, I no longer worked with her in classes but saw her often helping younger fellows - explaining them maths problems. Ann was ready and even interested to help students, and she was very active when her boy-friend needed a long, patient paper-and-pencil explanation of rather elementary mathematical ideas. Due to the frequency and repetition of that work, I am convinced that in this way, Ann had read mathematical courses not only for the second time, but maybe for the eighth time or so. She had clearly found out this is not boring but even highly interesting, with gaining deeper and deeper insight into maths! This was the "side-work" done to help her colleagues, but now, of new importance for her. Now, Ann is able to solve theoretical problems in a specialised area of maths in which she was interested and works in a group with other teachers and doctoral students. Ann contributes to scientific seminars arising from her own study and own results; she will be applying for doctoral study – and with a very high probability of success.

What happened and why values have been totally changed?

Teacher's roles

In the Czech Republic as similar to other European countries there are two official duties for university teacher, and we are regularly appraised in both of them: to teach and at the same time to do a scientific research. In teaching this means to read lectures on different mathematics majors, to run practical exercises, exams, diploma works etc. A lot of university teachers of mathematics recognize that as they are fully involved in mathematics they need not be deeply devoted to the study of special pedagogical methods on how to teach mathematics or at least, teaching is often perceived as an activity of secondary importance.

The common perception is that if we are professional mathematicians, having achieved a diploma and our own mathematical education, then there is no strong need to worry about teaching or even to be specially prepared for it. Of course, for those who are interested in pedagogy there are brilliant textbooks devoted to didactics of mathematics. There is a very wide conviction for teaching mathematics on university level, the best encyclopedia on how to do this are our own predecessors, e.g. our old teachers, that we liked and we tend very much to follow just them.

It was very interesting to find a concise book of summaries on teaching practices by Krantz (1999), showing the principles on how to professionally manage single aspects of mathematics lecturing. In our opinion, the main message of Krantz‘s book is that, for a teacher, it is necessary to be instructed, well-prepared, active, curious, totally involved. But, concerning learners, he supposes - mainly - that the learner in our audience is ready and fully prepared to be educated.

This fails to be true in general, we have experienced a dropping of learning skills in mathematics during recent decades, visual form of information is mostly preferable, and university audiences are not fully oriented to study and for students the principles of successful study have not been learnt. Recent trends of the Bologna process across European countries now impose learning outcomes as the criteria of main importance. Those are defined for certain cycles of high education study, but curricula or syllabus are suppressed by them.

Special topics in Krantz's book show that standard teaching methods are taken as an undoubtedly minimal requirement for the success, but very often they are not satisfactory, even when sophistically prepared. The main problem for the majority of teachers is that our audiences are not those students who had been chosen mathematics as their life‘s ambition. Simply lecturing, even qualified explanation, the use of special computer software and other classical, or very technical methods, are not enough. We are searching for some "added value" enriching our teaching, independently on its form. What might it be?

Even remembering our own old university professors, we recognize only some of them as extraordinary. Only some of them took us virtuously into mathematical world, and only the best ones are remembered such as those who were the most enthusiastic about teaching. Nowadays, a university maths teacher has to be curious, totally involved but first of all, he/she has to be an *active teacher*, and he/she has to recognize, based on personal experience, what does it mean. That is, the extraordinary feeling has to be given to us as students or learners first, either sooner or later and only then is there a chance to pass it to our audience.

What is active learning, active teaching?

According to Zweck (2006):

- active learning means getting involved — analyzing, synthesizing, evaluating,
- active learning involves students in doing things and thinking about the things they are doing,
- active learning usually results in generation of something new, e.g., a relationship between two ideas.

To learn in an active way means *to be engaged into the topic, to deal with a problem, to overthink it, to observe connections between notions* - not only to listen or „to be informed*“: to be forced to gain information himself/herself* (with conclusions from that information as well - and this is especially important at our faculty), *and to transform it into knowledge*. Active learning is based on learner‘s smaller or greater discovery, and discovery generates positive emotions. A question arises: it is possible to create the joy of acquiring knowledge as a positive emotion, even under conditions mentioned above? But trying to do this, first we, teachers, need to tackle the active attempt; a passive teacher probably would not meet an active audience.

Active learning begins first with the interest and motivation. We are convinced that each more or less experienced teacher is able to make use of facts from history of mathematics, starting, say, from ancient Egypt through Golden Ratio notion to fractals or three-dimensional surfaces in constructions, as tools for forcing study motivation, electronic multimedia support included. Experience suggests that teacher’s personal interpretation or enthusiasm plays the crucial role, and facts only follow them.

As an initial point of consideration, we accept the idea that teaching means helping students to learn. We cannot assume that any of our students is able to find his/her engagement by oneself in a given time. How to engage students to work, to learn in an active way?

Zweck (2006) provides a short list of types of active learning:

- Blackboard work by teaching assistant (TA) with continuous, active input from students.
- Think-pair-share: Students individually think for a moment about a question posed by TA, then pair up with a classmate next to them to discuss their thoughts. Finally, a few students are called on to share their ideas with the entire class.
- Pair summarizing/checking: Students work in pairs. One summarizes a concept, an approach to solving a class of problems, or a particular problem. The other listens and checks for errors, correcting them as they arise.
- Problem posing: Individual students construct a problem regarding a particular concept, and then exchange problems with a classmate for solving.
- Critiques: Students have short pair-wise or entire-group discussions to find flaws in an argument presented by TA.

Bressaud (1999) suggests to make students active participants to provide the most effective learning. He describes a common effort of 3-4 students trying to cope with an unfamiliar problem, taking it out of the class and continuing to work on it; sharing the knowledge in a productive and inspiring way. He also stresses the role of posing questions into lectures.

In summary, either guided or directed dialogue of any form seems to be crucial – historically, this was the main principle of teaching.

Ask questions – lead effective questioning

It is useful to cite Zweck (2006) recommendations concerning strategies for asking questions:

- Ask a lot of questions at low cognitive levels to help students shore up basic skills from previous courses or earlier in current course.
- Also ask some questions at high cognitive levels.
- Wait 3 - 5 seconds after asking a question.
- Encourage students to respond.
- Probe students' responses for clarification and to stimulate thinking.
- Acknowledge correct responses: "Praise should be used genuinely, sparingly, and it should be specific."
- Design questions so that about 70% are answered correctly.
- Balance responses from volunteering and non-volunteering students.

The reader of Zweck's (2006) remarks could find there very clear suggestions how to lead effective discussion with audience, including also managerial methods - those require e.g. also *Sense of timing: When to ask a question, when to offer a summary, when to be silent.*

The following observation shows our experience of another type. Practically all courses at Faculty of Informatics and Management are located as e-learning support on the university website in a WebCT environment. Environment tools enable, for example, collaboration with fellow students organized in discussion groups. Last semester, a very intensive discussion group was generated – students invented even the own mathematical notation, and helped mutually in solving problems ("the best way to learn a subject is to teach it to another“). The communication with teaching assistants was also very rich and fruitful and it was rather surprising that this indirect electronic tool enabled the breakdown of some barriers between students and lecturers/instructors; in this way, we got a lot of questions not readily posed directly before. After this, we made use of open discussion based directly on those topics, which helped us to recognize very substantial problems not treated before.

Active learning ability contains a very significant social aspect; it can be a base of an active attempt to life in general, or to life-long learning. This is very important in connection with phenomenon of ageing generations in European population.

How to tune the audience to active learning?

At the start, majority of our students shows very poor secondary school math skills. According to Zucker (1996): "students must be told immediately that they are about to face a big jump in level from high school“, and the basic task for new students is to reorder their thinking. University students are very often given a list of basic study demands required for their successful transition from secondary level to university study level, named "Survival Guides“ to new "Academic Orientation“. Their content depends on their branch of study, but it is worth announcing these prerequisites at the beginning of their study. Response to such demands could be summarized by teachers, or students as well (Zweck (2006)). In the University of Hradec Králové, the Faculty of Informatics and Management organizes special introductory lectures into study methods where psychologists are involved as authors or consultants.

Some items from Zucker’s (1996) demands list inform new students that they can expect university study materials covering two or three times the pace of high school, that it is up to the responsibility of students to learn them, and the role of a teacher is primarily to work as an instructor only. Any department could provide recommendations how to work with textbooks, but it is student’s responsibility to learn the material. And let us add that in European tertiary education framework, evaluation based on ECTS credit system directly defines one credit as linked to corresponding students workload.

So, these are students roles. Concerning those of teachers, let us cite again Zweck’s (2006) remarks as Cautionary Notes:

- Design your sessions so that students come fully prepared to participate.
- You need to continually explain why you want active learning.
- Explain the specific goal of each active learning exercise.

- Give clear and concise instructions.
- Match type of activity to content.
- If a strategy is not working, fix it and/or change it!

Provide some feedback to your work, formulate questions:

- How is your activity a good match to the content?
- How does activity generate meaning for students?
- What instructions will you give students?
- How will you measure success?

Conclusions

At university level, students must learn on their own, outside the class, and this is the main feature that distinguishes college from the high school (Zucker (1996)). In addition, this has to be also an active study, in a large extent, and our role is to involve students into learning activities in general.

Let us state this as a necessary starting condition. Then, we suggest to consider two strategies for active teaching implementation:

1. A change of conventional lecturing, using specific methods, into an active way of delivering knowledge.
2. Apply question-posing, in different and specific forms, depending on subject and audience as well, in teaching and learning.

Let us add that the use of questions in teaching belongs to the most powerful instructional strategies leading to active learning, as shown on special workshops delivered to active learning (Eison (2008)).

Appendix

In a personal conversation with Ann, the student from introductory "case study“, she confirmed that she forced herself to become immersed into a branch of mathematics, and due to her own effort, she became more and more engaged, and positively motivated by her success.

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Exploiting new technologies in mathematics support

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Abstract

In their pre-university education, teenagers in the UK are increasingly using the internet as a learning resource. Universities in the UK have responded to this phenomenon initially in an ad hoc manner by staff making learning resources available on their own web-sites and then in a more structured way by adopting virtual learning environments such as WebCT or Blackboard. In mathematics support, the mathcentre web-site was created to make a wide range of mathematics learning resources freely available. However, teenagers use of the internet is not restricted to (or even primarily) focused on learning. Many teenagers use the internet as a tool in their social lives – as a medium for communication and for collaborative gaming. Since the majority of new undergraduates are familiar with these technologies, it is worth considering if they may be used for educational purposes within universities. In this paper we will describe some initial steps being taken at Coventry University to explore the use of the social communication package Facebook and the multi-user virtual environment Second Life as tools in mathematics support. We will present some preliminary results from an embryonic Facebook community of students studying mathematics and demonstrate a prototype mathematics support environment in Second Life.

Introduction

Facebook was established in 2004 by Mark Zuckerberg, a Harvard undergraduate, as an on-line social space for Harvard students. Facebook is an internet-based communications tool allowing members to interact virtually with other users through messaging and playing games. Membership of Facebook is free and available to anyone with an email address. Members can use search facilities to find other members, and share material with friends via postings on a profile page. The postings range from messages written using a "wall" facility to pictures and videos. Facebook supports all of the common image and video file formats and also facilitates the creation of groups and external applications. A group can be created by anyone who is a Facebook member, that member consequently becoming the administrator of the group. Group membership is controlled by the administrator, who may choose to invite members or alternatively allow membership from anywhere in the Facebook community. Many thousands of applications have already been added to Facebook. Most of these are quizzes and games, with a few, such as the chess application, allowing on-line playing between members. Within four years, membership of Facebook has increased to 65 million active users (Facebook (2008)). Whilst the early growth in membership had been amongst college students, it appeared that the majority of the new membership has been drawn from the adult age group.

Second Life is one of many online experiences (such as World of Warcraft and Guildwars) in which participants can immerse themselves within a virtual environment and communicate with other users in real time using either voice or text chat. Users are represented in the world by an avatar which can be customised and made to interact with both the environment and other users. Linden Labs, the creators of Second Life, have made the environment open for public use since 2003. At the time of writing the total number of unique accounts in use has reached approximately 12 million. It should, however, be noted that actual usage is currently significantly lower than this: 11% of accounts have been used in the previous 60 days and 4% during the previous 7 days (Second Life (2008)). Data from Linden Labs does, however, suggest that usage is increasing on a monthly basis not only from individuals but also by businesses. Where Second Life differs significantly from alternative online experiences is in its focus on the individual user's ability to change his or her virtual world. Experiences such as World of Warcraft offer a narrow experience in the sense that users roles are pre-determined (i.e. exploration and completion of quests or fighting duels in a pre-created world). Users of Second Life potentially do not have these limitations placed upon them. Both the environment and the role of user can be freely defined. Those who choose to make use of the Second Life environment are able to build objects such houses and clothes.

This paper presents a study on the use of both Facebook and Second Life at Coventry University. Description and discussion is given of an embryonic Facebook community of students studying mathematics, followed by a demonstration of a prototype mathematics support environment in Second Life.

The study

Facebook groups at Coventry University

In the early years of this millennium, the main thrust in eLearning was based around virtual learning environments and course management systems such as Moodle (see http://moodle.org) (Everett (2007)). Little thought was given to the potential of social networking sites. However, the growth in popularity of this technology has forced educators to re-think their priorities. Facebook is now listed by the Centre for Learning and Performance Technologies as No.17 in the list of top 100 learning tools (Hart (2007)), with Moodle only a little ahead at No.12.

The rapid growth in membership of Facebook now means that students who are not members are sometimes considered to be outsiders by their peer group. Most educational establishments have at least one Facebook group associated with them. Some groups, such as those set up by local students' union members, have a membership composed almost exclusively of undergraduates. However, not all undergraduates are impressed with Facebook and prefer other tools. According to Conneely (2007): "*It's [Facebook is] so bad and boring it makes Bebo look half-decent – and I've seen maths teachers using that.*" At the other end of the spectrum, groups representing Centres of Excellence in Teaching and Learning (CETLs) draw membership from those employed to teach or carry out pedagogical research.

In May 2007, two second year mathematics undergraduates at Coventry University asked **sigma** staff to create a Coventry Maths (CM) Facebook group. The aim was to gain membership from across the faculty, in terms of both students and lecturers, in order to air views and discuss mathematical topics. Early membership of the group came from within the 2nd year cohort, with a few 1st year students joining after the existence of the group was mentioned to them. The 3rd years were, understandably, less interested since it was immediately before their final examinations.

Membership of the group increased during the summer months so that by September 2007 it had reached 32 members, distributed as shown in Table 1. The UG year classification indicates the stage of the course the student was about to commence.

Classification	Staff	2nd year UG	3rd year UG	Graduate
Number	5	10	15	2

Table 1: Coventry Maths Membership immediately before new intake, September 2007

In October 2007, the new intake of mathematics students were informed during their induction lectures of the existence of the CM group. After one week, only one student had joined. During the same week, five of the 1st year students formed their own Facebook group, calling it the Numeracy Team (NT). At the end of October, the group was entirely composed of 1st years. During this period, 1st year students had also started to join the CM group.

By the end of January 2008 the membership of the 2 groups was as shown in Table 2.

Classification	Staff	Randoms	1st year UG	2nd year UG	3rd year UG	Graduate
Number (CM)	8	0	10	10	15	5
Number (NT)	1	31	19	5	11	2

Table 2: CM and NT group memberships January 2008.

The figures for membership of the NT group are somewhat distorted by the addition of 31 members who are not actually associated with Coventry University and are simply friends of 1st year UGs who have been asked to join in order to increase membership. Predictably, most staff have remained members of CM and not joined the ‘unofficial’ group. Less predictably, several of the 2nd and 3rd years have joined the group. The total number of students in each cohort are: 1st years: 33; 2nd years: 36; 3rd years: 37.

The CM group had, by this time, posted 9 discussion topics. The topic “Help! I have a maths problem” has stimulated some varied question and answer strings. Amongst these was a question posted by a 3rd year maths student: “Linear Algebra – Why?” which elicited lengthy responses from 2 members of staff. A 1st year student also asked “when you use the dot product you get a number. What is that number?”. It is interesting that both questions were about conceptualisation of the maths. So far, questions asking explicitly how to answer coursework have not appeared. The latest update for any topic in the CM group is 5 December 2007. The CM wall numbers 109 posts. These range from students posting links to maths websites to staff arranging convenient times for focus groups.

The NT group had, in the period from October 2007 to January 2008 posted 26 discussion topics. These range from football talk to lecture quotes, with no serious discussion of maths problems. The only wholly maths related topic posted was the "Mathematics Name Game" where participants had to add names of mathematicians whose first names had the same initial as the family name of the previously added name. Whilst having little or no educational value, the various topics have kept the group active, with topic updates appearing on a regular basis. The latest topic update for the NT group is 17 January 2008.

Links to various maths websites have been posted on both groups. Some of the 2nd and 3rd year students have reportedly found them useful. Many of the students have also joined Facebook maths related groups such as "I wish I were your derivative so I could lie tangent to your curves" (which has over 100,000 members). The group "Tex, \LaTeX, and such \ldots" (over 2,400 members) has become particularly popular with 3rd year students who have been writing projects in LaTeX. The great advantage of these groups is their ability to give rapid responses to enquiries made by members.

Before the creation of the Coventry Facebook groups communication between the separate cohorts of maths students was minimal. Since the creation of the groups, there are now regular Friday evening social events encompassing all UGs for all three year groups, with the occasional inclusion of members of staff. The most noticeable connection has been between 3rd and 1st year UG students. The 1st years appear to find it reassuring that they can ask advice from more experienced students. It should be noted that formal mentoring is not implemented at Coventry University.

8 staff members of the Coventry University maths department have joined Facebook. These have all join the CM group, and many have also received and accepted "friend" requests from students. Occasionally, students ask staff members questions about mathematics or statistics problems, typically during the evening when the university itself is closed. When staff are on-line, these queries are usually be answered immediately.

Second Life at Coventry University

Several institutions in the UK are experimenting with using Second Life to provide an educational experience in a virtual environment to supplement what is on offer in the real world. For example, Imperial College (Virtual medical centre where students can practice diagnosing virtual patients) and University of Plymouth ('Sexual Health' Public Education and Outreach simulation) have established a Second Life presence aimed at enriching the learning experience already obtained on the real life campuses. A detailed survey of current Second Life usage in the UK has been carried out by Kirremuir

(2007). There are also attempts to use Second Life specifically for mathematics education (Caprotti & Seppala (2007).

Second Life offers a number of features that can be of great use to educators:

- Opportunity for students and staff in different geographic locations to meet.
- Virtual conferences.
- Use of the environment to perform simulations (Physics/ Engineering).
- Construction of virtual outreach centres that can provide support which is accessible anywhere in the world.
- Facilitation of collaboration between groups in different geographical locations.

The sigma Mathematics Support Centre forms part of the Coventry University virtual campus which is currently being developed. Figures 1 and 2 show the Mathematics Support Centre in Second Life.

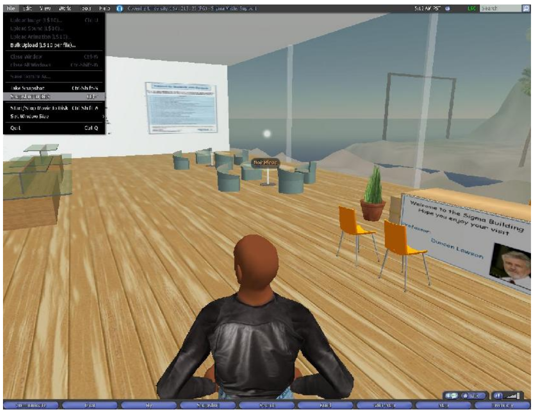

Figure 1: Reception and informal talk area located on the ground floor

Figure 2: Virtual conference room

The mathematics support centre in the virtual Second Life campus at Coventry University is currently under development with a view to supplementing the existing real life support centre and web based resources. With this in mind the goal of sigma is to try and utilise the uniqueness of the 3D environment rather than replicate materials which can already be downloaded from the internet. Sigma's Second Life support centre, once completed, will offer services for both students and staff that may or may not be separated geographically. Upon completion it is envisaged that students and staff will be able to visit the virtual support centre and be greeted by a 'virtual' member of staff who can assist with queries regarding services on offer (in real life and second life). Amongst the services that will be offered are:

- Virtual conference room (see Figure 2) - where users can attend virtual conferences and lectures. Presenters have the ability to use streaming video or images as part of the presentation. The unique feature here which is not available in other web conferencing systems is the facility for different users to interact with the same object. For example, if a presenter has created a conic object, other users who are in the conference room can interact with the object and modify it. Although cumbersome at present, in the future it maybe possible for users in different locations to construct virtual parts of a more complex machine and link them together in this environment.

- Community Area - where students and staff can mingle and chat. This is similar to the virtual chat rooms which are currently found in abundance on the internet. However the anonymity often associated with chat rooms does not necessarily transfer to the Second Life environment, as the community area is visible and seems tangibly real. In addition, users are able to uniquely customise their avatar.
- Physics and Games Laboratory. Currently under development, this will enable students to engage with virtual experiments exploring various topics such as forces and moments in 3 dimensions.
- Dissemination and other materials – It is also possible to supply users with hyperlinks or documents to further information about specific services e.g. notes for a virtual conference or instructions for a virtual experiment.. Current research carried out by staff can also be disseminated to staff, students and external visitors.

Conclusions and discussion

Although discussion is frequent and vibrant in the NT group, very little is specifically concerned with mathematics. Conversely, the “official” CM group now receives very few posts, although these tend to be on maths related subjects. More importantly, perhaps, social interaction between student cohorts has taken place. This has not occurred in previous years, and may lead to informal mentoring of students by those in higher years. Whether this results in better overall engagement and retention rates remains to be seen. At the moment the majority of existing Coventry University staff over the age of 50 are not members of Facebook. It would be unrealistic to insist that all staff join, and most argue that students can contact them by conventional e-mail if they need to. Perhaps the biggest advantage in using Facebook compared with other media in mathematics education is the facility to communicate informally with other students in the worldwide mathematics community via Facebook groups.

The potential of Second Life is only just beginning to be appreciated. Much of the work being carried out by various institutions is exploratory and as such the effectiveness of it at this time is not easy to assess. However, it would seem that there are many exciting possibilities for using Second Life as an educational tool.

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University teachers' perspectives about difficulties for engineering students to understand the Laplace transform

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Abstract

The Laplace transform is an important tool in many branches of engineering, for example electric and control engineering, but also one of the most difficult topics to master when learning electric circuit theory. We have interviewed 22 university teachers from five different universities in three countries, asking questions such as "What is the importance of the Laplace transform in engineering education?", "What are the difficulties hindering students from learning the Laplace transform?", "What is the use of the Laplace transform for solving real world problems?" and "What is its importance in students' future professions?". The answers were transcribed and analysed. Strikingly, the teachers did not have a unified view on either the difficulties involved in learning, or the importance of, the Laplace transform. In addition, the teachers were asked about their views on how the aspects of mathematics, physics and technology / application were related in the process of learning the Laplace transform. We have developed diagrams for illustrating the responses to this question that we call 'diagrams of relations'. Such diagrams will be presented from the analysis of answers from five teachers representing very different views. Although many different and sometimes conflicting views were expressed in the responses to the questions we asked we claim that our study shows that when teaching the Laplace transform it is important to focus on its applications, and teaching it as a separate topic seems to hinder the learning.

Introduction

In 1940 it was noted by Müller (1940) that: "*There is little evidence to show that the mind of modern man is superior to that of the ancients. His tools are incomparably better*". Mathematics is one of the tools modern humans are using and the Laplace transform is an important mathematical tool in many branches of engineering and science, for example electric and control engineering. However, as pointed out in several works by Grubbström and co-workers e.g. Grubbström (1967) it also has applications in other fields, e.g. economics, production technology and management science. Despite the importance of the Laplace transform it is not uncommon to hear students express questions like "*Why do I have to use the Laplace transform to solve an electric circuit?*" (González Sampayo (2006)). One of the reasons that such views are expressed may be that students find the Laplace transform very difficult to understand. Indeed, we (and co-workers) have previously found that "*In many engineering programs at college level the application of the Laplace transform is nowadays considered too difficult for the students to understand…"* (Carstensen & Bernhard (2004)), and concluded that the Laplace transform is one of the most difficult topics for students to grasp when learning electric circuit theory (González Sampayo (2006)).

However, the view mentioned above, and other results in our studies, show that it is important to study students' and teachers' views regarding the difficulties involved in learning, and the relevance of, the Laplace transform in engineering education. Here we focus on teachers' views.

This study is part of a larger series of investigations of learning, and the use of 'physical' and 'symbolic' tools for making sense and modelling, in engineering education, especially electrical engineering. In Holmberg's doctoral work (González Sampayo (2006)) both teachers' and students' views of the Laplace transform were analyzed. Students' understanding of the application of basic concepts, and the Laplace transform, in solving electric circuit problems were also studied. In other work we have also addressed aspects such as students' understanding of AC-circuits and complex numbers (Bernhard & Carstensen (2002), their understanding of the Laplace transform in the context of transient responses in electric circuit theory, the design of a laborative learning environment to enhance student understanding (e.g. Bernhard, Carstensen & Holmberg (2007); Carstensen & Bernhard (2004)), and the role of the Laplace transform in key and threshold concepts (e.g. Bernhard, Carstensen & Holmberg (2008); Carstensen & Bernhard (2008)). A common finding in all our studies is that students have problems establishing relationships between the 'object'/ 'event' 'world' and the 'theory'/'model' 'world'. It is important to make explicit these links in engineering education see for example Kaput (1987), Roth & Bowen (2001) and Tiberghien (1999).

Methodology and sample

We interviewed 22 university teachers from five universities, Linköping University, Instituto Politéchnico Nacional, Universitat de Barcelona, Escuela Politécnica Superior de Mondragon, and Universitat Politècnica de Catalunya, in three countries that were teaching topics related to the Laplace transform. Questions such as "What is the importance of the Laplace transform in engineering education?", "What are the difficulties hindering students from learning the Laplace transform?", "What uses does the Laplace transform for solving real world problems?" and "What is its importance for students' future professions?" were asked and the answers were transcribed and analysed.

In the analysis we were especially interested in teachers' views on the relationships between three aspects of the process of learning the Laplace transform:

Mathematics: The role/importance of the Laplace transform in mathematics.
Physics: The role/importance of the Laplace transform as a part of describing nature.
Technology and/or application: The role/importance of the Laplace transform in different fields, e.g. as a tool in economics or automatic control.
To analyze and summarise these aspects from our interviews with university teachers we have construed 'perspective dependent relational diagrams', using the method developed by Holmberg (González Sampayo (2006)). To illustrate the method we present one transcript below, together with the relational diagram construed from the interview.

> **Transcript from interview 15:** *"I think that is very difficult to divide in 3 aspects. I think that are all connected and you cannot you divided very clear I meant, you need 1st to understand what the Laplace transform is to be able to use it; you most then understand…I cannot see very much the physical aspect of the Laplace transform, honestly, I meant, I cannot see very much the difference*

between physical and technological aspect at the least from my point of view I mean the Laplace transform is the transform so it is a mathematical definition with all the proprieties and then it makes it possible to change, to study some problems in Laplace domain instead the time domain that it can be very much more difficult for us because the transfer function becomes much easier to study the proprieties of the system items of Laplace. I cannot see very much for me the difference between physic and technology? could be(program)? One that I have understood how you can use in some… then of course you can use also any program, I can think only Matlab for me because is one that I use more then I can use Matlab but is only way of computation. Matlab makes only faster computation that you don't have to do by hand but steel one is to understand what is behind, in my opinion, other wise the risk is that you can give a wrong interpretation of the results if you just do not have clear what the Laplace transform represents. This is so there are…. A mathematical definition more understand why and how they can use and then of course you can also using in the program that is the easier step."

To construct the resulting diagram shown in Fig 1. we extract the most important points from the interview:

1. I think that is very difficult to divide it into three aspects.
2. I cannot see much difference between physical and technological aspects.
3. The Laplace transform ... is a mathematical definition.
4. I can use Matlab but it is only a means of computation.

:

Interview I15

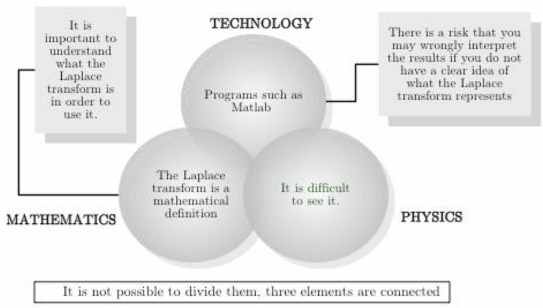

Figure 1. Relational diagram for one teacher

Results

We will here first briefly describe answers from university teachers to some of the questions asked in the interviews. One of the questions given to the teachers was: "*Is the Laplace transform a difficult topic to learn for engineering students?*" We identified three groups of perspectives in teachers' answers:

A. Teachers that consider the Laplace transform to be a difficult topic for students to learn. Typical answers in this group included: "The Laplace transform is an abstract concept"; "The change from the real time to frequency domain produces confusion"; "Engineering students do not see any real application"; "It is difficult to give it a physical interpretation".

B. Teachers that consider the Laplace transform to be either difficult or not difficult to learn. In this group the answer was that "it depends". It was considered to be difficult "if the Laplace transform is studied in depth" or "for engineering students without a strong mathematical background”. However, the Laplace transform was not considered to be difficult “if it is considered as just a table to solve differential equations” or “for students with a mathematically intensive background”.

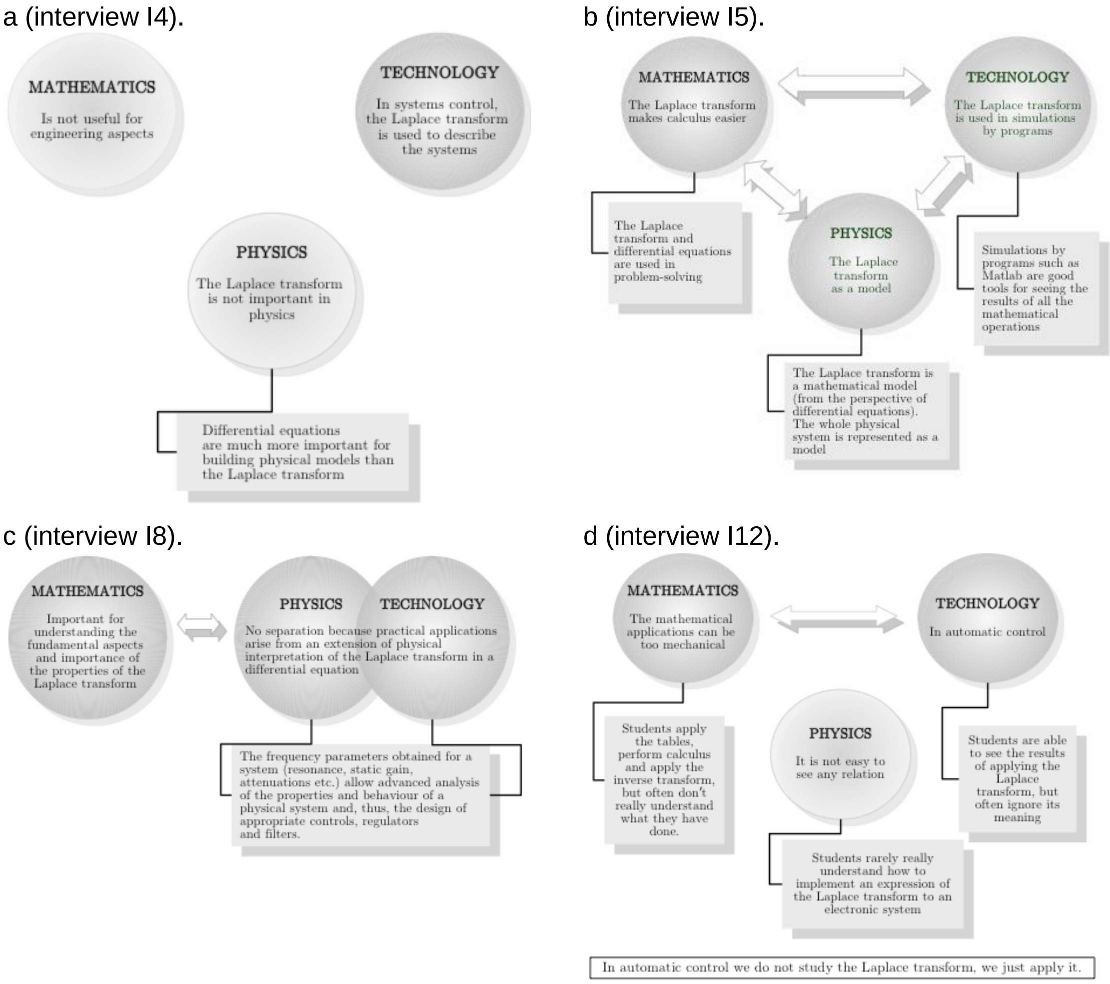

Figure 2. Diagrams of relations for four different teachers regarding the relationships between mathematics, physics and technology for the Laplace transform

C. Teachers that consider the Laplace transform is not difficult to learn. Typical answers in this group included: “It is just a procedure to solve differential equations”; “It is only a table to solve differential equations”; “If the students fail using Laplace transforms it is because they make errors in basic mathematical calculations”.

The teachers were asked to comment on actual statements made by students. One statement was “To use the Laplace transform formulas it is not necessary to solve real world problems, it is just a requirement to pass the course” and another was “The Laplace transform is just a requirement to pass the course and unnecessary for my future job”. We found that teachers’ responses to these statements corresponded very well to their grouping into the categories A, B and C listed above. Teachers in group A did not explicitly disagree with the first statement. However, teachers in groups B and C disagreed with it and gave reasons for their point of view. Regarding the second statement teachers in group A expressed the view that “it depends on the type of job the student will do". In contrast, teachers in groups B and C expressed the views that "it is a misguided idea" and that "it is not true", respectively.

Teachers were also asked about their views on the relationships between physics, mathematics and technology (see above section regarding methodology). The diagrams of relations construed from the interviews with five different teachers are presented in figures 1 and 2. These diagrams clearly show that the teachers expressed very different views. One teacher (I15) regarded all aspects as integrated and impossible to separate while another (I4) did not see any relation between the aspects.

In the interviews several teachers indicate beliefs that it is essential to stress the scope for applying the Laplace transform, but that use of many different transforms is confusing for students.

Conclusion and discussion

A striking conclusion of this study is that university teachers teaching or using the Laplace transform, did not have a unified view about either the difficulties involved in learning, or the importance of, the Laplace transform. Although it is well-known from educational research that it is extremely important in teaching to make links explicit, it is apparent from the diagrams of relations construed in our study that teachers themselves have established very different links between, for example, mathematics, physics and technology.

It should be noted that, with few exceptions, we did not detect any national differences in our interview data. However, we did detect differences in views among teachers at the same institution. This implies that students are likely to meet teachers with many different views on the relationships between different aspects of such topics. In another study Sundlöf, Carstensen, Tibell & Bernhard (2003) noted that "confusion might ensue [if teachers' and students'] conceptions of representations' ontological and epistemic status [were in conflict]". We here extend this claim and argue that confusion might also ensue among students if different teachers' conceptions are in conflict.

Educational research often focuses on students' different conceptions and so-called mis-conceptions. Our study points to the importance of studying the conceptions of teachers as well. When designing teaching-learning environments we must be equally aware of the many different conceptions held by teachers as well as students. The findings also show the importance of teachers explicitly discussing these matters with their fellow teachers and being aware that their colleagues may hold different views from their own.

Although many different and sometimes conflicting views were expressed in this study, we claim that our study shows that when teaching the Laplace transform it is important to focus on its application, and teaching it without making links between the 'world' of 'objects' and 'events' seems to raise obstacles for learning.

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An active learning approach to teaching mathematics at Kaunas University of Technology

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Abstract

The paper presents some active learning methods applied to the study process of engineering mathematics at Kaunas University of Technology (KUT). Promotion of active learning techniques in teaching mathematics at KUT was launched a few years ago, following the decision of the university authorities to implement active learning strategies (with the use of ICT facilities) in organizing tutorial classes in mathematics throughout the university. Therefore eleven specialized classrooms for teaching mathematics have been equipped at KUT.
In the paper, applications of active learning methods are discussed. During tutorial classes, mathematics software and interactive means are available. Students are enforced to change from passive listeners to active learners. To prepare teaching material, to promote active learning in discussion sessions, to monitor the study process, to interact between students and lecturers, to support brief question-answer sessions, a virtual learning environment WebCT and EDU Campus System are used.
Some advantages and weaknesses of the implemented active learning methods are presented too.

Introduction

Involvement in engineering studies in Lithuania has decreased considerably in the last few years. The majority of school-leavers prefer social or business studies at the university level rather than engineering studies. Every year more and more gifted school-leavers give priority to social or business studies and it means that the mathematical knowledge of first year engineering students at KUT (and not only!) fall gradually down. The first year engineering students cannot answer the theoretical questions during various tests on mathematics, because the secondary schools do not pay much attention to the theory of mathematics - they focus mainly on solving practical exercises. The results of first semester exams at KUT in 2002-2003 showed that nearly 30 percent of all first year students passed an exam in mathematics on their first attempt (Table 1). After some additional preparation (and several attempts) the percentage increased to 62. The rest of the first year students dropped out from the university or took the course on mathematics repeatedly, because they did not reach the required standard (the SEFI MWG recommendations for technical universities (Mustoe & Lawson (2002)) in a prescribed time period.

Summing up the situation, the university authority has decided to improve the curriculum of mathematical studies and the quality of the teaching process itself by introducing the following activities:

- Implementation of active learning strategies (using IT facilities) in order, and organizing tutorial classes on mathematics.

- Expansion of the usage of mathematical software in the study process of mathematics.
- Application of virtual learning environments in order to make teaching and learning of mathematics more student-friendly.
- Providing students with the opportunity to study themselves, to test their knowledge on mathematics, to develop their mathematical abilities.

