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Introduction

It is one of the main goals of SEFI's Mathematics Working Group to provide a forum for the exchange of views and practices regarding the mathematical education of engineers in Europe. The main means for pursuing this aim is to hold a seminar at an attractive place in Europe every second year. In 2014, we hold the seminar in Dublin because there is a very strong and active Irish community of lecturers interested in the mathematical education of engineers.

Since the last seminar in Salamanca, Spain, in 2012, there was one major achievement which is the third edition of the group's curriculum document which is called " A Framework for Mathematics Curricula in Engineering Education". This document is based on the concept of mathematical competence which is defined as the ability to master the mathematical challenges in situations where mathematics could be helpful. Besides helping in setting up mathematics curricula, the document is also meant to summarize and provide links to former seminar contributions which dealt with important topics in the mathematical education of engineers. The document can be freely downloaded from the group's web site at http://sefi.htw-aalen.de.

The group's 17th seminar in Dublin will further discuss the competence concept and other important issues. The response to the corresponding call for papers was very satisfying such that a rich programme resulted from this call which is reflected in the proceedings of the seminar.

There are three invited speakers two of which report about different aspects of the use of technology. Schramm presents a minimum requirement catalogue for beginning students and how students can be supported to check and improve their competence regarding this catalogue by using web technology. Sangwin provides the "big picture" of 400 years of educational technology. Moreover, Joyce gives and industry view of the mathematical education of engineers.

The paper presentations are grouped in seven sessions:

1. Competencies and Attitudes
2. Transition from School to University – Offerings for Students with Deficits and for Bright Students
3. Projects
4. Support Measures
5. Teaching/learning Methods
6. Using Technology
7. Assessment.

Moreover, there are special discussion sessions on the topics:

- What are the important issues in the mathematical education of engineers?
- How can we use technology to improve teaching and learning?

The programme is completed by software demonstrations and poster presentations giving a rich overview of tendencies and developments all over Europe. Most contributions are accompanied

by a paper in the proceedings such that the latter provide an excellent summary of the topics dealt with at the seminar. The author would like to cordially thank the local organizers for doing the language editing that makes the proceedings much more readable.

Aalen, June 2014

Burkhard Alpers

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Contents

How does Problem Based Learning fit with Cognitive Load Theory?

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Abstract
This paper reports on an investigation with first year undergraduate Product Design and Management students within a School of Engineering. The students at the time of this investigation had studied fundamental engineering science and mathematics for one semester. The students were given an open ended, ill formed problem which involved designing a simple bridge to cross a river. They were given a talk on problem solving and given a rubric to follow, if they chose to do so. They were not given any formulae or procedures needed in order to resolve the problem.

In theory, they possessed the knowledge to ask the right questions in order to make assumptions but, in practice, it turned out they were unable to link their a priori knowledge to resolve this problem. They were able to solve simple beam problems when given closed questions. The results show they were unable to visualise a simple bridge as an augmented beam problem and ask pertinent questions and hence formulate appropriate assumptions in order to offer resolutions.

Introduction
The majority of learners come to university with predominantly procedural knowledge, they know how to apply a procedure to a set of variables and constants and obtain a result but with little understanding of what the result implies or means. They do not seem to possess the conceptual knowledge necessary to be able to make assumptions or an informed judgement as to how sensible their result is or indeed be able to interpret the outcome. This is not surprising since in the UK education system, schools are judged on their academic performance by a regime of league tables. The learners at level three of the NQF (National Qualifications Framework) are mainly assessed via formal examinations which predominantly assess knowledge and skills. In the consultation on the new A level regulatory requirements, OFQUAL (Office of Qualifications and Examinations Regulation) deemed assessment via coursework as unreliable (OFQUAL, 2013).
One of the major challenges facing university engineering schools is to enhance this procedural knowledge into conceptual knowledge and to develop the skills required by a contemporary engineer. The vast majority, if not all, undergraduate engineering programmes feature a project, group or individual, as a final year module. In this restricted form of PBL (Problem Based Learning), learners are in some cases, given a teacher-selected problem which is based upon the learners' a priori knowledge and skills (Heitman, 1996). Although this approach embraces the notion of problem solving learning, it represents a small fraction of the curriculum, is time restricted and contrived and cannot be accurately described as PBL. In order to fully develop the knowledge and skills required by a professional engineer, a more holistic and curriculum wide approach is necessary.

Problem Based Learning
For the purposes of this paper, PBL is defined as 'Problem Based learning for professional action according to Savin-B (Savin-Baden, 2000)aden (2000, 2007). This model emphasises the important point that PBL is a combination of a learning methodology, knowledge construction and scientific approach (Kolmos, et al., 2009). It defines knowledge as 'know-how', learning is for the workplace, real life problems are used with the students solving these problems in order to undertake practical action. The facilitator acts as a

demonstrator of practical skills with assessment taking the form of assessing skills for the workplace and the necessary supporting knowledge.

In order for learners to be effective problem solvers they must be able to make sensible assumptions, be comfortable with the notion of resolutions and be prepared to adopt trial and improvement techniques. In addition to these skills the students must also be knowledgeable, confident and competent within the subject disciplines.

One fundamental question is how are knowledge and skills organised within the human cognitive system? One model suggested by Skemp (1986) and Sfard (1991) is the idea of schemata. A schema is defined to be how thought processes and the relationship between them are organised. For example, an arithmetic schema would involve a grammar (the rules) and a lexicon (the symbols) and how they interact.

Schemata construction is a process that starts from birth. They are constructed in order for us to acquire language, make sense of the world we live in and to survive. They provide a mechanism for working short term memory to have fast access to processes and knowledge stored in long term memory. This is an efficient way of accessing processes and knowledge since working short term memory can only deal with, approximately, seven items at a time. In terms of mathematics, schemata are created for arithmetic, fundamental algebra, solving simple equations etc. These schemata form the basis on which the learner interprets and makes sense of mathematical procedures. It is vital at this stage the fundamental aspects of mathematics are correctly learnt since it is extremely difficult to 'unlearn' a schema at a later stage (Skemp, 1986).

The knowledge and skills learners bring with them to university are at best incomplete and tend to be procedural in nature. The language used and the complexity of mathematics studied at level 3 NQF (National Qualifications Framework) is designed to be appropriate for that stage of the learners' education. In order to move towards a more conceptual knowledge base, these schemata need to be enhanced to accommodate more precise language and definitions. They also need to be enhanced in order for students to begin to understand the basis from which the procedures they apply are formulated so they can apply them to novel situations.

The next stage in the learner's journey towards competency and proficiency in problem solving is to bind the existing enhanced schemata with schemata from different domains. For example, vector calculus relies upon the calculus, knowledge of vectors, coordinate systems. The hierarchical nature of mathematics and the way it which it becomes more abstract, means that as the learner progresses, mathematics becomes more powerful but harder to learn. Therefore if the existing schemata are not resilient, the learners' will demonstrate their inability to apply and interpret these higher order procedures but, in actual fact, the difficulty could lie with the knowledge and skills the learner is assumed to have from a previous stage.

To complete their journey, learners needs to be nurtured to look beyond their particular field and recognise the resolutions they require to solve problems, could exist in a different discipline. It is also important they have schemata in place to deal with open ended, ill formed problems. In addition to the technical schemata, engineering undergraduates also need to acquire skills in leadership, team working and industrial practice.

Cognitive Load Theory

CLT is premised on five principles: The information store principle, the borrowing and reorganising principle, the randomness as genesis principle, the narrow limits of change principle and the environment and linking principle (Sweller, 2010). The information store principle states that long term memory is central to human cognition. It is not only a repository of facts but also it is where, for example, problem solving procedures are stored. How this information is acquired can be accounted for by the 'borrowing and reorganising principle' which states the learners' acquired knowledge and skills come from imitation, reading, and listening. This information tends to be reorganised and reconstructed in terms of the learners' individual experiences. In terms of schema theory, learning occurs when schemata are constructed and automated. The construction of schemata relies upon the storage of facts and procedures in long term memory. The underlying process involves the initial use of short term working memory to organise the information before passing it to long term memory in the form of disjoint schema. The automation of schemata occurs when information is processed unconsciously. Dehaene (1992) coined the phrase 'asemantic processing' to indicate that at this level of information processing, there was no conscious involvement of short term working memory.

The construction and automation of schemata does not generate new information. It essentially allows communication and the combination of facts. In order to create new information, the problem solving and the randomness as genesis principle states the random generation of resolutions to a problem followed by tests of effectiveness is how this phenomenon occurs. The information held in long term memory can indicate potential resolutions, provided the learner has encountered similar problems before but in a totally novel situation, the learner has to adopt a 'trial and improvement' approach where a potential resolution is tried, tested for effectiveness and adopted if successful but discarded if not. This procedure of testing for effectiveness and how it fits with a learner's existing schema also applies to the borrowing and reorganising principle. If the new information is deemed to be beneficial, existing schemata are updated and if deemed to be non-beneficial, discarded. In this sense, the contextualisation of information increases the probability the learner will see the 'usefulness' of the new information and consequently update their existing schemata.

A consequence of the randomness as genesis principle and the borrowing and reorganising principle is alterations to existing schemata must be incremental to avoid information overload. It is very easy to introduce a plethora of new ideas in a learning session which results in the learner's cognitive system becoming overloaded and existing schemata being destroyed. Short-term working memory is capable of processing approximately seven items of information (Millar, 1956). To test the effectiveness of,say, four elements results in 24 possible permutations to test, but if ten items are presented, 3,628,800 permutations require testing, which is beyond the scope of short-term working memory capabilities (Sweller, 2010).

The environment organising and linking principle offers an explanation why experts can process large amounts of information. This principle suggests that, providing information is organised into schemata in long-term memory, experts are able to transfer asematically the necessary schema to solve a problem within their particular environment.

The Participants

The students who participated in this investigation were first year undergraduate product design and management students. A typical qualification profile was: A levels in Product Design, Humanities subjects and rarely Mathematics or Physics. Prior to the investigation the students had studied Mathematics and Engineering Science for one teaching period. These lessons covered such topics as resolutions of forces, beams, algebra and solving equations.

The Investigation.

The participants were given an open-ended, ill-formed problem which focussed on them designing a simple bridge to ford a river. The only information they were given concerned the width of the river and the height of its banks. They were also given a talk about using a problem solving rubric. They were not instructed on any formulae they would need or how to go about resolving the problem. The participants were asked to work in pairs and the investigation ran for three sessions.

The Task

An outward bound company has set up a new campsite for young people in the Brecon Beacons National Park. There is a river running through the site which effectively separates the main camp site from the cook house. The river is 3m wide and the mean height of the river bank is 1m. During periods of heavy rain the river can overflow the banks. The management team have decided that they need to build a safe, simple bridge at minimal cost which would enable the young people to access the cook house all year round. Your task is to investigate and report back to the management team how you would resolve this issue. You will need to investigate resolutions which incorporate different designs and recommend a solution which is cost effective and fit for purpose.

Problem solving rubric

Exploring the problem:

1. What information is given by the problem?
2. What is the problem asking me?
3. Is there additional information I need to get started?

Resolving the problem

1. Have I resolved a similar problem before?
2. Do I know the mathematics to solve this problem?
3. What assumptions, if any, do I have to make?
4. Can the problem be broken down into smaller, more manageable problems?
5. Can the problem be looked at from a different perspective?

Reviewing my resolution

1. Is my resolution acceptable?
2. What have I learned from the resolution?
3. Could I use this resolution to resolve other problems?

Discussion of results

It was evident from the start of the investigation that the students found it extremely difficult to form assumptions. They had studied the loading of beams and Newton's third law but seemed unable to apply this knowledge to the problem in front of them. They also seemed incapable of simplifying the problem to that of a 'plank of wood joining the two banks' even though the problem solving rubric asked the question, 'have I resolved a similar problem before'. The students sketched bridges based on their conceptions of what a bridge should look like and seemed to have missed the affordance a bridge offers. After listening in on the group discussions, the investigator decided to intervene by initiating a class discussion on the nature of bridges which led to a discussion of the mathematics and engineering science required to offer an initial resolution.

In the second session many of the students had adopted, as a starting point, a wooden beam laid between the two banks. They proceeded to identify and signify the forces acting upon their bridge. Although they had correctly identified the forces they were unable to proceed since they had not considered factors which would influence the design and loading of their bridge. Once more a group discussion ensued which considered assumptions on the loading model for the bridge ie. Should a point or distributed load be considered. This initial discussion led to considering the weight the bridge would have to support which meant having to make an assumption of the number of people who would use the bridge at any one time. The final factor discussed was the amount of flexion that was permissible of the platform to make walking comfortable and safe.

The final session of this investigation should have been where the students performed the necessary calculations in order to decide on the materials they would use and the dimensions of their bridge. Again, after listening in on the group discussions and in order for them to proceed, the investigator decided to initiate a class discussion. They were shown formulae needed to calculate the loading and flexion of their bridges. It was evident from their reaction that they could not interpret, what they perceived to be extremely complicated formulae. For example, they were shown the formula for a distributed load and had to be guided through its interpretation.

By the end of this final session, none of the groups had been able to offer a reasonable resolution.

Conclusions

Since its emergence at McMasters University (Woods, 1994), PBL (Problem Based Learning) has had mixed responses. Many advocate its benefits in terms of education, yet others report of little benefit to learners (Van Barneveld & Strobel, 2009). In fact, there is very little evidence in general for 'constructivist' based approaches, to support the notion of an increase in student knowledge (Kirscher, et al., 2006). If the principle aim for introducing PBL is to increase student knowledge, then Savin-Baden's model 1: 'Problem based learning for epistemological competence' (2007) would be more appropriate, but if the aim is to equip students with the knowledge and skills required by many modern industries, then the model used in this investigation (Model II: Problem-based learning for professional action') is appropriate with the caveat a way has to be found to provide the students with the necessary technical knowledge and skills to support their problem solving activities.

Although this was a small scale study and could possibly be more accurately described as problem solving learning using an open-ended, ill formed problem, the results do indicate a disconnect between theory and practice. Although the students, theoretically, had the a priori knowledge and skills, they were unable to form an overarching schema incorporating the technical knowledge and problem solving skills. All the way through the investigation, they were expecting to be given the mathematics and engineering science required to present a resolution. In terms of Cognitive Load Theory their behaviour can be characterised as having a number of disjoint schemata which have been learnt during lessons; evidenced by the fact they were able to solve typical closed, well formed problems. They did not possess the confidence to adopt a trial and improvement approach (randomness as genesis principle) and subsequently were at a loss as to how to even begin to offer a resolution. Although, in principle like many learning philosophies, Problem Based Learning does meet the needs of contemporary engineers, there needs to be a clear and well-articulated reason for introducing it into the curriculum and its implementation requires careful and detailed planning.

References

Dehaene, S., 1992. Varieties of numerical abilities. *Cognition,* 1(2), pp. 1-42.

Heitman, G., 1996. Project-oriented study and projects-organized currilcula:A brief review of intentions and solutions. *European Journal of Engineering Education,* 21(2), pp. 121-131.

Kirscher, P., Sweller, J. & Clark, R., 2006. Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential and inquiry-based teaching. *Educational Psychologist,* Volume 41, pp. 75-86.

Kolmos, A., de Graff, E. & Du, X., 2009. Diversity of PBL - PBL learning principles and models. In: *Research on PBL Practice in Engineering Education.* Rotterdam: Sense, pp. 9 - 22.

Millar, G., 1956. The magical number seven, plus or minus two:Some limits on our capacity for processing information.. *Psychological Review,* Volume 63, pp. 81-97.

OFQUAL, 2013. *Consultation on New A level Regulatory Requirements,* Coventry: Office of Qualifications and Examinations Regulation.

Plas, J. L., Moreno, R. & Brunken, R., 2010. *Cognitive Load Theory.* New York: Cambridge University Press.

Savin-Baden, M., 2000. *Problem Based Learning in Higher Education: Untold Stories..* s.l.:Society for Research into Higher Education and OU Press.

Savin-Baden, M., 2007. Challenging Models and Perspectives of Problem-Based Learning.. In: E. Graff & A. Kolmos, eds. *Management of Change:Implementationof Problem-Based and Project-Based Learning in Engineering.* Rotterdam: Sense Publishers, pp. 9-29.

Sfard, A., 1991. On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics,* Volume 22, pp. 1-36.

Skemp, R. R., 1986. *The Psychology of Learning Mathematics.* 2nd ed. London: Penguin Books.

Sweller, J., 2010. Cognitive Load Theory: Recent theoretical advances. In: *Cognitive Load Theory.* New York: Cambridge University Press, pp. 29 -47.

Van Barneveld, A. & Strobel, J., 2009. Problem Based Learning: effectiveness, drivers, and implementation challenges. In: X. Du, E. de Graff & A. Kolmos, eds. *Research on PBL in Engineering Education.* Rotterdam, The netherlands: Sense Publishers, pp. 35-44.

Woods, D., 1994. *How to gain most from Problem Based Learning.* Hamilton, Ontario: McMaster University press.

Learning Analytics and Learning Tribes

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Abstract

Traditional mass education is characterized by relatively static learning materials and environments, a slow feedback cycle, and indifference to the variety of learning styles. Improving the learning process is slow and corrective interventions often come too late. Also, we do not take full advantage of the rich potential of individual learners. The quality of learning outcomes is less than ideal and there are too many failures, especially in difficult subjects such as mathematics. We offer students a new interactive learning environment, provided by TribaLearning, which the students can personalize themselves according to their interests and tastes. The TribaLearning application provides teachers with a tool for predictive analysis of factors influencing learning results as well as ways to improve them at personal and group levels. The TribaLearning tool gathers data that is used to build mathematical models to analyze and predict factors with positive and negative impacts in learning. The results indicate problem areas in learning processes and quantifies their importance from learning results perspective. Learners are divided into 'learning tribes', that is, groups defined by a shared learning behaviour. Presenting results in an understandable graphical way helps teachers to support learning and intervene in the case of predicted learning problems early enough.

Personalized Learning – strive for enhancing education in the future.

Personalised Learning (also called Adaptive Learning) is one of the most interesting visions for the future in the field of education. The topic has been taken up on multiple discussion platforms around the world and it is something that teachers in different parts of the world are interested in. Personalised Learning will be a norm in the future and currently different stakeholders are researching different ways to produce something that would fulfil the promise of Personalised Learning. New technologies are emerging and the current trend of digitalization of education can be seen as a base for the future developments that will ultimately bring forth a disruptive change in the field of education.

Learning Analytics is one of the areas investigated by researchers and innovative teachers in their attempts to bring this change to education to the practitioner level. According to Horizon Report (Johnson et al., 2014), the following is the best way to describe what Learning Analytics can be used for: “Learning analytics research uses data analysis to inform decisions made on every tier of the education system, leveraging student data to deliver personalized learning, enable adaptive pedagogies and practices, and identify learning issues in time for them to be solved. Adaptive learning data is already providing insights about student interactions with online texts and courseware.”

Teachers willing to test new technologies on their courses and in classrooms are of key value for the development of the technologies tied to Personalised Learning.

Cooperation models between education technology companies, teachers and educational institutions are the base for piloting and bringing in new innovations available for users, namely the students.

What does the future look like – and what are the challenges?

The United States Department of Education (2012) describes Personalised Learning as follows: "Education is getting very close to a time when personalization will become commonplace in learning. The instructor is responsible for supporting student learning, but her role has changed to one of designing, orchestrating, and supporting learning experiences rather than "telling". Rather than requiring all students to listen to the same lectures and complete the same homework in the same sequence and at the same pace, the instructor points students toward a rich set of resources, some of which are online, and some of which are provided within classrooms and laboratories. Thus, students learn the required material by building and following their own learning maps."

There are still challenges that need to be addressed however. Challenges can be found in a lack of teacher to student engagement, a lack of hardware (computers) in schools, a lack of financial resources and so on. According to the research paper "Teachers Know Best" (2014), the technologies brought to market do not cater to the needs of the teachers or the students particularly well. Either they are too simple, performing simple tasks, or they are too complicated, becoming too difficult to use. The needs of the teachers are quite simple when it comes to new educational technologies. Teachers identified six instructional purposes for which digital tools are useful:

1. Delivering instructions directly to students,
2. Diagnosing student learning needs,
3. Varying the delivery method of instruction,
4. Tailoring the learning experience to meet individual student needs,
5. Supporting student collaboration and providing interactive experiences,
6. Fostering independent practice of specific skills. (Teachers Know Best, 2014).

Many companies are trying, as noted above, to bring something new to the educational technology market, but the products that are launched are somehow lacking important elements. This means that, while the products are good as technology, they are not good enough to serve the purpose of education, especially for the teachers and students.

TribaLearning and learning theories

All Learning Analytics providers have data gathering and data presentation as the base of their system. They state that they are providing data that will help the teachers and students to understand their knowledge level which, in turn, will give them system-aided feedback. These learning analytics systems give the students either recommendations on how to proceed with their studies or they intervene when the system detects that the student is lacking in knowledge. In the latter case, in order to proceed to step M, the system tells the student that he or she must return to step N.

TribaLearning, established in 2013, also tries to bring 21st Century Skills into the field of Learning Analytics, namely cooperation, communication, creativity, innovation and problem solving. Therefore the company develops new algorithms from pedagogical theories that are based on social activity and these algorithms are incorporated into the personal learning environment. The analytics will be developed to also include motivation, emotions, collaborative knowledge building, stress and anxiety during the learning process. (Litmanen, 2012).

The Triba Learning Environment consists of the student user interface (UI) and the analytics interface for the teacher. The student interface is arranged topically as “boards” that include articles, namely documents and web resources. The student can include any board or article in their collection of favourites. The student can highlight and make comments on the articles, down to the level of a phrase in the article.

Analytics - Algorithms and Models

In its present state the Triba analytics tool gathers data on how often and when the students read the articles, how much they comment or highlight the articles and how much they make private notes. Results can be analyzed and visualized in several ways.

Two analytics views: Timeline and Tribes

Personal learning analytics:

1. Time series: Hours studied (per student, per day or hour), article views, commenting activity. See the section “An example: Metropolia, 10.3.-27.4”.

2. Learning orientations and moods. Students’ learning orientations are estimated using a 5-factor model developed at the University of Helsinki. Individual learning orientations affect moods and feelings of satisfaction, which in turn are known to affect learning outcomes. For example, “social learners” may thrive in groupwork settings, elevating their mood and learning outcomes. “Target-oriented learners” or “individually oriented learners” are often independent and they organize their study track themselves. “Cookbook learners” need clear

instructions and are primarily interested in just passing the course. (Litmanen, 2012).

Teacher analytics supporting interventions and course design:

1. Time series: Hours studied (per student, per day or hour), article views, commenting activity. See the section "An example: Metropolia, 10.3.-27.4".
2. Cluster analysis, grouping students into "tribes" sharing similar characteristics, for example similar time series, tastes (favoured articles), and interactions (commenting and replying "binds" the participants together; outliers may be isolated socially).
3. Learning orientations and moods. See above (personal learning analytics). Knowing the students' psychological orientations may help in designing the appropriate studying environment, for example "social learners" may be satisfied in groupwork settings while students with high "individual orientation" may be less satisfied by this.

Peer-to-peer tutoring:

Student comments can be tagged as "open questions". Teachers and other students can use topic based filtering to find open questions related to topics they are comfortable with. The student who posted the original question gets to decide whether to close the question (an answer was satisfactory) or keep it open. Interactions that result in questions being closed generate topic-specific "expertise points" for those that provided the satisfactory answers.

In the future, estimated topic-specific expertise will be used in clustering the students, so that, amongst other steps, the potential "tutors" (assistants) can be found in larger groupwork settings.

Personalised recommendations

Topic (via tags) and article level difficulty (challenge) ratings from students enable personalised recommendations (within topic). For example, if a student finds an article about linear algebra very difficult compared to his peers, the system can recommend related books that his peers rated as easier. Depending on the student's motivation, this may help him in catching up with his peers.

An example: Metropolia, 10th March to 27th April

The Triba learning environment was used during a Metropolia Industrial Management calculus course between March 10th and May 8th in 2014. The students ($n = 40$) could attend lectures and laboratory or practice sessions, or they had the option to study at home. All study material and instructions (roughly 50 documents) can be accessed in the Triba learning environment. On the average, 25 students attended the lectures and laboratory or practice sessions regularly. They relied mainly on classroom notes and only partially on Triba documents. The other students used the documents in Triba more (excluding drop-outs).

Time spent reading (in hours). Daily sums were taken over all 40 students and all articles during the period from 10.3. to 27.4.:

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
10.3.-16.3.	11.46	5.84	9.57	9.95	0.28	0.00	0.00
17.3.-23.3.	32.46	4.87	0.39	37.80	5.40	2.64	*27.23
24.3.-30.3.	35.97	2.88	0.26	17.92	3.58	0.01	0.31
31.3.-6.4.	10.71	3.05	9.40	8.12	4.03	3.47	4.19
7.4.-13.4.	8.86	0.08	0.01	3.35	0.03	0.00	0.00
14.4.-20.4.	0.59	0.05	0.00	0.05	0.00	0.02	0.00
21.4.-27.4.	4.02	0.01	6.32	8.66	0.03	0.18	1.43

Some observations:

- Based on measured reading times, Triba's usage appears to have declined over time. Possible reasons could include usability problems, reading and exercising offline and the Easter holiday around the week 14th to 20th April.
- Most activity tends to occur on Mondays and Thursdays when students have dedicated laboratory practice hours in their weekly schedule. Sunday 23rd March (*) was the deadline for the first course project. The second project workload was distributed more evenly across the week 31st March to 6th April.
- As the final test and the third project deadline is 8th May, daily sums will probably become higher during the two weeks that are not included in the table.

* A more detailed view: an hourly breakdown of Sun 23.3.:

	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-00
23.3.	0.98	2.24	1.00	0.36	2.44	4.76	3.22	*4.68	2.11	2.99	2.10	0.34

* The most popular hour, 19-20, had about 4 active readers contributing to the sum of 4.68 hours.

Suggestions for further analysis:

- In the future, identify the students who tend to stay up late the night before a lecture / an exam / an exercise session. Try to encourage healthier studying habits.
- Generate a separate table or figure for each course chapter, that is, they do not take sums over all articles to see whether students progress as planned. For example, if the intended schedule is two chapters per week, the data could show students abandoning earlier chapters and moving on.
- The analysis above examined reading times only. The same analysis can be done with article views and commenting activity.
- Using the official UI, individual students are always compared to peer averages.

Applications

- Course design: Perform a similar analysis for several courses in parallel. Try to design courses so that students' workload is distributed evenly over weekdays.
- Course design 2: Compare several courses to rank them based on how time consuming they are compared to the credits earned by that course.
- Interventions: Once usability / utility problems are solved, declining student work hours may be a sign of student disengagement – interviewing a subset of students based on these statistics may be more economical than interviewing them all.
- Interventions 2: Examining individuals odd working hours (for example working late at night on weekdays and an emphasis on weekends) might indicate planning problems (such as having a job instead of being a full time student).
- Administrators are able to look at detailed data across different classes to examine progress for all students attending a given institute to easily distinguish factors that promote success. Using the data, administrators can set, implement, and adapt their policies and programs to improve learning results.

References

Litmanen, T. et al. (2012) "Capturing teacher students' emotional experiences in context: does inquiry-based learning make a difference?" *Instructional Science*, 40: 1083-1101.

Johnson, L., Adams Becker, S., Estrada, V., Freeman, A. (2014). *NMC Horizon Report: 2014 Higher Education Edition*. Austin, Texas: The New Media Consortium.

Teachers Know Best: What Educators Want from Digital Instructional Tools. (2014). Bill & Melinda Gates foundation.

U.S. Department of Education, Office of Educational Technology. (2012). *Enhancing Teaching and Learning Through Educational Data Mining and Learning Analytics: An Issue Brief*. Washington, D.C.

MapleSoft Demonstration

Teaching Concepts with Computational Algebra and e-Assessment Modern engineering design processes rely on skills in problem definition, path to solution and investigative skills. These skills are often developed later in engineering education through project based work. Computational Algebra seeks to introduce these skills at an early stage through a concept based learning system which can intelligently assess a students understanding of a mathematical concept. In this talk we will demonstrate how computational algebra can improve student comprehension, provide better feedback, cut marking time and improve the overall teaching and learning experience.

Lagrange Multipliers as Quantitative Indicators in Economics

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Abstract

The quantitative role of Lagrange multipliers is under consideration. Applications in economics are examined. Illustrative examples are presented

Introduction

The *Lagrange multipliers method* is readily used for solving constrained extrema problems. Let us concentrate on the rationale for this method. Recall that for a function f of n variables the necessary condition for local extrema is that at the point of extrema all partial derivatives (supposing they exist) must be zero. There are therefore n equations in n unknowns (the $x's$), that may be solved to find the potential extrema point (called *critical* point). When the $x's$ are constrained, there is (at least one) additional equation (*constraint*) but no additional variables, so that the set of equations is overdetermined. Hence the method introduces an additional variable (the Lagrange multiplier), that enables to solve the problem. More specifically (we may restrict to finding a maxima), suppose we wish to find values $x_1,...,x_n$ maximizing

$$y = f(x_1,...,x_n)$$

subject to a constraint that permits only some values of the $x's$. That constraint is expressed in the form

$$g(x_1,...,x_n) = 0.$$

The Lagrange multipliers method is based on setting up the new function (the *Lagrange function*)

$$L(x_1,...,x_n,\lambda) = f(x_1,...,x_n) + \lambda g(x_1,...,x_n), \tag{1}$$

where λ is an additional variable called the *Lagrange multiplier*. From (1) the conditions for a critical point are

$$\begin{aligned} L'_{x_1} &= f'_{x_1} + \lambda g'_{x_1} = 0 \\ &\vdots \\ L'_{x_n} &= f'_{x_n} + \lambda g'_{x_n} = 0 \\ L'_{\lambda} &= g(x_1,...,x_n) = 0, \end{aligned} \tag{2}$$

where the symbols L', g' are to denote partial derivatives with respect to the variables listed in the indices. Of course, equations (2) are only necessary conditions for a local maximum. To confirm that the calculated result is indeed a local maximum second order conditions must be verified. Practically, in all current economic problems there is on economic grounds only a single local maximum.

In a standard course of engineering mathematics the Lagrange multiplier is usually presented as a clever mathematical tool („trick") to reach the wanted solution. There is no large spectrum of sensible examples (mostly a limited number of simple "well-tried" school examples) to show convincingly the power of the method. The economic meaning of the Lagrange multiplier provides a strong stimulus to strengthen its importance. This will be central to our next considerations.

Economic milieu

To grasp the issue we will notice two useful meanings $1^0, 2^0$ of the Lagrange multipliers.
1^0 Rearrange the first n equations in (2) as

$$\frac{f'_{x_1}}{-g'_{x_1}} = ... = \frac{f'_{x_n}}{-g_{x_n}} = \lambda. \tag{3}$$

Equations (3) say that at maximum point the ratio of f'_{x_i} to g'_{x_i} is the same for every x_i and moreover it equals λ. The numerators f'_{x_i} give the **marginal contribution** (or **benefit**) of each x_i to the function f to be maximized, in other words they give the approximate change in f due to a one unit change in x_i. Similarly, the denominators have a marginal cost interpretation, namely, $-g'_{x_i}$ gives the **marginal cost** of using x_i (or marginal "taking" from g), in other words the approximate change in g due to a unit change in x_i. In the light of this we may summarize, that λ is the **common benefit-cost ratio** for all the $x's$, ie.

$$\lambda = \frac{marginal\ contribution\ of\ x_i}{marginal\ cost\ of\ y_i} = \frac{f'x_i}{-g'x_i} \tag{4}$$

<u>Example</u> Let $Q = Q(l, k)$ be a production function, where *l* is a labour and *k* capital. The cost to the firm of using as input *l* units of labour and *k* units of capital is

$$P_l l + P_k k,$$

where P_l and P_k are the per unit costs of labour and capital respectively. If the firm has a fixed amount, M, to spend on these inputs then the cost constraint is

$$P_l l + P_k k = M.$$

In order to maximize the function $Q(l, k)$ subject to this constraint we set up the Lagrange function (rewriting constraint condition to $M - P_l l - P_k k = 0$)

$$L(l, k, \lambda) = Q(l, k) + \lambda(M - P_l l - P_k k).$$

Due to (2) it holds

$$L'_l = Q'_l - \lambda P_l = 0$$
$$L'_k = Q'_k - \lambda P_k = 0$$
$$L'_\lambda = M - P_l l - P_k k = 0,$$

but $Q'_l = M_{P_l}$ is the marginal product of labour and $Q'_k = M_{P_k}$ is the marginal product of capital. Then first two equation can be rearranged (according to (3)) to give

$$\lambda = \frac{MP_l}{P_l} = \frac{MP_k}{P_k}$$

which states that at the maximum point the ratio of marginal product to price is the same for both inputs and it equals λ.

Example A farmer has a given length of fence F and wishes to enclose the largest possible rectangular area. The question is about the shape of this area. To solve it, let x, y be lengths of sides of the rectangle. The problem is to find x and y maximizing the area $S(x, y) = xy$ of the field, subject to the condition (constraint) that the perimeter is fixed at $F = 2x + 2y$. This is obviously a problem in constraint maximization. We put $f(x, y) = S(x, y)$, $g(x, y) = F - 2x - 2y = 0$ and set up the Lagrange function (1)

$$L(x, y, \lambda) = xy + \lambda(F - 2x - 2y). \tag{5}$$

Conditions (2) are

$$L'_x = y - 2\lambda = 0, L'_y = x - 2\lambda = 0, L'_\lambda = F - 2x - 2 = 0.$$

These three equations must be solved. The first two equations give $x = y = 2\lambda$, i.e. x must be equal to y and due to (5) they should be chosen so that the ratio of marginal benefits to marginal cost is the same for both variables. The marginal contribution to the area of one more unit of x is due to (4) given by $S'_x = y$ which means that the area is increased by y. The marginal cost of using x is $-g'_x = 2$. It means value 2 from g; but since $g(x, y) = F - 2x - 2y$, the value 2 is taken from the available perimeter F. As mentioned above, the conditions (4) state that this ratio must be equal for each of the variables. Completing the solution (substituting $x = y = 2\lambda$ in $F - 2x - 2y = 0$) we get $\lambda = \frac{F}{8}, x = y = \frac{F}{4}$. Now let us discuss the interpretation of λ. If the farmer wants to know, how much more field could be enclosed by adding an extra unit of the length of fence, the Lagrange multiplier provides the answer $\frac{F}{8}$ (approximately), i.e. the present perimeter should be divided by 8. For instance, let 400 be a current perimeter of the fence. With a view to our solution, the optimal field will be a square with sides of lengths $\frac{F}{4} = 100$ and the enclosed area will be 10 000 square units. Now if perimeter were enlarged by one unit, the value $\lambda = \frac{F}{8} = \frac{400}{8} = 50$ estimates the increase of the total area. Calculating the "exact" increase of the total area, we get: the perimeter is now 401, each side of the square will be $\frac{401}{4}$, the total area of the field is $(\frac{401}{4})^2 = 10050{,}06$ square units. Hence, the prediction of 50 square units given by the Lagrange multiplier proves to be sufficiently close.

Example Let an individual´s health (measured on a scale of 0 to 10) be represented by the function f,

$$f(x, y) = -x^2 + 2x - y^2 + 4y + 5,$$

where x and y are daily dosages of two drugs. It may be verified, that this function attains its (local) maximum for $x = 1, y = 2$ with the corresponding value of $f(1,2) = 10$. So, at that point is the best health status 10 possible. Now we want to maximize f under the constraint that this individual could tolerate only one dose per day, i.e. $x + y = 1$. We put $f(x, y) = -x^2 + 2x - y^2 + 4y + 5$, $g(x, y) = 1 - x - y = 0$ and set up the Lagrange function

$$L(x, y, \lambda) = -x^2 + 2x - y^2 + 4y + 5 + \lambda(1 - x - y).$$

Conditions (2) are

$$L'_x = -2x + 2 - \lambda = 0, L'_y = -2y + 4 - \lambda = 0, L'_\lambda = 1 - x - y = 0.$$

Applying Lagrange multipliers method we get the solution $x = 0, y = 1, \lambda = 2.$ with the corresponding value of $f(0,1) = 8$. The value 2 may be interpreted as the remainder to the maximum value of health status 10. Now we reduce the restriction altering the constraint equation to $x + y = 2$. We expect f to increase. Finding the new solution as before we have $x = 0,5; y = 1,5; \lambda = 1$ with $f(0,5; 1,5) = 9,5$. So, there is still some remainder (approximately λ=1, precisely 0,5) to the optimal health status. Further reducing constraint to $x + y = 3$ leads to the solution $x = 1, y = 2, \lambda = 0$ which is the maximum of f (without constraint). For higher sums of $x + y$ (overdose) we expect negative values of λ.

2^0 Rewrite constraint condition $g(x, y) = 0$ as $c(x, y) = k, g(x, y) = k - c(x, y) = 0$, where k is a parameter. Then the Lagrange function is of the form

$$L(x, y) = f(x, y) + \lambda\left(k - c(x, y)\right).$$

For the partial derivative of L with respect to k we get $L'_k = \lambda$. From the interpretation of a partial derivative we conclude, that the value λ states the approximate change in L(and also f) due to a unit change of k. Hence the value λ of the multiplier shows the approximate change that occurs in f in response to the change in k by one in the condition $c(x, y) = k$, ie. $c(x, y) = k + 1$. Since usually $c(x, y) = k$ means economic restrictions imposed (budget, cost, production limitation), the value of multiplier indicates so called the **opportunity cost** (of this constraint). If we could reduce the restriction, ie. to increase k by 1, then the extra cost is λ. If we are able to realize an extra unit of output under the cost less than λ, then it represents the benefit due to the increase of the value at the point of maxima. Clearly to the economic decision maker such information on opportunity costs is of considerable importance.

Example The profit of some firm is given by $PR(x, y) = -100 + 80x - 0{,}1x^2 + 100y - 0{,}2y^2$, where x, y represent the levels of output of two products produced by the firm. Let us further assume that the firm knows its maximum combined feasible production to be 500. It represents the constraint $x + y = 500$. Putting $g(x, y) = 500 - x - y = 0$ we set up the Lagrange function

$$L(x, y, \lambda) = -100 + 80x - 0{,}1x^2 + 100y - 0{,}2y^2 + \lambda(500 - x - y).$$

Applying the Lagrange multipliers method we get the solution $x = 300, y = 200, \lambda = 20$ with the corresponding value of the profit $PR(300,200) = 26900$. Now we reduce the restriction altering the constraint equation to $x + y = 501$. Finding the new solution as before we have

$x = \dfrac{902}{3} = 300{,}666, y = \dfrac{601}{3} = 200{,}333, PR(\dfrac{902}{3}, \dfrac{601}{3}) = 26919{,}933$. We see that the increase in profit brought about by increasing the constraint restriction by 1 unit has been 19,933 - approximately the same as the value λ that we derived in the original formulation. It indicates that the additional increase of labour and capital in order to increase the production has the opportunity cost of approximately 20.

Example The utility function is given by $U = U(x, y) = 4x^{0.5}y^{0,25}$, where x or y is the number of units of a good X or Y respectively. Suppose the price of X is 2,5 USD per unit, the price of Y is 4 USD per unit. To calculate the optimal combination for an income of 50 USD we employ Lagrange multipliers method. The constraint is given by $2{,}5x + 4y = 50$. We put $g(x, y) = 50 - 2{,}5x - 4y$ and form Lagrange function

$$L(x, y) = 4x^{0,5}y^{0,25} + \lambda(50 - 2{,}5x - 4y).$$

Applying this method we get $x = \frac{40}{3}, y = \frac{25}{6}, \lambda = 0{,}313$ with the corresponding value of utility $U\left(\frac{40}{3}, \frac{25}{6}\right) = 20{,}867$. Now we moderate the constraint to $2.5x + 4y = 51$. Applying the method again we obtain the solution $x = \frac{68}{5}, y = 4{,}25$ with the corresponding value of utility $U\left(\frac{68}{5}; 4{,}25\right) = 21{,}180$. We see that the increase in utility equals the value of λ.

Conclusion

In the instruction of engineering mathematics the Lagrange multipliers method is mostly applied in cases when the constraint condition $g(x, y) = 0$ cannot be expressed explicitly as the function $y = f(x)$ or $x = h(y)$. When solving constrained extrema problems in economics the bulk of the constraint conditions may be expressed explicitly, so the reason to use the Lagrange multipliers method would seem to be too sophisticated regardless of its theoretical aspects. With a view to the crucial importance of the economic interpretations of Lagrange multipliers is the use of the method primarily preferred. Concrete applications of the presented interpretation principle may be developed in many economic processes. A deeper study on the role of the Langrange multipliers in optimization tasks may be found in Rockafellar (1993).

References

I. Jacques, *Mathematics for Economics and Business*, Addison-Wesley, Reading, Massachusetts, 1995

I.Mezník, *On Economic Interpretation of Lagrange Multipliers*, Proc. of the 11th International Conference "Turning Dreams into Reality: Transformations and Paradigm Shifts in Mathematics Education", Rhodes University, Grahamstown, South Africa, September 11-17, 2011, 229-232

W. Nicholson, *Microeconomic Theory*, The Dryden Press, New York, 1998

R. T. Rockafellar, *Lagrange Multipliers and Optimality*, SIAM Rev., 35(1993), pp.183-238

M. Wisniewski, *Introductory Mathematical Methods in Economics*, McGraw-Hill Comp., London, 1991

Investigating the Engagement of Mature Students with Mathematics Learning Support

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Abstract

The Mathematics Learning Support Centre (MLSC) in the Dublin Institute of Technology (DIT) provides free mathematical support to all DIT students. This support is primarily delivered through a drop-in service, where students can receive one-to-one tuition, without an appointment, in any area of mathematics. In the first semester of the 2013/14 academic year a significant proportion (approximately 42%) of students that availed of this drop-in service were mature students enrolled in Engineering programmes. This is of particular interest as mature students constitute a relatively small proportion of the total student body, motivating a deeper study of the reasons for the high levels of engagement in this cohort. To this end two focus groups were conducted, involving both those who did and did not attend the MLSC. Particular interest was paid to the motivations for attendance/reasons for non-attendance. The motivations of mature students were found to be multifaceted while the reasons for non-engagement given were mostly in line with the literature. In addition some quantitative analysis was carried to determine what effect the MLSC had on student's academic performance.

Introduction

In recent years an increasing number of students in Irish Higher Educational Institutions (HEIs) are taking courses with mathematical and statistical elements. This is in part due to the widespread recognition that mathematics underpins many other disciplines (such as Science, Technology and Engineering) and the emphasis placed by the Higher Educational Authority on producing graduates who are highly literate in mathematics (EGFSN 2008, HEA 2004).

Hand in hand with this increase however has come the so called 'Maths Problem'- that is a decline in the mathematical proficiency of incoming first year students across HEIs in Ireland and elsewhere (Gill 2008, Almeida et al. 2012, Carr et al. 2013 & 2013). This in turn is having a detrimental effect on enrolment and retention levels in science and technology courses in HEIs (OECD 1999). In fact, it is widely acknowledged that the absence of a solid foundation in mathematics can be one of the key inhibitors for student progression in higher education (HEA 2008).

As part of the response to this problem, Mathematics Learning Support Centres (MLSCs), defined by Lawson et al (2003) as 'a facility offered to students (not necessarily of mathematics) which is in addition to their regular programme of teaching through lectures, tutorials, seminars, problems classes, personal tutorials, etc.' have been set up in the majority of HEIs in Ireland (Gill et al. 2010). In the UK over 85% of HEIS surveyed offer some form of Mathematics Learning Support (MLS) (Perkin et al. 2012), up from 62.3 % in 2004 and 48% in 2001 (Perkin et al. 2004, Lawson et al. 2001). It is therefore clear that MLS has now become an integral part of the higher educational framework, both in Ireland and the UK.

However despite this, MLSCs in several HEIs exist precariously, often lack permanent funding and are regularly in the 'front line' for spending cut backs (Macgillivray et al. 2011, Mac an Bhaird et al. 2013). To ensure that the limited funding available for MLS is put to the best possible use and to establish 'Best Practice', much time and resources have been put into researching methods of evaluating MLSCs' activities. This evaluation can be undertaken using quantitative (usage figures, diagnostic testing, exam results etc.) and qualitative methods (focus groups, surveys, student feedback etc.) (Macgillivray et al. 2011). In a study on evaluation of the MLSC in Dublin City University, Ní Fhloinn found that a combination of both types of methods gave a more complete picture (Ní Fhloinn 2009). An extensive review of the literature on the evaluation of MLSCs can be found in (Matthews et al. 2012).

One important issue that arises from these evaluations is the non-engagement with MLS of so called 'at risk' students - those who are most in need of extra support. In a recent paper by Mac an Bhaird et al. (2013), details of a large scale study on student non-engagement with MLS across several Irish HEIs are given. The study found that the main reason students gave for non-engagement was that they did not need help. However this was more likely to have come from a student with a strong mathematical background. For the weaker 'at risk' students, issues with the structures of the MLS such as unsuitable opening hours or a lack of information were more likely to be cited as a reason for non-attendance. Symonds (2008) questions whether these reasons are valid and wonders if implementing the requested changes in structures would actually serve to increase the engagement levels of these students. This suggests that a deeper study into the reasons of student non-engagement with MLS, in particular for those 'at risk' students, is required to get to the root of the non-engagement problem.

In this paper, the authors seek to further this investigation by looking at the engagement levels of mature students with the MLSC in the Dublin Institute of Technology (DIT). In the DIT a mature student is defined as being 'any Irish or EU citizen who will be 23 years of age on the 1st of January of the proposed year of entry' (DIT Website). The authors examine qualitatively the reasons behind both the engagement and non-engagement of this cohort of students with the MLSC, as well as performing a quantitative study on the effect of the MLSC on these students' academic performance.

Methods

This study seeks to examine the reasons behind both the engagement and non-engagement of students with the MLSC in the DIT, as well as investigating how the MLSC has influenced the academic performance of mature students who have regularly availed of its services. The authors decided to use a mixed method approach by combining both quantitative and qualitative methods of research. Qualitative researchers are interested in understanding the meaning people have constructed from their lived experiences (Merriam, 2009). Hence, qualitative methods of enquiry and analysis are more suitable when humans are the instruments of enquiry. This is why the authors decided on a study of this nature. However, in order to evaluate the academic progress of mature students who have been attending the MLSC a quantitative measure is needed. Much research supports this integration of quantitative and qualitative research.

The use of multiple methods reflects an attempt to secure an in-depth understanding of the research and allows for broader and better results (Denzin and Lincoln, 2005).

<u>Participants</u>
The participants for this study comprised of mature students in their first year of an Engineering undergraduate programme in the DIT. As mentioned previously, in the DIT a mature student is defined as being '*any Irish or EU citizen who will be 23 years of age on the 1st of January of the proposed year of entry'*.

<u>Qualitative Data</u>
In order to get feedback regarding why students attend/do not attend the MLSC, two focus groups were conducted. The first group (Focus Group 1) was made up of mature students whose attendance in the MLSC was constant throughout the year. The second group (Focus Group 2) was made up of mature students who had never attended the MLSC. Each student was coded to ensure confidentially. There were ten students in Focus Group 1 (P1 – P10) and four students in Focus Group 2 (P11 – P14). Their responses were transcribed and analysed using NVivo software and arranged into themes by the authors.

<u>Quantitative Data</u>
In order to get a quantitative measure of how the MLSC influenced the academic performance of mature students who regularly availed of its services, the authors decided to compare the grades of mature students who attended the MLSC with those who didn't. The objective was to investigate if the MLSC had any effect on their grades. The authors understand that there may have been other variables which may have affected the students' grades throughout the year.

Results And Findings

<u>Focus Group Findings</u>
In this section the main themes that arose during the focus groups are outlined. There will be particular focus on the three topics most relevant to this paper namely what motives mature students to attend the MLS, the reasons given by these students for non-attendance and their attitudes towards traditional students.

Motivation
During the course of the focus group, it became clear that the motivations of mature students who attend the MLSC were multi-faceted.

The initial motivations that were raised were of a practical nature, such as financial motivation (not being able to afford private tuition) or simply a lack of availability of any other form of support
P1: *I didn't even do a Junior Cert and I'm doing mechanical engineering maths and I've had straight A's through and that's through the Learning Centre you know. I can't afford grinds you know.*
P2: *There's no other, no other help available. That's what I found. If you're looking for extra help as well, every door would be closed.*

An interesting theme that arose was the concept that it was the nature of mathematics itself, and its difference from other subjects, which motivated students to seek extra help. They experienced difficulties with self-study and keeping up with the pace of lectures.
P1: *Whereas maths, you have something at the start of a page and something at the end and if you don't understand the bit in the middle, unless somebody points their finger at it and says to you "this is what's happening". If you don't get it you don't get it.*
P1: *I find with maths in particular of all the subjects........Unless you get a hold of the stuff in September and you're doing October's work you haven't a hope, if you don't understand the basics of stuff you haven't a hope. So I found going to the learning centre each week, staying on top, learning whatever was current, you'd go in and you'd actually learn from the lecture as well.*

Related to this theme, some students stated that while they found self-study aids (such as textbooks or online mathematical resources) useful, it was their belief that these aids are not a replacement for one-to-one support, such as that offered in the MLSC.
P2: *They're all fairly good but you still need the one-on-one. Because you can keep pausing and rewinding and going backwards and forwards but you need the one-on-one....When you've got no basic level there's only so much a video or a book can teach you*

A widely held view among the participants was that the mature students' life experiences serve to motivate them to seek out the extra support offered by the MLSC.
P7: *because I'm guessing most of us have experienced what it's like to struggle through jobs and that kind of stuff and realise the importance of getting a decent qualification behind you and doing something you actually like.....*
P7: *it's that experience of having been at the bottom, you know and having to try and survive at the bottom, that you realise that when you get an opportunity like this, just how important it is to really avail of all the services, in my opinion the Maths Learning Centre being the most important that I've come across so far as an extra aid on top of your coursework and stuff like that.*

Finally, the participants noted that they are not just interested in passing the exams, but that they wish to gain a deeper understanding of the subject. They recognise, again possibly based upon their life experiences, the importance of possessing more than just a surface level knowledge of their chosen subject area.
P9: *but I want to be able to understand it you know, I want to be able to like if I go to a job interview and somebody puts a problem in front of me I want to be able to know what it's about…….I want to comprehend it basically and if I need that extra bit of support, which you do get in the Maths Support Centre then I'll take advantage of it.*

Reasons For Non-Engagement
This section outlines the main reasons given for the non-engagement of mature students with the MLSC. In a recent large scale survey on the issue of student non-engagement with MLS in Irish HEIs, it was found the main reason given by students who did not

avail of service was that they did not believe they needed it (Mac an Bhaird *et al.* 2013). This finding was supported in our study.
P13: *I haven't really had a problem that I couldn't track down an answer to myself with google, youtube or any of that.*

Mac an Bhaird *et al* (2013) found that the second most common reason given for non-attendance were issues with the structural organisation of MLS in their HEI e.g. opening hours, room size etc. This theme also arose during our study.
P13: *if it was at a different time during the day that would suit me.*
P7: *I found that the only thing that kind of stopped me from going was the size of the room and at certain times because of how packed it is*

In Irish HEIs, several programmes are run to ease the transition of mature students back to education. During the focus groups, it was noted that mature students who have attended one of these transition programmes, appear to have less of a need for the services of the MLSC then those who have entered directly into their undergraduate programme.
P13: *Some mature students have a problem I think. Since they finished the LC and come back into college it has been 5-10 years. Not studied anything... I did last year mechanical engineering, this year I am ok.*
P11: *I wasn't too bad because I did Fetac 5 last year and it had engineering maths in it as well.*

Quantitative Findings

The study focused on one particular group of students, who were undertaking their first year of an ordinary degree in mechanical engineering and compared the end of semester exam results of those in this group who did and did not attend the MLSC in that semester. There were 20 mature students in this cohort. Of these students 8 had attended the MLSC and 12 had never attended. Of the 8 students who attended, 2 dropped out of the course after the first few weeks so there was no data on their performance. For the 18 students who remained, their performance in the semester 1 mathematics module was compared (See Table 1).

The average mark of those who attended the MLSC was higher but not significantly so ($p=0.25$). It is not possible to determine if the two groups were the same or different to begin with as many of these students are international students, and many of the Irish students had not finished secondary school. Hence there is no single metric to compare their mathematical ability on entry. There is a DIT mathematics diagnostic test given to many students on entry but it was not given to this cohort.

Attended MLSC	N	Mean	Standard deviation
Yes	6	80.6	18.9
No	12	68.4	23

Table 1: A comparison of end of semester exam results of those who did/did not attend the MLSC.

In addition, the proportion of both groups of students that achieved a grade of more than 60% was examined (See Table 2). Using a two proportions test, it was found that the difference in in these proportions was significant (p=0.046).

Attended Centre	N	>60	< 60
Yes	6	6	0
No	12	9	3

Table 2: A comparison of the proportion of students who did/did not attend the MLSC that achieved a mark higher/lower than 60%.

It is a limitation of this study that this analysis was only for a small number of students in one course. The two students who attended but dropped out early are excluded and there is no metric for ranking the students on entry.

Conclusions and Future Work

In this paper the authors investigated the reasons behind both the attendance and non-engagement of mature students with the MLSC in the DIT. Two focus groups were conducted with some interesting qualitative findings. The motivations of mature students were found to be multi-faceted, ranging from practical reasons, such as financial motivation, to more complex reasons such as their life experiences as adults motivating them to seek out extra help. The notion that mature students are interested not just in passing their exams, but also in gaining a deeper understanding of their chosen subjects was raised. The importance of one-to-one support in a student's development as an independent learner, even with the widespread availability of online resources, was also stressed.

For those students who did not avail of the services offered by the MLSC, the reasons given were mostly in line with the literature (Mac an Bhaird *et al* 2013), for example a lack of need for the service or issues with the structures of the MLSC. An interesting point raised was that mature students who have had a transition year prior to beginning their programme may have less need for extra support than those who have not attended such a course.

On the quantitative side, the authors examined the end of semester exam results of one group of students. They found that while the mean grade of those who attended the MLSC was higher than those who did not, the difference was not statistically significant (p=0.25). However there was a significant difference (p=0.046) in the proportion of both groups that achieved over 60% in the end of semester exam. These results must be viewed with a certain amount of caution however, as there was no common baseline for comparison of students' exams scores (e.g. diagnostic test results) and the sample size was small.

The authors intend to conduct focus groups involving traditional students to investigate the non-engagement further. The authors also wish to extend the quantitative analysis of this study to a much larger group of students, including tradational students, and to benchmark students on entry using the DIT mathematics diagnostics test, in line with Carr *et al.* (2013).

References

Almeida, B .D, Fidalgo, E., Rasteiro C., D.M.L.D., Projeto "ACAM - Avaliação de Competências / Ações de Melhoria",*XIX Colóquio AFIRSE: Revisitar os es tudos Curriculares : onde estamos e para onde vamos ?, 2012.*

Carr, M., Bowe, B., & Ní Fhloinn, E. "Core Skills Assessment to improve mathematical competency", *European Journal of Engineering Education, pp.1-12, 2013.*

Carr, M., Ni Fhloinn, E., Murphy, E. & Bowe, B. Addressing continuing mathematical deficiencies with advanced mathematical diagnostic testing.*Teaching Mathematics Applications (2013) 32 (2): 66-75.*

Denzin, N.K. and Lincoln, Y.S. (2005). *The SAGE Handbook of Qualitative Research,* USA: Sage Publications

Dublin Institute of Technology (DIT) [online], Accessed via http://www.dit.ie/study/mature/prospective/ (April 2014)

Expert Group on Future Skills Needs. (2008) Statement on Raising National Mathematical Achievement. Accessed via http://www.skillsireland.ie/media/egfsn090616_statement_on_activity.pdf (April 2014)

Gill, O., Johnson, P. & O'Donoghue, J. (2008) An Audit of Mathematics Support Provision in Irish Third Level Institutions. CEMTL (Regional Centre For Excellence in Mathematics Teaching and Learning), University of Limerick.