Faculty	The number of first year students	Successful ’’passes’’ (first attempt)		Successful ’’passes’’ (after first and repeated attempts)	
Computer science	489	183	37,42%	362	74,03%
Telecommunications and Electronics	259	111	42,86%	196	75,68%
Electrical and Control Engineering	244	54	22,13%	130	53,28%
Chemical Technology	220	67	30,45%	125	56,82%
Civil Engineering and Architecture	139	20	14,39%	81	58,27%
Art and Design Engineering	275	75	27,27%	152	55,27%
Mechanical Engineering	328	28	8,54%	112	34,15%
Social Sciences	249	47	18,88%	147	59,04%
Economics and Management	233	91	39,06%	191	81,97%
Fundamental Sciences	150	87	58,00%	115	76,67%
Total	**2586**	**763**	**29,51%**	**1611**	**62,30%**

Table 1. Mathematics exam outcomes (First year students at Kaunas University of Technology; autumn semester, 2002-2003)

Learning environment

The aim of an active learning environment is to maintain and encourage students’ motivation to learn, to make them feel confident and ambitious during their studies (Jonassen & Land (2000)). The active learning environment at KUT was built according to the following principles:

- Students have to demonstrate active performance in their studies, i.e. they are not encouraged to gain their experience only from the books or sources provided by university.
- Some part of tedious mathematical calculations during tutorials should be replaced with calculations using appropriate mathematical software. Every time, attempts should be made to separate mathematics, which does not require much understanding and which can be done with the computer, from mathematics, which requires deep understanding and careful studying.
- Activeness of students should be increased by applying interactive approach (Croft & Davison (2004)).
- Students should be taught to solve real life problems and should understand that learning of mathematics is important and will benefit them in the future.
- Lecturers should encourage students to give feedback about their studies.
- Discussions on the matter should be organised, because they not only motivate students to be more active in their studies, give them an opportunity to develop

various theoretical and practical skills, but also this is a very effective feedback (MacKnight (2000), Crouch & Haines (2004)).

To achieve the goals of the active learning environment, 11 mathematics teaching classrooms were set up. Each classroom was equipped with 25 computers and the Internet connection, mathematical software and audio-video devices. This environment was created on interactive learning basis, which is focused on the learner-centered approach.

Mathematical software. For the basic mathematical courses, which are provided in the computer classes, the mathematical software MathCad has been chosen. First of all, the MATHCAD script is very similar to one used in the mathematical textbooks and, consequently, can be easily accepted by the students.

To say more, using MATHCAD it is possible to write a text, formulae, perform interactive calculations, use plots and diagrams, etc., i.e. easily create "live" electronic books. MathCad also has advanced features for document processing, which allow you to create complex mathematics documents faster and easier than using a word-processor or LaTeX.

MathCad software is used to create electronic tutorials - worksheet sample problems and solutions. These tutorials cover all major topics on mathematics. Students can easily edit these worksheets (change parameters, tables, graphs) and adapt them to solve standard and non-standard mathematical problems on differential equations, linear algebra, probability, statistics, etc. Using MathCad worksheets helps students to gain more knowledge and experience in mathematics and maximizes benefits of the course.

Obviously, technology-based learning materials have to be transparent and enable their users to inspect, test and modify them (Kadijevich, Haapsalo & Hvorecky (2005))**.** MathCad worksheets contain these features. In the opinion of many students using MathCad this software makes learning of mathematics easier and more attractive.

A virtual learning environment (VLE) WebCT. Students can access the course materials during and after the computer-based tutorial classes using WebCT. During the tutorial classes, students use a wide range of interactive websites designed for learning mathematics and statistics. There exists a wide range of websites focused on learning mathematics, which can be perfectly used in mathematics tutorial classes. WebCT is used for distance and face-to-face teaching. Students can always be in touch with the latest information if they have missed a class: all handouts and course related information can be found in the WebCT website. For instance, using WebCT students can find videos (created and uploaded using software VIP - product of KUT) of lectures from the course "Probability theory and statistics". WebCT also provides many links to additional mathematical resources for students, who need extra support. WebCT is mainly used for the preparation of learning materials for various mathematical courses, for promotion of active learning using discussion sessions, monitoring the study process and interaction between students and lecturer.

The virtual learning environment (VLE) enables students to work at their own places and invites them to pay more attention to things which need more knowledge and practice than others. University staff and students evaluate the WebCT positively and point out many benefits of VLE. Factors increasing motivation of students are as follows: more relevant module content, additional support, some schedule changes, new assessment schemes, more intense use of ICT.

We agree with the point that "The class-based tutorials are the cornerstone of the well-prepared mathematical courses. It is only a little possibility that at present or in the near future, such tutorials can be efficiently replaced by the on-line tutorials. Many features of the traditional "face-to-face" teaching cannot be properly implemented in WebCT" (Foster (2003)).

Simulation exercises. Integration of simulation into the learning process breaks the gap between the theory and real life problems, explains and demonstrates how real life processes can be transferred to mathematical models. Simulation is used for developing logical thinking and problem solving skills. By combining simulations with assessment technology the same tools can be used for learning and assessment. This can be particularly beneficial where learning outcomes require more than the demonstration of knowledge (Thomas & Ashton (2005)).

The interactive simulations provide students with a better understanding of more difficult problems. For instance, consider the influences of extreme observations on the regression line of a particular data set. Using an applet, one can see the effects of the extreme observations to the regression line.

It is the usual thing that students often have to master theory (e.g. regression analysis) before they start using its outcomes. In order to avoid such confusing and demotivating practice, technology should be used continuously along learning paths, provoking further learning of mathematics. On the other hand, when technology produces inappropriate results it is a great opportunity to gain more knowledge of the applied tool (MacKnight (2000), Crouch & Haines (2004)).

Assessment methods. WebCT and EDU Campus System (EDU Campus (2007)) are used to support of brief question-answer sessions, self-assessment and assessment during the semester. The assessment methods, currently available in WebCT and other VLEs, are not adequate for all mathematical courses. Multiple choice and similar types of questions do not test adequately basic mathematical skills, which are compulsory for students to progress satisfactorily in mathematics courses (Foster (2003)). Therefore the EDU Campus system was chosen for the creation of the active learning environment. This software has few shortages assessing the students compared with WebCT and many other tools. EDU campus system allows building of assessment documents with MathML 2.0, Maple, LaTex or WebEQ. Questions then can be exported to WebCT. EDU campus system maintains a wide range of question types: multiple choices, multiple selection, short answer, fill-in-the-blank or clickable image. Clickable image questions present an image with a number of "hotspots", and students are required to identify the correct image element by clicking on the corresponding hotspot. Lecturer can combine these question types within a single question depending upon the complexity of the concept tested and intent of assessment. EDU Campus can automatically generate content in any of its question types. EDU supports several Applet Interaction Questions.

Use of computer aided assessment system consolidates students' learning and learning results. Students at KUT gain their final grade in Mathematics by taking appropriate computer tests (70 %) and passing theoretical exam (30 %).

The influence of the active learning approach on the study results

Applying mathematics software, virtual learning environment and active methods to teaching and learning of engineering mathematics at KUT during the last five years (from 2002 to 2007; Figure 1) increased the number of students, who succeeded in getting a "pass" on time, by 27.1 percent, i.e. from 29.5 percent (the entrance year 2002) to 56.2 percent (the entrance year 2007).

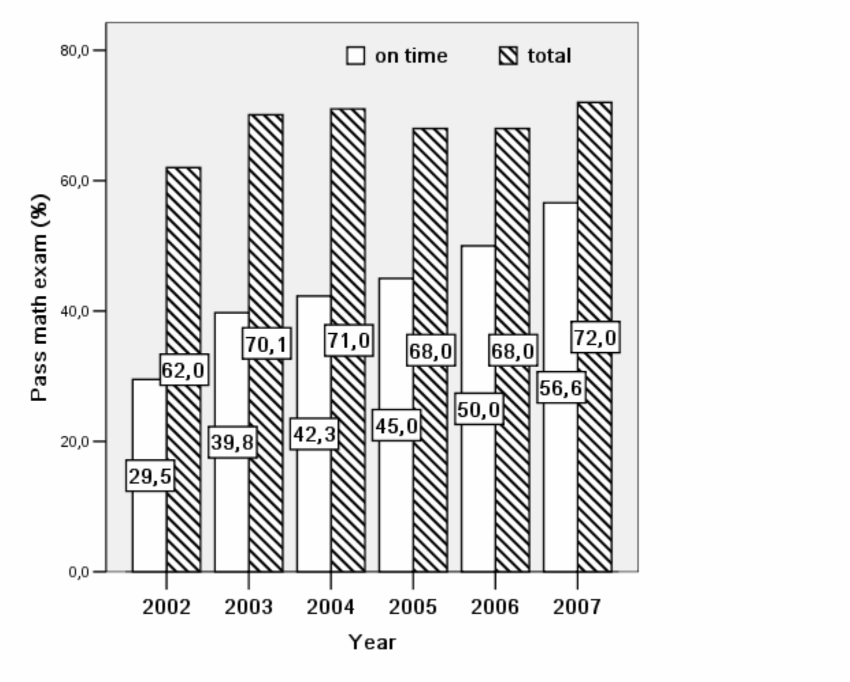

Figure 1: Distribution of successful ''passes'' mathematics exam (first attempt-on time) and successful ''passes'' after first and repeated attempts (total).

The total number of first year students passing exams (on time or repeated attempts) remained unchanged (70%). Additional analysis is needed to answer the question – why 30% of first year students are unable to pass mathematics exam?

Conclusions

Using the active learning environment undergraduate students are more involved in the learning process, their learning outcomes evidently improve. Some advantages of the discussed learning environment are listed below:

- It facilitates and improves students' understanding of basic concepts (through the use of new instructional materials).
- It provides students with a flexible learning medium.
- It provides students with individualized feedback and common interface, both being helpful and intuitive.
- It provides support to faculty and students in developing and expanding diversified mathematical materials.
- It facilitates monitoring of the student progress.
- It develops a more positive attitude towards learning mathematics.

Among the weaknesses of the environment we mention the following ones:

- Lecturers of mathematics are forced to develop additional problem-oriented material for the active learning of students. Because of rapid changes in the mathematics software development, the used learning material should be permanently updated.
- Some of mathematics lecturers do not have sufficient ICT knowledge and skills, so they cannot realise the real power of computer-based modelling. It keeps down further implementation of ICT in the study process.
- It takes more time to write mathematical expressions using computer's software in comparison with hand-writing.

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Helping Engineers Learn Mathematics: a developmental research approach

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A mathematics module in the undergraduate programme for first year engineers aims to enable those with low mathematical qualifications to understand and use efficiently calculus and related topics. The teaching approach is designed to develop student's fluency, understanding and responsibility through creating an inquiry community, encouraging students to engage with materials and support opportunities, and extending their thinking through investigative problems. Research is exploring the provision and outcomes of the module. Findings so far indicate highly variable patterns of attendance at sessions and scores on class tests. Attitudes to the module, relationships with students attending sessions and appreciation of materials and support are generally good. Achievement of the aims of the module, however, is relatively low.

(The full paper will be published elsewhere.)

Use of Voting Systems in Lectures at Loughborough University - A Review of Staff Experiences

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Abstract

Academic staff from the Mathematics Education Centre (MEC) have recently been using electronic voting systems (EVS) to teach Mathematics to undergraduate Engineering students. Staff from other departments at Loughborough University have also been using EVS to teach non-Mathematics subjects. This study was designed to investigate the views of affected staff about the use of EVS in lectures and associated pedagogic implications. The results show that EVS is generally seen as an effective teaching tool but that its value for Engineering Mathematics teaching requires further investigation.

Introduction

There has been a university-wide initiative at Loughborough University to make lectures more interactive and get students more engaged by, among other things, introducing electronic voting systems (EVS) into lectures. Consequently, a number of Loughborough academic staff have incorporated the use of EVS into their lectures. The staff come from a variety of disciplines, and among them, are two staff from the Mathematics Education Centre (MEC) who mainly teach Engineering Mathematics to undergraduate students. Although there is a perception that mathematicians are often reluctant to move away from traditional methods of teaching, this is not the case at Loughborough as MEC staff have embraced the use of technologies as tools to deliver lectures to Engineering undergraduates. Apart from EVS, electronic whiteboard-enabling devices such as tablet PC are also being used. However, this paper will focus only on the presentation of the findings of a study on EVS use in teaching Engineering Mathematics and other subjects at Loughborough University.

Electronic voting systems (EVS) are portable software systems that a lecturer may use to ask students to respond to a multiple-choice question (MCQ) during a lecture. The EVS used at Loughborough University was Turning Point (TP), 2006 version. This study reports on the use of Turning Point (TP) voting systems at Loughborough University, specifically during the first term of the 2007/2008 academic session. Consequently, this study was designed to provide insights into the following research questions: (1) What are the perspectives of staff on the use of EVS in lectures? (2) What are the implications of EVS use on teaching and learning? The outline of the paper is as follows: Sections 2 and 3 focus on literature review and methodology respectively, while Section 4 is an analysis of the findings. The conclusion and suggestions for future work are the focus of Sections 5 and 6 respectively.

Literature Review

Some of the earliest reports of EVS use in classrooms include those of Cue (1998), 1998) and Hake (1998). The single, most important benefit of EVS use, identified from literature review, is its capacity to enhance, catalyse or increase student engagement during lectures together with associated pedagogic applications like Peer Instruction (Mazur, 1997) and Just-in-time Teaching (Novak et al., 1999). Some of the EVS-related publications have been subject specific – Physics (Dufresne at al., 2004), Economics (Elliot, 2003), Engineering (Nicol et al., 2003), Philosophy (Stuart et al., 2004), and Medicine (Collins, 2007); while others have been more generic including Draper (2002), Duncan (2005), Caldwell (2007) and to some extent, Hake (1998) and Crouch & Mazur (2001). The main distinction of this study is that it will focus on staff perceptions on the use of EVS with a view to highlighting any peculiarities that may arise based on the use of the technology at Loughborough University.

Methodology

This section describes the methodological procedures adopted for this study.

Sample

Six members of academic staff at Loughborough University who have used EVS to deliver lectures during the first term of the 2007/2008 UK academic session participated in this study (going by current available data, only 10 members of staff at Loughborough University are known to have used EVS in their lectures). Two of the participants are Mathematics staff from the Mathematics Education Centre and teach Mathematics to Engineering undergraduates. The other four participants are from non-Mathematics departments. The participants are split equally between those who, according to their submissions, easily take to new technologies and those who do not. The participants mostly teach first or second year undergraduate students, with class size ranging from 12 to 300 students.

Methods

This study was conducted based on a triangulation approach and consists of a blog, observations, informal feedback, questionnaire and interviews.

I. Blog: A blog was created by MEC staff for them to 'journal' their thoughts and experiences about the use of the technologies that had just been introduced into Mathematics lectures for the 2007/2008 session. Only the two members of staff (of the six respondents for this study) from the MEC contributed to the blog.

II. Observations: One of the authors sat in on classes where voting systems were used and monitored staff and students' attitudes towards the use of the systems

III. Informal Feedback: This consisted an author discussing with staff and students who had used or been in lectures where voting systems had been used about their views on the use of the systems

IV. Questionnaire: All six participating staff completed a Bristol online questionnaire. The questionnaire had been designed with input from the findings obtained from the blog postings, observations of classes and informal chats with staff and students.

V. Follow-up interviews: This consisted of interviewing all six participating staff after they had completed the questionnaire and further clarifying their responses to the questions posed

Results, Analysis and Discussion

The focus of this section is the presentation and analysis, under the relevant categories, of a selection of findings from the study.

Pedagogical considerations

A voting system such as TP is basically a way of using interactive software to pose questions, usually MCQs, to students and get their feedback in real time. What questions are used for, how they are used, when they are used and the quality of the questions used are thus important elements to consider in order to evaluate the pedagogical impact of EVS use on the teaching and learning process. One of the main goals of the questionnaire and interviews was to get feedback on the use of questions with EVS. The corresponding submissions, reported in Table 1, show that MCQs used with EVS fall into three broad groups and these are: ice breakers, mini-tests and ConcepTests (Crouch & Mazur (2001)). The type of MCQ used and the goal(s) for using it subsequently determine the pedagogical implications. The relationship between EVS use and pedagogy is not presented in this paper.

Impact on Teaching Practice and Style

Participant response to survey showed that all respondents struggled initially with the increased preparation time associated with creating MCQs and learning how to use the TP software in order to use EVS in class. The preparation time however tended to decrease as the term progressed and staff’s confidence levels in using EVS increased. One participant noted that it was a challenge to cover lecture material in classes where EVS was used. This was due to a number of factors including the number of MCQs used (using more questions reduces the time available for a lecture); the difficulty level of an MCQ – tougher questions take longer to solve; student response time allocation; equipment setting up and closing down time; and (individual) lecture class management approach.

Analysis of data from respondent submissions indicates that EVS use has impacted the teaching practice of the participating staff in a number of ways. Two staff reported that they had revised their course notes based on feedback during EVS use in class. Two other participants also noted that use of EVS has made them put more thought into how lectures may be made more interactive and how to select the MCQs that will achieve this goal. But the most common observation is that EVS use has helped staff to identify the topics or areas that students find challenging. One participant remarked that often lecturers only get to know the areas students are struggling with when marking the end-of-term assessment; by which time it is too late to address student problems. Use of EVS enables student feedback in real time during the lecture phase of an academic semester. A similar finding is the remark by one of the participants that it was surprising to discover, via EVS feedback, that students were struggling with material the lecturer had assumed they would find easy to understand.

MCQ type	**What questions are used for i.e. goals**	**When (in lecture) they are used**	**Pedagogical implications**
Mini-test	1. To determine knowledge level	Beginning	Contingent teaching
	2. To test recall of previous lecture/material	Paced	Formative assessment/ Identification of problem areas
	3. To maintain student interest throughout lecture	Paced	Class management
	4. To encourage interaction or discussion	Paced	Interactivity
	5. To get students thinking beyond material taught in the classroom	End	
Ice breakers	1. Lighten up the mood in class for better student receptivity to lecture 2. Test if the EVS handsets and system are working 3. To maintain student interest throughout lecture	Beginning, otherwise paced/end	Student Motivation
Concep Tests	Determine student understanding of a topic	Paced	Group interaction and discussion (Peer Instruction)

Table 1: The role of questions (MCQs) in EVS use at Loughborough

Another surprising finding is an observation by one of the respondents that EVS use can, instead of promoting engagement, actually distract students. Student distraction may occur when the MCQ is too easy, or if some of the handsets do not work, as has sometimes been the case – students respond quickly and then use the remaining response time allocation as a window of opportunity to chat with friends. It should be noted that a group of second-year students who seemed to have shown some signs of being distracted when EVS was used had previously been exposed to EVS, having been taught in their first year via the technology. It might thus be possible that they are either slightly bored or no longer so enthralled with the technology. This is a behavioural issue that the authors will investigate in future studies. However, review of pertinent literature suggests that boredom or student familiarity with the technology has not been a problem where EVS has been in use for longer periods. For instance, EVS has been in use at Glasgow University for over five years (Draper et al. (2004)) and none of the publications from the EVS project has reported explicitly any major downside that is purely due to a regular and prolonged use of EVS.

Perceived Usefulness

All participants rated EVS as either “useful” or “very useful” on the questionnaire. However, the responses to a question on whether EVS is appropriate for teaching Mathematics drew an interesting selection of responses. All but one of the four staff from a non-Mathematics background thought that EVS is appropriate for teaching Mathematics. However, the MEC staff seemed ambivalent about the appropriateness of EVS use for Mathematics. This is in contrast to the evidence in literature which suggests that EVS has been successfully used in Mathematics lectures – see, for instance, the GoodQuestions project at Cornell University.

The ambivalence conveyed through the MEC staff responses may be partly due to the difficulty of finding the right (Engineering) Mathematics questions to use with EVS. It is also difficult to set ConcepTest-type questions which require calculation and, at the same time, stimulate discussion. Another factor that may explain the ambivalence is that the two Mathematics staff have different motivations for using EVS. One initially embraced EVS so that it could be used to set ConcepTest MCQs, but has not been able to do so thus far due to reason cited earlier. The other staff uses EVS to mainly ask non-demanding questions. Thus the capabilities that EVS can offer have not been fully utilized in (Engineering) Mathematics lectures. It is the aim of the authors to further investigate the ambivalence surrounding the use of EVS in the teaching of Engineering Mathematics at Loughborough in future studies.

The **key benefits** of EVS use that participants identified are:

- EVS can be used as a formative assessment tool, which in turn can help identify the areas where students are struggling
- EVS use promotes interactive engagement including student-to-student interactivity
- Increases student participation and contribution levels (one key feature is anonymous voting which encourages students, such as shy international students whose first language is not English but are studying at an English university, who otherwise would not participate in class to contribute)
- Its use can catalyse student motivation and interest (e.g. use of ice breakers)
- It can be deployed for contingent teaching purposes

Key Requirements for Effective Use

The following were identified from participant responses as key requirements for maximising the effective use of EVS in Lectures:

- The selection and use of good questions which should include appropriate distractors
- Creation of a bank of relevant, subject-specific EVS questions
- Allocation of adequate time for student response and/or subsequent discussion
- Need to use EVS for stimulating thought and reflection and not just to test memory
- Not overusing the technology
- Creation of a university-wide support forum for sharing tips and ideas on how to use EVS glitch-fee and effectively

Barriers

Analysis of the responses to an open-ended question about the barriers to the use of EVS identified the following as being the most important:

1. Provision of adequate technical support and personnel – this was particularly important for the Mathematics staff
2. Time constraints which include (length of) equipment setting up and closing down time; increased lecture and MCQs' preparation time
3. The confidence level with which EVS is used by staff support and time constraints)

Implication of Findings for Future EVS Use

The findings from this study and the observation of actual EVS use in classes suggest that the following will be valid for future EVS use in Mathematics lectures and for other subjects at Loughborough University:

1. The confidence level with which EVS is used by staff will (continue to) increase with time
2. Technical issues will be less of a problem, partly because staff will be more adroit at fixing technical glitches
3. The EVS questions that have been created from scratch will form a pool which staff can draw from, thereby reducing time spent on question preparation time
4. The effect (if any) of overuse and previous student exposure to EVS will become more evident
5. Staff conviction about the impact of EVS use on student performance and their teaching in general will also become more evident
6. Staff preparedness to, if required, change fundamentally or significantly their pedagogic practices to maximise the effectiveness of EVS use in classes

Conclusion

This paper reported the findings of a study aimed at obtaining the views of Mathematics and other subject staff who use EVS at Loughborough University. The study also sought to evaluate the impact the use of EVS has had on pedagogy. The findings indicate that the participating staff generally see EVS as a useful teaching tool, with some having adapted their teaching methods based on feedback obtained from EVS use. In general, further research needs to be undertaken to evaluate the significance and effectiveness of EVS use on student learning and achievement. It is expected that pedagogic issues would be addressed more in future when staff have had time to reflect on initial EVS use.

Future Work

- Another phase of the research study described in this paper will focus on getting student views and perspectives on the use of voting systems
- Future study will also seek to measure the impact or influence of EVS use (if any) on student performance

- In addition, future studies will involve a closer investigation of the pedagogical issues that have been highlighted in this study
- This study represents the first phase of a longitudinal study which hopefully will lead to a comprehensive examination of the effectiveness of EVS use in Mathematics at university level

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Increasing Engineering Students' Awareness to Environmental Issues through Innovative Teaching of Mathematical Modelling

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Abstract

This paper presents the results of two studies on using an innovative pedagogical strategy in teaching mathematical modelling and applications to engineering students. Both studies are dealing with introducing non-traditional approaches for engineering students in teaching/learning of mathematical modelling and applications to the environment and ecology. The aims of using these approaches were: to introduce students to some of the techniques, methodologies and principles of mathematical modelling for ecological and environmental systems; to involve the students in solving real-life problems adjusted to their region emphasising the aspects of both survival (short term) and sustainability (long term and to encourage students to pay attention to environmental issues. On one hand, the contexts are not directly related to engineering. Most of the graduates in engineering will be dealing with mathematical modelling of environmental systems in one way or another in their future work since nearly every engineering activity has an impact on the environment. The first study is a parallel study conducted in New Zealand and Germany with first-year students studying engineering mathematics. The second study is a case study based on the experimental course Mathematical Modelling of Survival and Sustainability taught to a mixture of year 2-5 engineering students in Germany by a visiting lecturer from New Zealand. The models used with the students in both studies had several special features. Analysis of students' responses to questionnaires, their comments and attitudes towards the innovative approach in teaching are presented.

Introduction

There are many papers devoted to investigating undergraduate students' competency in the mathematical modelling process. We mention some recent research. A measure of attainment for stages of modelling has been developed in Haines & Crouch (2001) These authors expanded their study in (Crouch & Haines (2004)) where they compared undergraduates (novices) and engineering research students (experts). They suggested a three level classification of the developmental processes where the learner passes in moving from novice behaviour to that of an expert. One of the conclusions of that research was that "students are weak in linking mathematical world and the real world, thus supporting a view that students need much stronger experiences in building real world mathematical world connections" (Crouch & Haines (2004)). It echoes with the findings from a study of 500 students from 14 universities in Australia, Finland, France, New Zealand, Russia, South Africa, Spain, Ukraine and the UK (Klymchuk & Zverkova (2001)). The study indicated that the students felt it was difficult to move from the real world to the mathematical world because of the lack of practice in application tasks. An investigation of undergraduate students' working styles in a mathematical modelling activity has been done in Maull & Berry (2001) and a study on the development of transferable skills in undergraduate mathematics students through mathematical modelling in Nyman & Berry (2002). Some relationships between students' mathematical competencies and their skills in modelling were considered in Galbraith & Haines (1998) and in Gruenwald & Schott (2000). Kadijevich pointed out at an important aspect of doing even simple mathematical modelling activity by new coming undergraduate students: "Although through solving such … [simple modelling] … tasks students will not realise the examined nature of modelling, it is certain that mathematical knowledge will become alive for them and that they will begin to perceive mathematics as a human enterprise, which improves our lives" (Kadijevich (1999)).

In this paper we consider engineering students' feedback on introducing non-traditional approaches in teaching / learning of mathematical modelling and applications to the environment and ecology. The approaches are not directly related to engineering but the expectation is that most graduates entering engineering will be dealing with mathematical modelling of environmental systems in one way or another in their future work because nearly every engineering activity has an impact on the environment. By involving the students in solving real-life problems adjusted to their region with emphasis on survival (short term) and sustainability (long term) we encourage them to pay attention to environmental issues. We present the results of two studies spanned over the period 2005-2007. The models used with the students in both studies have the following features:

- Each model is environmental. Environmental issues are getting more and more important for many human activities worldwide. This non-traditional area of application for engineering students will help them to broaden their vision and prepare them to take *ethical responsibility* in future because nearly every engineering activity relates to the environment.
- Each model is adjusted to the region where the students study by selecting a nearby lake or an island, putting its name in the title of the model and entering the corresponding values of the parameters into the model. We assume that this psychological strategy will help students to relate to the models in a *personal* and an *emotional* way and increase their motivation and enthusiasm.
- Each model is developed by professional mathematicians working in industry and is based on a real practical problem.
- Each model is adapted and presented in a way that is understandable to engineering students.
- Each model is a little bit beyond the scope of the mathematics course the students study and so they need to learn on their own some new concepts. We assume that this *discovery learning strategy* can help the students enhance their investigation and research skills.

In both studies, practice was selected as the basis for the research framework and, it was decided 'to follow conventional wisdom as understood by the people who are stakeholders in the practice' (Zevenbergen & Begg (1999)). The students' mathematical and modelling activities in the class as well as their attitudes were the research objects.

The First Study

The first study was conducted simultaneously with the first-year engineering students studying mathematics courses in the Auckland University of Technology, New Zealand and the Wismar University of Technology, Business and Design, Germany. Two environmental models were given to the students as a project/assignment. The total number of students who completed the project was 147 in both countries. Participation in the study was voluntary. The number of students who answered the anonymous questionnaire was 63, i.e. a response rate of 43%. One of the models is given below.

<u>Model of Water Quality Control in Taupo Lake</u>

Polluted water enters Taupo lake from a recently built factory at a constant rate N. A mathematical model of concentration of pollution has been developed under certain assumptions including the following:

- the upper levels of water are mixed in all directions
- change in mass of pollution is equal to the difference between the mass of the entering pollution and the mass of the pollution which is being decomposed
- the rate of decomposition of pollution is constant
- decomposition of pollution takes place due to biological, chemical and physical processes and/or exchange with the deeper levels of water.

Under the assumptions the equation of the balance of mass of pollution on any interval of time Δt can be written in the form:

$$V\Delta C_N = N\Delta t - Q\, C_N\, \Delta t - KV\, C_N\, \Delta t, \qquad (1)$$

where V – volume of the upper levels (constant), Q – water consumption rate (constant), $C_N = C_N(t)$ - concentration of pollution at time t, K – decomposition rate (constant).

Questions:

1. Set up a differential equation for the concentration of pollution from equation (1) by dividing both sides of equation (1) by Δt and taking a limit when $\Delta t \to 0$.
2. Solve that differential equation to find the concentration of pollution as a function of time provided that the initial concentration of pollution was zero.
3. Determine the equilibrium concentration of pollution C_{Ne} (that is the concentration when $t \to \infty$ or $\frac{dC_N}{dt} = 0$).

4. Determine time needed to reach p portion ($p = C_N(t)/C_{Ne}$) of the equilibrium concentration. Will it take more time to reach p portion of the equilibrium concentration in case when there is no decomposition of pollution?
5. Set up a differential equation for the concentration of pollution from the differential equation in question 1 in case when the initial amount of pollution entered the lake was C_o and after that pollution is not coming to the lake anymore (that is $N = 0$).
6. Solve the differential equation from question 5.
7. Determine time needed to reach the $(1–p)$ portion ($C(t)/C_{\mathrm{o}} = 1 - p$) of the initial concentration C_{o}.

After completing the project the students were asked to answer the questions:
Question 1. Do you find the project to be practical?
a) Yes Please give the reasons:
b) No Please give the reasons:
Question 2. Do you find the project to be relevant and useful for your future career?
a) Yes Please give the reasons:
b) No Please give the reasons:

The results in both universities were so similar that we decided to combine them. Brief statistics and students' comments are presented below:

Question 1. Practical? Yes – 48%
Selected students' comments were: "The models describe the real world", "A good way of increasing students interest in the subject", "It was so helpful for my other subjects", "I didn't realize modelling is used for fishing quotas. It also helped me realize the effects of sneaky illegal fishing (which most of us have done)".
Question 1. Practical? No – 52%
Selected students' comments were: "It is not possible to calculate the nature", "It did give a practical situation but you barely think about that at all when doing the assignment".
Question 2. Relevant for your career? Yes – 35%
Selected students' comments were: "Mathematics is the base needed to go into the Engineering World, so it will help a lot", In engineering, we will be dealing with these kind of situations", "We are more motivated to solve such real problems than working with dry examples", "Everything you learn is bound to be beneficial at some point".
Question 2. Relevant for your career? No – 65%
Selected students' comments were: "I don't see how it relates to mechanical or electrical engineering (most common comment)", "I don't compute formulas, I have to calculate beams…".

The Second Study

The second study is a case study based on the experimental course Mathematical Modelling of Survival and Sustainability taught to a mixture of year 2-5 engineering students in Germany by a visiting lecturer from New Zealand in 2006 and 2007. The course has a multidisciplinary character and is very practical. Matlab is used throughout the course. The expected students' benefits (learning outcomes) after completing the course are:

- Improving students' generic mathematical modelling skills
- Developing students' skills of analysing aspects of survival and sustainability of ecological, environmental, socio-economic and military systems
- Improving students' multidisciplinary and interdisciplinary competence
- Increasing students' confidence in using computer software
- Improving students' conceptual understanding of related mathematics topics
- Enhancing students' ability to deal with real practical problems
- Developing students' team work skills
- Creating students' ethical responsibility to environmental issues
- Making connections to the local industry
- Enhancing students' problem solving skills

The students solved a number of sophisticated environmental models in their individual and group projects and also on the exam. After completion of the course the students were asked to answer the question "Do you think this course is suitable for engineering students and if so, why?" There were 11 students in 2006 and 14 students in 2007 in the course. Participation in the study was voluntary. The response rate was 100%. The results in both years were so similar that we decided to combine them. All 25 students answered 'Yes' to the above question. The main two reasons were:

- Improving knowledge in mathematics, Matlab and mathematical modeling that is useful for engineering – 23 (92%). Typical students' comments were: "You consolidate your mathematical knowledge", "Raise knowledge about differential equations and especially how to build them", "Increasing skills in Matlab", "In my opinion many problems or predictions in the 'engineering world' could be handled/solved with the techniques that you can learn here", "Because you learned how to put some problems into a mathematical system", "To see new ways (models)", "In the course you can better make a statement for normal problems about the life", "Because I could improve my understanding for differential equations", "The mathematical models all around us and the true way for an engineer is to understand how a model from the nature react if you change one parameter".
- Practical and interesting – 10 (40%). Typical students' comments were: "To get practical problems", "It is very important to use practical part in the course as it is done here to help students to understand what are they going to do in their future jobs", "Of course it deals not with typically engineering problems but after all it was an interesting subject", "Engineering students can apply their knowledge and broaden their horizon", "It is nice to see we can use differential equations in other areas", "I think that every subject which have a lot of practical things is very useful. This mathematical course was very useful for me and I think, that in our university everyone must study mathematics in this way".

Conclusions from Both Studies

There is a big difference between the students' responses in the two studies on the relevance of the suggested context of applications. Only 35% of the first-year students from the first study indicated that the environment/ecology context is relevant for their future career whereas 100% of year 2-5 students from the second study commented that the course was suitable for engineering students. One of the reasons for such a difference might be the difference in maturity. Another reason might be the difference in students' mathematics background. It was reflected in the exam performance. In the first study, the pass rate of the first-year students in their maths courses was around 50%. In the second study, the pass rate of the year 2-5 students in the modelling course was 100%. Moreover, all 25 students from the second study received excellent or very good final grades. Their learning was measured through two individual projects, one group project and the final exam. Their oral group presentations were independently assessed by several lecturers. From informal talks to the students from the second study we received a strong indication that their enthusiasm and positive attitudes towards the course significantly contributed to their high performance in the course and very positive attitudes towards the unusual approach. It was a risk to offer such non-traditional course to engineering students. In spite of concerns of some engineering staff and to their surprise the students were very positive about the course. They were mature enough to value the new knowledge in mathematics and modelling they received from the course that can be applied in engineering (92%). They also enjoyed the practicality of the course that enhanced their problem solving skills (40%). From informal discussions with the students we learnt that all expected learning outcomes were achieved to a significant extend. The main lesson for us as teachers was that student feedback should be taken into account when designing curricula for their study.

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Using a computer-animated graphical approach to teaching differential equations

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Abstract

We give a short survey on a method applied in the training of engineers to increase the effectiveness of the teaching process in the field of differential equations. The basic ideas of the method are to show the connection between theory and practice via carefully selected examples and to give visual information in connection with the concepts studied to help motivate the students. Sequentially, we propose a technical problem with a differential equation in the background, we determine a parameter which has an essential role in the solution to the problem and finally, using animation, we visualize the dependence of the solution function of the differential equation on this parameter. An approximate solution to the problem is obtained from following the steps of the animation.

Introduction

The effectiveness of a course in mathematics can be characterized by the students' ability to discover the different occurrences of concepts and ideas in the investigation of technical problems and to apply mathematical methods to solve them ((Kocsis, Sauerbier & Tiba (2006)).

It is difficult for the majority of students to find the connection between the mathematical language and the technical way of thinking. One of the disadvantages of classical blackboard-based presentation technique that the possibilities for visualization are limited, while it is widely accepted that visualization is a powerful tool in the learning process of engineering subjects (Park & Gittelman (1992)). Up-to-date IT helps us to find new methods and suitable approaches to bridge this difficulty.

Well-planned representations assisted by special software may have a strong impact on the lectures, showing the concrete meaning of the concepts discussed in a course. Combining the abstract concepts with the practical technical problems can also increase the effectiveness. Our example shows a possible way.

An example

The background of the example

In the subject of second order differential equations, we can investigate as an example the second order differential equation $y''(x)=\sqrt{1+y'(x)^2}$. Students can learn the solution method and can determine the solution functions, but limiting ourselves to calculations in the mathematical model is not motivating enough for them (Kocsis (2007)). At this point we can present the well known problem of a flexible heavy cord fixed at its ends. If $x\to y(x)$ is the function describing the shape of the heavy cord as shown in Fig.1, it is known that with the assumption y'(0)=0 the function y satisfies the second order (nonlinear) differential equation

$$y''(x) = k \cdot \sqrt{1 + y'(x)^2} \ ,$$

where k>0 is a parameter depending on geometrical and physical characteristic data. By this interpretation the students can visualise the investigation of the equation from technical a point of view.

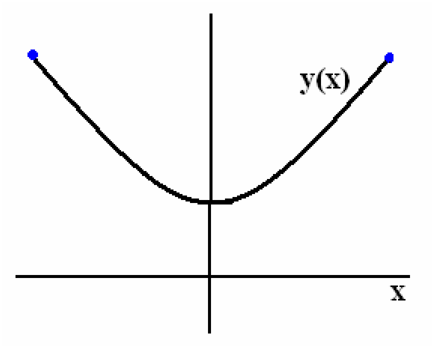

Fig. 1 Heavy cord fixed at its ends

Presentation of a more concrete problem in connection with the investigated differential equation

Our aim is to show the connection between theory and practice. To do this we continue reducing the level of abstraction. In our experience, the following example is concrete enough to motivate the learners. Consider the ends of a flexible heavy cord at two points at the same height with 10 [m] distance between them, as shown in Fig. 2. Suppose that the maximum admitted dip is 3 [m]. Determine the maximum length of the cord.

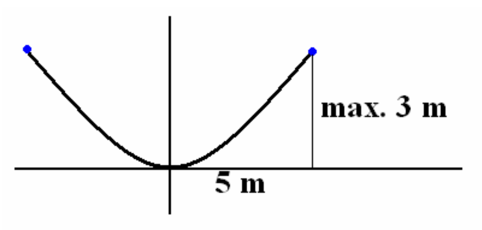

Fig. 2 Sketch of the problem

A possible way to determine the answer is first solving the differential equation (with or without computer) and investigating the cosine hyperbolic function provided as a solution. In this case a numeric method is needed to solve a hyperbolic equation, and finally we get an approximate answer to our question.

Here we follow another way. We use animation (created by Maple) to get an approximate value for the maximum length of the cord.

Solving the problem with animation

We manipulate with two Maple commands; `DEplot` to provide the graph of the solution function with given parameter value for k, while `animate` generates a series of pictures according to the changing value of the parameter k. To study our problem we are change the value of k and check the value of the solution function at x=5, y(5).

Increasing the value of parameter k in the differential equation increases the value of y(5). We fix the increment size ($\varepsilon>0$) and run the animation with values k = ε, 2ε, 3ε, .etc. Controlling the display of graphs we can determine an approximate maximum value of k satisfying $y(5)\leq 3$. It is clear that the smaller ε is, the more accurate the approximation is.

Simplifying the calculation we assume that y(0)=0 [m], that is, we consider the following initial value problem:

$$y''(x) = k\sqrt{1+y'(x)^2}\,,\quad y(0)=0,\quad y'(0)=0$$

on the domain $x\in[-5,5]$.

To construct the animation we apply the following MAPLE commands

```
> restart:with(plots):with(DEtools):
> DE:=diff(y(x),x$2)=k*sqrt(1+(diff(y(x),x)^2));
> animate(DEplot,[DE,y(x),x=-5..5,[[y(0)=0,D(y)(0)=0]],
linecolor=black,stepsize=0.1],k=0..0.3,frames=31,background=
plot(3,t=-5..5,colour=blue,thickness=3));
```

The value of k is changed gradually in increments of 0.01 and Figures 3-6 show some of the results. We can see four steps of the animation: $k_1=1$, $k_2=1.5$, $k_3=2$, and $k_4=2.2$ and estimate that $k_{max}=2.2$, approximately.

From the maximum value of k (k_{max}) we can calculate the maximum length of the cord (L_{max}) using the following relation [1]:

$$s^2+\left(\frac{1}{k}\right)^2=\left(y(x)+\frac{1}{k}\right)^2$$

where y is the solution function (the shape of the cord) for the parameter value k, and s is the length of curve AB. (Fig.7)

Thus we have

$$L_{max}=2s_{max}=2\cdot\sqrt{\left(y(x)+\frac{1}{k}\right)^2-\left(\frac{1}{k}\right)^2}=2\cdot\sqrt{\left(3+\frac{1}{0.22}\right)^2-\left(\frac{1}{0.22}\right)^2}=12.04$$

So the maximum length of the cord is L_{max}=12.04 [m].

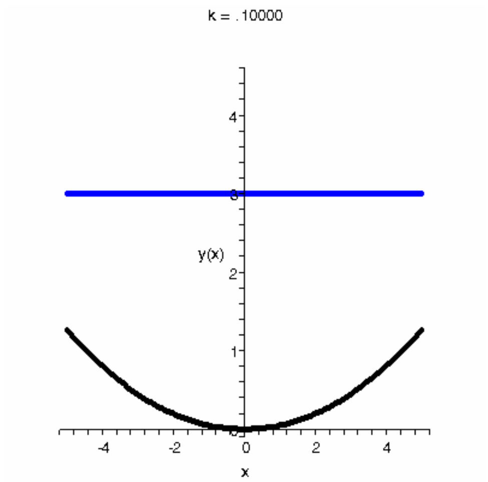

Fig.3 The solution (k=1)

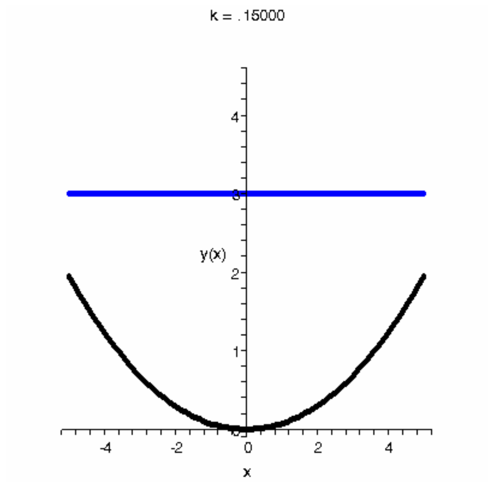

Fig.4 The solution (k=1.5)

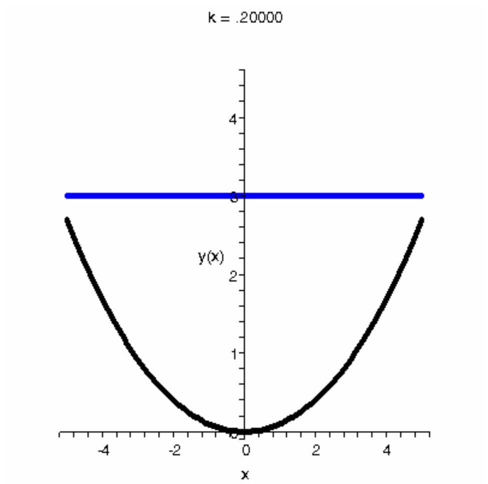

Fig.5 The solution (k=2)

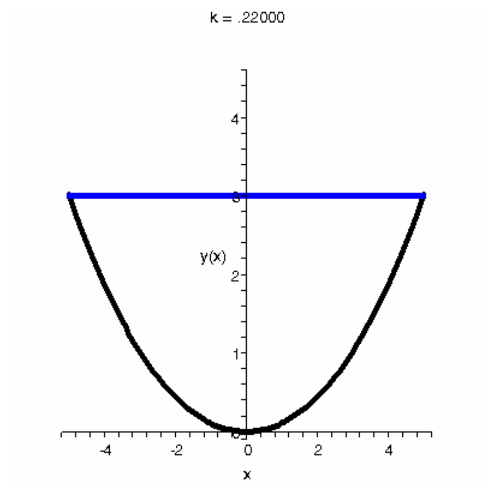

Fig.6 The solution (k=2.2)

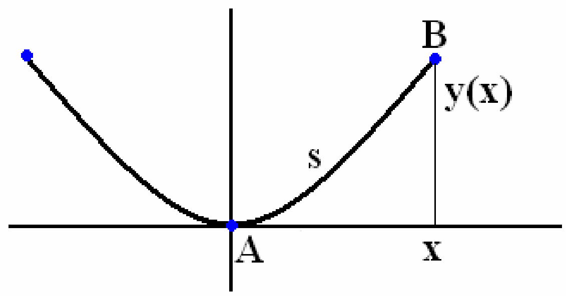

Fig.7 Calculation of the length of the cord

Conclusion

In our opinion the special facilities provided by computer needs new approaches in teaching mathematical subjects. There are topics where animation can greatly increase the effectiveness of the teaching process. It is especially true in the theory of differential equations.

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Computer Algebra System Supported Environment for Teaching and Learning Calculus I

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Abstract

The paper presents a computer algebra system (CAS)-supported approach for teaching and learning Mathematics applying Matlab as a symbolic, numeric and graphic tool. It is used in education of some specialties of the bachelor's degree studies at the Technical University (TU) – Varna, Bulgaria. The students' activities in the Calculus I laboratory exercises to evaluate limits and derivatives are discussed. The evaluation of the students in laboratory exercises is considered.

Introduction[1]

The development of computer technologies allowed the possibility of providing interactive software for CAS - Matlab, Matlab, Mathematica and others. They have been widely adopted in the academic community to make education more authentic and tangible. More than 3 500 universities worldwide apply CAS as training and research tools. The great power and potential of CAS are addressed to mathematics lecturers' and students' activities. However the implemented symbols, notions, functions and terminology in Mathematics cannot help to automatically transfer mathematical knowledge to students and to enhance their feeling for Mathematics. Methodological principles and lecturer's capabilities to put these principles into practice in a CAS-supported environment can give CAS vitality.

CAS is also are used in the Mathematics study process during the Bachelor's Degree in the TU. Almost all first and second-year students at the university have laboratory exercises (labs) as part of their high mathematics (HM) education. The students learn three parts of mathematics. These are realized with the assistance of CAS - Matlab and Matlab (for the specialism of Electronics). The university provides a thorough base for conducting education. It holds specially-equipped laboratories. The Internet link allows access to Internet-based resources for the Mathematics tuition. The aims of labs are to enable students to solve given problems during lectures and seminars using a ready-made program product on the computer. One of the most recent versions of CAS is in use. It allows the application of a dialogue regime of operation, a convenient interface and mathematical calculation media. There is a help menu for the applied calculation commands, the graphic visualization of the calculations and for programming.

Since the academic year 2004/2005 tuition for Turkish students has been launched at TU. The Bachelor's degree tuition has been carried out in the following specialties: Computer Science and Engineering (CSE), Electronics (E), Industrial Management

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[1] See Company Mathworks (Matlab) (2008), Kovacheva (2004, 2007).

(IM), Ship Construction and Marine Equipment (SCME) and Social Management (SM). Foreign students are taught to the Bulgarian curricula for the respective specialism, designed in the Mathematics Department. Mathematics classes are scheduled for 3 semesters for CSE, SCME and E, for 2 semesters for IM. Students in SM do not study Mathematics. Students in their first and second year attend separately designed lectures, seminars and lab classes in English. Labs are carried out utilizing CAS: Matlab for CSE, SCME and E, and Matlab for E.

Some Topics in Calculus I labs using Matlab[1]

The Symbolic Math Toolbox of Matlab is used to realize the Calculus I labs. As examples the basic students' activities in two labs for evaluating limits and derivatives of a one variable are given. At the beginning of each lab a short theoretical review is presented.

Limit of a function of one variable Methodology

In this lab students learn how to use Matlab for finding limits of functions of one variable. The system performs the calculation of most limits very quickly by the command **limit**. Function graphics are used to verify the correctness of the defined limits. The activities of the students are the following:

Activity 1. Evaluating limits exploring $\varepsilon-\delta$ definition

The students can understand the evaluating process if they solve the problems using the $\varepsilon-\delta$ definition. For example they determine a value of δ so that if $|x-1|<\delta$ then $\left|\frac{x}{x+1}-\frac{1}{2}\right|<\varepsilon$, $\varepsilon=0.001$. The students solve the inequality $\left|\frac{x}{x+1}-\frac{1}{2}\right|-0.001<0$ near $x=1$.

Activity 2. Evaluate limits using the table of function values near the limiting point and the function graph

The students consider two fundamental limits: $\lim_{x\to 1}\frac{\sin(x)}{x}=1$ and $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$. They plot graphs of the functions $y=\frac{\sin x}{x}$ and $y=\left(1+\frac{1}{x}\right)^x$ (Figure 1, Figure 2) and find the tables of functions values near the points 0 and *e*, respectively (Table 1). NaN ("Not a Number") value opposite the point 0 denotes that the function value is undefined. It

[1] See Calculus with Matlab (2008), Educational Mathematical Site Exponenta (2008), Nikolaev & Chakarov (1999), Tonchev (2005).

appears from the Table 1 that if x is near 0 then $y = \frac{\sin x}{x}$ is close to 1. The same result is obtained for the graph of the function.

x	$y = \frac{\sin x}{x}$	x	$y = \left(1 + \frac{1}{x}\right)^x$
-0.03	0.999850	50 000	2.718254646126732
-0.02	0.999933	51 000	2.718255179101404
-0.01	0.999983	52 000	2.718255691602668
0	NaN	53 000	2.718256184737094
0.01	0.999983	54 000	2.718256659595320
0.02	0.999933	55 000	2.718257117232703
0.03	0.999850	56 000	2.718257558471690

Table 1

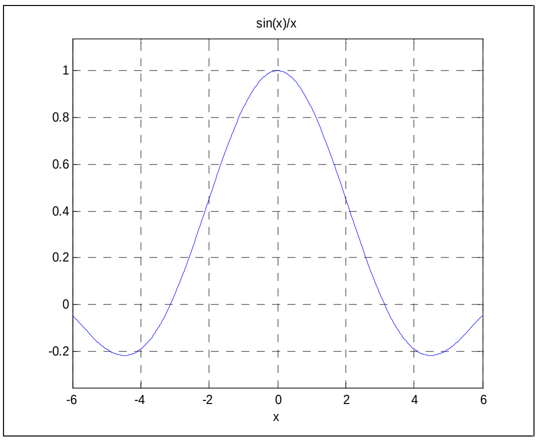
The students will see what happened near the origin of the function $y = \left(1 + \frac{1}{x}\right)^x$ (Table 1, Figure 2). The limit exists and its value is one of the most important numbers in mathematics. To 15 digits the correct value is 2.718281828459045.

Activity 3. Evaluate limits using ***limit*** *command*

The students evaluate the limits of different groups of functions: rational, irrational, trigonometric, etc. At first they declare *x* as a symbolic variable and use the **limit** command to evaluate the limits. For example: $\lim_{x \to pi/2} \sin(x)$

>> syms x

>> limit(sin(x),pi/2)



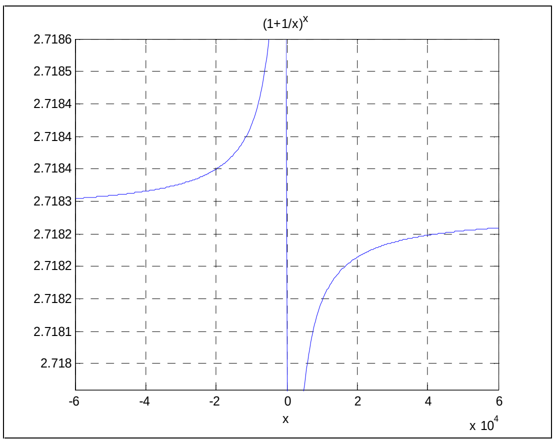


Figure 1

Figure 2

Activity 4. Evaluate some non existing limits

In the case when the limit at a specific point has not been defined, the Matlab cannot find it and the value is undefined - NaN. For the functions $f(x) = 1/x^2$ and $f(x) = 1/x^3$ using the commands

>>limit(1/x^2,0)

>>limit(1/x^3,0)

the students obtain the answer infinity in the first case and NaN in the second. They plot the graph of the functions under consideration (Figure 3, Figure 4).

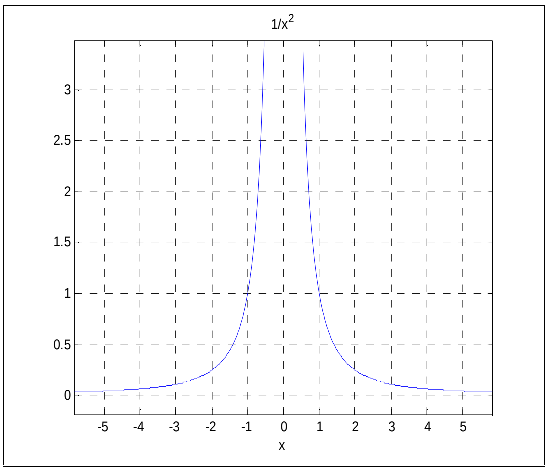


Figure 3

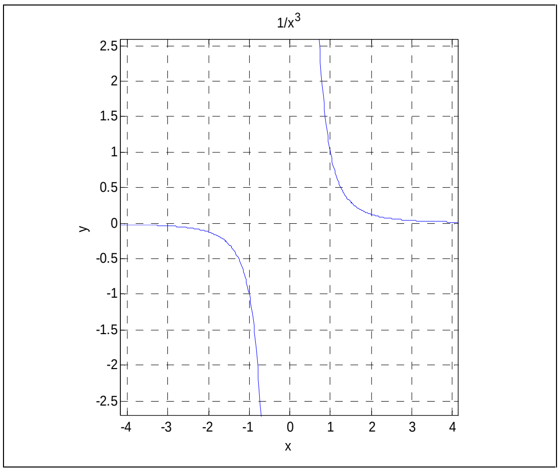


Figure 4

Activity 5. Evaluate one-sided limits of the function

The students investigate what happens to the function $f(x) = 1/x^3$ when x approaches 0 on the left and on the right. They specify the direction of approach. The commands are:

>>limit((1/x^3,x,0,'right')

>>limit((1/x^3,x,0, 'left')

The left limit is minus infinity and the right is infinity (Figure 4).

It is well to show of the students similar problems as in Figure 5. The first three functions have limit (-5) as *x* approaches 1 and the fourth function has different left and right limits at 1, and so the limit does not exist.

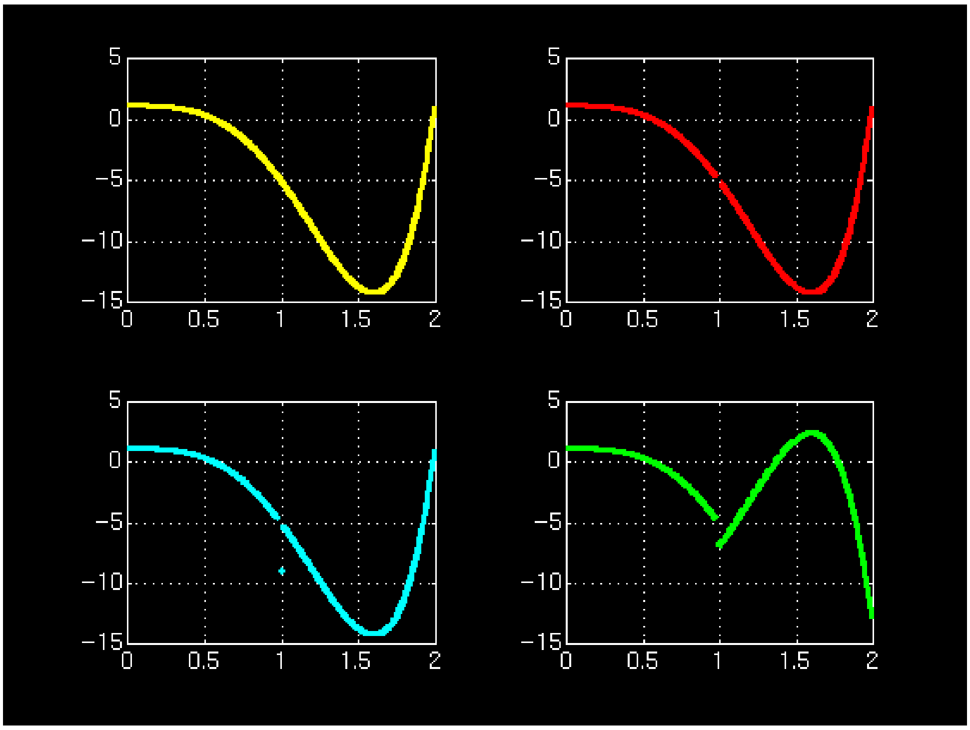


Figure 5

Symbolic differentiation

The other important notion for the students is derivative of a function. The main idea is they to learn the definition of derivative using **limit** command and the Matlab **diff** command. The basic activities for the students are:

Activity 1. Define analytically the derivative of simple function and its value at a fixed point

Activity 2. Calculate the values of the derivative numerically

Consider the function

$$F(x) = \frac{f(x) - f(a)}{x - a} \quad (1)$$

Using Matlab command **inline** function $g(x)$ is created for the function (1) and evaluated for series of values close to a fixed value a.

Activity 3. Calculate the derivative using the definition

The students evaluate the derivative using the **limit** function

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad (2)$$

*Activity 4. Calculate the derivative using the Matlab **diff** command*

Using the command **diff** the students evaluate different elementary and complex functions.

Activity 5. Compare the obtained values by the different approaches

Activity 6. Repetition of activities 2 to 3 for different value of a, resp. x and representation the results graphically.

Activity 7 Geometrical interpretation of the derivative

The value of (2) is the slope of the tangent line to the graph of *f* at the given point of ($x_0, f'(x_0)$). The equation of the tangent is $y = f'(x_0).(x - x_0) + y_0$.

The results of the above students' activities for the function $f(x) = x^3 + 2.x^2 + 1$ and x=1 are the following. They students construct an inline function for the F(x)

>g=inline('(x^3+2.x^2+1)-4)/(x-1)')

The next step they evaluate the function *g* for several values of x and find g(1.1)

>g(1.1)

ans = 5.71

Then using the **limit** or **diff** the students determine exact value of (1) or (2), when x=1 with the commands

> limit((x^3+2.x^2+1)-4)/(x-1),x,1)

> d=diff(x^3+2.x^2+1,x) ;subs(d,1)

and obtain that derivative of a function is $f'(x) = 3.x^2 + 4.x$ and $f'(1) = 7$.

After that they find the values of *g* for given values of *x* close to *1*. Analogy are imputed the inline functions for F(x) when x=2 and 3, respectively and are evaluated their values for given values of x close to 2 and 3 (Table 2, Figure 6).

a=1		a=2		a=3		a=4		a=5		a=6	
x	F(x)	x	F(x)	x	F(x)	x	F(x)	x	F(x)	x	F(x)
1.01	7.05	2.01	20.08	3.01	39.11	4.01	64.14	5.01	95.17	6.01	132.20
1.10	7.51	2.10	20.81	3.10	40.11	4.10	65.41	5.10	96.71	6.10	134.01
1.30	8.59	2.30	22.49	3.30	42.39	4.30	68.29	5.30	100.19	6.30	138.09
1.50	9.75	2.50	24.25	3.50	44.75	4.50	71.25	5.50	103.75	6.50	142.25
2.00	13.00	3.00	29.00	4.00	51.00	5.00	79.00	6.00	113.00	7.00	153.00
3.00	21.00	4.00	40.00	6.00	81.00	6.00	96.00	7.00	133.00	8.00	176.00
4.00	31.00	5.00	53.00	6.00	81.00	7.00	115.00	8.00	155.00	9.00	201.00

Table 2

x	f'(x)
1	7
2	20
3	39
4	64
5	95
6	132
7	175

Table 3

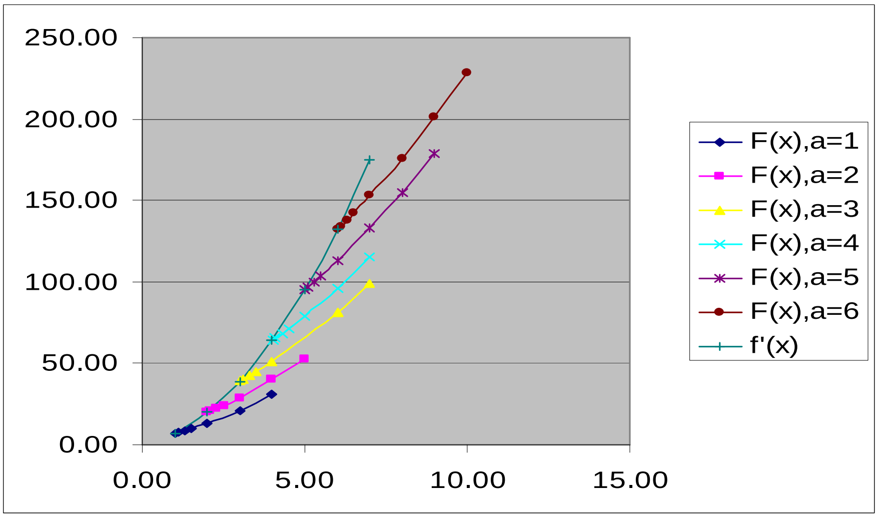


Figure 6

The students find the equation of the tangent to (2) at the point (1,f(1)) - $y = 7.(x-1)+4$ and plot the results.

Evaluation the Students' Activities in the Labs

The lecturer gives to students similar problems to solve and these are graded at the end of the lab. During the lab he ensures that the study process is carried out in the right manner. Checks and instructions are carried out for the whole group to economize time. Discourse and discussions facilitate the build-up of long-term and in-depth knowledge.

At the end of lab each student has an individual check with the lecturer on the validity of his solutions to the given problems; students are then aided in the drawing of conclusions and the evaluation of their work. The final assessment of each student is complex and takes into account their progress during labs, marks from the progress checks and/or written assignments.

To achieve better results from the execution of the laboratory classes and to make the system and exercises more appealing to the students, the lecturer can use different materials, such as a CD with the Matlab version, printed matter of the system operation, problems to be set aside for homework, assigned term papers.

Conclusion

Combining traditional modes and CAS-supported approaches to undergraduate mathematics during labs enhances the educational effectiveness.

Matlab holds great potential in assisting the study process of the chapters included in the mathematics tuition. Every lecturer aims at being most collaborative with their students and in order to achieve efficiency he/she has to creatively make use of the system's capabilities. With the lecturer's consideration, depending on experience, various exercises to suit the purposes of the labs can be performed. Such exercises could only enhance the in-depth comprehension of the material taught and likewise motivate students' interaction.

The application of Matlab in labs allows for a greater amount of solved problems among students, helps them determine certain properties unaided and thence comprehend the content of notions more thoroughly, their properties, theorems and the relations between them. Students learn to independently solve problems with the help of a CAS and to select methods for their solving. Mastering Matlab during the first years at University is beneficial to students not only in solving mathematical problems but also any complex applied technical problems they will encounter.

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Testing and Teaching Mathematics with Interactive Online Technology

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Abstract

First year engineering students in Finnish polytechnics have great motivation problems and very variable backgrounds. This leads to a high drop-out percentage and low study achievements. First year problems tend to concentrate on theoretical studies, especially mathematics. The Information Technology programme in Helsinki Polytechnic has decided to give special emphasis to the start of the studies by placing the students in small groups appropriate to their school background and their mathematical abilities.
The engineering faculty has started using an interactive online mathematics testing and teaching system for the first year students. The system is based on algorithmically generated numerical and symbolic problems and it gives immediate grading and detailed feedback for students. The system is used by all students commencing information technology and industrial management. The system is also used in remedial mathematics courses in other engineering programs.
The diagnostic tests of the system are used to assess the level of basic mathematical skills of the new students, both for students themselves and instructors, and also to place students in appropriate study groups. The results show that the diagnostic test correlates well with the achievements in the mathematics class in the first study period.
In industrial management we have used the system also for continuous online self-tests and homework assignments. Students use the system regularly and they are using its feedback to monitor their mathematical progress, with minimal intervention by the teacher. The results in the assignment system and the tests correlate even better with the achievements in a regular class test.

Why Automatic Testing and Assignment Systems?

First Year Study and Motivation Problems

The Finnish educational system provides higher education for about two thirds of all young people. This means that universities and other higher education institutes compete heavily with each other for good students. The competition has been especially detrimental to engineering departments in polytechnics (universities of applied sciences).

Young people applying for engineering studies in polytechnics have very varying backgrounds and often great motivation problems. Roughly one third of the accepted students have a senior high school background with extensive mathematics and some physics. They have, however, mediocre grades in mathematics on the average, the best students going to universities. The other thirds of the accepted students have studied only general mathematics and no physics in the senior high school or they come from vocational schools. These groups take usually somewhat slower and broader mathematics courses during the first semesters, but even then they have lower study achievements.

First year study problems concentrate on theoretical studies, especially mathematics, which is the essential basis for serious engineering studies. Young people who have had little success in mathematics and other theoretical studies in school would rather study something else, especially something more practical or application oriented. This, together with other reasons, leads to low motivation and failures and high drop-out percentages, which have lately been from 30% to 50%, highest in information technology, in the Helsinki Polytechnic engineering study programs.

The information technology program has tried to deal with the first year study problems taking several different measures. In addition to special tutoring, students have been placed in small study groups (20-25 students) according to their mathematical abilities. They use a special supporting pillar system, and they have ample opportunities to do online mathematics problems and self-tests getting immediate feedback. They are also tested in the beginning and after the first study period.

Diagnostic and Placement Tests

Information technology and Industrial management students (240 students yearly) take a basic mathematics online test just before the actual studies start. The test consists of 35 problems in algebra, equations, trigonometry and functions. The problems are parametrically and randomly generated in real time from carefully selected problem prototypes, which means that there are always different problems in each test or assignment set. The testing system is based on the MapleTA computer aided assessment (CAA) system and it is provided by WebALT Inc.

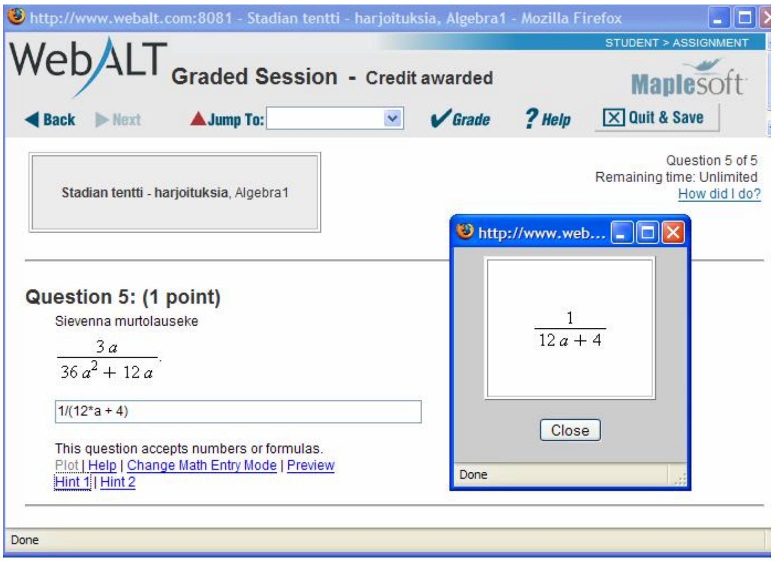

Figure 1. A test item ('simplify a fraction') and a student's answer. The Preview window is also shown.

Tests are used to diagnose the level of basic mathematics for each student. The test result and school background are used to place the student in an appropriate study group. Feedback gives teachers information on what can be expected and which remedial actions must be done. Students get detailed feedback that includes automatically generated detailed solutions.

The test is repeated after the first study period (8 weeks). This gives information on how the basic mathematical skills of individual students and study groups have developed. Automatic tests can be used to measure progress and the effect of various pedagogical and other interventions. However, it is essential to validate the automatic tests by comparing results with other tests and grading methods.

Supporting Student Work

The same MapleTA system is used for automatic online homework assignments and continuous self-testing. A homework assignment set typically consists of about 5 problems that are related to the topic students are currently studying. There are typically 20-30 assignment sets for an eight-week mathematics module. The assignments can be compulsory and integrated into course pedagogy, or they can be voluntary and independent, depending on the teacher.

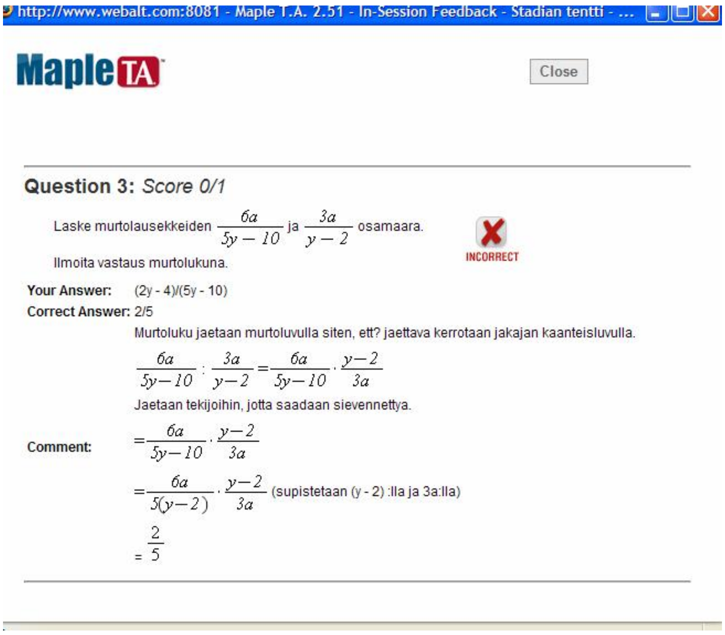

Figure 2. Feedback from a problem (division of two fractions) with intermediate stages and comments. The student’s answer is not accepted because it has not been simplified enough.

Feedback from each problem gives students valuable and critical support for self-study. The automatic feedback is immediate and detailed and it is available online whenever the student has time to do the assignments. Students can monitor their progress, with minimal intervention by the teacher.

Remedial Courses

Due to the first-year difficulties, there are large numbers of students who have failed their first-year mathematics courses, despite several attempts. Since many students are about to graduate and some of them are already in work, they are not especially motivated to take the basic course in school again. For this reason there is a possibility of wastage even at this point of studies.

The electrical, mechanical and automotive engineering programs have decided to give special first year mathematics remedial courses for students that are otherwise in advanced stages in their studies. The courses are based on self-study, a few tutorials (partly online), and compulsory online MapleTA assignments. The exam, however, is a conventional supervised written exam. Students have found this arrangement very motivating.

Results Found

Diagnostic Tests and School Background

Traditionally students have been placed in parallel study groups according to their school background, namely senior high school with extensive mathematics, senior high school with general (short) mathematics, or vocational school.

The diagnostic tests show that there is substantial overlap in the mathematical abilities of the three groups and a high variance in each group. Therefore we have decided to use both the school background and the test result as a basis to place the students in parallel study groups.

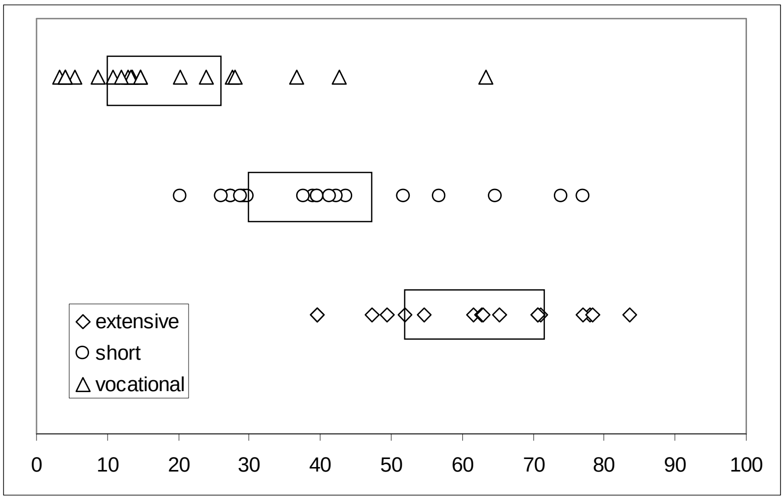

Figure 3. Diagnostic test results and school background.

Predicting Achievements

Results of the regular class test after the first study period have been compared to the results of the diagnostic tests and other factors that might explain the results of the class tests (which are the basis for the course grade). We have found that the diagnostic tests have consistently high correlations with the class tests. Similar results have been found elsewhere, e.g. Heck & van Gastel (2006a, b). For those groups that have used the MapleTA assignments during the first period, the assignments activity has even higher correlation with the class test. Similar results have been found also in the Helsinki University of Technology (Rasila, Harjula & Zenger (2007)) and in Finnish primary schools (Lehtinen (2008)).

	Diagnostic Test 1	Diagnostic Test 2	Online assignments	Regular assignments	Class attendance
Class test	0.63	0.69	0.73	0.59	0.39

Table 1. Correlations of diagnostic tests and other factors with regular class tests. N = 70, two separate study groups.

Effect of Assignments

The impact of the MapleTA assignments integrated into the first period mathematics course is clearly seen when we compare the average group test scores of a group that uses assignments with groups that do not use the assignments. For those who used the assignments the average score improved by 35 points, but for those who did not use the assignments the score improved only 18 points on the average (see Figure 4).

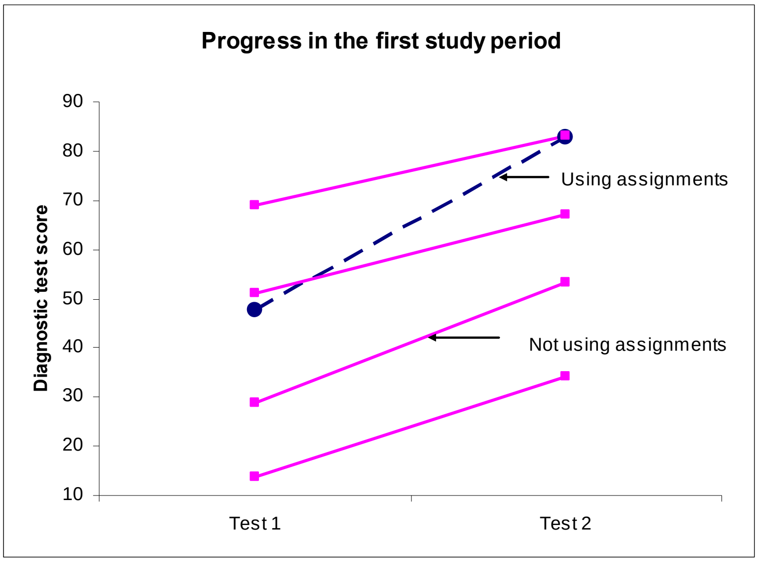

Figure 4. The effect of MapleTA assignments to the development of diagnostic test scores.

Practice and Feedback

Although the MapleTA system is quite straightforward to use and easy to learn, there is however a threshold for its use. Generally, students do not take any assignments voluntarily. The assignments should be integrated into the course. With some initial encouragement and push, students will use the system independently and actively and they start to see its benefits. During one eight-week study period the Helsinki Polytechnic Industrial management students did, on average, 3.7 assignment sets per week. To get good scores many students did the same set over and over. As the assignment problems are always different, this promotes the learning of general principles, not just memorizing the right answers. The feedback is generally positive, while the usability and consistency of the system must be improved.

Discussion

To conclude, we maintain that the main benefits of computer aided test and assignment systems are:

- To place students in proper study groups, together with other criteria.
- To minimize the number of drop-outs.
- To minimize unnecessary review courses.
- To construct an effective and economic study track.
- To support and speed up students' self-study and practice.
- To measure the effect of various pedagogical and other interventions.

Automatic testing might augment or possibly replace traditional testing in mathematics. Both synchronic and diachronic comparisons become possible because the same standardized tests can be used over and over.

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Implementing a Key Skills in Mathematics Initiative

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Abstract

The drive towards mass education, and the year-on-year fall in popularity of technical based subjects in Ireland (and other western countries), has created cohorts of students in many technical courses who are ill-equipped to succeed on those courses. Compounding this problem is the fact that most students in Ireland are now on semesterized courses, which seems to encourage students to learn enough for the regular examinations, without necessarily taking time to reflect on what they have learnt.

Whilst addressing the needs of these students is multi-faceted, we would like to describe the construction of a Moodle-based initiative at IT Tallaght called Key Skills Testing in Mathematics, with the aim of helping students bring knowledge with them from one semester to the next. Using the Moodle platform, Key Skills consists of;

1. creating many categories of multi-choice question which we believe our cohorts of students MUST be able to do. Each question comes with feedback and reference to a book chapter and an electronic resource.
2. creating tests which draw randomly from particular categories of questions. These tests may be repeated several times over a semester and only a high mark is rewarded with credit.
3. different tests run for different groups and in different semesters, reflecting the Key Skills of previously taught material required for that semester.

The aim is for students to expect Key Skills in each semester, re-enforcing and repeating their learning.

This project is in its first year, making evaluation premature. However, we would like to describe the development process and to present our material as a "how to do it" kit. Each question category together with upload instructions, additional files like images, and any tools used in the creation of questions, will be presented as learning objects. The whole Moodle course complete with tests will be presented as a learning object, as will detailed documentation on creating and managing such materials in Moodle.

Introduction

The Institute of Technology Tallaght Dublin (ITTD) is located in South County Dublin and was established in 1992. The Institute caters for a student population of approximately 2,300 full-time and 1,200 part-time students and offers a wide range of programmes from Higher Certificate, Ordinary and Honours Degree to Masters Degree and Doctoral level.