Gill, O., O'Donoghue J., Faulkner, F. & Hannigan A. (2010) Trends in performance of science and technology students. *International Journal of Maths Education, Science and Technology, 41, 323-339.*

A Study in Progression in Irish Higher Education, HEA 2010. Accessed via http://www.hea.ie/sites/default/files/study_of_progression_in_irish_higher_education_2010.pdf (April 2014)

National Strategy for Higher Education to 2030, January 2011 Accessed via http://www.hea.ie/sites/default/files/national_strategy_for_higher_education_2030.pdf (April 2014)

Lawson, D., Croft, T. & Halpın, M. (2001) Evaluating and Enhancing the Effectiveness of Mathematics Support Centres. Final report of a project funded by the LTSN Maths, Stats and OR Network. Accessed via http://www.academia.edu/2715644/Evaluating_and_Enhancing_the_Effectiveness_of_Mathematics_Support_Centres (April 2014)

Lawson, D., Croft, A.C. & Halpın, M. (2003) Good practice in the provision of mathematics support centres, learning and teaching in mathematics, statistics and operational research. LTSN Maths, Stats & OR Network, Accessed via http://www.mathcentre.ac.uk/resources/Good%20Practice%20Guide/goodpractice2E.pdf). (April 2014)

Mac An Bhaırd, C., Fıtzmaurıce, O., Ní Fhloınn, E. & O'Sullıvan, C. (2013) Student non-engagement with mathematics learning supports *Teaching Mathematics and Its Applications (2013) 32, 191-205*

Macgıllıvray, H. & Croft, A. C. (2011) Understanding evaluation of learning support in mathematics and statistics. *Int. J. Math. Educ. Sci. & Tech., 42, 189–212.*

Matthews, J., Croft, T., Lawson, D. & Waller, D. (2012) Evaluation of Mathematics Support Centres – A Review of the Literature. sigma, Coventry: Coventry University. Accessed via http://www.mathcentre.ac.uk/resources/uploaded/52487-evaluation-of-msc-7.pdf (April 2014)

Merriam, S.B. (2009). *Qualitative Research: A Guide to Design and Implementation,* San Francisco: Jossey – Bass Publishers.

Mathematics Learning Support Centre- Aims [online] Accessed via http://www.maths.dit.ie/mlsc/aims.html (April 2014)

Ní Fhloınn, E. (2009) The role of student feedback in evaluating mathematics support centres. *CETL-MSOR Conference 2009 Proceedings, 94–98,* Accessed via http://www.mathstore.ac.uk/headocs/Proceedings_2009_Upload_0.pdf (April 2014)

Organısatıon For Economıc Co-Operatıon And Development (OECD). (1999) Measuring Student Knowledge and Skills: A New Framework for Assessment. Paris: OECD.

Perkın, G. & Croft, T. (2004) Mathematics Support Centres – the extent of current provision. MSOR Connections, 4 (2) 14-18.

Perkın, G., Lawson, D. & Croft, T. (2012) Mathematics Learning Support in Higher Education: the extent of current provision in 2012. sigma, Coventry: Coventry University. Accessed via http://www.mathcentre.ac.uk/resources/ uploaded/52789-mls-in-uk.pdf (April 2014)

Symonds, R. (2008) Evaluating Students' Engagement with Mathematics Support. PhD Thesis. Loughborough University.

Application of Instructional design in a b-Learning course of mathematic for engineers.

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Abstract

The principles of instructional design have been developed in order to create a virtual learning environment, according to pedagogical models during the preparation of the course. This work intends to analyze a virtual learning environment course based on the set of 8 instruction design principles to distance education. The instruction design are available in Calculus I course designed for 1st year students of Electrical and Electromechanical Engineering in the Coimbra Institute of Engineering. The major purpose of this environment is to innovate the process of teaching and learning, exploring technologies as pedagogical forms, in order to improve students' motivation and increasing the levels of success in mathematics.

Introduction

With the development of new technologies in distance education, virtual learning environments have emerged, new demands of teaching materials for the courses and a new generation of students. Most courses in virtual learning environments do not explore their potential and use the features in an appropriate way (Burgstahler (2007)). The virtual content can be made available in any way? Are there some technical support for the development of activities in virtual environments? What methods are most appropriate in the preparation and planning of such content and activities? The resources used appropriately can guarantee participants new experiences and better learning outcomes (Camargo et al. (2010)). In this sense, instructional design (ID) principles have been developed, in order to generate virtual learning environment, according to pedagogical models during the course preparation (Dabbagh and Bannan-Ritland (2005)). Based on the universal instructional design principles (Conell et al. (1977), Burgstahler (2001) Scott et al. (2002)) a set of eight principles tailored to distance education were identified: (1) Equitable use; (2) Flexible use; (3) Simple and intuitive; (4) Perceptible information; (5) Tolerance for error; (6) Low physical and technical effort; (7) Community of learners and support and (8) Instructional climate.

This work intends to analyze a virtual learning environment course of Calculus I. The course was designed for 1st year students of Electrical and Electromechanical Engineering in the Coimbra Institute of Engineering. The students involved have different math skills and different ages. They have some knowledge of computers but it is the first time that they use a computer for learning with virtual tools. The course was organized to be applied in b-Learning mode, as a complementary activity to the curriculum unit based on eight principles of instructional design.

Instruction design project

The course was organized as a complementary activity in the course of Calculus I for Electrical and Electromechanical Engineering. The ID project of this course was designed by the teachers responsible Cristina Caridade and Maria do Céu Faulhaber. Figure 1 represents some activities of the virtual learning environment of the Calculus I course.

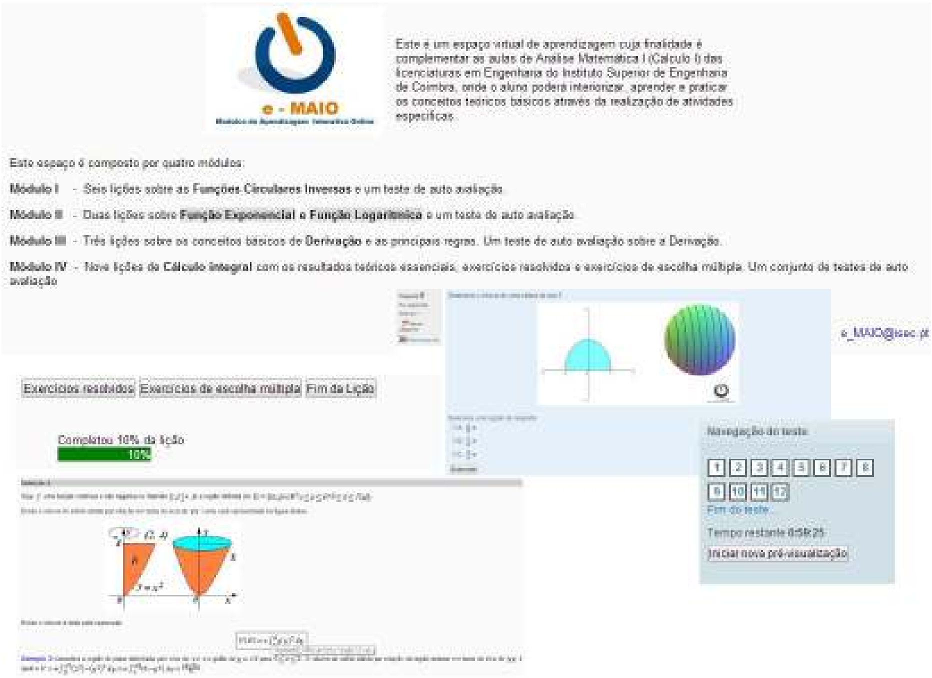

Figure 1: Virtual learning environment of Calculus I course.

Analysis of the course

The overall objective of the course is to teach the essential bases of Calculus I. The specific goals are to enable students to identify the importance of mathematics in engineering. Also, to familiarize the student with the tools of information technology currently used and motivate student during the learning process.
Table 1 presents the main characteristics of the target audience of the course based on Franco (2007).

Characteristics	Description
Students	First year of Electrical and Electromechanical Engineering
Number of students	68
Mean age	23.79 (from 18 years to 55 years)
Motivation	Students already know the importance of information systems in engineering.
Computer literacy	Some domain in computer use.
Support needed	Access to computer with internet, chat, forum, e-mail.
Location	At home and/or school.
Dedication	The student must maintain at least two hours a week for achievement of course activities.
Communication between the group	By email, forum and chat, but mostly in classes.
Communication with the teacher	The virtual learning environment and by personal contact during the classes.

Table 1 – Information about the main characteristics of the students.

During the 14 weeks of class, students will have 14 lectures and 14 practical sessions, one each week, performing activities that correspond to 4 hours per week, i.e. 46 hours in total. The syllabus is divided into 4 units: Trigonometric and hyperbolic functions; Derivatives; Antiderivatives and Integral calculus and its applications.

During the course, the students conducted the evaluation activities in different ways through diagnostic testing, training tests, and evaluation tests in class and at home. This type of summative evaluation (multiple choice questions), formative evaluation (solving problems and exercises) and diagnostic evaluation (tests which allow the teacher diagnose and identify the level of knowledge the student) promotes a critical analysis of the course objectives.

Design instructional recourses

The course identifies a set of 8 universal instruction design principles appropriate to distance education (Elias (2010)):

(1) *Equitable use:* The course is accessible to students with two different engineering characteristics (Electrical and Electromechanical Engineering) and can be accessed anywhere. All students have the same resources.

(2) *Flexible use:* The contents of the course are presented in different ways (lessons, tests, images, videos, files, training and multiple choice exercises) providing students the choice of learning methods.

(3) *Simple and intuitive:* The design of the course interface is easy to understand, regardless of experience knowledge and technical skills of the student.

(4) *Perceptible information:* The design communicates effectively all necessary information for the student to become active with reflective and critical thinking.

(5) *Tolerance for error:* The design minimizes the risk of accidental or unintended actions.

(6) *Low physical and technical effort:* The design can be used efficiently and comfortably, either in the classroom or at home.

(7) *Community of learners and support:* The learning environment promotes interaction and communication among students and between students and teachers through forums, e-mail or in the classroom, which can be a bridge for some doubts that may exist. The motivation of students during the course can be measured by their participation or attendance, the work conducted and their marks.

(8) *Instructional climate:* Comments and feedback are via email as well as during the class.

There are some features of instructional design that can facilitate the understanding of the course, from its planning to its evaluation by everyone involved. The first feature is presented in Activities Map. The Activities Map is represented in Table 2, for the specific example of applications of the Integral calculus and its applications unit. This table gives an idea of the planning of theoretical and practical activities provided during the course.

Lesson	Unit	Objectives	Theoretical activities (b-learning course)	Practical activities (b-learning course)
Lesson 1	Definite Integral.	- Know the basic concepts. - Understand and know how to use the concepts. - Distinguish the different concepts. - Make sure the concepts are well used.	- Presentation of concepts and some illustrative examples in lesson activity.	- Resolution proposed exercises. - Resolution of a diagnostic quiz with multiple choice, online. - At the end of the unit an evaluation test.
Lesson 2	Definite Integral: Applications to calculating the area between 2 curves.			
Lesson 3	Definite Integral: Applications to calculating the volume between 2 curves.		- Presentation of a video. - Presentation of concepts and some illustrative examples in lesson activity.	
Lesson 4	Definite Integral: Applications to calculating the length of a curve.		- Presentation of concepts and some illustrative examples in lesson activity.	
Lesson 5	Indefinite Integral.			

Table 2 – Activities Map for applications of the definite integral unit.

Figure 1 shows a Storyboard that complements the Activities Map (Filatro (2008)) as a graphical outline, showing the sequence of activities that students must follow in each lesson to perform the requested tasks. This example represents the third lesson of unit 4, Applications to calculating the volume between 2 curves.

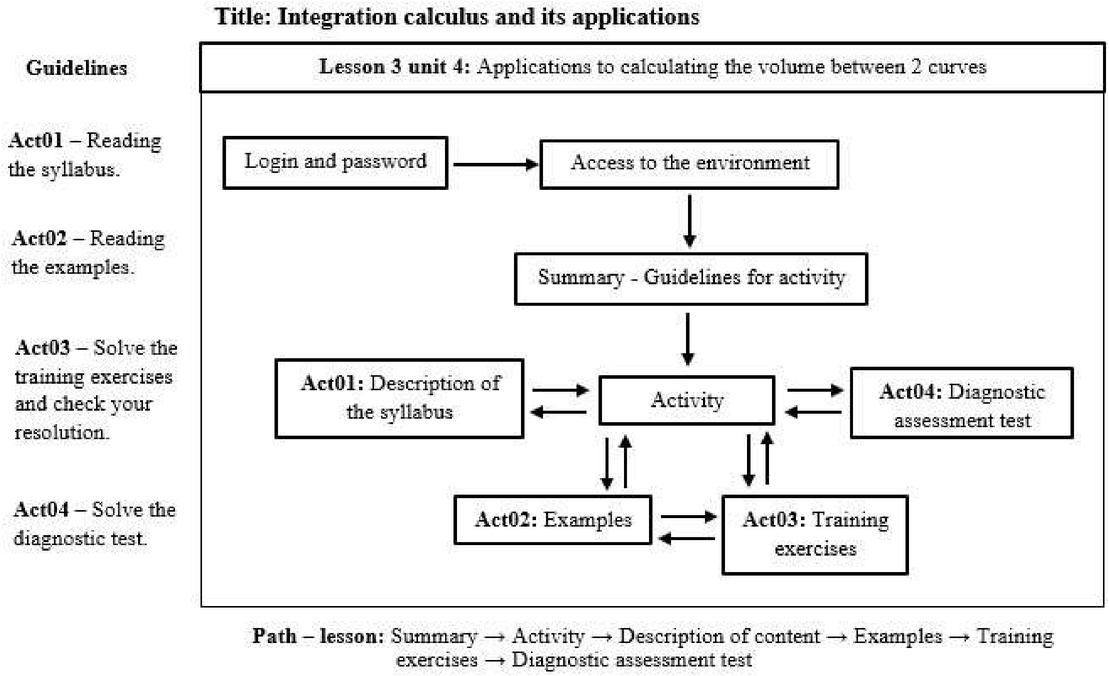

Figure 1: Storyboard Lesson.

Finally, in Table 3, it is presented the Instructional Design Matrix used to indicate in detail the activities during the course. This matrix should indicate date to be executed, its objectives, the students work and the evaluation criteria.

Identification the activity	APPLICATIONS OF THE DEFINITE INTEGRAL Area between two curves
Description	Students should read the text of the activity and follow the path indicated throughout the lesson.
Objectives	Compute the area between two curves with respect to the x and y axes.
Evaluation criteria	Consistent and well organized Answers.
Interaction with teacher	The teacher helps the student to define his learning path.
Date (duration)	1 week.
Tools	Computer, internet and Moodle.
Supporting Content	The text from the lesson activity of Moodle.
Student work	The student must do the activity on time.
Feedback	The activity will be discussed and evaluated in the next week.

Table 3: Instructional Design Matrix.

Development environment of the course

The development environment of the course used the following steps: Analysis, Design, Development, Implementation and Evaluation giving emphasis on training and development of skills such as the encouragement of autonomy, meaningful learning and critical approach.

Development phase

In planning the course, the analysis seeks to detail the development of the project considering several important aspects that serve as solid foundation for the success of

the course, in their preparation and their integration into the Moodle environment (Moodle 2014). The project plans to implement it as a complement to the course in order to enhance the learning of new skills and knowledge.
In this stage it was proposed to have activities using the various tools and media, in order to motivate the student and cover the potential that the virtual environment provides. The course is developed with the virtual learning environment Moodle, which has the necessary tools to create tasks that enable collaborative learning and interactivity learning between students and teachers.
The project aims to support students in the learning process, allowing the student to be an active, reflective and critical subject, participant research, inferring and testing hypotheses, interacting with peers and teachers in building their knowledge.

Implementation phase

The project can be accessed by students in their homes, places of work, study rooms and computer labs during classes. Teachers should check the constant motivation of the student, checking the progress of activities, participation in discussion forums, email or in classes. So, communication with students will take place in various ways.

The course includes three types of student assessment: diagnostic, formative and summative. The diagnostic evaluation at the beginning of each unit, allows to identify the knowledge that students have about the content. To assess formatively, the assimilation of the contents by the student, there are activities, exercises and assignments that students perform during the training. At the end of each unit, summative evaluation is done through exercises and tests.

Evaluation phase

In the end, the course is assessed in order to study the possibility of continuing the project and the improvements that will be required to implement in the course. The first analysis refers to filling by the students of the questionnaires that were applied at the end of the course. The motivation of students during the course can be measured by their participation and frequency, the work conducted and the marks obtained. The negative aspects indicated by the students in the questionnaire should be checked if problems are isolated or are the problems of most students in the class.

In the evaluation phase, the teaching methods and techniques used during the course are analysed by teachers in order to assess whether the pedagogical theories used in the course helped students to build their knowledge.

Conclusions and future work

Can these methods/resources in a virtual environment facilitate students' learning in a mathematics course for engineers? And will they allow plan and evaluate with a solid foundation student success? The results obtained by the students and inquiries made at the end of the course can convey more clearly these doubts and help us to integrate the technical tools of virtual learning more efficiently.

The students accessed the b-learning course on a voluntary basis, mostly more than 2-3 times per week (30.8 % 2-3 times and 23.1 % daily), which shows the interest and motivation for this type of environment learning (Caridade and Faulhaber (2013)). Although the course is used as a system of b-learning education, most students still prefer classroom learning (60%) than b-learning. Regarding working and teaching organization, students agree that the activities, examples, exercises available and proposed tests are useful for their learning. However, even the students who prefer classroom learning agree that activities are relevant in their teacher and learning process. With respect to the benefits of the project, students agree that it is clear and requires little effort and can be used anywhere and anytime. The access is quick, simple and easy and is always available. The main advantages from the point of view of students is the fact that this is available anytime and anywhere; be an incentive to study and problem solving; be a complement to regular classes, facilitating their monitoring and be quick and easy.

As future work it is necessary to analyze data obtained during this school year with a larger number of students in order to identify the benefits and drawbacks to improve the teaching/learning process. It is also of great interest to extend the project to other mathematical courses such as Calculus II and Linear Algebra from other engineering in Coimbra Institute of Engineering.

References

Burgstahler, S. (2001) Equal Access: Universal design of instruction methods. DO-IT. University of Washington. Accessed via www.washington.edu/doi/Brochures/PDF/equal_acess_udi.pdf

Burgstahler, S. (2007) Universal design of instruction: Definition, principles, and examples. University of Washington. Accessed via www.washington.edu/doi/Brochures/Academics/instruction.html

Camargo, F., Holland, M., Montrezol, A. and Nogueira, M. (2010) Trabalho de Pós-graduação em Design Instrucional Ambientes Virtuais 3D. *Centro Universitário SENAC*, São Paulo, Brasil.

Caridade, C.M.R. and Faulhaber, M.C. (2013) B-leraning no ensino da matemática para engenheiros, *Challenges 2013*, 855-872.

Connell, B. Jones, M., Mace, R., Mueller, J. Mullick, A., Ostroff, E. (1997) The principles of universal design. Accessed via http://www.design.ncsu.edu/cud/about_ud/udprincipleshtmlformat.html

Dabbagh, N. and Bannan-Ritland, B. (2005). *Online learning: Concepts, strategies, and application*. Upper Saddle River, NJ: Prentice Hall.

Elias, T. (2010) Universal Instructional Design Principles for Moodle. *International Review of Research in Open and Distance Learning*, 11 (2): 110-124.

Filatro, Andrea (2008) *Design instrucional na prática*. São Paulo: Pearson Education.

Franco, L.R. et al. (2007) Abordagens Teórico-Pedagógicas de cursos de EaD. Livro Digital. Curso de Design Instrucional para EaD Virtual. Itajubá: UNIFEI.

Horto, W. (2012) *E-learning by design* (second edition). Pfeiffer Publishing.

Moodle (2014). Accessed via https://moodle.org/

Scott, S., McGuire J., Shaw, S. (2003) Universal design for instruction: A new paradigm for adult instruction in postsecondary education. *Remedial & Special Education*, 24(6):369-379.

Benefits of cross modular tutorials for first year students

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Abstract

Given the increasingly diverse student body now entering third level, there has been an increased focus on student retention. At the School of Computing and Intelligent Systems cross modular tutorials with smaller class sizes have been introduced in an attempt to address the issue. In each session there is material from Engineering Mathematics, Programming and Electronics modules. Following employment of these sessions students have welcomed the opportunity to be able to engage more in a smaller group in terms of participating in discussion. Preliminary evidence showed that students felt that the re-enforcement of topics covered in the large lecture classes really helped them to gain a better understanding of the subjects. The integration of all modules in the one tutorial also gave them a better overall understanding of the learning objectives and outcome of the course. In addition this has proved successful in giving lecturers taking the large groups valuable feedback on student understanding.

Introduction

This paper explores a speculative initiative into teaching and subsequently evaluates the effectiveness this had on student engagement and retention. Figures show in the UK in 1997 that only 14% of lower social classes (CVCP, 1999) participated in higher education, thus the Government introduced a strategy for widening participation. The aim of this policy was to enrol students from disadvantaged backgrounds (HEFCE, 2001). The target of getting 50% of all 18-30 year olds in the UK to participate in higher education reached 38 % by 2013 (HEFCE, 2013). Students from these groups tend to need more support to successfully complete their studies (National Audit Office, 2007). As these students may be used to a very different learning style (HEFCE, 2001) it may require a rethink on how higher education should be delivered, i.e. a move away from the traditional lecture model (HEFCE, 2002). A study by Blanden and Machin (2004) showed that a considerable gap exists between students from advantaged and disadvantaged backgrounds dropping out of university in the first year of study.

Separating the link that exists between widening access and retention is not feasible due to the disparate student population, resulting in students arriving at university from the less traditional routes and finding it difficult to fit in with the traditional methods of delivery of courses (National Audit Office, 2007). In addition students who enter higher education from non-conventional routes, e.g. GNVQ, have different expectations about the type of teaching and support they will receive. These expectations are based on their previous experience of teaching methods. The high failure rate in Mathematics in first year appears to be as a result of the perceived lowering standards of A Levels, a reduction in entry requirements on some courses with a strong mathematical component and the wide-ranging educational backgrounds of many of the students (Bamforth 2007).

Information gathered from surveys carried out by the School of Computing and Intelligent Systems (SCIS) at University of Ulster during enrolment show that there is an increase in the number of students arriving from different routes, e.g. GNVQs.
For some time now management at colleges and universities have become increasingly concerned by the rising number of students either leaving or failing to progress in their studies. As a result retention and engagement have become a major priority and in an attempt to address these issues an initiative of providing cross modular tutorials with smaller numbers to all first year students in SCIS has been implemented. These tutorials were delivered to small groups with a maximum of 15 students in a group. Consistency was maintained by using the same member of staff to deliver all the sessions.

During the course of the semester evaluation took place, using informal discussions and student and staff questionnaires. The tutorials were subsequently implemented across all the first year courses within the school and the effectiveness.

Literature Review

The first year experience is crucially important in retaining students (zepke, 2013). Many universities have addressed this with the employment of some of the following initiatives such as

- Drop-in-centres
- Developing VLEs (virtual learning environment)
- Encouraging social engagement
- Put the best teachers on first year modules

Mathematics Support Centres (MSCs) have been established at universities in the UK and number of other countries. There is a growing body of research examining the operation and impact of MSCs. Evidence shows that these type of centres do improve retention (Matthews, 2013).

The use of VLE's, such as blackboard or Moodle, can help students from a wide range of backgrounds to cope with the demands of higher education. Use of the VLE is compulsory for all students. Through the use of quizzes and tests students are able to assess their individual progression. Students can work at a pace that suits them and revisit material as often as necessary (Vogel, 2010).

According to (Andrews, 2012) social engagement is important in relation to student retention. Teaching and learning can be made more interactive and social. This may be achieved by using more group work and by creating group study areas where students can work in collaboration with others. Peer teaching/support can also promote engagement; social media can also play a part in encouraging social interaction.

Research indicates that the best teachers should be allocated as coordinators for first year modules. This is a practice that has been adopted by the authors' school.

The Problem In the School of Computing and Intelligent Systems (SCIS) there have been concerns at the numbers of students failing to progress or dropping out from their studies as well the lack of student engagement. This was particularly problematic among first year students and had an effect for the subsequent years. The figures for first year students are usually higher than for the following years, and generally peak in the first semester. Table1 shows the first year retention and progression figures after the June 2008 exam board for the school. This table shows that only 71.8 percent of first years progressed to second year.

%	June exam board	August exam board
Early leavers	13.6	-
Failed outright	7.6	9.8
Required to repeat year	3.0	4.8
Re-sit elements	20.5	-
Progress to Year 2	55.3	71.8

Table 1. First year retention figures after the June and August exam board, Staff Development, UU

Students withdraw from courses due to a variety of reasons, e.g. personal, financial, institutional, or course related, some of which the college has no control over. Consequently students find it difficult to manage their time and settle into their studies.

In SCIS the main reasons for high attrition rates include lack of attendance, engagement and motivation. Students turning up for class not prepared i.e. no notes, books etc. was also cited as a reason. A diverse student cohort and students finding it difficult to make the transition from school to higher education also contribute the retention problem and the lack of A level mathematics as an entry requirement for the courses also adding to the problem.

Some of the reasons given for non-completion could be addressed by providing prospective students with clearer information on what is expected from them prior to arriving at university so that they are in a better position to make a more informed choice. While other reasons could be addressed by providing better induction and support to the student once they start a course. The wide gap in students' ability would suggest that lecturers needed to reconsider the methods they use when teaching as the traditional lecture model does not suit all students, particularly those students entering university from the vocational routes.

In the School of Computing and Intelligent Systems the modules that presented the greatest difficulty for students were those on which computer programming or mathematics is a major component. The difficulty was trying to find methods in which the numbers of students failing were reduced while at the same time maintaining and improving standards. Following consultations of all staff at school board meetings it was agreed that additional support was needed in first year in order to get them through the transition period successfully.

Statistics show that first year computer science students at SCIS shows that there is a recurring problem of 'high' failure rates within computer programming modules. This may be because for many of these students this is their first encounter with writing algorithms or computer programs. Mathematics also showed high failure rates perhaps as only GCSE mathematics is required for entry to the courses.

Additional support was implemented by introducing cross modular small group tutorials for students to enhance the retention rates. The idea was to supplement the mathematics and programming modules with extra questions covering the fundamentals and split large classes in groups of 10-15 students. The same tutorial would be delivered to each group and taught by the same person thus ensuring consistency. With approximately 150 students taking the module there is the potential for the tutorial to be delivered up to ten times weekly.

Design and Implementation

After reviewing the retention rates for the 2008 academic year, it was agreed that the first year cohort would be split into groups of between ten and fifteen students and each group would attend a cross-modular one hour tutorial which would typically be prepared by lecturers from each of the first year modules for each of the two semesters (McCourt,2010). Tutorials are set up so that fundamentals from each module are re-enforced with a question from each module in the tutorial each week typically consisting of three questions. The module co-ordinators decide on the topics which require more re-enforcement. The material in the tutorials runs one week behind the material that is being delivered in the lectures. Each question was designed so that it would take the student roughly twenty minutes to complete. The tutorials are taught by the same member of staff which ensures consistency and that all students are getting the same benefit from the tutorials. A record of student attendance is taken at the beginning of each class so that attendance can be monitored. As the group sizes are smaller it means that each student's attendance, contribution and understanding of the topics is monitored, it also provides an opportunity to record the names of students who are having trouble coping with the material. Upon identifying students who are struggling they are subsequently offered individual support in the form of one to one sessions. These sessions also helped to identify probable cases of students with high risk of dropping out at an earlier stage and facilitate remedial action to be taken.

In addition a first year teaching team was established which is made up of the course director, the staff delivering the tutorials and the lecturers of the first year modules. The first year teaching team meet on a weekly basis to discuss student's general attendance and progression. Students who miss timetabled classes on a regular basis or who are struggling are highlighted at these meetings.

Printed copies of the tutorial questions are distributed at the start of the class, the questions have not been seen by the students before class and are not available online. The tutor briefly explains what is expected from the questions and which notes need to be used to do the questions and gives instructions on how the questions are to be completed. Students are encouraged to work with their peers and discuss solutions with each other. While the students worked on the questions it gives the tutor an opportunity to engage with students to provide advice and direction and to answer any questions they have or to explain material they don't understand which was covered in the lectures or practical. Towards the end of the tutorial when everyone has attempted to complete the questions, the solutions to the questions are considered and discussed by the whole group and worked out on the white board. While members of staff delivering the tutorials are provided with both the questions and model solutions for each of the topics, it often happens that students have used different methods and alternative solutions; this is especially true for the programming and mathematics topics. This is encouraged as it proves that there is no one correct method to complete many tasks, and leads to further discussion which is the most efficient answer.

Towards the end of the semester the focus of the tutorials shifts towards the exam revision and exam techniques. The students are anxious to get practice doing exam questions so they can understand marking criteria and what they are expected to do in the written exam. This was done as feedback from staff/student consultative meetings suggested that some students had limited experience of timed written exams. This lack of exam experience for some students was due their vocational backgrounds. In order to provide this support sample exam questions and solutions with a marking scheme were prepared for students. Student feedback was positive on this initiative. They believed that it was highly valuable as it gave them the experience on how to provide an answer that could maximize the mark achieved.

Data Analysis

Formative evaluation about the tutorial sessions is conducted on an ongoing basis; this is carried out through casual conversations at the end of class both with individual students and groups of students and from feedback from student/staff consultative meetings.

Summative evaluation is performed by using questionnaires. Closed questions are used wherever possible in order to facilitate analysis. A free response question is also used so that students are given the opportunity to highlight any concerns they have or suggest what they would like to see improved. The questionnaire replies are anonymous so that students are not inhibited and give open and candid responses. In order to maximise the number of responses the questionnaire is distributed to the students during a lecture when they are in large group, in the last week of the semester.

The first year teaching team meet on a weekly basis to discuss the overall student attendance and performance of the first year cohort. The success of the cross modular tutorials is measured by using pass rates as at the end of the semester as well as gaging student engagement and performance in weekly tests which are used typically for mathematics and contribute to the coursework mark. Students reported that the tutorials helped to improve their marks in weekly class tests. This is especially true for mathematics where a relationship exists between the mark achieved and attendance . In general when a student misses a session the mark for that particular week is lower than when they have attended. However exceptions to this rule will also be true. Attendance figures also indicate that capable students will attend everything that is put on for the group but sadly some students that really need to attend do not. If a student is seen to be struggling in weekly class tests they are contacted personally and advised that an improvement in attendance is compulsory. There are also students who attend all sessions but still struggle with weekly tests, students can be identified in the small groups as their lack of understanding is evident when they attempt questions. These students are subsequently offered personal tuition where the tutor spends doing supplementary examples and encouraging the student to also attempt other examples. They tend to feel less threatened when receiving one to one assistance and make good progress.

Quantitative evaluation is carried out by examining the improvement in retention rates over a 4 year period, the tutorials have been running since 2009 and the difference in retention from 2009 to 2013 rates are used to measure the success of the cross modular tutorials as this is initiative was introduced to combat the problem

Survey results

On average each year roughly 80 responses are received, below is an analysis of the responses. Reasons given for non-attendance range from ‘did not feel the need’, to class times not being suitable and admissions of just being ‘too lazy’. In response to complaints regarding times a huge effort has been made to make sure tutorials are on at a time when students are already on campus for other classes.

Over three quarters of students either agreed or strongly agreed that they felt more at ease to participate in the discussions and ask questions in small groups. One of the main reasons being that they were known in first name terms by the tutor and felt more involved and feel that their progress is being monitored.

Students felt the personal help they receive is one of the main advantages as the tutor had more time to spend with individual students and this provides reassurance when they are finding any particular topic difficult. They also felt that examples used re-enforced topics already covered in lectures making it easier to understand complex topics.

Mature students expressed apprehension about returning to education and were worried that they would experience difficulty keeping up to date on work but felt the tutorials helped them to settle into their studies due to the nature of the one-to-one help they received.

They also felt that they got to know other people in their class better, as in the large groups people find it more difficult to get to know each other, the tutorials gave them the opportunity to communicate with people a more informal environment.

Overall the students felt that by having cross modular tutorials that they could see the links between modules and have a better overall understanding of the course. |In particular they could see the value of doing mathematics as it links into the other modules. Some students felt the tutorials should last at least another half hour longer and that they would like to see the same thing offered in second year, however at present staffing constraints would allow this.

Resulting Retention Rates

Table 2 below shows the progression rates for the Faculty of Computing and Engineering over a four year period. These statistics were provided by the staff development unit. Upon examination of the retention rates of the period from 2009 to 2013, it can been seen that the attrition rate has decreased at a steady rate to achieve an overall attrition rate of 15% in 2012-2013. This is a vast improvement from the statistics in 2008 which show that only 71.8 % of students progressed from year 1 to year 2. This improvement can be attributed in part to the success of the cross modular tutorials. Given the overall positive response from students and the resulting improved retention rates it can be seen that the small group tutorials have a significant role in both the student experience and student progression.

Comp & Eng	Early leaver & non-returner %	Proceed %	Fail %	Repeat %	Attrition %
2012-13	7.23	77	7.9	4.3	15.1
2011-12	9.7	76.0	5.2	6.0	14.8
2010-11	11.4	71.5	6.9	7.2	18.3
2009-10	16.2	67.6	6.5	7.2	22.8

Table 2. First year retention figures for faculty of Computing and Engineering from 2009-2013, Staff Development, UU

Conclusions

In an attempt to address the problem of high attrition rates cross modular tutorial were introduced in the School of Computing and Intelligent Systems at University of Ulster.

Attendance was monitored and recurrent absenteeism was immediately highlighted by the tutor and following no immediate improvement the matter was passed onto the courses coordinator for a follow-up. Students were informed of how their attendance was essential in order to successfully complete the course, attendance improved as students were receiving support on an individual basis. Students reported that as a result they felt they were getting a better understanding of complex topics.

Motivation and engagement improved as students felt that the tutor was recording progress and they were not being left to their own devices. They also felt that seeking help and asking questions was easier due to the nature of the small groups. The informal atmosphere during the sessions improved engagement. Weaker students could be easily identified and therefore provided additional support to those students. Working in smaller groups also allows more involvement with fellow students which greatly improves engagement.

The tutorials were devised such that a focus was placed on the previous weeks material and reinforce the topics in an informal setting. An improvement was noted upon examination of the student scores for weekly mathematics tests for students who engaged in the tutorials.

Students also stated that these sessions help them make new friends among the group. This was an important additional benefit that was not originally considered. The relationship that developed between the tutor and students meant that they were much more likely to seek help if they were experiencing problems in the understanding of complex topics.

Figures over the four year period showed that attrition rates have been reduced to 15% from 28% which shows a massive improvement over a relatively short space of time. However it must be noted that the improvement cannot be attributed to the tutorials alone but also on the additional one-to-one support received by students and also awareness by lecturing staff on the issued faced by students. Increasing retention is a worthwhile objective but it requires a significant effort and commitment staff to tackle the needs of a diverse student body.

References

Bamforth, S. E., Robinson, C. L., Croft, T. and Crawford, A. (2007) "Retention and progression of engineering students with diverse mathematical backgrounds."Teaching Mathematics and its Applications, Vol 26 (No. 4): pp156-166.

Blanden, J. and Machin, S. (2004) Educational Inequality and the Expansion of UK Higher Education. Scottish Journal of Political Economy, 51 (2), pp. 230-249 doi:10.1111/j.0036-9292.2004.00304.x

CVCP (1999) Briefing Note: widening participation. London: Committee of Vice-Chancellors and Principals.

HEFCE (2001) Strategies for Widening Participation in Higher Education – A Guide to Good Practice. London: HEFCE

HEFCE (2002) Funding for Widening Participation in Higher Education: New Proposals 2003-2004 to 2005-2006. London: HEFCE

(HEFCE,2013) New HEFCE data show how higher education participation has changed over time and by region. *http://www.hefce.ac.uk/news/newsarchive/2013/news83564.html*

National Audit Office (2007) Staying the Course: The Retention of Students in Higher Education. London: National Audit Office.

Vogel, M. (2010) Using VLEs effectively.
www.rsc-london.ac.uk/cms/fileadmin/images/events/Tools_Technology/toolsntech_jisc.ppt accessed January 14 2010.

(McCourt, 2010) Improving student engagement and retention through small group tutorials, *ITALICS Volume 9 Issue 1 February 2010 ISSN: 1473-7507*

(Andrews, 2012) Andrews, A., Clark, R. and Thomas, L. Compendium of effective practice in higher education retention and success.
http://www.heacademy.ac.uk/assets/documents/what-works-student-retention/What_Works_Compendium_Effective_Practice.pdf [27 June 2012].

(Matthews, 2013) Evaluation of mathematics support centres: a literature review Janette Matthews, Tony Croft, Duncan Lawson and Dagmar Waller *Teaching Mathematics and its Applications(2013) 32, 173-190.*

(Zepke,2013) Student engagement: A complex business supporting the first year experience in tertiary education ,The International Journal of the First Year in Higher Education Volume 4, Issue 2, pp. 1-14 August 2013.

Interpreting A-level Mathematics grades – as easy as A, B, C?

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Abstract

Many concerns have been expressed that students' basic mathematical skills have deteriorated during the 1990s and there has been disquiet that current A-level grading does not distinguish adequately between the more able students. This study reports the author's experiences of teaching maths to large classes of first-year engineering students and aims to enhance understanding of levels of mathematical competence in more recent years. Over the last four years, the classes have consisted of a very large proportion of highly qualified students – about 91% of them had at least grade B in A-level Mathematics. With a small group of students having followed a non-traditional route to university (no A-level maths) and another group having benefitted through taking A-level Further Mathematics at school, the classes have contained a very wide range of mathematical backgrounds. Despite the introductory maths course at university involving mainly repetition of A-level material, students' marks were spread over a very wide range – for example, A-level Mathematics grade B students have scored across the range 16 – 97%. Analytical integration is the topic which produced the largest variation in performance across the class but, in contrast, the A-level students generally performed well in differentiation. Initial analysis suggests some stability in recent years in the mathematical proficiency of students with a particular A-level Mathematics grade. Allowing choice of applied maths modules as part of the A-level maths qualification increases the variety of students' mathematical backgrounds and their selection from mechanics, statistics or decision maths is not clear from the final qualification.

Introduction

Traditionally, students studying for an engineering degree at a UK university will enter university directly from school where they will have studied three subjects (A-levels) in their final two years. These subjects often include mathematics and physics. Our university selects students prior to enrolment on their degree programme by considering their A-level grades against published thresholds.

The current system of grading A-level mathematics has been criticised for failing to permit effective differentiation between the more able students (MEI, 2012). For example, 49% of A-level Mathematics students in Summer 2013 were awarded either grade A or A* (CCEA exam board). Suggestions for improving this situation include making exam questions less structured and predictable to encourage development of problem-solving and other transferable mathematical skills, restricting resitting of individual modules, and reducing the weighting of AS modules compared to A2 modules (MEI, 2012).

Furthermore, a decline, throughout the UK, in students' basic mathematical skills during the 1990s has been described with deficiencies in numerical and algebraic manipulation and simplification, in particular, being evident (Hawkes and Savage, 2000). Diagnostic tests showed that students entering university in 2001 with a certain A-level Mathematics grade had lower levels of competency in basic mathematical skills than students who attained a much lower A-level Mathematics grade ten years earlier (Lawson, 2003). Changes in the A-level maths syllabus and the move to a modular structure, making revision easier, were offered as explanations for this pattern (Todd, 2001).

The current author began teaching mathematics to first-year engineering undergraduates in 2002. This paper reports some observations gained since then with the aim of improving understanding of A-level maths grading, in terms of what can be expected of students, in recent years.

First-year maths – topics and students

Two courses, Mathematics 1 in the first semester and Further Mathematics 1 in the second semester, represent the maths teaching (20 CATS points in total) for first-year aerospace and mechanical engineering students in Queen's University Belfast. First-year maths had been redesigned for the 2001/02 academic year with a new syllabus implemented as previous exam results had been disappointing and changes to A-level mathematics were occurring. Since then, some minor adjustments have been made to Mathematics 1 while the Further Mathematics 1 syllabus has remained largely unchanged apart from being supplemented by the topic of Laplace transforms, previously taught in second year, in 2012/13.

Thus, Mathematics 1 currently involves mainly a repetition of A-level topics – logarithms, polynomial equations, trigonometry, complex numbers, differentiation and integration – although effort has been made to demonstrate engineering applications. Further Mathematics 1 covers differential equations, matrices, vectors, Laplace transforms and descriptive statistics. It is aimed to develop skills in the basics (eg, applying methods for solving differential equations, matrix operations, calculating vector products) in a range of topics and to enhance students' confidence in their mathematical ability.

Figure 1 shows the A-level Mathematics qualifications of the first-year mechanical engineering classes over the last 11 years. (The small number of international students has been omitted when calculating the proportions.) An opportunity for lesser-qualified students to do a preparatory year before joining first year of the degree programme was removed in 2010/11. Furthermore, with the degree becoming increasingly popular in recent years, the A-level entrance qualification requirements have increased.

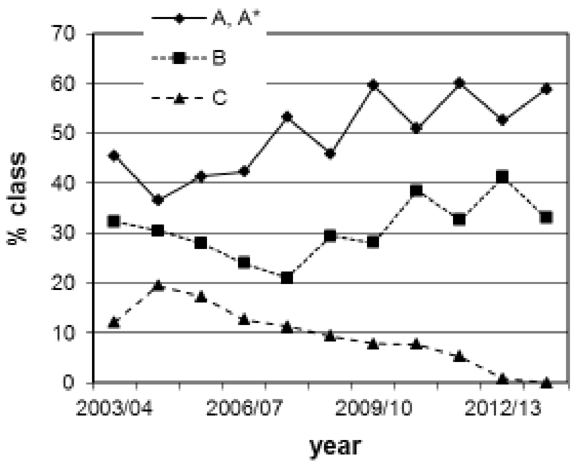

Figure 1. Proportions of the first-year mechanical engineering classes having certain A-level Mathematics grades.

These factors have ensured that classes over the last four years have consisted of a very large proportion of highly qualified students (about 91% of the class had at least grade B in A-level Mathematics) plus a small group (about 5%) who did not study A-levels but followed a non-traditional, alternative route to university (eg, BTEC National Diploma).

Student performance in introductory maths course

The author has been responsible for the introductory course, Mathematics 1, since 2010/11. The class size has increased from about 145 to 200 students over this period. In addition to the small group who did not study A-level maths, another small group (about 5% of the class) has benefitted through taking the more advanced A-level Further Mathematics at school. This means that, overall, a very wide range of mathematical backgrounds exists within the class.

Figure 2 illustrates a moderate correlation between A-level Mathematics grade and performance in Mathematics 1. However, it is surprising that, despite this course being largely a repeat of A-level material, students' marks were spread over a very wide range (for example, 16 – 92% for grade B students, 41 – 93% for grade A students in 2012/13). Some allowance can be made for a few students not being fully engaged, perhaps regretting their choice of degree programme.

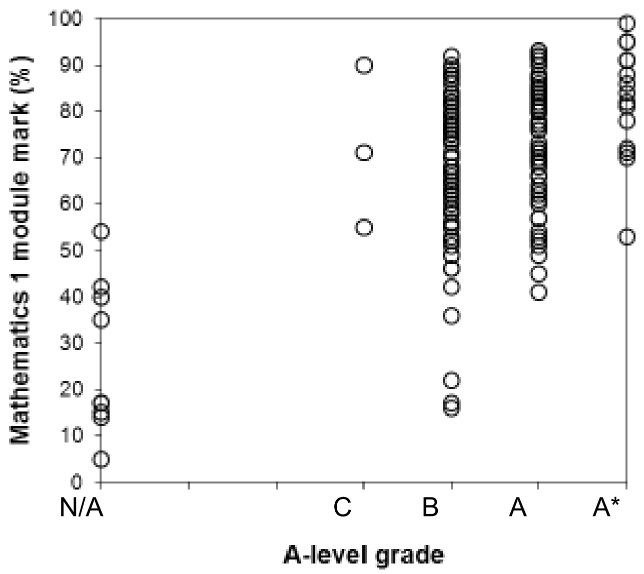

Figure 2. Students' marks in Mathematics 1 in 2012/13 compared with their A-level Mathematics grade.

As expected, grade A* students and those with A-level Further Maths performed very strongly across all topics. Only one exam question in the last four years gave difficulty to this group – use of integration to calculate the centre of mass of a lamina. The grade A students generally performed strongly across all topics but lagged noticeably behind the grade A* students in analytical integration. This topic and complex numbers proved most difficult for the grade B students who lagged noticeably behind the grade A students in these areas. In contrast, there was little difference between grades A and B students in numerical integration (Simpson's rule) and in trigonometry.

Thus, it was very clear that the topic which produced the largest variation in performance across the class was analytical methods of integration. Some students with good grades in A-level mathematics have not mastered certain integration techniques and there is sometimes confusion in taking a rule for differentiating (product rule) and applying it to an integration problem. It was apparent that the A-level students generally performed well in topics in differentiation and there was little variation in this area between students with different A-level grades. Solving triangles and manipulating polynomials were the other topics where there was relatively uniform performance across the A-level group. However, experience suggests that, for most students, some revision of A-level material is necessary and beneficial within first year at university.

In general, students with no A-level maths lagged well behind the rest and performed poorly across most topics. They struggled even with some of the less analytical aspects of calculus (Simpson's rule, finding the stationary points of a cubic function) and their strongest topics were solving triangles and polynomial equations.

Variation in mathematical ability over time

An investigation of first-year mechanical engineering students from 2007 and 2013 involved comparing their performance in identical exam questions in differential equations, matrices and vectors. Thus, with students being required to develop their solutions via a more lengthy analysis, this study has an advantage over diagnostic testing restricted to more basic skills (Todd, 2001). No significant differences in mean marks achieved in the questions by corresponding grade A and grade B students in the two years were observed. This suggests some stability in recent years in the mathematical competence of first-year engineering students with a particular A-level Mathematics grade. Moreover, for a particular year, grade A students generally performed significantly better than grade B students in the questions with the degree of significance being greater in differential equations and vectors compared to matrices. This might be expected given that less analytical processing was required in the latter topic.

A-level mathematics module choice

Interpretation of A-level maths grades is also complicated by the variety of modules available. The A-level Mathematics qualification is based on a compulsory set of pure maths modules (representing two-thirds of the qualification) and a selection of applied maths modules which cover mechanics, statistics and (sometimes) decision maths. This permits up to six different combinations of applied modules and, therefore, two students could have an A-level maths qualification, with the same grade, but have studied different topics. In an extreme example, a student could begin an engineering degree course having the required A-level maths qualification, but without having studied mechanics during A-level maths at school. In highlighting this issue, Hawkes and Savage (2000) noted that the choice of statistics in preference to mechanics provides much less reinforcement of the pure maths. Figure 3 indicates the relative popularity of particular module combinations for all students following the Edexcel exam board in 2006, demonstrating that a mixture of mechanics and statistics was most popular while uptake of decision maths was relatively rare.

A combination of mechanics and statistics was strongly the most common choice of optional modules in A-level maths for aerospace and mechanical engineering students in our university, meaning that only about one quarter of the class had studied mechanics beyond the basic module within school maths. Interestingly, an analysis of student performance in two core, first-year engineering courses, which build on a mechanics foundation, indicated that any benefits for students who studied the extra mechanics at school were small (Cole, 2014).

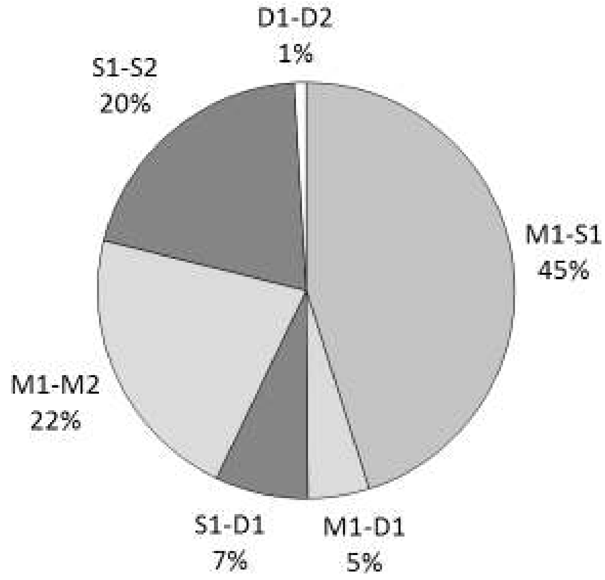

Figure 3. A-level Mathematics module combinations, Edexcel exam board, 2006 (QCA, 2007). D – decision maths, M – mechanics, S – statistics

Conclusions

This paper has reported experiences of teaching mathematics to first-year engineering undergraduates in order to improve understanding of A-level maths grading and what can be expected of students in terms of mathematical knowledge. A student's grade in A-level Mathematics was not a good predictor of performance in an introductory maths course, even though this course involved largely a repeat of A-level material – for example, grade B students scored across the range 16 – 92% in the introductory course in 2012/13. For most students, some revision of A-level material is necessary and beneficial. The topic which produced the largest variation in performance across the class was analytical methods of integration. In contrast, the A-level students generally performed well in topics in differentiation and there was little variation in this area between students with different A-level grades. A comparison of student performance in identical exam questions in 2007 and 2013 suggests some stability in recent years in the mathematical competence of students with a particular A-level Mathematics grade. The opportunity to select modules in applied maths as part of the A-level maths qualification hinders understanding of students' mathematical backgrounds – alongside the pure core, a student can study mechanics, statistics or decision maths, or some combination of these, but their choice is not apparent from their final qualification.

References

Cole, J.S. (2014) A-level mathematics module choice and subsequent performance in first year of an engineering degree. *MSOR Connections*, in press.

Hawkes, T. and Savage, M. (2000) *Measuring the mathematics problem*. Engineering Council, London, UK.

Lawson, D. (2003) Changes in student entry competencies 1991-2001. *Teaching Mathematics and its Applications*, Vol. 22 (4): 171-175.

Mathematics in Education and Industry (MEI) (2012) "How might A Level Mathematics be improved?" Accessed via www.mei.org.uk/files/pdf/Improving_A_Level_Mathematics_final.pdf (11 April 2014).

Qualifications and Curriculum Authority (QCA) (2007) "Evaluation of participation in GCE Mathematics." Accessed via www.ofqual.gov.uk/719.aspx (11 April 2014).

Todd, K.L. (2001) An historical study of the correlation between GCE advanced level grades and the subsequent academic performance of well qualified students in a university engineering department. *Mathematics Today*, 152-156.

Developing mathematics support for first-year engineering students with non-traditional mathematics backgrounds

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Abstract

A maths support system for first-year engineering students with non-traditional entry qualifications has involved students working through practice questions structured to correspond with the maths module which runs in parallel. The setting was informal and there was significant one-to-one assistance. The non-traditional students (who are known to be less well prepared mathematically) were explicitly contacted in the first week of their university studies regarding the maths support and they generally seemed keen to participate. However, attendance at support classes was relatively low, on average, but varied greatly between students. Students appreciated the personal help and having time to ask questions. It seemed that having a small group of friends within the class promoted attendance – perhaps the mutual support or comfort that they all had similar mathematical difficulties was a factor. The classes helped develop confidence. Attendance was hindered by the class being timetabled too soon after the relevant lecture and students were reluctant to come with no work done beforehand. Although students at risk due to their mathematical unpreparedness can easily be identified at an early stage of their university career, encouraging them to partake of the maths support is an ongoing, major problem.

Introduction

Deficiencies in the basic mathematical skills of some students entering higher education have prompted the development of mathematics support strategies (Croft et al., 2009). Maths support can include revision material sent to students just before starting university, lunchtime workshops, a drop-in centre where students can access resources and bring queries, and online maths resources, and it is advised that support needs to be "student-centred" (Croft et al., 2009).

Bamforth et al. (2007) described a voluntary, four-day, preparatory course provided for engineering students whose maths background was believed to be insufficient. Frequent use of additional support over the semester was necessary for success in first-year maths and, while uptake of additional support by students present at the pre-course might have been expected to be higher compared to non-attenders, this was not always so. In general, barriers to accessing maths support include a lack of awareness of the need of support due to low motivation and failing to keep pace with lecture material, students becoming overwhelmed by their workload, and feelings of embarrassment or intimidation (Symonds et al., 2008).

Maths support was found to improve students' confidence in addition to enhancing their skills (Parsons et al., 2011). Useful features of maths support to increase confidence included working in small groups, working with friends, offering achievable tasks to permit success and providing some more challenging problems to encourage students' confidence to grow, while the importance of verbal encouragement was emphasised (Parsons et al., 2011).

In 2012, it was established that at least three-quarters of UK universities had some form of maths support but further work was needed to investigate why some students don't recognise they need support or don't take advantage of the available support (Matthews et al., 2012).

The author of this paper assumed responsibility for an introductory mathematics course for a large class of about 145 first-year aerospace and mechanical engineering students in 2010/11. The majority of the class had followed the traditional route to university, doing A-level study in their final two years at secondary school and gaining a high qualification in maths – about 90% of them had at least grade B in A-level maths. However, there were four students without A-level maths – they had taken the BTEC National Diploma route to university, typically gaining at least 15 distinctions, but they failed this first-semester maths course by some margin. To help new students with similar, non-traditional entrance qualifications arriving in subsequent years, the lecturer desired to put in place a structured system of maths support to help the students' transition to university and enable them to cope with the high level of mathematical content throughout their engineering degree. This paper reports the implementation of a maths support system over the three years from 2011/12.

Introductory maths module – structure and assessment

The introductory mathematics course involves mainly a repetition of A-level topics – logarithms, polynomial equations, trigonometry, complex numbers, differentiation and integration – in order to provide students with a good grounding in a range of fundamental topics relevant to engineering. Teaching occurs over 12 weeks with a two-hour lecture and one-hour exercise class per week. Students attend lectures as a single large group but are divided into smaller groups of 40 – 50 students for exercise classes. Numerous worked examples are included in the lectures and engineering applications help to illustrate the usefulness of mathematics to engineers. In the more informal exercise classes, the students begin working through a sheet of practice questions and are expected to complete these in time for the following week's class. A continuous assessment based on these weekly practice questions encourages engagement with the material and contributes towards 15% of the module mark with the remaining 85% of the total available from the final exam (Cole, 2012). After the weekly assessment, solutions to the practice questions are posted on the university's intranet to allow the students to review their work.

Implementation of maths support system

Each year, at the beginning of the semester, the lecturer established the qualifications of the incoming students by checking the university records. Those students deemed to have a non-traditional mathematics background (no A-level maths) were explicitly contacted by email in week 1 of the semester to highlight the existence of maths support and to invite them to a brief meeting in order to check their maths background. The students generally seemed keen to partake of the support.

The intention was to hold an extra tutorial class for these students each week where they would actively work on the practice questions. There would not be any formal teaching but the students would have one-to-one assistance with their work in an informal setting. Thus, the practice was structured to correspond with lecture topics but students could discuss mathematical problems from other modules also.

The module information presented to the whole class at the first lecture contained the website address for mathcentre, the online resource bank containing formula leaflets, video tutorials and practice exercises (Williamson et al., 2003), while the HELM workbooks (Davis et al., 2005) were also advertised. Furthermore, the university operates the Learning Development Service (LDS) which aims to make academic support available to all students. Those with mathematics difficulties can avail of the drop-in service, make a one-to-one appointment or attend a workshop (typically a 90-minute afternoon or evening class on a specific topic such as algebra, differentiation, integration). In the School of Mechanical and Aerospace Engineering, it is usually left to students to take the initiative in contacting the LDS for assistance.

Thus, the support system demonstrates attributes of good practice (Croft, 2001; Croft et al., 2009; Parsons, 2005; Parsons et al., 2011) – a variety of support existed, the weekly tutorials involved working in small groups with student activity dominating and one-to-one help, while the mathcentre resources and LDS drop-in/appointment/workshop facilities were available at other times.

Evaluation of maths support system

In 2011/12, support consisted of a weekly, one-hour session for two groups containing four aerospace and four mechanical students. The aerospace students met on Tuesdays, 11 am – 12 noon, the mechanical students on Mondays, 10 – 11 am. The module lecturer supervised the groups and a postgraduate student was sometimes assisting also. Student attendance averaged 56% but varied greatly – three students each attended only one of the ten sessions, three came at least nine times (Figure 1). Also, it wasn't until week 6 when two students made their first and only appearance.

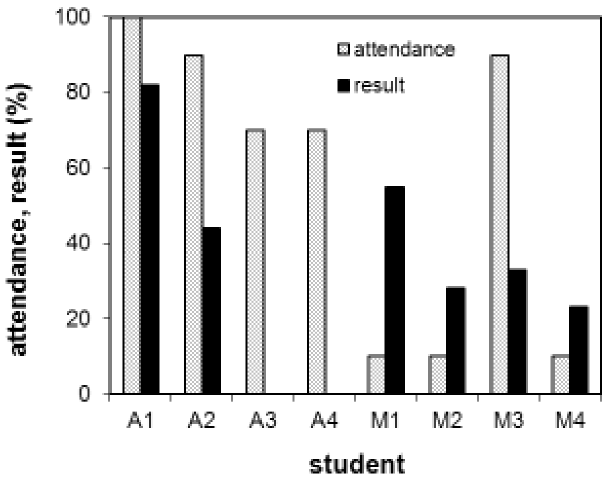

Figure 1. Individual student attendance rates at support classes and module results in 2011/12. aerospace students A1 – A4, mechanical students M1 – M4

It was noted that three of the four mechanical students each came only once but attendance by the aerospace group was high. The aerospace class as a whole was much smaller than the mechanical class, allowing the students to get to know each other well. The four aerospace students worked well together and probably benefited from a mutually supportive environment, knowing they all had similar problems with maths. They worked hard, took an interest in their work and there was some enjoyment of the class. One mechanical student suggested the class would be more attractive if working alongside familiar people.

Module results also varied greatly with only three of the eight students passing and no correlation with attendance at the support classes apparent (Figure 1). Two of the students, who attended reasonably well throughout the first semester, were absent from all of their first-semester exams and withdrew from the university. If students are considering withdrawing, they are entitled to be well informed as to their situation and the likelihood of progressing. Devoting resources to maths support in the first semester of first year is an important contribution in this regard and the high drop-out rate among the eight students is not necessarily a negative outcome.

Students indicated that they found the pace of university lectures much greater than that experienced during their previous education. This emphasises the need to have well-advertised support structures in place at the earliest opportunity to ease students' transition into university and minimise the feeling of being left behind from the beginning. The classes having a basic structure (eg, a set of questions) was believed to be important in establishing a work pattern and it was suggested a homework could be set for the following week. A more detailed analysis of the first year of maths support is provided elsewhere (Cole et al., 2012).

Following this experience, the lecturer thought that more time should be allocated each week to maths support. Therefore, extra maths support sessions were arranged in 2012/13 with each student having two one-hour sessions available each week, one on Monday and the other on Wednesday. In theory, this would allow some practice to be done early in the week, the students could then be given targets in terms of which questions to have attempted by the Wednesday class, and feedback and more practice could continue at this second class. However, average attendance at 22% was much worse than before with only one of the ten students frequently using both weekly sessions and seven students each attending less than one quarter of the classes (Figure 2). Attendance rates for the two days were very similar. Factors discouraging attendance seemed to include embarrassment at not having lecture notes up to date and reluctance to come with no work done beforehand. Again, there was little correlation between attendance and exam performance.

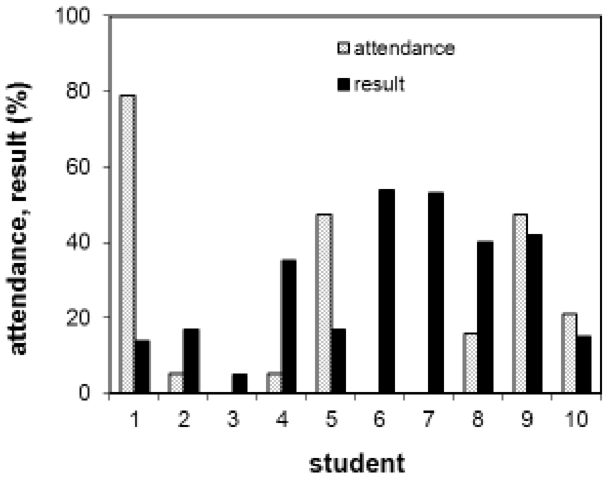

Figure 2. Individual student attendance rates at support classes and module results in 2012/13.

One hour of maths support was available each week (Mondays, 10 – 11 am) to 13 students in 2013/14 and attendance averaged 35% (Figure 3). The lecturer was encouraged by the attendance being relatively high initially and by a few extra students requesting to attend. He speculated whether a critical mass of students was influential in promoting attendance. However, only one student indicated on the evaluation questionnaire that a factor encouraging her attendance was the opportunity to meet others. Students appreciated the individual help, having time to ask questions, and believed the classes boosted confidence. Class timing was very important – many suggested it was too soon after the corresponding lecture – they wanted more time to attempt the questions before coming to the support class.

Two students scored very high marks in the introductory maths module. It was evident that they were very highly motivated and came to the support classes having done a lot of work in advance. The maths support probably contributed only a little to their exam success.

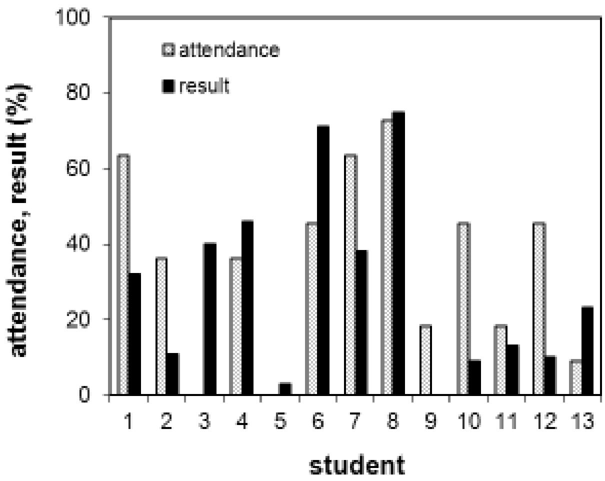

Figure 3. Individual student attendance rates at support classes and module results in 2013/14.

Having developed this system of maths support over three years, some aspects of good practice can be suggested:

- Students believed to have a much weaker mathematical background than the norm were explicitly contacted in the first week of their university studies and informed about the extra maths support. A general announcement to the whole class that support was available might not have had as much impact.

- The classes had an informal, non-intimidating atmosphere with much one-to-one help. There was a familiar face (the module lecturer) present.

- The classes were structured alongside the module teaching schedule and students were active, working through practice questions. The weekly question sheet serves to highlight whether students are keeping pace with the material.

- Grouping students with common backgrounds or similar interests may encourage a mutually supportive working environment.

The time of the support class needs to be considered carefully as students wanted sufficient time after the corresponding lecture to attempt the questions before attending the support class for review of their efforts. Students from non-traditional maths backgrounds generally notice a big increase in the pace and difficulty of the maths in university first year. Nevertheless, encouraging them to engage with the maths support available is a major problem.

Conclusions

This paper has reported the implementation of a maths support system to assist students with non-traditional entrance qualifications as they began the first year of their engineering degree. Extra classes in which students actively worked on practice questions structured to correspond to the main lecture material were held in an informal atmosphere with much one-to-one help. Student attendance was relatively low, on average, but very variable. Students appreciated the personal help, having time to ask questions, and believed the classes boosted confidence.

Working alongside others from a similar background (same degree programme), and perhaps having reassurance that others have similar academic difficulties, might be an important factor promoting attendance. Barriers to attendance included inappropriate timing of the class and there was reluctance to come with no work done beforehand. While students at risk due to their mathematical unpreparedness have been easily identified at the beginning of their university career, encouraging them to partake of the available maths support remains a major problem.

References

Bamforth, S.E., Robinson, C.L., Croft, A.C. and Crawford, A. (2007) Retention and progression of engineering students with diverse mathematical backgrounds. *Teaching Mathematics and its Applications*, Vol. 26 (4): 156-166.

Cole, J.S. (2012) "Using continuous assessment to generate continuous learning in engineering maths." In P. Kapranos, D. Brabazon, eds. *ISEE2012: 4th International Symposium of Engineering Education: Educating the Engineers of Tomorrow*, University of Sheffield, UK, 258-265.

Cole, J.S., Crawford, T.J. and Zubairi, M.S. (2012) "Implementing a maths support system for first-year engineering students." In P. Kapranos, D. Brabazon, eds. *ISEE2012: 4th International Symposium of Engineering Education: Educating the Engineers of Tomorrow*, University of Sheffield, UK, 235-242.

Croft, A.C. (2001) "A holistic view of mathematics support in higher education." Accessed via 78.158.56.101/archive/msor/workshops/maths-support/croft.pdf (17 April 2014).

Croft, A.C., Harrison, M.C. and Robinson, C.L. (2009) Recruitment and retention of students – an integrated and holistic vision of mathematics support. *International Journal of Mathematical Education in Science and Technology*, Vol. 40 (1): 109-125.

Davis, L.E., Harrison, M.C., Palipana, A.S. and Ward, J.P. (2005) Assessment-driven learning of mathematics for engineering students. *International Journal of Electrical Engineering Education*, Vol. 42 (1): 63-72.

Matthews, J., Croft, A.C., Lawson, D. and Waller, D. (2012) *Evaluation of mathematics support centres: a review of the literature*. The National HE STEM Programme, Birmingham, UK.

Parsons, S.J. (2005) Success in engineering mathematics through mathematics support and changes to engineering mathematics modules at Harper Adams. *MSOR Connections*, Vol. 5 (1): 31-34.

Parsons, S.J., Croft, A.C. and Harrison, M.C. (2011) Engineering students' self-confidence in mathematics mapped onto Bandura's self-efficacy. *Engineering Education: Journal of the Higher Education Academy Engineering Subject Centre*, Vol. 6 (1): 52-61.

Symonds, R., Lawson, D. and Robinson, C.L. (2008) Promoting student engagement with mathematics support. *Teaching Mathematics and its Applications*, Vol. 27 (3): 140-149.

Williamson, S., Hirst, C., Bishop, P. and Croft, A.C. (2003) "Supporting mathematics education in UK engineering departments." *ICEE 2003: International Conference on Engineering Education*, Valencia, Spain.

University Students Are Changing – And What about Bright Students?

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Abstract

During the last two decades access to higher education increased considerably. One of the effects caused by this rapid expansion is that the students entering a HE institution do not have similar knowledge, background and/or willingness to study. A lot of research has been carried out concerning teaching students with not enough knowledge and/or skills from their secondary education. Less attention has been paid to the question how to educate bright students. The paper brings some experience gained at the Czech Technical University in Prague concerning teaching bright students.

Introduction

According to OECD (OECD 2013a, 2013b/6) during the last two decades access to higher education increased considerably; between years 1995 and 2011 entry rates increased from 39 % in 1995 to 60 % in 2011 (in 2013 in Australia, 95 % of corresponding cohort entered HE education). At the same time, the age when students enter HE is increasing, the average age in countries belonging to OECD is 20, ranging from 19 in Belgium and Japan to 25 in Island, New Zealand or Sweden, see (OECD 2013a). One of the effects cause by this rapid expansion is that the students entering a HE institution do not have similar knowledge, background and/or willingness to study.

Engineering education has a special role concerning this phenomenon. The number of students entering engineering has remained nearly the same, according to OECD (see OECD 2013a) the average of OECD only 15 % of the cohort enter engineering, manufacturing and construction HE (more than 20 % only in Korea, Finland, Russia, Mexico, Slovenia, Israel). In some countries there is not enough students willing to study natural sciences, mathematics or engineering, (see e.g. (OECD 2013a)). So it is not surprising that students entering HE institutions in the field of engineering and computer engineering have very different background. A lot of research has been done concerning the problem how to teach students with not enough knowledge and/or skills from their secondary education. Less attention has been paid to the question how to educate bright students.

The following questions can be laid:

1. How to motivate/select bright students? Are results from the secondary schools sufficient for such selection?
2. Does math play role in the process of selecting/recognizing bright students?
3. Should bright students be taught separately?
4. Does the education focused on teaching competencies help with education and/or selection of bright students?

We will focus on experience gained at Faculty of Electrical Engineering, the Czech Technical University in Prague concerning the last two questions.

Three attempts held at FEE CTU

Three different attempts have been tried at FEE.

1. **Premium form of a programme**

A group of up to 10 students from the programme Communications, Multimedia and Electronics students was selected for premium form of the programme. Students learned about the premium form in three different ways: 1. Students with very good results in the first semester were contacted and informed by e-mail message from the vice-dean. 2. Teachers of compulsory subjects informed students during lectures. 3. Information about it was placed on the web. Students interested in the Premium form had to apply for admission; a part of the application was a motivation letter. All students who applied for admission into the premium form had to pass an interview where his/her motivation and interests were searched. If accepted, each student got his/her personal tutor to help him to start work on a project and/or with choosing elective courses.

Students from the premium form had for most of the subjects joint lectures with "ordinary" students of the programme; only tutorials were separated and they were led by lecturers. Tutorials were focused on more advanced problems and extended theory. Moreover, instead of two rather practical subjects the group passed two extra courses, one from math and one engineering course.

2. **Special study program**

A special program called Open Electronic Systems designed for academically oriented students was accredited by the Ministry of Education, Youth and Sports of the Czech Republic at the beginning of spring 2013. First students were accepted from the academic year 2013/2014. The aim was to attract good students who might continue their studies in the master and doctoral programmes.

Also we hope that programme Open Electronic Systems will attract students who otherwise will go to study maths and/or physics at the Faculty of Mathematics and Physics, Charles University. There is not enough experience with the programme till now. Since the programme was accredited in the spring the existence of such a programme was not known at secondary schools. But from the first experience it seems to be less successful than the premium form. One of the reasons may be that sometimes freshmen overestimate/underestimate their ability and/or their interest in studies.

3. **Minor specializations**

Seven years ago Faculty of information technology (FIT) was founded as the eight faculty of the Czech Technical University. Even though more than half of the teachers and the researchers from the Department of Computer Science left FEE for FIT, at FEE remained enough teachers and researchers, especially young once, who were interested in applied informatics. Hence they prepared new study programme called Open Informatics. One of the new concepts incorporated into Open Informatics was the concept of minor specializations (minors). There are 7 different minor specializations in the bachelor programme. They are:

- Economics
- Computer Graphics
- Artificial Intelligence
- Software Engineering
- Computer Networks
- Embedded Systems
- Mathematics

There is no special enrolment into the minors; minors are recognized retroactively: when a student successfully passes three subjects (out of four or five, depending on the minor) of the minor specialization.. For mathematics minor there are 4 courses that are declared as the Mathematics minor ones. Two of them are from advanced calculus, two from discrete mathematics. Three of them serve to the whole faculty as elective courses for not only bachelor students but also for master students. One of the courses is a obligatory course from the master programme Cybernetics and Robotics.

During bachelor studies of Open Informatics students have to take 6 elective courses, two of them from humanities and/or economics and management. Hence each student has to take at least four elective courses among which there are all minor specializations courses. If a student is not successful in an elective course he/she only must pass another elective course so that the number of credits gained for elective courses is reached.

Teaching and Learning

1. **Premium form of a program**

This was the most demanding of the three attempts; mainly on teachers who were responsible of both lectures and tutorials. The teaching had to be organized so that lectures were understandable for majority of the students of the programme but still it had to be deep enough to allow extending and/or deepening of the material during tutorials. Tutorials did not contain solving exercises (these were left to students as homeworks); they contained partly “further lecture”, partly problem solving session. In this way the teaching was made more project-oriented which put more demands on both the teachers and students’ tutors. Learning process of the students from premium for was better with deeper understanding of mathematical concepts.

Since there was up to ten students in the premium for in one year, the group was small enough that even though during tutorials they had to cover more material than students of the standard form, active participation of students from premium form was rather high. Under a guidance of the teacher students tried to find out their own arguments and defend them not only during hours dedicated to their projects but also during ordinary tutorials.

2. **Open electronic system (a new study programme)**

This special study programme was so far run only once, therefore there is not enough experience with it. But from the first year it seems that the teaching was rather traditional with advantage of having a small group of students where the teaching could be more effective and where is not difficult to activate studnets.

3. **Minor specializations**

Elective courses of Mathematics minor are courses with up to 25 students in each (the average being 20). So the number of students allowed the teacher to make experiments with both teaching and organization of the course. For instance, the course Graph Theory consisted of 2 hours of lecturing per week (during the 14 weeks semester) and one hour of tutorial was replaced by consultation, one hour weekly with possibility to meet on request also in other time when needed. There were app. 25 small problems/tasks to be solved; each solution has to be written down and justify (in written form). The assessment consisted of presenting solutions in written form and the teacher chose two of the tasks for oral presentation.

Acquiring competencies

The following eight competences are taken from [Alpers et all] and they are based on the KOM project [Niss]. The competencies are: Thinking mathematically; Reasoning mathematically; Posing and solving mathematical problems; Modelling mathematically; Representing mathematical entities; Handling mathematical symbols and formalism; Communicating in, with, and about mathematics; Making use of aids and tools.

1. **Premium form of a program**

Progress was made in majority of competencies; it is not surprising considering the form of teaching and learning in the premium form. Competencies that were mostly acquired were: Posing and solving mathematical problems, Modeling mathematically, and Making use of aids and tools. Since in the premium form students with good basic knowledge of math were involved, less stress was put on the competency handling mathematical symbols and formalism.

2. **Open electronic systems**

Here we do not have enough experience to make definite conclusion, but we can say that acquiring competencies of students will depend on external factors:

a) Number of students enrolling this programme. It is easier to involve students in small groups into active participation during teaching process. At the same time, it is easier to use project learning in a small group than in a class of two hundred students.
b) When there are not enough applications for this programme, there is a danger that the faculty will accept also students with insufficient background. This might cause problems and there is a danger that in that case Open Electronic Systems will become a new but “common” electrical engineering programme.

3. **Minor specializations**

The elective courses of Mathematics minor are used to try new ways of organizing a math course for future engineers. Such courses could focus on acquiring different competencies. For instance, the Graph theory course is focused especially on Reasoning mathematically, Communication in, with, and about mathematics (in written and oral form), Modelling mathematically, and Thinking mathematically.

Conclusions

Each of the attempts that were (and are) tried at FEE CTU how to educate bright students has its advantages and disadvantages.

Premium form: Advantages: Students were chosen after finishing the first semester, so they knew better what university study in engineering meant and their decision was based on their own experience and not on their expectation. Students were not separately from other students. Most of the students of premium form achieved better understanding and they also enjoyed their study better than students of the “ordinary” form. Interviews with graduates showed that students had also valued the personal contact with teachers and the time they spent with their tutors..

Disadvantages: Success of premium form depends heavily on the personality of the teacher. It also requires more resources than the standard form.

Open electronic systems: Advantages: It is a small programme with small number of students (app. 20). Therefore, it is considerably easier to activate students not only during tutorials but also during the lectures. Similar background of the students allows the teacher to leave technicalities out and put stress on reasoning, modelling and arguing.

Disadvantages: Rather difficult engineering programme for which there does not need to be enough applicants. If this should happen, it will be one of many “the same” engineering courses only with a new name.

Minor specializations: Advantages: Students who would like to learn more and who want to know also why (and not only how) can try minor specializations courses without official enrolment in “premium form”. Hence a student does not need to feel embarrassed if he/she is not successful. Good students are not separated from the group of all students of the programme. So they are still in the tutorial groups for other, especially compulsory subjects.

Disadvantages: It is not difficult for a student to leave a difficult elective course. So the passing rate for Mathematics minor courses is not very high. Compared to the premium form and/or Open electronic systems, students do not get a thorough education in majority of subjects.

Final remark: When a more demanding education process is in preparation, make sure that the information about it can be written in the diploma student gets when graduating. Students from FEE made it clear that they want their effort to be recognized.

References

Alpers,. B., Demlova, M., Fant, C-H., Gustafsson, Th., Lawson, D., Mustoe, L., Olsson-Lehtonen,B., Robinson, C., and Velichova, D. (2013), „A Framework for Mathematics Curricula in Engineering Education“, SEFI, Brussels.

Niss, M. (2003) Mathematical competencies and the leasing of mathematics: The Danish KOM project. In A. Gagatsis, S. Papastravidis (Eds.), *3rd Mediterranean Conference on Mathematics Education,* Athens, Greece: Hellenic Mathematical Society and Cyprus Mathematical Society, 115-124

OECD (2013a), *Education at a Glance 2013: OECD Indicators,* OECD Publishing. *http://dx.doi.org/10.1787/eag-2013-en*

OECD (2013b/6), How Are University Students Changing?, *Education Indicators in Focus*, http://www.oecd.org/edu/skills-beyond-school/EDIF%202013--N %C2%B015.pdf

Future Technology for Engineering Maths

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Abstract

"Mathematics wars" have been raging throughout the XX century, and technology has only added fuel to fire: should mathematics be taught as poetry, requiring an inordinate amount of memorising and practising or should teachers concentrate on abstract concepts, with the tedium of calculations left to calculators and computers? Can ordinary learners grasp abstract concepts at all? On top of that, a modern University maths teacher teaching STEM students, particularly, future engineers has to cope with large classes, much larger than most European teachers had to deal with in the past. Can any of the teaching approaches be implemented in such environment in an effective manner? The advent of the XXI century saw mathematics teachers cajoled into employing the "evidence-based" technological solutions that had been shown to work when training University administrators, business managers or technicians. Many resisted, arguing that maths learning is a different process to learning a few words and procedures. Now it is all about MOOCs and Flipped Classrooms. Can ordinary engineering students learn mathematics by watching MIT or Khan Academy videos? Can ordinary mathematics teachers facilitate the process by "flipping" in an effective way?

Introduction

"Mathematics wars" have been raging throughout the XX century, and technology has only added fuel to fire: should mathematics be taught as poetry, requiring an inordinate amount of memorising and practising or should teachers concentrate on abstract concepts, with the tedium of calculations left to calculators and computers? Can ordinary learners grasp abstract concepts at all? On top of that, a modern University maths teacher teaching STEM students, particularly, future engineers has to cope with large classes, much larger than most European teachers had to deal with in the past. Can any of the teaching approaches be implemented in such environment in an effective manner? The advent of the XXI century saw mathematics teachers cajoled into employing the "evidence-based" technological solutions that had been shown to work when training University administrators, business managers or technicians. Many resisted, arguing that maths learning is a different process to learning a few words and procedures. Now it is all about MOOCs and Flipped Classrooms. Can ordinary engineering students learn mathematics by watching MIT or Khan Academy videos? Can ordinary mathematics teachers facilitate the process by "flipping" in an effective way?

MOOCs stand for Massive Open Online Courses. These are *free online courses* offered by universities around the world (e.g. Stanford, Harvard, and MIT) to anyone who has access to internet. Several popular MOOCs providers have emerged, such as Coursera, Udacity, EdX, and NovoEd that collaborate with universities to offer MOOCs on their platforms. To date, ten large public university systems have formed a partnership with Coursera: the State University of New York system, the Tennessee Board of Regents and the University of Tennessee systems, the University of Colorado system, the University of Houston system, the University of Kentucky, the University of Nebraska, the University of New Mexico, the University System of Georgia and West Virginia University. Some systems plan to blend online materials with faculty-led classroom sessions. Other leading online providers, too, have begun projects with public universities: edX, the non-profit collaboration founded by Harvard and the Massachusetts Institute of Technology, has teamed with the University of Texas and some California State University campuses, and Udacity, another Stanford spinoff, with San Jose State University. Others plan to offer credit to students who take the courses online followed by a proctored exam on campus.

The enthusiasm for MOOCs is tempered by reservations. Some faculty resistance has emerged recently against using online materials, even if they are blended with classroom work. Recently, 58 Harvard professors wrote a letter seeking the creation of a new committee to consider the ethical issues related to edX and its impact on higher education. In this paper they discuss pedagogical issues and opportunities associated with using MOOCs to deliver engineering maths, particularly to students from disadvantaged backgrounds.

Below general pedagogical issues are discussed associated with using MOOCs for teaching engineering maths: first, “maths wars” are described that have led to many changes in school curricula and a gap in mutual expectations of academics and undergraduates and then a view is offered on whether and how MOOCs can pave way to narrowing this gap.

General pedagogical issues associated with MOOCs

Math wars

Like many other wars math wars that have been fought particularly fiercely in the USA from the mid XX century onwards can be classified as “religious” and have been fought mainly between traditionalists and constructivists. In many cultures the traditional approach to teaching the subject is based on the myth that intelligence is mostly memory and ability to recognise basic patterns. In this context memory is understood as a declarative memory, ability to retain disjointed facts, without conscious awareness (see Anderson and Libiere, 1998), and since the “drill and kill” approach and coercion went out of fashion, many students have found maths manipulation out of their reach. A constructivist revolt — a natural reaction to traditionalists — rests on other myths (Budd *et al.* 2005), the main three being

1. only what students discover for themselves is truly learned;
2. there are two separate and distinct ways to teach mathematics,… conceptual understanding through a problem solving approach and through drill and kill;

3. maths concepts are best understood and mastered when presented "in context"; in that way, the underlying math concept will follow automatically.

The first myth has been confronted in recent large-scale studies, and it has been shown that in some cases, Direct Instruction improved school achievement from the 16th percentile to above the 90th percentile (Rebar 2007). Hattie (2008) summarizes the results of four meta-analyses that examined Direct Instruction. These analyses incorporated 304 studies of over 42,000 students. Across all of these students, the average effect size was .59 and was significantly larger than those of any other curriculum Hattie studied. Despite the constructivist hopes, the child-centred, cognitively focused, and open classroom approaches tend to perform poorly on all measures of academic progress.

The second myth is a logical fallacy known as a false dichotomy, and it has been argued e.g. by Wu (2006) that conceptual advances are invariably built on the bedrock of technique. The same author addresses the third myth by pointing out that *w*hen story problems take centre stage, the math it leads to is often not practiced or applied widely enough for students to learn how to apply the concept to other problems (Wu 2011).

As an outcome of this war school curricula have been changed many times resulting in a significant gap between expectations of academics and college entrants: while the disadvantaged students have limited maths background, limited memory and limited proficiencies in explanatory reasoning, the former expect them to have an ability to use traditional lectures/tutorials, memorise numerous disjointed facts and ability to discover explanations — all by themselves! Having spent 16 years on the “front line”, teaching engineering maths at the University of widening participation, the author has been advocating a blended approach based on the Soratic dialogue that can be classed as “guided teaching” of Vygotsky (Fradkin 2010 - 13). Others have been experimenting with different types of support classes (often manned with non-academics or inexperienced academics). The latest highly publicised possible solution is MOOCs.

Educating disadvantaged students

In 2012 Gary S. May, Dean of the College of Engineering at the Georgia Institute of Technology wrote: “MOOCs offer a huge opportunity to investigate how to use technology to more effectively educate students. They could potentially serve as laboratories to conduct experiments that might reinvent education. How can student learning be optimized in an online environment, and what is the best role of the faculty member in such an environment? Is the "flipped classroom" — i.e., using online lectures as preparation work for in-person interactions at multiple locations — a viable approach?” (May 2012). Would it not be great to have every student introduced to the subject content by a top professor? Fuelled by this thought, Coursera initially recruited as partners only the Ivy League members in the Association of American Universities. Houston Davis, the chief academic officer of the University System of Georgia, said that while the system would start with just a handful of Coursera courses next fall, he hoped a full menu of general education courses – the gateway classes usually taken in a student’s first two years — would eventually be available online through Coursera, for sharing by all the campuses (Lucas 2013). Moreover, Stanford Professor Thrun, one of the creators of Udacity suggested that “in 50 years there will be only 10 institutions in the world delivering higher education" (Lekhart 2012). These unrealistic initial claims would be acceptable if left to academics and journalists to discuss. Unfortunately, they supported the fallacy popular with many managers in HEIs (Higher Education Instituions), who find it expedient to claim that no local professors are needed to teach undergraduates, this can be done using teacher assistants and/or on-line courses.

On the other hand, many academics would argue that it is precisely in the first two years of their academic studies that students — and particularly disadvantaged STEM (Science, Technology, Engineering and Mathematics) students — need intensive guided teaching. This can be delivered only in person and only by the most experienced teachers capable of not just "talking" — delivering content, but also of "listening" — and readjusting their delivery depending on the student immediate feedback.

"Our first year, we were enamoured with the possibilities of scale in MOOCs," said Daphne Koller, one of the two Stanford computer science professors who founded Coursera. "Now we are thinking about how to use the materials on campus to move along the completion agenda and other challenges facing the largest public university systems." The company is eager to work with a broader range of institutions, to see how its materials can help more students complete their degrees (Lewin 2013). Despite Professor Thrun's predictions, while some Universities still intend to use existing Coursera materials developed by faculties at elite universities, others begun to say that they would expect that their own faculties will develop materials for the Coursera platform, making them available at campuses system wide and beyond. Faculty members will be able to customize existing courses, adding their own lessons and refinements, the company said. What led to this change of tack? The first sets of MOOCs data have shown that the initial expectations were unrealistic. The University of Edinburgh has reported in 2013 that "Thirty-three per cent of respondents (a subset of their MOOC users — LF) were between 25-35 years old and were mostly in the "teaching and education" field or students at university. 70% reported having completed an academic degree, a larger percentage than organisers expected." (Custer 2013) Similar reports have been produced by Harvard and MIT: "Average course certification rates are …6% among all registrants in the course (841,687 registrants with 597,692 unique users – LF)…The most typical course registrant is a male with a bachelor's degree who is 26 or older …(31%)…33% report a high school education or lower; 6.3% report that they are 50 or older; and 2.7% have IP or mailing addresses from countries on the United Nations list of Least Developed Countries."

It is not clear whether any certificates of attainment have been secured by those who had no prior degree. Of course, the low success rates do not diminish the value of MOOCs, not for the learners genuinely interested in learning, having reached the pre-requisite level and not able to attend the classes in person. However, for the first time academics at the top institutions — just like those who work in the institutions of widening participation had to do for a long time — have to draw managers' attention to the fact that educating the disadvantaged is much harder than educating students with strong prior backgrounds, and for the first time they have to start questioning the pedagogy required to deliver higher education to those who received no good education prior to registering for their course.

The questions are particularly pertinent when addressing maths education of STEM students. It is well known that a significant number of STEM freshmen do not understand mathematical symbols. Even the equality sign is a problem (Robison *et al.* 2007 - 10) and few know the difference between an expression and equation. The Sheffield Hallam University has recently reorganised their whole calculus curriculum, devoting each lesson to a particular mathematical symbol and treating the standard calculus concepts as examples of mathematical symbols' use. Given this ignorance of mathematical language, coupled with the well-known phenomenon of maths aversion, is it realistic to expect that many STEM students could follow Stanford, MIT and Harvard professors, who are used to delivering and deliver material suitable for students with strong mathematical background, in a traditional lecture style? Only proponents of the discovery learning could think that the answer could be positive, but this popular educational approach has been repeatedly shown to be ineffective with novice learners (Kirschner *et al.* 2006, Rebar 2007, Hattie 2008). Anyone who taught maths to such learners would give a negative answer even before they saw any MOOCs data — simply because they know how many hurdles have to be overcome before such learners are turned into competent engineering students: they have to be guided towards understanding the mathematical language, connections have to be established between the neural pathways that would allow them to use this language, engage in logical reasoning and critical thinking and master schemae for integrating new information with their prior knowledge. As argued in several previous publications (Fradkin *et al.* 2010 – 13) none of this can be achieved without a dedicated teacher giving (Delgado and O'Malley 2013) and receiving (Hattie 2008) a constant feedback. Ideally, the findings of such dedicated maths teachers have to be correlated with those of the rest of the faculty working with the same students.

Conclusions

At present only a minority of academics utilize modern pedagogical principles when teaching engineering maths. While there is no need to concentrate on these principles when educating students well prepared for college work, the issue becomes important with the students from disadvantaged backgrounds. The first MOOCs have been developed by professors from elite Universities who have not had to think about these issues in the past, so cannot be expected to solve the problem, but MOOCs platforms can be used in future by local faculty to provide courses based on sound pedagogical principles and more suitable for those students who are in need support teaching.

References

R.J. Anderson and C Libiere (1998) *The Atomic Components of Thought*, Maywah, New Jersey: Lawrence Erlbaum.

K. Budd, E. Carson, B. Garelick, D. Klein, R. J. Milgram, R. A. Raimi, M. Schwartz, S. Stotsky, V. Williams and W. S. Wilson (2005) "Ten Myths About Math Education And Why You Shouldn't Believe Them", 4 May 2005." **http://www.nychold.com/myths-050504.html**. Last accessed 2 February 2014.

T. Bulch, F. Dellaert, N. Feanster, J. Foley, A. Howard, G. Lebanon, A. Orso, K. Ramachadran, P. Traynor, R. Wuduc and B Waters (2013) "MOOMS proposal", 28 February 2013. **https://www.documentcloud.org/documents/703594-mooms proposal-2-28-13.html#document/p11/a103918**. Last accessed 2 February 2014.

S. Custer (2103) "U of Edinburgh releases first ever MOOC data", The Pie News, 12 June 2013. http://thepienews.com/news/u-of-edinburgh-releases-firstmooccomprehensive-data/. Last accessed 2 February 2014.

A. Delgado and K. O'Malley (2013) "Rapid Feedback Ties to Brain Science Research: Teaching in a Digital Age". http://researchnetwork.pearson.com/wp-content/uploads/DigitalAge_BrainScienceOverview_062713_final.pdf. Last accessed 2 February 2014.

S. Lekhart (2102) "The Stanford Education Experiment Could Change Higher Learning Forever", Wired, 20 March 2012. http://www.wired.com/wiredscience/2012/03/ff_aiclass/all/. Last accessed 2 February 2014.

T. Lewin (2013) "Universities Team With Online Course Provider", The New York Times, 20 May 2013. http://www.nytimes.com/2013/05/30/education/universities-team-with-online-course-provider.html?_r=0. Last accessed 2 February 2014.

H. C. Lucas (2013) "Can the Current Model of Higher Education Survive MOOCs and Online Learning?, *Educausereview*", 7 October 2013, http://www.educause.edu/ero/article/can-current-model-higher-education-survive-moocs-and-online-learning. Last accessed 2 February 2014.

G. S. May (2012) "MOOCs' Missing Pieces", *Inside Higher Ed.,* 11 September 2012. http://www.insidehighered.com/views/2012/09/11/essay-what-moocs-are-missing-truly-transform-higher-education. Last accessed 2 February 2014.

L. Fradkin, V. Zernov, C. Crisan & S. Lerman (2010) "First Year Engineering Mathematics: The London South Bank University experience," *MSOR Connections*, 2010 Spring.

L. Fradkin (2010) "Socratic dialogue: Learning to teach a modern learner, "*Learning Dialogues, The University of Northampton Learning and Teaching Conference*.

L. Fradkin (2010) "Teaching algebra and calculus to engineering freshers via Socratic Dialogue and Eulerian sequencing," *International Conference on Engineering Education,* ICEE-2010, Gliwice, Poland.

L. Fradkin (2012) "Issues surrounding teaching pre-calculus to engineering freshers", *International Conference on Engineering Education,* ICEE-2012, Turku, Finland, 771–8.

L. Fradkin (2013) "Efficient teaching of differential calculus to engineering freshers", *International Conference on Engineering Education,* ICEE-2013, Marrakesh, Morocco [to appear].

J. Hattie, *Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Achievement,* Routledge, Abingdon, 2008, 392 pp, ISBN 978-0-415-47618-8.

P. A. Kirschner, J. Sweller and R. E. Clark (2006) "Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching", *Educational Phychologist*, 41(2): 75–86. http://www.cogtech.usc.edu/publications/kirschner_Sweller_Clark.pdf. Last accessed 2 February 2014.

M. Rebar (2007) "Academic acceleration in first grade using the Direct Instruction model", Report No. 2007–1. Cheney, WA: Eastern Washington University.

M. Robison, M. M. Thomlinson and N. V. Chalis (2007 -10) "Staff views of students". In M*aths at Uiversity: Reflections on experience, practice and provision* (Eds. M. Robison, N. V. Chalis and M. M. Thomlinson, More Maths Grads project), 70 – 80.

http://www.mathscareers.org.uk/viewitem.cfm-cit_id=383010.html. Last accessed 30 April 2014.

H.-H. Wu (2011) "The Mathematician and Mathematics Education Reform", University of California, Berkeley.

H.-H. Wu (2013) "Basic Skills vs Conceptual Understanding; a Bogus Dichotomy", Department of Mathematics, University of California, Berkeley.

E-Assessment and Mathematical Learning: A Spanish Overview

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Abstract

Assessment can be used to enhance student learning (formative assessment) or to measure derived or accrued learning within the particular subject area (summative assessment). These strategies can be employed either separately or in combination according to the requirements or needs. Nowadays, the student must be the protagonist of their learning, and it is important that they receive timely feedback of the learning process. Advances in technology tools available have increased the capacity to provide fast efficient performance feedback to students. Therefore, technology allows the development of new models of assessment which are easy to implement. Technology-supported assessments are already being used as in formative assessment and to some extent in the summative sense of grading student achievement in some Spanish universities. After some general ideas concerning the learning process, this paper analyses the various ways in which technology is used in assessment activities of basic mathematical subjects in Spanish engineering programmes. The results are supported by the construction of a database of Learning Guides (information provided by the universities to the students).

Introduction

Teaching and assessment strategies are inseparable parts of the educational process. All teaching innovation should consider the implications on the efficacy of the assessment methods employed. New degrees in engineering, adapted to the guidelines of the Bologna process, involve the assessment of the acquisition of competencies, and the technology should help in this task. Some ideas on the new forms of assessment are included in SEFIMWG (2013).

The assessment models might be closer to "the real-world". So if in the real-world the Engineer is continuously using technology (including mathematical software) we have to promote the use of technology in the assessment tasks. Therefore, e-assessment has become a common educational strategy in higher education.

In its broadest sense, e-assessment is the use of Information Technology for any assessment activity. Actually, the term e-assessment is becoming widely used as a generic term to describe the use of computers within the assessment process. In this paper we accept this definition and we will analyse any assessment task using computers.

The benefits of online formative assessment have been identified in numerous research papers. In Gikandi et al. (2011) a report on on-line assessment from research literature is provided. On the other hand, Computer Algebra Systems (CAS) have been used in the teaching and learning of mathematics for decades (see García et al., 2000; Meagher, 2000; García et al., 2009; Marshall et al., 2012), and its use in different assessment activities has more recently been discussed (see Brown, 2001; Sangwin, 2004 and García et al., 2014).

In the first section of this paper we will present general ideas concerning teaching, learning and assessments according to the Bologna process. In the second part we present the results of a study conducted with a sample of mathematics subjects from Engineering and Computer Science degrees from different universities, analysing the use of technology-supported (formative and summative) assessment.

General ideas for teaching, learning and assessments

A new scenario for teaching and learning

The Bologna Declaration and the creation of the European Higher Education Area have promoted a structural change in Spanish Universities. This change, which has been increasingly addressed in recent years, has not been limited to a mere restructuring of the academic curriculum. The process has served to change the paradigm of university teaching. In this new scenario, the advance of constructivist methods in teaching practices seems to have become the hegemonic method. Some of the characteristics of such methods that are currently being imposed in general, not only within the field of mathematics, are as follows:

- Student-focused teaching.
- An increase in participation by the students themselves, favouring discussion between students and instructors and departing from the traditional method of teaching through lectures.
- The use of educational technology.

The process is now beginning to impact on the traditional methods of assessment.

The role of assessment

Assessment is a core component for effective learning. The change from teacher-centred instruction towards learner-centred instruction and competencies-based learning implies the development of new assessment methods. If teaching and learning are based on acquiring competencies, then assessments must determine the acquisition of these competencies. There is a strong relationship between learning and assessment: *what is assessed strongly influences what is learned.*

Arguments for introducing CAS in assessment activities are well established (Meagher, 2000). Some authors have described the impact of the introduction of CAS in examinations (MacAogáin, 2002; Brown, 2001), and they concluded that using CAS gives students the opportunity to be more responsible for their own learning.

Moreover, assessment must be more than a summative assessment. A formative assessment with feedback is an important strategy that can help students take control of their own learning and develop critical thinking. Virtual learning environments such as Moodle provide many opportunities for high quality feedback and formative assessment (see Limniou and Smith, 2014). Furthermore, CAS may play an important role in any model of formative assessment in mathematics courses among engineering students. For example, CAS is a very useful tool for problem-based learning in mathematics.

Many instructors continue to feel that good mathematics assessment must be restricted to a traditional exam (a collection of problems to be solved with pencil and paper). However, a good model of formative assessment about mathematical competencies, consistent with the literature on student-centred learning (see Baartman et al., 2006; Niss and Højgaard, 2011), should include:

- Team work for solving problems and doing projects, because collaborative learning has a higher efficiency than individualistic learning method (Hsiung, 2010; García et al., 2011).
- Online quizzes with feedback, using the corresponding Learning Management System, mainly MOODLE.
- Solving written exercises or problems related to the real world, using aids and tools (Díaz et al., 2011).
- Exams with free use of mathematical software, since it allows evaluation of more realistic mathematical competencies.

E-assessment has many advantages over traditional assessment such as: flexibility, efficiency, lower cost and instant feedback for students.

A Spanish overview

The use of different CAS in Spanish engineering degrees has undergone several iterative stages. Initially, its use was restricted to the subjects of Numerical Analysis as a programming tool. Later, with the emergence of new, more versatile and friendly versions, its use spread to mathematics laboratories, performing different practices related to all subjects of mathematics (see García et al., 2000 or García et al., 2009). At present, in some Spanish universities, CAS are used in an integrated way in different approaches: For experimenting and developing teaching resources (Botana et al., 2012); for learning based on competences (Díaz et al., 2011); for developing small projects (García et al., 2011), etc. In this context, the students can use the available technology in all teaching scenarios. Our previous study, García et al. (2012) analysed the use of technology in the new teaching scenario according with the Bologna process.

The next step has been to analyse the use of the technology in the assessment of the learning. In the literature there are very few references to works by Spanish authors, therefore it has been necessary to conduct a field study.

The research work

The aim of our research was to examine the use of technology in assessment activities in mathematical topics of Spanish engineering degrees.

The methodology adopted involved a quantitative study using a sample of 44 Spanish universities, chosen according to the following criteria: All universities are public universities and they are disseminated between all Spanish regions. For each university we selected some engineering degrees, and analysed the use of technology in assessment activities primarily within the topics of Linear Algebra and Calculus.

In this step we have analysed the Learning Guides (LG). The LG's are documents that provide general information to the students including: competencies and learning objectives, contents, planning and chronology, teaching methodology and evaluation models. It is compulsory to prepare the LG to be offered to the students at the beginning of each academic course. An example of an LG can be seen at
http://www.fib.upc.edu/en/estudiar-enginyeria-informatica/assignatures/M2.html

Taking into account the large number of different engineering degrees offered by Spanish Universities, we have focused the study on programmes within the general realms of Information Technology (ICT), Industry in general (IND) and Construction and Civil Engineering (CCE). The following table outlines the data used in our study.

Degrees related with	LG analysed	LG including technology	LG including assessment tasks with technology
ICT	80	48	36
IND	71	52	39
CCE	50	34	32

For each LG, the information regarding assessment activities with the use of technology was analysed. We paid special attention to the following items;

- Evaluation of Laboratory sessions or reports of practical sessions
- Exams with computers
- Projects (in general and including the use of technology)
- Quizzes online

The following table lists the information obtained after the analysis of the LG

Degrees	ICT	IND	CCE
Laboratory sessions	26	30	30
Exams with computer	14	23	12
Projects	3	6	5
Quizzes online	6	3	6

We also have analysed the percentage in the final grade taking into account all assessment activities using technology. The figures are not homogeneous because the percentages are oscillating between 0 and 50%.

To analyse the differences between the target groups a study of proportions was made with a confidence level of 95%. Significant differences were noted: Computer examination between ICT and IND (p-value 0.0336) and evaluation of laboratory sessions between ICT and CCE (p-value 0.002).

Remark: Our research is focused on the use of technology in student assessment activities. Technological resources (such as optical or similar reader) are not considered because these resources only provide the final grade of the tests performed by students with pencil and paper.

Conclusions and Future Work

The following conclusions are draw:

- The use of technology in mathematical learning of engineering students is increasing.
- The most used technology is CAS, usually working in practical laboratory sessions.
- The use of computers in assessment activities is less frequent in the group of ICT degrees.
- Multiple technology-based assessment strategies exist. The most common ways are reports for practical sessions or exams with computers. Projects, quizzes online and other activities are less common.

The next step in our research will be a qualitative analysis. We are preparing a survey to be filled for a target group of selected teachers. The items of the survey are related to the way of teaching or assessing using technology.

References

Baartman, L.K.J.; Bastiaens, T.J.; Kirschner, P.A. and Van der Vleuten, C.P.M. (2006) The Wheel of Competency Assessment: Presenting Quality Criteria for Competency Assessment Programs. *Studies in Educational Evaluation,* 32: 153-170.

Botana, F.; Abánades, M.A. and Escribano, J. (2012) Using a Free Open Source Software to Teach Mathematics. *Computer Applications in Engineering Education*, DOI 10.1002/cae.21565

Brown, R. (2001) Computer Algebra Systems and the challenge of assessment. *International Journal of Computer Algebra in Mathematics Education,* 8 (4): 295-308.

Díaz, A.; García, A. and de la Villa, A. (2011) An example of learning based on competences: Use of Maxima in Linear Algebra for Engineers. *The International Journal for Technology in Mathematics Education,* 18 (4): 177-181.

García, A.; García, F.; Rodríguez, G. and de la Villa, A. (2000) Learning calculus of several variables with new technologies. *The International Journal of Computer Algebra in Mathematics Education*, 7 (4): 295-309.

García, A.; García, F.; Rodríguez, G. and de la Villa, A. (2009) A toolbox with DERIVE. *The Derive Newsletter*, 76: 5-13.

García, A.; García, F.; Rodríguez, G. and de la Villa, A. (2011) "Small Projects: A Method for improving Learning." In *3rd International Research Symposium on Problem-Based Learning*, Coventry (UK).

García, A.; García, F.; Martín, A.; Rodríguez, G. and de la Villa, A. (2012) "Learning and Assessing Competencies: New challenges for Mathematics in Engineering Degrees in Spain," In *Proceedings of the 16th SEFI MWG Seminar Mathematical Education of Engineers,* Salamanca (Spain). ISBN: 978-84-695-3960-6.

García, A.; García, F.; Martín, M.; Rodríguez, G. and de la Villa, A. (2014) Changing Assessment Methods: New Rules, New Roles. *Journal of Symbolic Computation,* 61-62: 70-84.

Gikandi, J.W; Morrow, D. and Davis, N.E. (2011) Online formative assessment in higher education: A review of the literature. *Computers and Education,* 57: 2333-2351.

Hsiung, C.M. (2010) An experimental investigation into the efficiency of cooperative learning with consideration of multiple grouping criteria. *European Journal of Engineering Education*, 35 (6): 679-692.

Limniou, M. and Smith, M. (2014) The role of feedback in e-assessments for engineering education. *Education and Information Technologies*, 19 (1): 209-225.

MacAogáin, E. (2002) A CAS-Index. *The International Journal of Computer Algebra in Mathematics Education*, 9(4): 305-313.

Marshall, N.; Buteau, C.; Jarvis, D. and Lavicza, Z. (2012) Do mathematicians integrate computer algebra systems in university teaching? Comparing a literature review to an international survey study. *Computers & Education*, 58: 423-434.

Meagher, M. (2000) Curriculum and Assessment in an Age of Computer Algebra Systems. *ERIC Digest*.

Niss, M. and Højgaard, T. (eds) (2011) *Competencies and Mathematical Learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark.* IMFUFA 485-2011, Roskilde University.

Sangwin C. (2004) Assessing mathematics automatically using computer algebra and the internet. *Teaching Mathematics Applications*, 23(1): 1-14.

SEFIMWG (2013): "A framework for mathematics curricula in engineering education" (3rd Edition). Accessed via http://sefi.htw-aalen.de (28 April 2014).

How to evaluate bridge courses? – The risk of false positives

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Abstract

We analyse students' answers to a set of tasks designed to gain information about their ability to reason proportionally. These tasks have been particularly designed in order to control for false positives, where students arrive at the correct answer for the wrong reasons. We describe the rationale behind this design and discuss implications for bridge courses and test designs in general.

Introduction

Many universities offer mathematical bridge courses for first year students before the actual lectures start. A common rationale for such courses is to remedy perceived deficits in secondary maths education. A central question is whether and to what degree such courses actually remedy such deficits.

Over the past decades work in various branches of discipline-based education research has shown that deficits in student learning can quite often be traced back to characteristic difficulties inherent in the subject matter (Bransford, Brown & Cocking, 2000). In such cases learners typically use or even develop alternative conceptions (often called misconceptions) rather than scientific concepts. The concept group of ratios and proportions is an example of this in mathematics (Tourniaire & Pulos, 1985).

Many students have considerable difficulties recognizing whether, in a given situation, the ratio or product of two quantities is invariant, in other words that the quantities are proportional or inversely proportional to each other. A typical and frequent misconception in such cases is that the sum or difference of the involved quantities is considered to be invariant. We will follow an established terminology and refer to this as additive reasoning (AR) and proportional reasoning (PR). Of course, AR is appropriate if a situation calls for it. It is well known from cognitive science that humans acquire ability for AR earlier than for PR.

Many instructors recognize that a noticeable number of students have difficulties in applying PR. Therefore there are regular demands for ratios and proportions to be covered in bridge courses, at least in Germany. However, given that successful intervention programs which address students' development of PR stretch over a time period of more than one year (Adey, 1999), it seems to be quite challenging to develop such reasoning over the duration of a bridge course of only about two weeks. It is also quite challenging to assess students' learning gains with respect to PR as we will describe in this paper. This, of course, is not a matter of the duration of the intervention, but is strongly related to characteristic students' answers on test items requiring AR and PR, respectively.

In the next section we will give background information on the bridge course that provided the data underlying this work. That section will also describe the process that led to those test items which will be discussed in subsequent sections. We will close with conclusions and implications for evaluating learning gains in general.

Testing for proportional reasoning within the bridge course

The two-week bridge course takes place at the beginning of every semester before the start of lectures. The course covers fractions and proportions, among other topics of secondary school mathematics, with an improvement in students' ability to reason proportionally being one goal of this course. Taking this course is not mandatory for beginning students, but usually a large fraction of the incoming student population enrols on this course, in particular those studying in a STEM department. The bridge course is offered every semester with a typical total enrolment of almost 1000 students per year.

The students are required to take a test at the beginning and at the end of the course; these tests consist of about 30 multiple-choice questions and cover many aspects of the course content. The pre-test serves to inform instructors of the mathematical abilities of the participating students. Pre- and post-test together serve to evaluate the efficacy of the bridge course.

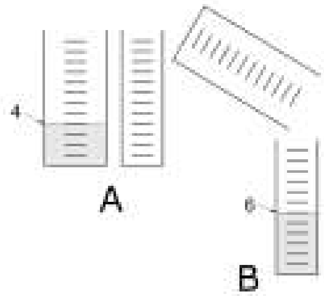
In assessing students' ability to reason proportionally using the cylinder task (to be described below) in the pre-test we have observed over several semesters that between 50% to 60% (approximately) of students seem to reason proportionally. This performance seems to be disappointing and confirms instructors' concerns about students' mathematical abilities. On the other hand, these numbers are quite remarkable in the light of research which investigated the transition from AR to PR. Several investigations (Karplus, Karplus, Formisano & Paulsen, 1977; Shayer & Adey, 1987) suggest that only about one third of the population in this age bracket will have developed the cognitive ability to reliably reason proportionally while another third reasons additively on items requiring PR. The last third is in a transition phase between AR and PR.

In the light of this research the performance of our students is quite impressive, but this might be deceptive. It is quite plausible that some students choose a proportionality based solutions strategy because they had previously experienced intensive training in solving problems requiring PR, e.g. in secondary school. Note that tasks requiring PR often involve two pairs of quantities with three of these quantities given numerically and the fourth being unknown. Some students might have become quite proficient in detecting this pattern in the tasks formulation and use this as a trigger for subsequent successful computations, but might not have yet developed the ability for PR. Although following proportionality based solutions strategies, they should then still be more inclined to reason additively. In order to scrutinize this hypothesis we have designed test items which follow the described pattern of typical tasks requiring PR, but which actually call for AR as a solution strategy.

Pre-test items testing for additive and proportional reasoning

The test items we use are designed as pairs. For the pre-test the first item in this pair is the following task:

To the right are drawings of a wide and a narrow cylinder. The cylinders have equally spaced marks on them. Water is poured into the wide cylinder up to the 4th mark (see A). This water rises to the 6th mark when poured into the narrow cylinder (see B). Both cylinders are emptied (not shown) and water is poured into the wide cylinder up to the 6th mark. How high would this water rise if it were poured into the empty narrow cylinder?



(A) to about 8
(B) to about 9
(C) to about 10
(D) to about 12
(E) none of these answers is correct
because
(A) the answer cannot be determined with the information given.
(B) it went up 2 more before, so it will go up 2 more again.
(C) it goes up 3 in the narrow for every 2 in the wide.
(D) the second cylinder is narrower.
(E) one must actually pour the water and observe to find out.

Subsequently we will refer to this as the cylinder task. This item has been taken from the Classroom Test of Scientific Reasoning (Lawson, 1978), and is characterised by two parts or sub-tasks; the answer to the numerical question and the reason why the test taker chose their numerical answers. The distractors are based on the typical and frequent wrong answers. The combination of answers (A) and (B) for the two sub-tasks is indicative for AR.

The cylinder tasks served us as a model for the design and layout of the paired item, which we will subsequently call the bicycle task. It requires AR as solution strategy.

Alice and Greta each went for a cycle. They started at different times and then cycled at the same constant rate. By the time Alice had gone 6 km, Greta had already gone 8 km. How far will Alice have gone when Greta has gone 12 km?
(A) 12 km
(B) 10 km
(C) 14 km
(D) 9 km
(E) None of the above
because

(A) one can compute the answer
(B) the distance which Greta cycled has increased by a factor one and a half
(C) the answer cannot be determined with the information given
(D) the distance between Greta and Alice does not change
(E) Greta and Alice cycle to the same place

The cylinder and bicycle tasks are superficially similar to each other, and this is deliberate. Both give the numerical values of three quantities and ask to determine a fourth one. Via this design we want to probe whether students who have selected the correct answer (B) on the first part of the cylinder task did so because they realize that the situation at hand involves proportions, or whether they simply computed the result using proportions triggered by a pattern in the formulation of the task.

Table 1 shows data obtained in the autumn term of 2013. In general we observe that the data does not considerably vary from semester to semester. In this table students' answers are classified as proportional or additive, as appropriate, if a student answered an item correctly both numerically and for the justification. For the cylinder tasks this would correspond to the answer combinations (B) & (C) for PR and (A) & (B) for AR. All other students' answers have been classified as inconsistent.

N = 446		**cylinder task (PR)**		
bicycle		proportional	additive	inconsistent
task	proportional	10.5%	1.3%	4.0%
	additive	22.6%	11.2%	11.7%
(AR)	inconsistent	14.6%	6.5%	17.5%

Table 1

Table 1 clearly indicates that there is a considerable fraction of students (10.5% in this case) who reason predominantly proportionally although the bicycle task calls for AR. We interpret this as supporting our starting hypotheses in that some students might show PR because the formulation of the tasks triggers them to do so. Hence, one has to be careful when interpreting test items like the cylinder task in isolation. The corresponding data of the isolated task would lead a much more favourable interpretation of students' ability to reason proportionally than is actually warranted.

The analysis becomes even more pronounced, when one ignores the justification parts of the test items. In table 2, student answers have been classified as "other" if the chosen numerical answer is not (A) or (B) for the cylinder task and is not (B) or (D) for the bicycle task. As can be seen from the data, in the numerical part student choose predominantly those answers which indicate AR or PR. In fact this also holds for students whose answers have been classified as "inconsistent" in Table 1. Most of them had chosen the "uninformative justifications" (D) for the cylinder task and (E) for the bicycle task in order to support their numerical solution indicating AR and PR. Based on these data and the fact that data has been pretty stable over the past semesters we argue that only about one third of the participants of our bridge course show the desired behaviour, that is, answering the cylinder task using PR and the bicycle task using AR. Therefore helping students to acquire PR is an issue.