This decade has seen a gradual trend of falling numbers applying for Engineering and Technology based courses in Ireland. Compounding this fall has been a fall in the preparedness of students for their course, in terms of ability in mathematics and physics. Table 1 below shows the number of first preference choices through the Central Applications Office (the equivalent of UCCA within the Irish Education System) for Engineering and Technology Courses from 2000 to 2007 at level 6/7 (two or three year courses not including honours Degrees). Also shown is the total number of preferences (1^{st} to 10^{th}) and the total number of students sitting the Leaving Certificate Examinations over the period.

Year	1st Preference (1st February Figures)	All Preferences (1st February Figures)	Candidates
2000	15055	99607	Not available
2001	14312	89674	59537
2002	11066	66893	58522
2003	10208	58983	59536
2004	9262	50894	58742
2005	9109	48945	57422
2006	9004	45270	40403
2007	8700	40451	39727

Table 1: Student Course Preferences.
Data available from Central Applications Office at http://www.cao.ie and the State Examinations Commission at http://www.examinations.ie

The number of candidates *available* to do Engineering courses (Candidates) has only dropped significantly in 2006 and 2007. This is due to demographic changes at school leaving age and due to a higher proportion of students enrolling on level 8 (Honours Degree) courses. The numbers who really want to do engineering (1st preferences) fell by over a third from 2000 to 2005 and has recovered a little in 2006/07 as a proportion of the candidates, but Engineering and Technology based courses at level 6/7 have fallen hugely out of favour as a secondary option (All Preferences).

Table 2 below shows the proportion of engineering students at ITTD with reasonably good maths (B or better in Ordinary Leaving Certificate maths, roughly the equivalent of English O Level and combining B1, B2 and B3) and reasonably poor maths (C or less at OLC, combining C1 to C3 and D1 to D3)

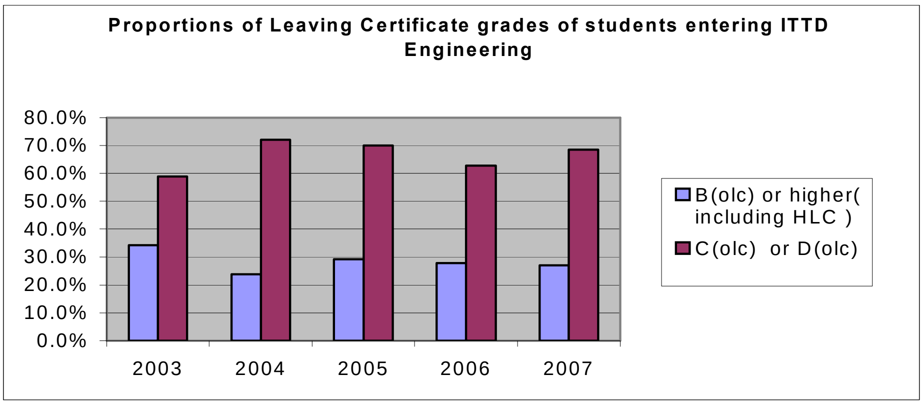

Table 2: Mathematics ability on entry at ITTD on Engineering Certificate courses, Robinson et al (2007).

Table 3 below shows the proportion of students who had done a Leaving Certificate physics course before coming to ITTD.

	Known to have studied LC Physics	Haven't studied LC Physics
2003	37.50%	62.50%
2004	36.00%	64.00%
2205	36.36%	63.64%
2006	25.00%	75.00%
2007	8.00%	92.00%

Table 3**:** Physics preparedness on entry at ITTD on Engineering Certificate courses, O'Sullivan et al. (2007)

Tables 1, 2 and 3 show that engineering is becoming less popular, and that the mathematical knowledge and facility of those students that do enter our programmes is changing too, at least at ITTD. These trends are well documented elsewhere – see for example Bamforth et al. (2007).

Several initiatives to improve basic mathematics ability have been tried at ITTD, Robinson et al. (2007) and Marjoram et al. (2004). These initiatives have targeted first - year students with the aim of improving first-year retention. Unfortunately, while these initiatives appear to have been moderately successful in the fight to retain students, there is still a worrying lack of mathematical ability and knowledge amongst most second and third year students. The semesterized examination system does not help, leaving students with little time to reflect on their learning. Typically, the weak student does not bring key mathematical knowledge with him/her from one semester to the next. How can we help them to do that, and continuously refresh key skills?

In October 2006 we applied for an internal seed fund grant of €3,000 to implement a project which we call Key Skills. The idea is to continuously test key mathematical skills over a semester until a high mark is achieved (a high threshold competency based test). Such tests need to be randomized so that adjacent students do not see the same questions. They need to be repeatable, automatically marked, and provide immediate feedback on learning resources that students might go to in order to do better next time.

The tests must be reasonably difficult to cheat on and marks gained must go towards the continual assessment for their maths module – a currency all students understand.

The Moodle Quiz Platform (http://www.moodle.org) was chosen to implement the Key Skills initiative. This platform has proved to be extremely stable, with no performance issues or corrupted data to date. It possesses all the features we require, as well as keeping excellent student record data that can be further processed in Excel, for instance. It is also open source and has a very active Quiz component user group (http://moodle.org/mod/forum/view.php?id=737).

Implementation of Key Skills

Implementation was done in two phases. The longer-term goal was to carry out frequent Key Skills testing for all groups in years 1, 2 and 3 in Mechanical, Electromechanical and Electronic Engineering (9 groups comprising 316 registered students). *Phase 1* was a pilot process involving just one group in semester 2, 2006 intake (Jan to June) and 3 groups in semester 1, 2007 intake (Sep to Dec). *Phase 2*, involving all groups in years 1 to 3, is ongoing for the semester 2 2007 intake (Jan to June).

Our Key Skills initiative has been set up as one Moodle course that all students enrol on. This cuts down on explanatory material for students and also gives lecturing staff a single location and log-on process to describe. The idea is that students will do Key Skills tests in all semesters, so that this single location is valuable for continuity of student access. All student records are also kept in the same location, which will allow us to do a longitudinal analysis of student performance over several semesters. This in turn will provide insight into something that we really want to know;

does frequent Key Skills testing promote deeper learning and lasting knowledge retention in students?

To create a moodle quiz, one first creates catagories of question. Each quiz is then built from questions chosen from categories. Most categories were created in Phase 1, with others created as we needed them. At the time of writing there are some 50 categories comprising about 600 questions. Each question is multichoice with 3 distractors, and each answer option comes with feedback to a CALMAT lesson (Computer Assisted Learning in Mathematics) and a book chapter. Tests of 15 questions each are then built from questions in categories. Each question is randomly selected from each category required, with its answer options randomized and questions in random order in the quiz. Together with lecturer set password, duration and start and stop time we are confident that cheating can be minimized in the medium term. As students become more familiar, we can provide further security in terms of restricted IP addresses and secure browser windows.

Tests for different groups are, in general, made up of questions from different categories. This is to reflect the different Key Skills that students need for this semester's work. All questions test material from previous semesters, and the first Key Skills test is in semester 2, testing semester 1 material. There is an overlap of material between semesters and we want the student to appreciate that, semester after semester, they will be required to know certain things. Each test also has a partner Practice Test. This test is always available and comprises 15 fixed questions similar to the ones in the real test. Students can review or repeat this test anywhere they have a web browser. When students complete a real test they can also review it later and see question feedback. All real tests are supervised and students may not repeat a test for at least 24 hours, preferably longer.

We want students to be active learners. The delay between tests is there to allow students to review their test attempts and question feedback. We would like them to have a little time to reflect and seek out information. During tests, no formula sheets are allowed and between tests we supply no supplementary notes to help with particular topics. We do not want frequent tests to be so frequent that students simply attempt to learn the form of each question and its distractors.

The best mark in the semester's Key Skills test is allocated to half of the continuous assessment for that semester (typically 15% of the module mark). At the moment, for a 15 question test, a student gets 5% for 11 or 12 correct, 10% for 13 correct, 15% for 14 or 15 correct and 0% for 10 or less correct. We will probably drop the requirement for 5% to 10 or 11 correct and change the 10% mark to 12 or 13 correct. We want to avoid students becoming demoralized by repeatedly getting 0%.

The most difficult aspect of implementation in Phase 2 has been to provide sufficient test opportunities, both in terms of time and lab space. There is the added complication of providing testing opportunities for groups that the named authors do not teach. Luckily, the mathematics staff at ITTD are well disposed to this initiative, in part due to the fact that no administrative effort is required from them! We have tried to create two test opportunities for each group within their own class slot and with their own lecturer taking part. The lecturers have found this useful, and we think it is also crucial that students see Key Skills as being embedded within their maths module. Students are emailed about the coming test opportunity and also texted.

After the first two test sessions students will be emailed and texted about further opportunities to do a test outside their normal lecture times. Labs are deliberately overbooked with groups as we found in Phase 1 that attendance is typically less than 30 % of each group, often much less. Currently, the first and fourth named authors are coordinating test sessions, contacting available student groups and contacting postgrads to do supervision. This work requires about 2 hours per week each, on top of the initial work in the semester of identifying free lab slots for each group.

Test Uptake and the testing process

Table 4 below shows the uptake of tests so far in the 2008 Spring Semester, between the start of teaching on the 19th of Jan and the present 4th Mar.

Test	1 Test	2 Test	3 Test
Elec_Semester 6	9	7	1
Mech_Semester 6	26	19	10
MechElec_Semester 2	39	8	
MechElec_Semester 4	5		

Table 4: Numbers of students and their test frequency

This table shows that 120 students have taken 180 tests so far this semester (there were 131 tests in total for the whole of the previous semester's pilot phase). This is probably about half the students who are actively on each course. In this table, the Semester 2 and 4 tests are common to electronics and mech/electromech engineering, while the Semester 6 tests are different. The testing process is yet to get in full swing in semester 4, but we would anticipate 600 to 800 tests completed by the end of May 2008.

If the reader wishes to repeat our Key Skills initiative they should keep the following points in mind:

1. As early as possible in the semester identify all free computer lab slots, lab slots that coincide with mathematics classes and all slots that can be used to run a test for each group. Review this frequently as timetables change.
2. Obtain as many student mobile phone numbers as possible and invest in an sms texting facility from your PC. Most students now give their mobile number to student records. Email to students is also useful, but is checked less frequently.
3. Involve each group's maths lecturer in the test process. Show them how to log into Moodle and change test times and passwords.

The main difficulties with the testing process is providing *timely* warning that a test is available and coordinating time, group and test supervisor. A further ongoing problem, as our seed funding runs out, will be payment of supervision for tests that run outside a groups normal maths classes.

Conclusions

Without a longitudinal study of student performance over several semesters, conclusions are premature as to whether this initiative really does promote lasting knowledge and skills in weaker students. Further investigation of student attitudes to Key Skills, and feedback from them on improving the process, is also required. That said, Key Skills seems to have captured student attention somewhat. The latest Key Skills test session on 4th March had 28 students in a lab from 5pm to 6pm repeating their test. Anecdotally, students appreciate that they should know how to do test questions and like the immediate feedback on test completion.

Readers are welcome to all of the material we have produced. As well as the complete course with its categories and tests, we have detailed documentation on Moodle quiz creation and quiz management. We have also bundled each category as a learning object, together with any files used to help create such questions and upload instructions to Moodle. Please contact the fourth named author if you would like this material.

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Supporting Engineering Students within a Maths Learning Centre Environment

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Abstract

In 2002, a study conducted within Dublin Institute of Technology (DIT) examined the numerical skills of first year engineering students to ascertain the impact of these skills on their ability to successfully pass the year. The results showed a strong likelihood that students with low numerical skills would withdraw or fail to pass the year. DIT is a multi-level institute, offering engineering programmes at levels six, seven and eight, and also incorporating a "ladder system", which allows students who perform sufficiently well at one level to proceed into the next. As a result, students of different educational backgrounds often find themselves in the same mathematics module. Anecdotal evidence indicated that particular difficulties were experienced by students who, having completed a level seven programme, proceeded directly into third year of a level eight programme, where a significantly higher level of maths was suddenly demanded. As a result of these, and other, issues, the Students' Maths Learning Centre (SMLC) was established to provide additional mathematical support for DIT students. Last year, one in ten students from the Faculty of Engineering availed themselves of the SMLC's drop-in service, with almost a fifth of engineering students making significant use of the SMLC's online resources. In this paper, we identify the most common problem areas in maths for DIT engineers at various levels, along with the most useful resources for these topics. We also consider the possible effects of semesterisation on the learning of mathematics for engineers, based on patterns of attendance over the past couple of years.

Introduction

In 2002, the Retention Office in Dublin Institute of Technology (DIT) completed a study (Costello (2002)) aimed at determining whether poor numerical skills on entry were a strong predictor of failure to pass first year engineering. The results unsurprisingly showed that those with low scores in this area were highly likely to withdraw before the terminal examinations, or fail to pass the year overall. In interviews conducted with some of these students, a significant portion professed themselves to be taken aback at the strong mathematical content of an engineering programme, and felt themselves to be completely unprepared for the level of mathematics they would be studying.

DIT is in a somewhat unique position in Ireland, as it offers a wide range of engineering programmes (both full-time and part-time) at levels six, seven and eight (namely Higher Certificates, Ordinary Degrees and Honours Degrees), as well as incorporating a "ladder system" which gives students the opportunity to progress from one level to the next, provided they have performed to a sufficiently high standard. As a result, engineering students within a single mathematics module will often be from varying educational backgrounds, even in relation to their third-level mathematics learning, which creates an even greater challenge for educators. Anecdotally, it had been observed that students who had successfully completed a level seven programme and subsequently entered directly into the third year of a level eight programme experienced considerable difficulties with the mathematics component, although they tend to cope well with the more practical elements of the programme.

As a result of these, and other, concerns, DIT followed in the footsteps of numerous other third-level institutes (Lawson (2003)), both in Ireland and the United Kingdom, and established the Students' Maths Learning Centre (SMLC) during the academic year 2004-2005 (Ní Fhloinn (2006)), with the aim of providing additional mathematical support for students from any faculty in DIT whose programme contained a mathematics module.

Students' Maths Learning Centre: Supporting Engineers

DIT is a multi-campus institute, with the main campus buildings located on different sites around Dublin city centre. In addition, resources for the SMLC were limited, so a two-fold approach was deemed the most effective, incorporating one-to-one help through the form of drop-in sessions, along with e-learning support via a WebCT site. There are three main campus sites at which one-to-one support is provided, with between six and nine hours drop-in service available in each, in three-hour blocks. In order to best facilitate engineering students, who have a full timetable of lectures and labs, drop-in sessions are held during lunchtime hours or from 4:00-7:00 in the evening, when most students are available.

Due to budgetary constraints, there is usually only one advisor present in a drop-in session to help students. As a result, easy availability of relevant resources within the centre has been crucial, allowing students to be more independent in their learning: as such, the revision sheets from the Engineering Maths First Aid Kit from the Mathcentre website (http://www.mathcentre.ac.uk) have proved invaluable. Mathcentre is an online collaboration between the Universities of Loughborough, Leeds and Coventry, the Educational Broadcast Services Trust and UK Learning and Teaching Support Networks. The Engineering Maths First Aid Kit consists of a series of two-page summaries on various important topics from first-year maths, including examples, exercises and solutions. Having assessed the students' needs, the SMLC advisor can provide them with relevant revision sheets to work through in the centre, allowing them to gain in confidence as they successfully complete certain exercises, while still having the reassurance of an advisor present should they struggle with any concept.

As the SMLC became more established, the number of students using the service continued to rise. In the academic year 2006-2007, over half the students who used the centre's drop-in facility were from the Faculty of Engineering, with one in ten engineering students availing themselves of the service. This amounts to a total of almost three hundred engineering students, from all years and programme types – an increase of more than one hundred students on the previous academic year.

In addition, almost one fifth of engineering students made significant use of the WebCT service (with "significant use" being defined as ten or more hits on the site). A separate WebCT site was designed for each faculty, so that the resources and information could be tailored to best suit the specific needs of students in that discipline. Each site contains revision notes, self-tests on problem areas, recommended textbooks and websites, relevant mathematical articles and general information about the centre. For the Faculty of Engineering, the Engineering Maths First Aid Kit was included, grouped by topic (with the kind permission of Professor Tony Croft). Selected articles about the use of maths in engineering applications, taken from Plus magazine (http://plus.maths.org) were also uploaded, to emphasise to students the importance of the maths they were studying, and also expose them to some more unusual and interesting applications.

Most Common Problem Areas

Each time that a student attends a drop-in session, the advisor present makes a note of the topics covered. As a result, the most common problem areas for which engineering students seek help in the SMLC can be determined. Table 1 below shows the top fifteen problematic topics, along with the number of visits in which these topics were addressed, during the academic year 2006-2007.

Problem Area	Number of Visits
1. Laplace Transforms	75
2. Basic Integration	68
3. Basic Differentiation	58
4. Differential Equations	55
5. Matrix Arithmetic	49
6. Fourier Series	47
7. Eigenvalues	40
8. Transposition of Formulae	39
9. Complex Numbers	34
10. Logs	30
11. Basic Trigonometry	28
12. Binomial Theorem	24
13. Normal Distribution	23
14. Partial Fractions	23
15. Runge-Kutta	20

Table 1: Most common problem areas for engineers who sought help in the SMLC during the academic year 2006-2007.

From this list, it is clear that certain areas which are entirely new to students in third-level, such as Laplace transforms or Fourier series, cause considerable difficulties; but it is striking that the majority of the areas listed above are ones with which students should be highly familiar prior to coming to third-level (for example, basic calculus, transposition of formulae and basic trigonometry).

Effects of Semesterisation on Patterns of Attendance

As well as considering the topics with which engineers have difficulty, it is also of interest to look at attendance patterns in the drop-in centre over the past two years. Recently, DIT has changed to a fully modularised, semesterised calendar, with two exam periods per year, in January and May. This change was undertaken on a phased basis over two years; in 2005-2006, two teaching semesters of twelve weeks (with an additional "review week" after week six) were introduced; however, some programmes did not introduce modularisation or semesterised exams until the following year. Most engineering programmes took the latter approach, allowing us to review any differences between the patterns of attendance at the drop-in sessions between the two years, and offer possible explanations for these differences. Although more data would be necessary to draw any firm conclusions about the effects of semesterisation on attendance or study patterns, an initial analysis is of interest at this point.

The SMLC drop-in sessions ran every week (including review week) for both semesters. In addition, special sessions were run three days a week during the two-week Easter break in second semester, as well as the week prior to the January exams, and for the full two weeks of each exam period.

It should be borne in mind that there was a natural increase in the number of engineering students who attended the SMLC, as well as the frequency of their visits, between 2005-2006 and 2006-2007. However, although the number of visits increased by 61%, this drops to an increase of 21% if exam weeks are excluded. Therefore, we begin by considering the exam weeks in isolation.

Attendance during Exam Periods

The January exam period was the busiest time of 2006-2007; students were on holidays from mid-December, returning in early January to face exams, but without having had contact with their lecturers in the meantime. The SMLC was open for three days prior to exams commencing, during which there were 117 visits from engineers, with a further 76 visits the following week. Some students were well-prepared and attended only to clarify small points, but the majority sought considerable help. It is not particularly relevant to compare these numbers with those of the previous year, as there were far fewer exams at that stage; but it is striking that the average number of visits per week from engineers in the first semester of 2006-2007 was just thirteen.

The May exam period showed a less extreme variation (though still busier than the previous year), with 46 visits the first week and 26 the second week. This can largely be attributed to the fact that students had full access to lecturers, tutors, library facilities and the SMLC in the direct run-up to these exams and seemed to begin their study at an earlier stage, perhaps due to this being ingrained as a more traditional exam-time!

Attendance during Semesters

Figure 1 overleaf compares attendance during semesters by week, omitting exam weeks.

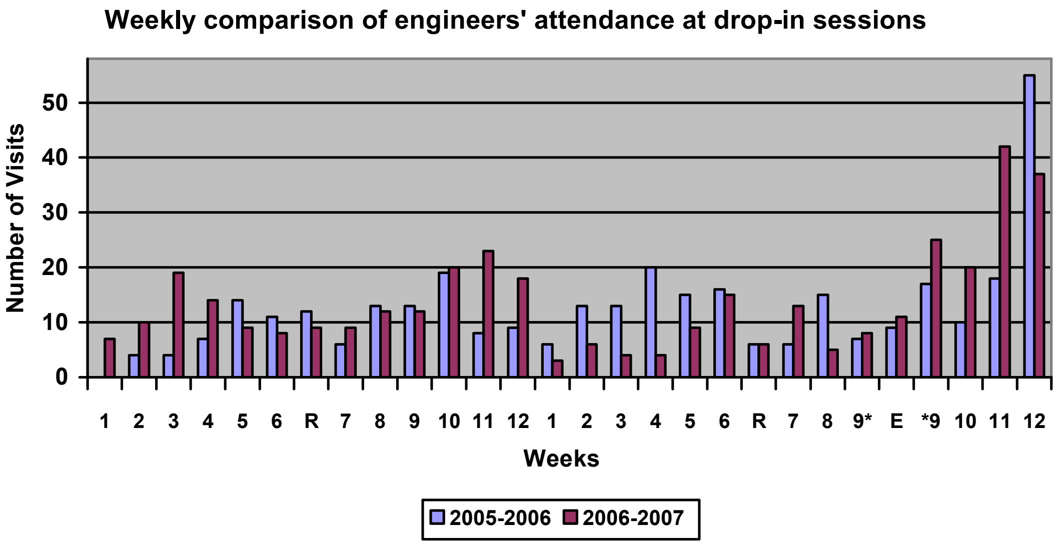

Figure 1: Weekly comparison of attendance of engineers at drop-in sessions during 2005-2006 and 2006-2007. “R” is “review week”, when students have no lectures; “E” is Easter holidays; “9*” is week 9 in 2005-2006, but week 1 of Easter holidays in 2006-2007, while “*9” is week 2 of Easter holidays in 2005-2006, but week 9 in 2006-2007.

As we are specifically interested in changes in patterns of attendance, Figure 2 is useful in showing exactly that; it was generated by subtracting the weekly visits in 2005-2006 from those in 2006-2007. Therefore, the portions of the graph in which the blue line is above the red zero line represent an increase in 2006-2007 over the previous year.

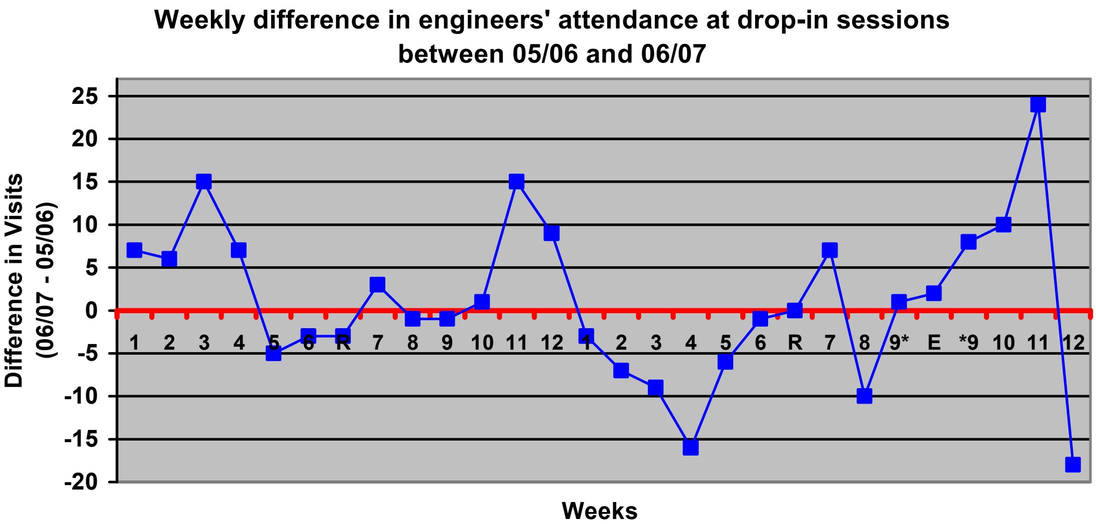

Figure 2: Weekly difference between attendance of engineers at drop-in sessions during 2005-2006 and 2006-2007. The same abbreviations apply as for the previous figure.

It should be stated that the weekly differences are generally not that extreme, as we are looking only at engineering students and subsequently, weekly numbers are not that high. In addition, some spikes can be explained by in-class tests, for example, in week 7 of the first and second semesters, and week 11 of first semester. However, it would have been expected that 2006-2007 would have shown higher attendance almost every week, given that students were better aware of the facility, and many would have attended the previous year. Instead, taking both graphs together, a distinct drop can be seen, most notably at the start of the second semester: this can partially be explained by the fact that some students would have started new modules at this point; however, the majority of engineering maths modules are year-long ones, which suggests that it may take students some time to settle back into study following the January exam period.

Many third- and fourth-year students sat their terminal maths exams in January; however, although they attended in huge numbers immediately prior to the exam, the majority had not attended during the first semester, suggesting a reliance on last-minute studying, even for students in the later years of their degrees. Anecdotally, the previous year, these students attended for several weeks prior to exams, allowing deeper learning to take place. This is not to say that semesterised exams have a negative impact on student learning of mathematics, but merely to suggest that some students may need a shift in thinking now in order to make best use of the support systems in place.

Conclusion

In this paper, we have described the daily operation of the Students' Maths Learning Centre in DIT, and specifically, its work with engineering students within the institute. We have identified the most common problem areas for engineering students, along with the resources we have found most useful. In addition, we have looked at the possible effects of semesterisation on attendance patterns in the SMLC; these effects may become more discernable in the coming years as more data becomes available.

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Reviewing the Effects of Revision Packs and Streaming on First Year Engineering Maths.

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Abstract

In June 2000 The Engineering Council released their report 'Measuring the Mathematics Problem' (Savage & Hawkes (2000)) which presents evidence of the 'serious decline in students' mastery of basic mathematical skills' and highlighted that this was a problem facing students 'at all levels'. The paper also predicted that these problems would increase over the next decade with undergraduates increasingly struggling with mathematics. This, in combination with reports citing the general short fall in number of graduate engineers (Spinks, Silburn & Birchall (2006), The Times (2007), The Higher Education Academy Engineering Subject Centre (2005), Henley (2006)), has meant that many engineering departments are reviewing curriculum content and delivery. This paper describes how one engineering department reorganised its maths provision in order to better support current and future students. Three things were done - bring the maths teaching into the department, revision packs and streamed classes.

Introduction

In June 2000, the Engineering Council (UK) published their report 'Measuring the Mathematics Problem' (Savage & Hawkes (2000)). Contained within this report were a number of observations and recommendations for the engineering community including:

> 'The decline in skills and the increased variability within intakes are causing acute problems for those teaching mathematics-based modules across the full range of universities.'
>
> 'There is a need for greater institutional awareness of the problems and for adequate resources to deal with them'
>
> 'What ever problems universities now face are likely to get worse rather than better over the next decade.'

In April 2001 the Learning and Teaching Support Network carried out an in-depth survey to investigate how departments were responding to the issues raised by this report (The Higher Education Academy Engineering Subject Centre (2008a,b)). A number of projects to look specifically at mathematics support for engineering students have been developed including creating a web-based UK Mathematics Learning Support Centre (MLSC (2008)). More recently the Centre for Excellence in the University-wide Provision of Mathematics and Statistics Support, sigma, has been funded by Higher Education Funding Council for England (HEFCE). These projects have resulted in a broad range of resources available to academics to assist them in addressing the teaching of mathematics to their students (Sigma (2008)).

In parallel with this research around the area of engineering and the issue of mathematics, several further reports have been produced highlighting the general shortfall in the number of graduate engineers (Spinks, Silburn & Birchall (2006), The Times (2007), The Higher Education Academy Engineering Subject Centre (2005), Henley (2006)). After reviewing such reports, on the position of engineering in the UK's Higher Education Institutes and schools, and engaging in discussions with many of the key groups providing support and promotion for Science, Technology, Engineering and Mathematics (STEM) across the UK, the Royal Academy of Engineering developed a proposal for a 'National Engineering Programme'. This programme was designed and written to build on present best practice work and to bring together active groups of people. Its key aim was to increase and widen participation in engineering across the UK whilst promoting engineering as a sound career option. The proposal was that this would be achieved through a partnership of regional schools, universities, STEM organisations and industry in selected neighbourhoods around the UK that had low participation rates in HE, using engineering as a vehicle to widen participation. This proposal was submitted to the Higher Education Funding Council for England (HEFCE) in 2005. It was at this point that the need for a pilot project of activity was highlighted and the London Engineering Project (LEP) was formed.

In light of the published research on the issues of mathematics and engineering it was decided that one of the areas of focus for the LEP would be the first year engineering mathematics experience in University College London and the initial results of this work are presented here.

The Study

Background

UCL is a selecting university with a thriving mechanical engineering department which runs accredited undergraduate courses for both B.Eng. and M.Eng. with entry A' level offers in the region of AAB or ABB. All students must have A' level mathematics or equivalent to be eligible for any course. Previously the mathematics courses were the responsibility of the mathematics department of the university. For a number of reasons it was decided that delivery of this would come from within the engineering department. As a result of this the Modelling and Analysis 1 module, that is sat by all first year engineers within the department, is designed not only to teach the basic mathematical tools required for engineering, but also, applications for each of the topics covered are discussed fully rather than the traditional approach of teaching just the topics as abstract mathematics. The module is assessed by a combination of five open book tests, sat through the year, and one end of year examination that will assess all topics covered throughout the year. Even with the high level of entry and a more engineering related mathematics course, not all students were achieving the required standards and this was having a detrimental effect on progression and retention and as such a two strand support strategy has been piloted over the last two academic years.

Strand 1 - Revision Packs and Lectures

Method

After a number of informal discussions with students it was discovered that many of them, whilst having obtained a grade B or above in mathematics at A' level, had done little, if any, mathematics since sitting their final examinations in June. This often led to a gap of up to three months where students had not practised their mathematical techniques leading them to arrive at University 'rusty' and not as mathematically confident as when they sat their A levels.

Initially what was piloted in the first year was a set of revision lectures for all the students which covered topics previously studied at A' level with no new work being introduced prior to their first assessment. While this was found to be successful in improving the performance of some of the students it was still felt that more could be done to give the students the chance to arrive at university and 'hit the ground running'. With this in mind a 'revision pack', based on work previously piloted at Warwick University (Savage & Hawkes (2000)), was developed for the students, that they could attempt prior to their arrival. Once students had received their A' level results and confirmed their place at UCL they were sent a set of worked examples of mathematics that they would encounter in their first term plus sets of questions to attempt themselves.

Results

Whilst it has not yet been possible to monitor the direct effect these 'revision packs' may have on the students' attainment, feed back from the students has been very positive: they felt better prepared and overall more confident about their personal mathematical ability when they arrived. This change in attitude has had a positive effect for the students, improving their initial experience in the university and even, if later research shows there is no difference in the attainment of the students, the 'revision packs' will remain in place to support retention and improve the match between students' expectations and the actual experience of university.

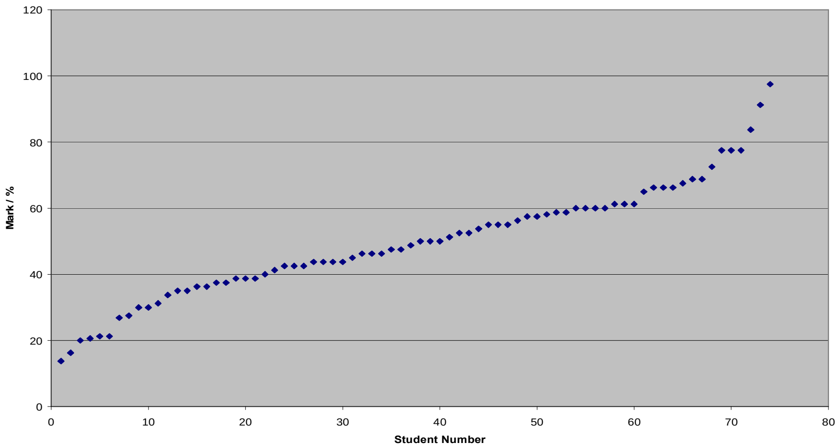
Some of the issues encountered with this practice have included logistics, with the window of time between the confirmation of student places and the start of term being quite short and so only just allowing time for the dispatch of the packs and completion of the problems. Should there be any minor problems with the dispatch or situations where students have taken a long vacation prior to beginning university this can result in some students not having completed the work. In addition to the logistical issues the whole project does rely on the students themselves being proactive and attempting the work, although the results appear to show that most students were engaging with the activities. Some students also fed back that they would have liked some further support should they need to ask questions or seek guidance, it is feasible that in future years the revision package may be supported by further on-line assistance available to the students prior to their arrival at UCL.

Strand 2 - Booster Sessions

Method

This work built on previous trials at UCL where students were tested on their mathematical ability on arrival at the University and those below a set level were given an extra hour of tuition on each subject but still remained being taught with the bulk of the class. This method was found to be unsuitable as it built resentment amongst those students required to sit further sessions. The decision was made to facilitate two completely separate groups that would be taught the same materials but in a different style and at a different pace, but with the final result being that the students sat the same assessment papers and end of year exam.

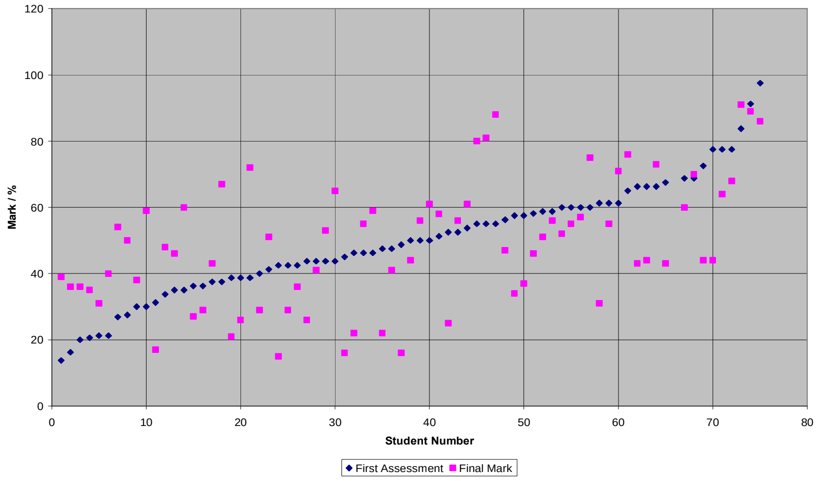
The first year that the booster sessions were introduced it was to students who had received their 'revision packs' and had taken a set of revision lectures together as a group. At the end of the revision lectures the students had the first of their assessments and as a result of those assessments were allocated a teaching group. The total group size was 75 with the distribution of marks obtained in the first assessment shown in Graph 1. From this graph it can be seen that 20 of the students obtained a mark lower than 40% for their first assessment with a further 40 students falling in the range 40% - 60% with the final 15 students gaining marks of 60% or above. From this the two groups were formed as follows: the first group consisted of students whose mark was 40% or below and they were taught using three one hour lectures per week and one and a half tutorial sessions per week. The second group consisted of students whose mark was above 40% and they were taught using two one hour lectures per week and one tutorial session per week. Both groups covered the same material, had the same final revision sessions, sat the same assessment papers throughout the year and sat the same end of year exam.



Graph 1 First Assessment Marks

Results

Graph 2 shows the comparison between the students' final examination marks and the marks they obtained in the first assessment prior to them being streamed.



Graph 2 Comparisons of final marks and first assessment

What can be seen from this is that in both teaching groups some students improved their grades and others did not maintain the same standard as their first assessment. However when these results are examined in more detail what can be seen is that 65 % of the students in the first group (first assessment 40 % or below) significantly improved their grade (significant meaning an improvement of 10 % or above) but only 14 % of the second group significantly improved their grade. When examining the results of those students who did not maintain the same standard, what can be seen is that in the first group only 3 students suffered a significant decline in their mark (significant meaning a decline of 10 % or more) compared to 19 students in the second group.

There are many variables in this work that could account for the differences in the performance of the students, including: the first group being much smaller in student numbers than the second, each group being taught by a different academic, the first group had additional lectures and seminars. Due to this it is impossible to draw a specific conclusion as to why the two groups performed in differing ways. But what can be concluded is that the work showed a significant enough difference between the groups to warrant further investigations.

Conclusions

Bringing the teaching of mathematics into the department of engineering has allowed for the development of a more ‘engineering relevant’ mathematics module which incorporated engineering examples and problems into its teaching. The impact of this has been that the students were able to develop a deeper understanding of the importance of mathematics to them as engineers. Feedback from the students has suggested that they appreciated having the module taught in the department as they felt it easier to access the academics.

The ‘revision packs’ gave the students the opportunity to enhance their mathematical abilities prior to starting university. Interestingly one of the key impacts of this work has been to narrow the gap between students' expectation of the mathematical level at university and the students' experience of mathematics at university. Further work for this project will be to examine the feasibility of developing such packs for other topic areas within engineering.

Streaming the student and altering the teaching style has proved a successful method of supporting the mathematically weaker students and the results justify a further more detailed qualitative study.

Acknowledgements

We would like to acknowledge the hard work of Dr E Stride who originally prepared the revision packs and Dr R Moaleij and Dr I Eames who assisted in the teaching on the Modelling and Analysis 1 course.

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A Poor Mans TA –
Testing and Assessing Mathematical Skills by CAS-Scripts

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Abstract

The ever decreasing level of mathematical skills of students beginning their engineering studies calls for radical countermeasures. To gain time for developing high-order competencies we have to unload crude drill & practice exercises to a computer program. We propose a combination of an open source learning platform together with an open source computer algebra system, CAS, in order to generate custom-made exercises by CAS-scripts which in turn assess the answers of learners. By coupling a CAS and a learning platform the learning platform states the problem generated by a CAS script and processes the scripts assessment of and its feedback to the answer of the learner, saving it for later evaluation.
This division of labour allows for efficient programming of (classes of) exercises and for fully utilising the learning platforms bookkeeping, evaluation, and statistics features.
We will discuss and illustrate our approach by relevant examples.

Tendency of Students Mathematical Skills to Fall and Countermeasures

Over the years students at the start of their engineering degree courses show more and more severe deficits in their mathematical skills – deficits as monitored by many observers at least in Germany. These deficits comprise difficulties in calculating and in giving rough estimates as well as problems in modeling and troubles in checking plausibility, etc. At the same time students have little background knowledge in science. Especially in the mathematics education there seems to be a need for crude drill and practice exercises which we consider necessary to enable students to work their way from simple calculus (with most of the time unrealistic problems) to more demanding tasks like modeling and simulating real problems. Students should train their basic, low level mathematical skills by running lots of exercises. Such exercises have to be marked, and there has to be some reasonable feedback to the student in case of failure. The high demand for such training cannot be met by traditional means. There are not enough resources to do all the marking and to give the necessary feedback. Therefore, a computer program is needed to generate parameterisable questions and problems which the students have to answer and to solve – a computer program which at the same time assesses the answers and solutions providing a right/wrong decision and some feedback on errors. (The question of which types of mathematical skills can be developed by which type of computer assistance is addressed in general by Risse (2002) and for graphical input by Risse (2005).)