N = 446		cylinder task (PR)		
bicycle		proportional	additive	other
task	proportional	25.9%	5.1%	3.3%
	additive	29.7%	16.1%	6.9%
(AR)	other	6.7%	3.1%	3.1%

Table 2

Post-test items testing for additive and proportional reasoning

For the post-test we have designed a similar pair of test items. The tractor task involves a situation which calls for PR and is isomorphic to the cylinder task. Correspondingly the evaporation task is isomorphic to the bicycle task and is intended to control for false positives in the tractor task.

Data on this pair of tasks is given in Table 3. Overall it shows the same picture as Table 1; a fraction of students again answer predominantly additive on both tasks and a considerable fraction of students reason proportionally on both tasks, indicating problematic application of PR.

N=308		tractor task (PR)		
evaporation		proportional	additive	inconsistent
	proportional	12.3%	4.5%	8.1%
task (AR)	additive	26.3%	8.8%	15.9%
	inconsistent	9.7%	4.9%	9.4%

Table 3

Investigating the development of individual students from pre-test to post-test we observe that almost no development has taken place. Firstly those students who reason predominantly AR in both test items of the pair of tasks in the pre-test, do so in the post-test as well. Secondly, students who show PR on the first task and AR on the second task on the pre-test keep doing so on the post-test. Finally and most disappointingly, students who seemingly are capable of PR, but apply it to additive situations as well, stick to that behaviour.

Conclusions

We draw several conclusions on several levels from the data reported here. First of all we interpret the data as supporting our hypothesis that there is a considerable fraction of students who are seemingly capable of PR, but are actually blindly applying a solution algorithm triggered by the superficial structure of the task. Furthermore we conclude that if bridge courses aim to improve the participants' ability in PR, they need to particularly address the observed inability of students' to recognize whether a given situation calls for AR or PR, or something else. We certainly see room for improvement with respect to that in our own bridge course. At the same time we continue to view such an endeavour as particular challenging due to the very limited duration of bridge courses.

On the level of bridge courses we conclude that one has to be careful when interpreting test data in order to evaluate such courses. Apparent learning gains might be traced back to an increased capability to infer the mathematical computations needed to solve the task from superficial features of the task. We suggest controlling test items for such false positives. This is particularly warranted in situations where characteristic misconceptions are known. In such cases a test design comparable to the cylinder-bicycle and tractor-evaporation pairs might be useful.

References

Adey, P. (1999). The Science of Thinking, and Science for Thinking: A Description of Cognitive Acceleration through Science Education (CASE). Innodata Monographs 2.

Bransford J.D., Brown, A.L., & Cocking, R.R. (2000). How people learn: brain, mind, experience, and school. Washington, DC: National Academy Press.

Karplus, R., Karplus, E., Formisano, M., & Paulsen, A. C. (1977). A survey of proportional reasoning and control of variables in seven countries. Journal of Research in Science Teaching, 14(5), 411-417.

Lawson, A. E. (1978). The development and validation of a classroom test of formal reasoning. Journal of Research in Science Teaching, 15(1), 11-24.

Shayer, M., & Adey, P. (1987). Towards a science of science teaching. London: Heinemann Educational Books.

Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. Educational studies in mathematics, 16(2), 181-204.

Mathematics: Creating Value for Engineering Students

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Abstract

While students' attainment in mathematics and their attitudes about mathematics are strongly inter-related, value is an important concept in mathematics education. It is arguable that lecturers, especially in engineering faculties, know little about the relationships students form with mathematics; for example what value do engineering students place on mathematics learning?

Mathematics is often perceived as a difficult subject and it is associated with certainty and with being able to get the right answer. However the narrowness of the assessment process overshadows predictors of achievement behaviour: expectancy (am I able to do the task?) and value (why should I do the task?). At the same time lecturers are tasked with mathematically preparing students for an increasingly technological world, however for many students, the nature of a career involving mathematics is not at all clear. A significant difference between engineering education and practice is the social aspect of work compared to education. In particular engineers' difficulty communicating mathematics is a significant weakness of engineering education.

While engineering mathematics curricula often prescribe a fixed body of mathematical knowledge, this study takes a different approach; second year engineering students are additionally required to investigate and document an aspect of mathematics used in engineering practice. A qualitative approach is used to evaluate the impact students' investigations have on their mathematics learning and whether this approach creates greater value for students compared to curriculum mathematics learning. This paper contains an account of students' engagement with and their emotional responses to their investigations of professional engineers' mathematics usage.

1 INTRODUCTION

Mathematics is important; mathematics is required for successful functioning in society (Ernest, 2010), professional engineers use a broad range of mathematics in their work (Goold and Devitt, 2012) and mathematics achievement is a strong predictor of third level persistence generally (Mooney et al., 2010).

It is claimed that a society of lifelong learning requires individuals with well–developed learning dispositions (Falsafi, 2010). There are different perspectives of mathematics relationships; sociocultural, discursive and psychoanalytic factors that influence people's relationships with mathematics. Research literature indicates that mathematics can be made more accessible in classrooms which encourage exploration, negotiation and ownership of knowledge due to enhanced relationships (Black et al., 2009). Experienced learners seek and engage life experiences with a learning attitude and they believe in their ability to learn. The

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primary focus of people who see themselves as learners is not on immediate performance or goal achievement but on the on-going process of learning (Illeris, 2014).

Motivation is a major factor in mathematics teaching and learning, however there is some evidence to suggest that mathematics is a special subject compared to other school subjects. (Smith, 2004). Characteristics of classroom mathematics include: tedium; isolation; rote learning, elitism; and depersonalisation (Nardi and Steward, 2003).

Mathematics is often associated with certainty whereby "doing" mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question and a mathematical "truth" is determined when the answer is ratified by the teacher" (Lampert, 1990). Consequently for many students, the nature of a career involving mathematics is not at all clear (Petocz et al., 2007). Similarly adjusting to the workforce can be problematic for many students as they discover what they learned in university needs to be contextualised for work (Wood, 2010). Trevelyan maintains that engineering practice relies on applied engineering science, tacit knowledge (unwritten know-how carried in the minds of engineers developed through practice and experience) and an ability to achieve practical results through other people (Trevelyan, 2010).

This paper presents the findings of a study investigating the impact of student-directed learning on students' relationships with mathematics.

3. METHODOLOGY

In this study mathematics relationships are studied in the context of Wigfield and Eccles' social cognitive expectancy-value model of achievement motivation. This theory posits that predictors of achievement behaviour are: expectancy (am I able to do the task?); value (why should I do the task?); students' goals and schemas (short- and long-term goals and individuals' beliefs and self-concepts about themselves); and affective memories (previous affective experiences with this type of activity or task) (Wigfield and Eccles, 2002, Schunk et al., 2010). Expectancy-value research has substantiated that students with positive self-perceptions of their competence and positive expectancies of success are more likely to perform better, learn more and engage in an adaptive manner on academic tasks by exerting more effort, persisting longer and demonstrating more cognitive engagement. Students who value and are interested in academic tasks are more likely to choose similar tasks in the future. Interest refers to the liking and wilful engagement in an activity. Interest can be: personal (personal enjoyment or importance of specific activities or topics); situational (interestingness of the context e.g. novel versus textbook) or psychological (heightened interest when personal interest interacts with situational interest) (Schunk et al., 2010, Wigfield and Eccles, 2002, Wigfield and Eccles, 2000, Wigfield, 1994).

While engineering mathematics curricula often prescribe a fixed body of mathematical knowledge, this study takes a different approach; in addition to their curriculum mathematics learning a class of second year part-time energy engineering students are required to select an aspect of mathematics they had studied in their engineering course and, using whatever resources available to them, investigate how professional engineers use this mathematics in engineering practice. The class group of seventeen students comprises a diversity of students many of whom work part-time in engineering and trades environments. A survey questionnaire was used to capture the students' feelings about the exercise before and after conducting the task. The data was analysed qualitatively using a system of open coding (Miles and Huberman, 1994, Silverman, 2010).

4. FINDINGS

There are five main findings:

- The majority of the class of engineering students are neither confident in their mathematics ability nor demonstrate any value of mathematics other than for the purpose of passing examinations.
- Students present mathematics learning as a chore, they have no mathematics goals and their mathematics learning comprises of repetitive memorisation of solutions to past examination questions.
- There is no evidence of social interaction in the students' mathematics learning; students only discuss how irrelevant mathematics is to their lives.
- Students have difficulty communicating mathematics as evidenced by their reluctance to engage in documenting their investigation of mathematics used in engineering practice. They also show a reluctance to depart from rules or "set procedures" and from seeking a precise goal such as the "correct answer" in traditional mathematics learning.
- Following completion of the assignment, students show improved relationships with mathematics. In particular they demonstrate an increased awareness of the usefulness of mathematics in their future careers. The new awareness of the relevance of mathematics in life and work energises students to "work harder". Another observation is the increased classroom discussion about mathematics arising from the assignment.

Prior to engaging in the exercise it is observed that the majority of students' feelings about mathematics are negative, for example:

- *"I like it [mathematics], I just wish I understood it better"*
- *"I don't like it as I find it difficult and hard to understand"*
- *"I'm not confident ... I often make mistakes"*
- *"I don't like maths at all, probably because I find it difficult"*
- *"It is my least favourite subject; most of questions and theories are very difficult to understand unless they are well instructed"*
- *I am really interested in maths, sometimes I find it difficult but I understand it"*

Only one of the seventeen students likes mathematics:

- *"I like maths, I like the logic behind it"*

Students do not exhibit positive mathematics relationships: they do not show commitment to the on-going process of learning and they do not demonstrate any mathematics learning goals; the only learning strategy is practising past examination questions:

- *"I don't have a strategy, I just go by past exam questions".... repetition but it is hard because there is so much to cover"*
- *"I have not got a maths strategy ... just questions and past exam papers"*
- *"I try to understand the step by step solutions to exam questions , repetition is essential"*
- *"practice, practice, practice ...doing past exam questions with detailed steps and solutions"*

Students' mathematics discussion is sparse and negative**:**

- "*I never discuss maths outside of lectures"*

- *"I often discussed how irrelevant maths topics are to our future careers"*
- *" We discuss how bad maths actually is and question why do we need it"*
- *" Not many discussions, nobody wants to talk about maths"*

Students are challenged by the departure from "set procedures" and getting the "correct answer." When asked if they liked the exercise, students' responses include:

- *"Not all of it, parts of it were too abstract, not sure what relevance it has to course"*
- *"It's different, questions are too abstract"*
- *"Not particularly, I found it difficult to write about something which I didn't particularly like"*
- *"Difficult to put mathematical equations into a word document"*
- *"I did not enjoy this assignment as I did not fully understand what was being asked"*
- *"I didn't because I didn't know how to do it properly ... what is the expectation?"*
- *"It was very difficult to do it, I didn't even know how should I start"*

While some students are uncomfortable with the concept of an investigative approach to mathematics learning and report writing, each student state that they benefitted from the exercise. In particular it is noted that the students' insight into mathematics used in engineering practice creates value for them. This new learning about mathematics in engineering practice motivates the students to engage with mathematics that is important to them. Students have acquired a sense of wanting to learn mathematics; they now see themselves as learners preparing for their future careers. Students' responses include:

- *"The one major finding I got was when I looked up potential career opportunities in environmental engineering ... in all of the job opportunities in environmental engineering, statistics and probability are fundamental in gathering and presenting your data"*
- *"When I was given this task to write a report on mathematics in engineering, I never realised just how much it would open my eyes"*
- *Before this task I took for granted the amount of engineering and maths in the world but I now understand a lot through the eyes of maths"*
- *"Although before the task I wasn't too sure about its relevance to the course, I enjoyed it largely because of my own interest in the subject and I look forward to learning more in this area in the future"*
- *"The thing that I learned most of all is how useful a tool EXCEL really is. I didn't realise how much maths was to be involved in doing a job like this because usually someone else does it ... it was good to see how engineers use maths in day to day tasks"*
- *"I was sceptical as to how much of the maths we are studying would come into use again ... I saw real life ways of applying maths ... I saw the relevance of trigonometry and how it can be used to calculate many different things in relation to engineering ... maths is one of the key ingredients of engineering and I now know to try my hardest to master the discipline so that I can become the best engineer I can"*
- *"Before this report I thought maths was a bit dry and difficult to understand how mathematics can be applied to benefit my job in the future ... but after this report, I now have a much better understanding that mathematics is almost everywhere in the engineering world ... almost every new invention uses mathematics and mathematics*

is also used in everyone's normal lives ... this report helped me develop many new skills and boost my confidence in creating a report"

- *"After doing this assignment I now have a better understanding and respect for maths and algebra in particular than I had before the assignment. Before I began the assignment I felt like it could be a waste of time but on finishing the report I've changed my mind .... My understanding of algebra has improved and so has my respect for the subject ... I can now see the relevance of algebra and its close ties to areas of engineering I will be involved with"*
- *"This assignment certainly opened my eyes and helped to understand things a bit better"*
- *"Having done this report I was surprised just how essential statistics are to successful engineering ... it was good to see that some of the course topics will be so important in a future engineering career"*
- *"This assignment highlights the importance of arithmetic"*
- *"This assignment changed my thinking about mathematics because I can now see how engineers can use it in different ways ... I can see how engineers use mathematics to connect to their knowledge ... I can now see a clearer picture of what we are doing now and in the future"*
- *"This was a challenging and enjoyable assignment that will benefit my future career"*
- *"This assignment was not my most favourite part of maths but before I didn't realise how important maths is especially in the field of engineering practice ... I have learned a lot from doing the assignment"*
- *"Mathematics has been one of my more difficult modules since starting this course ... I have found maths the most difficult subject to self-learn ... I have now realised that at a touch of a button and from the help of Google and similar websites, the information I need and more is available ... I have been introduced to a whole new way of learning and retrieving information"*
- *"I found this assignment interesting and it gives me a feeling for what I can expect after college"*
- *"I learned how some topics in the course and also EXCEL and MATLAB are used in engineering"*

5. CONCLUDING DISCUSSION

The findings in this study highlight the challenges of mathematics education whereby engineering students who have negative feelings about mathematics engage in repetitive learning of mathematics often at the expense of understanding. Students' goals are to learn sufficient mathematics to pass their examinations and they show no desire to learn mathematics outside the curriculum. Students also show low mathematics task value; they are reluctant to investigate and write about useful mathematics (e.g. interesting mathematics or applications of mathematics in engineering practice) or to discuss mathematics generally. They are uncomfortable with the ambiguity of an investigative approach; they prefer the certainty of following set rules in order to achieve the "correct answer".

This study illustrates the impact of students' investigations' of mathematics usage in engineering practice; students, having completed the exercise, show increased mathematics task value; in particular they now see how a variety of mathematics topics and applications can benefit engineers' work. This in turn generates increased student interest in mathematics and particularly in mathematics that is useful to students' future careers in engineering practice.

It is reported that graduate engineers' difficulty communicating mathematics is a significant weakness of engineering education (Goold, to be published 2014). This study introduces students to the concept of communicating mathematics and its relevance. It is also noted that one student who found mathematics difficult to "self-learn", discovered, from the exercise, that mathematics learning can take place outside the classroom given the availability of a variety of mathematics learning resources on-line.

The study illustrates the positive changes in students' mathematics attitudes arising from their insight into engineering practice and how mathematics is used in the workplace. The study also illustrates that feelings about mathematics are an important factor in mathematics learning and that mathematics communication skills benefit engineers.

It is concluded that engaging students in exercises that do not solely rely on the "precise" rules of mathematics and the "correct answer" improves their relationships with mathematics and their motivation to engage with mathematics generally.

REFERENCES

BERNSTEIN, B. & SOLOMON, J. 1999. Pedagogy Identity and the Construction of a Theory of Symbolic Control *British Journal of Sociology of Education,* 2**,** 265-279.

BLACK, L., MENDICK, H. & SOLOMON, Y. (eds.) 2009. *Mathematics Relationships in Education,* New York: Routledge.

ERNEST, P. 2010. Add It Up: Why Teach Mathematics? *Professional Educator* 9**,** 44-47.

FALSAFI, L. 2010. *Learner Identity a Sociocultural Approach to How People Recognize and Construct Themselves as Learners.* Doctorado Interuniversitario de Psicologia de la Educacion.

GOOLD, E. to be published 2014. Putting Mathematics "into a Form that a Non-Engineer will Understand". *SEFI 2014 Annual Conference: Educating Engineers for Global Competitiveness.* Birmingham, UK.

GOOLD, E. & DEVITT, F. 2012. *Engineers and Mathematics: The Role of Mathematics in Engineering Practice and in the Formation of Engineers,* Saarbrücken, Germany, Lambert Academic Publishing.

ILLERIS, K. 2014. *Transformative Learning and Identity,* Oxon, Routledge.

LAMPERT, M. 1990. When the Problem is Not the Question and the Solution is Not the Answer: Knowing and Teaching Mathematics. *American Educational Research Journal,* 27**,** 29-63.

MILES, M. B. & HUBERMAN, A. M. 1994. *Qualitative Data Analysis: A Expanded Sourcebook,* Thousand Oaks, London, New Delhi, Sage.

MOONEY, O., PATTERSON, V., O'CONNOR, M. & CHANTLER, A. 2010. A Study of Progression in Irish Higher Education. Dublin: Higher Education Authority.

NARDI, E. & STEWARD, S. 2003. Is Mathematics T.I.R.E.D? A Profile of Quiet Disaffection in the Secondary Mathematics Classroom. *British Educational Research Journal,* 29**,** 345-367.

PETOCZ, P., REID, A., WOOD, L. N., SMITH, G. H., MATHER, G., HARDING, A., ENGELBRECHT, J., HOUSTON, K., JOEL, H. & PERRETT, G. 2007. Undergraduate Students' Conceptions of Mathematics: An International Study. *International Journal of Science and Mathematics Education,* 5**,** 439-459.

SCHUNK, D. H., PINTRICH, P. R. & MEECE, J. L. 2010. *Motivation in Education: Theory, Research, and Applications,* Upper Saddle River, NJ, Pearson Educational International.

SILVERMAN, D. 2010. *Interpreting Qualitative Data,* London, California, New Delhi, Singapore, Sage.

SMITH, A. 2004. Making Mathematics Count: The Report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education. London: Department for Education and Skills (DfES).

TREVELYAN, J. 2010. Mind the Gaps: Engineering Education and Practice. *Australasian Association for Engineering Education (AAEE) Conference.* Sydney.

WIGFIELD, A. 1994. Expectancy-Value Theory of Achievement Motivation: A Developmental Perspective. *Educational Psychology Review,* 6**,** 49-78.

WIGFIELD, A. & ECCLES, J. S. 2000. Expectancy-Value Theory of Achievement Motivation. *Contemporary Educational Psychology,* 26**,** 68-71.

WIGFIELD, A. & ECCLES, J. S. 2002. The Development of Competence Beliefs, Expectations for Success and Achievement Values from Childhood through Adolescents. *In:* WIGFIELD, A. & ECCLES, J. S. (eds.) *Development of Achievement Motivation.* San Diego: Academic Press.

WOOD, L. N. 2010. Graduate Capabilities in Mathematics: Putting High Level Technical Skills into Context. *International Journal of Mathematical Education in Science and Technology,* 41**,** 189-198.

Linear algebra at work: simulating multibody systems

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Abstract

Finding the equations of motion of a mechanical system is a difficult task for students even in simple cases - much more for the complicated three-dimensional systems studied in multibody dynamics. Using a modern simulation program this task is transferred to the software. Two simple examples will show how the equations of motion are built up systematically by using the equations of the components and simple connection rules. With enough perseverance students can do this manually. Besides basics in mechanics they need a good working knowledge of "abstract" linear algebra, which proves here to be of eminent practical use.

Multibody simulation

Simulating mechanical systems with standard programs that are based on signal processing block libraries is an interesting and basically simple task for students – as long as one knows the underlying equations of motion. To find them students usually have a rather restricted toolbox: They mainly rely on d'Alembert's principle in combination with a free body diagram. This is often not sufficient even for apparently simple examples, not to mention systems of three-dimensional solid bodies connected by joints, which are the main subject of multibody dynamics (Wittenburg (2008)).

Here modern simulation programs based on “physical modelling” come to the rescue: Their building blocks are models of simple physical systems, the connections between them are abstractions of real flanges, wires or pipes transporting physical properties in both directions. In the widely used Modelica language the components and their connections define equations, which are assembled by the simulation program to get the equations of motion automatically (Fritzson (2004)). The two examples presented in the following will demonstrate how this works in practice, even for multibody systems. This not only show the students how these programs work, but adds another method to their “equations of motion” toolbox.

Prerequisites

The physical relations that are used in the MultiBody library (basic dynamics and the Euler equation) are presented in mechanics lectures, a working skill with vector and matrix computations should result from linear algebra lessons. Usually lacking is a deeper understanding of rotation matrices. Particularly the following three relations are generally unknown to the students and have to be presented beforehand – and proven, if time admits:

- computing a rotation matrix from a fixed axis **n** and rotation angle φ

$$\boldsymbol{R} = \boldsymbol{n} \cdot \boldsymbol{n}' + (\mathbf{1} - \boldsymbol{n} \cdot \boldsymbol{n}') \cos\phi - \tilde{\boldsymbol{n}} \sin\phi$$

- writing the cross product as a matrix (i.e. hiding the Levi-Civita tensor ε)

$$\tilde{\boldsymbol{a}} := \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \Rightarrow \boldsymbol{a} \times \boldsymbol{b} = \tilde{\boldsymbol{a}} \cdot \boldsymbol{b}$$

- computing the angular velocity **ω** from a time variant rotation matrix **R**(t)

$$\tilde{\omega} = \boldsymbol{R} \cdot \dot{\boldsymbol{R}}'$$

Physical modeling with Modelicas MultiBody library

Simulation software implementing Modelica usually comes with a graphical editor that allows to build models by connecting predefined building blocks. They define a set of variables and equations between them together with parameters that are fixed during the simulation. The components have connection points ("Connectors") that define the physical quantities of the block that can be accessed externally. They come in (basically) two classes: potential variables are identical at connection points and flow variables add up to zero. These relations are added to the predefined equations of each block to make up the equations of motion for the model (Fritzson (2004)).

In the Modelica MultiBody library a connector describes a local coordinate system (`frame`) relative to a globally given inertial system `world`. For this purpose it defines as potential variables the vector **r** that connects the origins of `world` and `frame` and the orthogonal matrix **R** that rotates `world` into `frame`, both defined in `world`. Corresponding flow variables are the cut force **F** and the torque **M** at the connection, both given in `frame`. In the following we will count **R** simply as three independent variables. Internally it is given by its nine matrix elements together with six constraints given by its orthogonality (Otter (2003)).

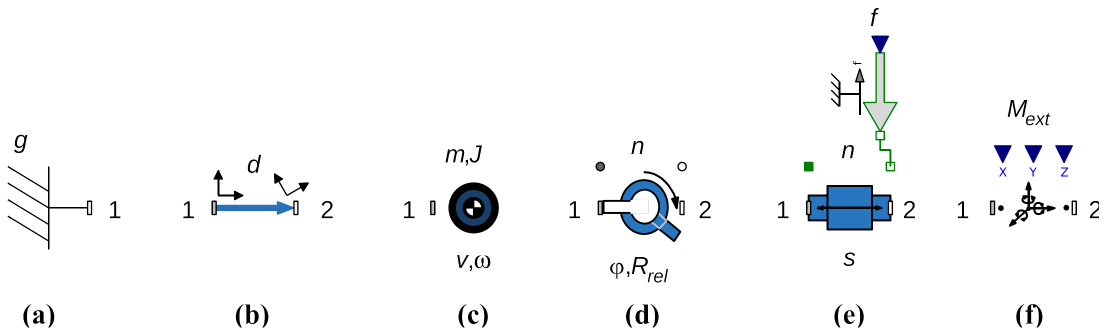

Figure 1: MultiBody components

For the two example models we need six different components (cf. Fig. 1) and their equations, which are given directly by the functionality of a block and basic mechanical relations:

a) `World` supplies the global `world` frame and the gravity acceleration **g**

$$\boldsymbol{r}_1=0 \qquad \boldsymbol{R}_1=\mathbf{1}$$

b) `FixedTranslation` contains two frames with a relative displacement **d**

$$\begin{aligned} \boldsymbol{r}_2&=\boldsymbol{r}_1+\boldsymbol{R}_1'\cdot\boldsymbol{d} & \boldsymbol{F}_2&=-\boldsymbol{F}_1 \\ \boldsymbol{R}_2&=\boldsymbol{R}_1 & \boldsymbol{M}_2&=-\boldsymbol{M}_1+\boldsymbol{d}\, x\, \boldsymbol{F}_1 \end{aligned}$$

c) `Body` describes a rigid body of mass m and moment of inertia **J**. Additionally it contains the internal variables **v** for the velocity and **ω** for the angular velocity.

$$\begin{aligned} \boldsymbol{v}&=\dot{\boldsymbol{r}}_1 & \boldsymbol{F}_1&=\boldsymbol{R}_1\, m(\dot{\boldsymbol{v}}-\boldsymbol{g}) \\ \tilde{\omega}&=\boldsymbol{R}_1\cdot\dot{\boldsymbol{R}}_1' & \boldsymbol{M}_1&=\boldsymbol{J}\,\dot{\omega}+\omega\times(\boldsymbol{J}\,\omega) \end{aligned}$$

d) `Revolute` is a joint that allows a rotation around a fixed axis **n**. Additional variables are the rotation angle φ and the relative rotation matrix $\mathbf{R}_{rel}$ between its two frames.

$$\begin{aligned} \boldsymbol{R}_{rel}&=\boldsymbol{n}\cdot\boldsymbol{n}'+(\mathbf{1}-\boldsymbol{n}\cdot\boldsymbol{n}')\cos\phi-\tilde{\boldsymbol{n}}\sin\phi & \boldsymbol{F}_2&=-\boldsymbol{R}_{rel}\cdot\boldsymbol{F}_1 \\ \boldsymbol{r}_2&=\boldsymbol{r}_1 & \boldsymbol{M}_2&=-\boldsymbol{R}_{rel}\cdot\boldsymbol{M}_1 \\ \boldsymbol{R}_2&=\boldsymbol{R}_{rel}\cdot\boldsymbol{R}_1 & 0&=\boldsymbol{M}_2\cdot\boldsymbol{n} \end{aligned}$$

e) `Prismatic` permits a linear displacement of its two frames along a fixed direction **n**. The internal variable s describes the amount of the displacement. An extra input can be used to bring in an external force **f** along the axis.

$$\begin{aligned} \boldsymbol{r}_2&=\boldsymbol{r}_1+s\,\boldsymbol{R}_1'\cdot\boldsymbol{n} & \boldsymbol{F}_2&=-\boldsymbol{F}_1 \\ \boldsymbol{R}_2&=\boldsymbol{R}_1 & \boldsymbol{M}_2&=-\boldsymbol{M}_1+s\,\boldsymbol{n}\, x\, \boldsymbol{F}_1 \\ & & f&=-\boldsymbol{F}_2\cdot\boldsymbol{n} \end{aligned}$$

f) `Torque` relays an externally given torque $\mathbf{M}_{ext}$ into the system.

$$\begin{aligned} \boldsymbol{F}_1&=0 & \boldsymbol{M}_1&=\boldsymbol{M}_{ext} \\ \boldsymbol{F}_2&=0 & \boldsymbol{M}_2&=-(\boldsymbol{R}_2\cdot\boldsymbol{R}_1')\cdot\boldsymbol{M}_{ext} \end{aligned}$$

Example: robot arm

The first example is a simple model of a robot arm that is driven by an external torque, supplied e.g. by a servo motor. A corresponding Modelica model can be easily assembled using the components described above (cf. Fig. 2). The parameters of the model are fixed as

$$\boldsymbol{g}=\begin{pmatrix}0\\-g\\0\end{pmatrix},\quad \boldsymbol{n}=\begin{pmatrix}0\\0\\1\end{pmatrix},\quad \boldsymbol{d}=\begin{pmatrix}d\\0\\0\end{pmatrix},\quad \boldsymbol{M}=\begin{pmatrix}0\\0\\M_z\end{pmatrix},\quad \boldsymbol{J}=\begin{pmatrix}J_x&0&0\\0&J_z&0\\0&0&J_z\end{pmatrix}$$

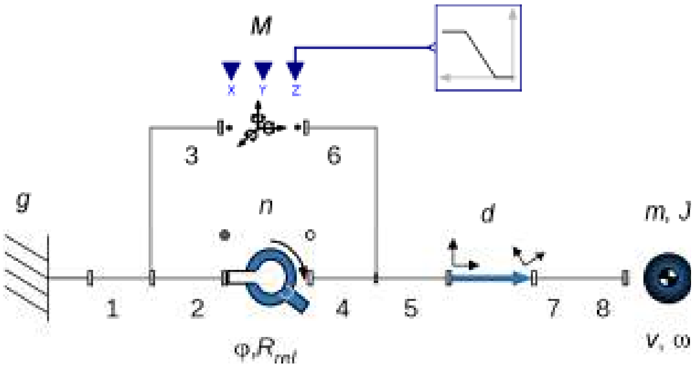

Figure 2: Model robot arm

Collecting all component and connection equations one gets the following rather huge system:

$$r_1 = 0 \quad (1)$$
$$R_1 = \mathbf{1} \quad (2)$$
$$F_3 = 0 \quad (3)$$
$$F_6 = 0 \quad (4)$$
$$M_3 = M \quad (5)$$
$$M_6 = -R_6 \cdot R_3' \cdot M \quad (6)$$
$$R_{rel} = n \cdot n' - \tilde{n} \sin\phi + (\mathbf{1} - n \cdot n') \cos\phi \quad (7)$$
$$r_2 = r_4 \quad (8)$$
$$R_4 = R_{rel} \cdot R_2 \quad (9)$$
$$F_4 = -R_{rel} \cdot F_2 \quad (10)$$
$$M_4 = -R_{rel} \cdot M_2 \quad (11)$$
$$0 = M_4 \cdot n \quad (12)$$
$$r_7 = r_5 + R_5' \cdot d \quad (13)$$
$$R_7 = R_5 \quad (14)$$
$$F_7 = -F_5 \quad (15)$$
$$M_7 = -M_5 + d \times F_5 \quad (16)$$
$$v = \dot{r}_8 \quad (17)$$
$$\tilde{\omega} = R_8 \cdot \dot{R}_8' \quad (18)$$
$$F_8 = R_8 \cdot m(\dot{v} - g) \quad (19)$$
$$M_8 = J\dot{\omega} + \omega \times J\omega \quad (20)$$
$$r_1 = r_2 \quad (21)$$
$$r_1 = r_3 \quad (22)$$
$$R_1 = R_2 \quad (23)$$
$$R_1 = R_3 \quad (24)$$
$$F_1 + F_2 + F_3 = 0 \quad (25)$$
$$M_1 + M_2 + M_3 = 0 \quad (26)$$
$$r_4 = r_5 \quad (27)$$
$$r_4 = r_6 \quad (28)$$
$$R_4 = R_5 \quad (29)$$
$$R_4 = R_6 \quad (30)$$
$$F_4 + F_5 + F_6 = 0 \quad (31)$$
$$M_4 + M_5 + M_6 = 0 \quad (32)$$
$$r_7 = r_8 \quad (33)$$
$$R_7 = R_8 \quad (34)$$
$$F_7 = -F_8 \quad (35)$$
$$M_7 = -M_8 \quad (36)$$

The total number of (scalar) equations is 35x3 + 1 = 106, the number of variables is 8x4x3 + 3x3 + 1 = 106. One starts by eliminating as many variables as possible using the trivial equations and gets

$$\boldsymbol{r}_1 = 0 \quad (1) \qquad \boldsymbol{R}_4 = \boldsymbol{R}_{rel} \quad (9) \qquad \boldsymbol{F}_7 = -m\,\boldsymbol{R}_{rel}\left(\ddot{\boldsymbol{R}}_{rel}{}'\cdot\boldsymbol{d}-\boldsymbol{g}\right) \quad (35)$$
$$\boldsymbol{r}_2 = 0 \quad (21) \qquad \boldsymbol{R}_5 = \boldsymbol{R}_{rel} \quad (29) \qquad \boldsymbol{F}_6 = 0 \quad (4)$$
$$\boldsymbol{r}_3 = 0 \quad (22) \qquad \boldsymbol{R}_6 = \boldsymbol{R}_{rel} \quad (30) \qquad \boldsymbol{F}_5 = m\,\boldsymbol{R}_{rel}\left(\ddot{\boldsymbol{R}}_{rel}{}'\cdot\boldsymbol{d}-\boldsymbol{g}\right) \quad (15)$$
$$\boldsymbol{r}_4 = 0 \quad (8) \qquad \boldsymbol{R}_7 = \boldsymbol{R}_{rel} \quad (14) \qquad \boldsymbol{F}_4 = -m\,\boldsymbol{R}_{rel}\left(\ddot{\boldsymbol{R}}_{rel}{}'\cdot\boldsymbol{d}-\boldsymbol{g}\right) \quad (31)$$
$$\boldsymbol{r}_5 = 0 \quad (27) \qquad \boldsymbol{R}_8 = \boldsymbol{R}_{rel} \quad (34) \qquad \boldsymbol{F}_3 = 0 \quad (3)$$
$$\boldsymbol{r}_6 = 0 \quad (28) \qquad \boldsymbol{r}_7 = \boldsymbol{R}_{rel}{}'\cdot\boldsymbol{d} \quad (13) \qquad \boldsymbol{F}_2 = m\left(\ddot{\boldsymbol{R}}_{rel}{}'\cdot\boldsymbol{d}-\boldsymbol{g}\right) \quad (10)$$
$$\boldsymbol{R}_1 = \boldsymbol{1} \quad (2) \qquad \boldsymbol{r}_8 = \boldsymbol{R}_{rel}{}'\cdot\boldsymbol{d} \quad (33) \qquad \boldsymbol{F}_1 = -m\left(\ddot{\boldsymbol{R}}_{rel}{}'\cdot\boldsymbol{d}-\boldsymbol{g}\right) \quad (25)$$
$$\boldsymbol{R}_2 = \boldsymbol{1} \quad (23) \qquad \boldsymbol{v} = \dot{\boldsymbol{R}}_{rel}{}'\cdot\boldsymbol{d} \quad (17) \qquad \boldsymbol{M}_3 = \boldsymbol{M} \quad (5)$$
$$\boldsymbol{R}_3 = \boldsymbol{1} \quad (24) \qquad \boldsymbol{F}_8 = m\,\boldsymbol{R}_{rel}\left(\ddot{\boldsymbol{R}}_{rel}{}'\cdot\boldsymbol{d}-\boldsymbol{g}\right) \quad (19) \qquad \boldsymbol{M}_6 = -\boldsymbol{R}_{rel}\,\boldsymbol{M} \quad (6)$$

For further simplification one chooses e. g. $\mathbf{M}_8$ and $\mathbf{M}_5$ as basic and eliminates all other torques:

$$\boldsymbol{M}_7 = -\boldsymbol{M}_8 \quad (36) \qquad \boldsymbol{M}_2 = \boldsymbol{R}_{rel}{}'\cdot\boldsymbol{M}_5 - \boldsymbol{M} \quad (11)$$
$$\boldsymbol{M}_4 = -\boldsymbol{M}_5 + \boldsymbol{R}_{rel}\cdot\boldsymbol{M} \quad (32) \qquad \boldsymbol{M}_1 = -\boldsymbol{R}_{rel}{}'\cdot\boldsymbol{M}_5 \quad (26)$$

This leaves the variables $\mathbf{R}_{rel}$, φ, **ω**, $\mathbf{M}_5$ and $\mathbf{M}_8$ together with the equations

$$\boldsymbol{R}_{rel} = \boldsymbol{n}\cdot\boldsymbol{n}' + (\boldsymbol{1}-\boldsymbol{n}\cdot\boldsymbol{n}')\cos\phi - \tilde{\boldsymbol{n}}\sin\phi \quad (7)$$
$$\tilde{\omega} = \boldsymbol{R}_{rel}\cdot\dot{\boldsymbol{R}}_{rel}{}' \quad (18)$$
$$\boldsymbol{M}_8 = \boldsymbol{J}\,\dot{\omega} + \omega\times\boldsymbol{J}\,\omega \quad (20)$$
$$\boldsymbol{M}_5 = \boldsymbol{M}_8 + m\,\boldsymbol{d}\times\boldsymbol{R}_{rel}\left(\ddot{\boldsymbol{R}}_{rel}{}'\cdot\boldsymbol{d}-\boldsymbol{g}\right) \quad (16)$$
$$0 = \boldsymbol{n}\cdot\left(\boldsymbol{R}_{rel}\,\boldsymbol{M} - \boldsymbol{M}_5\right) \quad (12)$$

Using the explicitly given parameter values the equations are simplified substantially and one arrives at

$$\boldsymbol{R}_{rel} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \omega = \dot{\phi}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \boldsymbol{M}_8 = J_z\ddot{\phi}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$
$$\boldsymbol{M}_5 = \left[(J_z + m\,d^2)\ddot{\phi} + m\,d\,g\cos\phi\right]\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (J_z + m\,d^2)\ddot{\phi} + m\,d\,g\cos\phi = M_z$$

The variable φ is the angle of the joint, it is zero at horizontal position. Introducing instead the angle θ against the vertical, one finally gets the well known result

$$(J_z + m\,d^2)\ddot{\theta} + m\,d\,g\sin\theta = M_z$$

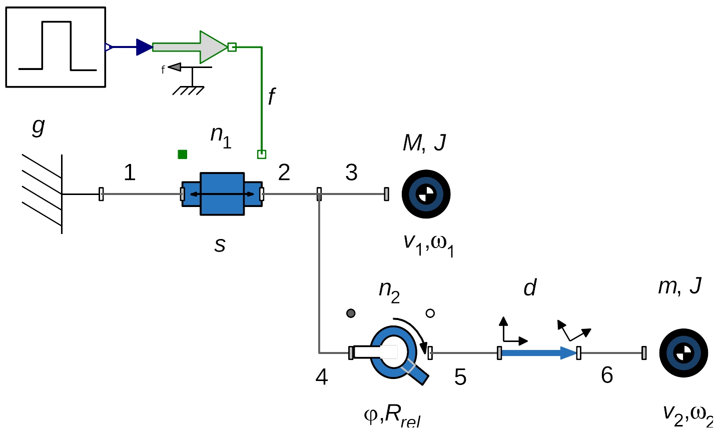

Figure 3: Model gantry crane

Example: gantry crane

The second example is a simple model of a gantry crane. Getting its equations of motion with a free body diagram is usually too difficult for the students, but a simple Modelica model can be obtained easily (cf. Fig 3). The parameters are given like before. Using the equations of the components and adding the relations defined by the connections, one gets 29x3 + 2 = 89 equations for 6x4x3 + 5x3 + 2 = 89 variables. After a simple, but tedious, computation along the lines of the first example, the equations can be reduced to

$$(m+M)\ddot{s}-(m\,d\sin\phi)\ddot{\phi}=f+m\,d\,\dot{\phi}^2\cos\phi$$
$$(-\sin\phi)\ddot{s}+d\,\ddot{\phi}=-g\cos\phi$$

By introducing the angle θ to the normal direction and isolating the second derivatives, one easily brings these equations into the standard form:

$$\ddot{s}=\frac{f+(g\cos\theta+d\,\dot{\theta}^2)m\sin\theta}{M+m\sin^2\theta}$$
$$\ddot{\theta}=-\frac{f\cos\theta+M\,g\sin\theta+(g+d\,\dot{\theta}^2\cos\theta)m\sin\theta}{d\,(M+m\sin^2\theta)}$$

If the students already know the Euler-Lagrange method, one can apply it here to obtain the same result in a shorter way. But this requires a deeper understanding of the physics, especially the formulation of the energies.

Conclusions

What is the use of such tedious computations? First of all they help to demystify the simulation software and make the “black box” translucent. Next they are a good exercise for the students who are often not used to cope with simple but long calculations and can get some practice here. Furthermore they show another method to obtain the equations of motion, which is often a difficult problem. Finally they prove that linear algebra is not just another abstract mathematical theory but has very useful applications in mechanical engineering.

References

Fritzson, P. (2004) *Principles of Object-Oriented Modeling and Simulation with Modelica 2.1*. Wiley & Son.

Otter, K., Elmqvist, H. and Mattsson, S. E. (2003) "The New Modelica MultiBody Library". In *Proc. 3rd International Modelica Conference*, Linköping.

Wittenburg, J. (2008) *Dynamics of Multibody Systems*. 2nd edition, Springer.

Tree-structured Online Mathematics Exercises for Civil Engineering Students

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Abstract

In the 2012/2013 winter semester at RWTH Aachen University, a new type of e-learning exercises has been introduced into first-year mathematics courses for civil engineering students. The project is intended to increase the amount of feedback given to learners – and thus improve their maths exam performance, which had been noticeably declining over the past years. The exercises are created as Moodle lessons. A single lesson covers one particular task, for example a principal component analysis of a given matrix. Via sequentially linked pages, users are guided through the solution process. Depending on their answers to questions, they are forwarded to different further pages. The resulting tree structure allows for adaptive, individual feedback. During the first year of use, the effectiveness of the exercises has been evaluated by a survey and a statistical analysis. Results have turned out positive, and the exercises continue to be used.

Motivation for developing new exercises

Over the past years, first-year mathematics lectures for civil engineering students at RWTH Aachen University have basically stayed constant in their choice of topics and exam tasks (calculus, linear algebra, differential equations etc.). At the same time, the students' exam performances have been noticeably declining ratio-wise. While this may or may not be caused by a change in first-year students' qualifications, measures were sought to counteract this development. High application numbers for civil engineering programmes – with roughly 1200 students registered for one maths lecture – have further contributed to a need for change in the way material is presented to students.

The mathematics lecture itself is supplemented by a large range of various learning activities. A major issue that arose was that most of the activities offered before winter 2012 lacked individual feedback. This was found to be a disadvantage to students, in particular those in the big group of average students. For example, presence-based supplementary courses are still largely based on frontal presentation of possible solutions. While such sample solutions are certainly useful, they leave learners passively receiving instead of actively "doing the maths" themselves. Learning mainly from sample solutions may be sufficient for good students, but it possibly demands too much of average learners. Due to a lack of direct feedback, it is further conceivable that many students did not recognise their own misconceptions and underdeveloped solving skills until they had received their exam results. Efforts for introducing autonomous student work into presence-based courses were and are made, but big group sizes of 100 or more leave these endeavours mostly cumbersome and unrewarding.

During the reading period, learners can work on special homework exercises (which are similar to exam tasks) and hand in their written solutions. These are then marked and commented on by teaching assistants, and finally given back to the students. The service is used by about 400 out of the approximately 1200 students per semester. While it is an effective means of feedback, this method has its drawbacks: apart from the associated workload of checking and commenting hundreds of written solutions, it offers no help to the students on how to exactly solve the exercises. For a large number of students, the exercises are too much to handle. Many learners have trouble approaching complex mathematical tasks in a rational, target-oriented way. Accordingly, many of the solutions handed in are far from acceptable. These students probably need comprehensible, small-step coaching – instead of frontally presented sample solutions – in order to learn how to solve exam-relevant tasks.

Simple online assignments have been introduced even before the exercises described in this paper. Students must still solve about half of these assignments correctly as a requirement for exam application. They are realised as Moodle quizzes (see Quiz module (2012)). Such a quiz consists of a couple of questions which are randomly selected from a question pool, and which are usually easier and less complex than exam tasks. Learners have to first solve on paper the assignments given in the questions, and then enter the correct results into the corresponding text boxes or as multiple choice answers. The only feedback here is "wrong" or "correct", there are no other responses or even progression changes that depend on learner actions.

Thus, two important demands concerning the new exercises' design were that they should provide individual feedback and offer coaching whenever the students might need it. Moreover, the high number of registered students suggested e-learning exercises to be the most practical approach.

Exercise design as tree-structured Moodle lessons

After looking through various e-learning platforms, Moodle was chosen as the one to use for implementing the e-learning exercises (see Moodle (2014) – the exercises currently run on version 2.3.1). A crucial factor for this decision was the "lesson" feature that Moodle offers (see Lesson module (2012)). It allows questions which – after a user has submitted an answer – automatically link to different further pages depending on the answer given. This means that the progression of a lesson can change according to the user's answers, making individual feedback possible. The mathematics exercises were thus devised as Moodle lessons. One exercise always corresponds to one particular task; the idea is to accompany users through the whole solution process. Starting with a task description, users are guided along sequentially linked pages until the exercise is finished and the task is completed. At numerous points, user input is required, which can change the path on which a user reaches the end of a lesson. The resulting exercise structure is vaguely linear, but with "path forks" at user input points. It can intuitively be visualised as a tree structure. (This is only a rough intuition. There are in fact no restrictions as to how pages can be linked with each other, and cycles are possible. Strict tree structures exist only locally.) An example structure for a curve sketching exercise is given in figure 3 at the end of this section.

It is important to note that this tree structure creates a kind of adaptivity, as the exercises do change behaviour automatically to suit user needs (for the concept of adaptivity see Oppermann (1997)). While they do not intelligently keep track of user attributes, the exercises immediately send users to appropriate further paths after input points. This can be called *local adaptivity.*

On a technical level, there are two basic page types in lessons: Content pages and question pages. Content pages consist of a body containing text, formulas, images etc., and of one or more link buttons at the bottom which link to different pages.

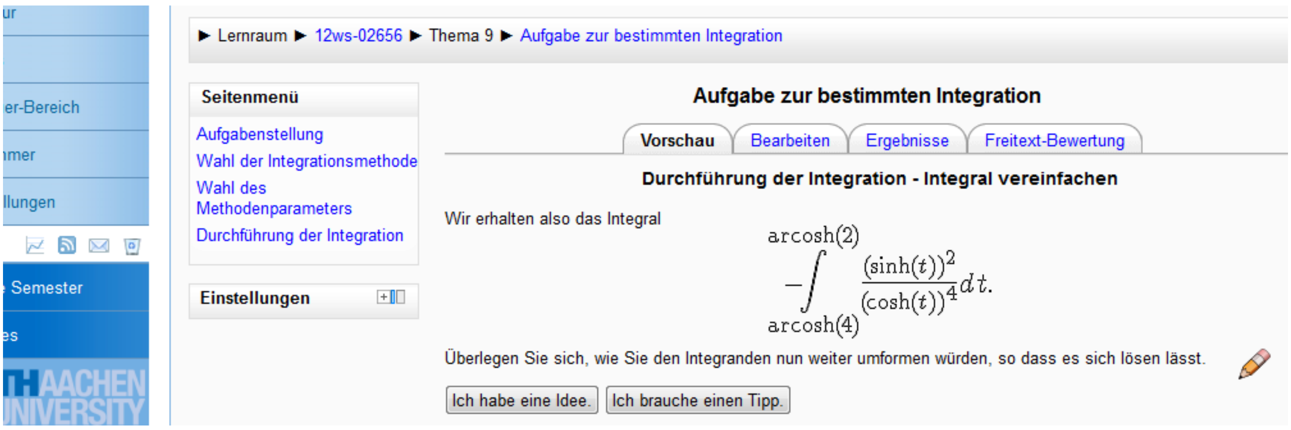

Figure 1. Content page in an exercise on integration – note the menu and pencil symbol.

For example, content pages might explain task-related thoughts, identify the next step, demonstrate sample calculations, or visualise a situation graphically. With these elements, coaching can be realised. Using two or more link buttons at the bottom, it is also possible to let the user decide how to continue: Go directly to the next subtask, or use an auxiliary path that offers tips or advice. An example is shown in figure 1 on the previous page. Unlike content pages, question pages always require input from the user, and can again be subdivided into different types. Question types used in the exercises are multiple choice, short answer (typing an answer into a text box) or matching (correctly associating entries from one list with entries from another). Participants' answers can be used to diagnose misconceptions or calculation errors, and to direct participants to explanation pages accordingly. As solving the exercises should also train students in working autonomously, there are some other design elements which are meant to improve learning behaviour and thorough argumentation. Graphical symbols at the right page margin tell users when they are expected to write/calculate on a piece of paper on their own (pencil symbol), to look up a mathematical concept (book symbol), or to pay close attention to pitfalls (lightbulb symbol). A menu at the left page margin shows important content pages as clickable links. It allows quick navigation through the exercise and acts as a reminder of major solution steps.

Content-wise, the exercises' designs are based upon typical exam tasks. Lessons developed up to now cover the following tasks:

- Sketching a set of complex numbers given by an inequation
- Solving an inequation of real numbers
- Convergence of a sequence and finding its limit
- Convergence of a series
- Sketching the curve of a univariate real function, which includes roots, relative extrema etc.
- Integrating a univariate real function
- Checking a univariate real function for continuity and differentiability at a specific point
- Solving a linear system
- Normalising and visualising a quadric, which includes a principal component analysis of the corresponding symmetric matrix
- Solving a Bernoulli differential equation

From winter 2012 onwards, the exercises have been made available on the standard online platform of RWTH Aachen University – the so-called L²P. As soon as the mathematics lecture reaches a certain topic, corresponding exercises are released for registered students. So far, usage of the exercises has always been fully voluntary.

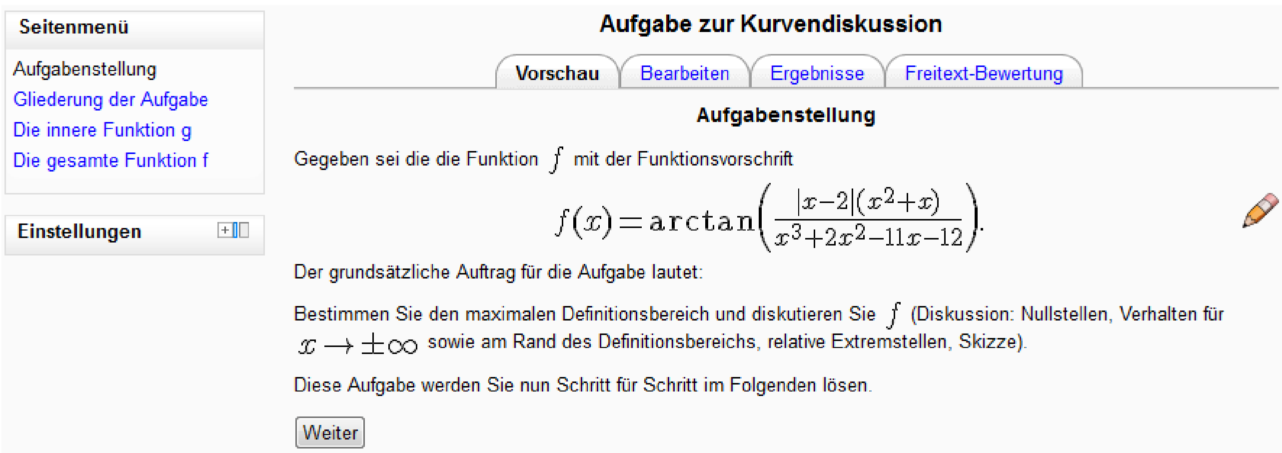

Figure 2: Task description page in an exercise on curve sketching. The structure of the entire exercise can be seen on the next page.

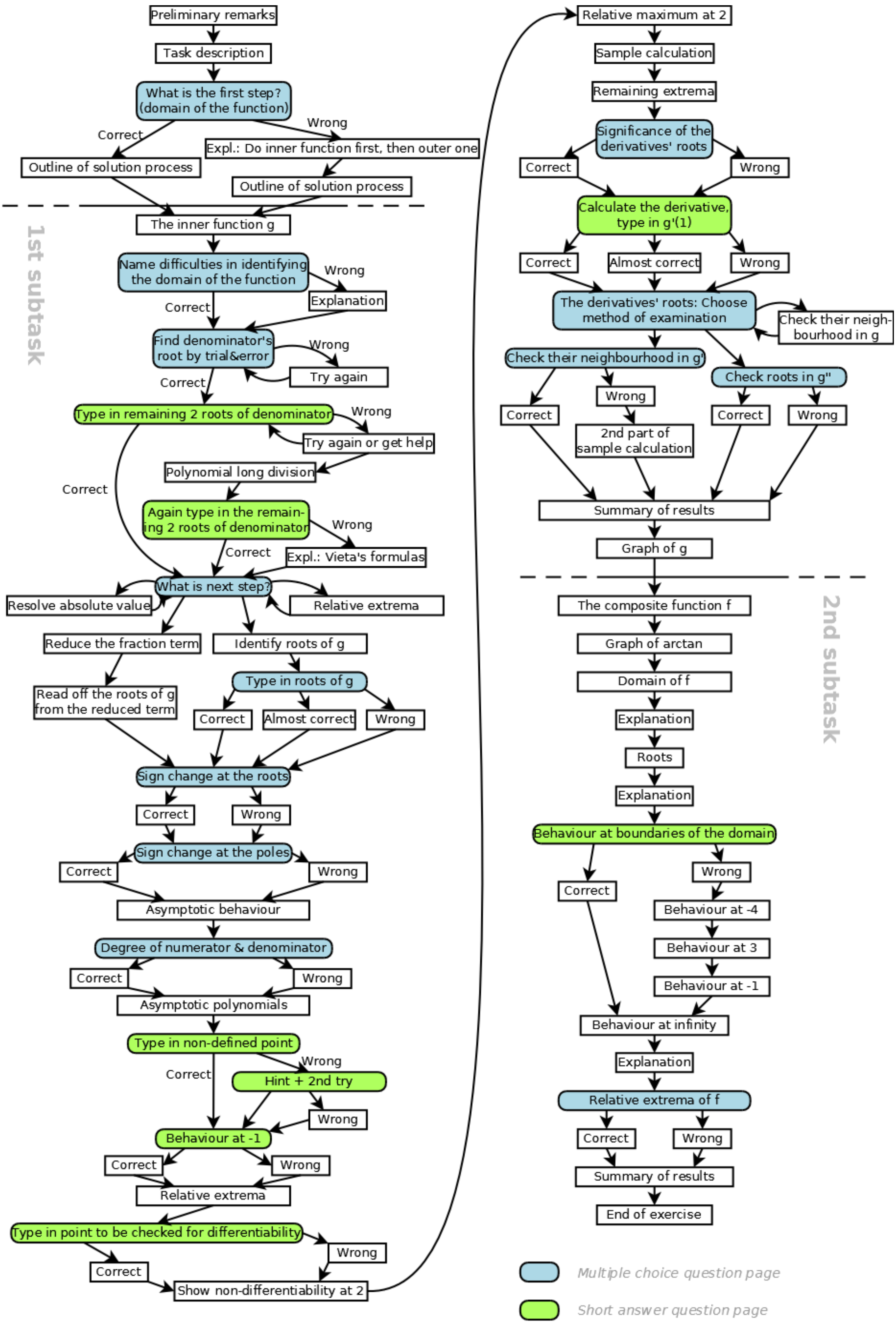

Figure 3. Structure of the exercise on curve sketching.

Assessment of the exercises' effectiveness

Despite being purely voluntary, the exercises have been used by about 45% of 1256 registered students in winter 2012/2013 and by about 35% of 1120 registered students in summer 2013. Exam results in winter 2012/2013 showed some improvement, and in summer 2013, grade distribution and pass rates were significantly better than in the year before. This does not necessarily have to be caused by the exercises' introduction, however. So in order to further assess the impact of the exercises during this first year of use, two methods were utilised: A questionnaire asked students for their opinion, and a statistical analysis investigated the association between exercise usage and exam grade. The results of both methods have turned out positive.

The questionnaire was handed out in a lecture session and also made available online on the L²P platform. It contained questions regarding thoroughness of usage, the exercises' general helpfulness, helpfulness for exam preparation in particular, and appropriateness of exercise design. 270 of around 500 survey participants stated that they had actually used at least one exercise; this group is used as basis for the following percentages. Numbers are rounded to the nearest integer.

- 69% have stated that they have used most or all of the exercises offered, and 51% said they have actually carried out calculations on paper when a lesson page asked them to.
- 67-83% found that the exercises helped them to better understand lecture contents, to apply those contents in mathematical tasks, to see solution structures as a whole, to compose written solutions, and to assess one's own level of knowledge.
- 58% have stated that the exercises were helpful for exam preparation, and 37% thought that without the exercises they would have performed worse in the exam. However, 52% still found the exercises to be too different from actual exam tasks.
- 74-82% found the exercises appropriate in terms of difficulty, length, step width and amount of coaching, respectively.