FiM – a Fitness Program in Mathematics not only for Engineering Students

We designed such a system as an addition to the open source learning platform ILIAS (Ilias (1997)), and used MATLAB/Maple (The Mathworks (1994-2008)) as the symbolic mathematics engine. (In the following, ILIAS stands for any suitable learning

platform and MATLAB/Maple for any suitable computer algebra system.)
Students formulate their answers using the prevalent syntax of mathematical expressions as in MATLAB/Maple or in any programming language. We demonstrate how by scripting, a wide variety of parameterisable problems can be generated and how MATLAB/Maple checks the correctness of the solutions of the students.

Programming FiM-Scripts by Using a Template

Together with some supporting functions we use a template for FiM-scripts to facilitate the programming of exercises. Here, we list its generic structure as a pidgin MATLAB m-file to shown how scripts work as well as how the learning platform and the computer algebra system are coupled.

In principle, when no answer is passed to a FiM-script it produces an exercise by normally generating variable names or constants at random. However, when an answer is passed, the FiM-script checks the correctness of the answer and produces suitable feedback.

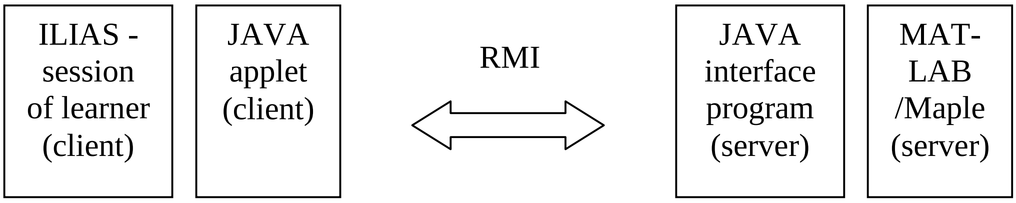
```
function result = Testname(answer)
if (nargin == 0)              % if no argument is given then generate test
                              % generate symbol names
  result = textoftest;        % parameters or coefficients at random
                              % and then, construct text of test question

else                          % if argument is given then assess test
  if (answer is mathematically correct)
     if (answer has the correct form)
        result = 'ok';
     else
        result = 'false';     % or some other appropriate feedback
     end
  else
     result = 'false';        % or some other appropriate feedback
  end
end
```

Of course, in this template for the sake of clarity we omitted the management of the functions workspace by the session ID, the localization (the same exercise in different languages), and the discrimination of exercise and examination mode.

Communication between ILIAS and MATLAB/Maple

The coupling between the learning platform ILIAS and the computer algebra system MATLAB/Maple provides the necessary services to the client application, i.e. the learner. The following figure illustrates the different interacting programs.

ILIAS allows the so called tests to be implemented by JAVA applets, cf. Schottmüller (2007). We implemented the communication between the JAVA applet on the client and the JAVA interface program on the server by Remote Message Invocation, RMI. The access to MATLAB/Maple is implemented using the JMatLink library, see Müller (2005).



1. Client: By choosing an exercise the learner makes the JAVA applet to call the m-file on the MATLAB/Maple server by RMI.

2. Server: The JAVA interface program calls the m-file on the MATLAB/Maple server which generates parameters at random and produces the exercise as a string. The JAVA interface program returns this string to the JAVA applet by RMI.

3. Client: The JAVA applet presents the exercise to the learner.

4. Client: The learner provides his or her answer as a string. The JAVA applet sends this string by RMI to the m-file which generated the exercise on the MATLAB/Maple server.

5. Server: The m-file on the MATLAB/Maple server checks the answer for correctness and produces appropriate feedback as a string which again the interface program sends to the JAVA applet by RMI.

6. Client: The JAVA applet presents the result to the learner. The learning platform ILIAS does all the bookkeeping for later evaluation and statistics.

Potential and Limits of FiM

There is practically no limit to the areas which can be addressed by FiM-questions and problems: algebra, calculus, statistics, etc. comprising literally all areas of scientific computing. We verified the potential of FiM by implementing a lot of the exercises of a typical exercise book for first year students, see Schäfer, Georgi & Trippler (2006). Right now, the pool of exercises comprises well over seventy exercises for students about to commence their engineering degree courses. Topics cover term manipulations, fractions, powers, percentages etc.

All exercise parameters like constants and variable names can be generated at random: programming one script can thus generate a variety of exercises.

By initialising the seed of the pseudo random number generator depending on the date of some test or examination it is even possible to generate exactly the same questions and problems for a group of students who are scheduled to be under examination on this very date.

A Real Open Source Version of FiM

Substituting MATLAB/Maple by octave/GiNaC (octave (1998), GiNaC (2001), Bauer, Frink & Krekel (2002)) in the future, will render the FiM system a complete open source solution for the generation and assessment of question/answer type tests – competing with commercial solutions like MapleTA (Maplesoft (1988-2008)) for example.

Expectations and Promotion of FiM

The necessity of reasonable support of the drill & practice part of mathematical education drives our development of the technical aspects of FiM. The means offered by FiM will be employed at the latest when the next generation of first year students has to brush up their mathematical skills. At the same time we co-operate with teachers of a handful of schools in Bremen who weekly specify exercises which we implement quickly enough so that their students can work on these tailored exercises. In this way, their ongoing learning process is supported.

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Positions to Mathematical Education of Engineers

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Abstract

The authors representing a network of active lecturers in northern Germany state the following current dilemma in the country: On the one hand competitive industrial regions need many well educated and highly motivated engineers. This should include profound knowledge in Mathematics, Physics and Computer Science. On the other hand there are not enough beginners in engineering studies and their knowledge in basic sciences is often poor. Similar situations are known from other countries. Some general causes of the dilemma are named: trend to poor entertainment, bad position of Mathematics in public and politics, quickly changing and ineffective reactions of politicians to proven deficits in basic education, negative attitude of many pupils and students towards Mathematics, bad working conditions of teachers, decreasing time budgets for mathematical education, obsolete teaching methods in schools and universities. The consequences of the dilemma will be social and cultural decline as well as loss of jobs and a bad trade position on the international market. Developing high technology needs more and more high Mathematics, especially Analysis and (numerical) Linear Algebra. While theoretical knowledge becomes more important to solve practical problems on the computer, elementary calculation skills are pushed into the background. Some general demands are postulated to reverse the negative development: upgrading of Mathematics in public and education (key technology of societal progress, general thinking and cultural technology), upgrading of teaching profession, statement of uniform (and possibly central) minimum requirements for mathematical competencies at beginning and end of engineering studies, securing of satisfactory staff and IT resources for mathematical education in Engineering. Last but not least, the influence of modern developments in education and of staff activities on the dilemma is mentioned.

Introduction

Engineers use in practice mathematical models and methods which mathematical lectures in engineering studies do not offer. Unfortunately these gaps between education and profession increase further.

Contradictory causes of maldevelopment:

- Many beginners to study have insufficient pre-knowledge in Mathematics.
- Therefore mathematics lectures are often adapted to a low level.
- Quantity and quality of important mathematical subjects increase.
- Nevertheless available time for mathematical subjects in schools and universities is reduced stepwise.

Berger and Schwenk (2001, 2006), Brüning (2004), Polaczek (2006), Strauß (2006).

Counter steps:

- Mathematical pre- and complementary courses for weak students,
- New teaching methods, means and media,
- Cooperation of school teachers and lecturers in the domain of Mathematics,
- Didactic workshops and conferences of lecturers in Mathematics.

The resulting problems increase so dramatically that the counter steps will not improve the situation essentially. The announced theses are not new but of increasing relevance. They are to put new life into the discussion about true reforms in the educational sector.

Young generation problem in engineering

Thesis 1: A competitive region needs well-educated and highly motivated engineers. Their number is an important parameter for the future prospective of a society. Already now this number of engineering experts is too low.

Thesis 2: There are not enough study beginners in the engineering disciplines. At the same time too many young people abandon engineering studies, often because of poor knowledge in Mathematics.

Müntefering (2006)

Consequence: Well-educated young engineers will be missing in the near future. The demographic change will aggravate the situation. It is only a temporary solution to import engineering experts from abroad. Without changing the educational conditions the problem will increase. A social decline would follow.

Key position of Mathematics in science and engineering

Thesis 3: Mathematics is a very old cultural and thinking technology in the heart of science and a global science language independent of ideology which is indispensable to ensure the future of mankind. In a modern society therefore the strengthening of Mathematics has many positive effects.

Thesis 4: The huge growth of knowledge enhances the importance of basic sciences, especially of Mathematics. This must be considered in engineering education. Mathematics is a key qualification, which determines the quality of an engineer essentially. Mathematics is necessary to model, analyze and simulate technological systems as well as to calculate and to evaluate solutions of model problems.
In the past engineers could contribute essentially to mathematical disciplines and thinking. This tradition has to be continued.

Grünwald, Kossow and Schott (2000), Pesch (2002)

Mathematical education in engineering studies

Thesis 5: In mathematical education a solid basic knowledge is crucial. Further, modern means (as computer, internet) have to be used as an important ingredient.

Schott and Grünwald (2001, 2004), Schott (2004, 2005a)

Thesis 6: Including computers in the mathematical education needs a change of teaching contents and a shift of priorities. In this process the demands on both mathematical basic knowledge and computer software knowledge increase.

Schramm (1998), Schott (2006), Strauß (2006)

Mathematics as a stepchild in society

Thesis 7: Mathematics has (for some decades) a bad reputation in public and only a small lobby in politics. It does not seem to fit in the spirit of this age.
Mathematics is a neglected subject in education at schools and universities. Negative attitudes of pupils and students towards Mathematics are often accepted and partly supported with pleasure. Without reason teachers are usually blamed for the misery.

Public reactions

The discussion to reduce the known deficits in basic sciences such as Mathematics is often covered by ideological reservations. Sometimes reasonable demands are ignored or even defamed as nonsense of backward directed persons. Many politicians react helplessly and ineffectively with quickly changing proposals and activities.

Conclusions

Thesis 8: Decreasing numbers of beginners in engineering studies and increasing drop out rates of students in engineering subjects have societal roots whose effects are already visible in schools. One crucial reason for the conflicting situation is the position of Mathematics in society and education. A deep reform of the educational system is urgently necessary.

Grünwald and Schott (2000), Schott and Grünwald (2001, 2004), Schott (2006)

Upgrading of Mathematics

Evidently, the counter steps already initiated at schools and universities named in the introduction have to be continued and developed further with full power. Below we list some of the activities we offer and practise.

Cooperation with schools

- Snooping weeks for pupils at university,
- Use of laboratories at university by pupils,
- Coaching of projects for pupils by university members,
- Further education of school teachers at university,
- Development of attracting events and materials to train imagination and creativity.

Support of study beginners

- Refreshing course in the introductory week of study,
- Entrance test in the first weeks of study,
- Consultation hours for students headed by lecturers or good students,
- Online courses to train basic mathematics.

Special offers

- Computer aided Mathematics (MATLAB, MAPLE), Schramm (2002a, 2002b, 2006, 2008), Schott (2004, 2005a)
- Project work in Mathematics (e.g. Dynamical systems in Engineering and Ecology), integration of interactive teaching units and team work, Risse (2001, 2002), Schott (2005b)
- Facultative special and complementary courses in Mathematics, Strauß (2003), Lutz and Lutz-Westphal (2005)

General demands

- Politicians have to recognize the decisive role which Mathematics is playing for the development of science, engineering and society. Mathematics must achieve its true position in public and society.
- In (engineering) education Mathematics has to be upgraded (satisfactory staff and IT resources as well as sufficient time budget with respect to the corresponding specialization, strong promotion and recognition of mathematically gifted and motivated young people).
- More special schools with focus on Mathematics, Natural Sciences and Engineering have to be established, but this should not lead to a new division of labour in engineering practice.
- Uniform (and central) minimum requirements for mathematical competencies at beginning and end of engineering studies have to be fulfilled which are orientated at the true requirements in society and practice.
- It is necessary to modernize mathematical education on all levels continuously.
- A central commission of (not too many) true experts should initiate and supervise the reform process.

Schott, Strauß, Schramm and Risse (2007)

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E-materials, E and B learning: a practical approach

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Abstract

After introducing different e-materials (files using CAS, selected web sites, Java applets, on-line calculators, Centres of Mathematics, etc.) we analyse some different possibilities for using these materials in an attempt to match the learning required by engineering students with the time devoted to mathematical issues.

Introduction

Taking into account the new reference framework of the European Area of Higher Education (EAHE) and the greatly increased availability of electronic learning materials, it is necessary, at least in Spain, to make a profound reappraisal of the form and mathematical content to be taught in engineering schools.

During the 1960s and 1970s, mathematics occupied a large part of the curriculum in engineering schools, since it was argued that mathematics developed the students' minds regardless of the specific material covered (which tended to be highly theoretical and in most cases divorced from the mathematical needs demanded by engineering studies). Instructors would give what their "personal inclinations" told them to and there was a trend towards teaching mathematical content that was disconnected from subjects with a more technological focus.

The current mathematics curriculum in engineering studies, which stems from the 1990s, underwent a brutal reduction in the time, in some cases up to 50%, devoted to the study of mathematical concepts. The initial reaction of instructors was either to continue giving the same content, but in less depth, or give a reduced content, in both cases using "traditional methods".

Inertia, a lack of motivation, the sparse recognition by Spanish universities of the teaching activities of its professors (because achieving promotion is based almost exclusively on research performance), together with students' attitudes (such as their unwillingness to give personal sacrifice, the scant interest they place in their studies and very poor studying habits) all work together to create a hostile environment for learning mathematical concepts, which requires tranquillity, understanding, and a mature outlook. This situation will become further exacerbated with the EAHE (Direction General For Education And Culture (2004), Garcia et al.(2006), Parlamento Europeo (2002)). The main milestones here are:

- A competency-based curriculum. It is then possible to plan and select the mathematical content for specific students: engineers, for example.
- A diversified curriculum (theoretical and practical teaching, supervised academic activities, independent individual work, and so on).
- Concern for the student's overall work load; as opposed to the current system in which the only measurement in certain European countries, such as Spain, is the number of lecture hours given in the classroom. This work should have a maximum annual volume estimated at 60 European credits (European Credit Transfer System, ECTS). An initial estimate suggests that one ECTS credit is equal to 25-30 hours of student work, including class attendance, laboratory work, workshops, individual and group tutorials, individual or group work and assessments.
- Prevalence of student learning over the lectures provided by teachers.
- A duration of study more adapted to reality.

The demands on the teaching staff, stemming from the European-wide policy of harmonising higher education studies, are basically as follows:

- Teachers should teach things that add value in the labour market (competencies).
- Teachers should adopt a different approach to teaching (methodological innovation).
- The basic criteria for the planning and development of teaching should be students and their needs (students as a reference point).
- All this involves a greater dedication by teachers to a student's learning process, which should include the teaching hours, planning, material preparation, suggested lines of work, guidance and supervision and, in general, overseeing the student's entire learning process.

This whole situation requires teachers to take special care when planning the following aspects:

- The competencies that students will acquire individually within the context of the full range of competencies that are taught in the subject in question.
- The academic scenarios in which the teacher wants the learning to take place, referring to specialized classrooms (computer rooms, laboratories, audiovisual rooms, etc.), libraries and other documentation centres, provided that the students can proceed independently within them. In addition, the proper use of e-learning can contribute to the actual learning process, taking into account:
    - The working process to be followed by the student for the acquisition of these competencies.
    - The system of tutoring-supervision that the teacher has adopted to control the acquisition of competencies on the part of students.
- The system for assessing the competencies acquired by the student on an independent basis, taking mainly into account: the assessment criteria, the assessment tools and the assessment dates.

The teacher will be required to provide students with material, give lectures, set supervised work or problems, arrange different kinds of tests, use the latest technologies to a greater or lesser extent, in order to generate the number of working hours corresponding to the credits assigned, which are furthermore to be controlled in an effective manner. However, concern over the overall work load of the student and optimisation of the time involves some dedication by the instructor to management and evaluation tasks, and the truth is that many of us fear that this will not be the case because such activity receives little academic recognition; although the tasks are more bureaucratic, they are equally necessary in teaching. In any case, despite the above remarked difficulties, it is necessary to consider the possibility of a profound methodological remodelling of the teaching of mathematics, using the whole technological arsenal now available to us.

The materials

When designing mathematical courses for engineering students it should be borne in mind that our students are users of mathematics and that they use them as underpinning to their specific technological studies. It is crucial that the materials given to them be of high quality so that we can take maximum advantage of the total time devoted by a student to his or her training in mathematics.

The materials provided to students start with text books adapted to their needs and that we have developed in collaboration with instructors from several Spanish Universities. These books contain a compendium of theoretical results that instructors can use appropriately in their theoretical classes. Students can check their level of understanding of the material by doing self-assessment tests with a true/false format. We later choose significant problems, sequenced in chronological order of the contents and level of difficulty, and offer a collection of proposed problems with the same level of difficulty as the worked examples.

The electronic materials (e-learning) are of very different types and can be classified as follows:

1. Files based on CAS (*Derive*, *Mathematica* and *Maple*) for use by students. Depending on the mathematical concepts analysed, these files use different CAS resources (graphic, symbolic or numerical). The authors have participated in the compilation of these materials, either as a complement to more conventional mathematical texts (Garcia et al. (2002), Garcia, Garcia, Lopez, Rodriguez & Villa (2006), Garcia et al. (2007)) or as other types of experiments (Alonso et al. (2001), Garcia et al. (2005), Roa & Villa (2007), Rodriguez & Villa (2005, 2007a)).
2. Materials developed under the umbrella of different European projects in which we are involved. The general philosophy of these projects consists of enhancing cooperative work with a view to optimising efforts in the generation of this type of resource. The projects in which the authors of this communication have been

involved can be found in (Calculus Course (2007), SEFI MWG (2006), EVLM (2008), EVLM-Slovakia (2007), Rodriguez & Villa (2007b)).

3. Materials freely accessible on-line, enhancing their correct use by our students This selection of Internet resources contains Java applets, e- books, exercises and self-evaluation tests, mathematical curiosities, etc.
4. All this material is given to students using B-learning (blended learning) techniques, since it should not be overlooked that we are working with students in a face-to-face teaching situation at the University. In our case, the *Moodle* platform serves as a vehicle to offer our students efficient guidelines that will allow them to take maximum advantage of the resources selected and enhance their participation and involvement in academic tasks.

The use of these materials will be governed by the credits assigned to the corresponding topics, the availability of mathematical laboratories, and the possibility of having tutors able to respond to students' doubts. In any case, it is necessary to mix conventional teaching through lectures with the use of the different materials selected and active supervision by teachers in the student learning process.

Some experiences

Under the auspices of this philosophy we have carried out certain experiments:

1. **A course on calculus at the Engineering School at the Pontificia Comillas University.**
   In this course, the basic material consists of a textbook (Garcia, Garcia, Lopez, Rodriguez & Villa (2007)) in which the authors of this communication have collaborated. The book is accompanied by a CD with the answers to the proposed (and unsolved) problems, most of them made using *Derive*, and tutorials on the concepts analysed in the book. The students are provided with guidelines to the issues addressed in the course, with information about the contents of topics, the problems to be dealt with in practical classes and those forming the personal task of each student. They also receive information about complementary tasks (normally consisting of doing tests and problems selected from the chapters of the textbook), and visiting certain web sites that may reinforce their learning. Students are recommended to use *Derive* and are expected to attempt self-study tutorials, which we anticipate will be easy for them to follow. After the tutorials, it is suggested that they do some exercises with *Derive* and that they use CAS as a check of exercises performed "with pen and pencil". All this information has been organised through the course web site for which an access code is required.

2. **A course using the *Moodle* platform at the Polytechnic University of Madrid.**
   At the School of Industrial Engineering of this University, the *Moodle* content manager is used in the course on calculus in one variable. This has been a pilot experiment since in the design of the course the number of hours for classes and practical work has been reduced to increase those devoted to a CAS (which has also been *Derive*). A textbook has been selected (Garcia, Garcia, Lopez, Rodriguez &

Villa (2007)), and on the basis of this guidelines to the topics addressed in the course (a standard course on calculus in one variable) have been generated. The *Moodle* platform has been used to organise the course and to facilitate the handing-in of work, self-assessment, etc. A detailed overview of the course can be found in Alvarez, Asensio, Garcia-Miguel, Velasco & Villa (2007a,b) and the course's web site is at (Calculus Course (2007)).

3. **A course on ordinary differential equations (ODEs) with a CAS.**
It is clear that concepts concerning ODEs are the paradigm of "applied mathematics". In all stages of engineering studies phenomena are emerging where it is necessary to study the "variation" of some variable against another variable. For instance, population models, radioactive decay, heat transmission, RLC circuits and mass-spring mechanisms, are examples of technical situations modeled by ODEs. Thus, we must analyze from the theoretical and practical points of view different concepts associated with ODEs. The use of the abilities of a CAS, in our case *Mathematica* and *Maple*, allows a new focus on the teaching of differential equations; mainly by avoiding heavy calculations and paying more attention to the concepts. Taking into account that Computer Algebra Systems (CAS) allow one to obtain the solution of ODEs, in exact form quicker than with "paper and pencil" it is possible to design an ODE course paying especial attention to concepts, structures and some aspects of solutions. CAS can be used to introduce or enhance theoretical aspects related to ODEs and as a calculator for solving ODEs (Rodriguez & Villa (2007a)). We can then focus our attention on analyzing the solutions and the main characteristics: bounds, stability, etc. CASs can also be used to experiment with and simulate different situations involving technical problems, such as RLC circuits and mass-spring mechanisms. The most important numerical methods for solving ODEs (Euler, Runge-Kutta, Adams-Bashforth, etc) can be programmed in the corresponding CAS, or otherwise special libraries can be used. The main goals of the course are: To learn the basic terminology, to use a few exact methods to solve specific ODEs (mainly linear ODEs because the structure of the solutions is very important in the analysis of certain technical phenomena), to interpret as much information as possible directly from the ODE (direction field, qualitative analysis, isocline lines, asymptotic behavior, etc.) and to be able to implement numerical methods and to interpret the results obtained. The use of the different capabilities of CASs allows this alternative. Some examples can be found in the CD in Garcia, Garcia, Lopez, Rodriguez & Villa (2006).

4. **The creation of the Centre of Mathematics at the University of Salamanca.**
Taking advantage of the experience of other countries such as that reported from the United Kingdom http://www.mathcentre.ac.uk/ with this Centre of Mathematics we offer a face-to-face and on-line tutorial service to which students can turn to resolve all their doubts relating to their training in mathematics. The centre organises what are known as zero-course introduction to university teaching and other activities in the field of basic training.

Conclusions

The strategy for the use of e-materials should be to improve their benefits and to diminish their pernicious effects. According to the European Area of Higher Education (EAHE) our task will be to optimize the time required by our students in learning to use mathematical tools. Regarding CAS, we promote their use only to those students who are able to solve the problem, posed in an academic way, using “paper and pencil”. We have produced tutorials for many topics: Linear Algebra, Calculus in one and several variables, and ordinary differential equations. In each topic we begin the tutorial by explaining, if necessary, the commands to be used. Then, the first exercises are solved “step by step” after which, on some occasions, we write a procedure to make the use of the CAS automatic. We have selected interesting web sites where it is possible to find Java applets, theoretical and practical concepts, exercises to be solved by students with immediate checking, etc. We are currently trying to introduce “Centres of Mathematics” where students can find tutorials on-line and immediately consult the instructors as to any doubts arising from the use of such tutorials.

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Linear Algebra as a Bridge Course for First-year Engineering Students

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Abstract

Linear Algebra is frequently the first course in abstract mathematics and, as such, a hurdle for students in engineering fields. Beginning students are generally ill-prepared for the independent learning required by higher education, in which feedback to one concept comes well after the introduction of the next topics building upon it. Incoming students are often overwhelmed by the new learning environment typical of (especially large) colleges and universities. The anonymity of such institutions and the demands on students, especially in the first year, can tempt students to copy homework solutions, a phenomenon that seems to have increased over the past years due to the internet.
The TUMULT (TUtorien MULTimedial) learning environment and its didactical implementation take advantage of technological possibilities in order to deal with the difficulties described above. Feedback to procedural skills through online training exercises and support materials for lectures and tutorials allow instructors to focus more time on helping students to understand concepts and to improve their problem-solving skills. Electronically corrected homework sets in which each student receives the same type of problem, but with their own personal numbers, encourage students to discuss solution methods with each other, yet require students to solve problems individually. The time saved in grading many routine exercises can be invested in correcting a few in-depth problems. Through the introduction of prelearning, students are prompted as to how to prepare themselves independently for this course, thereby providing them with basic tools that may prove instrumental in the successful completion of their studies.

Background

Linear Algebra for Engineers at the Technische Universität Berlin (TU Berlin) is a required course for approximately 2000 incoming freshmen engineering students per semester. Typically, students weekly attend one of the four to five parallel lectures as well as a tutorial, the size of which varies between 35 and 55 students. In a traditional tutorial, the tutor solves sample problems at the board, whilst the students frantically copy everything down. Homework exercises with problems not substantially differing from the tutorial exercises are then solved and submitted in groups of two students. This group work, which is not to be confused with group learning in cooperative groups, such as described in (Hagelgans et al. (1995)), was introduced over a decade ago to cope with the growing number of students to be taught by a reduced number of tutors (due to budget cuts). In our experience, only one of the students in the group actually solves and writes up the solutions. This is confirmed by the typical 45 - 55 % passing rate for students who have acquired enough homework points over the semester to be admitted to the final examination. Due to the sheer numbers, the variety of mathematical abilities of the students (and the tutors), as well as the abstract nature of the course, Linear Algebra is considered a major hurdle for and by our engineering students. The outcome is disastrous, with only about 30 % of students enrolled in a tutorial at the beginning of the semester actually passing the final examination. Students who fail two written examinations are then required to take an oral examination. Many of these students turn out to have quite a bit of ability. So where are we failing our students?

In an effort to remedy this situation, an e-learning environment was developed for the course in linear algebra for engineers using the MUMIE (Multimedial Mathematics in Engineering Education) platform (Mumie (2008)). The key goals were to provide training modules with instantaneous feedback as well as to motivate students to actively participate in mathematics during the tutorials. Electronically corrected exercises were also integrated into the system to alleviate tutors from grading routine exercises so that their time could be invested more wisely, for example in correcting a theoretical problem for each student per week.

During the first three semesters in which this system was used, experimental tutorials were held in order to observe how students learn with the materials developed as well as to identify and find solutions for factors contributing to the failure of our students. The TUMULT (TUtorien MULTimediale) tutorials arose as a result of this study and have been an integral part of the course over the past three semesters. The purpose of this contribution is to report on the intentions and the preliminary outcomes of TUMULT.

Challenges and Strategies in the Mathematical Education of Engineers

During a three-semester study, several problems were identified in the way linear algebra was being taught to and learned by our students. This section is devoted to a discussion of the main points, most of which do not appear to be specific problems of the TU Berlin, as well as our approach to rectifying these shortcomings.

Preparing for and attending class

Students do not come prepared for the lecture. Reading the new material prior to the lecture in order to identify areas of confusion would substantially help students to better understand the lecture. Generally the students attend the lecture (if they even go at all) with the thought that they can understand mathematics by osmosis: attending the lecture equals understanding. If they do not understand the lecture, then it is because the lecturers are ill prepared, talk in mathematical jargon, etc., which may be true some of the time, but certainly not all of the time. Especially freshmen students are unaware that they need to take responsibility for their own learning at the university. Students who were good pupils in mathematics at the high-school level often have a distorted perception of their own mathematical abilities when they enter the university, as they were previously comparing themselves only with others at the same high school. In a poll of incoming freshmen, only 15 % said that their knowledge of mathematics was less than average (Roegner et al. (2007)) even though assessment skills testing showed otherwise. Furthermore, the expectations of university studies are much different at this level than what beginning students are used to.

Working through examples before the tutorial is for most students just as unthinkable as reading the material before the lecture. The outcome is an inefficient tutorial, with most of the time being spent on a review of the lecture. For this reason, many students opt to skip the lecture, resulting in them being even less prepared for the tutorial. The first exercises in the tutorial, for which tutors spend most of the time, are generally of a computational nature that lead to the theory. Due to time demands, the theoretical problems at the end of the tutorial are rushed through. The result is that students have great difficulties in solving theoretical problems in the final examination, especially when it comes to understanding the connections between concepts. Furthermore, as students are generally forced into the role of passive consumers during the tutorial due to time constraints, questions in the examination with a slightly different wording lead to confusion amongst the students.

To combat these problems, prelearning was introduced as an integral part of the course. Although this is not at all new a new concept, it is for our students at the TU Berlin. The main point here is that with an e-learning system, the prelearning problems can be controlled electronically, so that instructors are not required to do more grading. Students are required to solve two relatively simple problems that relate the new material to school mathematics at the beginning of the semester or to previous concepts in the course towards the end of the semester. For example, before introducing the Gaussian elimination process, students are required to solve systems of two equations in two variables which demonstrate the fact that a system of equations can have no, one or infinitely many solutions. They are asked to reflect upon which operations were permissible, preparing them for the ensuing discussion on Gaussian elimination in the lecture and tutorial. In a later stage, when determinants are introduced, students are prompted to make connections with invertibility and linear independence.

Lecturers who have previously held this course before prelearning times are amazed that students now answer questions posed during the lecture. The acceptance among students is higher when the lecturer ties the prelearning problems in with the lecture, for otherwise students see prelearning as busy work. One half of the students say that they feel like the prelearning problems help prepare them for the lecture. About one half find the prelearning problems too easy. It is not foreseen to beef up these problems in the near future, as it is not the intention to demotivate weaker students from the onset. Perhaps optional problems that are more challenging could help to motivate more advanced students.

Working ample exercises and getting timely feedback

As mentioned previously, only about one half of our students worked enough exercises to pass the examination. Students tended to exchange answers through the internet Forum provided, in which the answer was the focus, not the solution strategy. Furthermore, the feedback from the tutor came often two to three weeks after the introduction of a topic in the lecture, so that students were quite often unsure if they understood the material so that they sought help too late in the course.

TUMULT provides students with many possibilities for visualization as well as training concepts and computations online with feedback at the click of a button. Not only routine exercises, such as matrix multiplication, can be practiced. It is also possible for students to learn and test their knowledge on theory and connections, such as what a coordinate vector is or the geometric interpretation of an eigenvalue for a linear mapping from $\mathbf{R}^2$ to $\mathbf{R}^2$.

The training modules have also been integrated into the tutorials. Each of the three to four themes per week begins with a learning check to identify where students are in terms of their understanding. After the ensuing discussion, students then train in small groups at a computer (often their own laptops), freeing the tutor from the board to control student progress. Of course, tutors used to be able to pose a question and have students work on problems. The advantage with the electronic training is that students can ask for another question if they finish ahead of the others. We have also found that the new media sparks more interaction amongst students. What one cannot figure out alone can often be solved through a discussion with fellow students. Ideally the tutor should be more of a coach than the central omnipotent figure in the tutorial. Time constraints, however, do not always allow for this.

Each training module is equipped with a demonstration and training environment. The related electronic homework problem is often stated somewhat differently than the problem in the demo and training portions in order to guarantee a certain amount of transfer of learning. The problems in the training and homework portions are individualized and have a mask in which free form answers can be entered. Each student has the same type of problem, but each has their own randomly generated values. Students therefore communicate with one another about solution strategies, not just solutions. Exercises can be practised and checked until enough confidence is gained for working the homework problem to be electronically graded. Copying, which used to be conceived as a major problem, is rendered useless.

The training possibilities are widely accepted amongst students. Polls taken (Roegner et al. (2007)) show that 85 % of our students use the online trainer on a regular basis. The electronic corrector is one of the least popular aspects of the TUMULT program, with only about a 45 % approval rate. The problem is that with free answers, it is difficult to award partial credit. If the automatic corrector recognizes that a plus-minus error was made, some points can be assigned. Other minor errors are not so easily detectable. One solution would be to require students to enter their steps in between, which could become annoying for students who are certain of their final answer. Furthermore, the prompt for the answer to a between step would also provide too many hints as to how the student should proceed and would require all students to use the same procedure for solving a problem, which is philosophically incompatible with TUMULT, in which we encourage students to develop and explore a variety of solution strategies.

Communicating mathematics

During the tutorials, students are expected to learn to communicate mathematics orally. Yet writing mathematics is also a task that is difficult, not only for engineering students. In the past, correcting final examinations was often frustrating. Many students would jot down a few numbers, and it was up to the graders to figure out what they were trying to say with them. A common question in tutorials was, “Okay. I think I understand. But how do I know what to write down?” It became clear that discussion and even presenting solutions for students is insufficient for developing their writing skills.

In order to develop written skills in mathematics, TUMULT students are required to turn in one written assignment per week, which is often an examination problem from a previous semester. In order to give students some orientation, the ideas set forth in Polya’s book *How to Solve It* (Polya (1945)) have been condensed into the following scheme: First students should reformulate the problem in their own words, bringing in data and definitions where necessary. They should then relate their plan of action. Which theorems are helpful? Which algorithms will they use? After carrying out their plan, they should then check their solution. In order to get students started, various examples are given to illustrate the scheme.

The time tutors save in not having to grade four problems for each group of two students is now used for grading this one written problem per student each week. It is expected that corrections also contain comments, not just checkmarks and x’s. Discussions between the tutor and individual students about written work are highly encouraged. Even a five minute consultation before or after a tutorial can be extremely beneficial for clearing up a misconception or for motivating students.

In the first semesters in which the Polya scheme was introduced, the students were very appreciative. It is helpful for students, as long as one is not a slave to the scheme and sees it as an *orientation*. Especially the reformulation of the problem annoys students, the major argument being that they will not have time in the examination to reformulate the problem. It is hard to convince them that by practicing reformulation consciously now could help them subconsciously in the examination, and the time it would take to convince them could be wisely used elsewhere. This semester, on the other hand, the assistants have been very diligent in having the tutors control the form as opposed to focusing on the mathematics. Needless to say, the Polya scheme is not very popular this semester and will need to be redirected for the next.

Tutor support

Previously, tutors were provided with an exercise set including solutions. The preparation for the tutorial generally consisted of perusing the solutions and jotting down a few remarks as to which “tricks” should be presented additionally.

The TUMULT tutorials are much more demanding on tutors. Training is provided for incorporating multimedia into the classroom as most have no experience in this direction. Apart from a two-day schooling for our tutors, a weekly meeting is held to give them insight on how to use the training modules and to discuss other aspects of the learning environment.

In order to aid them in their preparation for each tutorial, suggestions are given for which definitions and theorems are available in the learning environment and at which point in the tutorial it would be useful to project them. The training modules are also specified along with questions to help them get discussions going between students, along with tips concerning moderating discussions when appropriate. This is perhaps more challenging for tutors than incorporating the multimedia into their tutorial, for it is much easier to give answers than to elicit them. It is this aspect that will challenge instructors in the upcoming semester.

Discussion

The TUMULT learning environment, including its intricate didactical concept, has been an integral part of the course Linear Algebra for Engineers at the TU Berlin for the past three semesters. As with any system, there are obstacles that need to be overcome for the TUMULT learning environment to succeed in the long run. First and foremost it is crucial to gain student and faculty acceptance for this type of learning. Students do not like the picky electronic grader, and faculty members worry that they themselves will have to begin using multimedia in their lectures. Resources are scarce, so that time slots in computer laboratories need to be negotiated each semester until ample classrooms are equipped with WLAN and multiple electrical outlets, provided there are enough students willing to bring their laptops to the tutorial.

On the other hand, our 2000 students each semester come better prepared to the lectures and tutorials, which enables them to engage meaningfully in mathematical discussions. They take a more active and independent role in the tutorials, so that the tutor becomes more of a learning coach as opposed to the presenter of solutions. Not only has the oral communication of our students improved, their written work in the examinations is far beyond that of students from just a few semesters ago. We expect that the success rates of our students will begin rising once the introductory phase has been closed.

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Using games in mathematics teaching

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Abstract

The author experimented with the use of some games in mathematics lectures during 2006-07 and found that the games were both well received by the students and effective in the intended learning goals. However, the first games (Rossiter (2007)) had some weaknesses so improved versions were trialled in semester 1 of 2007-08. This paper explains the rationale behind using games, describes the games and provides some evaluation.

Introduction, motivation and background

Several aspects help student learning (Race, P. (2005)) but primarily students need to be actively engaged (Gallop, Bell & Barnes (2005), Guzman et al. (2006), Huxham (2005), Khan & Vlacic (2006)) and this is facilitated by positive experiences, enjoyment, encouragement, a need to learn, practise, feedback and several other factors. Critically, students must be active rather than passive and this may not happen with a didactic teaching model. Good teaching practise encourages lecturers to find mechanisms for increasing student alertness and participation and to find means of engaging more senses (touch, vision, emotion etc.) (Challis (2006), Pickford & Clothier (2006)) to improve learning (Foss et al. (2006), McKay (2006), Middleton, Mather & Diamond (2006), Thomas (2006)); examples include group debate, peer learning, risk taking, competition and games. A sense of fun helps with recall and games can also give immediate feedback on current understanding.

In the second section this paper presents an idea adapted from a management game (Hill et al. (2006)) that is easy for lecturers to adapt to their own topics. The 'model', has been used in two different modules for two years: (i) year 1 engineering mathematics and (ii) frequency response methods (FRM) (complex number algebra) and thus the third section reports on the student feedback and staff perception of the exercise and how reflections were used to modify the initial games. Some conclusions on efficacy are given in the fourth section.

The game – context, origins and modifications

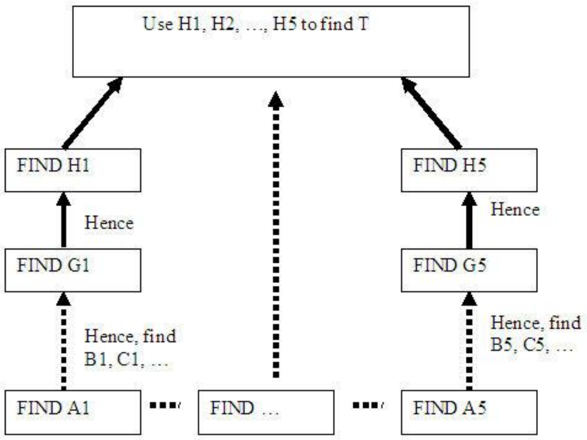
Typical student problems are associated: weakness with core mathematical skills, disengagement, not fully convinced of the importance, not fully aware of their own weaknesses. So, as part of wider departmental strategy, group based competitive games were introduced to: increase student awareness of their understanding; encourage peer assisted learning and making lectures more fun and thus improving the potential for deep learning (engaging emotion and attention). So far the games have been used in semester 1 for systems engineering mathematics (year 1) and FRM (year 2).

- FRM requires fluency with complex number algebra and the game used fun to expose the importance of a year 1 topic many had not learnt.

- Mathematics is core for new students and has to deal with transition issues such as the variability in the mathematical competence and aptitude of the intake. The 1st semester covers a quick review of basic algebra and functions before introducing trigonometry, calculus, solution of ODEs, curve sketching and optimisation.

Find-T game

The basic game concept used (Hill et al. (2006)) was developed for management and is intended to help develop team working skills; being a race the most effective team wins. The game itself requires teams to find the value of **T** by answering, in sequence, the questions on a number of cards; teams need to discern what this sequence is. Thus a team must first sort out the correct sequence and then distribute cards between team members to ensure effective use of everyone's time. A typical scheme is outlined in the figure: (i) solve A1, use to solve B1, use to solve C1, …., use to solve H1; (ii) solve A2, use to solve B2, use to solve C2, …., use to solve H2; (iii) etc and finally H1, H2, … to solve for T.



Modifications of find-T for engineering teaching

The basic concept requires 30-40 numeric/algebraic questions which build one upon another and are based on key learning from the module. The questions should have simple numbers to allow consistent computation and are printed onto small colour cards.

Use in teaching and evaluation

The game was used very early in semester with mathematics and FRM and late in semester with maths; an alternative game format was used late in semester for FRM. The intent was to engage students immediately in appraising their own preparedness for the modules and hopefully to encourage them to work hard to catch up if necessary.

Evaluation by students

Students were asked to select keywords, as many or as few as they liked, to comment on their perception of the games. Some comments and quantitative data are given next.

'*Showed me how much I forgot after the summer', `Out of practice from the summer', `Probably a bit much to begin with', `A good mix of topics', `Not just about maths but getting people to work in groups and get to know one another', `Made me realise how little I know', `Must study harder', `It showed me how much I had learnt in the semester and which parts I need to focus on for the exam', 'A good revision* '.

Time in semester	Fun	Good idea	Refreshing change	helpful	GBU	Difficulty about right
Week 11	50 and 46	85 and 57	65 and 54	90 and 63	85 and 44	90 and 33
Week 1	35 and 50	76 and 63	65 and 46	68 and 54	46 and 29	54 and 33
	Frustrating	Boring	OK	unhelpful	Too difficult	Better later
Week 11	25 and 33	0 and 13	55 and 71	0 and 8	5 and 13	10 and 30
Week 1	35 and 12	3 and 21	49 and	5 and 13	41 and 29	57 and 38

GBU (Good benchmark of understanding)

Week in semester	Fun	Good idea	Refreshing change	helpful	GBU	Difficulty about right
Week 11	50 and 48	59 and 79	46 and 52	65 and 75	54 and 73	39 and 38
Week 6	35 and 40	50 and 67	35 and 42	41 and 40	20 and 40	20 and 21
	Frustrating	boring	Ok	unhelpful	Too difficult	Better later
Week 11	8 and 10	2 and 4	35 and 40	0 and 4	0 and 6	2 and 8
Week 6	2 and 21	2 and 6	28 and 48	8 and 8	8 and 21	8 and 21

Table I,II: Student feedback on games within mathematics and FRM module. Numbers are percentage of returns marking selected keyword (2007-08 first and then 2006-07)

The majority of students were very positive marking either good idea or helpful or fun (or all three). A small minority found the quizzes difficult or frustrating although many of these still thought it was a good idea and helpful. Thus, there is certainly ample evidence that this type of exercise is worth repeating with future students.

Evaluation by staff and further modifications

The lecturer observed that in all sessions: (i) students seem to be highly engaged and animated; (ii) there was clear evidence of team working and group discussion; (iii) most students made a very obvious effort to complete the game and concentrated well and (iv) a small minority disengaged from the activity. However, in 2006-07 he felt that many groups struggled to progress due to lack of key knowledge and thus could not continue up the ladder of questions. No groups managed to complete the game and thus there was no winner! In the second year, he introduced a mechanism to release blockages and to ensure a winning team (enhance competitive aspect). In essence:

- added a scoring system with increasing points for each completed answer.
- added the option to purchase (with negative points) hints or answers.
- gave bonuses for the first team to compute a few pre-specified variables.

Although hard to unpick the effect from the data sample, the lecturer's view is that the games did run better overall with these changes. In fact, there does seem to be marginally better feedback with the second FRM quiz; in this case there was no sequencing of questions so it was easier for students to divide and conquer. The find-T format does require the team to work sequentially and many ran out of time (we allowed 45min) or were blocked by some topics. Finally, the quizzes late in semester seemed to be viewed slightly more positively, perhaps because these had the advantage of reflecting the upcoming exam as opposed to exposing lack of assumed prior knowledge.

Conclusions

The games have been successful in that the students both enjoyed the exercise and recognised their value in encouraging reflection on their abilities and needs. The format seems to be working reasonably well, although the reliance on sequencing seems to be an obstacle to effective progress for many and so this will be considered. The author's view is that the format is perhaps secondary; the novelty of the activity within lecture time and the group competition, based on key learning, is probably most important. However, he is planning further modifications for 2008-09 to facilitate better progress for groups where certain topics are a big barrier.

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Appendix A: Typical questions (Will happily email soft copy on request)

CARD	QUESTION	ANSWER
	SYSTEMS ENGINEERING METHODS	
B5	(7/5)A5x-C5	21x
A5	Hypotenuse with sides 9 and 12	sqrt(225)=15
C5	Min root of $(2x^3+6x^2+4x)$	0
F5	$\log[\sin(D5\pi/4)]+3[E5^6/E5^4]/4$	12
D5	Max. root of $(2x^2-6x+4)$	2
E5	$[3^4 2^6 4^2 9^{-1} 4^{-4}]/9$	4
	cos(B5)-F5/2	cos(21x) -6
	FREQUENCY RESPONSE METHODS	
$P5 = [B5/7]^3/(F5+4i)^2$	$[4\arg(60)]^3/[1-5i+4i]^2$	32arg(270)
$R = P1^2 P2P3P4/P5$	(16.16.3.2)/(32sqrt(2))arg (30+120+180-150-270)	24sqrt(2) arg(-90)

Integrating mathematics teaching into engineering modules

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Abstract

This paper discusses a project aimed at countering some of the deleterious effects of modularisation. Specifically the author’s department has considered how to embed the mathematics teaching more explicitly into the overall curriculum so that the students are better motivated to learn mathematics, have a stronger context to aid their learning and have material reinforced by appropriate curriculum design. Early evaluations demonstrate the efficacy of the project, as well as some risks.

Introduction, motivation and context

For a long time the author’s department had experienced year on year problems with the mathematics modules in year 1; some of this was discussed in the corresponding conference of 2006 (Rossiter (2006)). At that time, the author presented some ideas on his departmental plans for improving the engagement and performance of engineering students on first year mathematics modules. The proposed idea was implemented in the academic year 2006-07 and fully evaluated (by independent evaluators). Hence, this paper seeks to close the loop by reporting on the results of the evaluation and corresponding reflections as well as plans for further improvements and some more up to date reflections from 2007-08.

The basic idea was to make the links between the engineering modules and mathematics module much more formal and to ensure that there was effective synergy in the timing at which different topics were introduced. Hence, the topics were introduced within mathematics shortly before being used in the parallel engineering modules. This sequencing was planned to help with motivation by ensuring clear links to and explicit reference to, the applications of the mathematics; active staff engagement was critical for this later part. A secondary benefit was the rapid reinforcement of the mathematics content within the corresponding engineering modules.

The second section gives a little more detail on how the integration was managed, the third section discusses staff and student evaluation and the paper is finished by some conclusions. For completeness this paper will also include evaluation of other aspects of the delivery as these usefully reinforce ideas in the community.

Overview of organisation of mathematics teaching

In the author's department there is a feeling that poor student progress is far more complex than just providing better learning resources (including staff and tutorials) as students often do not make use of these. One essential key is to engage the students and this is a potential weakness with service teaching of mathematics: it does not tackle the issue of engagement directly. The topic may be separated from their other modules, the engineering department may have too little influence over the curriculum content or delivery and there is a danger that mathematics' lecturers may put too much focus on rigour and not enough on mathematics as a tool. Consequently relatively poor student performance is common.

The author's department wanted to tackle the issue of engagement and curriculum coherence together by integrating the mathematics teaching more explicitly into the parallel engineering modules, some of which had to move semester to make this work well. One aim was to enable the maths to be learnt and used in context as far as possible, without double teaching (i.e. teaching new maths and engineering concepts simultaneously). Our proposal was to build in the context while not removing the integrity and independence of the mathematics module. The proposal requires module lecturers to co-ordinate their lectures to ensure cross referencing, equivalent notation if appropriate and the use of similar examples, albeit from alternative perspectives.

In our case, the key parallel ACSE engineering modules in semester 1 are systems modelling and systems dynamics and in semester 2 are MATLAB (as a tool for solving engineering problems), introduction to control (including Laplace + Fourier) and measurement. In order to support semester one modules, students need to have an elementary knowledge of calculus, how to infer the behaviour of simple ODE models, use of logarithms, simultaneous equations and optimisation. So, the basic plan ensured that mathematical knowledge is wherever possible introduced shortly before its usage in the engineering modules (e.g. see Rossiter (2006)); this to aid motivation and reinforce learning of key concepts. Moreover, the mathematics module should make frequent use of engineering examples to explain the use of the mathematics as it is introduced and emphasis how it links into the parallel engineering modules.

Evaluation and student and staff feedback

The evaluation was prepared by a University evaluator (G. Diercks O'Brien) from the Learning Development and Media Unit, summarising the feedback from student and staff survey and focus groups and questionnaires. The overall summary is that apart from minor alterations, students are generally satisfied with the Maths module with regard to syllabus, learning resources, tutorial support and WebCT but improvements in the delivery of some lectures are required.

Student comments on integration

Learning of Maths is accepted as a 'necessary evil' and most students are resigned to the fact that there are no real exciting ways of teaching Maths. The application to the real engineering world is seen as interesting and motivating. Nevertheless many would like a less theoretical approach to teaching and more worked examples.

- 77% of students liked the integrated mode of delivery rather than modules being independent. Also liked having departmental staff rather than service teaching.

- Differences in teaching style among lecturers were a concern. They did not appreciate staff who over emphasised proofs/theory rather than application.

- They really liked the close linkages between Maths content and engineering curriculum (especially in semester 1), and explicit reference to these in lectures.
- They liked the smaller classes from not being on a joint department module.
- Seem to have properly grasped relevance of maths syllabus to engineering study.
- Suggested improvements included lectures with less complexity, more worked examples on the board and highlighting of key learning points.

Student comments on general delivery issues

For staff loading reasons, the department uses two staff per semester. Students did not like this, in particular commenting on the considerable differences in teaching style.

- One staff member was inconsistent in use of HELM workbooks and two over complicated their overheads (keen on rigour); this caused confusion.
- Tutorials (2-3 per week) are seen as an essential and useful part of the module.
- Overall positive feedback about the learning resources and in particular about HELM and mathtutor and the integrated use of the VLE.
- Regular in class tests (6% each) were seen as useful by the majority of students to provide formative feedback, although this does put a marking load on staff.
- Topics were pitched at right level, but nevertheless many students still found the module challenging although some of this is perhaps down to lecturer style.
- The majority of students indicated that the module helped to build their confidence although a few did say the opposite.

Student quotes include: `*Dr *** notes contained a lot of in class examples for us to do so we could get an idea of how well we were doing while there was someone around to explain it.' `Good we have regular tutorials, smallish classes (compared to previous years) and regular assessment.' `Good and relevant selection topic for engineers in semester 1 year 1.' `The tests helped with preparation towards the exam. The HELM workbooks were of a lot of help.' `I personally prefer to be taught by just one lecturer' , `The lecturers were good. But one went a little too fast. ' `I have done modules on the previous model (non integrated), this module was a big improvement* [Resit student]'.

Question	Likert scores (sem 1,2)	
I found the Maths content easy to understand.	3.61	3.3
This module has been useful in supporting my studies.	3.94	3.7
This module has helped me see the relevance of the mathematics topics to the engineering programme.	3.94	3.54

Independent teaching staff were involved in the student focus groups and commented that student expectations with regard to teaching delivery is an important factor in student perceptions of the module. We must provide greater cohesion of teaching approaches and styles in the module. This also reinforces some of the recommendations for use of the HELM workbooks.

Student performance

It is not possible to draw definite conclusions from a single set of student marks due to the variability in questions and cohorts. Nevertheless it is useful to compare average scores with previous years. Here, we focus on the relative performance over a number of modules. Historically, mathematics was one of the poorest scoring modules with the largest number of clear fails but in 2006-07 there is clear evidence that relative performance on maths is improved.

a) 2006-07: marks similar to main engineering modules and better than EEE modules.
b) 2003-06: marks 5-10% lower than main engineering modules and similar or worse than EEE modules.
c) The average for maths was marginally up (by 2%) whereas that for many modules was significantly down; it is believed that the cohort is weaker than previous years.

Conclusions

The students were positive about the delivery and notably repeat students said it was much improved on the previous year. Also, the relative performance of these students on mathematics as compared to other modules was also much improved on previous years. Nevertheless, this evaluation was based on just one year and we would hope to have more data to include by April 2008.

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Modelling of oscillators

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Abstract

Engineering students want to learn Mathematics in the context of practical applications. A suitable subject to follow this idea is to investigate oscillations in engineering. The topic can impressively show the interplay of different sciences as mathematics, physics, computer science and engineering science as well as the interplay of different disciplines within each of these sciences. In basic lectures students are often only confronted with the theory of linear oscillators. Nonlinear oscillators are then approximated by linear ones, which is possible for small displacements from rest position. Later the study of certain nonlinear oscillators is an interesting subject for interdisciplinary lectures, projects and thesis papers because nonlinear effects are important both for theory and practical applications. Besides, the students can learn a lot on interdisciplinary cooperation and teamwork.

Introduction

Setting up the model for an engineering system needs some physical and engineering basics. Here we consider oscillators which can generally be described by an initial value problem of a second order differential equation. Starting with given initial conditions a mass is displaced in time against restoring and damping forces and under the influence of a driving force. The investigation of the mathematical model needs some mathematical basics while the solution on a computer often uses both mathematical and software knowledge. Interpreting and validating the solution in the original frame again requires a lot of knowledge, especially engineering knowledge. Project and team work can integrate the following subjects:

- Elementary *functions* (e.g. trigonometric functions),
- *Differential and integral calculus* (stability, first integral, time period),
- *Differential equations* (model),
- *Numerical solution methods* for differential equations,
- *Linear Algebra* (vectors, matrices, determinants, eigenvalues),
- *Graphical* representations and elementary *geometry*,
- Computer *software* and software engineering,
- *Simulation*, experiments and tests,
- *Physics* and *engineering* background (interpretation and validation of results).

In the following sections we give some theoretical basics and some suggestions for investigations and experiments.

Harmonic Oscillators

The theory of *harmonic oscillators* is well-known. Since more complicated oscillators can be considered often approximately as harmonic, these oscillators represent the *basic* *model*. Here we restrict our attention to oscillators of one degree of freedom with the time-state function $x(t)$. The behaviour is described by an initial value problem of a *linear* differential equation of second order with constant coefficients, namely

$$mx''(t) + dx'(t) + cx(t) = f(t), \quad x(0) = x_0, \quad x'(0) = v_0 .$$

This differential equation results from the balance of inertial force, damping force, restoring force and external force. The initial conditions select the motion from the set of all solutions. For a *spring oscillator* in mechanics the mass $m > 0$, the spring constant $c > 0$ and the damping constant $d \geq 0$ are parameters while the time function $f(t)$ represents the excitation or control. But this simple model is also true for several other oscillators. *Mathematics is able to stress the essence of phenomena beyond the concrete context.*

The above model includes some assumptions on the oscillator, namely that the restoring force and the damping force are proportional to the state and its velocity, respectively. This is often approximately true if the absolute values of the initial conditions are small enough. For a spring oscillator this can be verified by experiments. Theoretical considerations can complement the experiments. *For applications it is important to prove (by experiments and theory) whether the model assumptions are fulfilled for a real oscillator or not*.

One didactic principle is to start with simple assumptions and to go step by step to more general ones. The following model levels can be distinguished:

- The damping force is neglected.
- Velocity proportional damping is assumed.
- A more sophisticated damping characteristic is given.
- There act external forces.
- Control the external forces to get a desired result.

From the mathematical point of view the differential equation can also be solved for negative parameters. The question is if such dynamical systems with predictable behaviour can be realized in practice and if they are of practical value. *Playing with parameters can open new fields of application.*

Conservative and damped harmonic oscillators

The simplest case for a harmonic oscillator is to assume that there is no damping ($d = 0$) and no excitation ($f(t) \equiv 0$). Then the solution is the *harmonic oscillation*

$$x(t) = x_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t, \quad \omega = \sqrt{\frac{c}{m}} = \frac{2\pi}{T}.$$

The constant ω is the *circular frequency* and the constant T is the *time period*. An equivalent representation reads

$$x(t) = A\sin(\omega t + \varphi_0), \quad A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}, \quad \tan\varphi_0 = \frac{x_0\omega}{v_0}.$$

Here A is the *amplitude* and φ_0 is the *phase* (shift). The extreme displacements $x_{\max} = A$ and $x_{\min} = -A$ are attained at times

$$t_{a,k} = t_k(x_{\max}) = \left(k + \frac{1}{4} - \frac{\varphi_0}{2\pi}\right)T, \quad t_{i,k} = t_k(x_{\min}) = \left(k - \frac{1}{4} - \frac{\varphi_0}{2\pi}\right)T.$$

Knowing the solution $x(t)$ the *velocity* $v(t) = x'(t)$ and the *acceleration* $a(t) = x''(t)$ can be obtained by differentiation. Both are again harmonic oscillations. Further, there are the important relations

$$v^2 + \omega^2 x^2 = C, \quad a = -\omega^2 x.$$

The first relation gives the *phase* equations containing a parameter $C \geq 0$. They represent ellipsoidal curves in the $x - v$ plane with the singular point $x = v = 0$ in the centre which is the only rest position (*equilibrium*) of oscillation. It is *stable* because small perturbations of rest position lead to neighbouring ellipses.

By the *energy conservation law* the sum of potential and kinetic energy is constant (and equal to the initial energy):

$$E = E(x,v) = E_{kin} + E_{pot} = \frac{m}{2}v^2 + \frac{c}{2}x^2 = \frac{m}{2}v_0^2 + \frac{c}{2}x_0^2 = E_0 = \frac{m}{2}C.$$

For the potential and kinetic energy the following explicit representations hold:

$$E_{pot} = \frac{cA^2}{4}\left(1 - \cos(2\omega t + 2\varphi_0)\right), \quad E_{kin} = \frac{cA^2}{4}\left(1 + \cos(2\omega t + 2\varphi_0)\right).$$

Hence, they oscillate harmonically, too.

If velocity-proportional *damping* is considered ($d > 0$) and assumed to be moderate, then the amplitudes are limited by a decreasing exponential function while frequency and phase are slightly modified:

$$x(t) = e^{-\delta t} A\sin(\omega_\delta + \varphi_\delta).$$

Nonlinear oscillators

The following can happen in the nonlinear case:

- Apart from oscillations other *motion types* are possible.
- The oscillations are *anharmonic*.
- The oscillations are *asymmetric*.
- There are several equilibrium positions which can be *stable* or *unstabl*e and belong to different *types*.
- The *time periods* depend on *initial conditions*.

Conservative nonlinear oscillators

We start with the nonlinear model

$$mx''(t) + F(x(t)) = 0, \quad x(0) = x_0, \quad x'(0) = v_0.$$

Here F is a nonlinear characteristic. In this case we can often give no analytic solution. We have to use numerical methods. If MATLAB is applied we need a formulation of the differential equation as a system of two equations of first order. This is naturally done by introducing the velocity $v = x'$ as second state variable. This concept is closely related to the mentioned phase representation and opens the door to study more general oscillations (e.g. predator-prey models in biology). Then we get the state space model

$$x' = v, \quad v' = -\frac{1}{m}F(x), \quad x(0) = x_0, \quad v(0) = v_0.$$

The energy conservation equation reads

$$E = E(x,v) = E_{kin} + E_{pot} = \frac{m}{2}v^2 + V(x) = \frac{m}{2}v_0^2 + V(x_0) = E_0.$$

Here V is a suitable primitive integral of F, called the *potential* of F. Geometrically the energy relation defines the *phase portrait* of the oscillator. By the way, *the numerical calculation of energy is a good estimator of the quality obtained by numerical solution of the model equation.*

Resolving the energy balance after the velocity gives

$$v = v(x) = v(x, E_0) = \pm\sqrt{\frac{2}{m}\left(E_0 - V(x)\right)}.$$

The formula represents a *first integral* of the model equation and an explicit form of the *phase trajectories*. The idea of solving the first integral numerically will not be successful. Here the uniqueness of solution fails although there are uniqueness statements. *Using mathematical theorems without checking their assumptions is dangerous*.

From the velocity formula the time-state relation follows:

$$t = t(x) = t_0 + \int_{x_0}^{x} \frac{du}{v} = t_0 \pm \sqrt{\frac{m}{2}} \int_{x_0}^{x} \frac{du}{\sqrt{E_0 - V(u, E_0)}}.$$

It is the inverse mapping of $x = x(t)$. If the motion is periodic, then $x(t)$ is bounded with minimum state $x_{\mathbf{min}}$ and maximum state $x_{\mathbf{max}}$. The time period is obtained by

$$T = T(E_0) = t(x_{\mathbf{max}}) - t(x_{\mathbf{min}}) = \sqrt{2m} \int_{x_{\mathbf{min}}}^{x_{\mathbf{max}}} \frac{du}{\sqrt{E_0 - V(u)}}.$$

It depends on the initial energy (initial conditions). If F is an odd function, then it follows that $x_{\mathbf{min}} = -x_{\mathbf{max}}$. Hence, the period is obtained by taking twice the integral in the limits 0 and $x_{\mathbf{max}}$. Apart from some special cases the integrals have to be solved numerically. The use of numerical methods involves a certain danger.

Often results of numerical calculations should not generally be trusted. A careful analysis is necessary to get correct results. This requires solid mathematical knowledge.

Equilibrium positions

The equilibrium positions of the oscillator are the zeros of force F, or in other words, the *local extremes* of potential V. Moreover local minima are stable, and local maxima unstable, equilibria. This can be illustrated by using the model of a ball rolling under the influence of gravity along the landscape of potential curve. Well-known criteria of the differential calculus can be applied to identify the equilibrium positions and their character. Another approach is using the theory of eigenvalues in Linear Algebra for the state space model (first order system).

There are interesting special cases which can be investigated by projects:

- *Cubic* oscillators (e.g. softening and hardening springs) $F(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$,
- *Asymptotically linear* oscillators (e.g. masses bonded by linear springs) $F(x) = c_0 + c_1 x + c_2 \frac{x}{\sqrt{x^2 + l^2}}$,
- *Trigonometric* oscillators (e.g. gravity pendulum) $F(x) = c\,\mathbf{sin}(ax + b)$,
- *Logarithmic* oscillators: $F(x) = c\,\mathbf{ln}\,x - \frac{d}{x} \quad (x > 0)$,

- Non-continuous oscillators (e.g. jump function characteristic).

Damped nonlinear oscillators

If, apart from restoring forces, damping forces also occur, they often can be split into two additive parts:

$$mx'' + G(x') + F(x) = 0, \quad x(0) = x_0, \quad x'(0) = v_0.$$

Nevertheless there exist a lot of possibilities which can occur often also in the engineering practise. Three damping cases are of special interest:

- *Dry* or *sliding* friction: $G(x') = d\,sign\,x'$,
- Velocity proportional (*viscous*) damping: $G(x') = d\,x'$,
- Velocity squared (*turbulent*) damping: $G(x') = d\,x'^2\,sign\,x'$.

These cases can be combined with the mentioned types of restoring forces. The energy balance has now dissipation caused by damping:

$$E_{kin} + E_{pot} = \frac{m}{2}v^2 + \int F(x)\,dx = E_0 - \int G(x')\,x'\,dt.$$

One task is to determine the returning points of oscillation where $v = 0$ and $E_{kin} = 0$. Further, one can discuss the ratio between consecutive amplitudes and the time sequence of similar function sections (pseudo periods).

Software Engineering

Project 1. Develop MATLAB files which solve model equations and produce time-state, time-velocity and phase relations.
Project 2. Develop MATLAB files which create phase portraits for a whole set of typical initial conditions.
Project 3. Develop MATLAB files which calculate time periods or pseudo periods of oscillations.
Project 4. Develop real time animations of certain oscillators.
Project 5. Develop user-friendly and flexible GUI (Graphical Unit Interface) which can be used for complex experiments.

Experiments using MATLAB or JAVA applets

Exercise 1. Play in the model equations with initial values and parameters and look at time state, velocity and phase diagrams. Find oscillatory motions. Study the types of equilibrium points.
Exercise 2. Detect equilibrium positions x_e by experiment. Try to check their stability. Find $x_{\min}$ for different values of $x_{\max}$ in case of oscillations around x_e. Are the displacements symmetric with respect to x_e? If not, try to measure the asymmetry by a

suitable number. Does the asymmetry grow monotone with the amplitude of oscillation? Find the time periods T for different values of $x_{\mathbf{max}}$. Does T depend on $x_{\mathbf{max}}$?

Exercise 3. Compare solutions of model equations and of their linearization around equilibrium positions. Under which conditions the linear model is acceptable?

Exercise 4. Find solutions for conservative model equations using different MATLAB solvers and different tolerances. Compare the respective numerical solutions by computing their total energy. Which solver has done the best work? Look for long time behaviour. Is the solution periodic or do the amplitudes increase or decrease?

Exercise 5. Solve the first integral of conservative model equations for certain initial conditions numerically. Plot the solutions and compare with the solutions of the original equations. What is going on?

Exercise 6. Look at models where the restoring force has singularities (e.g. logarithmic oscillator). What can happen near the singularities?

Exercise 7. Study models with damping or excitation forces. Investigate resonance effects for harmonic excitation.

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Numbolics: Applied Numerical and Symbolic Computations in Engineering Education

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Abstract

Engineers often have a misleading concept in the advantages and disadvantages of numerical or symbolic methods in Mathematics and the use of computational tools. We show some examples and introduce our concept of using computeralgebra and numerical tools as Maple or Matlab to combine both approaches in a synergetic way.

Introduction (Headers)

In short, for most practical purposes, it is not a question of whether to proceed symbolically or numerically; rather, it is a question of how far to proceed symbolically before turning to numerics. Sommese and Wampler (2005, p. 67)

Years ago, when working as "consultant for scientific computing" at a computer center I was sometimes confronted with very strange methodologies for solving mathematical problems. For example, numerical root-finding of third order polynomials, handwritten codes for solving systems of ordinary differential equations with hundreds of equations in Visual Basic, handwritten implementation of finite differences for the solution of partial differential equations in Excel. The strategies typically started with handwritten manipulations of formulae (preconditioning) to find explicit expressions, which are then implemented into a numerical problem-solving environment (PSE, programming language, spreadsheet or even Matlab).

With the advent of Computer-Algebra-Systems (CAS), the situation changed (a bit). Scientists learned that they can even handle high-order equations (or systems) symbolically and got the also misleading idea that it is always better to prefer a symbolical solution, if there is one (compare e.g. Schramm (1998, 2000)).

We show an example adapted from Hermann (2006, p. 31), using Maple, that illuminates the situation:

Compare the three functions

$$> P_1 := x \to x^8 - 8x^7 + 28x^6 - 56x^5 + 70x^4 - 56x^3 + 28x^2 - 8x + 1:$$

$$> P_2 := x \to 1 + (-8 + (28 + (-56 + (70 + (-56 + (28 + (-8 + x)x)x)x)x)x)x)x:$$

$$> P_3 := x \to (x-1)^8:$$

and evaluate them at a location between 0.99 and 1.01, e.g in Maple using 10 significant digits: $P_1(0.998123) = 0., P_2(0.998123) = -1.2\ 10^{-8}, P_3(0.998123) = 1.540686159\ 10^{-22}$

This result is quite puzzling because the three functions are equivalent in a symbolic sense, as it is easy to see in Maple.

> $P_3(x) = expand(P_3(x));$

$$(x-1)^8 = x^8 - 8x^7 + 28x^6 - 56x^5 + 70x^4 - 56x^3 + 28x^2 - 8x + 1$$

and P_2 is just the horner-form of P_1

> $convert(P_1(x), horner);$

$$1 + (-8 + (28 + (-56 + (70 + (-56 + (28 + (-8 + x)x)x)x)x)x)x)x$$

Moreover, the last result (giving a wrong sign) is quite frustrating, since we have been told very often that polynomial expressions must be converted to their horner form if evaluated numerically. In Matlab, the situation is even worse since we have a fixed double precision accuracy. We can reach Matlab from within Maple:

> $with(Matlab) : set\,\mathrm{var}("x", 0.998123) :$

> $evalM(convert(P_1(x), string)) : get\,\mathrm{var}("ans");$

$$-6.21724893790087663\ 10^{-15}$$

> $evalM(convert(P_2(x), string)) : get\,\mathrm{var}("ans");$

$$1.99840144432528176\ 10^{-15}$$

> $evalM(convert(P_3(x), string)) : get\,\mathrm{var}("ans");$

$$1.54068615878524454\ 10^{-22}$$

One should keep in mind that both systems use IEEE-standards to evaluate numeric results.

We can look at the situation using some plots in Maple (note the different scales).

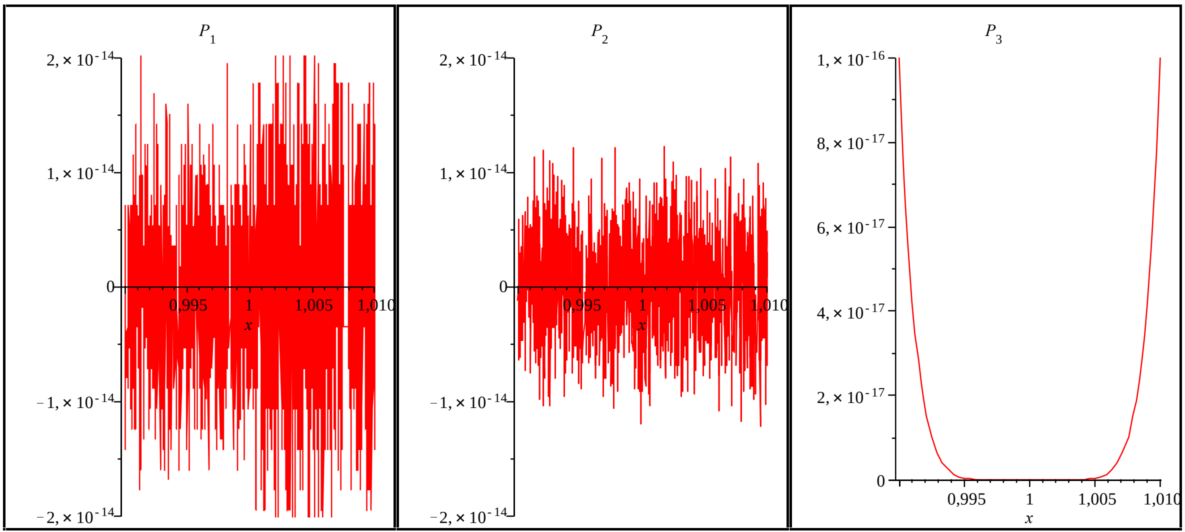
> $with(plots) :$

> $display(Array([$

$plot(P_1(x),\ x = .99\ ..\ 1.01,\ -0.2e\text{-}13\ ..\ 0.2e\text{-}13,\ title = typeset(P_1)),$

$plot(P_2(x),\ x = .99\ ..\ 1.01,\ -0.2e\text{-}13\ ..\ 0.2e\text{-}13,\ title = typeset(P_2)),$

$plot(P_3(x),\ x = .99\ ..\ 1.01,\ title = typeset(P_3))]));$



To analyze the situation we should try to understand the rules of machine computing, roundoff errors, catastrophic cancellations etc. (compare e.g. Hermann (2006)). However, using a CAS, we can do some experiments, e.g. we can change the accuracy and look when the result becomes stable. For example, the Maple command evalf(ex, n) evaluates the expression ex with n significant digits. Here a list of values for P_2 with ten to thirty significant digits.

> $seq(evalf(P_2(0.998123), i), i = 10 .. 30);$

$-1.2\ 10^{-8}$, $-1\ 10^{-10}$, $-1.4\ 10^{-10}$, $-4\ 10^{-12}$, $5\ 10^{-14}$, $5.9\ 10^{-14}$, $1.39\ 10^{-14}$, $5.1\ 10^{-16}$, $1.3\ 10^{-17}$, $3.9\ 10^{-18}$, $1.1\ 10^{-19}$, $1.29\ 10^{-19}$, $-4.\ 10^{-21}$, $5.3\ 10^{-22}$, $1.41\ 10^{-22}$, $1.539\ 10^{-22}$, $1.5430\ 10^{-22}$, $1.54060\ 10^{-22}$, $1.540673\ 10^{-22}$, $1.5406871\ 10^{-22}$, $1.54068593\ 10^{-22}$

What can we learn from this?

- It is not always a good idea to trust the numerical approximations of complicated symbolic expressions.
- Investigate the different forms of symbolic expressions.
- CAS can be used to analyse the effects of floating point arithmetic.

Numbolics

In the last section, we tried to explain that numerics or symbolics alone could lead to problems if one is note able to change the viewing direction. However, it occurs, that the preconditioning of formulae is too complex to give a canonical algorithm to be tackled by a numerical PSE. Here, only a combined approach of numerical and symbolic methods would apply. We show an example for the construction of averaging splines.

Averaging Splines

Splines are typically used for interpolation purposes. Datapoints are connected using (piecewise) cubic polynomials with connection conditions of identical slope and curvature in the datapoints. If the datapoints contain errors, it is senseless to run the splines exactly through the datapoints, i.e. we need an approximation scheme to find the connection points of the splines at the nodes of the datapoints. In the literature (Engeln-Müllges & Uhlig (1996, pp 287-297)), one finds such *fitting splines* with weighted datapoints introducing relations between artificial weights, datapoints and third derivatives of the spline function to have enough conditions to compute the connection points. A nice property of this approach is that one finds a regression line for small weights and an interpolations curve for large weights.

One can cast the conditions into a solvable linear system for the general case of a piecewise spline function for n datapoints and solve it using numerical PSEs.

However, there are some disadvantages:

- For n datapoints, we need $n-1$ splines. For dense datasets, these splines will mostly be nearly straight lines. A numerical overcast.
- The statistical properties of these splines are not very obvious.

Therefore, we use a different approach. We simply compute an avaraging spline for an artificial set of nodes in the datafield in the least squares sense. The algorithm goes as follows:

For a given set of n datapoints $x_i, y_i,\ i = 1..n$

1. Choose a set of intermediate nodes $X_j,\ j = 1..m,\ \ m < n,\ x_1 \le X_1, X_m \le x_n$
2. Construct a **symbolic** spline using the artificial nodes X_j and their **symbolic** function values Y_j.
3. Insert the datapoints into the symbolic spline function. The result is an overdetermined linear system of n equations in m unknowns Y_j.
4. Solve this system using the standard least squares method.
5. Replace the unknowns in the symbolic spline by their least squares approximation. The result is an averaging spline in the sense of least squares.