For the statistical analysis of association between exercise usage and exam grade, a table was set up for each of the two semesters in question (winter 2012/2013 and summer 2013). Rows containing the anonymous student data were combined with columns containing the corresponding number of exercises used and the corresponding exam grade. Correlation and contingency coefficients between exam grade and number of exercises used could then be determined. For the sake of brevity, we only present the findings for summer 2013 in this paper; results for winter 2012/2013 are similar. Numbers are rounded to the nearest hundredth. 683 students sat the exam in summer 2013, which form the basis for the following calculations. Distinguishing between students who have or have not used at least one exercise, and between students who have or have not passed the exam, the corrected contingency coefficient is 0.55. The contingency table is as follows:

Summer 2013	Exam passed	Exam not passed	Total
Has used exercises	296	49	345
Has not used exercises	154	184	338
Total	450	233	683

The Pearson correlation coefficient between exam grade and number of exercises used is -0.43 for summer 2013. Note that German grades go from 1 (excellent) to 5 (not passed), hence the negative sign. Using a *p*-value test, this result can be shown to be statistically significant. A point cloud, as a way to illustrate the association, has turned out uninformative because of discrete values. However, by visualising average grades depending on the exact number of exercises used, a sufficiently informative illustration can be achieved, seen in figure 4 on the next page.

Taking into consideration that more than 50% of students have received a grade between 5 to 3.7, it can be concluded that – on average – using the exercises has brought students to substantially higher rankings in the whole cohort. As a comparison, one can determine the correlation coefficient between exam grade and number of mandatory online quiz assignments solved. This coefficient is -0.19 for summer 2013 (again negative), suggesting a less strong association than with the exercises described in this paper.

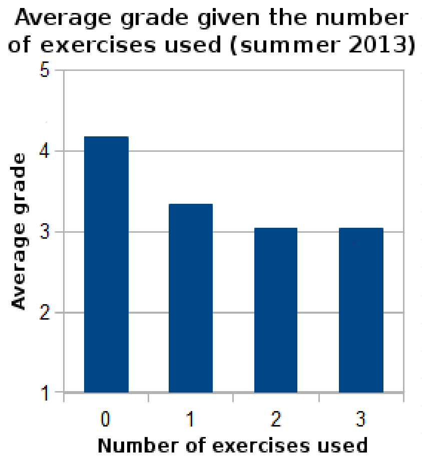

Figure 4. Association between exercise usage and exam grade.

Finally, it should be checked if higher exercise usage is merely an indicator for higher general diligence. As a presumably valid indicator for diligence, we used the amount of mandatory online quiz assignments solved: They test rather than coach, and students only need 50% of them for exam application, so learners who solve more than the required amount can be considered diligent. Percentages of diligent learners among all 683 exam participants and among exercise users in particular both depend on the exact threshold value used. Using various thresholds like 60% or 80% of quiz assignments solved, it has turned out that the proportions of diligent students in the two groups (among all exam participants or among exercise users only) never differ by more than about 8% in summer 2013. We can thus suspect that the above correlation is not significantly distorted by diligence traits.

Conclusion

While the new exercises did not solve all problems, their effects were markedly positive. This includes student opinions as well as effects on examination success. For this reason, the exercises continue to be used in mathematics courses for civil engineering students and have at least partly replaced written homework. The tree structure of the exercises has introduced an element of adaptivity which, up to now, we could not find in any other e-learning system.

Concerning future developments, there are still some aspects to be worked upon. Exercises do not yet cover all relevant topics, so more should be developed especially for the summer course. Because of their strong association with exam success in comparison to the online quiz assignments, it is also worth considering to make the exercises mandatory in some way. A reasonable method of validation would then be needed. For more extensive comparisons, it will be sensible to also determine the effectiveness of other learning activities offered, in particular written homework assignments. And lastly, in order to have a reasonable number of tests, more student data will be evaluated in forthcoming semesters.

References

Lesson module (2012). Available from: <http://docs.moodle.org/23/en/Lesson_module>. [30 April 2014].

Moodle, computer software (2014). Available from: <http://moodle.org>. [30 April 2014].

Oppermann, R., Rashev, R. and Kinshuk (1997) "Adaptability and Adaptivity in Learning Systems" In A. Behrooz, ed. *Proceedings on Knowledge Transfer*, University of London, pp. 173-179. Available from: <http://publica.fraunhofer.de/documents/970292.html>. [30 April 2014].

Quiz module (2012). Available from: <http://docs.moodle.org/23/en/Quiz_module>. [30 April 2014].

Active/Retroactive Feedback in Continuous Assessment of Mathematical Competences

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Abstract

The acquisition of mathematical competences is a difficult process, continuous and recursive, which feeds back to a time reference. After the process of acquisition of competences, the student is submitted to the assessment process. Essentially, it is similar to the acquisition process involving the execution of activities along a limited period of time within a controlled environment. The evaluation process is executed throughout a discrete number of measurement points called "moments of evaluation". In order that the discrete assessment process can be considered representative of the continuous process of acquisition, we define a "continuous assessment model" based on the existing partial order relationships between specific curricular domains of 'moments of evaluation'. In this sense it attempts, at every moment of assessment, to reassess the level of previous skills.

We propose an approach for improvement based on the fact that the acquisition of a competence creates a favourable framework for the acquisition of further competences thanks to the recursive and accumulative nature of the general process of learning. Conversely, a failed evaluation of a competence involves a lack of level in the competence and, quite likely, bad acquisition of previous competences, degradation of the level in the same ones or failure in the measurement process.

Introduction

Usually, the study programme of any given course imposes a temporary distribution of the contents and activities that give support to the learning process. In this sense, the profile of studies contains different topics that are split into units or groups of units linked by their temporal distribution.

The implementation of the plan consists of a set of activities to be run and assessed to measure the level of competence achieved throughout the execution period. In fact, following the Tuning Project (2006), p. 9, competences are developed in all course units and assessed at different stages of a programme.

The acquisition of mathematical competences is a difficult process, continuous and recursive, which feeds back to a time reference. It is parallel to the process of learning and evolves at the time that we run activities of study and transcends beyond the own university studies in synergy with the personal professional experience, regardless of some competences might not have been achieved in due time.

After the process of acquiring competences, the student is submitted to the assessment process. Essentially, it is similar to the acquisition process, i.e. by means of the execution of activities, but along a more limited period of time and within a controlled environment.

We execute the evaluation process throughout a discrete number of measurement points called “moments of evaluation”. Every moment of evaluation has a domain defined by the set of activities necessary for the development of the process (units of study, classes, practices, group work, etc…) associated with a specific curriculum material or collection of thematic units.

In this way, the Tuning Project (2006), p. 28, established that “competences can be placed on a continuum and can be developed thought exercise and education”.

The assessment process is the means to establish if the different competences have been achieved at the required level. This process affects teaching and learning.

According to Niss and Højgaard (2011), p. 190, “In all mathematics teaching the issue of assessment occupies a key position whether it concerns different forms of final assessment, including test and exams or continuous assessment attached to the teaching”.

In this article, we are going to propose a model of continuous assessment based on a concept of retroactive feedback.

Continuous assessment model

Taking into account the continuous nature of the learning process, the assessment process should also be of a continuous nature. The assessment process is similar to the process of acquisition, i.e. by means of carrying out activities, only concentrated in a short space of time and in a controlled and verifiable context.

The assessment process activities are supposed to be defined and representative of the competence to be measured: Its successful execution means a certification of the acquisition of the level required in order to conclude that the competence has been acquired.

The competences are achieved by means of carrying out activities. If we assume that the continuous assessment process may be embedded by the continuous acquisition of competences, we could establish that the carrying out of a number of activities generates a succession of the states approaching to the level of required competence.

An activity should be carried out successfully in order to increase the level of competence and for this it is necessary to define or to "measure" when a successful outcome is achieved. In general, the level of competence acquired should be considered as an estimate when we lose sight of the context in which the activity is carried out (the student is at his/her home, on campus, it might be done by another student, etc.). In fact, a major blockade is to guarantee the identity of the student who carries out online activities. This is the first step if we are to transfer many of the academic activities from the classroom to private university networks and we should try as much as possible to ensure the identity of the students who are running activities online (for some tips see Eplion and Keefe (2007), Qinghai (2012), Cabrera (2013)).

The evaluation process is basically the same, i.e., the completion of a set of activities but in a controlled context. In this case we get a more reliable measure instead of an estimate.

But there is one additional important difference. In the acquisition, the carrying out of activities contributes to increasing the value of the unitary competence. Through the evaluation activities the goal is to measure the level of competences achieved even though eventually they can also contribute to increasing the level of competence.

We carry out the evaluation process throughout a discrete number of measurement points called “moments of evaluation”. Every moment of evaluation has a previous domain defined by the set of activities necessary for the development of the process (units of study, classes, practices, group work, etc…) associated with a specific profile.

In order that the discrete assessment process can be considered representative of the continuous process of acquisition, we define a "continuous assessment model" based on the existing partial order relationships between specific curricular domains of 'moments of evaluation'. In this sense it attempts, at every moment of assessment, to reassess the level of previous skills (scope, maintenance or degradation).

In a real model of continuous assessment, each session of evaluation should assess the competences achieved at that very moment and earlier. As this alternative is not feasible because of time limitations, we propose an approach based on the fact that the acquisition of a competence is based on the acquisition of previous competences, owing to the recursive and accumulative nature of the general learning process.

Conversely, a failed evaluation of a competence involves an insufficient level in that competence and, quite likely, a bad acquisition of previous competences, a degradation of its level or a failure in the measurement process.

To increase the number of moments of evaluation means a qualitative improvement in search of continuity of the evaluation which obviously is impossible to achieve as a mathematical concept. There are several ways to increase this, for instance by transferring assessments to online systems or self-assessment, field investigated by Roberts (2006) among others, so that the number of exams or written activities does not increase significantly and by transferring assessment to certain activities like Lab Sessions as part of the assessment process. This alternative is indeed a common practice in use (see for instance Mínguez/Ferrer/Legua/Sánchez Ruiz (2013) and Mínguez/Moll/Moraño/Sánchez Ruiz (2014)). It is worth noting that the latter may require reversing somehow the process of teaching in lab classes in favour of a flipped approach as investigated by Toto and Nguyen (2009) among others.

Retroactive continuous assessment model

By its own definition, a model of continuous assessment should include carrying out all activities that take place during the learning process throughout the academic year. This is always complicated if not impossible and a selection of activities is tested.

On the other hand, the present classical models of assessment commonly take place at certain times throughout the course. These models require grouping several thematic units corresponding to several different mathematical skills in the same test.

The competences to be evaluated have got a natural partial order so that it could be represented by means of a graph. We will consider each node of this graph to represent a moment of the evaluation of competences.

Let us draw the aforementioned graph as a directed graph where the measurement of evaluation of every node has three sets of arches:

- An incoming arch which represents the contribution of the evaluation of the specific competence to be assessed in that moment of evaluation to the general grade.
- A set of outgoing arches towards the predecessors' competences related to the competences evaluated at that moment of evaluation. They represent a component of retroactivity toward previous assessments with a positive or negative contribution depending on the success of the moment of evaluation.
- A set of incoming arches representing the contribution of specific activities to be taken into account in the evaluation process.

In order to clarify the second item above, we may think that not to overcome the evaluation of calculus of surfaces of revolution may be due to a poor ability in the process of calculation of antiderivatives and not to a deficiency in the approach to the calculation of surfaces of revolution.

This failure should be identified in the process of evaluation (to know where the lack is) and propagate descending arches toward the predecessor's competences and even correct downward the levels previously measured as acquired.

Conversely, successfully overcoming the calculus of surface of revolution implies a good ability in the resolution of antiderivatives, which indicates that the required level of this competence has effectively been achieved or improved. In this case, the graph propagates a null or positive contribution toward the predecessor's competences correcting the level previously acquired.

Conclusions for Education

We believe that this model represents a major approach to the concept of the process of continuous assessment by the properties of the model:

- The representation of the levels of competence in a graph is appropriate since there is a natural partial order between these levels. Partial order is more evident in some disciplines than others. It represents the model of continuous learning by successive approximations and deepening in the levels of complexity.
- A consequence of the partial order is the logical assumption to achieve higher competency levels necessarily implies the acquisition of previous levels, always taking into account the specific characteristics of each discipline.
- The retroactive contribution implies the modification of measurements of competences of previous moments of evaluation, i.e. the measurements have a new variable "time" in the learning interval.
- The partial order represented in graphs allows establishing more accurately the relationships of precedence between different learning units.
- The retroactive contribution model aims to be fairer in the grading of previous competences. It tends to be more reliable in the processes of measurement than the traditional averages, weighted or not, that homogenize measurements of levels of competence through compensation without taking into account the functional dependence between the levels assessed.

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References

Cabrera, Dan. (2013) "Tips to Reduce the Impact of Cheating in Online Assessment", Faculty Development and Instructional Design Center Blog, Northern Illinois University, Accessed via http://facdevblog.niu.edu/onlinecheating (1 May 2014).

Eplion, D.M. and Keefe, T. J. (2007) Practical Tips for Preventing Cheating on Online Exams. *Distance Education Report*, Vol. 11 (no 8), p. 4.

Mínguez, F.; Ferrer, J.R.; Legua, M. and Sánchez Ruiz, L.M. (2013) "Evaluating competences in Mathematics at Aeronautical Engineering." In *Proc. 40th SEFI Conference.* Leuven.

Mínguez, F.; Moll, S.; Moraño, J.A. and Sánchez Ruiz, L.M. (2014) "Mathematics Competencces Assessment at Lab sessions of Aerospace Engineering Freshmen." In *Proc. 44th Frontiers in Education Conference*. Madrid (submitted).

Niss, Mogens and Højgaard, Tomas (eds.). (2011) *Competencies and Mathematical Learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark*. Roskilde University, IMFUFA tekst nr. 485/2011, English Edition.

Qinghai, Gao. (2012) Using IP Addresses as Assisting Tools to Identify Collusions. *International Journal of Business, Humanities & Technology*, Vol. 2 (No 1), p. 70.

Roberts, Tim S. (ed.). (2006) *Self, Peer and Group Assessment in E-Learning*, Information Science Publishing.

The Tuning Management Committee. (2006) "Universities' contribution to the Bologna Process.The Tuning Project". Accessed via http://www.deusto-publicaciones.es/deusto/index.php/en/tuning-en (1 May 2014)

Toto, R. and Nguyen, H. (2009) "Flipping the work design in an industrial engineering course". In *Proc. 39th ASEE/IEEE Frontiers in Education Conference*, Session T4F. Oklahoma City.

To see Mathematics Useful

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Abstract

The paper presents experience with the experimental integration of project work into teaching Basics of Numerical Mathematics in bachelor studies at the Faculty of Mechanical Engineering of Slovak University of Technology in Bratislava (FME STU); and outlines the possibilities of broadening the project learning also on group of regular students within usual concept of course, based on traditional lectures and exercises aided by CAS *Mathematica*.

Introduction

Numerical mathematics is taught at FME STU in Bratislava as a compulsory subject in the summer semester of the second year at the Bachelor Degree of studies. It is scheduled in the range of 26/26 (2 hours lectures and 2 hours of exercises per week) with 4 allocated credits. Understanding of numerical methods directly depends on knowledge and skills achieved in preceding courses: Mathematics I, Mathematics II and Programming, and capability of their application. Besides the other aspects, it depends mainly on knowledge achieved in technical subjects, so this course does not belong to the easiest ones. The faculty does not adopt the system of prerequisites, and each year there are students who attend the course without finishing the preceding courses and without appropriate preparation. In addition, taking into account that credit system allows entering the third year of study already with half of previous year credits, and eleven from fifteen credited subjects in second year provide more credits than Numerical math, the result is that students, in case of need, do not hesitate and choose to skip the exam and repeat the subject in the next year (Fig.2). No wonder that becoming more familiar with credit system, students speculate on credits, and the number of students repeating course has increased. While in recent years, the number of repeating students was about 2 dozen, two years ago, it was 44 (15.5% of enrolment), and last school year it achieved even 84 (28 % of enrolment). Such a high number of students can no longer be simply redistributed to existing classes, and therefore we have since set up a special class for them with modified program. At ordinary exercises work, the emphasis is put on algorithmic processing tasks using computer algebraic software (*Mathematica*). Exercises for repeating students are not held in computer rooms, they focus mainly on understanding the methods and the principle calculations using calculator. Last year we emanated from quality home preparation: individual theory study, after which students were to set up three important questions with answers and to solve simple pencil tasks for each topic. Actually, the exercise was then led by means of discussions, guided dialogue with questions and reasoning, and problem solving examples with analysis. Furthermore, we experimentally involve project work with presentation.

Project learning is a form that requires a high degree of student's ability to work individually, and it is always associated with the highest aims of taxonomies of learning objectives like specific and non specific transfer of Nemierko's taxonomy, or analysis, synthesis and evaluation of Bloom's taxonomy; developing critical thinking skills and competences both in technical and social areas. Although projects already appear at secondary or even primary schools, this teaching method makes an important part mainly at university education, used e.g. for seminar works, and especially for final thesis elaboration. At Slovak University of Technology, project learning makes one of the standards in the Master's degree study.

Numerical Mathematics is a subject that combines knowledge and skills from different mathematical branches. it has an interdisciplinary character and among mathematical disciplines, besides statistics, it is very suitable for engaging project method. Till this time, at bachelor study degree, the method has been undertaken in subject sporadically, mainly on a voluntary basis, typically within the student's scientific competitions, etc. Reasons are prosaic: semester curriculum setting up for 2+2 teaching hours of lectures and exercises per week, high performance of teachers, and impending overloading of students. Since the usual concept of course does not provide much space for such kind of work we took advantage of forming special relatively large group of students retaking the course. As the students had been familiar with curriculum yet, it allowed us to slightly moderate the teaching plans and to assign a time for project work. Together with presentation, it was experimentally included as a compulsory part of the exam evaluation at the bachelor degree of study.

Experiment

In the first week of the course, the students were acquainted with the conditions of granting credits and exam, which included preparation and presentation of the project. Taking into account high number of students per teacher, and appropriate benefits, we made decision for the form of group projects. Mitigating non-equivalence of groups, the students were randomly selected into 16 groups involving ca 5 members. This, at the same time, also allowed us to monitor social skills of students as ability to work in pre-determined group, work allocation, division of roles, responsibility, etc.

The lecturer carefully selected eight main mathematical project topics covering common themes of the subject: equations in one variable, approximation, integrals, differential equations, and linear algebra problems. In the second week of course, the topics were randomly distributed among the working groups. Each group was represented by one voted leader, person responsible for communication with pedagogue, submission of work, division of work among group members and their personal involvement evaluation in percentage (20% normal). The task for each group was to find an application of the determined mathematical topic in problems presented in previously attended courses comprising mechanics, physics, mechanical engineering, thermodynamics, etc; then to describe the problem from both technical and mathematical point of view; to solve the problem using the numerical methods given by topic; to compare, and to interpret the results. Each project had to be submitted in written paper form as well as presented in front of the class. In the end, the students were asked to answer the questionnaire to analyse their opinions and attitudes.

Findings and Discussion

Overall, 77 students participated on the experiment (number of students at the end of the course), two of them were not involved in project work at all, and four of them participated at minimal rate (rated involvement up to 10%). Students of two groups did not manage cooperation in the predesigned group and each group disintegrated into two subgroups. Four students in great manner did the work instead of others (rated involvement higher than 30%). Totally, 18 seminar works were assessed.

- 2 works meet all attributes at highest level
- 7 works had minor shortcomings in numerical part
- 4 works had major shortcomings in numerical part
- 5 works were revealed as internet plagiarisms (including 1 work which partially, and one work which completely did not meet the requirements)

Generally, it could be stated that half of the works (50%) met the requirements of the task and their assessment was positive. This fact corresponds to the character of the class, participants of which repeated the course second or the third time and it is natural that a lot of them were not interested much in the subject.

Questionnaire survey was answered by 69 respondents, representing almost 90% of students at the end of the course. The questionnaire aimed to obtain the opinions and attitudes of students dealing with

- the numerical math course, its high demands, attractiveness and usefulness; the causes of course repetition
- the project, its time demands and attractiveness
- the group work
- project self-assessment

Most of the items in the questionnaire were closed, if necessary, semi-enclosed. In the last open item a respondent was free to give feedback or comments on the form and methods used in the course. Since the questionnaire was anonymous, it also contained items to ascertain gender, grades in relevant, previously attended courses Mathematics I, II, and Programming.

Twelve percent of all respondents were women. Subject Mathematics I had been retaken by 27% of respondents, subject Mathematics II by 12%, and Programming only by 4%. The most respondents achieved marks D or E: 73% in Mathematics I, 75% in Mathematics II, while in Mathematics I 47% had the worst mark E, and 7% had not succeed at all (Fig. 1). Only 18% of respondents considered their knowledge of mathematics weak and nobody insufficient (49% said their knowledge of mathematics was sufficient, 32% considered it to be good or excellent). As the main reason of repeating the course, 56% of respondents denoted prioritize other course, together with considerable part of 17% who denoted other (Fig. 2). For 47% respondents, Numerical Mathematics was difficult or very difficult, and only for 6%, it was easy or little demanding.

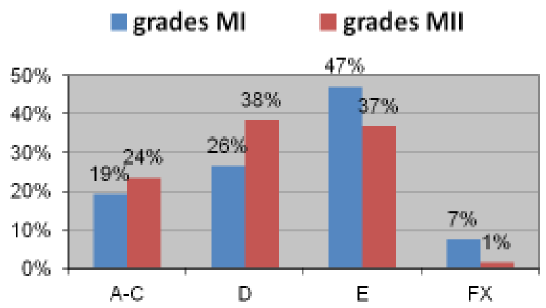

Figure 1. Grades statistics in the courses of Mathematics I and Mathematics II

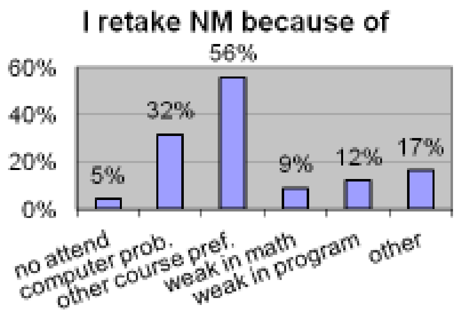

Figure 2. Reasons for retaking the course

Only 33% of respondents were aware of the utility of numerical mathematics, which explains lax approach to the development of the project, while 50% of the works had major deficiencies or were internet plagiarisms; 30% of the respondents were striving to spend "short time" and the other 19% were striving to find existing solutions. Only 31% of students dedicated to the project time longer than 6 hours, it was assumed that the project development will need time about 8–10 hours. 33% of respondents felt intrigued or very intrigued with work on the project.

Students considered cooperation with team members (64%) as the most demanding part of the project work, and admittedly almost half of the students (48%) would prefer to work on the project by themselves. The most demanding aspect of the collective work was considered different approach of group members (47%), time management (41%) and segregation of duties (35%). Second most demanding aspect on project work was judged to be the numerical processing (62%), what clarifies the fact that 50% of the projects were assessed as insufficient in numerical processing.

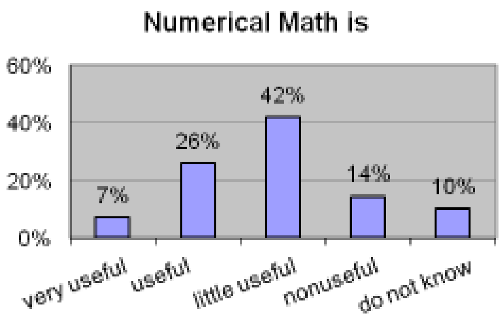

Figure 3. Numerical maths utility

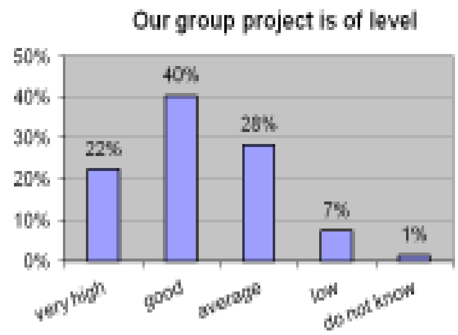

Figure 4. Self-assessment of the project

The most surprising item result of the questionnaire concerned self-assessment, where 62% of the students would evaluate their project to be executed on a very high level and good level, or 90% of students would evaluate their own work as on a very high, good or average level, and only 7% were aware of their low level of execution.

Conclusions and Plans

Insertion of projects into compulsory classwork allowed us to observe several parameters in terms of preparedness in the course Numerical Mathematics and, in a wider context, social preparedness for team work of potential graduates of the bachelor program.

It is possible to claim that half of the students retaking the course of Numerical Mathematics, out of which three quarters were third year students working towards their bachelor’s degree, were not able to create satisfactory project. We need to note that more than a quarter (28%) were not aware of the implications of plagiarism, even though the subject was brought to their attention. The principle of random selection of group members surprised students as they were expecting to create the group in their own preference. We have observed that some people were unable or reluctant to accept determination of the working groups, as two teams disintegrated. Positively, only two students refused to take part in the group work and only four students participated minimally, which together form less than 8%.

Important issues, revealed in the questionnaires, were insufficient critical thinking and inadequate self-assessment. These issues were especially pronounced in two cases. First, it was disproportionality in comprehending one’s mathematical competence: staggering 82% of students assessed their knowledge of mathematics as sufficient, while 54% obtained mark E (lowest passable) or did not pass the course of Mathematics I at all (Fig. 1); and secondly, it was perception of the project elaboration level, where 50% of the projects were deemed unsatisfactory; however, 80% of respondents would think their projects on an average level, or better (Fig. 4).

The findings of the experiment, imposed on students retaking the course, have persuaded us to extend it also for regular students. This school year we managed to include the project work with slight modifications into assessment for one of two main parallel groups of regular students.

In order to provide maximally pleasant encouraging atmosphere we gave up random manner in grouping of students and also in assignment of numerical topics. We let students to make groups consisting of 2-3 members on their own. For each numerical topic we set maximal number of groups and let groups to make their choice but covering all topics in predefined maximal number. The students have been provided with instructions how to elaborate a project and with available consultancy time. On the contrary to larger time consuming complex projects formulated as technical assignments (e.g. Alpers, 2002); at our faculty, these are usually worked out as thesis or large semester projects at technical departments; we have concentrated on basic technical and mathematical skills of students, but developing and enhancing their ability to identify mathematical concepts with at least possible numerical solution within problems presented in previously attended courses. We have aimed at minimalist form of project work focussing on ability

- to find and identify the technical problem as suitable for given numerical method,
- to formulate goal of the task from technical and from numerical point of view,
- to solve the problem using available numerical methods, compare results,
- to formulate and discuss a solution and interpret it in technical application.

Moreover, as a preparation for project work, after each lecture, students have been asked to find adequate application of particular method in technical previously attended courses. As a side effect we expect that students will have better overview of their study subjects and they will be better prepared to graduate.

Our primary motivation to insert the project method into the course at bachelor degree was to encourage students' better awareness of mathematics' connection to technical problems, serviceability of mathematics, what in succession activates the students, increases their inner motivation for Mathematics study, understanding and usage. Based on previous years experience we have come into conclusion that to give technical application examples only on lectures is insufficient. Students have to come over them by themselves and enjoy benefits that accrue from creativity, self understanding, and development of cognitive and social skills.

We suppose that results of this experiment tied with success at examination and questionnaire survey would provide us with sufficient material for a further deeper statistical analysis.

References

Alpers, B. (2002) "Mathematical application projects for mechanical engineers – concept, guidelines and examples." In M.Borovcnik, H. Kautschitsch, eds. *Technology in Mathematics Teaching. Proc. ICTMT 5 Klagenfurt 2001*, Plenary Lectures and Strands. Öbv&hpt: Wien, 393-396.

"Bachelor Degree Study Program for Automobiles, Ships and Combustion Engines." Available on: http://www.sjf.stuba.sk/sk/uchadzacov/akreditovane-studijne-programy/bakalarske-studium/studijny-program-bakalarskeho-stupna-studia-automobily-lode-a-spalovacie-motory.html?page_id=4144 (in Slovak)

"Quality Management Principles of Education at the Slovak University of Technology in Bratislava." Available on: http://www.stuba.sk/new/docs/stu/pracoviska/rektorat/odd_vzdelavania/SMKV/zasady_manazerstva_kvality.pdf (in Slovak)

Student Performance in the Quality Control of Assessment

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Abstract

This contribution focuses on a scheme developed to characterize the level of difficulty of an examination in the courses "Linear Algebra for Engineers" and "Analysis I for Engineers". The scheme depends heavily on experience gained by working with students and on an analysis of student performance in previous examinations. The relation to the competency framework is also considered.

Introduction

In the traditional German tertiary system before the introduction of bachelor degree programmes (by 2006), there were two main types of institutions for engineering education: (technical) universities and *Fachhochschule.* The engineer from a *Fachhochschule* generally had more practical, career-oriented training, whereas the engineer from a university generally completed a more theoretical programme with a greater emphasis in mathematics. The degrees awarded were in both cases a *Diplom* (similar to a master degree), but there was a distinction between a university *Diplom* and a *Fachhochschule Diplom.*

In this traditional system at the Technical University (TU) Berlin, grades in mathematics were not considered to be so important. The frequently changing teams (lecturers, assistants, tutors) for the first-semester courses "Linear Algebra for Engineers" (LinA) and "Analysis I for Engineers" (Ana), each of which at that time serviced well over 2500 students per year, led to varying examination difficulty levels. This variation resulted in little more than a handful of disgruntled students.

Around the time of the introduction of the bachelor and master degree programmes in Germany, the *Fachhochschule* were allowed to designate themselves as "universities of applied sciences" (following a law from the state of Baden-Württemberg 2005). As a consequence, there is no longer a formal distinction between the bachelor at the level of a university of applied sciences and that of a university. As grades play a major role for admission to a master degree programme and making a distinction between universities and universities of applied sciences is considered discriminatory, the students (at least) at the TU Berlin are often at a disadvantage when being considered for a master degree programme in part because of lower grades in mathematics.

In light of the aforementioned changes, the procedures for developing assessment instruments at the TU Berlin were scrutinized and revised. A method for determining the difficulty level of an examination has been realized for LinA and is currently being extended in a slightly different form to Ana. The goals of the scheme proposed in this contribution are to stabilize the difficulty level of examinations without standardizing the test, to provide support for assistants or instructors responsible for constructing the examination, and to ensure aligned assessment.

Preliminary Considerations

In addition to the situation described in the introduction, it is beneficial to have some knowledge concerning the difficulty level of test items. According to Leong (2006), varying the difficulty among items in the test can result in a wide spread of grades, and control over the difficulty level of individual items is useful for maintaining standards from year to year. How to determine the difficulty level of an individual item is a bit tricky, as there are several ways of referencing to provide a basis for the judgement, which will be summarized here according to McAlpine (2002). *Ipsative referencing* takes into account an individual student's previous performance to form a comparison. With the large number of students at the TU Berlin and data security issues, this type of referencing was considered impractical for the current work. *Criterion referencing* deals with a student's mastery of learning objectives, which was considered feasible in light of the standardization of the course contents and presentation in the tutorials. *Norm-related referencing* refers to comparing students with their peers. Considering the large number of students at the TU Berlin and the standardization just mentioned, this type of referencing was also deemed appropriate. McAlpine (2002) warns that this norm-related referencing does not lead to "maintaining standards across time" due to "curriculum and intake changes". However, with the standardization at the TU Berlin, it is possible to at least acquire guidelines using norm-related referencing with cohorts.

Description of Scheme for Course "Linear Algebra for Engineers"

In a first attempt to develop a scheme to judge the difficuly level of an examination, each individual subproblem in examinations for LinA was categorized as easy, medium, or hard according to experience based upon working with students studying for an oral examination (i.e. they had twice failed the written examination). The total points were distributed amongst the categories, and the outcomes were compared. This method worked fairly well in some semesters but not in others. A revision was thus necessary.

In a second attempt, weighting factors were assigned for the points awarded to each step in an expected student solution. Thirteen categories (based upon the German grading system) were taken into account and weighting factors assigned to individual steps of the skills learned in the course. The advantage of this approach, which uses a hybrid of criterion and pseudo-norm-related referencing, is that the scheme can be applied to new examination questions so that testing does not necessarily become standardized.

The scheme was then applied to thirteen examinations in LinA, whereby the points for each individual step or subtask were multiplied by the assigned factor. The results for an examination were then added together and divided by the total number of unweighted points to give the difficulty level of the examination. These levels were then related to student performance in order to obtain reference values for predicting passing rates. This second experience-based approach was rather good with a correlation of –.5646 and a p-value of .0558. One predicted outcome was quite close (2% difference between the predicted passing rate and the actual passing rate in October of 2011) but the other was unsatisfactory (10% difference in July of 2011).

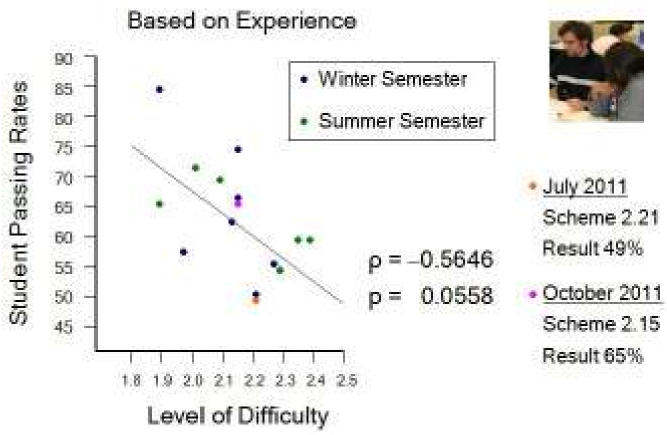

Figure 1. Passing rates according to the level of difficulty with weighting factors gained through experiences.

In a third approach, a cohort of over 100 students in each of several examinations was taken into account. The percentage of points earned for each subtask was recorded, the success of the students for each of these documented, and the weighting factors set forth in the second approach were adjusted to better reflect the actual examination situation. The correlation improved the *p*-values to –.7900 and .0022 respectively. With this scheme, the outcomes of several examinations have been predicted with quite accurate results (July and October 2013 examinations predictions: ±4%).

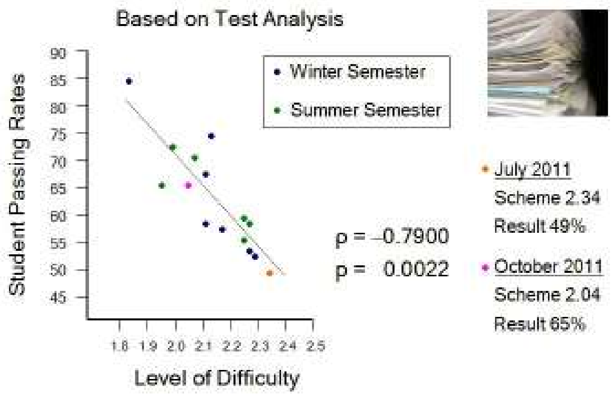

Figure 2. Passing rates according to the level of difficulty with weighting factors determined through previous examinations.

In order to provide a flavour of the scheme, consider the problem of determining eigenvalues of a matrix. If the matrix is triangular, then the weighting factor for the perhaps 1 point task is 1.0 (very easy for the students). If the eigenvectors are known for a “large” (say 4 x 4), concretely given matrix, the question of finding the eigenvalues becomes much more difficult for students. The students will often not relate the eigenvectors to the eigenvector equation, which is perhaps the intended solution method, but rather perform a cumbersome computation. During the course of the solution process, arithmetic errors can turn the problem into a much more difficult one, so that a difficulty factor of up to 3.7 results for a task worth about 2 points.

Here is an example illustrating why it is important to consider the student solution and performance on a problem instead of a mathematician's solution. The student is given an explicit orthogonal matrix *A* and an explicit vector *b*. They are first asked to determine A^{-1} and then to solve the linear system of equations $Ax = b$. Even though the first part of the problem only requires transposing the matrix, many students will use the Gauss algorithm to determine the inverse. In the second part, the Gauss algorithm is often applied anew.

The actual student performance in the statistical analysis of the individual problems in an examination does not take into account the time lost by solving tasks in such a cumbersome way. This may affect the weighting factors obtained for other problems for which students consequently had little time. One compensation measure is to match the difficulty level of the entire examination assigned by the scheme to the overall performance of the students. Another measure is to use the easier problems that frequently occur as a sort of calibration device. If anomalies occur by other examination questions, then the average outcome taken over all students compared to those who actually worked the problem may then be worked into the weighting factors. Moreover, the distribution of the weighted points, which loosely corresponds to the radius of coverage in terms of the SEFI "A Framework for Mathematics Curricula in Engineering Education" (2013), is also important but does not appear explicitly in the weighting scheme. There is thus still a great deal of subjectivity in the scheme, but it is not clear that the time required to further improve the scheme is worth investing. The results are used as a guideline and acceptably predict student performance, which is influenced by more factors than just the difficulty level of an examination as defined here.

Reliability of the Scheme

In order to test the reliability of the scheme, an assistant who was relatively new to the course and one who had been in the course for six semesters were given a preliminary version of the scheme. They and the designer of the scheme calculated the difficulty level of a new examination. The results were somewhat differentiated (2.67, 1.90, and 1.91 respectively). The relatively new assistant predicted a success rate of only about 35% whereas the more experienced individuals predicted a success rate of about 77% with the actual passing rate being 80%. It was therefore necessary to revise the instructions. With this revision, the second examination received the following estimations for the level of difficulty: 2.29, 2.30, and 2.33, which adequately described the difficulty of the examination (predicted: 54%, actual: 50%).

Prior to the summer of 2008, there were many anomalies in the grading of examinations at the TU Berlin. The tutors chose the problem they wished to correct. Deviations to the sample solution to a given question were not discussed, which led to quite a bit of subjective grading. Since then, several measures have been taken to remedy this situation in LinA. Tutors are assigned to problems based upon abilities demonstrated in previous grading situations and/or during facilitation of their tutorials. Instructors or assistants are delegated as problem leaders, and alternative solution strategies are discussed to ensure fair grading. During the analysis of examinations, the grading in

LinA proved to be more homogeneous than in Ana, so that these new measures appear to have had a positive impact. Meanwhile, these methods have been transferred to Ana.

Connections with Competencies and Discussion

The SEFI "A Framework for Mathematics Curricula in Engineering Education" (2013) sets forth competencies that are expected learning outcomes of the specific educational programme. Especially in first-semester courses, such as LinA and Ana, it is not expected that each student will master certain levels of each of the eight competencies. The knowledge and capabilities of the students are too heterogeneous, so that some students will need more time and motivation from other coursework to attain certain competencies. The focus here is thus on a selection of the competencies that are aligned with the competencies stressed in the respective course. The categories considered are "Thinking mathematically" (TM), "Reasoning mathematically"(RM), "Posing and solving mathematical problems" (PS), "Representing mathematical entities" (RME), "Handling mathematical symbols and formalism" (HMS), and "Communicating in, with, or about mathematics" (C).

A sample examination from 2013 containing 22 problems for a total of 60 points was investigated to determine which competencies had been tested and to what extent. The following table illustrates the outcome of the investigation. In this table, the number of test items, the number of points awarded, the average weighted points, the range of the weighted points, and the percentage of points a typical student would be expected to earn are depicted for each competency under consideration. Note that each test item generally involves multiple competencies, so that the total number of points, for example, exceeds the 60 possible points.

Competency	Number of Test Items	Total Points	Average Weighted Points	Range of Weighted Points	Expected Percentage of Earned Points
TM	6	13	2.20	1.35 – 3.30	72
RM	7	17	2.19	1.00 – 3.30	73
PS	15	45	1.96	1.00 – 3.30	76
RME	5	19	2.11	1.10 – 3.30	75
HMS	3	7	1.47	1.00 – 3.30	87
C	5	13	1.65	1.35 – 2.30	81

Table 1. Competencies Tested in April 2013 at the TU Berlin

The competency PS occurred most often and consisted only of problem solving. The average weighting of the points being 1.96 with the given range means that the typical student would earn approximately 75% of the 45 points. (Note that a student must earn at least 50% of the total points to pass the examination.)

The remaining competencies with the exception of HMS were of similar importance in terms of the total number of points. The weighted points were also close for TM, RM, and RME. Although C appears much easier since its weighting is only 1.65, it is not expected for the students to actually obtain 81% of the points. In these types of problems, the students are required to explain some mathematical aspect and the impression is that they are generally weak in this area. So again, there is some limitation to the scheme with respect to the competencies. The apparent underrepresentation of HMS may arise from the fact that this competency was only assigned to a problem when the point of the item included testing this specific competency. Naturally there is always some degree of handling mathematical symbols and formalism in any examination.

The expected percentages displayed in Table 1 are quite high, so that it appears as if the typical student earns at least 72% of the total points possible. The actual passing rate for the examination under consideration was only 61%, which matches the prediction (see Figure 2 with difficulty level 2.17) based upon a comparison of student performance in earlier years as described in the previous section. One reason for the discrepancy could be that students are not equally strong in all competencies. Further investigations will be necessary to understand how the competencies interrelate and affect performance in an actual examination situation.

References

Alpers, B. et al. eds. (2013) *A Framework for Mathematics Curricula in Engineering Education*. Accessed via http://sefi.htw-aalen.de/ (30 April 2014)

State of Baden-Württemberg, Germany (2005) *Gesetz über die Hochschulen in Baden-Württemberg.* Accessed via http://www.zuv.uni-heidelberg.de/imperia/md/content/einrichtungen/zuv/recht_u_gremien/gesetze/lhg_stand_02.03.09.pdf (30 April 2014)

Leong, S. C. (2006) On varying the difficulty of test items. Paper presented at the 32nd Annual Conference of the International Association for Educational Assessment, Singapore. Accessed via http://www.iaea.info/documents/paper_1162a1d9f3.pdf (30 April 2014)

McAlpine, M. (2002) Principles of Assessment, *Bluepaper Number 1*, University of Glasgow. Accessed via http://caacentre.lboro.ac.uk/dldocs/Bluepaper1.pdf (30 April 2014)

Self-Organized Learning in Mathematical Education of Engineers

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Abstract

The concept of Self-Organized Learning (SOL) according to M. and C. Herold is one of the most challenging approaches for sustainable learning with a clear neuroscientific foundation. Putting it into practice implies a tremendous restructuring in the teaching organization compared to the current conventional university lectures. However, the introduction of some of the SOL methods has already contributed noticeably to the activation of students.

Self-organized systems tend to avoid alterations. As learning requires changes for the learning subject its steadiness is worthy of consideration. The conditions of learning dispositions are explained using the model of the examination board. SOL concepts result in a teacher's attitude of being more of a companion than an instructor to the students.

Experiences with three of the SOL methods are presented here: competence list, advance organizer and the jigsaw method. Competence lists give the learning requirements in complete sentences and refer to appropriate exercises. By setting up the competence lists the SEFI MWG competency framework forms one of the most important media to define the requirements of the course. The advance organizer presents, within a map, links to the content and a sense of the course or part of it. The jigsaw method gives students the responsibility to acquire a theme and impart it to their group members.

Self-Organized Learning

The concept of Self-Organized Learning was developed in the 1980s by Martin Herold. The book on SOL describes the learning process, its neuroscientific foundations as well as the SOL teaching practice (Herold and Herold 2013). The main issue of the SOL concept is that students themselves organize their own learning process. The teachers' task is to support the students in this process and to provide opportunities for reflection. SOL is based on constructivism and models of self-organized living systems. These systems can be individuals as well as organisations or societies. The functioning of self-organized systems is enlightened by two models: the comfort zones and the examination board.

Steadiness of Self-organized Systems: Comfort Zones

Natural adaptive systems must demonstrate some resistance to new information to survive. On the other hand they have to react immediately to life-threatening events. Therefore they need mechanisms to judge new information on its relevance. Within a very short time the system decides to process the information or to ignore it. In the latter case the system remains unchanged. The processing is illustrated in figure 1. The stable situation (1) is jolted from outside. If the system decides to react an amount of energy is needed to change thinking or acting habits. The system runs through a phase of instability (2) where assistance may be needed. If not enough energy is spent the system will fall back to comfort zone 1. Otherwise it experiences the momentum (3) before reaching the new comfort zone 2, where the new information is integrated (4).

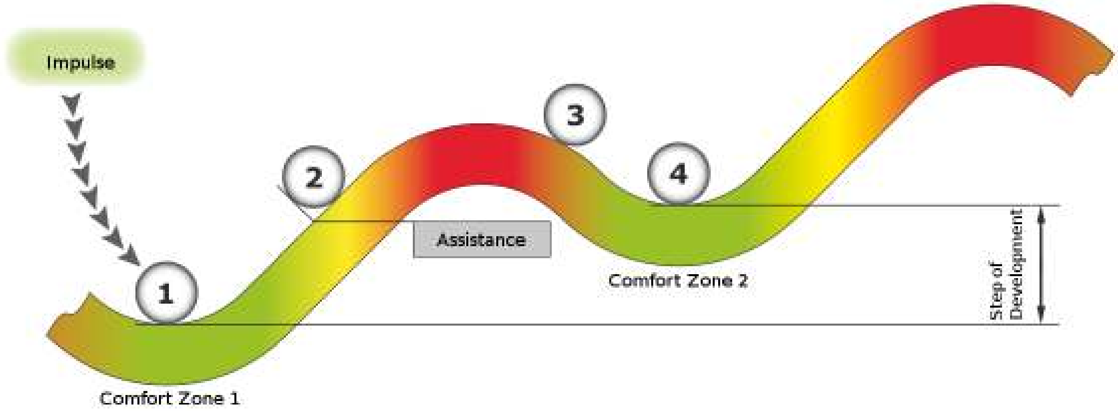

Figure 1: Comfort zones. Copyright: SOL-Institut (Herold). English version: M. S.-G.

The model of comfort zones illuminates important emotions in the learning process that should be reflected by students and teachers:

- The willingness to adapt impulses from outside is a prerequisite to learning.
- Successful learning processes need a minimum stability.
- It is helpful to know that an instable phase will accompany the learning process.
- Within the instable phase support rather than instruction is needed.
- The student’s own momentum while reaching a new comfort zone is most satisfying.

The best motivation is to remind students to former learning successes and their great emotions.

Readiness of Self-organized Systems: Examination Board

The model of the examination board in figure 2 illustrates the complexity in the internal regulation of self-organized systems. An external impulse hits the system. It is shown as a circle with a dashed border. The “examination board” makes a decision on either to process or ignore the impulse in a very short time. Within this period a consultation of intern system components which are the basic needs such as eating, drinking, sleeping and warmth, as well as previous experiences and emotions.

The decisions of the examination board are directed by three principles: target orientation, self-similarity and self-optimization.

If basic needs are unfulfilled, target orientation will reject external impulses if they are not important for survival. An impulse is compared with previous experiences. If known habitual reactions are triggered or if it is completely strange it will be rejected. For learning purposes impulses similar to known patterns are optimal. A state of mind with strong emotions has serious impact on the readiness to learn. Hüther underlines that enthusiasm mobilizes energy to overcome learning hurdles (Hüther 2011).

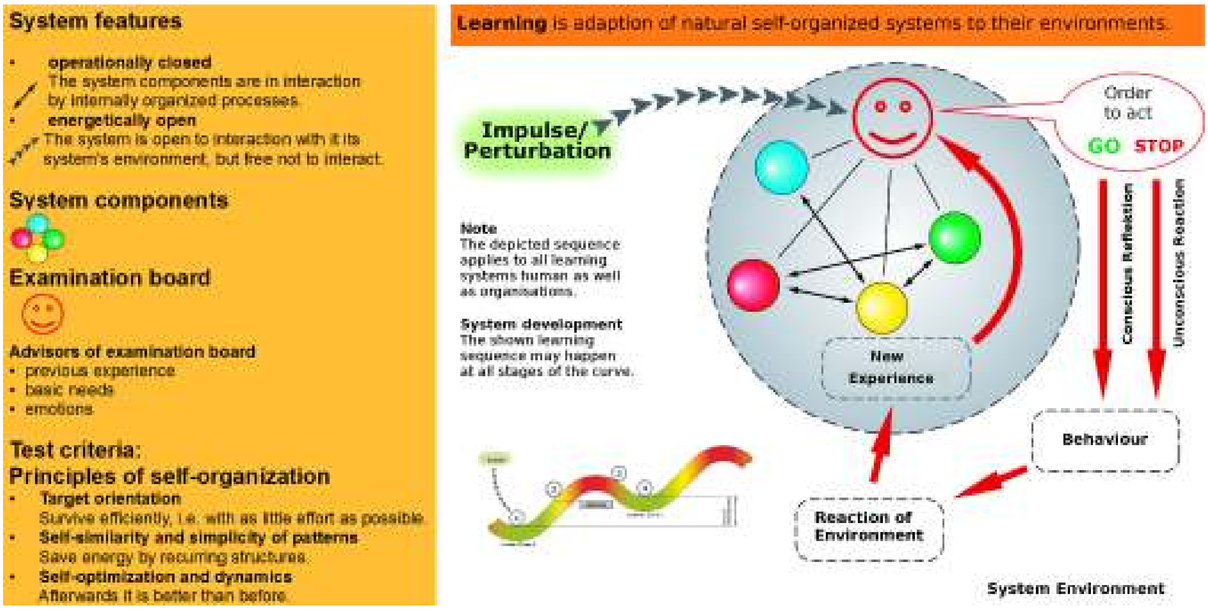

Figure 2: Learning system. Copyright: SOL-Institut (Herold). English version: M. S.-G.

Beside the internal system instances, the environmental response to the systems behavior plays a key role. An example from a mathematics class may illustrate this particular fact:

Three economic functions are to be worked out with the jigsaw method. Some students participate reluctantly. In the evaluation they claim that the themes were too difficult for them: “you must not examine the themes.” The teacher admits defeat. Two weeks later the teacher proposes the jigsaw method for another theme. Before starting the students provide conditions on their participation. Deviant from the teacher’s intention, the students have learned the lesson: “resistance is rewarded”.

The examination board model and the given example sort out the responsibilities: the teacher is not responsible whether the students have not learned nor whether they are not ready to learn. But teachers are responsible for the “system environment”. Clear rules for course and examination are the keys to the responsibility that students take for themselves.

The Teacher’s Attitude

The SOL-concept gives an attitude characterized by the following (Herold 2013)

- Respect the basic needs of the students.
- Accept that learning is self-organized and
- that teaching seen as a synchronous transfer of knowledge is impossible.
- Understand that teaching is the design of learning environments.
- Give up instructive teaching and don’t judge this as loss.
- Start being a learning companion and see this as a gain in quality.

If these attitudes are lived by then the common complaints of math lecturers become less important. The lack of positive learning results may not be explained by the laziness of students. This would make the teacher an enemy of self-optimization that evolution had once implanted into our brains. It is not the teacher who is responsible for the students learning. The real experts for learning are the students themselves.

Competence lists and the SEFI MWG Competency Framework

Competence lists are parts of a learning environment that enable the student to take the responsibility for his own learning process. For each single competence three parts of information are given:

- the competence in a complete sentence,
- at least one reference exercise with a focus to this competence,
- a taxonomy level.

Nr	Optimization on one variable, Newton I am able to …	Exercise	Tax	✓
90	determine extreme values by differential calculus	B12-1,	A	
91	obtain statements on monotony and curvature from the derivatives of a function	B12-2,	A	
92	quote the steps in an optimization problem.	B12-3, 4, 5, 6, 7	K	
93	solve an optimization problem	B12-1, 3, 4, 5, 6, 7,	P	
94	list critical areas for the Newton method for finding roots	B12-8-b	K	▪
95	approximate the solution of an equation with Newton's method	B12-11,12,13,14-c	A	▪
96	indicate the achieved accuracy with Newton's method	B12-8-d	U	▪

Figure 3: Excerpt of a competence list on analysis

The competences shown in figure 3 could be summarized with terms as: extreme value, monotony, curvature, optimization, Newton's method. In contrast to using a single word, the descriptions are given in full sentences to avoid possible misunderstandings. Referring to an exercise the learning requirements become clear. The taxonomy is defined in 4 levels: Know (K), Understand (U), Applicate (A), Problem solving (P). The taxonomy helps to understand the depth of understanding that is needed. The last column allows to check off the already learned competences and thus to make the learning progress visible.

Competence lists show the meta-level of a subject, which aids memorization.

The Curriculum of the SEFI Mathematical Working Group has been developed through two decades in a series of seminars and workshops on mathematical education for engineers (SEFI 2002, 2013). Its curricular part is organized in four core levels, from Core Zero containing the indispensable prerequisites for each engineering education, to Core 1 containing the basic knowledge for general Engineering Science, to Core 2 showing advanced knowledge essential to special engineering disciplines, and up to Core 3 which comprises highly specialist mathematical knowledge. Within each core the single themes are sorted along the five mathematical disciplines Algebra, Analysis and Calculus, Discrete Mathematics, Geometry, Statistics and Probability. With the exception of Core 3 the curriculum gives explicit learning objectives in great detail making it to an excellent foundation to plan courses with competence lists.

Core 0 has 215, Core 1 has 205 and Core 2 has 260 learning objectives. For my courses I defined 550 learning objectives. Half of which are translations of SEFI curriculum objectives. These are referred to, at least partially, in about 1100 exercises. A self-written software supports the creation of competence lists, once the exercises are assembled in a course definition. This reduced the time for establishing the competence lists to about a tenth of the time needed, compared to when this was done manually. The mathematics and statistic courses at MKT Lingen are 4 hours a week through a 14 week semester. 80-120 learning objectives are a realistic size for one course, so that a two semester course does not cover Core 1, provided that the students entering tertiary education have many deficiencies in Core 0 objectives.

Using the SEFI curriculum helped a lot in finding weak points in the prepared material especially the exercises. An examination of exercise assemblies showed the same danger: very often few learning objectives are repeated with exercises of the same type. The curriculum thus helps to ensure the quality of education.

Course evaluations point out that students are using the competence lists successfully. The acceptance of the examination requirements has essentially improved as well as the examination results. For the practice it is important to provide the competence lists at the beginning of the courses.

Advance Organizer and Jigsaw Method

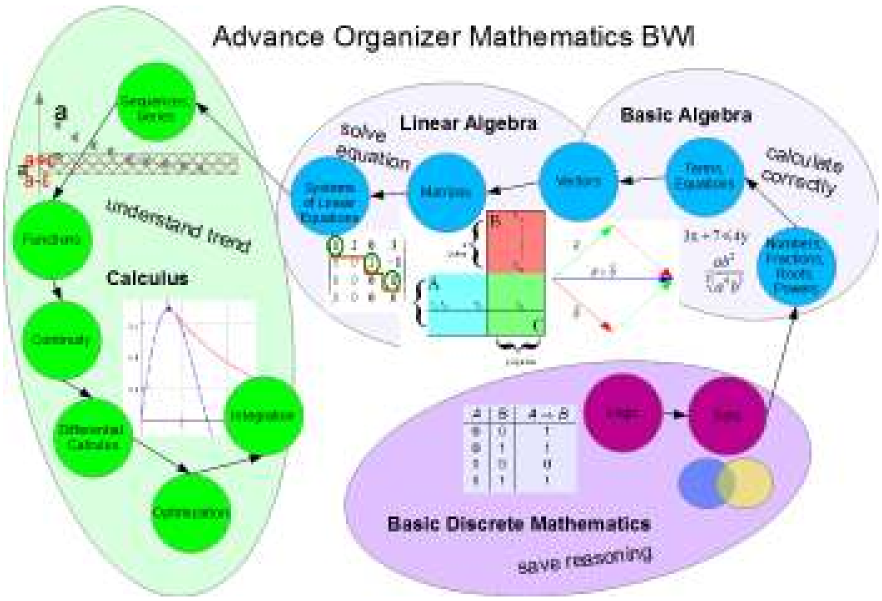

Figure 4. Advance Organizer for Undergraduate Mathematics course

The Advance Organizer and the Jigsaw method are presented in short. The Advance Organizer gives an overview on themes and mission of the course or a part of it. It is composed of four elements: pictures, terms, relations and messages. Students keep the overview through the course better by use of this simple landscape.