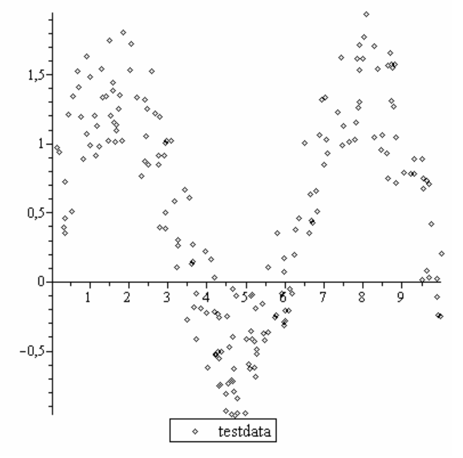
A working example shows how to implement this method in Maple:

Load and display a set of testdata

> $restart : with(plots) :$

> $TD := readdata(\text{"testdata.txt"}, 2, float):$

> $p_1 := pointplot(TD, legend = \text{"testdata"}); display(p_1);$



> $n := nops(TD);$

$$n := 200$$

Extract the x- and y-values

$x := TD[1..n,1] : \; y := TD[1..n,2] :$

Step 1: We decide to construct an averaging spline for ten equidistant nodes

> $m := 10 : \quad X := \left[x_1 + \frac{(x_n - x_1)}{m-1} \cdot j \$ j = 0..m-1 \right];$

X:=[0.131, 1.225333333, 2.319666666, 3.413999999, 4.508333332, 5.602666665, 6.696999998, 7.791333331, 8.885666664, 9.979999997]

Definition of the symbolic Y-values

> $Ylist := [Y_j \$ j = 1 .. m)];$

$$Ylist := [Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}]$$

Step 2: Construction of the symbolic spline function S where we use ξ as free variable

> $S := unapply(CurveFitting[Spline](X, Ylist, \xi),\xi)$:

Step 3: Construction of the overdetermined linear system as set of equations

> $sys := [S(x_i) = y_i \$ i = 1 .. n]$:

and in matrix form

> $LinearAlgebra[GenerateMatrix](sys, Ylist)$;

$$A, b := \begin{bmatrix} \text{200 x 10 Matrix} \\ \text{Data Type : anything} \\ \text{Storage : rectangular} \\ \text{Order : Fortran\_order} \end{bmatrix}, \begin{bmatrix} \text{1..200 } Vector_{column} \\ \text{Data Type : anything} \\ \text{Storage : rectangular} \\ \text{Order : Fortran\_Order} \end{bmatrix}$$

Step 4/5: with solution

> $Y := LinearAlgebra[LeastSquares](A,b)$;

$$Y := \begin{bmatrix} 0.713861487313643184 \\ 1.27752906020948488 \\ 1.19573843117451338 \\ 0.279316935006456156 \\ -0.559365367832887928 \\ -0.245539768515272788 \\ 0.665558850251661726 \\ 1.43528572834436163 \\ 1.12751943464574711 \\ -0.0629727008745847306 \end{bmatrix}$$

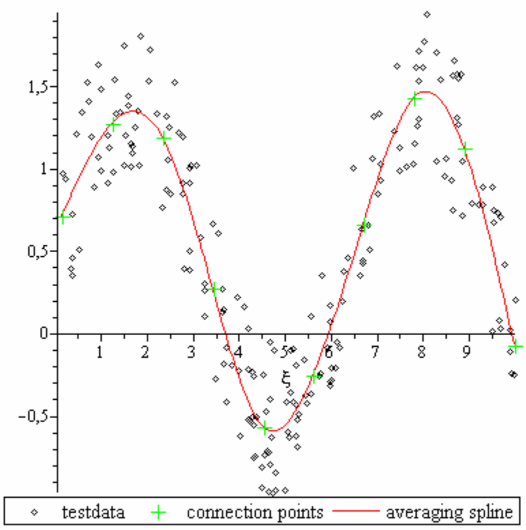
The spline function now reads

> $S(\xi)$;

$$\begin{cases} 0.6326190102+0.6171158460\xi+0.0349893475\xi^2-0.0890314189\xi^3 & \xi < 1.225333333 \\ 0.5559490056+0.8048280417\xi-0.1182034120\xi^2-0.047357655\xi^3 & \xi < 2.319666666 \\ -1.709147077+3.734252494\xi-1.381067811\xi^2+0.1341144541\xi^3 & \xi < 3.413999999 \\ -4.612589258+6.285607675\xi-2.128389181\xi^2+0.2070808056\xi^3 & \xi < 4.508333332 \\ 27.40550992-15.02033633\xi+2.597513364\xi^2-0.1423389761\xi^3 & \xi < 5.602666665 \\ 13.17013909-7.397874561\xi+1.237007335\xi^2-0.06139501919\xi^3 & \xi < 6.696999998 \\ 47.50310900-22.77773265\xi+3.533536826\xi^2-0.1757014004\xi^3 & \xi < 7.791333331 \\ -59.19824616+18.30689971\xi-1.739582735\xi^2+0.04989625543\xi^3 & \xi < 8.885666664 \\ -111.7013145+36.03311108\xi-3.734504667\xi^2+0.1247329552\xi^3 & \text{otherwise} \end{cases}$$

and can be used for plotting

> $p_2 := plot(S(\xi), \xi = x_1 .. x_n, legend = \text{"averaging spline"})$;
$p_3 := pointplot(X, convert(Y, list), color = green, symbolsize = 20,$
$symbol = cross, legend = \text{"connection points"})$;
$display(p_1, p_2, p_3)$;



In the last example, we used the symbolical features of a CAS to construct the system of equations that was solved using the numerical features. We used the artificial word "**numbolics**" to state that both features are important. Most CAS can do that und have (against the common belief) a similar numerical performance as dedicated numerical systems. However, it is a matter of taste, which systems are to be used. If you like e.g. matlab better, one can include the symbolic toolbox (which is a full CAS) to do the same task.

Conclusions

We showed some examples about the interaction of numerics and symbolics in mathematical problem solving. Engineering students should be aware of these implications. They should not know only about the theoretical background but also about the software systems to tackle their mathematical problems. They should be able to use both numerical and symbolic strategies to combine them to powerful methods.

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Mathematics Teaching Innovation in Engineering Education

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Abstract

The rapid technological development had started to change society and its attitudes towards education. This process is causing urgent needs to change the education environment, both at school and university. Mathematics is very much touched by this process. There exists a gap between mathematics offered by mathematics departments of technical universities and the mathematics needed to educate a modern engineer. A shift of the teaching-learning paradigm is a necessity.
Our challenge is to find synthesis different attitudes towards mathematics teaching. The purpose is to move away from mathematics as an isolated subject. Mathematics must be integrated into the study. For that purpose the dialogue between mathematicians and engineers is needed. We must also investigate mathematical culture of engineering. The other important issue: We know that in increasingly more countries the decrease in the mathematical ability of new entrants to the university degree programmes is a major problem in organizing adequate teaching. Consequently, a productive dialogue between mathematics teachers at school and university would be needed. The study of educational innovations has become increasingly important. It must lead to new didactics, with its own characteristics and teaching possibilities, different from the traditional way of teaching offered by mathematicians now.

Mathematics teaching and core mathematics for engineering

The aim of mathematical education at a technical university is not an efficient mathematical practice to support current technological innovation; rather, it is concerned with the transmission of basic ideas of existing traditional 'mathematical culture', considered by mathematicians as crucial. The values of such a traditional mathematics teaching approach are more social-cultural than practical. Like all other social values, the values of a phenomenon, named as "mathematical culture", have a stable core and this core is considered by mathematicians as a necessary instrument for interpretation of the surrounding world. It seems to be vital to reconsider this viewpoint. Any modern calculus course and modified mathematics teaching process must reflect the needs of new engineering disciplines and new technologies.

An important social value for the community of engineers is "an engineering mathematics basic" – a certain amount of core mathematical knowledge and ideas considered by the engineering community as natural. Under the influence of constantly modernizing technologies the subject domain of this "basic" has been in constant change. This continuously changing, but at the same time quite stable, amount of mathematical knowledge forms the 'core mathematics' for engineering. There exists a difference between the mathematics, taught at a technical university and mathematics really needed by engineers.

According to the SEFI Mathematics Working Group (2008) the mathematical topics of particular importance include:

- *fluency and confidence with numbers;*
- *fluency and confidence with algebra;*
- *knowledge of trigonometric functions,*
- *understanding of basic calculus and its application to real-world situations;*
- *proficiency with the collection, management and interpretation of data.*

The SEFI Mathematics Working Group also stresses the importance of using the elements of mathematical modelling in calculus:

- *It is important that the exposition of the modelling process should be introduced as early in the curriculum as is reasonable.*
- *The first models that are presented should be simple, so that the process is not obscured by the complexity of the problem, by concepts in engineering not yet encountered and by notation with which the student is unfamiliar.*
- *The mathematics used in the first models should be straightforward.*
- *The models must be realistic.*
- *The physical situation should be one to which the model has been applied in practice. It is of little value to apply a mathematical model to a situation to which it has never been applied, simply to make a pedagogic point.*

By using perfect teaching methods and having students with excellent mathematical background it would be possible to teach adequately all topics proposed by the SEFI core mathematics program. In theses circumstances the result would give students a good knowledge about traditional 'mathematical culture' as well as an understanding about mathematics, needed by engineers. Requirements for achieving these results are, as we mentioned above, the perfect calculus teaching for excellent students. But the reality is somewhat different. A decline in the mathematical ability of new entrants to the university degree programmes is a well-known global problem. For instance according to Mustoe (2004), *'The decline of mathematical ability of undergraduate entrants to undergraduate engineering courses in the United Kingdom has been well documented'*. According to a report from the Swedish National Agency for Higher Education, the decline in the mathematical ability is an international problem. A group of Russian mathematicians have also pointed to the dangerous tendency of declining knowledge of mathematics at school (ICMI Bulletin (1999)):

> *"... today we are painfully aware of the deterioration of mathematical education in our society and decline of its mathematical culture. A number of the so-called innovations can break down traditions of the Russian educational system bringing forward the worst Western models to follow. ... Within the educational system itself it is mathematics that appears to be in an unfavorable situation as an academic subject which is not consistent with market economy. ... The importance of mathematics and mathematical education in the modern world is difficult to overestimate. The sovereignty of a state, its security system, economics, science and technology depend on*

knowledge of mathematics by its citizens. It is necessary to emphasize the significance of mass mathematical literacy rather than a small elitist approach. Fundamental mathematics is a corner stone of modern science and engineering. ... Global computerization doesn't diminish the role of mathematical education, but quite the reverse, it has set science educational system new aims. Further deterioration of mathematical literacy and mathematical culture may turn man from a master of computer into a slave of it".

Clearly, it would be important to have new students with better mathematical literacy. Working with newcomers for years, staff of mathematics at university has been confident about developments at school, especially about positive and negative tendencies in mathematics teaching. It would be productive to achieve real cooperation between university lectures and mathematics teachers at school.

Of course, it would be impossible to start again to teach school mathematics in the way it was done earlier. But it seems reasonable to make a concise re-study of mathematics teaching methodology at school during the 1950^s and 1960^s. It must be done through the paradigm of recent mathematics teaching experiences. The results of such study would certainly have positive effects, helping to restructure mathematics teaching at school. We think that proper didactical analysis of methods of teaching mathematics by using different information technology facilities and advanced mathematics software would lead to much better teaching results.

Theoretical bases of curriculum development process

We are investigating some aspects of an engineering calculus syllabus designing process at present day's situation (see Sikk (2003, 2004b,c, 2005)). There is a need to investigate the effect of the use of computers in mathematics teaching. We are aware that achieving positive results by teaching mathematics in network environments requires much more than simply providing access to hardware, software and net. The computer packages typically used in the teaching process are designed for professional use rather than for learning. From our point of view it would be better to use packages explicitly designed for use of mathematics teaching. There is also a danger that the educational environment will become shaped by the technology, and enhancing student mathematics learning will become a secondary consideration. Our idea is to activate students by making the learning process more efficient. One of the purposes of our didactic research is to reorganize the teaching process of calculus, changing also the substance of the subject. We are aware that using mathematical models in a computerized environment would help students to gain deeper understanding about such a complex subject as mathematics. As a base of our considerations we follow some modern ideas of mathematics didactic theory. What do we want students to understand? There exist many different understandings about mathematics:

1. Mathematics as a subject of study sees mathematics as an obligatory part of the degree programme, to be studied via various teaching and learning techniques;

2. Mathematics as the basis of other subjects, both for study and in the world at large, sees mathematics as something existing in its own right, something to be tackled (learned or understood) for future appropriate use;
3. Mathematics as a tool for analyzing problems that occur in the world at large and hence solving them. This viewpoint considers mathematics as something which co-exists with other areas of knowledge and supports the study and development of that knowledge.

Our challenge is to find a synthesis for all these viewpoints and work out a common approach, satisfying general aspects and needs for both mathematicians and engineers. Mathematics must be integrated into the curriculum of study and into the world it describes. To start this process, the dialogue between mathematicians and engineers is needed. In order to save all rational, what we have in our mathematics syllabus, we must investigate, determine and estimate the existing mathematical culture of engineering.

According to J. L. Schwartz (Sikk (2004a)) the community of mathematics educators' goal is to make mathematics a real part of the cultural heritage of humanity. She adds:

> *"In contrast to the community of mathematics educators, in many ways, the community of mathematics-using disciplines has already embraced the opportunities offered by the new technologies. It has done this by changing the ways in which mathematics is used within the various disciplines, often without waiting for the mathematics community to sanction the changes. There are already universities in United States which have abolished their mathematics departments and where the responsibility for mathematics instruction is being assumed by engineers and economists, physicists and physiologists."*

The teaching of mathematics has always been dependent upon technical facilities available for computation. A technical development of computation had caused a change of mathematics teaching methods and subject of mathematics. As a consequence, a certain evolutionary process had lead to an 'equilibrium' situation between concepts of mathematics teaching and computation. Rapid advances in computation, linked with a development of mathematical software, have already started to liquidate the existing 'equilibrium'. The constant changes in computer technology would make it virtually impossible to estimate the rate of sophistication of facilities which will be available to engineering students after five years. This phenomenon is playing a major role as a motivator in a calculus syllabus redesign. Having in mind a calculus renewal process by using modern mathematical software we must have clear answers to the following questions (see Lawson (2004, p.7)):

- *What do we want students to understand?*
- *What do we want students to do with their understanding?*
- *What is the purpose of teaching?*
- *What are the goals for lectures and the students?*

One of the corner stones of our research of the syllabus design is ‘Realistic Mathematics Education’ (RME) (see Drijvers (1997), Freudenthal (1991)). The RME is a domain-specific instruction theory for mathematical education for secondary school students. It appears that it is possible to use the same ideas for calculus redesign. Below we will introduce some basic standing points of the theory by using the works of Drijvers (1997) and Freudenthal (1991).

The ‘Realistic Mathematics Education’ (RME) is a domain-specific instruction theory for mathematical education. One of the basic concepts of RME is an idea of mathematics as a human activity. Mathematics is not only an amount of knowledge; it is also the activity of solving problems and looking for problems, and, more generally, the activity of organizing matter from reality or mathematical matter. This organizing activity is called in RME as ‘mathematization’. According to Freudenthal, doing it can aid the learners of mathematics and mathematization is the core goal of mathematics education. Usually two types of mathematization are distinguished: horizontal mathematization and vertical mathematization.

In the case of horizontal mathematization, mathematical tools are brought forward and used to organize and solve a problem situated in daily life. Vertical mathematization stands for all kinds of re-organizations and operations done by the students within the mathematical system itself.

A second important characteristic of RME is a ‘level principle’. Students pass through different levels of understanding on which mathematization can take place. Essential for this level of theory of learning is that the activity of mathematization on a lower level can be the subject of inquiry on a higher level. This means that the organizing activities that have been carried out initially in an informal way, later, as a result of reflection, become more formal.

Within RME the term ‘model’ is not taken in a very strict way as in the theory of mathematical models. It means that materials, visual sketches, situations, schemes, diagrams and even symbols can serve as models. These models must have the following properties. They have to be rooted in realistic, imaginable contexts and they must have to be sufficiently flexible to be applied also on a more advanced, more general level. This implies that a model should support progression in vertical mathematization without blocking the way back to the sources from which a strategy originates. In the beginning of a particular learning process a model is constituted in very close connection to the problem situation at hand. Later on the context-specific model is generalized over situations and then becomes a model that can be used to organize related and new problem situations and to reason mathematically.

One can add to the ‘Realistic Mathematics Education’ principles the principles of anthropological approach in didactics. The anthropological approach shares with ‘socio-cultural’ approaches in the educational field the vision that mathematics is seen as the product of a human activity. Mathematical production and thinking modes are thus seen as dependent on the social and cultural contexts where they develop. As a consequence, mathematical objects are not absolute objects, but are entities that arise from the practice of given institutions. As regards the objects of knowledge it take in charge, any didactic institution develops specific practices, and this results in specific norms and visions as regards the meaning of knowing or understanding the object. Theory uses terms such as pragmatic value, epistemic value, the routinisation of techniques etc. For obvious reasons of efficiency, the advance of knowledge requires the routinisation of some techniques. This routinisation is accompanied by a weakening of the associated theoretical discourse and by a "naturalisation" of associated knowledge which tends to become transparent, to be considered as "natural". A technique that has become routine becomes "de-mathematicised" for the institution. It is important to be aware of this naturalisation process, because through this process techniques lose their "nobility" and become simple acts.

The SEFI Core Curriculum (Mustoe & Lawson (2002)) with its hierarchically structured and detailed list of topics of calculus provides us an excellent research material. By using principles of horizontal and vertical mathematization we can make didactical considerations for calculus teaching. In Core Curriculum each of its three lower levels (0, 1 and 2) are divided into many component parts in which lie the main topics and subtopics. Levels 0, 1 and 2 are representing hierarchical progression from school towards to the first two stages of university education. This 'structure of knowledge', by its levels and sublevels gives us a possibility to analyze all topics of calculus by paradigm of mathematization theory. By this analysis we can determine how to approach to different parts of calculus. The mathematical operations from lower level used at higher level would be considered as purely algorithmic and executed by students by using the computers and software. The time saved would enable us to shift students' attention to realistic problems related with the material considered and to model by using simple mathematical models. Clearly, in these circumstances the mathematics software would support the creative development of the student.

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A Diagnostic Followup Programme for First Year Engineering Students

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At the University of Manchester, students of Engineering (as well as other disciplines) take a diagnostic test at the beginning of the first year. There are several outcomes from the test (e.g. assignment to mathematics course units, tailoring lectures to the mathematical profile of the class as a whole) but one outcome is to set in place an individualised programme of work for each student to act as a refresher on either one or two topics where the student may have underachieved, perhaps through being 'rusty' after several months away from the subject or perhaps through incomplete original understanding. This program has been evolving over several years now and involves a wide variety of resources (various texts, Web-sites, HELM materials, support of the Mathematics Resource Centre etc). The programme culminates in a short computerised assessment for the student concerned which contributes to the assessment for the relevant mathematics course unit

New elements introduced for 2007-08 include pro-active support sessions from the Manchester Mathematics Resource Centre, and computerised practice assessments involving a number of computer packages accepting mathematical functions as input and offering detailed feedback relevant to the answer given by the student. This last element involves a multi-school investigation concerning the suitability of computerised assessment packages at the University of Manchester, funded by the Engineering and Physical Sciences faculty teaching development fund.

A possible future development may be to extend this scheme to Foundation Year Students.

Background

The University of Manchester (and forerunner UMIST) has a tradition in recent years of catering for a wide variety of input profiles of students entering degree courses with a mathematical background. This has included the extra resources given towards students identified as ‘at risk’ by a short diagnostic test at the beginning of the first semester and also includes the streamed system (Steele (1997, 2000, 2003)) where different courses in mathematics are aimed at the stronger, middling and ‘at risk’ students. Such schemes aided the majority of students in general but it remained the case that students were entering the middle (Q) stream despite having some weaknesses in critical areas of mathematics. External factors such as the changes in the Mathematics A-level system caused as a result of Curriculum 2000 also meant that students were undertaking courses in mathematics despite a severe lack of confidence in some areas of pre-university mathematics.

While the approach of the R-stream (for the weaker ‘at risk’ students was to assume that not a great deal from A-level modules had been absorbed and fully assimilated by the students, this approach could not be used with the Q-stream as students were fairly conversant with some of the areas from P2 and P3 (and later with areas from C3 and C4). Instead, it was necessary to use some tool for improving areas of weakness for individual students while allowing different students to concentrate on their own areas of weakness. It should be noted that when a student displayed a perceived weakness in a particular mathematical topic, it may well be explained by a lack of time since this particular topics was last used fully by the student or that time had not been available during the teaching of A-level mathematics to cover this topic fully. Explanations along these lines had been considered rather than assuming that particular students were not capable of understanding particular mathematical topics.

Over the last decade, the following procedure aimed at identifying and treating possible mathematical weaknesses at the school/university interface has been developed (and may well be developed further in coming years).

The diagnostic test and followup programme

Diagnostic Test

Students at Manchester/UMIST have sat a diagnostic test on arrival since 1996; since 1997, this test has been divided into sections with there being six sections of four questions each (40 minutes total) in 1997 to 2001 and twelve such sections (80 minutes total) since 2002.

The school of mathematics has had to work hard to dispel any notion that any 'mark' (counting towards assessment or not) is generated by this test. Instead, the test generates a series of 'mini-marks' out of 4 for each section and it is the profile of these 'mini-marks' that puts in place any outcomes of the test. Initially the test was created as a tool for streaming (i.e. dividing the students into the P, Q and R streams) but more sophisticated outcomes such as the followup have also been developed.

Section A

A1. Evaluate $a + b \times c$ where $a = -5$, $b = 2$, $c = 4$

A2. Solve the equation $6x + 4 = 22$

A3. If $x^8 \div x^3 = x^m$, find the value of m.

A4. Make x the subject of the formula $y = \dfrac{2}{x+1} - 7$

Please enter your answers in the grid below. Write your answer to the right of the question number. Keep the 3 columns on the right clear.

A1		A2				
A3		A4				

Figure 1 : Sections from the 2003 test showing the types of questions and format for entering answers.

Until 2004, the test was a written one where students gave short answers i.e. a number, a function etc. in a specified box (see Figure 1); these were marked quickly by a human marker but since 2005, the test has been marked by optical reader. The price to pay for this has been to make the test multiple-choice. An aspiration has been to make the diagnostic test administered and marked by computer; to date this has not been possible due to various local factors involving registration etc. but new developments *may* allow this to happen for 2009.

Some of the outcomes of the diagnostic test include assignment of students to courses within the streamed system, indication to students of 'weak' areas and indication to lecturers of strong and weak areas of the class as a whole. However, the outcome under review here is the followup process designed to focus the minds of students on mathematical topics where they may have under-achieved and, indeed, to help students improve their skills in such areas.

Diagnostic Followup Procedure

The diagnostic followup process was aimed initially at students on the Q-stream who were judged to be 'weak' in one or more of sections A to H of the diagnostic test. Initially students on the P-stream were not deemed to be in need of this help and for the R-stream, the course-unit itself acted as a followup procedure. A 'weak' area was defined as one where a student failed to answer correctly two or more of the four questions. Although some students on the Q-stream carried more than two weak sections, it was decided to cap at two the number of sections where the followup would be applied at two. Sections A (Arithmetic and Algebra) and D (Functions) were sections to which students were allocated preferentially (in cases where more than two sections were possible); otherwise, the allocation proceeded with the first sections being allocated alphabetically to students until the quota of two sections was filled. A few students were allocated only one section or were excused the procedure completely.

Subsequently the followup was introduced to course 1P1. One school, for a time, sent all students to 1P1 so there were students in 1P1 who were weak on a number of sections. Therefore, 1P1 students were directed initially to sections A to H for the followup and later resources were written for sections I, J, K and L.

At the inception of the scheme, students were provided with a list of questions that lay within their topic area. They were also provided with a list of references to where help could be found on these topics; this included the tutorial sessions dedicated to the mathematics courses and also paper- and web-based resources such as Stroud & Booth (2007), Croft & Davison (2004), Pledger et. al. (2004), s-cool (2008), and HELM (2008). The students were expected to hand in the completed questions by a specified date and the work was marked by a team of human markers.

When various forms of computerised assessment became available, this exercise was an ideal candidate for conversion for computerised marking. The ability to randomise parameters within questions meant that students could be given a 'practice' version of the test which could be done a large number of times with the students gaining understanding with repeated attempts and absorption of the feedback provided. When the students decided that the time had come to do the test for real, a fresh set of parameters could be used. The conversion to computerised marking also provided relief to the markers who reported that the hand-based marking of work of this form was particularly tiresome.

Initially, the questions were assessed using assessment tool ‘Question Mark for Windows’ but this switched to WebCT in 2003. In both cases, the question-types used were ‘calculated’ (the preferred type and used in conjunction with a specified tolerance) and, where calculated questions were deemed inappropriate, multiple-choice.

The pre-diagnostic mailing shot

From 2003 onwards, it was decided to contact prospective students in late August with a copy of a ‘mock’ diagnostic test. This contained a paper similar to the test that would be presented at the end of September but also contained answers to the mock-questions, a guide to the topics where questions could be asked, references to the explanations about the topics where questions had been asked, telephone and e-mail addresses for further questions etc. It must be said that the number of students who made contact was fairly low but they showed some interest in the scheme. The process was aimed at minimising the surprise that students would encounter on taking the test on the first day or two at university.

Developments for 2007

Two new developments were identified for the followup procedure for 2007: practice assessment using algebraic answers and sessions at the Manchester Mathematics Resource Centre.

A project funded by the Faculty of Engineering and Physical Sciences within the University of Manchester concerned the suitability of more sophisticated computerised assessment tools for the particular local conditions at the University of Manchester. The two main additional sophistications were i) the ability to receive, process and mark an algebraic expression rather than a number and ii) the ability to give feedback specifically aimed at the response presented by the student.

As a result of this project, students on the diagnostic followup were able to practise using the computer packages STACK (2008) and WeBWorK (2008) which satisfy the two criteria mentioned above. In fact, these packages were made available to the students at the time of the Mock Diagnostic test in August. Ideally, either STACK or WeBWorK would be used for the actual diagnostic followup but there are some registration issues that need to be addressed before this happens.

Spring 2006 saw the creation of the Manchester Mathematics Resource Centre, a drop-in centre for mathematical help and advice. The centre was able to act as a focus for events and several sessions were organised in the centre. Typically a given session would cover two of the sections. In practice, the organised sessions would act as an introduction to the topic and a tour of the resources available. Students were able to follow this up with individual visits to the resource centre to discuss the topics under consideration.

The University of Manchester, Faculty of Engineering and Physical Sciences, Foundation Year has over 200 students in 2007-08. These students take up to six mathematics courses depending on which school they wish to join the following year. There is some streaming of these students in mathematics but even within the main stream, there are some topics where the range of experience of students varies tremendously. Such a topic is trigonometry where some students have great difficulties finding general angles (outside the first quadrant) with particular trigonometric ratios while other students take this topic in their stride. An extension of the followup scheme to the Foundation Year may allow students to concentrate on topics where study may be rewarded most.

Conclusions

In 2007, the performances on the diagnostic test and the followup were as shown in Table 2. Note that performance is measures as the average score expressed as a percentage. The number of students is a measure of how many students were assigned to, and attempted, that particular section of the followup assessment.

Sect.	Diagnostic Perfor-mance	Followup Perfor-mance	Student numbers		Sect.	Diagnostic Perfor-mance	Followup Perfor-mance	Student numbers
A	83	90	75		G	89	53	3
B	67	84	114		H	69	83	12
C	38	90	184		I	49	88	18
D	65	89	163		J	49	87	24
E	70	93	41		K	29	87	87
F	60	95	24		L	69	92	92

Table 1 : The 'average scores' for students in the sections of the diagnostic test and followup assignment in 2007.

While most sections show an improved performance between the diagnostic test and the following, it is worth pointing out that there are differences in the form of assessment and in the sample spaces used. Even taking these into account most sections show a healthy increase in performance. Most extreme is section C where in 2007 there was a very poor performance in the diagnostic test. The 2006 situation of an increase from 61% to 92% was more like the other sections. On the other hand, section G, admittedly involving only 3 students, showed a decrease between diagnostic test and followup. In 2006 with 24 students, the average mark increased from 67 to 80 between the diagnostic test and the followup.

The diagnostic followup now forms an established part of the school/university interface for Engineering and other students in Manchester. Further developments are planned for coming years.

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Supplementary teaching of mathematics for engineers

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Abstract

Many basic mathematical subjects and procedures of great relevance for the engineering practice are not included in mathematics schedules of universities. The use of computers does not substitute these needed mathematical skills, on the contrary, it actually requires a more "higher mathematics"-knowledge than in the past. During the last four years the author offered interested students at Rostock University a supplementary course in mathematics for engineers and economists. The course is covering four additional hours of mathematics per week and results in no credit-points. Though the lectures were originally designed for the author's students only, they quickly evolved into lectures for all interested students who want to improve their mathematical knowledge. The presentation will demonstrate the concept of the supplementary course. The content matters seem to be theoretical at first sight but are strongly orientated on the most important tasks of mathematics in modern engineering practice.

Introduction

Universities nowadays are confronted with the problems of less resources and a decreasing number of students. However, universities have the task of educating a great number of high qualified engineers. Over the last years the mathematical starting knowledge of students has fallen to a very poor standard which persists at the moment (Bowen et al. (2007), Krätzig (1997)). In Germany this was started by reforming the school system in the late 1970-ties. Some universities court students to get engineer degrees with only little knowledge in mathematics or at least less mathematics than elsewhere. There is a real tendency to pull down standards and shorten mathematics requirements at many universities. This is seen as a way to increase rate of students studying successfully. It is our goal, too, to lead the new entrants successfully to their university degree. Therefore, whether we like it or not, mathematical lectures are often adjusted to a lower level. Nevertheless, it is almost impossible to fill the gaps in school mathematics at university level in regular lectures (Polaczek (2006)).

Four reasons for an experiment of teaching

The first idea in order to help students at entry level implied adjustment courses (Brüning (2004), Newman-Ford et al. (2007)). Besides such adjustment courses some universities such as Loughborough University established Mathematics Learning Support Centres to help students throughout their degree programmes with additional mathematics (Perkin et al. (2007)). Unfortunately, Rostock University does not have a corresponding support programme for engineering students. But in the slight adjustment specified courses can be a little counterproductive, too, because they help mainly on the low level side. Well-prepared students learn no new mathematics and might think they do not need to study for mathematics. Weak students have to learn first of all that they want to understand. We propose an alternative to the traditional university mathematics lectures. During the last six years the author offered interested students at Rostock University a supplementary course in mathematics for engineers and economists on an entirely voluntary basis in the beginning. There were only 12 participants. All of them knew me from regular lectures. Later the author got support from the economic faculties (but not from mathematicians). Now the course is a facultative one and can be found in the official schedule. However, the author can predict that he will be teaching more supplementary courses on a private non profit hobby bases soon again. This is due to the reorganization of courses from diploma to bachelor which entails further extensive cuts of mathematics lectures. The course the author offered students covered two additional hours of mathematics per week over a period of two terms and resulted in no ECTS-points. In standard lectures for business engineers and students of business informatics we teach analysis I (school-math, differential and integral calculus in one dimension, Taylor series, power series), analysis II (multidimensional calculus, extreme value problems for function with several variables, differential equations), linear algebra and geometry (school math, matrices, determinants, systems of linear equations, vector space, distance, vector products, elementary geometry), some discrete mathematics (groups, finite fields, relations of equivalence and order, linear difference equations), some numerical mathematics (iteration, integration in one dimension). This gives the first reason for the invention of a supplementary math lecture.

Reason 1: Students mostly are interested in passing the exams. They tend to study only as far as the exams require them to do. But many mathematical methods common for engineers like FEM, BEM are fully unknown to the students even after taking math lectures and passing exams. In order to prepare them for the actual demands in practice we need to teach more essential higher mathematics.

Reason 2: Even if students have mathematical knowledge, they usually need more experience using the mathematics in certain circumstances. The idea is to show how mathematics is usable for problems in the practical work of engineers.

This cannot always be satisfactorily achieved in standard mathematics lectures because of the lack of time. For example, students have seen infinite series and several types of differential equations. Nevertheless they may not know that series are good for solving differential equations and for other purposes. In lectures mathematical methods are often taught in an abstract manner. Their benefit for certain problems is not easy to recognize for beginners.

Reason 3: Engineers use computers for calculation and simulation, but we have not enough time to develop mathematical skills and a sensible use of computers.

In my regular lectures the computer is used for calculating complicated examples for saving time only. It is in my opinion a mistake to teach the mathematical software functions instead of mathematics itself. There is no time to teach high level mathematics and details of computer algebra systems for instance. Meaningful use of the computer requires more mathematics and particularly more “higher mathematics”. For this reason I use Maple in the supplementary lecture extensively. We compare Maple output with results calculated by paper and pencil. If it is necessary, programs are written for solving more complicate problems.

Reason 4 Lectures of engineering sciences use mathematical terms which have not been sufficiently introduced in the math lecture**.**

This is the reason for most students to take part in the supplementary math lecture. It really helps them. Some content matters are demanded by students. They informed me that they need more detailed information about Fourier series, Fourier transformation and Laplace transformation. They also wanted me to explain the Dirac function. Now these themes are main points in the supplementary lectures**.**

Content of the supplementary math course for engineering students

In many ways mathematics for engineers is different from mathematics as a science. Mathematics is from an engineer’s point of view only a toolbox for his scientific work. It is the task of mathematicians to maintain this toolbox effective, to teach as much mathematical methods as possibly necessary. So good teaching of mathematics is a real challenge which depends on technical innovation and gives simultaneously the general basis for future technical developments. What kind of mathematics is needed for tomorrow? We are not able to respond to this question with certainty. I want give two examples. In the 19-th century Riemann, Liouville, Grünwald, Letnikov and others developed integrals and derivatives of non-integer order and moreover of complex order. The broken derivative was forgotten at least in main stream of mathematics. It seemed to have no application. Only short time ago the applicability was recognized by engineers for modelling processes in viscous or porous media. Now a growing list of literature is available for this theme (Meinardi (1997), Samko et al. (1993)). Another example is Riccati's differential equation which is usually not included in mathematics lectures for engineers. Sometimes the special Riccati equation $y'=x^2+y^2$ is used as example of an equation with no closed solution, i.e. there is no solution in the set of elementary functions. This fact was proved by Liouville in 1841. But of course there are solutions in terms of special functions named after Bessel or sometimes called Neumann-functions, which are power series. The computer (Maple) knows it but the engineers do not. Moreover Riccati's equation has modern applications in transformation for particular differential equation and especially to Schrödinger's equation (Fraga et al. (1999)). We can find endless number of examples of higher mathematics which important applications have. Anyhow, such considerations do not answer the question what we have to teach our young engineers. The answer should be oriented at the expected professional tasks of the students. Which mathematical methods are useful for engineers cannot be known definitely, because professional profiles of engineers differ from each other and quickly change. But it is true for all that engineers use computers increasingly for simulation of real situations with mathematical models. These models are more and more complicated and complex. There are many equations for many unknowns. Large-scale problems are seriously difficult to solve. There are a number of mathematical methods taught in math lectures that are mainly of academic interest. For example, we teach eigenvalue problems with characteristic polynomial which are useful only for small sized matrices. But there are some main tasks for engineers that occur after linearization and discretization of almost all equations. They have to be taken into account by the teaching:

- Systems of differential, integral and difference equations are the central tasks in engineering problems
- Systems of linear equations are obtained for instance from the discretization of differential equations
- Large eigenvalue problems arise during the solution of homogeneous differential equations
- Least-square-methods are applied if great quantities of dates must be approximated by a function
- Systems of nonlinear equations are often solved by iteration methods. Often for this a sequence of systems of linear equations must be solved.

Only few students are able to solve other than small academic problems. They have no experience in numerical solution of practice-like tasks. Therefore, the most important issue the author intends to reach with the supplementary math lecture is to step from single equations to systems of equations while taking also large systems into account.

Hence, numerical linear algebra is the central matter of the supplementary lecture. The author believes that it is necessary to teach linear algebra as a key issue of mathematics for engineers. If we teach linear algebra first it is much easier to teach mathematics in a more effective way, since students will better understand the lecture notwithstanding the high level. Extending linear algebra helps to save time in teaching analysis, even. Linear algebra is a basis for many methods of applied mathematics realizable with the computer. In connection with numerical mathematics and analysis a good scientific basis for engineers is attainable.

In more detail the content matters of the supplementary lecture are:

- Matrices (special matrices, norm, condition, stability, factorization)
- Vector space
- Systems of linear equation (solving , forward errors, backward errors, bad condition, over-determined systems, QR-factorization)
- Systems of linear inequalities
- Linear maps in finite dimensional spaces (spaces of linear maps, dimension, matrix representation, bases, similarity of matrices)
- Eigenvalues, eigenvectors of linear maps (Gerschgorin circle, principal vectors, bases of eigenvectors, diagonalizable endomorphism, Jordan’s normal form, Schur factorization)
- Calculation of eigenvalues (Wilkinson, Householder, Hyman, von Mises).
- Polynomials of matrices (minimal polynomial, Cayley-Hamilton)
- Functions of matrices
- Solving first order linear systems of ordinary differential equation
- Interpolation (Lagrange, Newton)
- Iterative methods for solving systems of linear equations
- Scalar product and orthogonality (angles, norm, orthogonalization procedure, orthogonal projection, shortest distance)

- Hilbert space (orthogonal series, convergence, orthogonal polynomials)
- Fourier series
- Fourier transformation
- Discrete Fourier transformation
- Laplace Transformation
- Distribution (as limit of piecewise continuous functions)
- Application to differential equations

I was not able to teach the complete content matters in four hours a week. Therefore, the lecture will be continued during the next semester (2 hours a week). Then it will be supplemented by numerical methods for differential equations.

Conclusion

Although the lecture was originally designed for the author's students (business engineers and students of business informatics) only, it quickly evolved into a lecture for interested students who want to improve their mathematical knowledge. The students should have finished their whole mathematics education for their high school degree. The first group of participants have got very different grades in mathematics examinations. All of them understand that better mathematical knowledge will improve their future chances on the job market. They give a living example that more mathematics is needed in education of engineers. It is an open secret that the course is especially favoured by students preparing for mathematics examinations. Positive examination results of really weak students after visiting the lectures show that it improves mathematical knowledge of not only talented students. Weak students listen to the complicated themes without understanding it. But everything is imbedded in simple known facts which are explained again. In this way the weak students get at least helpful insight into standard facts from a new point of view. This helps them in the recognition process when taking the examinations. This is not a contradiction. Every kind of mathematics helps to develop positive achievement factors. The author is convinced that there is a very strong need for extending "higher mathematics"-teaching at our universities. Additional mathematics courses should be offered in order to improve the mathematical standard at universities and reduce the failure rate in mathematical examinations.

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Promoting Student Engagement with Mathematics Support

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Abstract

This paper reports the findings of qualitative research undertaken to seek to identify the key reasons why students are not engaging with mathematics support provided by Loughborough University. The research involved a number of focus groups and "on the spot" interviews with 'non-users' from across the campus. Barriers identified included a lack of awareness of the location of support and a fear of embarrassment. Further interviews were conducted with regular users of the support in an attempt to understand how some of these barriers to usage might be overcome. The paper will discuss actions that may be taken to improve student engagement with mathematics support and the issue of how student motivation may affect such action.

Background

It is widely accepted that there has been a decline in the mathematical preparedness of students on entry to universities in the UK and that many students embarking on a degree course lack some basic mathematical skills (L.M.S., I.M.A. & R.S.S. (1995), Sutherland & Pozzi (1995)). A strategy adopted by many universities to respond to this is the establishment of a mathematics support centre, whereby learning support is offered to students, which is additional to that provided by their normal teaching. In 2004, Perkin and Croft (2004) found that 66 out of 106 universities questioned provided mathematics support.

At Loughborough University mathematics support is offered by the Mathematics Learning Support Centre (MLSC). It provides a wide range of support mechanisms including one-to-one support on a drop-in basis, paper-based handouts and computer-based material. As a result of the MLSC's success in supporting students and similar work at Coventry University, both Loughborough University and Coventry University were jointly awarded Centre for Excellence in Teaching and Learning (CETL) status in 2005. A new centre, **sigma**, has been established between the two universities and the funding that the CETL award brings is currently being used to expand and enhance the provision of mathematics and statistics support.

Introduction

The MLSC at Loughborough University is highly valued by staff and students and recognised as an integral part of the University (Croft (2000)). The success of the MLSC is evident through its popularity amongst students, with 3926 visits recorded in 2005/6 (Mathematics Education Centre (2006)). However, analysis of recent MLSC usage data has revealed that a large proportion of Science and Engineering students who need mathematics support are not using the centre. In particular, data from 2005/6 reveals that of 626 Engineering and Physics students taking a first year mathematics module, 96 failed at the first attempt. Of those who had failed, it was found that over 90% (or 87 students) had never, or very rarely, accessed the extensive support available via the MLSC. Support provided by the MLSC requires students to be *proactive* and take the initiative in accessing the support available. Consequently, if students are unaware of their weaknesses or lack motivation to seek support, then the support will remain unused. Therefore, it is essential that the reasons behind the lack of uptake of support are identified so that appropriate action can be taken to improve it.