The Jigsaw method was developed by Elliot Aronson in the 1970s (Aronson 1996). From the many methods activating students it is one of the most common one. The course is divided into jigsaw groups made up of 3-4 members according to the number of themes that are to be worked out in the unit. Each member of the group is responsible for one theme. After the jigsaw groups have been formed and the themes decided on, members of different groups with the same theme form expert groups. In about 30 minutes they have to prepare a 5 minute reports. Everybody presents the report on returning to the original jigsaw group. The jigsaw method is favored by most students. Because of their increased engagement several groups need a longer time than planned. Material for the topics has to be carefully prepared. Additional summaries of the topics in a following session are advisable.

Conclusion

The SOL-concept gives an integral view on learning processes. Its methods are to be introduced step by step. The models on self-organized learning systems help to define the roles and the responsibilities within a school or university. Joint sights on teaching as comprised by the SEFI competency framework are of high value in the implementation of the SOL-methods. There are two exciting challenges to the teachers in mathematical education of engineers: to activate your students within the teaching and to keep your enthusiasm for mathematics alive and then share it with your students.

References

Aronson, Elliot and Patnoe, Shelley. (1996). *The Jigsaw Classroom: Building Cooperation in the Classroom*. Longman 2nd Edition

Herold, Cindy and Herold, Martin (2013) *Selbstorganisiertes Lernen in Schule und Beruf.* Beltz Verlag, 2nd Edition

Hüther, Gerald (2011) *Was wir sind und was wir sein könnten*. S. Fischer

SEFI Mathematics Working Group (2002): *Mathematics for the European Engineer - A Curriculum for the Twenty - First Century*
http://mwg.lboro.ac.uk//core.html

SEFI Mathematics Working Group (2013) *A Framework for Mathematics Curricula in Engineering Education.*
http://sefi.htw-aalen.de/Curriculum/Competency%20based%20curriculum%20incl%20ads.pdf

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Signal reconstruction as scientific project work in Electrical Engineering

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Abstract

In telecommunications engineering signals $x = x(t)$ are often not completely received. Causes can be noise or limitations in measurement. Nevertheless, often a-priori information allows satisfactory reconstructions of signals. Here it is supposed that the phase spectre of the signal and possibly some more information of convex type are known. The solution of the problem needs engineering mathematics including Fourier methods and some parts of convex analysis. There are already some research papers informing about the principal solution of the problem and giving experimental results based on mathematical software. The proposed mathematical methods contain some parameters which had to be optimized to get good reconstructions. The project framework offers a lot of free space for students' own creativity. Students can study research papers, can check the given results and can make new experiments. They can use MATLAB and develop MATLAB packages for experiments. They can modify the reconstruction algorithms and can study the effects on the quality of reconstruction. The students should work in teams in competition with other teams and work independently except for some guidance from supervisors. At the end of the project presentations can be given reporting the work done. In special cases publications can be written and submitted to an engineering journal. The project work stresses aspects of engineering which are partly neglected in classical classroom teaching, but also partly in modern teaching methods transforming engineering studies into vocational training. Students should gain a particular insight into scientific work.

Introduction

Nowadays a lot of engineering students start at university with unsatisfactory basic knowledge in mathematics and natural sciences. Where a failure to grasp the relevance and motivation for studying these subjects is a major issue, mathematical or interdisciplinary project work can be particularly beneficial for the students involved. Often students at universities of applied sciences believe that they need no theory in their future life. It may be that they can get a good job in engineering which is based only on practical abilities. But the big challenges of the future will demand as many highly educated engineers as possible. Therefore all students should have an opportunity to develop a certain understanding of scientific work. Teaching at universities of applied sciences must not be reduced to purely vocational training.

In this paper a project is elaborated which can motivate students of electrical engineering to learn more mathematics. This project also introduces some aspects of scientific work. Schott (2000) presents the mathematical background used in this project. Basics and former investigations in this field can be found in a paper of Hayes (1982), and in some papers collected and edited by Stark (1987) in book form. Schott

(2013) also discusses this project of signal reconstruction, but set up with other priorities.

Project work

Projects in general have important advantages compared with classical classroom teaching.

- They can be particularly *attractive* and *motivating*.
- They often have a *practical value*.
- They support *self-responsibility* and s*elf-initiative*.
- They need *cooperation* and *communication* in and between teams.
- They often feature *flat hierarchies* and run in a stimulating atmosphere.
- They are often *flexible* and *open ended*. Hence the level of understanding and difficulty can be adjusted appropriately.
- Advisers or *supervisors* are ready to help.
- Professional and *scientific work* can be trained at a certain level (job preparation).

Nevertheless good projects need special effort and engagement from the staff members involved, both in preparation and realization. The project of signal reconstruction presented here concerns in the first instance mathematics and telecommunication engineering. It can be embedded in both subjects or it can be an extra interdisciplinary project. The organization of the project depends on the prior knowledge of the students. If the project is a part of mathematical teaching then the contents can be developed at least partly around the project topic. Since the project is universal, teams of students can compete to achieve the best results. A project for engineering students with another topic (image reconstruction, computerized tomography) but with corresponding ambitions is outlined in Schott (2005).

Basics about the background of the problem

Signal reconstruction needs some knowledge about signals in general. Signals are given here as real or complex valued time functions *x* = *x(t)*. It is assumed that *x(t)* is quadratically integrable belonging to the Hilbert space $H = L_2(R)$. This setting is necessary to equip the class of signals with appropriate properties. E.g. signals can be represented by

$$x(t) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} \, d\omega \tag{1a}$$

as a continuous superposition of oscillations $X(\omega) \cdot e^{j\omega t}$ with (complex) *amplitudes* $X(\omega)$ depending on angular frequency ω. The imaginary unit *j* satisfies $j^2 = -1$. The Euler exponential function $e^{j\omega t} = \cos \omega t + j \sin \omega t$ combines cosine and sine oscillations. The amplitude can be given in polar form $X(\omega) = r_x(\omega) \cdot e^{j\varphi_x(\omega)}$ using its *amplitude spectrum* $r_x(\omega) := |X(\omega)|$ and its *phase spectrum* $\varphi_x(\omega)$. The *Fourier transform*

$$X(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt \tag{1b}$$

of *x(t)* supplies this amplitude function while formula (1a) expresses the *inverse Fourier transform*. Hence a signal has two faces, one in *time domain* and one in *frequency domain*. The faces are related to each other by the given transforms.

The Hilbert space of signals is a linear space such that addition and scalar multiplication of signals are defined. Further a scalar product

$$< x, y > = \int_{-\infty}^{\infty} x(t) \cdot \overline{y}(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot \overline{Y}(\omega)\, d\omega \tag{2}$$

of signals working in time and frequency domain analogously allows us to speak of orthogonality if $< x, y > = 0$. In (2) terms $\overline{y}$ and $\overline{Y}$ mean the conjugate complex values. Finally a metric or distance can be created by

$$d^2(x, y) = \| x - y \|^2 = < x - y, \overline{x} - \overline{y} > = \int_{-\infty}^{\infty} | x(t) - y(t) |^2\, dt < \infty\,. \tag{3}$$

Now an important class of examples is considered, namely

$$x(t) = x_0 \cdot e^{(-a+jb)\cdot t} h(t), \quad x_0, a, b \in R, \quad a > 0\,, \tag{4a}$$

where *h(t)* is the unit step function. These signals starting with value x_0 at time $t = 0$ produce *damped* sine and cosine *oscillations* using the Euler exponential function. Only in the special case $b = 0$ is there a pure exponential decline. The Fourier transforms are

$$X(\omega) = \frac{x_0}{\sqrt{2\pi}} \cdot \frac{1}{a + j(\omega - b)} = \frac{x_0}{\sqrt{2\pi}} \cdot \left[\frac{a}{a^2 + (b-\omega)^2} + j \frac{b - \omega}{a^2 + (b-\omega)^2} \right] \tag{4b}$$

with amplitude and phase spectra

$$r_x(\omega) = \frac{x_0}{\sqrt{2\pi}} \cdot \frac{1}{\sqrt{a^2 + (b-\omega)^2}}, \quad \varphi_x(\omega) = \tan^{-1}\left(\frac{b-\omega}{a} \right), \tag{4c}$$

respectively. It is easy to show that the values of $X(\omega)$ fill a circular orbit in the complex plane.

The amplitude spectrum reveals the dominating frequency $\omega = b$ with the maximal amplitude. Each of these signals has bounded range with absolute values up to $|x_0|$, but the carriers are unbounded. Often signals have bounded carriers. Then the integral (1b) becomes a usual definite integral.

Imagine you know only the phase

$$\varphi(\omega) = -\tan^{-1}(\omega) = \tan^{-1}(-\omega) \tag{5}$$

of a signal *x(t)*. This phase fits to the signal class (4a) for *a = 1* and *b = 0*. That is, all signals $x(t) = x_0 \cdot e^{-t}$ have this phase. The question is whether you already have all signals with this phase. Generally more a priori knowledge about the signal helps to reduce the number of candidates. If you know that the closed range of *x(t)* is the interval *[0,1]*, then only $x(t) = e^{-t}$ is possible in the detected class of signals. Is there a method which constructs a signal *x(t)* with just the known properties? In many cases it is not known before if the constructed signal *x(t)* is the right one. Hence, it is a reasonable strategy to experiment with a *test suite* of original signals which can simply be compared with the reconstructions.

The problem class and its solution

PROBLEM: The observer has got some *information* about a signal *x* (by measurement or by theory). Can the signal *x* be reconstructed?

A PRIORI INFORMATION: It is assumed that the phase $\varphi = \varphi(\omega)$ and the carrier interval *I* of *x* are known. Possibly also the range *J* of *x* is given.

CONVEX FEASIBLITY PROBLEM: The convex sets

$$C_\varphi = \{x : \varphi_x(\omega) = \varphi(\omega)\}, \quad C_I = \{x : x(t) = 0 \ \textit{for}\ t \notin I\}, \quad C^J = \{x : x(t) \in J\} \tag{6a}$$

are given representing known phase, carrier and range of the signal. The intersections

$$C = C_\varphi \cap C_I, \quad C' = C_\varphi \cap C_I \cap C^J \tag{6b}$$

are considered. The problem is solved if an appropriate $x \in C$ and $x \in C'$, respectively, can be determined. This is a special *convex feasibility problem*.

SOLUTION METHOD: Looking at a general convex feasibility problem an object $x \in X$ has to be determined in the nonempty intersection of *m* convex sets C_i (*i=1, ...,m*). For solution the iterative *method of relaxed projections* is available:

$$x_0 \in X, \quad x_{k+1} = T_{i(k)} x_k = x_k + \lambda_k \left(P_{i(k)} - I\right) x_k, \quad 0 < \varepsilon \le \lambda_k \le 2 - \varepsilon\,. \tag{7}$$

Here P_i is the *metric* (nearest point) *projection* onto C_i, *i = i(k)* is the *selection strategy* for C_i, e.g. a cyclic one, while the parameters λ_k introduce relaxation. This method and some modifications can be found in Stark (1987) as well as in Butnariu and Censor (1990).

REALIZATION: According to (6b) it is *m = 2* and *m = 3*, respectively, in this application. The sets C_I and C^J refer to the time domain while C_φ relates to the frequency domain. So we can start in each iteration cycle in the time domain with carrier and range correction, apply the Fourier transform and then continue with phase correction in the frequency domain. Finally the cycle is closed by the inverse Fourier transform coming back to time domain. The metric projections P_I, P^J and P_φ are given in Schott (2000) and in Schott (2013).

The numerical calculation needs a finite time interval [*a,b*] and time discretization. Hence the signal $x(t) \in L_2[a,b]$ is replaced by a sequence of points $(t_j, x(t_j))$ with $t_j \in [a,b]$ for all *j = 0, 1, ..., n*. The simplest way is to choose the discrete times t_j equidistant. It is important to continue the signal sequence by further times t_j with $x(t_j) = 0$ for *j = n+1 ,..., N*, where $N \geq 2(n+1)$ is a power of *2*. Otherwise the structures of continuous signals are lost in the discrete framework. The Fourier transform and its inverse are modelled by their discrete analogies in form of FFT and inverse FFT.

A stopping rule will finish the iteration of relaxed projections after a certain period. Then the resulting signal sequence can be transformed into an approximate signal $\tilde{x}(t) \in L_2[a,b]$, e.g. by linear spline interpolation. Now $\tilde{x}(t)$ can be compared with the original *x(t)* if it is known.

EXPERIMENTS: There are a lot of possibilities to start investigation: Selection of different signal types, change of a priori information, modification or change of solution methods, change of parameters (for relaxation), change of set selection strategy, selection of starting signals in iteration, change of stopping rules in iteration, influence of changes on the quality of reconstruction and optimization of the reconstruction process. Some results of experimentation are contained in Schott (2000) and Schott (2013).

Project steps and findings

The following steps are recommended for planning and realizing the presented project work in signal reconstruction:

- *Problem description* and model assumptions,
- Provision of *theoretical basics*,
- Understanding of *discretization process* and error propagation,
- Collection of *material* concerning the topic (papers, web links),

- Selection of *solution methods*,
- Selection of appropriate *test signals* for later experiments (signal test suite),
- Selection of *software* and software development (e.g. MATLAB, toolbox with GUI and files, correctness tests and test reports),
- *Experiments* (tests with competing solution methods, representing parameter studies using the signal test suite),
- *Communication* (brainstorming and sharing experiences in teams, between teams and with supervisors or other experts),
- *Evaluation* (software tests, proving quality of reconstructions, recommending methods and parameter choices),
- *Presentation* of results (PowerPoint presentation by all teams, examination by supervisors, organizing a workshop, writing research papers or submitting papers to journals, offering the software toolbox for other users).

Conclusions

Summarizing, the described project has the following advantages:

- Students learn about mathematics.
- Students learn about applications of mathematics in engineering.
- Students learn about mathematical and engineering software and software development.
- Students learn some aspects of scientific work.
- Students learn self-study and team work.

There are a lot more interesting and demanding engineering projects which can increase the interest and motivation of students in engineering education, see e.g. Schott (2005). But mathematical basics are crucial for their realization.

References

Butnariu, D. and Censor, A. (1990) On the behavior of a block-iterative projection method for solving convex feasibility problems. *Internat. J. Computer Math.,* Vol. 34: 79-94.

Hayes, M. H. (1982) The reconstruction of a multidimensional sequence from the phase or magnitude of its Fourier transform. *IEEE Transactions on Acoustic Speech and Signal Process,* Vol. 30: 140-154.

Schott, D. (2000) Signal reconstruction from given phase of the Fourier transform using Fejér monotone methods. *Disc. Math. Diff. Incl. Contr. Optimiz.,* Vol.20:27-40.

Schott, D. (2005) Bildrekonstruktion am Computer als Studentenprojekt. *Global J. Engng. Educ.* Vol. 9 (No. 3): 267 – 274.

Schott, D. (2013) Signal reconstruction - a project for students of electrical engineering.

WTE&TE Vol. 11 (No. 4): 438-444.

Stark, H., ed. (1987) *Image Recovery: Theory and Applications.* Academic Press.

Feedback and formative assessments in mathematical lectures

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Abstract

The competence approach describes the mathematical competence as "the ability to understand, judge, do and use mathematics in a variety of intra - and extra -mathematical contexts" (Education, 2013). We feel that it is the understanding and use of mathematics that has to be put in the foreground, by integrating it into the courses. Lectures have to be adapted in order to make the students use and understand mathematics. At the same time we are bound to the traditional way of giving lectures, because this is the common form of teaching a large number of students (groups of 50 and more participants). To improve the understanding, feedback and formative assessments are used as well as voting methods. The implementation of methods such as "one-minute-paper", self-learning with online-assessment and online-feedback, peer-instruction using clicker into the lectures is presented. The authors are using these methods on a regular basis during their mathematic lectures. The paper describes how these are included into a traditional lecture. The authors want to share their experience in teaching mathematics to engineers. First evaluation results are presented.

Introduction

In the past, the main focus/goal of the mathematics lectures was the teaching of calculus. The conceptual understanding was left to the mathematicians. However it is evident that the understanding of the mathematical concepts is very important for its application in the engineering sciences.. The wide availability of powerful calculators renders pure calculus tasks less and less important in engineering practice. Some university mathematics lectures still have to take account of this.

The workgroups, with whom we work, consist of more than 50 students, with very different mathematical backgrounds.. Traditional lectures are not well suited in this context to learn the concepts of mathematics: only a few students are actively involved in the lecture. The remaining ones only "passively consume". Active, independent learning and processing of mathematical content and concepts takes place outside of the course. In this process, calculation techniques are learned through repeated use, but the understanding of the contexts and mathematical concepts doesn't happen.

The lecturer doesn't get an overview of the students' knowledge state. In traditional lectures there is no way to determine whether the concepts were understood or if only the rules were learned by heart. The lecturer can't find out if these concepts were incompletely internalized or completely wrong (so called misconcepts).

To understand the mathematical concepts, the students must actively deal with the content (Breidenbach, et al., 1992). This should be done during the lecture, to respond promptly to questions and misconceptions.

In the position paper of Lehre and Kolleg (Kolleg, 2014), general principles for improving the mathematical education of engineers are represented:

- Students should learn from the lecturer in an active process
- Intensifying the contact between the students and the lecturers
- Promoting the contact of students among themselves
- Immediate feedback of the learning success

These goals seem easy to implement for small groups of students with homogenous qualifications. We present, in this sequel, methods we have applied to try to improve the situation in those points in our own situation.

The main goal of the authors is to present methods that enablethe implementation of the above principles for large heterogeneous groups of students. Later in this paper, the examined methods will be presented. An important selection criterion of the methods was that they do not require structural changes in the lecture or in the curriculum. The methods we have studied can be applied independently on individual lectures. It will be explained how to mesh the different methods and this will be illustrated by an example.

The scientific evaluation of the process requires a large amount of resources and is not easy to implement. Therefore, in the context of this work, we limit ourselves to qualitative evaluations.

The paper concludes with final comments and a summary.

Method of Investigation

First of all we have to encourage the students to participate actively at the lecture.Thereby the contact between the students and the teacher is significantly intensified. Some of the methods give the students and the teacher feedback about the current teaching and learning achievement. Students are encouraged to prepare for the lecture and this can be checked by the use of LON-CAPA, the e-learning platform.

An essential method that is used in our lectures is Peer Instruction with the use of Clickers (Mazur, kein Datum). Here, students are asked questions that require an understanding of the mathematical concept.

The answers are given as a multiple choice. The students choose an answer using the Clicker (with a small keyboard). The first time the students answer this question, they do it on their own; there is no further discussion. The distribution of the students' answers can be displayed immediately. This provides an immediate feedback to the lecturers, if / how the topic has been understood. Frequently mentioned wrong answers also provide evidence about the students' state of understanding and possibly misconceptions. Subsequently, two students discuss their responses exchanging mathematical arguments. They practice the formulation of mathematical arguments, as well as listening to and understanding someone else's arguments. Again they choose an answer using the Clicker. Another vote shows if a large part of the students understood the matter through the discussion.. The plenary discussion is then used to clarify remaining uncertainties.

Additionally, we use the One-Minute Paper (Waldherr & Walter, 2014). The students are asked at the end of the lecture to state which topic was particularly important in this lecture, which topic they have not understood and to give additional comments. As the name suggests, these questions should be answered in a short time. Filling the One-Minute-Paper motivates the students to reflect, which topics are important and where they have difficulties. The lecturer gets the important feedback, if the students understood what the main subjects are and if they are able to mention the most important points.

The results of the One-Minute-Papers are used in the following lecture. For example, the difficulties are picked up. This can be done by formulating clicker-questions for Peer-Instruction. The lecturer has also the possibility to point out important topics. The students feel that their answers have a direct impact on the course. This provides motivation to proceed filling the One-minute-Paper and to participate actively during the lecture.

Further, students are encouraged to work out selected parts of the lecture material independently. In LON-CAPA the students can answer small surveys related to the topics. They can control by themselves if the material is well understood. The topics are not further presented in detail during the lecture. The students can ask questions online (in LON-CAPA) (Henderson & Rosenthal, 2006). During the lecture the questions of the students are discussed. The advantage is that topics which are widely understood don't have to be dealt with during the lecture. There is more time left to clarify basic understanding questions and to draw the attention of the students to misconceptions.
The methods are embedded in interactive designed lectures and tutorials.

Explanation with an example from linear algebra

The procedure and coordination of the different methods will be illustrated with an example. For this purpose, we choose an example from the introduction to linear algebra. The introductory contexts are explained in a frontal lecture. Scalar and vector product methods are then prepared by the students within a reading question. For these methods they find accompanying tasks in LON-CAPA. Parts of the tasks involve only calculation of the products. Other questions relate to the concepts of the scalar and vector product. Here, the focus is to check whether the relationship between scalar product and included angle, or vector product and torque has been understood.

The LON-CAPA results show where the students have difficulties. We observe that the methodological tasks are often solved without problems. Students take the vector product as a calculation rule. Many students have difficulties in conceptual work. In the lecture there is the opportunity to actively work on the conceptual deficits of students. Together with the students the questions asked in LON-CAPA are discussed. This is (of course) done anonymously. The questioners are not mentioned. From the responses of the students,clicker questions can be formulated. Here you can determine to what extent the concepts are understood. The students discuss the answers with each other. Possible questions are for example:

What can you conclude for the vectorsa and b from the following expression:

$$\vec{\boldsymbol{a}} * \vec{\boldsymbol{b}} = |\vec{\boldsymbol{a}}| * |\vec{\boldsymbol{b}}|$$

Here, students need to realize that a scalar product is essentially determined by the angle between the vectors. The task cannot be solved by using calculus rules. Calculating the scalar product is an easy task for the students. Therefore it is not a main topic in the lecture, but only used.

At the end of the lecture the students fill the One-Minute-Paper. The lecturer can notice if the relation between scalar product and angles are clarified. The lecturer can conclude how to proceed with the topic in the following lectures.

This kind of proceeding can be found in all parts of the course. Peer-Instruction and One-Minute-Paper is also used, when there is no reading question.

Evaluation

As mentioned above, a quantitative evaluation of how far the methods improve concept understanding is difficult. The application of the evaluation questionnaire BeVaComp (Braun, et al., 2008) turned out to be not very meaningful. The students had difficulties to assess whether an increase in competence is due to the current course. However, from the analysis of the questionnaire, it could be interpreted that the students are much more active in the course. Depending on the evaluated lecture, between 60% and 90% of the students answering the questionnaire indicated that they had actively taken part to the lecture either by word contributions or by asking questions.

The impact of the methods is evaluated by examining various observations and comments. A group discussion with a representative selection of participants was carried out for one lecture (without the lecturer).

In general, the methods are very well accepted by the students. The students perceive the use of Peer Instruction and Clicker as a break from the pure listening. It is perceived as a deceleration. Apparently, only a few students recognize that an essential part of the learning takes place, as they are actively involved. There is an ambition to be able to explain and to give good answers, as the following comment from a student illustrates:

> *I have to watch out better in the lecture, so I may answer the Clicker question well.*

The students feel they are being taken seriously and are involved in the lecture. Also, during the semester, the participation in the Clicker questions and the One-Minute-Paper is high. The students have fewer concerns to ask questions and to admit mistakes. The following semester's lecturers reported that the students significantly differed from normal courses. They were more open-minded and asked questions immediately, if something essential was not clear. The lecture is clearly more interactive.

We observe that the majority of the students prepare the reading question and answer the question in LON-CAPA. They are prepared for the lecture. Some students mention that here an important part of learning happens.

It is evident that the methods need to be very well coordinated. The structure and the main topics have to be very clear for the students. The students need to understand that all parts of the course are essential, including content that had to be prepared independently.

There are also students who reject a more active lecture and concept understanding. In general, students are not (yet!) used to the need to understand the background of the mathematical method. Comment of a student:

Too many discussions among the students are stimulated by the lecturer. Lecturer should simply recite the material and teach.

All together there is a positive picture of the changes from lectures and students. We cannot expect that all students understand the improvement of such a way of teaching. Students cannot always recognize how and where learning takes place.

Summary

The methods described are applicable in the mathematics lectures for engineers. The implementation requires time. Especially the formulation of new questions for LON-CAPA und clicker has to be done very carefully. The method only works if the questions are appropriate. There are collections of questions, but they are unfortunately incomplete when it comes to testing concept understanding in mathematics.

It is important that questions, suggestions and problems formulated by students in One-Minute- Paper or LON-CAPA are quickly incorporated into the lecture. The lecturer has to be prepared to modify the following lectures, if the content is not sufficiently understood. If the students observe that their questions are immediately answered, they are motivated the take an active part and ask questions.

The methods used can be implemented together or separately. An implemented method should be used more than once in the course. The students need time to get used to new structures. If more than one method is used, it is essential that they are combined in a clear way. If the course is moved to more conceptual understanding, the final exam has to be correspondingly adapted. Only this leads to more understanding of the structure of the course and the importance of all parts.

All together, we see these methods can create a good possibility to increase the understanding of mathematical concepts and to increase the active participation of the students in the lecture. The combination of classical frontal lecture, reading question, LON-CAPA, Peer-Instruction and One-Minute-Paper gives us the possibility to guarantee the high amount of subjects and to motivate students to take an active participation in the lecture.

References

Braun, E., Gusy, B., Leidner, B. & Hannover, B., (2008). Kompetenzorientierte Lehrevaluation - Das Berliner Evaluationsinstrument für selbsteingeschätzte studentische Kompetenzen (BEvaKomp). *Diagnostica*, Vol. 54, pp. 30-42.

Breidenbach, D., Dubinsky, E., Hawks, J. & Nichols, D., (1992). Development of the process conception of Function. *Educational Studies in Mathematics Vol. 23*, pp. 247-285.

Henderson, C. & Rosenthal, A., (2006). Reading Questions: Encouraging Students to Read the Text Before Coming to Class. *Journal of College Science Teaching*, Vol. 7, pp. 46-50.

Kolleg, L., (2014). *Mathematik in der Ingenieurausbildung,* Accessed via http://www.lehrehochn.de/fileadmin/user_upload/mathing/LehreN_MathIngBrosch%C3%BCre2014.pdf. (8. April 2014)

Mazur, E., *Peer Instruction- A Users's Manual,* s.l.: Upper Saddle River: Prentice Hall.

Waldherr, F. & Walter, C.,(2014). *didaktisch und praktisch: Ideen und Methoden für die Hochschullehre.* Stuttgart: Schäffler-Poeschel.

TEACHING MATHEMATICS TO STUDENTS WITH NEURODEVELOPMENTAL CONDITIONS

SEÁN MCGARRAGHY AND MILENA VENKOVA

1. Introduction

This work investigates the awareness among mathematics lecturers in Ireland of certain neurodevelopmental conditions, and what strategies they use to address such conditions. To this end, we carried out an online survey, "Teaching Mathematics to Students with Neurodevelopmental Conditions", through `www.surveymonkey.com`, inviting responses through the Irish mathematics lecturers' MATHDEP mailing list. This survey consisted of six questions, covering three main topics: awareness of the existence of these conditions and their implications for learning; institutional provision of information on these conditions to lecturers; and teaching strategies directed towards students with these conditions. In this paper, we discuss these three main topics; for each, we first give necessary background, then give the results of the relevant survey questions with analysis and comments.

We conclude by commenting on the significance of the results, and suggest some future avenues on the use of smartphones and modern technologies to provide improved teaching and learning strategies.

2. Neurodevelopmental conditions and awareness of them

In this section, we first describe the neurodevelopmental conditions we consider. We then discuss awareness of them among mathematics lecturers in Ireland, as indicated by our survey.

The conditions we cover are

- Autistic Spectrum Disorder (ASD) is a *pervasive developmental disorder*. People with ASD tend to have communication deficits, such as responding inappropriately in conversations, misreading nonverbal interactions, or having difficulty building friendships appropriate to their age; in addition, people with ASD may be overly dependent on routines, highly sensitive to changes in their environment, or intensely focused on inappropriate items (APA, 2013). The symptoms of people with ASD will fall on a continuum (spectrum), with some individuals showing mild symptoms and others having much more severe symptoms. It is believed that 1 person in 150 is on the autism spectrum, although there are some estimates that claim it is 1 out of 80. In DSM-5, ASD now includes the previously separate Asperger's Syndrome (AS) (APA, 2013).
- Attention Deficit Hyperactivity Disorder (ADHD). ADHD symptoms are divided into two categories: inattention; and hyperactivity and impulsivity; that include behaviors like failure to pay close attention to details, difficulty organizing tasks and activities, excessive talking, fidgeting, or an inability to remain seated in appropriate situations (APA, 2013). ADHD affects about 6–7% of children when diagnosed via the DSM-IV criteria.

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- Dyspraxia is a neurological disorder beginning in childhood that can affect planning of movements and co-ordination as a result of brain messages not being accurately transmitted to the body. People with dyspraxia have problems with both gross motor skills (difficulty remembering the next movement in a sequence, problems with balance, problems with spatial awareness) and fine motor skills (slow writing speed, problems with establishing the correct pencil grip) (APA, 2013).
- Dyscalculia is a learning disability which involves such difficulties as understanding and using basic mathematical concepts (such as number, quantity and time), which in return bring difficulties with manipulating numbers or number facts (e.g., the multiplication tables) (Butterworth, 2010). Estimates of the prevalence of dyscalculia range between 3% and 6% of the population (Butterworth, 2010).
- (Developmental) dyslexia is defined as a specific and significant impairment in reading abilities, unexplainable by any kind of deficit in general intelligence, learning opportunity, general motivation or sensory acuity (WHO, 1993; Habib, 2000).

Dyscalculia and Dyslexia are *learning disorders* (also called *learning disabilities*). When a student's cognitive ability is much higher than his/her academic performance, the student is often diagnosed with a learning disorder. Many of these conditions are co-occuring. Approximately 25–75% of individuals with ASD also have some degree of learning disability (O'Brien, 2004). A quarter of children with dyscalculia have ADHD (Shalev, 2004). Learning disabilities have been found to occur in about 20–30% of children with ADHD. Although each disorder occurs in approximately 5% of children, 25–40% of children with either dyslexia or ADHD meet the criteria for the other disorder.

One common theme for these conditions is difficulty in sensory processing. Sensory processing was defined by Ayres (1972) as "the neurological process that organizes sensation from one's own body and from the environment and makes it possible to use the body effectively within the environment". Sensory processing problems are suspected to be the root of many of the conditions we listed: for example, many people with dyslexia describe the letters of a written text as moving on the page. There are different known techniques which can alleviate such problems, and we discuss them in Section 4.

Although these conditions are usually diagnosed in children under 10, they are lifelong conditions and (for example) the child with ASD becomes an adult with ASD. They also may have behavioural aspects (Hughes *et al.*, 2007), but this is outside our scope.

Table 1 gives the responses to Question 1 of the survey; Table 2 gives those to Question 2.

TABLE 1. Responses in numbers and percentages to Question 1: "*Which of the following neurodevelopmental conditions are you aware of?*"

	Yes	I've heard of it	No	Total
Autistic Spectrum Disorder (ASD)	43	8	5	56
	76.79%	14.29%	8.93%	
Attention Deficit Hyperactivity Disorder	48	6	2	56
(ADHD)	85.71%	10.71%	3.57%	
Dyspraxia	19	29	8	56
	33.93%	51.79%	14.29%	
Dyscalculia	24	19	12	55
	43.64%	34.55%	21.82%	
Dyslexia	50	6	0	56
	89.29%	10.71%	0.00%	

TABLE 2. Responses in numbers and percentages to Question 2: "*Are you aware of the implications these conditions have for students' learning of mathematics?*"

	Yes	Vaguely aware	No	Total
Autistic Spectrum Disorder (ASD)	12	33	11	56
	21.43%	58.93%	19.64%	
Attention Deficit Hyperactivity Disorder	20	28	8	56
(ADHD)	35.71%	50.00%	14.29%	
Dyspraxia	4	23	29	56
	7.14%	41.07%	51.79%	
Dyscalculia	17	22	15	54
	31.48%	40.74%	27.78%	
Dyslexia	27	22	6	55
	49.09%	40.00%	10.91%	

Although awareness of conditions, especially ASD, ADHD and dyslexia, was high, awareness of implications for learning of mathematics was noticeably lower.

3. INSTITUTIONAL PROVISION OF INFORMATION ON THESE CONDITIONS

Neurodevelopmental conditions are commonly met at the college level, as more and more students with such conditions are incorporated into mainstream secondary education and are better prepared to enter third level (Moon *et al.*, 2012, p. 94). Webb (2011) notes that ASD and ADHD are "invisible" disabilities: educators, especially at third level, may not be aware that students have these conditions unless they identify themselves. Statistics are hard to come by, but Hibbert (2004) states that in the University of Nottingham Engineering faculty, from 2000-2003, between 8–10% of students had a disclosed disability, and of these, almost half were "invisible" disabilities. It has been stated (Hughes *et al.*, 2007) that 1–4% of Physics undergraduates have AS, and it seems reasonable to assume that a similar proportion of Engineering undergraduates have the same condition. Also, students may never have obtained a formal diagnosis and so not be registered with Disabilities Support. It thus seems reasonable to assume that in a class of 50 or more, there will be at least one student with neurodevelopmental disorders.

Irish third-level institutions have been making efforts to provide information on these conditions to lecturers. Some (non-exhaustive) examples of this include: UCD and TCD provide online factsheets on the conditions and guidelines on exam and academic accommodation; DIT have occasional lunchtime training sessions for staff on topics such as ASD/AS.

Questions 3 and 4 of our survey sought to establish the timeliness and degree of institutional support and provision of information on these conditions, as perceived by lecturers.

There were 56 responses to Question 3, '*Have you been contacted by Disabilities Support Centre to inform you of one or more students with neurodevelopmental conditions in a class you teach?*' Of these, 30 (53.57%) answered "yes, at the beginning of term", ten (17.86%) answered "yes, notified just before the examination" and 16 (28.57%) answered "no".

This would appear to indicate a reasonable degree of provision of information by institutions to lecturers, though there is some room for improvement.

There were 32 responses to Question 4, '*If you answered "Yes, at the beginning of term", were you...*'. Of these, two (6.25%) answered "offered training about the nature of the condition and how to accomodate it in your teaching"; 14 (43.75%) answered "given on-line or other materials and guidelines"; and 16 (50%) answered "not offered assistance".

This finding appears to show that although faculty may be informed of the presence of students with particular needs, they are not so well informed on how to accomodate them, particularly in terms of mathematics teaching.

4. Teaching and learning strategies addressing these conditions

In this section, we address the various in class techniques that could help students with such disorders. Various authors have suggested a range of such techniques, including

- Providing lecture notes, especially in electronic form, in advance of the class, helps students mitigate the effects of poor handwriting skills (Webb, 2011); Moon *et al.* (2012) discuss this in the context of ASD but it will also apply to cases of dyspraxia. Electronic formats are particularly helpful, as students can read them using accessibility software (Moon *et al.*, 2012) and "listen" to print materials as a learning strategy. Further, Trott (2003) recommends this approach for students with dyslexia.
- Using different colours to highlight different parts of text or of mathematical expressions. Webb (2011) suggests such highlighting, e.g., within multiple integrals.
- Using mind-maps in lecture notes. Webb (2011) suggests this may improve accessibility and flexibility and gives an example of solving two-dimensional linear systems with complex eigenvalues. Trott (2003) gives an example of how use of a mind-map helped a student better organise the process of partial differentiation.
- Using visuals (sketching graphs, Venn diagrams, etc.) whenever possible. Friend and Bursuck (2006) point out that this provides additional ways to reinforce important concepts. However, clear explanations should be provided with such graphics (Moon *et al.*, 2012, pp. 97-98), and students with ASD benefit from clear precise directions, especially if given both orally and in writing. Students with ADHD were challenged by tasks primarily using symbolic, analytic and verbal representations and showed a preference for graphical or pictorial approaches to problem solving (Judd, 2008).
- Allowing students to use their laptops in class. Moon *et al.* (2012, p. 95) suggest that this can mitigate the effect of poor handwriting skills, which is a major problem for students with these conditions (Webb, 2011). Draffan (2001) describes the benefits of other technologies such as tactile technology and speech to maths formatting.

Table 3 gives the responses to Question 5, while Table 4 gives the responses to Question 6. We see that use of visuals and allowing laptops are almost universal, while pre-class provision of notes, use of different colours and permitting photographs are widely used; however, mind-maps are hardly used at all. Together, these responses show that although 54 respondents use such strategies, at most 24 use them specifically to aid students with neurodevelopmental conditions; of these, the majority used pre-class provision of notes and allowing laptops.

5. Discussion

The broad findings from this work are that although mathematics lecturers are mostly aware of the existence of neurodevelopmental conditions, and are informed by Disabilities Support of the presence of students with such conditions, they are less aware of in class strategies, particularly as used to aid students with these conditions. At the same time, many of the strategies suggested in the literature are already being used in the classroom, while some others are fairly easy to implement, e.g., using mind-maps instead of a numbered list of points.

With the advent of technology such as smartphones/tablets, the authors have experience of cases where students ask permission to photograph boards with notes and/or worked examples.

TABLE 3. Responses in numbers and percentages to Question 5: "*Which of the following do you use in your teaching? (Select all that apply.)*". Total number of respondents: 54.

Answer Choices	*Responses*	*Responses* (%)
Providing lecture notes in advance of the class	31	57.41%
Using different colours to highlight different parts of text or of mathematical expressions	27	50.00%
Using mind-maps in lecture notes	3	5.56%
Using visuals (sketching graphs, Venn diagrams, etc.) whenever possible	46	85.19%
Allowing students to take a photograph of the board before cleaning it	25	46.30%
Allowing students to use their laptops in class	41	75.93%

TABLE 4. Responses in numbers and percentages to Question 6: "*Of the teaching strategies, if any, that you selected in the previous question, which do you use specifically to aid students with neurodevelopmental conditions? (Select all that apply.)*". Total number of respondents: 24.

Answer Choices	*Responses*	*Responses* (%)
Providing lecture notes in advance of the class	19	79.17%
Using different colours to highlight different parts of text or of mathematical expressions	5	20.83%
Using mind-maps in lecture notes	1	4.17%
Using visuals (sketching graphs, Venn diagrams, etc.) whenever possible	11	45.83%
Allowing students to take a photograph of the board before cleaning it	12	50.00%
Allowing students to use their laptops in class	16	66.67%

We posit that allowing this may be particularly helpful to students with neurodevelopmental disabilities, as they are not then under pressure to assimilate the information and reproduce it in their own notes before the board is cleaned; they may do so at their own pace. We believe that this can go some way to mitigating the effect of poor handwriting skills (as discussed in (Moon *et al.*, 2012, p. 95)), and inability to quickly process and reproduce detailed information.

Use of such technology by lecturers may also be beneficial in that colour overlays may be used to reduce screen glare of black type on white background, which is known to cause problems for students with dyslexia, ASD and other conditions (Trott, 2003). We suggest that a non-white background colour on presentation slides may benefit such students.

One respondant suggested use of online tests as a form of assessment; this reduces many of the issues experienced by a student with neurodevelopmental disorder, such as sensory overload, distraction, noise and inability to concentrate.

6. CONCLUSIONS AND FUTURE WORK

The significance of this work is that, for the first time we know of, it investigates the awareness among Irish mathematics lecturers of neurodevelopmental conditions, the support and information provided by third level institutions and the teaching and learning strategies used to address these conditions. It finds that awareness is high in a general way, but that specifics of how to address the conditions are less widely known. Many of the specific strategies that may be used do not require great effort, but may still make a difference to the student. Our hope is that this work may lead to a wider understanding of what can be done, and more effective and thorough provision of information to lecturers on useful strategies.

This piece of work has concentrated on the topic of neurodevelopmental conditions from the perspective of mathematics lecturers. Future work aims to extend to the perspective of the student with one or more neurodevelopmental conditions, and the use of new technology such as tablets and smartphones in improving the student's learning.

Further investigation is required into smartphone/tablet apps which convert speech or handwritten text into print, particularly for the kind of formulae encountered in mathematics education, where the positioning of elements (e.g., superscripts) is not well represented by common speech: this makes these formulae especially difficult to convert.

REFERENCES

American Psychiatric Association (2013). *Diagnostic and Statistical Manual of Mental Disorders (DSM-5)*, fifth edition. Arlington, VA: American Psychiatric Association. url: `dsm.psychiatryonline.org` [access date: 1 June 2013].

Ayres, A. J. (1972). *Sensory integration and learning disorders*. Los Angeles: Western Psychological Services.

Butterworth, B. (2010). Foundational numerical capacities and the origins of dyscalculia. *Trends in Cognitive Sciences*. **14**(12):534–541.

Draffan, E. A. (2001). Accessing Maths and Stats by other means. *MSOR Connections*. **1**(3):10–12.

Friend, M., and Bursuck, W. (2006). *Including students with special needs: A practical guide for classroom teachers*. Boston, MA: Pearson Education.

Habib, M. (2000). The neurological basis of developmental dyslexia: An overview and working hypothesis. *Brain*. **123**(12):2373–2399.

Hibbert, S. (2004). MELEES — an environment to support dyslexic students? *Engineering, Maths and the Dyslexic student: DDIG event*.

Hughes, M., Pepper, S., McCall, A. and Milne, V. (2007). Supporting students with Asperger's Syndrome in the Physical Sciences. HEA supporting material.

Judd, A. D. (2008). *Function visualization and mathematical thinking for college students with Attention Deficit Hyperactivity Disorder*. PhD thesis. University of Northern Colorado.

Moon, N. W., Todd, R. L., Morton, D. L., and Ivey, E. (2012). *Accommodating students with disabilities in science, technology, engineering, and mathematics (STEM): Findings from research and practice for middle grades through university education*. Atlanta: Center for Assistive Technology and Environmental Access, College of Architecture, Georgia Institute of Technology. url: `http://www.catea.gatech.edu/scitrain/accommodating.pdf`, [access date: 30 April, 2014.]

O'Brien, G. Pearson (2004). Autism and learning disability. *Autism*. **8**(2):125–140

Shalev, R. S. (2004). Developmental Dyscalculia. *Journal of Child Neurology*. **19**(10):765–771.

Trott, C. (2003). Mathematics support for dyslexic students. *MSOR Connections*. **3**(4):17–20.

Webb, S. D. (2011). Accessibility of University Mathematics. *MSOR Connections*. **11**(1):42–47.

World Health Organisation (1993). *ICD-10. The international classification of diseases, Vol 10: Classification of mental and behavioural disorders.* Geneva: World Health Organisation.

The SEFI Maths Working Group – Current Offerings and Future Tasks

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Abstract

In this discussion paper we firstly summarise the current offering of the SEFI Mathematics Working Group with regard to orientation for those who are interested in the mathematical education of engineers. Based on this summary we identify directions for further work. Finally, we present some ideas of how progress might be made in these directions.

Introduction

For over 30 years the Mathematics Working Group (MWG) of the European Society for Engineering Education (SEFI) has provided a forum for the exchange of views and ideas amongst those interested in engineering mathematics and has created several documents to capture the state of the art in learning, teaching, assessment and curriculum development regarding the mathematical education of engineers. In this paper we firstly give a brief overview of the current offerings of the MWG which are freely available on the group's website (http://sefi.htw-aalen.de). Then, we specify those topics which in our view will be of great importance in the near future. This includes a better understanding of the competence concept and ways to acquire and assess competencies as well as possible reactions to changes in the learning environment and the learning behaviour and technology use of incoming students. Finally, we outline potential further activities of the MWG in order to address these issues.

Current Offerings

In the last ten years the Working Group has held five seminars in Vienna, Kongsberg, Loughborough, Wismar and Salamanca. Contributions to these seminars and discussion sessions were concerned with questions such as

- What are the essential issues regarding the mathematical education of engineers?
- What is the role of technology?
- Which forms of assessment exist in Europe and are they adequate?
- How can we activate students?
- How can we achieve higher-level learning goals like mathematical understanding?
- How can we improve the attitude of engineering students towards the mathematical part of their education?

- What is the impact of the Bologna agreement on the mathematical education of engineers?

All the seminar contributions and reports from the discussion sessions are available from the group's web page at http://sefi.htw-aalen.de. These documents provide valuable experience from many European countries without claiming to offer a comprehensive and systematic overview of developments in Europe. Discussions at the seminars indicate that there is broad agreement on the importance of the topics and questions listed above but there are clear differences regarding the answers. This was particularly evident in discussions on the role of technology.

The second main means (in addition to the two-yearly seminars) for providing orientation is the core curriculum document. The third edition of this document was issued in September 2013, called "A Framework for Mathematics Curricula in Engineering Education" (Alpers et al. 2013). This document adopts the concept of mathematical competence, from the Danish KOM-project (Niss & Højgaard 2011) as the major goal of mathematics education. Higher-level learning goals that have been a topic of many discussions in the seminars are captured by this concept which is specified in more detail by identifying eight so-called competencies. The curriculum document is to be understood as a framework document, not as a specific one-size-fits-all curriculum. For a concrete curriculum for a specific type of engineering study course, the competencies need to be specified in more detail. The KOM-project provided three dimensions in relation to each competency (degree of coverage, radius of action, technical level). The third edition of the core curriculum document retains the lists of content-related learning outcomes from the second edition (Mustoe & Lawson 2002), although some have been slightly modified. Again, for specifying a concrete curriculum, one has to choose from these lists (and possibly make a few additions if necessary). The latest edition of the core curriculum document also contains chapters on learning and teaching arrangements and on assessment which take into account appropriate contributions and discussions at the seminars and also other relevant literature. Therefore, the document could equally be seen as a "framework" for the important questions listed above. It gives an overview and points the reader to further relevant literature.

There is also a special curriculum for a practice-oriented study course in mechanical engineering written within the framework (Alpers 2014). There, the competencies have been specified in more detail based on the experience of the author. This document should be seen as a first attempt to specify such a curriculum and it is likely that several iteration cycles will be required to improve this document. Nonetheless, the document can act as an example and inspire other people to write a similar curriculum for their type of study course.

The dissemination of the MWG's outputs outlined above occurs via different routes:

- the MWG's mailing list
- the national contact persons who are encouraged to disseminate the information to national, regional and local bodies and individuals interested in the mathematical education of engineers

- presentation at conferences (like the SEFI Annual Conference or national conferences)
- contacts with other bodies (e.g. ASEE Mathematics Division).

Important topics for future work

The development of the framework document is by no means the “end of history” in the mathematical education of engineers but rather an organizing scheme for further activities. One major area of future work is the concept of mathematical competence itself. A more precise competence description for different types of engineering education is required (i.e. an identification of those aspects of the eight competencies which are important). This could be done with respect to

- specific mathematical topics in context (for example, Laplace transforms, Dirac delta function, convolution) in order to recognize which kind of mathematical understanding is required to use a certain mathematical concept for solving certain kinds of application problems (usually this will be different from the kind of mathematical understanding a mathematician needs to understand and further develop mathematical theory)
- specific application subjects treated more generally, for example, which aspects of the competencies are important for machine element dimensioning (including the usage of tools like a machine element dimensioning program)
- workplace activities (also including the usage of tools available at engineering workplaces).

The first two points are dealt with in the German KoM@ING project (see http://www.kom-at-ing.de/ where an English description is available; see also Schreiber & Hochmuth 2013). For mechanical engineering and electrical engineering, the usage of mathematics in textbooks, lecture notes and assignments has been investigated, using qualitative research methods, to identify the required competence components. Since such a project must be restricted to a small sample of topics within the set of relevant applications subjects in a study course, further studies of this nature would be beneficial in order to build up a broader understanding. This could lead to the development of a sound knowledge base of competence components for different types of engineering study courses. This could be the basis for setting up additional curricula or for improving and enhancing the existing curriculum for a practice-oriented study course in mechanical engineering.

The crucial goal of mathematics education in engineering study courses is to enable graduates to use mathematics to solve problems in their daily work. In order to capture the competence aspects that are important for this goal with respect to a range of jobs, workplace studies are necessary. One might argue that if students are successful in their application subjects then this is a good indicator for success in their later jobs. But it is by no means certain that lecturers in application subjects really capture the competencies necessary for successful usage of application concepts in engineering jobs. Therefore, workplace studies are necessary but also very time-consuming to pursue (Alpers 2010).

In order to really have an impact on the capabilities of graduates, one has to find suitable learning scenarios for competence acquisition. For this, having a pool of competence related example tasks (assignments, problems, projects) with corresponding learning environments would be helpful. Such example tasks are also needed for convincing colleagues that the competence approach is helpful. When a colleague thinks that the tasks are interesting and important and that his/her students should be able to tackle such example tasks, then he/she is more likely to seriously consider the competence concept as a means for understanding and promoting student development.

Having a pool of competence-related example tasks also helps to assess competence since such tasks could be used not just for competence acquisition but also for assessment. But this is certainly not sufficient to address the assessment topic. Given the sometimes large number of students in classes, one also has to think about assessing aspects of competencies in smaller tasks performed in written exams. In the ICTMA community there is already a substantial amount of work on assessing the modelling competency which should be taken into account (see http://www.ictma.net).

The mathematical education of engineers cannot be considered in complete isolation from the other educational elements of engineering students' education. In recent years, there has been a rise in new approaches to teaching engineers. Pedagogies such as CDIO (see http://www.cdio.org/), Problem-based learning and Project-based learning (Graham, 2010) are becoming increasingly widely used. There is a need to consider if the mathematical education of engineers should be integrated into these methodologies and, if so, how this can be achieved. A common characteristic of these pedagogies is their motivation of students to engage in active learning by presenting them with tasks which require them to use fundamental engineering principles, some of which they may not have met before. These approaches offer potential for development of some of the mathematical competencies outlined in the framework document provided tasks with suitable mathematical content can be developed. One area where the MWG might address future activity is in exploring ways of integrating mathematics education more closely with other elements of engineering education.

Another open question is how we react to changes in the learning environment. Incoming students have access to a wealth of sources of software and hardware to supplement face-to-face teaching. Smartphones, laptops and tablets are commonplace and enable students to access information which their lecturer may have hosted on their VLE, but also provide access to computer algebra systems, MOOCs, lecture material from other universities, revision material and, increasingly, Mathematical Apps. Moreover social networking is enabling students to share information and work together in a way not dreamed of even 10 years ago. Many experience such technologies at school and they expect continued usage at University. An area for future work could be to address how best to teach and encourage conceptual understanding and active learning in this new environment. Perhaps also we could address how to take advantage of social media and other advances in technology to share resources/exchange ideas, etc. and, as a community of educators, we could also benefit.

It is important that not only do undergraduate engineers acquire mathematical competencies whilst at university but that they maintain these competencies throughout their working lives and this long-term "sustainability" of the competencies is a crucial element of engineering education in developing engineers to meet the expected future challenges of society (see Come et al. 2013). Graduate engineers will need professional competencies to reflect all society changes, including not only engineering knowledge, problem analysis, investigation and applied research, solution design, project development and use of various high-tech tools, but also ethics, communications, social behaviour, project management and open capacities to use resources for life-long learning. Changing modes of knowledge production, dissemination and application are creating increased demand for skills in the inter-disciplinary team-working, the use of ICT and the ability to learn for oneself and from peers. Transformation of engineering education is necessary in order to provide environments and curricula facilitating such high demands on future engineers. Future challenges of the SEFI Mathematics Working Group might therefore also be focused on finding innovative teaching strategies that might lead to both a deeper mathematical conceptual understanding and enjoyment of solving mathematically based applied engineering problems, thus strengthening basic professional characteristics and mathematical competencies of engineers ready to work in the competitive environment of the future decades.

Potential future activities of the Mathematics Working Group

There are several ways to achieve results in relation to the directions stated above:

- We should monitor respective developments in other projects like KoM@ING and ask them to give presentations at the group's seminars to make the results better known.
- We should encourage the SEFI MWG community to specifically work on the above topics by formulating seminar calls for papers accordingly.
- We should provide a framework for organizing contributions and putting them into perspective such that accumulation, progress and remaining deficiencies become evident and inspire future work. This could happen in the following ways:
    - By regularly updating the curriculum document to include new contributions
    - By keeping a (hopefully growing) set of curricula for special types of study courses
    - By developing databases of competence-related tasks (assignments, projects) for competence acquisition and for competence assessment (such as the MAPS server for mathematical application projects (Alpers, 2003) and the question bank for electronic voting systems (Robinson 2010)).

By following the directions stated above, the SEFI Maths Working Group should retain and extend its role as a valuable source of information and an interesting place for exchanging views on the mathematical education of engineers.

References

Alpers, B., Demlova, M., Fant, C.-H., Gustafsson, Th., Lawson, D., Mustoe, L., Olsson-Lehtonen, B., Robinson, C. and Velichova, D. (2013) “A Framework for Mathematics Curricula in Engineering Education”, SEFI, Brussels.

Alpers, B. (2014) “A Mathematics Curriculum for a Practice-oriented Study Course in Mechanical Engineering”, Aalen University, Aalen.

Alpers, B. (2010) “Studies on the Mathematical Expertise of Mechanical Engineers”, *Journal of Mathematical Modelling and Application*, 1: 2-17.

Alpers, B. (2003) “MAPS – A Mathematical Application Project Server.” In In S. Hibberd, L. Mustoe, eds. *Proc. IMA Conference on Mathematical Education of Engineers, Loughborough*. Engineering Council, London.

Come, F., Fouger, X., Hawwash, K. and Van Petegem, W. (Eds.) (2013) “SEFI@40. Driving Engineering Education to Meet Future Challenges”, SEFI, Brussels.

Graham, R. (2010) *UK Approaches to Engineering Project-based Education*, Gordon-MIT Leadership Program Report. (Available at http://web.mit.edu/gordonelp/ukpjblwhitepaper2010.pdf accessed 31 January, 2013).

Mustoe, L., Lawson, D. (Eds.) (2002) Mathematics for the European Engineer. A Curriculum for the Twenty-First Century. A Report by the SEFI Mathematics Working Group. SEFI, Brussels.

Niss, M., Højgaard, T. (eds.) (2011) Competencies and Mathematical Learning. Ideas and inspiration for the development of mathematics teaching and learning in Denmark, English Edition, Roskilde University, Roskilde.

Robinson, C. (2010) “Using electronic voting systems for active learning”, Alpers, B. et al. (Eds.): *Proc. 15th SEFI MWG Seminar*, Wismar 2010 (available at; http://sefi.htw-aalen.de/Seminars/Wismar2010/SEFI/papers_pdfs/MWG2010_Robinson_C.pdf , accessed Dec. 4, 2013).

Schreiber, S., Hochmuth, R. (2013) “Mathematik im Ingenieurwissenschaftsstudium: Auf dem Weg zu einer fachbezogenen Kompetenzmodellierung.“ *Beiträge zum Mathematikunterricht 2013*. WTM-Verlag, Münster, 906-909.

Has the introduction of "Project Maths" at post-primary level affected the attitudes of first-year higher education engineering students in Ireland?

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Abstract

Since 2008, the National Council for Curriculum and Assessment have led a reform of the mathematics curriculum in post primary education in Ireland, known as "Project Maths". It aims to support students through a new teaching/learning style to enhance their thinking and mathematical skills. In this paper, we report upon an attitudinal survey on students attitudes and beliefs towards mathematics, as part of a broader study underway to investigate the impact of Project Maths on first year engineering students.

1 Introduction

Mathematics is increasingly a focus of educational studies nationally and internationally, due to the growing need of mathematical skills in today's technological, economical, and industrial world (European Commission 2011, Conway and Sloane 2005). It is widely known that mathematics is a core subject for science and engineering disciplines. As Project Maths was implemented on a phased basis over a four-year period, our study is directed upon testing the mathematical skills and investigating first year engineering students' attitudes towards mathematics over the course of the implementation.

In this paper, we give a detailed overview of the results of a pilot attitudinal survey conducted in 2012 on a cohort of students who studied phase one of Project Maths. Overall, the results show quite a negative attitude towards mathematics, a fact that is naturally of concern among a cohort of engineering students who will rely heavily upon mathematics for the duration of their studies and beyond. Possible reasons for these attitudes will be further investigated in the following years, along with comparisons as to whether there are any improvements in students' mathematical skills and attitudes evident in the data we collect.

2 Background

2.1 The Irish Education System

Post primary education in Ireland is called secondary level (Department of Education and Science 2004). Students spend five or six years in secondary level, depending on whether they take an optional transition year after their third year or not. Two major state examinations are taken by second level students; the Junior Certificate (JC) upon completing their third year, and the Leaving Certificate (LC) upon finishing secondary school. Even though taking mathematics at LC is not mandatory, most of the students who take LC study mathematics (Breen, Cleary and O'Shea 2009) as it is a core requirement for entry into higher education. Mathematics at LC is offered in three levels: foundation level, ordinary level, and higher level.