This paper will describe a study conducted in the academic year 2006/7 which sought to identify the reasons why these failing students do not use the MLSC. It will give details of the study itself including the participants and the methodology used. Data from the focus group and interviews will then be analysed and the results of these will be discussed in detail. The paper will then use these findings to suggest possible action to improve the lack of uptake of support.

The Study

Methodology – Phase 1

In the first stage of this research, undergraduate students from Loughborough University who had failed a mathematics module during their first year (in 2005/6 and 2006/7) *and* who had never or rarely used the MLSC were targeted. 179 students met these requirements and were contacted via e-mail (on three separate occasions). Seven students responded, and they were interviewed individually, in a group setting or via a focus group. All sessions were led by one of the authors of this paper, Symonds, and the discussions were recorded using a digital voice recorder.

To obtain additional data, "on the spot" interviews were conducted with a variety of students across the university campus. Students were recruited on three separate occasions and from two locations, namely the Students' Union and the campus library. 85 students (who had a mathematical component in their course) were questioned in this manner, of which 10 met the original requirements (and so were part of the 179 targeted students). Of the remaining 75 students, 60 had never or rarely used the centre but had passed their mathematics module, two had used the centre but failed their mathematics module and 13 had used the centre and passed their mathematics module. For all 85 students, their responses were recorded in writing by the same author.

Methodology – Phase 2

The second stage of the research was conducted in a similar manner. Students who were identified as being regular users of the centre (10 or more visits) in 2006/7 were targeted. 105 students met these requirements. However, 27 of these students were no longer studying at Loughborough University. The remaining 78 students were contacted via e-mail, and nine responded.

A further eight participants were recruited by approaching students in the MLSC on several occasions. The seventeen students were interviewed individually (by the same author, Symonds) and all sessions were recorded using a digital voice recorder.

Barriers preventing students using the centre

Analysis of the Phase 1 data reveals that a number of factors may have contributed to the lack of uptake of mathematics support by failing students. Theses reasons can be seen in Table 1 below. A more detailed discussion of these reasons is presented in Symonds, Lawson & Robinson (2007).

Reason	Total number of responses (77)			
	Focus Group / Interviews	**"On the spot" Interviews**		
	Non-user and failed (7)	Non-user and failed (10)	Non-user and passed (60)	**Tot.**
Lack of awareness of the location of the MLSC	4	2	21	**27**
Lack of awareness of the facilities available in the MLSC	0	4	17	**21**
Lack of awareness of the need of mathematics support	8	0	10	**18**
Too many problems that need addressing	2	0	0	**2**
Fear of embarrassment / intimidation / demoralisation	5	4	10	**19**
Mathematics support perceived as not appropriate for non-STEM* students	0	0	8	**8**

* STEM covers science, technology and engineering and mathematics

Table 1: Reasons given for non-use of the MLSC

It can be seen that some of the barriers preventing students from using the centre are relatively 'simple', for example a lack of awareness of the location of the MLSC. Although the centre is regularly advertised, it appears that many students are still unaware of its location and, therefore, do not access its support facilities. However, there also appear to be more complicated issues that act as a barrier. In particular, the data reveals that many students do not appear to be monitoring or directing their own learning and, consequently, they are unaware that they need support. From the focus group and interview data it appears that this is caused by two main factors. The first is a lack of motivation by the students. The second is that students are failing to manage their time effectively in order to cope with the demands and workload of their courses.

How regular users overcome barriers

To understand how some of the barriers discussed above might be overcome, regular users of the centre were asked specifically if these barriers had influenced their usage. A discussion of their responses is given below.

Lack of awareness of the MLSC's location and its facilities

For the majority of the regular users (12 out of 17) awareness of the MSLC was not an issue, they had already known where it was before they needed to use it. Those from the Mathematics and Physics departments indicated that they were aware of the MLSC's location because the centre is within their department building and so they pass it on a day-to-day basis when attending their lecture/tutorial sessions. Others were aware of its location due to the rigorous advertising of the centre.

The five students who felt that they did have to overcome this barrier said that they had actively sought out the MLSC's location - they took the initiative to find the centre. These students regarded themselves as generally motivated individuals and so when they felt the need for mathematics support, they went out to find the centre.

In terms of awareness of the MLSC's facilities and resources, 14 students agreed they had not known such details about the centre before they had used it. However, these students felt that this was not a barrier, since they were aware that some type of support in mathematics was available and this information was enough to motivate them to investigate the centre.

Lack of awareness of the need for help

The 17 regular users of the centre were asked if they were ever unaware of their need for mathematics support, which may have prevented them from using the centre at some point. The majority of the students (15) expressed that this was never the case. Their responses indicated that the students interviewed were academically engaged and motivated, since they had attended their lectures/tutorials regularly and had frequently completed problem sheets. This suggests that, unlike the non-users of the centre, these students were monitoring and directing their own learning and were aware of the need for help. In addition, five of the students said that since they had felt weak in mathematics during their prior education, they were aware that they would need support at university and had therefore intended to use the MLSC from the outset.

Too many problems

For most regular users this was not an issue. There were two students who had at times felt that the amount of problems they were encountering was overwhelming. However, unlike the non-users, this had motivated them to seek out help from the MLSC as they felt that without it they would undoubtedly fail. These students indicated that once they had made their first visit they had felt welcome to come back with their problems, despite being behind in their work.

Perceived to be not appropriate for non-STEM students

Of the 17 students interviewed, only four were from non-STEM departments. Of these four, three students indicated that initially they had felt that the centre was not for them because of their discipline. These students overcame this barrier largely due to encouragement from MLSC staff and friends. In particular, all three students said that a tutor from the centre had advertised the centre during one of their lecture slots, encouraging students from their department to use the support. It was also indicated by the students that they had felt it was easier to come to the centre with a group of friends, since these provided moral support.

Embarrassment, intimidation and demoralisation

Only four students felt that they had had to overcome feelings of embarrassment before using the centre. They had initially felt too intimidated to ask for support but their need for help and the advantages of receiving the support outweighed their misgivings. In particular, such students felt that the pressure of the amount of work and the fear of failure were more important to them than feeling embarrassed. Two of the students also indicated that the encouragement of a friend helped them to overcome such feelings.

The remaining 13 students said that they did not mind asking for help for a number of reasons. Some students were familiar in asking and receiving extra support from their experience prior to university. Others indicated that they preferred to ask for help from a tutor in the centre, since they perceived the MLSC staff as more friendly and approachable than their own lecturers.

Discussion

From student feedback, at face value there are a number of straightforward explanations as to why some students are not accessing the support provided by the MLSC. Based on these reasons, as outlined above, we suggest that the MLSC needs a more extensive advertising campaign to engage students in using the support facilities. In a previous paper (Symonds, Lawson & Robinson (2007)), possible suggested action to improve the uptake of support included increased advertising via posters, leaflets and lecturer recommendation (particularly within non-STEM departments), actively seeking out students who need mathematics support and recruiting staff members who are familiar to the students (lecturers from other departments, besides Mathematics, and post-graduate helpers).

However, analysis of the responses from the regular users indicates that such reasons had initially prevented a number of these students from using the centre. Nonetheless, these students were able to overcome these barriers in order to avail themselves of the support facilities. This poses the question; would simply implementing the above suggestions be enough to improve the uptake of support amongst failing students?

A common theme that emerged from the analysis of the regular users' responses was that of motivation and engagement. Generally, students who use the centre regularly tend to be frequently attending timetabled lecture and tutorial sessions and regularly monitoring their own learning by completing problem sheets. Consequently they are aware of any mathematics difficulties and the need of support.

In addition such students are motivated to seek help by a desire to improve their performance. These students are aware that they must work hard to achieve their goals; indeed, many aspire to the top grades. Whilst, on one level, all the students interviewed wanted to pass their mathematics module, among the non-users of the centre, it appears that their motivation to pass was not enough to make them take action.

In terms of encouraging students to use mathematics support, the issue of motivation needs to be considered. If a student is not intrinsically motivated then it may be possible to provide extrinsic motivation. Since the desire to succeed does not seem to be a sufficiently strong extrinsic factor in improving engagement with the support provided, then we must consider alternative methods of extrinsically motivating students. Such methods could involve changing the general teaching approach of mathematics, for example by introducing problem-based learning (Pedersen (2003), Bragg (2005)) or an inquiry-based approach (Crabtree (2004)) which are claimed to significantly enhancement motivation and engagement.

It is acknowledged that further research is needed to investigate if such action would be successful in motivating students to engage with mathematics support. Our findings suggest that simple actions (such as improved advertising) could bring some improvement in the uptake of support; however, for many students the reasons for not accessing the support are complex and need deeper analysis.

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Multivariable Calculus with Understanding and How to Assess It

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Abstract

Some ideas are presented in this paper on how to use geometric interpretation for better understanding of selected problems related to multivariable calculus. Samples of a possible approach to assessing this understanding are given, and several examples of test problems used at the Slovak University of Technology in Bratislava are included.

Introduction

Multivariable calculus and the behaviour of functions of more variables is an advanced topic, extremely important at the technical universities, which can be understood properly with a certain effort of abstract thinking only. Presenting the concepts with a proper geometric interpretation might help a lot. Obviously, this is not an easy task. For instance, to achieve real students´ comprehension of the concept of a function of two variables and its behaviour, we must use some of the computer algebra systems to visualise its graph that is a surfaces in the three-dimensional space. Understanding of this representation requires a certain capability of 3D geometry and knowledge about the projection of the space to the plane. Problems are easy to appear as the graphs are often of rather peculiar and not easy-to-convey shapes. Particularly, special knowledge is tacitly assumed and required about tangent plane to the surface in an arbitrary point. This must be a surface regular point, and with respect to its type – elliptic, parabolic, or hyperbolic, the form of the tangential superposition rather differs. In a hyperbolic point the most "uncertain" situations can be expected.

Another problem can arouse with detecting extrema points of a function of two variables. If the tricky criterion based on the value of the zero Hesse determinant fails (the second derivative test), geometric interpretation and intuitive approach could be very helpful. But we can hardly expect our students, who most likely have not heard much about geometry in 3D and intrinsic properties of surfaces, to be able to sketch graphs of functions in two variables, and to solve related problems successfully. Therefore we usually simply assess the calculation skills and mastering of the mechanical usage of formulas learnt by heart without proper understanding and knowledge of their powerful meaning and decision power. The question is: Can we somehow overcome these unpleasant consequences of the recent development in maths education?

Elementary problems on functions of two variables

Multivariable calculus is a mathematical subject that appears within the study subject Mathematics II, and it is compulsory for all specialisations in the second semester of the bachelor study programmes at the Slovak University of Technology in Bratislava. The contents and difficulty level of the multivariable calculus course might differ for different specialisations. Students at the Faculty of Mechanical Engineering are supposed to cover and understand the theoretical backgrounds of the following topics: concept of the *n* dimensional Euclidean space and its basic properties, functions of *n* variables - domains of definition, graphs and limits, continuity, partial derivatives and total differentials. More advanced topics of local, constrained and global extrema are studied on practical examples for functions of 2 variables. Almost nobody among our students have heard before about 3D geometry and some of the projection methods that must be applied to illustrate simple graph of a function of 2 variables. To be able to sketch these graphs means therefore to explain students some of the basic principles of the orthographic projection, while there is no time and space reserved in the curricula for these topics. Generally, we simply use CAS and their commands to plot the graphs on practical exercises in computer laboratories during the semester, and we try to avoid this question at the examination tests, as these are not organised in laboratories with computers available. The variety of problems to be assessed is therefore very limited. Further problems dealing with function extrema are assessed mostly on the numerical basis, which means that students can learn by heart some formulas and calculate some data without proper understanding and ability to represent and apply these, for instance geometrically. A study has been carried on at the faculty about how students are able to solve certain problems on functions of 2 variables using system Mathematica.

For example, the problem to find the equation of the tangent plane to the graph of function $f(x, y) = y\cos(3x + 2y)$ at the tangent point $T = [0, \pi, ?]$ can be solved easily by simple calculation of function partial derivatives and their values in the given point from the well known formulas, while resulting equation of the tangent plane is $y = z$. From the illustration in Fig.1 it is clear that this plane is tangent to the surface not only at the point *T*, but the tangency superposition is more specific, as point *T* is a parabolic point of the surface. Tangent plane $y = z$ is tangent to the surface in all points on lines with equations $y - z = 0$, $3x + 2y = 2k\pi$, $k \in Z$. Students could not analyse this important property straightforwardly from the calculations themselves, but they were able to prove it by solving the system of two equations, namely $y = z$, $z = y\cos(3x + 2y)$, as they tried to find common points of the graph of function and the determined tangent plane in the given point.

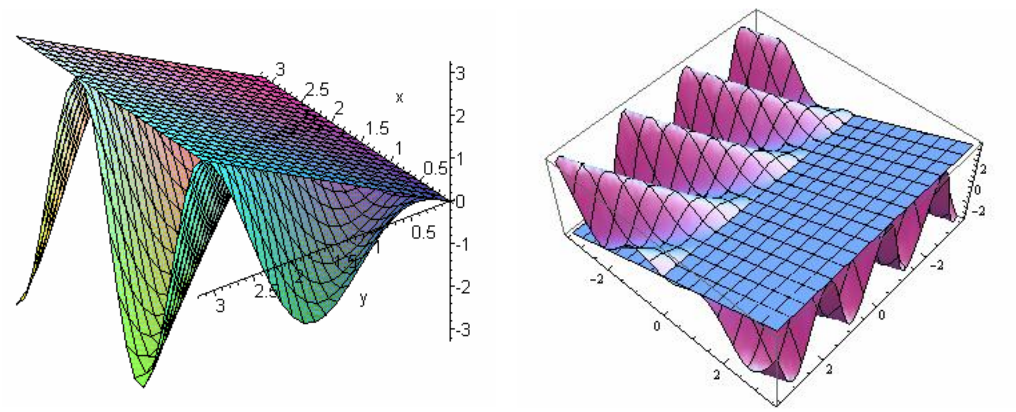

Fig. 1 Graph of function f(x, y) = ycos(3x + 2y) with tangent plane in parabolic points

Anther example isfinding the equation of the tangent plane to the graph of the function $f(x,y)=1-\sqrt[3]{x^2+y^2}$ in the point T = [-2, 2, ?] leads to the plane determined by the equation $x - y + 3z - 1 = 0$. Tangent point T = [-2, 2, -1] is the surface hyperbolic point and the determined tangent plane intersects the surface – graph of the function, in planar curve represented by the implicit equation $x-y-3\sqrt[3]{x^2+y^2}+2=0$.

An orthographic view of the tangent plane and surface configuration in the image plane $x + y = 0$ passing through the coordinate axis z clearly illustrates the situation, whereas tangent plane appears in the edge view, see fig. 3. Students are able to sketch the configuration, as from the condition $x = -y$ simply yields that equation of the surface and image plane intersection curve is $z=1-\sqrt[3]{2x^2}$ and the edge view of the tangent plane in the defined image plane is line determined by the equation $z=\dfrac{1-2x}{3}$.

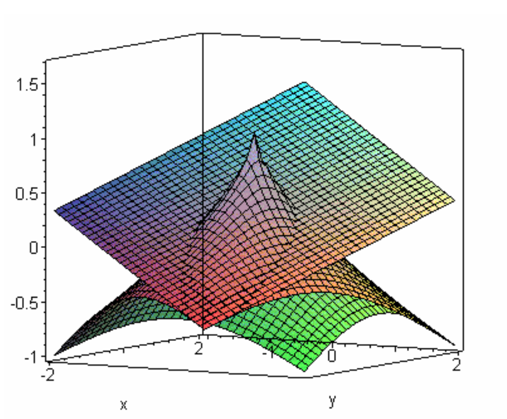

Fig. 2 Graph of function $f(x,y)=1-\sqrt[3]{x^2+y^2}$ with tangent plane in hyperbolic point

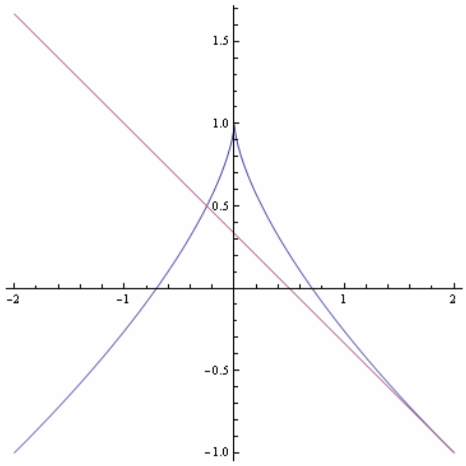

Fig. 3 Orthographic view of the tangent configuration

Local, constrained and global extrema of functions of two variables

Part of the assessment in the course Mathematics II is performed in the form of focused projects elaborated by students on specific topics. Complex problems about extrema of functions of two variables were solved as particular tasks delivered at the end of the testing period. Projects aimed to evaluate the understanding of the basic concepts as local, global and constrained extrema and their relation to the behaviour of the function of two variables. Students used the Mathematica system for illustrations and some calculations. Several examples of problems solved in the projects are presented in the following.

Problem 1.

Consider the functions a) $f(x,y)=\dfrac{3x}{x^2+y^2+1}$, b) $f(x,y)=4-x^2-2xy-3y^2$.

Using CAS sketch the graph of the function $f(x, y)$ of two variables on the domain

a) $D=\{(x,y)\in R^2:-4\le x,y\le 4\}$, b) $D=\{(x,y)\in R^2:-2\le x,y\le 2\}$.

Calculate the gradient of function $f(x, y)$, sketch the curves $f_x(x,y)=0,\ f_y(x,y)=0$, identify the existence of critical points of the function f, and determine these points exactly by solving the system of equations. Use the second derivative test to classify each critical point, and calculate the extremal values of the function f and equations of tangent planes to the graph of function in the points of the strict local extrema. Compare differences and similarities of the behaviour of the two functions in a) and b).

Critical points are the intersection points of the two sketched curves (hyperbola and two lines) located in coordinate plane xy, K_1 = [-1,0,$-\frac{3}{2}$] and K = [1,0,$\frac{3}{2}$], while the strict local minimum that is equal to $-\frac{3}{2}$ can be determined at the point [-1,0], and the strict local maximum equal to $\frac{3}{2}$ at the point [1,0]. Students use the second derivative test to prove this fact.

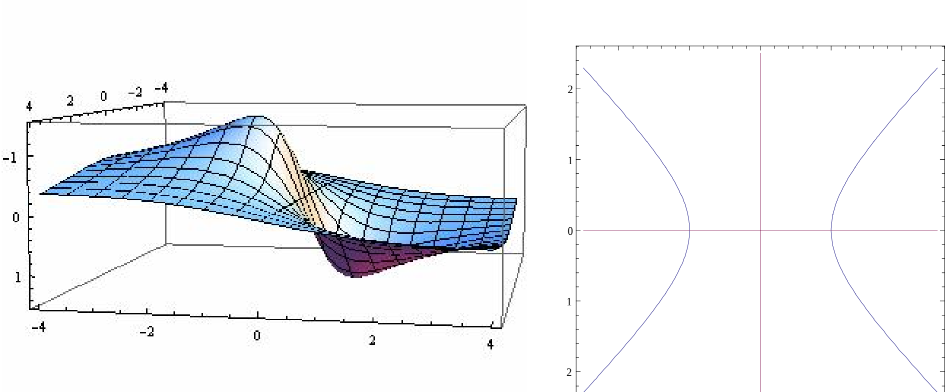

Fig. 4 Graph of function f and curves f_x = 0, f_y = 0 in a).

For the problem b) there exists a unique critical point [0, 0] determined as the intersection point of two lines, while the strict local maximum at this point is equal to 4.

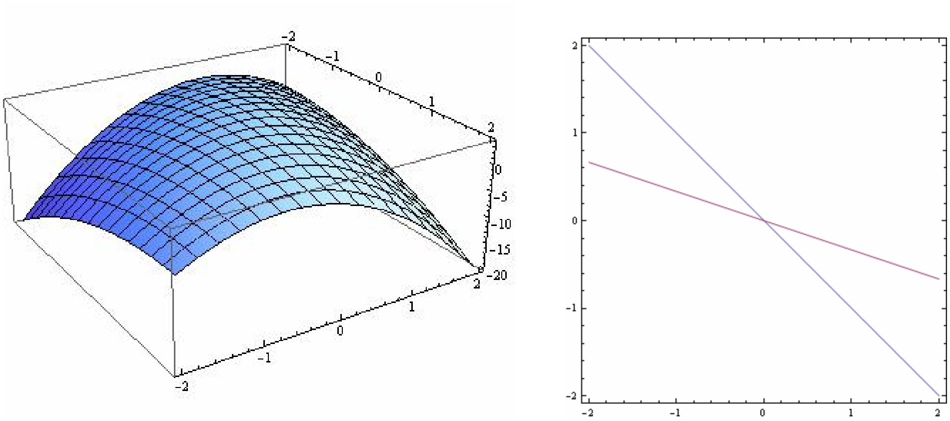

Fig. 5 Graph of function f and curves $f_x = 0$, $f_y = 0$ in b).

Students are rather successful in solving this sort of problem, and they understand the geometric representations of the analytic methods and algorithms behind them. Projects with problems related to constrained extrema of functions of two variables and global extrema of functions defined on closed sets in R2 were regarded as more difficult.

Problem 2.

Find global extrema of the function $f(x, y) = x^2 + y^2 - 12x + 16y$ on the set of points determined by the inequality $x^2 + y^2 \leq 25$. Solve the problem in the four steps:

a) Find local extrema inside the set M determined by the inequality (calculate function values at all critical interior points of set M).

b) Find the least and the greatest values of function f on the boundary of the set M (use the method of Lagrange multiplier).

c) Identify the greatest and the least of all the found values of function f in a) and b), which is the global maximum and the global minimum of function f.

d) Plot the graph of the function $f(x, y)$ and curve on this surface that is determined by the points of the circular boundary of the set M and characterize this curve geometrically.

Solution of the problem can be estimated after the plot required in the point d) is performed, and illustrated graphical information and be geometrically analysed (Fig. 6). To perform the plot, it is suitable to find parametric equations of surface that is the graph of the function f (paraboloid of revolution with equation $z = (x - 6)^2 + (y + 8)^2 - 100$, with axis in the coordinate axis z and vertex at the point [6, -8, -100]) and circular cylindrical surface with axis in the coordinate axis z and radius 5 intersecting this graph in the ellipse located in the plane determined by the equation $12x - 16y + z - 25 = 0$.

Solution $x = 6$, $y = -8$ of system of equations $2x - 12 = 0$, $2y + 16 = 0$ determines the critical point [6, -8], which is not the point inside the set $M = \{[x, y] \in R^2 : x^2 + y^2 < 25\}$ and therefore there exist no local extrema of function f on the interior of the set M.

Using the method of Langrange multiplier *l* two critical points [-3, 4] and [3, -4] can be calculated for values *l* = -3, *l* = 1, where constrained maximal value 125 of the function *f*(*x*, *y*) is achieved in the first point and function constrained minimum -75 in the second point. Function global extrema on the set *M* coincide with the constrained extrema.

More examples and solved problems can be found in materials cited in the references below and in the on-line database of e-learning educational materials linked to the Central EVLM portal of the European project European Virtual Laboratory of Mathematics at the address www.evlm.stuba.sk.

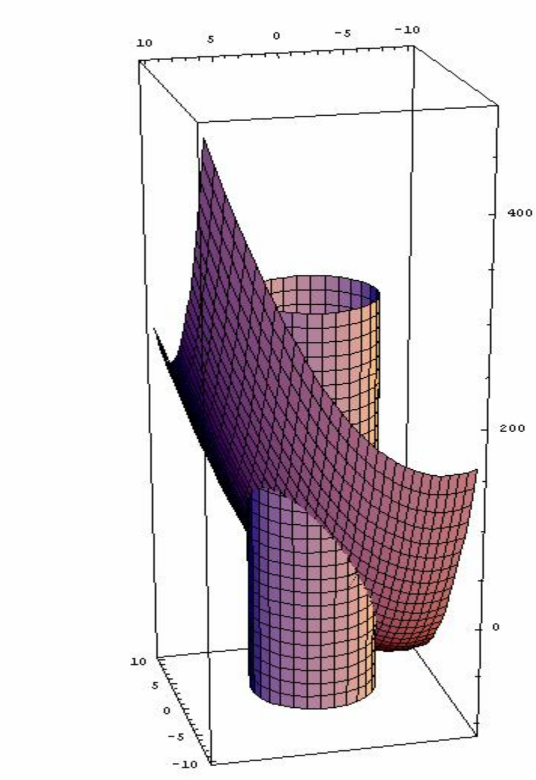

Fig. 6 Global (and constrained) extrema of function of two variables

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On the Nature of Mathematical Education of Engineers: Identifying Hidden Obstacles and Potential for Improvement

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Abstract

Mathematical literacy and competence in mathematics of engineering students are defined and analysed through the structure of conceptual understanding on both levels. The paper provides a view on technology integration in the teaching process, in particular, the ways how technology can contribute to learning needs of engineering students and what difficulties we can expect on that way. Examples that show relationships between different mathematical concepts and using technology are discussed. One of the most important concepts of calculus, limit is considered in detail with applications in MATLAB to identify possible obstacles in learning and students' misconceptions. Potential of conceptual understanding in mathematics for development of students' abilities in engineering applications and computerised calculations is shown. The current situation of the mathematics education of engineers in Australia in the context of the questions raised in the paper is briefly outlined.

Introduction

A high level of prospective engineers' mathematical skills and knowledge is the necessary condition for their future successful work and career. There is a permanently increasing demand for engineering specialists in industry throughout the world. In such circumstances the role of mathematics as provider of any kind of special knowledge for engineering students cannot be underestimated or neglected. It is widely recognised as a specific feature of mathematical education of engineers that to be a good engineer means more than just knowledge and understanding of mathematical theory; it is also the ability to use such knowledge and skills to model different applications with practical outcomes. However, even most of beginning engineering graduate students have not had a good experience with mathematics or have diverse mathematical backgrounds (Bamforth et al. (2007)). Absence of conceptual understanding, which refers to the student's comprehension of mathematical concepts, operations, and relations, leads to a fragmentation of mathematical knowledge in the student's head. In this respect Mitin et al. (2001) emphasised the situation: "As a result, in many specialized engineering courses, such as Control Systems, Solid State Electronics, or Communication Theory, instructors have to begin with a review of the mathematics they to use in the course…Vector analysis stays forever uniquely associated with electromagnetic waves, set operations with computer architecture, and the Laplace transform with circuits and signal processing. The mathematical methods encountered in each course do not evolve into unified patterns which the future engineer would be able to recognize and use universally"(p.vii). The other big concern was raised by Mustoe (1988), who pointed out that "most students lack the ability to apply their mathematical knowledge to non-standard problems. Whereas they may be capable of performing simple exercises in manipulation they fall down badly when required to use the same skills in a different or unfamiliar setting. 'If only I could see how to start' is an all too common remark made by students"(p.ix). Like many other researchers, who deal with engineering students we strongly believe that this situation can be and should be radically improved. There are different views on the methods that should be used to achieve students' better understanding in mathematics. However, we do not have the intention of covering such methods in depth in the paper. Furthermore, this paper does not present specific results of a particular case study or a teaching project aimed on the improvement of students' mathematical skills and comprehension. On the contrary, the paper highlights the key moments in teaching engineering mathematics, which we believe determine the strategic way Mathematics Education of Engineers will follow in 21st century. Moreover, we believe the strong emphasis of the paper on students' conceptual understanding combined with appropriate using and support of technology and strong background in proper mathematical structures, referring to engineering disciplines but not relying on them, will contribute to further discussion to work out the actual tools for radical improvement the current state-of-arts in the near future. As much as possible we tried to demonstrate particular examples in support of the claims and ideas that could be seen too broad or insufficiently described in terms of conference paper. We found this approach workable in many situations and hope readers will appreciate this mixture of global and particular ideas, structures and patterns. At the end of the paper we provide readers with brief outline of the current situation in the mathematical education of engineers in Australia (Mathematics & Statistics (2006)). We have no doubts this last example will serve as one more evidence that the situation in the whole needs to be improved.

Mathematical competence of engineering students

Speaking about mathematical competence of engineering students, we start with definition of mathematical literacy. We understand engineering students' mathematical literacy as the capacity to identify, to understand, and to engage in basic mathematics concepts and make well-founded judgements on mathematical statements. Mathematical competence of engineering students is defined as the ability to work with advanced mathematics concepts by considering the world through a "mathematical frame of mind" (Schoenfeld (1985)). Mathematical competence is related to the process of activating resources (knowledge, skills, strategies) in a variety of mathematical contexts. The way forward from mathematical literacy to competence in mathematics does not seem to be easy for most students. Quite often the competence level remains beyond their highest possible learning achievement that inevitably carries a negative impact on their professional skills and career. On the other hand, students themselves, not all, but quite a significant number of them (Gynnild et al. (2005)), perceive competence in mathematics as the highest level in learning engineering mathematics which not necessary should be achieved. Those students' misconception has, unfortunately, even older roots and origin than anyone could have thought. The core of this challenging transition from literacy to competence lies in development of students' conceptual understanding in mathematics that interacts with technology. Indeed, it is not a difficult task for any student to differentiate or integrate a certain function or evaluate, for example, a volume of any specific 3D-solid using multiple integrals. The difficulties appear in the students' comprehension on any of the mathematical concepts, e.g. what the concept of derivative is, how it can refer to the concept of limit, which physical or geometrical interpretation the differential of a function can have or what uniformly convergent series actually means, etc. In the case of mathematical literacy (Fig.1, left side) student's conceptual understanding is concentrated on separate concepts whose links to each other cannot be identified by students.

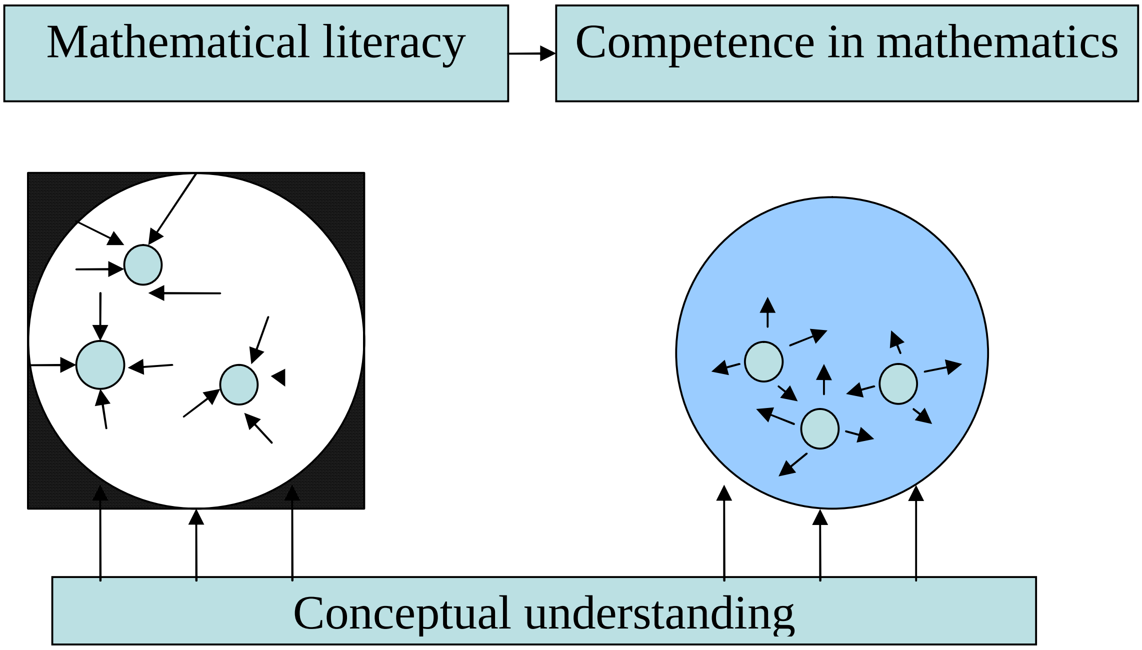

Figure 1. Scheme of the structure of conceptual understanding on two levels

As a consequence, students cannot analyse any significant part of theoretical material as it was initially designed in the course and experience difficulties with most concepts. However, in the competence case (Fig.1, right side) different concepts are considered by students with their relations to each other that leads to further development of conceptual understanding and improvement of the students' competence level. As an example we would like to demonstrate a calculus (functions of one variable) conceptual framework that can be used to motivate students' conceptual understanding in the first year calculus course for engineers. Every student has got the following scheme (Fig.2) on the first week of the semester. During the semester students worked on the scheme regularly to provide their comments and description on every component of the scheme, i.e. on different mathematical concepts and relations between them. At the end of the semester students were involved in the summary session, where the work of each student on the scheme was analysed. Despite the visual simplicity of the scheme, its impact on students' conceptual understanding was even higher than it could be expected, though interaction with technology was observed in both, positive and negative, directions. Now we turn to the analysis of technology influence on literacy and competence level.

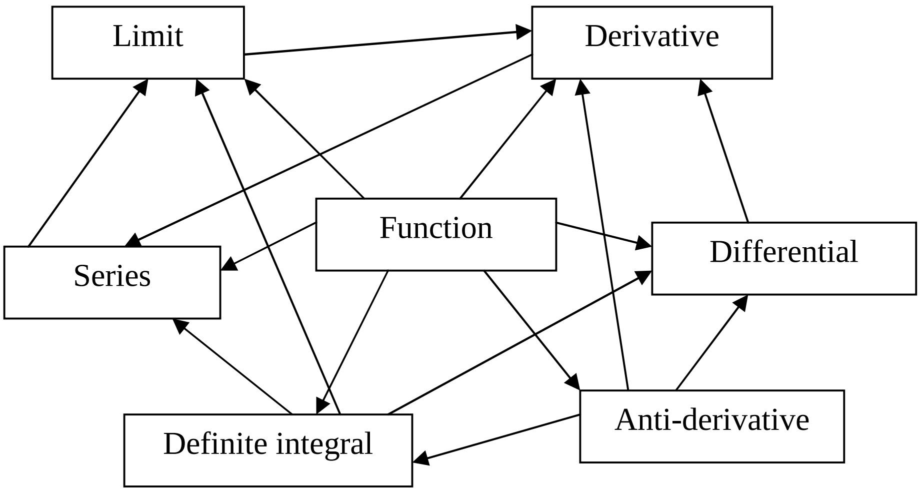

Figure 2. Calculus conceptual framework

Technology: pro and non-contra?

One of the most important issues to deal with in the 21^{st} century will be technology. Despite the role of technology is recognised and looks extremely important nowadays, its influence will be strengthening in the near future. It will require a new look on the integration technology in teaching and learning engineering mathematics. CAS, DGS, statistical software, handwriting using Tablet technology, computer-based learning environments, different mathematical and engineering software will influence the teaching process more and more. Will theoretical frameworks in mathematics education in general and in the mathematical education of engineers in particular be changed dramatically due to the new breakthrough in technology that, apparently, can happen in the next 10-15 years? This question still remains open. Engineering mathematics widely uses the advantage of different mathematical software like MATLAB, Maple, etc. We would like to show two examples of students' obstacles in using technology. The first one refers to the literacy level, the second one deals with the competence level and use MATLAB possibilities for limits evaluation. Both examples emphasise the importance of conceptual understanding in mathematics that can be recalled for use in a wide range of engineering questions.

Example 1

Solve the equation $3^{x+1} = 27$

Student X's write-up

The student used a standard calculator to get the following result:

$$x+1=\log_3 27=$$
$$=\frac{\log_{10} 27}{\log_{10} 3}=\frac{1.4314}{0.47712}=3$$

that provided him with the correct answer *x*=2. But what a sacrifice that method was where preference was given in the direction of technology even on such a basic level as a standard calculator.

Example 2

Find $\lim_{x\to 0} \frac{6^x - 2^x}{x}$.

Student Z's write-up

The best way to estimate this limit is numerically, e.g. in Matlab, or using a calculator. Take *x* in small steps either side of *x = 0*.

<u>MATLAB Code</u>

```
h=0.01;
x=[-3*h:h:-h,h:h:3*h];
y=(6.^x-2.^x)./x
y =
1.0585 1.0717 1.0851 1.1124 1.1263 1.1404
```

We can repeat these calculations for *h=0.001*, *h=0.0001*, etc. to estimate the limit at *x = 0*. (The middle two numbers are at −*h* and *h*, closest to *x = 0*.) We should get an answer around *1.0986*.

Again, the answer was correct, though it looked like an approximation. However, if we consider the same limit from pure mathematics point of view, that wouldn't be difficult to obtain exact value *ln 3* as the answer.

$$\lim_{x\to 0} \frac{6^x - 2^x}{x} =$$
$$\lim_{x\to 0} \frac{2^x\left(3^x - 1\right)}{x} =$$
$$\lim_{x\to 0} \frac{2^x\, x \ln 3}{x} = \ln 3$$

that follows from the following fact:

$$a^x - 1 \approx x \ln a \left(x \to 0\right), a > 1.$$

Both examples demonstrate a possibility of conflict between conceptual understanding in mathematics and using technology. Despite the technological aspect being taken into account at least the last 20 years (many significant results were achieved and published), the question of compatibility mathematics and technology, i.e. teaching mathematics using technology in the most effective way requires further research and consideration. The same question with focus on the needs of engineering students looks to be one of the most important questions today and in the near future.

Concluding remarks

Unfortunately the lack of space in the paper for conference proceedings does not give the opportunity to provide the full version of the material. Nevertheless, even the brief description of the questions raised in the paper and approaches to their further investigation and analysis presents for readers those important and significant directions that research on the Mathematics Education of Engineers will address and further develop. All over the world 21st century engineering students differ from their predecessors and that difference will intensively increase, first of all due to the huge impact of technology. The examples mentioned above on obstacles in using technology show the great potential in studying engineering mathematics that students can achieve combining the power of innovative technology with understanding the concepts of fundamental mathematics which slightly changed for the last 100 years. Probably, the supporting balance between engineering students' conceptual understanding in mathematics and the appropriate use of technology will be one of the main challenges in Mathematics Education of Engineers in 21st century. Moreover, we believe that the research on students' successful transition to the competence level over the territory of conceptual understanding and technology, where both components have a huge impact on each other will form a new theoretical framework of the question. We believe it will inevitably happen due to the further and deeper technology penetration in research on the teaching and learning of mathematics as well as in industry development and job requirements for future engineers.

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