2.2 Project Maths

Project Maths is a reform of mathematics teaching and assessing in second level in the Irish education system, set by the NCCA. *"It involves changes to what students learn in mathematics, how they learn it and how they will be assessed."* (Project Maths Development Team 2014). Project Maths began as a result of educational concerns about mathematics education in Ireland. Conway and Sloane (2005), for example, addressed many concerns regarding mathematics education nationally and internationally. In particular, they emphasised the lack of students' capacity to apply mathematics in practical 'real world' contexts. In addition, a report by the NCCA (2011) declared that a significant number of students in post-primary level are lacking the skills needed in their academic and professional lives. Moreover, Scanlan (2010) stated other concerns including: students' performance levels in PISA tests; the small number of students taking mathematics at higher level in LC exams; the difficulties with mathematics illustrated by third level students; the lack of problem solving skills highlighted by employers of Irish students; and the general need for qualified mathematical and scientific graduates for the knowledge economy.

Project Maths was first implemented on a pilot basis in 24 schools (who volunteered to participate) from September 2008. These schools were chosen to run the project over three years, along with the associated changes to the examinations which commenced in 2010 for LC and 2011 for JC. The overall feedback from the participating pilot schools resulted in adjustments to the syllabus subsequently rolled out on a nationwide basis. The rollout was then applied in three main phases: in September 2010, phase one began nationwide, with phases two and three following in subsequent years. The first national LC examination to contain Project Maths material took place in June

2012, with the JC following in June 2013. The fully revised examinations containing only Project Maths-type questions will be in place from June 2014 and June 2015 respectively.

3 Attitudinal Survey

The attitudinal survey used in this study is based largely upon the work of Breen, Cleary and O'Shea (2009). However, in our case, two open-ended questions were added following each part of the survey in order to better explore any further opinions or ideas expressed by the students.

3.1 Survey Design and Administration

The questionnaire used collected personal information (including gender, year of birth, level of mathematics at LC) from the participants as well as recording responses to sets of rating scale items relating to Confidence, Anxiety, Theory of Intelligence, Goal Orientation (Learning/Mastery and Performance) and Persistence (Breen, Cleary and O'Shea 2009). In addition, two other scales were included in the study, known as Approach and Prior experience. All rating scale items were presented using a five-point Likert scale where (1) represented 'Strongly agree, (2) 'Agree', (3) 'Not sure', (4) 'Disagree' and (5) 'Strongly disagree.

3.2 Survey Analysis:

In 2012, 34 students were included in the pilot study. The pilot survey included 44 Likert-scale questions (referred to as Q1…). After a preliminary analysis, eight questions were dropped from the main survey. In this paper, only the questions used in the main study are explained in detail.

3.2.1 Confidence Scale:

The survey started with six questions examining students' confidence regarding mathematics, all of which are adopted from the study of (Breen, Cleary and O'Shea 2009). While the first three questions (Q1-Q3) in the confidence scale address positive statements regarding confidence in mathematics, the following three questions (Q4-Q6) address negative confidence statements about mathematics. Students' responses regarding confidence in mathematics were mainly negative. The responses show that most of the students, more than 64%, "strongly disagree" with Q1*: "I can learn mathematics quickly"* and Q2 *"I feel confident in approaching mathematics"*. On the other hand, about 20% of the students are "not sure" whether they could get *"good marks"* in mathematics or not, but 50% of the students strongly disagreed with that. The main survey is run at the beginning and the end of first year, which will allow us to take a closer look at their attitudes to compare whether their uncertainty about getting good marks in mathematics will be changed in any way after taking mathematics exams during that year in higher education. Furthermore, the majority of students, more than 82%, "agree" or "strongly agree" on Q6 which stated: *"I am just not good at mathematics*". What is more, when students were confronted with the statement: *"Q5. Mathematics is one of my worst subjects.",* strikingly, students only responded negatively, with more than 61% agreeing with that statement, and more than 35% strongly agreeing with that. It is particularly concerning that engineering students would respond thus.

3.2.2 Anxiety Scale:

Since the anxiety scale is also adopted from (Breen, Cleary and O'Shea 2009), and giving that (Q11) was dropped off their scale due to Rasch analysis results, we excluded the same question from the main study, even though it was included in the pilot survey, and for that reason Q11 does not appear on the anxiety results in this paper. Unlike the confidence scale, the most common responses to anxiety questions were "not sure". However, a considerable number of students (more than 26%) felt helpless, uneasy or worried about mathematics shown in the responses to Q9: *"I often feel helpless when doing a maths problem"*; Q10 *"Mathematics makes me feel uneasy and confused"*; and Q12 *"I usually feel at ease doing mathematics problems"* respectively. In our main study, we will take a closer look at the anxiety levels of the students compared with the individual's maths test results in order to determine whether their mathematical level affected their anxiety towards mathematics or not, with particular focus upon the very few students who showed no worries about mathematics.

3.2.3 Theory of Intelligence:

There are seven items in the theory of intelligence scale, which showed a variety of responses regarding students' beliefs in intelligence in general, and in terms of mathematics in particular. What is significant here is that the majority of the responses (79% of the students) disagreed or strongly disagreed with: *"Q16.You can succeed at anything if you put your mind to it."*. Again 44% of the students strongly disagreed with the statement: *"Q17.You can succeed at maths if you put your mind to it."* and with *"Q18.It is possible to improve your mathematical skills."*. Moreover, more than 55% of the students strongly disagreed with the last question on the scale which was: *"Q19.Everyone can do well in maths if they work at it."*. However, a considerable number of students did not respond to many of the theory of intelligence related questions and possible reasons for that will be examined and discussed later on the study.

3.2.4 Persistence Scale:

There are seven persistence questions in the survey. In terms of persistence attitudes towards mathematics, the responses varied from agreement and uncertainty to strong disagreement with persistence in mathematics-related statements, with the exception of Q25: *"When presented with a mathematical task I cannot immediately complete, I give up"* which got a striking level of agreement in student responses, with percentages of 64% agreed and an extra 23% who strongly agreed with that statement . Also, more than 58% of the students strongly disagreed with Q23 which stated: *"When presented with a mathematical task I cannot immediately complete, I increase my efforts"*. It is also worth mentioning that a couple of questions received fewer responses than the total number of students. In general, responses to the persistence-in-mathematics questions gives an overall impression of consistent failure to persist when encourtering a mathematical challenge, great or small, again a worrying trait in engineering students.

3.2.5 Learning Goals Scale:

The learning goals scale consists of five questions investigating students' goals in learning mathematics. Unfortunately, the questions of learning goals scales are missing a considerable number of students responses (over than 58% on each question), ending up with only 20 responses or slightly more, which hopefully will be avoided in the following surveys. Nonetheless, the majority of students who responded to those questions reflected a negative point of view regarding their mathematical learning goals. The majority of responses maintained that the goal of working at mathematics is not necessarily for the possibility of learning, figuring things out, or finding new methods or ideas. The most interesting points from the learning goals scale were that almost all the responses to Q29 were strongly disagreeing with the statement: *" I work at mathematics because I like figuring things out"*. Again almost all the responses to Q31 were strongly disagreeing with the statement: *" I work at maths because it is important for me that I understand the ideas."*.

3.2.6 Approach Scale:

The approach scale attempts to investigate students' approaches to learning mathematics and determine whether it is by memorizing mathematics rules or understanding the principles of mathematics. Students' responses to the scale showed an overall negative response to both questions. Looking at the first item on the scale, which stated: *"I learn mathematics by understanding the underlying logical principles, not by memorizing the rules."*, the majority of responses showed uncertainty along with a definite negative approach to learning mathematics. Specifically, 26% were not sure and 32% strongly disagreed with the statement. However, it is worth mentioning that more than 35% of the students did not answer that question. The second question on the scale illustrates a absolute negative student views to approaching mathematics, with more than 64% of the students strongly disagreeing with the statement: "*If I cannot solve a mathematical problem, at least I know a general method of attacking it"*.

3.2.7 Prior Experience Scale:

There are four items questioning mathematical prior experience. They are specially designed to investigate students' experiences with mathematics in school and specifically in second level, in order to determine whether the phased implementation of Project Maths over the period of the study is making any difference to students' experiences and feelings in relation to post-primary level mathematics. Question one on this scale obtained a variety of responses with only 2% strongly agreeing that mathematics was always "enjoyable" in school; continuing with comparable responses (around 14%) who either agree, not sure or disagree; but ending with a majority of 44%, who strongly disagree with that statement: *"Q41:Mathematics is a course in school which I have always enjoyed studying"*. Furthermore, when focusing on mathematical enjoyment in secondary school on the fourth question on the scale, comparable results were shown with 38% strongly disagreeing. The second question on this scale also resulted in variable responses; on the one hand, 40% of the responses agreed about forgetting mathematical concepts learnt in secondary level, while on the other hand 26% of students strongly did not agree with that statement. These responses will be looked at in comparison with the following years of the implementation of Project Maths, exploring the long-term recall memory of mathematics. What is significant in Question four on this scale is that 50% of the responses strongly disagreed with having a good background in mathematics, and 17% are not sure, so an overall negative response to the question: *"I have a good background in mathematics"*.

4 Conclusion

By investigating students' attitudes towards mathematics in this pilot survey, an overall negative response to the subject was strongly shown by the first-year engineering students who responded. The confidence scale showed low levels of students' confidence in mathematics, which was also seen in the mathematics test results which are currently being analysed. Furthermore, the persistence scale showed a significant lack of persistence in learning mathematics. Many of the responses given are particularly concerning the case of engineering students. However, it must be remembered that these students had only experienced two years of the first phase of Project Maths (so two out of five topic "strands" had been changed, but only for their final two years in secondary school). In the coming years, it will be of interest to compare whether students with greater exposure to Project Maths display more positive attitudes towards the subject, and to find out whether Project Maths has made any improvements to students' beliefs in mathematics and their abilities to learn and achieve high goals and scores in mathematics.

5 References

Breen, S., Cleary, J. and O’Shea, A. 2009. Constructing and validating an instrument to measure students’ attitudes and beliefs about learning mathematics. *IN:* Corcoran, D., Dooley, T., Close, S. and Ward, R. (eds.) *Proceedings of the Third National Conference on Research in Mathematics Education.*pp.332-342.

Breen, S., Cleary, J. and O'Shea, A. 2009. An investigation of the mathematical literacy of first year third-level students in the republic of Ireland. *International Journal of Mathematical Education in Science & Technology,* 40(2), pp.229-246.

Conway, P.F. and Sloane, F.C. 2005. *International trends in post-primary mathematics education.* National Council for Curriculum and Assessment.

Department of Education and Science. 2004. *A Brief Description of the Irish Education System.* Report ed. online: Communications Unit Department of Education and Science.

European Commission 2011. *Mathematics education in Europe: Common challenges and national policies.* Brussels: Education, Audiovisual and Culture Executive Agency.

National Council for Curriculum and Assessment 2011. *Project Maths: Reviewing the project in the initial group of 24 schools - report on schools visits.* Dublin: NCCA.

Project Maths Development Team 2009. *Overview of Project Maths* [Online]. Available from: http://www.projectmaths.ie/overview/ [Accessed 05/05 2014].

Investigating engineering students' mathematical problem solving abilities from a models and modeling perspective

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Engineers are expected to solve complex problems in their daily work. In order to do so, it is essential for them to translate real life problems to mathematical problems, i.e. mathematical modeling, and solve those problems in a systematic manner, i.e. structured problem solving. In this study, we have used a qualitative approach to investigate how engineering students develop their modeling abilities. We have analyzed students' modeling processes when engaged in solving mathematical modeling problems. The theoretical framework that underpins this study is the *models and modeling perspective* developed by Lesh and collaborators (Lesh & Doerr, 2003), where 'modeling' is seen as an important component in developing mathematical abilities. This study is a part of a larger research project that intends to improve teaching and learning mathematical problem solving in engineering education.

As one of the authors of this paper has developed a course on mathematical modeling and problem solving for computer engineering students (see Wedelin & Adawi, 2012), this course was the natural context for our study. We provide a brief description of the course in terms of content and design in the paper. The empirical data used in the study is comprised of 1) semi-structured interviews with eight students early on in the course – based on two problems that they had solved during the course; and 2) reflective reports submitted by all 103 students at the end of the course, describing the most important impacts of the course and their problem solving pathways. We found significant developments on students' conceptual understanding of modeling and problem solving, and the structure of their problem solving process.

Among others, students indicated significant developments on their problem solving abilities especially in metacognitive aspects (Flavell, 1979). In this poster presentation we intend to describe the course, which has been awarded a pedagogical prize in 2012, introduce the models and modeling perspective as well as present our preliminary findings from the data analysis.

References

Flavell, J. H. 1979. Metacognition and cognitive monitoring: A new area of cognitive–developmental inquiry. *American psychologist*, 34(10), 906.

Lesh. R. and Doerr H. 2003. *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching*. Lawrence-Erlbaum.

Wedelin, D., & Adawi, T. (2012). Bridging theory and practice: An inquiry-based course in mathematical modelling and problem solving. Paper presented at the ICEE conference, Turku, Finland.

400 years of educational technology

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This talk will examine the nature of mathematical technology, arguing for a broad definition which includes software such as computer algebra, and also the underlying algorithms and techniques such as place value. In 1614 John Napier published his "description of the Wonderful Canon of Logarithms", which revolutionised science. Logarithms also had an effect on education, but also generated arguments about the effect of this technology on teaching and learning mathematics. I shall discuss what I term the "fundamental tension of educational technology", which is the extent to which understanding a concept is necessary for the effective and successful use of an algorithm, or automatic procedure. I shall contrast the development of algebra teaching in England between 1800 and 1960, with the current changes induced by contemporary computer algebra. I shall argue that the developments of the last 25 years are as significant as those of Napier.

From The Individual Towards The Collective

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The first classes under the new Bologna program were taught at the University of Zagreb in the 2005-2006 academic year.
That same year, we launched the e-learning system with Moodle LMS (at the Faculty of Geodesy) and Virtual Sharepoint (at the Faculty of Architecture). At the same time, the University E-learning Support Center was established. It provides the main support for harmonizing the implementation of information and communications technology (ICT) in university teaching. In 2009, the University Senate passed a resolution to facilitate the recognition of e-learning levels. This document proposes three such levels, each defined by its purpose, scope and method of applying ICT in teaching.
We would like to emphasize that we introduced elements of e-learning into our courses much earlier than 2005. We quickly achieved the first e-learning level and we are presently fulfilling the requirements of the second level. At the same time, the higher percentage of students who have passed our exams, and their positive and encouraging comments in student surveys, have prompted us to go even further.
The project "Introducing 3D Modeling into Geometry Education at Technical Colleges (3D GEOM TECH)" has thus begun. It brought together four university faculties: the Faculty of Architecture, the Faculty of Civil Engineering, the Faculty of Geodesy and the Faculty of Mining, Geology and Petroleum.
Our goal was to enhance the collaboration between the teachers of mathematics/geometry courses, and to improve teaching methodologies and harmonize the standards of educational materials for their further implementation in the e-learning systems.
The project was approved as a development project and supported by the Fund for the Development of the University, University of Zagreb, for the 2011-2012 academic year.
We have made an effort to compile our educational materials to make them available to students and other users interested in a particular content. The materials have been gathered in a common basic repository. It is the first such repository in university and polytechnic centers in Croatia.
At the 17TH SEFI MWG Seminar we would like to present parts of our individual teaching content and to describe their transformation from an individual into a collective one.

References

[1] J. Beban-Brkić, S. Gorjanc, Ž. Tutek: New Challenge for Developers of e-Infrastructure, 15TH ICGG,

1-5 August, 2012, Montreal, Canada, Book of Proceedings/CDRom

[2] B. Divjak et al.: Ishodi učenja u visokom školstvu (Learning Outcomes in Higher Education), TIVA Varaždin, Fakultet organizacije i informatike, Varaždin, Croatia, 2008

[3] Sveučilište u Zagrebu; Odluka o oblicima sveučilišne nastave prema razini primjene tehnologije e-učenja (Resolution on Forms of University Education According to Application Level of E-learning Technology). Retrieved 20. 1. 2014.

Internet sites

[I] http://e-ucenje.geof.unizg.hr/
[II] http://virtual.arhitekt.hr/
[III] http://www.grad.hr/geomteh3d/index-eng.html

Lehre^{n} Kolleg: Transferring Reform Approaches in Mathematics for Engineers

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The Lehre^{n} Kolleg was initiated by the Bündnis für Hochschullehre (Alliance for College Teaching) for the exchange of ideas and experiences gained within reform projects in mathematics courses for engineering students. The Bündnis provides a platform for this exchange, bringing the participants in contact with key players in higher education in Germany. This contribution presents an overview of the Kolleg and the six participating projects. Themes considered include incoming students' potential lack of mathematical skills (Technical University Wien), need for assimilation to college learning (Fachhochschule Aachen), combining practical experience and theory (Ruhr-University Bochum), transdisciplinary courses (University of Applied Sciences Hamburg), feedback and formative assessment implemented into traditional lectures (Ostfalia University of Applied Sciences Braunschweig/Wolfenbüttel), and learning and teaching ressources for the transition period to the university (Technical University Berlin). This poster contribution provides an overview of the Kolleg activities on an individual as well as a collective level.

Mathematics and Statistics Support – the Experience of the Sigma Network

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In 2011 the (UK) Advisory Committee on Mathematics Education

> estimate[d] that of those entering higher education in any year, some 330,000 would benefit from recent experience of studying some mathematics … at a level beyond GCSE, but fewer than 125,000 have done so.

In 2012, the (UK) House of Lords Select Committee on Science and Technology reported that post-16 mathematics is not taught at a level that meets the needs required for undergraduate study in STEM subjects. Furthermore, STEM graduates do not always have the skills and knowledge required by employers.

This "mathematics problem" impacts every UK university and almost every discipline, from engineering to nursing. The majority of universities in the UK, and many in other countries, now provide some form of mathematics and statistics support – extra and non-compulsory, designed to assist students in developing the mathematical and/or statistical confidence, knowledge, skills and understanding necessary for success in their chosen discipline.

The **sigma** Network is the government-funded UK community of practice for professionals working in mathematics and statistics support in higher education. From pioneering beginnings at Coventry and Loughborough Universities, it now engages with more than 80 UK higher education institutions. This paper will review the recent activities of the **sigma** network in

- The development and sharing of resources and good practice guides;Promoting rigorous evaluation of mathematics and statistics support;Provision of staff development and training;An annual conference, attended each year by over 100 delegates;Annual **sigma** prizes ;The **sigma** advisors scheme;Continuing to host and develop **math**centre and the **math**centre community project;Developing **stats**tutor in response to the growing need for statistics resources.

The paper will also seek to convey

- how much students value mathematics and statistics support provision in enabling them to achieve their full potential in their chosen discipline;
- some observations on UK HEI practice in relation to the mathematical content of STEM programmes arising from the experience of those involved in the **sigma** Network and other observers.

What is troublesome knowledge in Mechanics for mature students?

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As the landscape of Higher Education changes, increasing emphasis is brought to the area of widening participation in higher education. One example of this is pathways to allow mature learners from non-traditional backgrounds into Higher Education. At the same time education is also becoming more and more globalized. For many international students who intend to study undergraduate degree, a one-year foundation programme is often a necessary first step. This is due to the gap between their high school qualifications and the admissions criteria of UK universities.

Some Foundation Centres, such as that at Durham University, deliver courses that help mature students and international students to develop the skills and knowledge necessary for successful study in UK higher education. After successfully studying on the one year program, students can progress to a degree course in any subject at Durham University, including all the STEM disciplines. Mathematics is a core subject for the students who progress to sciences degree courses. It is an intensive course, consisting of two twenty credit modules, taught through three terms, six hours a week. The unique combination of student cohorts, i.e., home mature students study alongside younger international students, brought many challenges to the teaching due to the difference in educational background and experiences. For example, each group may have different misconceptions, and the threshold concepts for one group may not be as troublesome for the other. This highlights the importance of differentiating barriers to their understanding in order to deliver effective teaching.

This paper compares mature home and younger international students' troublesome knowledge when they are learning alongside each other, one of the most challenging mathematical module - mechanics. The study shows that both home mature students and younger internationals students have similar troublesome concepts in Newton's Law to many undergraduate students in other countries such as America, suggesting the alien nature of this knowledge to students. More intervention is needed to define in detail these threshold concepts in order to transform their understanding.

Both mature and international students have different strength and weakness when solving a problem in Newtonian mechanics. The course has helped mature students to improve understanding of Newton's Law, however, they are still not equipped with sufficient mathematical modeling skills.

It can be difficult for mature students when study together with younger international students who are generally younger and very competitive in algebra and have good analytical skills. However, there is also a benefit when they work together to help each other.

Further research is needed to understand what exactly the troublesome knowledge in mathematics is when solving a mechanics problem, and this investigation is under way.

Reference

Docktor, J. and K. Heller. (2003) *Gender differences in both Force Concept Inventory and introductory physics performance*. in AIP Conference Proceedings.

Meyer, J. H. F. and Land, R. (2003) *'Threshold Concepts and Troublesome Knowledge 1 – Linkages to Ways of Thinking and Practising' in Improving Student Learning – Ten Years On.* C.Rust (Ed), OCSLD,Oxford

Land, R., Meyer, J.H.F. and Baillie, C. (2010) *Editors' Preface: Threshold Concepts and Transformational Learning*, in: Threshold Concepts and Transformational Learning, Land, R.,

Meyer, J.H.F. and Baillie, C., (eds), Sense Publishers, Rotterdam, pp. ix-xlii, [book details].
Land, R., Cousin, G., Meyer, J.H.F. and Davies, P. (2005), *Threshold concepts and troublesome knowledge (3): implications for course design and evaluation*, In: C. Rust (ed.), Improving Student Learning - diversity and inclusivity, Proceedings of the 12th Improving Student Learning Conference. Oxford: Oxford Centre for Staff and Learning Development (OCSLD), pp 53-64.
Meyer, J. H. F. and Land, R. (2003) *'Threshold Concepts and Troublesome Knowledge 1 – Linkages to Ways of Thinking and Practising' in Improving Student Learning – Ten Years On.* C.Rust (Ed), OCSLD, Oxford
Perkins, D. (2006) *Constructivism and troublesome knowledge*,in: Overcoming Barriers to Student Understanding: threshold concepts and troublesome knowledge, edited by Jan H. F. Meyer and Ray Land, Routledge - Taylor & Francis Group, London and New York, pp 33-47 .

Foundation Year Engineering Mathematics. The triple identity crisis.

CDC Steele, School of Mathematics, University of Manchester.

An important part of the school/university interface is the concept of the Foundation Year which runs at many universities. Students can join a Foundation Year for several different reasons e.g. school system finishing short of university entry standard, desire to change direction with existing qualifications in inappropriate subjects, narrowly failing to gain admission to year 1 etc. Once on a foundation course students take course units in mathematics and science and possibly skills or projects units. On successful completion, students are suitably qualified to start in year 1 of an Engineering or other programme.
Foundation Years follow several different models. One distinction is between those where the teaching is carried out within the university and others where teaching is contracted to a local college etc. Different foundation years will have different balances between the various subject taught. These considerations can lead to some students taking mathematics within a foundation year with a view to progressing to an Engineering degree can face some questions concerning their identity on the course e.g. am I indeed part of the university ? Am I really part of my subject discipline ? When studying mathematics, am I a mathematician or an engineer ? This last question is, of course, common to the great majority of Engineering students but the presence of the other questions may lead to an amplification of issues surrounding this last question.
The EPS (Engineering and Physical Sciences) Foundation Year at the University of Manchester is taught from within the university and has an intake each year of over 300 students with slightly over half of them intending to progress to Engineering degrees. This talk will address various initiatives past, present and future (options structure, HELM, Class examples, Computerised Assessment) to give Engineering students on the Manchester Foundation year identities as Engineering students studying mathematics.
The benefit that students have had on the Manchester Foundation year with many mathematics courses is clearly shown in their subsequent studies.

Consequences of using technology in mathematics education

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The international assessments TIMMS and PISA had a big impact on the mathematics education at school in Germany. In general the results admit a high level in mathematics education but deficits in the way of teaching.[1] A guideline for teachers at school the so called "SINUS Transfer" project was established. The SINUS experts suggest: "Reduce the curriculum to basics. Reduce the predominance of teaching the formal skills therefore improve the understanding." Also the aim is "an individual learning instead of drilling lock-step the formulas".[2] They emphasize the role of the learners to be self dependent, and the role of the teachers as a moderator. They try to reduce formal exercises. Doing exercises until the rules can be used automatically is defamed as a *plantation of exercises*. Mathematics should be linked to real world problems. At school the focus is now on modelling using technology. But what are the consequences at universities?

It seems that an active discussion between teaching staff at school and university has not taken place up to now. Now at university we can observe the results of this development. Written exams show terrible elementary mistakes we rarely have seen before. And even relative well doing students show up these mistakes. We will give examples and little statistics of some of these problems.

We also compare the level of books for mathematics used at German schools over years.

How shall university react? What do engineers think should be the outcome of the mathematical education? What is the need of industry? Or are we mathematicians dinosaurs?

[1]Baptist, P.; Raab, D.: SINUS Transfer - Auf dem Weg zu einem veränderten Mathematikunterricht. http://sinus-transfer.uni-bayreuth.de/fileadmin/MaterialienBT/sinus-transfer.pdf
[2]See 1

The End Product of Engineering Practice Depends on Society Understanding the Maths!

Tim Joyce, Chartered Engineer, Office of Public Works, Ireland

The modern Engineer must be a social beast. Otherwise, Engineers will be fated to backroom positions in organisations where other disciplines control both management of the organisation and interactions with society. Instead of accepting this fate, this paper puts forward the mathematically well educated Engineer as being the best to see through risks and, because of that, to clearly communicate them and win stakeholder support. All we have to do is improve the first two of the three 'R's and add in a dollop of psychology. What could be easier than that?

As a backdrop, the talk looks at the study of urban flooding. First, it aims to help you investigate a problem, show you some of the required information, statistical models for flood peaks and how hydraulic modelling converts this to the profile of a flood that causes damage; that can then be evaluated. And, all that mainly communicated through graphs! Second, it will show you how a constant regard for 'the basics' guides the creation of a solution to the flooding problem tailored to the individual needs of the local environment and society. If this is not economically feasible, environmentally acceptable, does not comply with Health and Safety or the Public do not accept it, it will not be built. What could be easier than that?

You also get to meet some of my mathematical heroes along the way.

ACAM - Competency Assessment / Improvement Actions: Diagnose to guide

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GIDiMatE - Research Group on Didactics of Mathematics in Engineering

Keywords: Mathematic skils, Engineering, Mathematics, Diagnostic Test.

1. FRAMEWORK

Are increasingly frequent the debates about the failure of mathematics in the teaching of engineering and its relationship to the knowledge obtained in high school. It is well known the poor performance of primary and high school Portuguese students on national exams and also on *Programme for International Student Assessment (PISA)*. According to PISA 2012 report, those have been closer to the average observed but the improvements do not change the fact that students who access to higher education have, in general, difficulties on basic and elementary math contents, which may be assumed as a cause of abstention to classes and assessment and therefore the high failure rates.

The insufficient preparation that students have when they arrive to higher education is not exclusive of the Portuguese education (in 1998 the *European Society for Engineering Education (SEFI)*, meeting in Finland, have already addressed this fact) but is compounded by the heterogeneity of the undergraduate students in engineering.

Motivated by the huge budget cuts and the increasing reduction of candidates to higher education, Portuguese institutions used alternative processes to attract new students, which increase the heterogeneity of personal and motivational features of students, that access through very different ways (Scientific – Humanistic graduates, Professional undergraduates Courses, Technological undergraduates courses, Technological Specialization undergraduate courses (CET), over 23, etc). This multiplicity of students has originated heterogeneous mathematical skills, asymmetries in the essential mathematics knowledge and difficulties on integration into higher education, and motivate the definition of alternative paths that allowed those students to follow a positive learning process [2].

The Department of Physics and Mathematics (DFM) of Coimbra Institute of Engineering (ISEC) has promoted and developed a set of strategies to reverse this situation, reorganizing the operation and evaluation of courses, building tools that facilitate the learning process and implementing strategies for student engagement. Despite all this effort, students have not met the expectations and continue to exhibit a high failure rate and high abstention rates (to classes and also to assessment tests).

Assuming that students are not learning what they should learn, because of the enormous gap in the basic knowledge, we should construct pedagogical tools that can contribute to the diagnosis, acquisition and consolidation of mathematical knowledge and skills needed in engineering, as well as develop resources that will give engineering students the best possible learning experience.

Differential and Integral Calculus (DIC) courses have been referred in many studies, and the difficulties experienced by students in elementary and basic contents, essentials for full integration in higher education, are major concerns expressed by many teachers, leading to adaptations of curricular reorganization and definition of actions that allow modify this situation.

GIDiMatE (Research Group on Didactics of Mathematics in Engineering), a group formed within the Mathematic Scientific Area of DFM has, since 2011, implemented *ACAM - Competency Assessment / Improvement Actions*, developing teaching tools and instruments that contribute to the acquisition and consolidation of basic and complementary mathematical knowledge, essentials for the Differential and Integral Calculus.

The aim of this paper it to describe the experience carried out in the last three academic years, presenting the results obtained and the consequent improvement of actions which have been introduced, motivated by *Mathematics for the European Engineer - A Curriculum for the Twenty - First Century (SEFI, 2002)*, and also the joint discussion with teachers and students and the partnership established with the Dublin Institute of Tecnhology.

The analysis and conclusions obtained from the *Diagnostic Test* motivated the creation of *CeAMatE* (Support Centre for Mathematics in Engineering), which intend to monitor individual student work and help them to overcome their difficulties within the basic and elementary knowledge and to allow the definition of custom tasks that fit the learning style, and methods of study, of each student.

2. METHODOLOGY

This research aimed to study, describe and interpret the results obtained at Mathematics Diagnostic Test from ISEC, based on the following question:

Q – "What is the level of knowledge of mathematic contents of students placed in ISEC?"

To study this question, since the school year 2011/2012 has been held annually implemented a Diagnostic Test in the first week of classes of the first semester. First year students who attended to classes of IDC courses were considered.

To answer to the research question, we chose a quantitative approach, according to an interpretive paradigm. To study the level of mathematics knowledge of students when they arrived into higher education, we design a case study.

A common type of research, namely in mathematics education, is the case study [10]. According to Kilpatrick, cited by Ponte [10] a case study is essentially a research design. It is not in itself a well defined research methodology and can be applied in the context of positivist paradigms, interpretive or critical. As stated by Coutinho and Chaves [1] many authors prefer to use the term strategy, whose purpose is always holistic (systemic, broad, integrated) for preservation and understanding the "case" as a whole and its uniqueness.

According to Ponte [10] a case study "may have a well-creased theoretical orientation, serving as support for the formulation of the respective issues and selection of instruments for data collection and constitutes a guide the analysis of results".

2.1. The sample

All studies have been focused in IDC, of the different degree courses of Coimbra Institute of Engineering: Biological, Biomedical, Chemical, Civil, Computer Science, Electrical, Industrial Management and Mechanical.

Every year ISEC give the opportunity to attend optional courses of undergraduate inserted in ISEC to students who have not joined in Higher Education. The goal of this initiative, named "Year Zero" is to allow this students to deepen knowledge in basic Engineering subjects (Mathematics, Physics and Chemistry), and to prepare them to the qualifying exams needed to access to Higher Education in the following year. These students are also considered in the sample.

In this context GIDiMatE carried out a data collection at the beginning of the first semester of each academic year, using a *Diagnostic Test,* to analyze the mathematical knowledge of the students placed in ISEC and reflect on the skills of new students in higher education. This diagnostic test occurs in practical classes according to the availability of teachers of each IDC course, who have provided full cooperation in this work.

3. DIAGNOSTIC TEST

Diagnostic test have been suffering successive alterations. Nowadays we consider a stabilized version that will be used for future comparisons. The results could lead to the implementation of tasks that lead to overcoming the shortcomings detected, towards the full integration of students in IDC courses.

3.1. Scholl year 2011/2012

The multiple reflections that ISEC teachers of Mathematics have maintained throughout their academic career, it has been found that students bring large gaps in elementary knowledge necessary for successful integration in the mathematics courses of Engineering degrees [3,4].

In the first edition of diagnostic test, placed in 2011-2012 school year, topics were addressed that, based on the experience accumulated by the teachers, should be the most relevant in the acquisition of knowledge and skills in mathematics in higher education, particularly for degrees in engineering. Five topics were considered: Equations, Functions, Rationals, Geometry and Trigonometry and Derivatives.

The questions were of multiple choice, with six options each, in order to assess the most common shortcomings of students. The 36 questions of the test were distributed as showed in Table 1. The results were obtained by arithmetic mean of correct answers in each of the 5 topics.

Area / Topic	Number of questions	Question ID
Equations	8	1, 2, 7, 12, 20, 27, 29, 35
Functions	5	4, 11, 15, 17, 26
Racionals	7	3, 9, 19, 24, 25, 32, 34
Geometry / Trigonometry	10	5, 6, 8, 13, 14, 18, 21, 23, 28, 31
Derivatives	6	10, 16, 22, 30, 33, 36

Table 1: Distribution of the 36 questions by GIDiMatE area / topic.

3.1.1. Mathematics for the European Engineer – A Curriculum for the Twenty-First Century- Core Zero

SEFI, through its Mathematics Working Group (MWG), aims to provide a forum for discussion and guidance to all those who are interested in mathematics education of engineering students in Europe. In this context, in 1992, was established the first curriculum guidance document that sets out a detailed and structured list of topics which correspond to specific core content for the learning of mathematics in undergraduate engineering. Subsequently, in 2002 the MSW revised the report *Mathematics for the European Engineer - A Curriculum for the Twenty-First Century (SEFI, 2002)* aimed the learning outcomes rather than a simple listing of topics. Regarding the minimum knowledge of higher education engineering degree, these are detailed by areas and topics identified in Core Zero section [12].

Faced with the existence of such relevant document, our research group decided to compare the structure of the diagnostic test with the structure suggested by MWG. Among the suggested areas and according to the program of the Elementary and Secondary Education of Portugal, GIDiMatE gave special attention to Algebra, Analysis and Calculus, Geometry and Trigonometry. Those areas were considered as the most significant, because they are essential for most of the mathematic courses and because they already integrate the diagnostic test proposed by GIDiMatE. According to the guidelines of SEFI the 36 initial questions included in the diagnostic test were regrouped in the areas listed in Table 2.

Area / Topic	Number of questions	Question ID
Arithmetic of real numbers		
Algebraic expressions and formulae	3	3, 25, 27
Linear laws	3	2, 19, 20
Quadratics, cubics, polynomials	2	7, 13
Arithmetic of real numbers	2	1, 35
Analysis and Calculus		
Functions and their inverses	8	4, 9, 11, 15, 17, 24, 32, 34
Logarithmic and exponential functions	3	12, 26, 29
Rates of change and differentiation	6	10, 16, 22, 30, 33, 36
Complex Numbers	2	18, 23
Geometry and Trigonometry		
Geometry	2	5, 8
Trigonomteric functions and applications	4	6, 14, 21, 28
Trigonometric identities	1	31

Table 2: Distribution of the 36 questions by SEFI area.

3.1.2. Summary analysis, conclusions and recommendations

The sample is formed by 232 students of the following engineering degrees: Biological (24), Biomedical (28), Civil (64), Electrical (17) and Computer Science (99). It also includes the results of 40 students from “Zero Year”[5].

In terms of the question "What is the level of knowledge of mathematic contents of students placed in ISEC?” it was found that students of Biomedical Engineering are the best performers while students of Computer Science and Electrical Engineering exhibit the worst results.

It should be noted that in both classifications presented by GIDiMatE and SEFI group, the students present maximum results of 53.24% and 56.40%, respectively, with modal classes with values about 40%.

The best average results were presented in Derivatives (GIDiMatE classification), where rules of elementary derivation were evaluated, without form consulting. According to the guidelines proposed by SEFI, the best performances were performed in Algebra.

The worst results were observed in Rationals (GIDiMatE classification), where simplification of rational expressions, power properties, identification of natural and rational exponents, identification of remarkable limits and limit calculation were evaluated. According to the guidelines proposed by SEFI, the worst performances were obtained Geometry and Trigonometry.

Since the guidelines proposed by the two groups were differently categorized, no comparisons were possible to be made.

Taking as starting point this preliminary study, the GIDiMatE members reflected on the results and it has emerged the need to, among others, reduce the diagnostic test both in size and in time, and changing some issues in order to contribute to a better understanding of the difficulties faced by students. It was proposed the creation of a competence centre that monitoring the individual student work, encourages self-employment of students and suggests the tasks that best suit the learning style of each individual student. Additionally this Centre can create a learning environment to research studies concerning how students learn mathematics.

3.2. Scholl year 2012/2013

The work in this school year was centred on the reduction of the number of questions (from 36 we pass to 25 questions), the uniform distribution on the topics proposed by GIDiMatE, the reducing to 4 possible answers and the elimination of the option "none of them". It was also decided to remove all the question about complex numbers, since it was found this content was not uniformly addressed in secondary education, putting students in unequal situation in the diagnostic evaluation. From the 36 original questions of 2011/2012 diagnostic test only 12 were inserted in this new version.

From contacts with Dublin Institute of Tecnhology and the joint reflection that was done, we found out to be important to introduce some issues in common with the Irish diagnostic test in order to be able to make comparisons of results and proposed actions in partnership. With this purpose, 3 questions were modified, according to Table 3 (change of questions 1 and 26 and introduction of question 3).

Area / Topic	Number of questions	Question ID
Equations	5	1, 2, 6, 21, 24
Functions	5	4, 10, 12, 13, 15,
Racionals	5	3, 8, 18, 19, 22
Geometry / Trigonometry	5	5, 7, 11, 16, 20,
Derivatives	5	9, 14, 17, 23, 25

Table 3: Distribution of the 25 questions by area / topic.

3.2.1. Summary analysis, conclusions and recommendations

The sample is formed by 338 students of the following engineering degrees: Biological (29), Biomedical (32), Civil (16), Electrical (35), Electromechanics (18), Mechanics (44), Industrial Management (40) and Computer Science (124).

In terms of the question "What is the level of knowledge of mathematic contents of students placed in ISEC?" it was found that students of Biomedical Engineering are best performers while students of Electrical Engineering exhibit the worst results.

The best average results were presented in Equations (GIDiMatE classification), where quadratic formula, mental calculation, logarithms and exponentials properties, manipulation of inequalities, power rules, module properties and solving equations, were evaluated.

The worst result was observed in Rationals (GIDiMatE classification), where simplification of rational expressions, power properties, identification of remarkable limits and limit calculation were evaluated.

No comparative analysis with the classification proposed by *Mathematics for the European Engineer was held - A Curriculum for the Twenty-First Century-Core Zero* was made, because it was decided to develop the collaborative research with DIT partnership, and to stabilize the Diagnostic Test questions for further investigation studies.

3.3. Scholl year 2013/2014

As a result of the cooperative work with DIT, it was decided to approach Diagnostic Tests of both groups, so that comparative studies can be conducted in the two countries [6]. Therefore, 9 common questions were constructed or modified, and the number of questions in both tests was reduced to 20.

According to SEFI *Mathematics for the European Engineer was held - A Curriculum for the Twenty-First Century-Core Zero*, the final 20 questions were regrouped by the areas considered in the 2011/2012 school year approach (3.1.1.). The covered areas are listed in Table 4.

Area / Topic	Number of questions	Question ID
Álgebra		
Arithmetic of real numbers	2	7
Algebraic expressions and formulae	4	3,4,5,6
Linear laws	1	2
Quadratics, cubics, polynomials	1	1
Analysis and Calculus		
Functions and their inverses	3	9,10,11
Logarithmic and exponential functions	1	12
Rates of change and differentiation	3	13,14,15
Geometry and Trigonometry		
Geometry	2	16,17
Trigonomteric functions and applications	1	18
Trigonometric identities	1	19

Table 4: Distribution of the questions by area / topic.

Additionally, Danish KOM project led by Niss organized a detailed and systematic description of what we should expected to obtain with the teaching of mathematics, using the concept of *competence* which influence the description of the learning objectives reflected in studies of the OECD-PISA [11]:

“Possessing *mathematical competence* means having knowledge of, understanding, doing and using mathematics and having a well-founded opinion about it, in a variety of situations and contexts where mathematics plays or can play a role.” [7]

KOM project identified a list of mathematical competencies such as “the ability to ask and answer questions in and with mathematics, focus on mathematical thinking, problem handling, modelling and reasoning” and “the ability to deal with mathematical language and tools, focus on representation, symbols and formalism, communication competency” [7,8].

In this context we decided to integrate a question to evaluate the competence in mathematical modelling. For that purpose we selected a statement of a problem which reflects a linear system of two equations and two unknowns.

3.3.1. Summary analysis, conclusions and recommendations

The sample is formed by 371 results of students of the following engineering degrees: Biological (7), Biomedical (23), Civil (25), Electrical (42), Electromechanics (23), Mechanics (65), Industrial Management (15) and Computer Science (146).

The best average results were presented in Algebra (54,04% of correct answers), where the questions related to “Arithmetic of real numbers” and “Algebraic expressions and formulae” recorded percentages of correct answers up to 70%. In the topic "Analysis and Calculus" students revealed serious gaps, registering the worst results with an average percentage of correct answers of 39.66%, with percentages less than 35% in the issues related to "Functions and their inverses".

In terms of a specific question, the best results where on mathematically modeling, with a percentage of 75.54% correct answers.

We conclude that students show lack of knowledge on essential topics to perform well in IDC courses, so it is urgent to define strategies that help us to invert those results.

4. RESULTS

In this work we chose to compare the results of the questions that remained common in the three years of the GIDiMatE Diagnostic Tests. According to *Mathematics for the European Engineer was held - A Curriculum for the Twenty-First Century-Core Zero*, we obtained the following distribution areas / topics (Table 5).

Area / Topic	Number of questions	Question ID In Diagnostic Test 2013 / 2014
Álgebra		
Algebraic expressions and formulae	1	5
Analysis and Calculus		
Functions and their inverses	3	9,10,11
Rates of change and differentiation	3	13,15
Geometry and Trigonometry		
Geometry	2	16

Table 5: Distribution of the questions by area / topic.

We observe that this selection was not pre-planned in order to make any comparative study. It was only based on intuition, sensitivity and teaching experience of researchers. However, we observe that the remained questions are those that are more directly linked to IDC syllabus, since “Functions and their inverses” and “Rates of change and differentiation” are essential topics for a full integration of a student that access to an engineering degree.

To analyse data we considered an average weight (weights 1 to a right answer, 0 to a wrong answer and -1 for a blank question blank were used). We used Cochran's test (nonparametric test for proportions), which allows to compare three paired proportions where each variable is expressed in dichotomically success and failure was used.

The hypothesis tested for the 7 issues under consideration were:

H0: the proportion of right / wrong answers is the same in all tests
H1: the proportion of right / wrong answers is not the same in all tests

Questions 11 (domains), 13 and 15 (rates of change and differentiation) reveal a constant rate of right / wrong answers over the years under study. This confirms the apparent lack of knowledge in these essential themes for a correct integration into IDC courses.

Questions 5 (radicals), 9 (limits), 10 (inverse function) and 16 (geometry) show a non constant proportion right / wrong answers over the three years studied.

When the hypothesis of equal proportions in all paired samples it rejects in Cochran Q test, it is necessary to identify the groups that differ. For this purpose we considered two groups based on three samples and then we compared samples in each subgroup. We concluded that only question 9 (limits) kept the proportion of right / wrong answers on 2012 and 2013 years. This analysis supports the general opinion that students who access to higher education must be annually submitted to a test that diagnoses their basic and complementary knowledge, allowing to implement strategies for individual support and develop educational resources that may assist and overcoming the identified gaps.

5. CONCLUSIONS AND RECOMMENDATIONS

After these three years designing, implementing, sharing reflections and assessing students, we think we have built a tool that can help us to identify emerging gaps of students in higher education. To solve this problem researchers intend to implement CeAMatE, a support centre for mathematics in engineering that will allow to define educational pathways depending on the objectives and the learning profile of each student, and stories success or failure. This centre will support students in classroom and / or in distance.

The custom support offered in CeAMatE will induce behaviours of self-effiency, avoiding demotivation for self-study that leads to the abandonment of classes and academic failure. The developed learning objects will be organized according to the reference Mathematics for the European Engineer - A Curriculum for the Twenty-First Century (SEFI, 2002), and will be adapted to the Portuguese education.

The main objective will be "learning by doing". The system that we will intend to implement will learn through success and failure episodes and the learning objects will be selected according to the relevance they have to the users, according to their learning process and their level of cognitive development. The proposed activities stem from a review that student will do of his own work, which will involves his co-responsibility in the educational process and the construction of the learning environment.

We expect that CeAMatE enhances an environment of personalized learning where all stakeholders of the educational process (teachers and students) will act co-responsibility, responding according to the differences of each student that access to Coimbra Institute of Engineering, in terms of cognitive development and in style of learning of each stakeholder. This environment will enhance investigation work to redesign and improve education and training projects of higher education institutions that offer degrees in engineering, educational practices and training policies for teaching staff and will also serve to provide feedback to schools of elementary and secondary education in order to provide them a guidance to prepare students that intend to continue their studies in engineering.

The application of Diagnostic Test will allow the development of a *PIT* (Single Plan of Work), a document that will observe the learning progress of the student. The evaluation of the work done by each student that visits CeAMatE and the self-proposed tasks according to PIT will be the set of instruments that will monitor the student work. Additionally, periodic achievements of Diagnostic Test will be made with subsequent review and reformulation of the Individual Work Plan until student reaches the minimum required to be considered able to integrate the syllabus of IDC courses in ISEC.

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6. BIBLIOGRAFY

[1] Coutinho, C. & Chaves, J. H. (2002), *O estudo de caso na investigação em educação*, Revista Portuguesa de Educação - Cied Universidade do Minho, 221-243.

[2] Bandura, A. (1993), *Perceived Sel-Efficacy in Cognitive Development and Functioning*, educational Psychologist, 28(2), 117-148.

[3] Bigotte, E., Fidalgo, C., Rasteiro, D.M.L. (2012). “Understanding the Difficulties in Mathematics of Engineering Students in the Transition from High School to Higher Education”, ATEE Winter Conference - Professional development of teacher educators: Bringing together policy, practice and research, Coimbra.

[4] Bigotte, M.E., Fidalgo, C., Rasteiro, D.M.L. (2012) *The teaching of Mathematics in Engineering:The ACAM – Assessment of competencies/improvement actions project*, in Proceedings 16th SEFI MWG Seminar Mathematical Education of Engineers SEFI [ISBN: 978-84-695-3960-6]

[5] Bigotte, E et al (2012), *Projeto ACAM-Avaliação de Competências/Acções de melhoria*, in Actas XIX Colóquio da Secção Portuguesa da AFIRSE:Revisitar os Estudos Curriculares. Onde Estamos e Para Onde Vamos? Lisboa: EDUCA/Secção Portuguesa da AFIRSE. [ISBN 978-989-8272-14-0]

[6] Fidalgo, C. et al (2013) *Maths Diagnostic testing in engeneering: Dublin Institute of Tecnhology and Coimbra Institute of Engeneering*', in Actas 1st International Conference of the Portuguese Society for Engineering Education (CISPEE 20113)[ISBN 978-1-4799-1221-6/13/$31,00(CISPEE2013)]

[7] Niss, M. (2003), *Mathematical Competencies and the learning of mathematics: The Danish KOM project*, In A. Gagatsis, S. Papastravidis (Eds.), 3rd Mediterranean Conference on Mathematics Education, Athens, Greece: Hellenic Mathematical Society and Cyprus Mathematical Society, 115-124.

[8] Niss, M. (2003), *Quantitative literacy and mathematical competencies,* In B. L. Madison, L. A. Steen (Eds.) Quantitative literacy: why numeracy matters for school and colleges. Princeton: National Council on Education and the Disciplines, 215-220.

[9] Ponte, J. (1994), *O estudo de caso na investigação em educação Matemática*, Quadrante, 3 (1), 3-18.

[10] Ponte, J. (2006), *Estudos de caso em educação Matemática*, Bolema, 25, 105-132.

[11] SEC (2008). http://eurex.europa.eu/LexUriServ/LexUriServ.do?uri=COM:2008:0425:FIN:PT:PDF, consultado em 28 de Dezembro de 2011.

[12] SEFI (2002). *Mathematics for the European Engineer – A Curriculum for the Twenty-First Century. SEFI Mathematics Working Group*.

Experimentation with web lectures as part of the mathematics education for engineering students

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Abstract

New educational learning tools and resources online are growing in scope and extent. To support the possibility of taking an online engineering degree program, Narvik University College has chosen to facilitate a streaming service of all lectures conducted by the college. At the college campus Bodø in the academic year of 2012/2013 we chose to use these online lectures as a central component in a didactic innovation project. The aim was to test and evaluate the capabilities and limitations that web-based communication provides both academically and socially. The theoretical reference is linked to a social constructivist facilitation of students' learning (Vygotsky, 1978). A survey amongst the students was performed in the mid-semester. The results of the analysis show that the renewal of the teaching was well received by the students in the group. However, the social learning environment suffered, since students failed to participate in the sessions created in connection with the web lecturing. Thus, the social learning context that we initially sought to strengthen, were fragmented and faced lacking momentum as a result of the change.

Introduction

This article will deal with my exploration of a specific teaching method tested in the first year of engineering studies at Bodø, where I taught Mathematics 1 (Autumn 2012) and Mathematics 2 (Spring 2013) . The courses were given under the auspices of Narvik University College.

Since I had the opportunity to follow the same class over two semesters, I had ample possibility to collect information about how the teaching worked in practice, devise and listen to suggestions with regards to improvement potential, and perform such improvements.

After we had conducted the first term, we launched an anonymous survey in which the students were asked to evaluate the program. The survey highlighted the fact that many felt they had too little time with the teacher in a clean problem solution setting. It pointed out that too much time was spent on the review of the theory on the blackboard, while the students would like to have an example-driven teaching with a focus on problem-solving.

To accommodate the wishes of the students we implemented a fairly radical change of how the mathematics teaching was performed. It turned out that Narvik University College was voted the best in the country on the use of podcasts in their education (Arnstad, 2012). Due to the fact that all courses that we taught in Bodø had the same syllabus as in Narvik, we could easily adapt these podcasts, or streamed lectures, in our own course material. The idea was to use these presentations instead of tailoring my local version of them, and instead use my resources as a teacher in a more direct handson interaction with the students. This forms the background for my examination. I have chosen to formulate the research problem as follows:

Can the combination of online lectures and local tutoring lead to a better quality of the teaching of mathematics for engineering students?

Theoretical foundations

Learning strategies can roughly be divided into so-called teacher-centred or student-centred strategies (Kember, 1997). In the teacher-centred strategies, the tutor will have more or less complete control over how the material is presented and processed, while in a student-centred strategy, the student plays the central role. The teacher then has less direct control over how the learning occurs, and it will be more up to the students to take responsibility for their own learning process.

Student groups may have different backgrounds, and one strategy may be more suitable for them than the other. However, there has emerged an understanding that in contexts where the teacher is able to facilitate a so-called social learning (Jahr, 1998), it usually increases engagement, motivation and capacity for deeper understanding of the material taught. This type of learning strategy is usually associated with a student-centred approach, meaning that group work, discussions and various activities have priority. This could be viewed in contrast to a more teacher-centred strategy that involves a more lecture-based teaching approach, where the tutor presents theory in the form of theorems, evidence, methods and examples. In this scenario, the students play a rather marginal role (Mascolo, 2009).

This student-centred learning strategy places higher demands on both teacher and student. Teachers need to think more deeply about the discussion questions and activities that will encourage greater involvement of students. Students themselves have to be more active and responsive to these suggestions from the teacher, otherwise the dialogue-based learning would be less successful.

The way I chose to implement this student-centred learning in Bodø, was thus to take advantage of the online streamed lectures, and then be able to spend more time with students in a social constructivist context. In this way they could get help with conceptual understanding and problem solving through social interaction. In retrospect, I have come across a term in the international educational literature that is appropriate for this learning method, namely “Flipped Classroom” (Bergmann, 2012), hereafter abbreviated FC. “The Flip” here points towards a more student-centred learning, where most of the time spent on campus is centred towards learning from solving problems in a social context.

The innovation aspect of the FC model is mainly that it presents the bulk of the material through small video lectures recorded in advance. The idea is that students should study these *before* attending class as a kind of homework. When students arrive to class, the material would first be recapitulated. The teacher would first take a brief summary, run a quiz, etc. This is both to get students into the right “mood” to work with the material, while both teachers and students get the opportunity to diagnose acquired knowledge. The rest of the lesson would be mainly spent on various forms of social learning (Bergmann, 2012). In mathematics this is done most often by using the time to solve puzzles of varying difficulty, while getting help and discussing substance / tasks with the teacher and fellow classmates.

By making use of the online lectures I wanted to facilitate the students' own participation in their education (Skott, 2010: 185-186). I wanted to create room for discussion, questions, and cooperation on the basis of newly acquired knowledge and by doing so I would therefore contribute to a more active student learning. Research on the effectiveness of this type of cooperative learning strategy as opposed to traditional classroom teaching is voluminous (Mascalo, 2009). An overwhelming majority of studies indicate better learning through such arrangements. However, there are many challenges if one chooses to use a more pupil/student- centred learning method. It is not intended to disarm the teacher from teaching, rather the teacher is challenged to a greater extent both purely professional and as a leader. Teachers need to pitch tasks that at the right level for the students, as it must be taken into account that there are varying levels of mathematical understanding among the student body, which means that exercises and instruction need to be adapted to a large extent. The teacher also has to deal with students' different levels of ability to explain and communicate when problems are to be formulated.

In addition, the cooperative relationship between parts of the group may vary greatly. Some individuals may tend to want to dominate and this tendency could have a negative impact on individual learning. Other individuals may have difficulty contributing in a social context. Others may have a tendency to want to talk too much about extra-curricular topics, which can also be disruptive to others in the group. All in all this contributes to classroom management becoming a more important component than in a purely lecture-based context (Mascalo, 2009).

Implementation

I arranged it so that the students should meet on campus to jointly watch the streamed lectures on a video projector as it was performed in Narvik. I was present during these sessions, so that the students would have an opportunity to ask questions about the topic to a professional directly.

After the session, I posted recommended tasks, and the students usually worked in self-arranged groups as they wanted to complete these tasks. The work session on assignments was always arranged immediately after the lecture online so that they had the topic fresh in mind when they were working with the assignments.

Although the students should play an active role in their own learning process, the FC model requires quite some interaction from the teacher during the classroom activity. In my simplified form of this model, I chose to let the students work more or less on their own with the recommended tasks for the day. Only on the occasions when several groups got stuck on some task, I went through it on the blackboard.

Sometimes it was necessary to take corrective measures if the technology did not work as it should. On a few occasions it was necessary to contact the IT department of Narvik to get the sound working, otherwise it went surprisingly smoothly.

Method

After having run this form of teaching during half of the spring term, I conducted a survey in which I asked the students for the opinion on the change of the educational approach. This survey consisted of a total of 24 questions, of which 21 were multiple choice questions, usually using with 5 grading options to measure student agreement with each given statement. The class consisted of 19 people, of which 14 responded to the survey, (which is a response rate of 74%).

In brief, the background for my quantitative analysis is to consider this survey as a sample collected from a population of possibly all first year engineering students in Norway. The results were analysed in SPSS and Excel, where I performed a two-sided hypothesis testing on each of the questions with a grading 1-5, using the calculated mean and standard deviation on each question. My hypothesis test was:

H_0: It is not possible to draw any significant conclusion in positive/negative direction based on the answers on the question, ie. $\mu = \mu_0 = 3$

H_1: There exist a clear trend in the results, either in positive or negative direction, ie. $\mu \neq 3$

According to statistical literature (Lysø, 2010), one should always use Student t-test for an unknown population mean, a category in which this case clearly falls into. Also, sample size is of course very small, again advocating the use of the t-test. Degrees of freedom would be 13 in this case, and I chose a significance level of $\alpha = 0.05$.

Results

Since I do not have space here to delve into details on all categories, I will only highlight a few of the most interesting results. Firstly Table 1 below lists the results that falsified the H_0 hypothesis in either positive or negative direction:

Statement	Mean	Std. dev	Test
It was hard for me to understand the online lectures.	1.50	0.52	False
Too much time was dedicated to activity after watching the online lectures.	2.38	0.96	False
I felt well prepared to solve tasks after having watched the online lecture.	4.29	0.91	True
I felt that watching the lectures online and concentrating the classroom activity on solving problems was a fruitful use of the course time.	3.93	1.07	True
If a quiz had been run about the topic of the lecture in the session following the lecture, I believe that the learning outcome would have been better achieved.	2.57	0.65	True

Table 1. Survey questions in which student responses show clear trends.

Among the statements in the survey questions that failed to elicit a clear false/true conclusion from the students were the following:

- I prefer to use the time in the classroom for problem solving rather than listening to a lecture
- My understanding of the topic were enhanced by the additional time we spent working on problem solving after watching the online lecture
- I prefer to listen to lectures face-to-face (“live”)
- I felt that more time could have been used to repeat the content of the lecture in the session afterwards

Discussion

As we see from the results above, there was statistically significant agreement among the students with the statement that it would be fruitful to run lectures online and rather work on assignments in groups. They felt well equipped to solve problems from watching the online lecture, a result which is probably rooted in the fact that they expressed the opinion that that they thought the quality of these lectures was good.

Usually we met to watch the online lectures before lunch, followed by a working session immediately after lunch. From the data it seems a little more difficult to draw clear conclusions about this arrangement. The students seemed to agree on the amount of time that was allocated to activity after the online lecture, but it cannot be concluded whether they think that this session increased their understanding of the topic. It seems to be paradoxical that this same group of students gave this response, given that much of the feedback in the previous study indicated a desire for more assistance on problem solving, and less on lectures.

Quite rapidly we saw that the students failed to meet on campus to follow our program of teaching. On average, approximately 75% of the class was not present, resulting in quite some loss of momentum in teaching here. Since it was possible for the students to log on to the streaming service from their own PC, many chose to sit at their home PC, and work privately instead of participating at campus.

It turned out that many of the students had difficulty in actively contributing to a social learning process. We had discovered this already quite early in the first semester and tried to remedy this by inviting the students to social events to create better cohesion and acquaintance. However, it was difficult to get commitment for such activity. Many of the students had family obligations and many had a job in parallel to their studies so as to be able to fund their education. In addition, there was also a group of students from out of town, many of whom could not spare time for such activities.

Several students also showed early signs of falling behind in the review of the curriculum. The fact that many chose not to attend classes, suggests to me that this type of education sets higher demands on the students’ progress. It was no longer possible to sit back and passively receive lessons from the lecturer; one had to be up to speed on the topic, and to be prepared to actively work on it immediately after having it presented. There were statements from some of the students indicating that they felt it uncomfortable not being updated adequately on the curriculum. Thus, they felt it was difficult to work effectively together with the other students. Mathematics as a field of learning is typically very cumulative in nature, each brick of knowledge usually resting on previous knowledge, thus making it difficult to step into a topic if sound progress on previous chapters hasn't been made. Usually, the engineering students also tend to give less priority to the math and physics subjects, compared to what they perceive to be their main subjects; this results in a more burst-like progress, which is not very suitable for the FC model.

Summary

The starting point of this study was to undertake an innovative modification of mathematical education in the engineering studies, by including online lectures as a key component. The quantitative analysis of the responses to the survey indicates that the majority of students felt that the quality of teaching was better. However, one can question whether any progress was gained in the social learning aspect. Many students chose to sit at home most of the time to watch the lectures/work with tasks, and in the end I lost completely contact with as much as 50% of the students. Thus, one could actually state that this experiment acted **against** its primary objective which was an enhancement of the social-constructivist way of learning.

References

Arnstad M. (2012, 7. november). *HiN best på podcast*. Collected 5th June 2013 from http://norgesuniversitetet.no/ikt/hin-best-pa-podcast

Bergmann J., Sams A. (2012). Flip Your Classroom, Reach Every Student in Every Class Every Day, Washington DC: ISTE.

Fredriksen H. (2013). Mathematics Teaching on the Web for Student Teachers: Action Research in Practice, In *Proceedings of the Eight Congress of the European Society for Research in Mathematics Education*

Jahr E. (1998). Piaget og Vygotskij. I Tufteland G. (red): *Matematikk 1 for allmennlærerutdanningen, Bind 1.* Oslo: Universitetsforlaget.

Kember, D. (1997). A reconceptualisation of the research into university academics conceptions of teaching. *Learning and Instruction* 7(3), 255–275.

Lysø, K. (2010). Sannsynlighetsregning og statistisk metodelære, Oslo: Caspar Forlag.

Mascolo M. (2009). Beyond student-centered and teacher-centered pedagogy: Teaching and learning as guided participation. *Pedagogy and the Human Sciences*, 1(1), 3–27.

Skott J., Jess K., Hansen H. (2010). *Matematikk for lærerstuderende, Delta, Fagdidaktikk*. Frederiksberg, Danmark: Forlaget Samfundslitteratur.

Vygotsky L. (1978). *Mind in Society,* Cambridge: Harvard University Press.

Zappe S., Leicht R., Messner J, Litzinger T, Lee H. W. (2009). «Flipping» the classroom to explore active learning in a large undergraduate course. In *American Society for Engineering Education Conference Proceedings.*

Activity "lesson" on Moodle for the teaching and learning of mathematics to engineers

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Abstract

Moodle is a class management system to promote interactive and collaborative learning activities. One of the most complex and interesting activities in Moddle is "Lesson" which facilitates the formation of theoretical thinking, based on reflection, analysis and planning, which in turn leads to psychological and intellectual development.

This Moodle activity was used in Calculus I on the Electromechanical Engineering Course in Coimbra Institute of Engineering. The main purpose of its use was to improve the students' motivation and increase the success in Calculus I. By using this activity in a creative way, it is possible to build contents that really overcome the normal limitations of contents pages and transform them into motivating and effective tools.

Introduction

It is widely known that, currently, demotivation, lack of interest and educational underachievement levels in higher education are very high (Arman 2008, Marçal 2009, Wagner 2008, Woodill 2004). That is why it is essential to increase the motivation of each student. For that propose, along with the intention of increasing the teacher-student interaction by enabling a more flexible learning, the project e-MAIO (Interactive Online Learning Modules) was created. The e-MAIO is a project developed over the Moodle platform for mathematics teaching and learning, and was applied to the engineering courses in Coimbra.

The activity "Lesson", available in e-MAIO, is a set of pages containing not only text and multimedia material but also a number of questions concerning the relevant content. The development of the lesson is based on the answers given by the students to the questions. These lessons have a number of alternative pathways, through which certain content becomes available, depending on the student's answers. This allows for the tailoring of the content to each student's study rhythm. For example, students with greater knowledge are conducted through a shorter pathway, without having to go through the more elementary topics. Each question has different answer options, each one connected to a different progress phase of the lesson. This leads the student either to another page or returns to the same page, so that the answer choice determines the sequence of the lesson. As not all the students have the same study rhythm or even the same way of interiorizing the discussed themes, the use of this tool encourages them, enhancing the learning and increasing its success.

This article describes the application of the activity "Lesson" in Mathematics teaching to Engineering courses as well as the feedback received from the students involved in this experience.

At the end of the first semester of the school year 2013-14, an online inquiry was given to the students from the Electomechanical Engineering course, aiming to get to determine the satisfaction level of the students regarding the utilization of the activity "lesson" on e-MAIO.

Activity "lesson"

More and more, motivation is a key factor in student achievement (Lourenço 2010). Because students do not have the same learning pace, nor the same way of absorbing the contents, the use of different teaching methodologies according to each student may be an important motivation element and therefore a promoter of the students' learning success (Cury 2000, Zhang 2006). The use of Moodle (Modular Object-Oriented Dynamic Learning Environment) provides its users (students) an individual learning environment that can be customised (Penny 2011).

Many learning models, based on Moodle, have been developed for the teaching and learning of mathematics in engineering (Coan 2011, Fujimoto 2010, Madeira 2012, Reali 2012, Rodrigues 2010).

"Lesson" is one of the most interesting Moodle activities and is similar to a textbook with pages and exercises. The big difference between "lesson" and a textbook is the available resources. A Moodle lesson may have audio, video and web links, whereas a book does not.

A "lesson" allows the presentation of contents in an interesting and flexible way. Basically, a "lesson" is made by a set of contents pages where there is a theoretical exposition of the subject matter and pages of questions about the subject matter on contents pages. These questions may be, for example, of the true/false type, short answer type, multiple-choice, formative or numerical.

The questions are on pages which may include different response options, each option being associated with an advance (or retreat) in the lesson. This advance (or retreat) guides the student to another page, or back to the same one. It is the student's answers that define the lesson sequence.

The student goes forward in a sequential way or is guided onto different paths, so that the content is always adapted to the student performance.

The figures below illustrate examples of this potential:

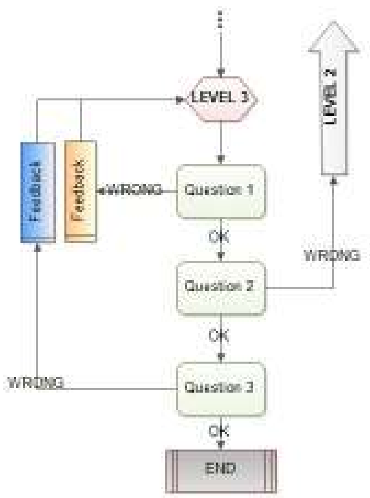

Figure 1 - Lesson navigation

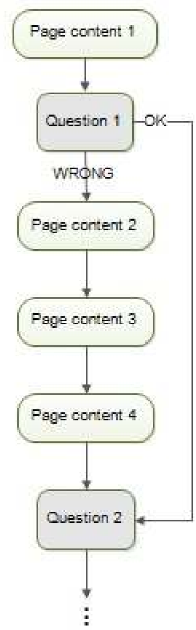

Figure 2- Lesson navigation

In Figure 2, if the student answers correctly the question on page 1, it will lead him to page 2, skipping the entire set of intermediate pages. As the student verifies the knowledge that he has acquired in a particular subject matter, the student then progresses along several pages related to this same subject matter. So it possible to adapt the learning path of each student to the level of knowledge that he demonstrates he has already acquired.
The level of complexity that a lesson can reach is unlimited and the professor can create lessons with a much more complex navigation, as is illustrated in Figure 3.

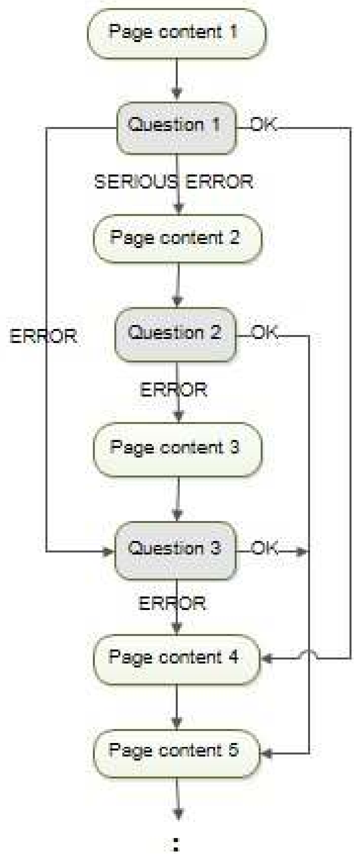

Figure 3 - Lessons with complex navigation

In a much more complex scenario, each answer option to a question may guide the lesson to a completely different page sequence. These possibilities offer several pedagogical advantages, as the content fits entirely to the students' knowledge and performance.
Another capability of the content page is to use it as a menu, so that the student can choose, according to his needs, different sets of lesson pages, as shown in Figure 4 below.

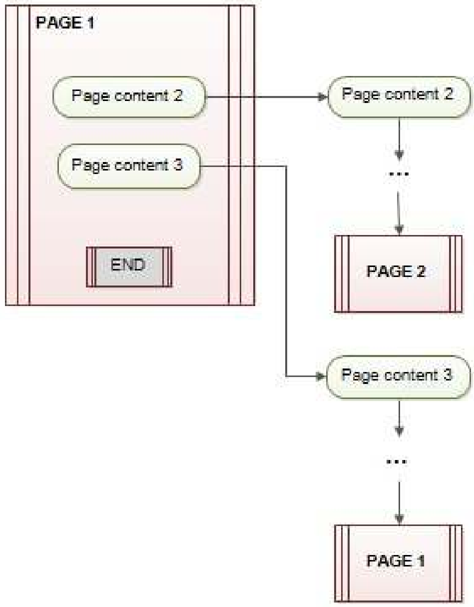

Figure 4- Content page used as a menu

Page 1 is lesson's first page and it presents three options. The first two allow movement to different sets of pages, whereas the third option ends the lesson. Note that the last page of each set of pages, is set to go to page 1, to allow the student to return to the initial choice where he can then make new choices.

One of the purposes of this kind of structure is to allow the student to choose the order that he wants to consult the pages and present him with alternative ways of learning so that he can choose the one that best fits his way of learning. It also makes the student more responsible for his learning process.

Feedback from students that use Lesson on e-MAIO

The lessons on platform e-MAIO were used by Calculus I students from the courses of Electromechanical Engineering during the first semester of 2013-14 as a complement to regular classes. Students used them optionally, most of the times more than 2 to 3 times a week, which shows the interest and the motivation for this kind of activity.

For the Lesson operation and pedagogical organization (Table 1) students evaluated all the items very positively. It was adopted to a Likert scale, where each item was evaluated with a degree of concordance between 1 to 5, as follows: 1-Strongly Agree, 2 - Agree, 3 - Neither agree nor Disagree, 4 - Disagree 5 - Strongly Disagree.

Table 1: Evaluation of the operation and organization of pedagogical activity Lesson

Question	Average	Standard Deviation
The activities are relevant to learning	1.73	0.452
The texts available in lesson are useful in clarifying the content of the discipline	2.08	0.628
The solved / proposed exercises in lesson are useful in the consolidation of subject contents learning	1.62	0.496
The proposed tests are useful for the self-evaluation of the acquired knowledge of the subject contents	1.85	0.543

It is clear that the students valued this activity. As for the lesson benefits on e-MAIO, it is possible to see that students had a very positive appreciation to the questions (Table 2).

Table 2: Evaluation of the benefits of existing lesson on e-MAIO

Question	Average	Standard Deviation
e-MAIO lessons are clear and require little effort to deal with its structure	2.23	0.863
e-MAIO lessons allow student to use it anywhere	1.73	0.452
e-MAIO lessons allow the student to use it at any time	1.65	0.485

The main advantages of the existing Lessons on e-MAIO, from the students' point of view are: the fact that they are always available (anytime and anywhere); being an incentive to study and solve exercises; being an excellent addition to taught classes.

Future Work

For future work, it is necessary to analyse the results of the surveys during the school year with a more representative sample of students to better identify the advantages and disadvantages and try to better adapt this moodle tool to the needs and expectations of the students, and so contribute to the increasing improvement of the teaching/learning process. It would also be desirable to apply this tool to other disciplines beyond Calculus I.

References

Arman, A.M., El-Arif T., El-Gazzar, A. (2008). A suggested E-Learning model based on Moodle-LMS for implementing a course in Biomedical Engineering. Asian Journal of Information Technology, 7(10), pp. 442-448.

Caridade, C.M.R, Faulhaber, M.C. (2012a). TIC como suporte ao Ensino e Aprendizagem da Matemática no ensino Superior. ProfMat2012. Coimbra-Portugal.

Caridade, C.M.R, Faulhaber, M.C, Rosa, P.M., Silva, P.M., Baeta, N.S. (2012b). Teaching Calculus using e-Learning in a Moodle Platform. TicEduca2012. Lisboa-Portugal.

Caridade, C.M.R., Faulhaber, M.C., Rosa, P.M. (2012c). Teaching & learning in Calculus: is e-learning really useful? ETEN2012, Coimbra, Portugal.

Caridade, C.M.R. and Faulhaber, M.C. (2013) B-learning no ensino da matemática para engenheiros, Challenges 2013, pp.855-872.

Coan, L.G.W; Viseu F. (2011). A utilização do Moodle pelo professor de matemática de cursos do PROEJA do IF-SC. Revist Eletrónica de Investigação y Docencia (REID), número monográfico, pp.135-151.

Cury, H.N. (2000). Estilos de aprendizagem de alunos de Engenharia. XXVIII Congresso Brasileiro de Ensino de Engenharia. Ouro preto.

Fujimoto, M., Watt, S.M. (2010). An Interface for Math e-Learning on Pen-Based Mobile Devices. Proceedings of the Workshop on Mathematical User-Interfaces.

Madeira, R.N., Silva, B., Palma, J. (2012). Helping Math Learning. Proceedings of the 2012 IEEE Global Engineering Education Conference, pp.1-7.

Marçal, J.M. (2009). Blended learning em Portugal: Situação actual e tendências futuras. Lisboa:ISCT. Tese de Mestrado.

Penny, K.I. (2011). Factors that influence student e-Learning participation in a UK higher education institution. Interdisciplinary Journal of E-learning and Learning Objects. 7.

Reali, G.A., Santana, A.M., Lopes, D., Oliveira, J.B., Denadai, P.E., Maximiano, R.C.P. (2012). Ferramentas mediadoras no ensino da matemática: Moodle e geogebra a favor do aprendizado de saberes matemáticos. Atas de la Conferencia Latinoamericana de Geogebra, Uruguay, pp. 245-251.

Rodrigues, P.A., Brandão, L.O., Brandão, A.A.F (2010). Interactive Assignment: a Moodle component to enrich the learning process. Frontiers in Education Conference (FIE), pp.T4F-1-T4F-6.

Vilela,A., Moodle2 para professores, ED-ROM, 2012.

Wagner, N., Hassanein K., Head M. (2008). Who is responsible for E-learning success in higher education? A Stakeholders' Analysis. Educational Technology & Society, 11(3), pp. 26-36.

Woodill, G. (2004). Where is the learning in e-learning? Operitel Corporation. http://www.operitel.com/lib/pdf/wp_elearning_analysis.pdf

Zhang, D., Zhou, L., Briggs, R.O. (2006). Instructional video in e-learning: Assessing the impact of interactive video on learning effectiveness. Information & Management, 43, pp. 15-27.

Easing the transition to higher education for adult learners in an Access to Engineering course

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Keywords: Mathematics, transition to higher level, learning support, e-learning,

Abstract

It has been identified [1] that proficiency and achievement (or the lack thereof) in mathematics is a strong predictor of academic success with respect to the first year of third level, and more generally the undergraduate experience. There is evidence to suggest that students who progress fastest in their studies have typically completed first year mathematics courses according to the recommended schedule, while students who face problems studying mathematics more often progress slowly with their studies in general (Pajarre, Lukkari, Lahtinen, 2010).

With this in issue mind, during the summer of 2013 the College of Science, the Adult Education Centre (AEC), the School of Mathematical Sciences and the School of Electrical, Electronic and Communications Engineering (all at UCD) came together in a collaborative process to redesign both The Access to Science and Access to Engineering programmes (a pre-entry foundation year programme designed to give students the pre-requisites to study an honours degree in Science or Engineering at UCD). The outcome of these discussions were three-fold in that

(1) All incoming Access students would study *both* the modules offered in mathematics: whereas in the past only those wishing to study engineering would do the second, more involved maths module.
(2) It was also agreed that more time be given over to the instruction of mathematics modules at the expense of other modules including chemistry and IT skills.
(3) The maths curriculum would be broadened to include topics in basic Probability and Statistics.

While many European universities have adopted similar strategies to address the competency issue in mathematics, a widening of the curriculum is not par for the course. It was also stressed that there should be no reduction in syllabus content, that no topic should be diluted and that more attention be given over to the use of real world applications as opposed to quasi-engineering examples deployed in the past.

To support these developments, we at the UCD Maths Support Centre (MSC) proposed to hold a pre-entry workshop for all students (approx. 50) who wished to be considered for the in-coming 2013/14 Access class. During the last week in August these prospective students were offered nine hours of mathematical literacy support in the MSC. These prospective students were then asked to take a one-hour diagnostic test directly after their interviews with the AEC. We felt that this process made it clear to students from the outset that a good grounding in some basic mathematics was essential if they were to enter an Access to Science and Engineering Programme.

As part of the suite of academic supports offered to the students who gained a place on the course (n=28) was the 40-day use of an online adaptive learning tool [2] developed here in Dublin. After the 40 days the students were then asked to take the same diagnostic test that they took at the pre-interview stage. The students' performance on these pre and post tests as well as their qualitative feedback on the online support tool will be discussed here.

Introduction

The transition from secondary to tertiary level education is a well studied issue with many working groups now dedicated to the first year experience at third level in both the US and here in Europe, (The National Resource Center for the First Year Experience and Students in Transition (http://www.sc.edu/fye/) and The European First Year Experience Network (http://www.efye.eu). The transition from working (or unemployed) life to third level for the adult learner is a less studied phenomenon and undoubtedly a more challenging transition. Couple this with the predominant challenge that the discipline of mathematics poses to many non-traditional learners and you have a melting pot of issues confronting the third level educator. A few ideas on how to confront these issues including maximising lecture time, efficient use of technology both inside and outside the class room and further support structures to ease the transition to higher education for adult learners are discussed in this contribution.

Background

For the past 12 years the Access to Science and Engineering programmes at UCD have been run in essentially the same format. From approximately a pool of 60 prospective candidates seeking a return to education around 30 are accepted on to the course.
The criteria for acceptance include, no formal 3rd level education (exceptions for interrupted learning – life changing circumstance etc), some knowledge of the course, self-motivation and a progression plan following the courses' completion, interest in reading popular science books, English competence etc**.** Evidence of learning in a formal setting in the previous 3-5 years is a strong indicator of success on the programme. Prospective students are asked to submit a writing piece of a maximum of 1500 words based on a Science lecture they attend in UCD. Students are interviewed two weeks later, whereby the essay and acceptance criteria are discussed.
The candidates are then asked to take a maths diagnostic test following the interview, which assesses knowledge of the basics of arithmetic, algebra and statistics. Acceptance on to the course is based on the student displaying evidence of how they meet the above criteria and while the maths score is not a potential disqualifier the student will be flagged if their score was very poor.
After gaining entry on to the programme students study modules in Biology, Chemistry, Study Skills, Digital Literacy and Mathematics with both the mathematics modules being mandatory. In total students will receive 72 hours of mathematics tuition.

Students who achieve an average of 60% or more across all modules are guaranteed entry to programmes of Science, Agriculture or Engineering, but in addition Engineering students are required to score a minimum of 70% in each of the mathematics modules.

Online Academic Supports

The students are all set up with *Blackboard* accounts which host their lecture notes for chemistry, study skills and maths. In addition to this years students were invited to attend an orientation workshop hosted by the UCD Maths Support Centre on the usage of the *RealizeIT* system whereby students were given a URL and login and stepped through a demonstration on how to use the e-learning platform. One of the principle rationale behind this teaching intervention was to reduce the teacher input time for the course in terms of remedial support and as such, this was the first and only face-to-face instructor/learner session on the e-learning support component of the course.

As the system is quite straightforward to navigate even for the novice computer user the students used this time to test themselves using the *Determine Knowledge* facility which asks the student a minimum number of questions based on how they rated their own knowledge of that topic in advance. All students were asked to start with the *indices* node and this then prompted a discussion of the topic and some instructor intervention time.

Pre and post-test comparisons

While all 28 students attended the orientation session on the e-learning platform only 7 students engaged with the system on a consistent and substantial basis. Consistent here meaning that the student used the tool at least every other day, and substantial meaning for at least 35 minutes at a time. It should be stressed that the Access course is a very demanding part-time programme of study and that the e-learning facility was in no way mandatory or pushed by the tutors, lecturers or administrators involved.

As the e-learning facility was intended to get the students up to scratch with the basics of mathematics in as fast a time as possible only 4 nodes of the system were used, namely:

1. Indices
2. Linear Equations
3. Functions
4. Statistical Models

The following table details the scores achieved by the 7 consistent users of the online support for the pre and post tests as well as their overall score for both maths modules.

Student	Pre-Test	Post-Test	Final Score
1	8	**62**	61
2	40	**67**	56
3	45	**93**	60
4	50	**85**	70
5	65	**80**	87
6	70	**88**	51
7	90	**100**	94

Table: Pre/Post-Test Results

The class average final mark was 66% and the average of the 7 who used the system was 69%. The following questions were asked and discussed at a semi-structured focus group facilitated by a member of staff not known to the students or directly involved with the running of the Access programme or its learning elements:

1. How often and for how long did you use the system?

2. Did you find it useful? If so, how was it useful? If not why not?

3. Did you find the system easy to use/navigate? Explain

4. Do you think it improved your knowledge/confidence of the basics of mathematics? Explain

5. Do you think you would prefer to have this resource alongside other maths related subjects?

Upon analysis of the free response section of the focus group interviews 2 themes emerged, namely (1) Usefulness and (2) Improved knowledge/confidence in math basics

(1) Usefulness

"I did find it useful as I found that the explanation and the practice questions made the subject stick much better in my mind. I think I'm a visual learner and its beneficial to have a system that allows you to take part in the demonstration to really break down the subject material to a level where I can understand it. "

"It was great for the algebra and all the Cartesian plane stuff, pretty much

*everything we did with ******. I found it very easy to follow and the explanations of how to do things were great"*

"I found the system really useful when starting a new topic in maths or when revising a topic to highlight and fill in the gaps. I didn't find it as much use when I had a specific problem or when I was trying to get more information on a specific section of a topic. This type of information could be in there but I had a few problems finding it."

"It broke topics into easily managed chunks and you could only progress after reaching required level in basic topics. I liked that it gave you a colour coded chart of how well you had performed on each topic."

"*I found the math software extremely helpful, my math skills were very poor and I hadn't done anything math related since my GCSE's in 1996 so I had forgotten pretty much everything except how to add! "*

(2) Confidence/Knowledge in math basics

"Yes, it greatly improved my ability and confidence in quite a short space of time, my husband has always made fun of my maths abilities and now I can do things he can't! I definitely put it down to this programme. I don't think I could have achieved such a good mark in my maths assignments without it. To have it at my fingertips was great. You can only do the practice sheets so many times before you remember the answers but for the most part the programme had a large catalogue of questions."

"Yes, definitely, when I used it for topics I knew I was generally weak at, I noticed that after a while, I was able to answer the questions faster and more easily."

"Yes, I do think it helped me to better understand some of the material as mentioned earlier, I found the demonstration coupled with the practice questions half way through helped me to understand the method of working out a problem, and if I got stuck I could return to the places I was getting stuck"

"Yes it increased my knowledge and understanding of indices and helped me to get my head around the basics."

"Yes I do believe it improved my knowledge and confidence. When having an issue you could always use this resource and while sometimes it didn't leave you with a full understanding it certainly helped and improved my knowledge."

E-learning and b-learning

The RealizeIT system enabled a more flexible learning approach. Comments pertaining to this point are that the students enjoyed the 24/7 access of the platform as well as the 'at home' style of learning. While there was only one session face-to–face with a tutor the online element gave the students a sense of self-directed learning with one student commenting “I used the system to pre-empt any issues I might have with the next part of the course so I would try to skip ahead therefore getting more from actual class time”

Findings and Discussion

While we make no claim that the observations arising from this pilot are significant we do now have the confidence to run amore substantial pilot with a larger group of adult students (both control and experimental) making the transition from working life to third level, including distance learners.

Based on 10 years of teaching on the access course I can report that the initiatives employed by the MSC for the academic year 2013/14 did result in an effective utilisation of support staff time, effective use of new technology, and an effective identification of weak students.
A different pedagogical approach to teaching (Flipping the classroom; the students diagnose their problems ahead of time using the tool and then meet with the lecturer for one hour in the MSC every week to work in a small group on their reported problems) is certainly suggested by this trial.

Conclusions for Math Education

Obviously one has to be very careful when evaluating a teaching innovation and also drawing conclusions concerning such an intervention based on such a small sample size. As stated in the introduction the experiment conducted here was to see if there was a reduction in teacher time in dealing with the very basics of mathematics thus allowing more time to support student learning in class and also to gauge the adult student experience with an online adaptive learning tool. The results of the investigation suggest an emphasis on embedding the e-learning into the module would prove beneficial.

References

[1] America’s Promise Alliance. 10 Indicators of academic achievement and Youth success July 2011. Beth Gifford, Kelly Evans, Lisa Berlin, and Yu Bai.
Center for Child and Family Policy at Duke University

[2] RealizeIt, CCKF, Tallaght Dublin 24, Ireland.

[3] Mustoe L., and Lawson D., (eds.) (2002). Mathematics for the European engineer. A curriculum for the twenty-first century. SEFI Mathematics Working Group.

[4] Pohjolainen, S. & Silius, K. (2011). Teaching and learning mathematics at the university level - with and without technology. In H. Silfverberg & J. Joutsenlahti (Eds.) Integrating Research into Mathematics and Science Education in the 2010's. Annual symposium of the Finnish Mathematics and Science Education Research Association. University of Tampere Press, pp. 25–45.

HOW WELL PREPARED MATHEMATİCALLY ARE OUR ENGİNEERİNG STUDENTS WHO TRANSFER FROM AN ORDİNARY DEGREE İNTO AN HONOURS DEGREE

Keywords: Engineering Mathematics, Honours degree
Subject: Technology Education
Abstract:

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Students who have received a C3 (55%) or higher in Higher level mathematics in the Irish Leaving Certificate (the terminal secondary examination in Ireland) may enter directly onto a 4-year Honours degree in engineering. Students who have not achieved this level of mathematics have the option of entering onto a 3-year Ordinary degree(Level 7). Upon completion of this students may progress to the third year of the Honours degree. Relatively little work has been done on the transition(articulation) from an Ordinary degree to an Honours degree and in particular the mathematical preparedness of these students. In the third and fourth year of many Honours engineering courses within the DIT it is not unusual to have 30-50% of the students coming from an Ordinary degree background. The majority of these students come from within the DIT while others transfer in from other Institutes of Technology in Ireland. Previous work has shown that students from an Ordinary degree background are more than twice as likely to fail mathematics in their third year of the Honours degree when compared with students who have proceeded directly through an Honours degree programme. In this study we analyse students' performance across all subjects and examine if there is a relationship between mathematical performance in the final year of the Ordinary degree and overall performance across all subjects in the third and fourth year of the Honours degree. In addition, a similar comparison is made with these students mathematics grade on entry to first year and whether this is a determining factor in their success in the in the Ordinary degree and their ability to transfer to the Honours degree.

Introduction

There are two distinct routes to an Honours degree (Level 8) in engineering in the Dublin Institute of Technology (DIT). Students with a C3 (55%) or higher in Higher level mathematics in the Irish Leaving Certificate (the terminal secondary examination in Ireland) may enter directly onto a 4-year Honours degree. Students who have not achieved this level of mathematics but have a pass in ordinary level mathematics may enter onto a 3-year Ordinary degree(Level 7). Students who successfully complete this award may apply to progress to the third year of the Honours degree. Up until relatively recently an upper merit (60%) was the minimum required to make this transition. In recent years this requirement has been relaxed with many students with lower marks being offered the possibility of transition upon successful completion of an interview.

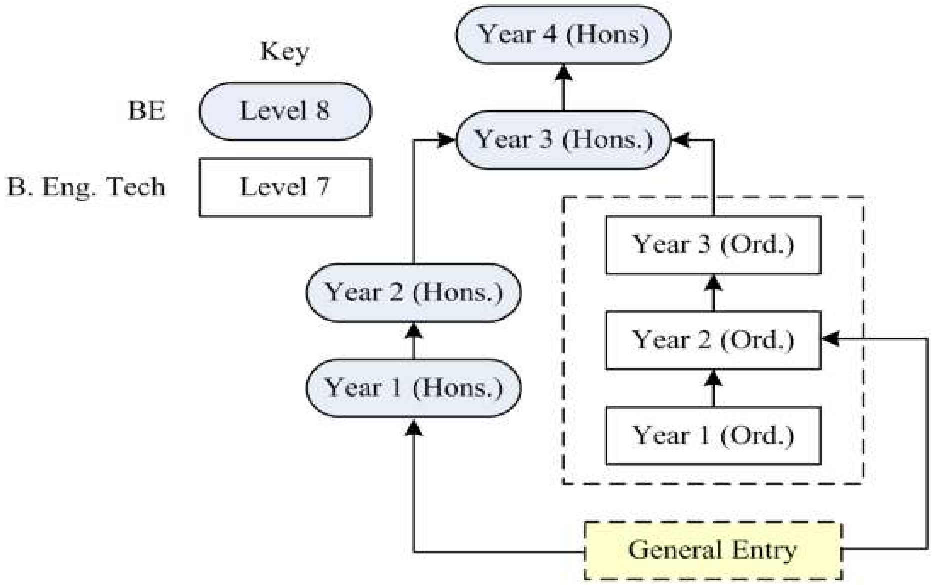

Figure 1: Schematic of the alternative routes to an Honours degree in Engineering in Ireland

Previous work has shown that students from an Ordinary degree background are more than twice as likely to fail mathematics in their third year of the Honours degree when compared with students who have proceeded directly through an Honours degree programme(Carr 2013). In this study we examine the performance of the group of students from the Ordinary degree in mechanical engineering who entered the third year of the honours programme in 2007 and 2008 and who subsequently graduated in 2009 and 2010 respectively.

Results

2009 and 2010	Direct Entry to level 8	Entry via Level 7 course
N	85	33
Average mark (Standard deviation)	53.4(18.8)	62.1(8.1)
Number with grade of more than 60%	37/85	27/33
Graduated on time(Complete pass)	62/88	32/33

Table 1:Comparative performance of students who transfer onto an honours degree programme and those who enter directly from secondary school

In table1 above we show a combined analysis for the combined mechanical engineering classes of 2009 and 2010. There were a total of 85 students who graduated who came from an Honours degree background i.e they had entered the course directly from secondary school. In contrast 33 students graduated who had entered the Honours degree programme after having completed the 3 year Ordinary degree. The average mark of the direct entry students was 53.4 % with a standard deviation of 18.8. In contrast the students who had entered via the ordinary degree had an average of 62.1% with a standard deviation of 8.1%. A two sample t-test was applied to this data and the average mark of the Ordinary degree students was found to be significantly different with p=0.000.

In addition we measured the proportion of students who achieved a 2.1 degree or higher. Of the direct entry students 37/84 achieved a 2.1 degree or higher in comparison with the ordinary degree students where 27/33 achieved a 2.1 degree or higher. This difference was found to be significant using two proportion test(p=0.000) and the Fisher exact test(p=0.000).

Of the students who entered from the ordinary degree background 32/33 graduated on time in comparison with 62/88 who had come through the direct entry route. Again this is significantly different using both the two proportion test(p=0.000) and the Fisher exact test(p=0.002)

Maths results

The original motivation for this study was the failure rate in the 3rd year Honours mathematics module. We now show the performance of these students in the mathematics module.

Results for the 2009 and 2010 graduating class

Correlation Coefficient(R^2)	3rd Level 8 maths R^2 (p value)	4th Level 8 Maths (p value)	4th Level 8 Overall (p value)
3rd year Level 7 Maths	0.139(0.454)	0.533(0.001)	0.57(0.001)

Table 2: Correlation between 3rd level 7 maths grade, 3rd level 8 maths grade, 4th level 8 maths grade and 4th year level 8 overall

What we see here is little or no correlation between the 3rd year level 7 maths grade and the third year level 8 maths grade with a correlation coefficient of R^2= 0.139 and p=0.454. This is rather worrying. But when we look at the relationship between the 3rd year level 7 maths grade and the 4th year level 8 grade we see a strong correlation (R^2=0.57), that is highly significant (p=0.001). We are also seeing a strong relationship between the 3rd year level 7th maths grade and their overall performance in the 4th year (R^2 =0.57, p=0.001).

Maths grade as a predictor of success.

Given the strong correlation we see between the maths grade and the overall grade in fourth year should we use the 3rd year Level 7 maths grade to select students for entry onto the honours programme. In this section we compare whether we should use the overall 3rd Level 7 average grade, 3rd year Level 7 maths grade or the 3rd year Level 7 project grade. We see from table 3 below that the 3rd year level maths grade is as good a predictor of overall success in the honours degree as the 3rd year level 7 overall grade.

Correlation Coefficient(R^2)	3rd Level 7 maths R^2 (p value)	3rd Level 7 Overall (p value)	3rd year Level 7 project (p value)
4th year Level 8 overall	0.57(0.001)	0.585(p=0.000)	0.308(p=0.08)

Table 3: Correlation between overall 4th year performance, 3rd year level 7 maths grade, 3rd year level 7 overall grade and 3rd level 7 project mark

Conclusion

Several researchers in the U.S. have identified a phenomenon known as “transfer shock” (Cejda, 1994; Lanaan, 2001; and Hills, 1965). Through transfer shock, community college students who transition to a university typically experience a drop in grades for the first semester or two immediately after transfer. Grade point averages will typically recover by the time that students graduate and the dip in grades is typically attributed to the effort it takes to transition from one educational setting to another. We seem to be observing a similar phenomenon in the DIT, whilst there is a temporary dip in the performance of transfer students in the first semester these student quickly recover and there is a very strong correlation between their performance in the ordinary degree and their final performance. The American literature recommends that well-defined articulation agreements between the community college and the university as being critical to transfer student success. At DIT, the faculty teaching the ordinary and honours programs are typically in the same department and, in fact, most faculty teach in both programs. Thus, it appears that conditions are ripe at DIT for successful transition of students between the programs.

In addition we have noticed that these transfer students are outperforming their direct entry comparators , both in overall grade and the percentage who complete the course on time. Further work is required in this area and we hope to follow up this work with focus groups of students who have articulated in the past, along with a focus group of staff who have taught these students on both the ordinary and honours programmes.

References

Carr,M. Ni Fhloinn.E, & Bowe,B. (2013) Core skills assessment to improve mathematical competency, , European Journal of Engineering Education, pp1-12

Carr,M , Ni Fhloinn,E., Murphy,E. & Bowe,B. (2013)Addressing continuing mathematical deficiencies with advanced mathematical diagnostic testing.M.Carr, E.Ni Fhloinn, E.Murphy & B. Bowe *Teaching Mathematics Applications 32 (2): 66-75.*

Cejda, B. D. (1994). Reducing transfer shock through faculty collaboration: A case study. *Community College Journal of Research and Practice, 18, 189-199.*

Hills, J.R. (1965). Transfer shock: The academic performance of junior college transfer. *Journal of Experimental Education, 33,* 201-215.

Lanaan, F. S. (2001). Transfer Student Adjustment. *Transfer Students: Trends and Issues. New Directions for Community Colleges, Number 114. The Jossey-Bass Higher and Adult Education Series, pp. 5-13.*

Teaching and Learning with Learning Outcomes for Mathematical Competencies

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Abstract

This paper presents the core experience gained from managing the 3-year ESF project *REFIMAT* at the University in Hradec Králové, Czech Republic, initially at its Faculty of Informatics and Management (FIM UHK). The main goal of the project was to invent and to implement innovative teaching/learning approaches and methods for improving knowledge and skills in mathematics, in 12 subjects with mathematical content - the mathematics base for students of informatics, managerial and economic branches (bachelor and master level studies) at FIM UHK. They cover calculus, linear algebra, discrete mathematics, operations research methods up to statistics and stochastic modeling. The overview of the project solution phase enables us to relate its conclusions with knowledge and recommendations provided in educational theories, and also to compare the experience gained with that of recognized specialists or teams, with specific aspects of the study programmes at FIM UHK.

Introduction: project starting point, learning outcomes as a key concept

The purpose of the project has been stated by the faculty as the real need to improve students' knowledge and skills in subjects with mathematical content. Failure in these subjects is one of the main reasons why students leave their studies (although all reasons are not yet fully documented - "mathematics is not primarily responsible"). The level of success at mathematics is also important, as mathematics is a pre-requisite for subsequent study programmes. Innovative approaches in teaching/learning of mathematics were connected also with those in different programmes, to drive more effective methods. In the last decade most HE institutions have adopted a *Learning Outcomes (LO)* approach; LO in teaching/learning are the key concept in the innovation and design of the project *REFIMAT*. The concept of LO turns the focus of education strategy away from the teacher and onto the learner, and puts the main emphasis of education on its outputs (Kennedy, Hyland and Ryan (2007), Adam (2009), Gehmlich (2010)):

> *Learning Outcomes mean statements of what a learner knows, understands and is able to do on completion of a learning process and are defined in terms of knowledge, skills and competence* (Gosling and Moon (2001)).

Here LO as *knowledge, skills and competences* are (Adam (2009) in his "Prague lecture"):

- *knowledge* is the ability to aquire, to process and to use information, and it is the outcome of the assimilation of information through learning; it consists of facts, principles, theories and practices that are related to a study branch/subject;
- *skill* means the ability to apply the aquired knowledge and perform it, using appropriate tools, in an appropriate way for solving appropriate problems;
- *competence* means the ability to use knowledge and skills; such ability has to be proven.

After adopting LO, it follows that teachers are expected to be able to show how:

- the educational outcomes for a programme and LO for a module are being achieved;

• the assessment methods are appropriate to test the achievement of the intended LO;
• the criteria used to judge achievement are aligned to the intended LO.

Crucially the project team agreed: *LO enable a shift from the surface approach to learning to the deep approach,* that is, from
• *surface level learning*: fact memorisation to
• *deep level learning*: understanding underpinning theory and concepts

(Marton and Säljö (1984)). Studies more recently have shown that deeper approaches to learning are related to higher quality LO (Ramsden (1992), Prosser and Millar (1989), Trigwell, Prosser and Waterhouse (1999)).

In maths (Kahn and Kyle (2008), Krantz (1999)), start from *learning objectives* first: *Having taken this subject, students should have:*
• *knowledge of the subject matter, terminology, techniques and conventions covered in the subject, as well as understanding of the underlying principles,*
• *the ability to solve problems involving understanding of the concepts.*

There is a common agreement on the learning objectives (SEFI (2013), Curricula):
1. thinking mathematically,
2. reasoning mathematically,
3. posing and solving mathematical problems,
4. modelling mathematically,
5. representing mathematical entities,
6. handling mathematical symbols and formalism,
7. communicating in, with, and about mathematics,
8. making use of aids and tools.

Hence, LO in mathematics, independently on a programme/subject/module level, mean: *upon completion of the programme/subject/module students should be able:*

KNOWLEDGE *(what does it mean)*	to acquire knowledge on mathematical concepts, on their substance, structure, properties and importance; to gain knowledge on methods applicable in the corresponding branch, and on specific theories
SKILLS *(how does it work)*	to demonstrate an appropriate level of problem-solving skills using analytical reasoning; to manipulate with concepts/objects in an effective and consistent way, applying corresponding formal mathematical procedures (rules, methods, operations), including possible technological or software means – to demonstrate proficiency with various technological tools; the ability to justify the procedure due to its mathematical substance, and the ability to recognize correct/incorrect steps; to recognize the range of applicability of the procedure(s) used
COMPETENCE *(the purpose and how to apply it for the purpose)*	to express a simple problem in the form of a mathematical model, using mathematical symbols for concepts, objects, relations and operations – to provide a transition from concrete to abstract thinking; the ability to decide the use of appropriate mathematical tools for solving the problem and apply them; using logical argumentation, the ability to interpret its solution, or provide reasons for the non-existence of a solution; the ability to formulate statements in the formal (mathematical) language

In mathematical subjects at FIM UHK, the work begins with defined learning objectives and then with LO identified in subjects; these are then analyzed in detail for each module to be implemented into teaching/learning and assessments. This concerned also study supports (printed books, electronic supports – "e-courses" in LMS system BlackboardLearn 9.1 used at FIM UHK; e-course means a complete learning/communication unit related to any subject).

The implementation has been substantially supported by knowledge and activities following from the Bologna process in EHEA, while a very large experience base (see references) enabled the work. Educational theories (especially Biggs and Tang (2011), also Ramsden (1992)), and activities of HEIS in education of engineers in mathematics provided important and very comprehensive project guidance (e.g. UK: UCE Birmingham, Open University; also those in Australia, several European consortia, MWG SEFI, EQANIE), Association for Computing Machinery (ACM). Two Czech all-nation projects QRAM and "QUALITY" have to be quoted: QRAM (oriented towards building up the national qualification framework of tertiary education) recommended the use of LO, and one of the key activities of the project "QUALITY" considers use of LO to be a pre-condition for all quality assurance of HEI. The implementation of LO has been the official and required output of the project *REFIMAT.*
Some of our project experience on implementing LO (with more than 5000 students involved) is listed:

- LO in module/subject/programme should be written by its own teacher(s), based on syllabi.
- Teaching/learning proceeds within a strong, explicit, relevant correspondence with LO.
- The learner's role is to facilitate ways in which to acquire LO (a student: "…give me time …").
- Knowledge, skills, competences, and their goals must be explicitly presented in teaching by the learner *himself/herself* (on a principle "explain – show – use").
- The teacher *identifies with learning methods and accepts them* - preconditions of success.
- In study supports: it is necessary to declare which knowledge, skills, competences are actually developing; thus, it is necessary to provide materials at different levels to work with, containing also showcases/showsteps/instructions leading to knowledge or skills.
- Formative and summative tests must be prepared in a structured form and they must contain a subject/programme "overview" at a level in question; aim: to prove that LO are achieved.
- The learner is focused on conditions for his/her success in learning; there are possibilities to improve failures (based on study regulations, and also the ethics code).

Immediate success on introducing LO cannot be presumed; as a rule, modified variants will follow. The adjustment of the whole institution environment to this teaching/learning style is required (compare also with *ECTS Study Guide*), with

- study supports (classical or e-supports) available and accessible, fully covering syllabi;
- effective IS, study administration, IT equipment for teaching and self-study; "smart" exams system, transparent and functional;
- mutual communication and social contacts available, official study guidance (a kind of a body consisting of tutors or "e-tutors"); ethical principles known and respected.

Assessment; assessment in mathematics

Surveys documented the importance of the key step in involving LO: the implementation of LO into assessment. Well managed assessment should guide and encourage the learner's approach to learning in general; the formative, ungraded assessment serves as a support and provides a feedback on learning progress; then the summative, graded assessment documents his/her final result. The assessment approach, its methods and tight alignment with education and learning are steps of crucial importance; all components – curriculum and intended LO, the teaching methods used, the resources to support learning, and the assessment tasks and criteria for evaluating learning – have to be aligned to each other and facilitate the achievement of the intended LO. Thus, the next step in the project problem was: how to organize assessments forcing *a shift from the surface approach tolearning to the deep one?* Approaches are known and well described (web page of Macquarie University, Sydney (2013)). In our project, the principles in James, McInnis and Devlin (2002) were important:

- Variety in types of assessment allows a range of different LO to be assessed. It also keeps students interested.
- Students need to understand clearly what is expected of them in assessed tasks.
- Criteria for assessment should be detailed, transparent and justifiable.
- Group assessment needs to be carefully planned and structured.
- Systematic analysis of students' performance on assessed tasks can help to identify areas of the curriculum which need improvement.

It was necessary to change the assessment in its content, methods and even verbal form, applying a special wording in communication; thus, the constructive alignment means:

- the learner constructs his/her own learning system through relevant learning activities (in a sense of student centred learning); and the teacher's job is to create a learning environment that supports the learning activities appropriate to achieving the desired LO;
- formative assessment is included in learning *as an obvious, standard part of the learning, enabling stress-free feedback* (preparing the way to summative assessment);
- to relate assessments to LO: formally, verbally, in a tight relation with credits or grades.

There is also an extra added value of assessment (James, McInnis and Devlin (2002)): this is often a final consideration in the curriculum planning. In contrast, students often work 'backwards' through the curriculum, focusing first and foremost on how they will be assessed and what they will be required to demonstrate. The concept of *the aligned curriculum* is required (Biggs and Tang (2011)): an effective assessment task is one which assesses students' attainment of LO, and these are represented in the form of tasks, together with the definition of performance levels (in the project, multiple choice quizzes, based on reasoning but not guessing, not omitted in the formative assessments, see, e.g. Macquarie University (2013)). Due to concepts in the project *REFIMAT,* assessments in subjects with mathematical content were formulated mostly in written form (redefined in alignment with LO), followed by oral form (assessing the depth of understanding and knowledge checking); PC are allowed; formulations requiring argumentation and a time schedule did not allow guessing of solutions only. There was a need to change formal wording as well (it turned out that some partial themes have been omitted in tests). Final assessments are based on a cumulative principle, middle-semester tests counted. The whole operational procedure has to be assessed; for feed-back reasons, we provide them as a "step-by-step analysis".

Project surveys and questionnaires

Several *REFIMAT* project surveys were provided: on motivation and on approaches of learners to the study and to LO, on preferred learning styles, taking into account also the change in life approach of generations (Generation X, Y, Z etc.), on student's prerequisites to studies. In the first semester study, surveys documented

- a lack of basic skills in mathematics for HE studies; its perception is often as a vague feeling only,
- that learners are confused: even with the belief of their inner motivation, they are not able to achieve improvement in the study.

In the final project phase, we tried to measure the effect of the innovations; a questionnaire consisting of 22 questions was submitted to students of 7 subjects, 273 respondents, only 147 responses of total 6006 rated. Likert's 5-degree scale was used; we present here some reactions only, made after a reduced analysis (first two levels of agreement taken):

- *Assessment fully proved the level of LO achievement, followed by credits award:* mean 2,307, agreement nearly 65%.

- *My ability to understand a scientific lecture/text with mathematical content increased:* mean 2,247, agreement more than 60%.
- *My knowledge of principles, concepts, methods increased:* mean 2,247, agreement nearly 75%.
- *My practical and computational skills of how to solve a problem have increased:* mean 2,289, agreement nearly 70%.
- *My ability to communicate how to solve mathematical problems increased:* mean 2,523 agreement 56%.
- *I have been led to organize my worktime and its effective use:* mean 2,625, agreement more than 51%.
- *My competences as to how to organize work/the study increased:* mean 2,667, agreement nearly 50% (however, the response *I strongly agree* only 10%).
- *I got a deeper insight into roles and methods of mathematics:* mean 2,603, agreement more than 50% (however, one third of responses were *neither agree nor disagree*).
- *I gained a positive attitude to the study of mathematics*: mean 3,165, agreement nearly 33%. However, *disagreement or strong disagreement* were expressed in 40% of responses, neutral responses at the rather high level of 28%.

One remark: this innovation has been applied to learners in their first year of study; so later on, after wider experience of LO, it may appear "success not yet, but possibly only postponed".

Conclusions

In spite of the lack of initial knowledge and skills in mathematics, surveys showed that our freshmen are motivated and prepared – though the failure percentage is (and remains) high. Let us provide the main project conclusions; several (but not all!) go beyond mathematics education:

- do not lose learners: identify learners' level of pre-knowledge, pre-skills as soon as possible;
- inform them on the level identified; they only *feel* the lack, but a large proportion are unable to identify the level precisely, and unable to find and to accept the proper steps to solution;
- the lack of learning skills and elementary computation routine implies that it is necessary *to learn how to learn mathematics;*
- as teachers, include LO into teaching, and *demonstrate such inclusion*; disperse them in teaching applying the scheme knowledge – skills – competences, and use appropriate terms and language;
- the teacher's effect becomes a stronger one where he/she is able to make an emotional experience out of the recognition, finding or discovery – "show discoveries, and force them";
- include a great number of tools, based on diverse study approaches, for formative self-assessment, *as a standard part of the learning; provide evaluation of it*; the lack of these, or not providing its well-timed results, lead to a decline in the interest to learn;
- relate assessments to LO: formally, verbally, close to credits/grades; return and improve;
- check LO even in the follow-up subjects (reduce non-functional concepts, skills which are not useful);
- return several times to LO, use the cumulative principle of mathematics to reinforce them, do not rely on "immediate understanding and acceptance";
- reduce the volume if this is necessary for the lower study workload, but repeat concepts of knowledge and ways to gain skills – accept LO and identify your own learning style;
- long-term (magister) studies enable the better structuring of the study workload.

Surveys made by the project team confirmed that improving skills in mathematics is the task of the whole institution: mathematicians themselves do not manage the job alone. The cosre of the task is based on the approach to the work: creating a complete, favourable learning environment is a significant priority. The approach of the whole society to education requires changes.

References

Adam, S. (2009) "LEARNING OUTCOMES, CREDITS (ECTS), QUALITY ASSURANCE AND QUALIFICATIONS FRAMEWORKS". Accessed via http://bologna.msmt.cz/images/docs/konference/23_6_2009/K090623Adam.pdf (29 April 2014)

Biggs, J. (1993) From theory to practice: a cognitive system approach. *Higher Education Research & Development*, 12(1), pp. 73-85.

Biggs, J. (2003) *Teaching for quality learning at university: What the student does* (2nd ed.). Buckingham: Society for Research into Higher Education and Open University Press

Biggs, J., Tang, C. (2011) *Teaching for Quality Learning at University.* Open University Press, McGraw Hill Education.

Fry, H., Ketteridge, S. and Marshall, S. (eds.) (2008) *A Handbook for Teaching and Learning in Higher Education: Enhancing Academic Practice.* Routledge, New York.

Gehmlich, V. (2010) "Plenary Session III: Exploring Learning Outcomes, 5th European Quality Assurance Forum Lyon Nov 18 – 20 2010". Accessed via http://www.eua.be/Libraries/EQAF_2010/Plenary_III_Volker_Gehmlich.sflb.ashx (29 April 2014)

Gibbs, G. (1992) *Improving the Quality of Student Learning.* Plymouth UK: Technical and Educational Services Ltd.

Gosling, D. and Moon, J. (2001) *How to Use Learning Outcomes and Assessment Criteria.* London: SEEC.

James R., McInnis, C., Devlin, M. (2002) "Assessing Learning in Australian Universities". Accessed via http://www.cshe.unimelb.edu.au/assessinglearning (29 April 2014)

Kahn, P. and Kyle, J. (eds.) (2008) *Effective learning and teaching in mathematics and its applications.* Taylor & Francis, London and New York, pp 92 – 105.

Kennedy, D., Hyland, Á., Ryan, N. (2007) *Writing and Using Learning Outcomes: A practical guide.* Cork: Quality Promotion Unit, UCC.

Krantz, S. (1999) *How to Teach Mathematics.* AMS, Providence, Rhode Island.

Macquarie University, Sydney, Australia (2013) "Evaluation and TEDS". Accessed via http://staff.mq.edu.au/teaching/evaluation/resources_evaluation/developing_unit/assess_achievement/, http://staff.mq.edu.au/teaching/curriculum_development/assessment/toolkit/ (29 April 2014)

Marton, F. and Säljö, R. (1984) "Approaches to learning". In Ference Marton, Dai Hounsell and Noel Entwistle (eds.*) The Experience of Learning: Implications for teaching and studying in higher education*. 3rd (Internet) edition. Edinburgh: University of Edinburgh, Centre for Teaching, Learning and Assessment, pp 39-58.

Prosser, M. and Millar, R. (1989) The "How" and "What" of learning physics. *European Journal of Psychology of Education,* Volume 4, Issue 4, pp. 513-528.

Ramsden, P. (1992) *Learning to Teach in Higher Education.* Routledge, New York.

SEFI (2013) " Framework for Mathematics Curricula in Engineering Education. " Accessed via http://www.sefi.be/wp-content/uploads/Competency%20based%20curriculum%20incl%20ads.pdf (29 April 2014)

Trigwell, K., Prosser, M. and Waterhouse, F. (1999) *Relations between Teachers' Approaches to Teaching and Students' Approaches to Learning*. Higher Education, Springer, Kluwer Academic Publishers, Vol. 37, No. 1.

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“A minimum requirement catalogue for beginners - Using technology for supporting and testing students”

Many beginners in math, natural and engineering sciences as well as in economic sciences have big problems starting their studies because of their lacking mathematical competencies. We all know this. Neither ignoring and waiting for better students helps, nor one-week preparatory crash courses. So, a lot of different programs are under way to mitigate the situation. Since we all have the same problems there is a high potential for concentration, standardisation and most important for cooperation. In Baden-Württemberg colleagues from schools and universities developed a catalogue which defines the minimal standard of mathematical skills needed to begin the above mentioned study programmes. The TU9 - the German association of the 9 most important technical universities - decided to support a project to build up an online-eLearning platform on the base of this common standard including a self-diagnostic of the students. Meanwhile, a lot of other universities and universities of applied science share this project which will be released this year. We report about the project, the problems and the perspectives.