Teaching inspires research and vice versa
case study on matrix/vector methods for 3D kinematics

T De Laet[1]
Postdoctoral researcher
Department of Mechanical Engineering, KU Leuven
Leuven, Belgium
E-mail: tinne.delaet@kuleuven.be

J De Schutter
Professor
Department of Mechanical Engineering, KU Leuven
Leuven, Belgium
E-mail: joris.deschutter@kuleuven.be

Conference Key Areas: Engineering education research, Integration of research in engineering education, Novel education tools for engineering programs

Keywords: Mechanics, conceptual learning, semantics, 3D motion

INTRODUCTION

Teaching matrix/vector methods for 3D kinematics and statics to undergraduates is challenging. While the geometric concepts of three-dimensional relative motion (translations and rotations) between rigid bodies can already be hard to understand, there are a lot of different coordinate representations (position vector, rotation matrix, homogeneous transformation matrix, Euler angles, etc.) each with its own advantages and disadvantages. Moreover, these coordinate representations are used in geometric calculations (compositions like adding position vectors, integrating twists, etc.) and each of the coordinate representations has its own calculation rules (composing rotation matrices requires multiplication, while composing position vectors requires addition, etc.). Finally, the coordinate representations have never been standardised such that students are exposed to different notations and different assumptions (for example, a twist six-vector is composed of the angular and linear velocity three-vectors in either this or the reverse order). While teaching we noticed that students make a lot of errors. On the one hand these errors result from a lack of conceptual understanding. On the other hand, since a lot of reasoning and assumptions are made implicitly and do not appear in the notation, students are unaware of them during exercises. The teachers observed that these errors are not only made by students, but also by researchers in robotics, for instance while

[1] Corresponding Author
T De Laet
tinne.delaet@kuleuven.be

programming robots. Therefore, they started research on the semantics needed for standardizing the geometric relations between rigid bodies. This research resulted in a proposal for the complete semantics underlying geometric relations and in software preventing commonly made errors. While this research was originally inspired by teaching, the result was much appreciated in the robotics research community, as evidenced by two published journal papers on this subject. Finally, the insights obtained from research are now supporting teaching. The proposed semantics help to explain the conceptual ideas and offer an exhaustive notation, making all underlying assumptions explicit, which helps students to prevent errors. We believe that this case study proves that teaching can inspire research and vice versa.

1 RELATED WORK

1.1 Conceptual understanding and threshold concepts

As any engineering faculty member teaching undergraduates knows, students possess a wide variety of misconceptions about fundamental engineering concepts [1]. While engineering professors often succeed to learn students how and when to apply the appropriate equations, lifting the students to conceptual understanding is not so straightforward [1]. This is illustrated by the bibliography of Duit [2], which shows that the problem of conceptual understanding is widespread in Science Technology Engineering and Mathematics (STEM) education.

Threshold concepts (TC) are “akin to a portal, opening up a new and previously inaccessible way of thinking about something” [3]. TC are characterized by five properties: initially troublesome to students, transformative of understanding, integrative, irreversible, and bounded [3]. TC have been studied in engineering education for several purposes: to discover what materials engineering students find difficult or uninteresting in troublesome [4]; to investigate if threshold concepts influence a student’s retention in an electronics engineering program [5]; and to identify threshold concepts in early electronics [6]. Furthermore, Meyer and Land list different reasons for student difficulties: ritual, inert, alien or tacit knowledge, conceptually difficult, and troublesome language [3].

1.2 3D kinematics and statics

3D kinematics and statics require a good conceptual understanding of *relative motions* between different bodies. A number of misconceptions have already been documented in earlier research. Kozhevnikov et al. [7] state that the problems of relative motion are mostly related to the lack of understanding that different observers, each with their own frame of reference, can observe the rigid bodies’ motion. In particular, students have difficulty in understanding the fundamental relativity of motion, i.e. that a motion is only defined with respect to a reference frame, and that it therefore depends on the observer’s frame of reference [1,8-11]. Additionally, Gray et al. identified and reported eleven primary concepts that cause difficulties in teaching dynamics [12]. The first five difficulties are related to 3D kinematics and statics: (1) Different points on a rigid body have different velocities and accelerations, which vary continuously; (2) If the net external force on a body is not zero, then the mass center must have an acceleration, which must be in the same direction as the force; (3) Angular velocities and angular accelerations are properties of the body as a whole and can vary with time; (4) Rigid bodies have both translational and rotational kinetic energy; (5) The angular momentum involves translational and rotational components and requires a reference point.

Different educational approaches have been explored to enhance the understanding of relative motions. A first approach uses *computer simulations* in which the motion of different objects is simulated with respect to different reference frames [11,13]. A second approach uses *virtual environments*. Kozhevnikov et al. [7] found that the "first-hand," egocentric experiences in a virtual and immersive environment significantly contribute to the sense of "presence" students feel, hereby enhancing the conceptual understanding even better than desktop simulations for 3D relative motion problems. A third approach uses *model-eliciting activities* to help repair the misconceptions in dynamics by providing the students with a real-world context [1].

As indicated above the main topic of educational research related to 3D kinematics and statics concern the conceptual understanding and learning of relative motions. Additional to the difficulties in understanding relative motion, matrix/vector methods in 3D kinematics and statics introduce even more conceptual problems. To our knowledge no educational research provides an overview of additional problems or has suggested solutions for them.

2 PROBLEM ANALYSIS

2.1 Teaching 3D kinematics and statics

KU Leuven is a university in Flanders, Belgium, offering Bachelor, Master, and Doctoral programs in a wide variety of disciplines. The engineering curriculum consists of three Bachelor and two Master years. The Bachelor is divided in two consecutive phases of three semesters. The first phase is common for all engineering disciplines. In the second phase the students choose a Major and Minor discipline, which prepares them for the subsequent Master's program. The course "Applied mechanics – part 3" is part of the second Bachelor phase (fourth semester) and is mandatory for all students with major and minor "Mechanics". The number of students attending the course is around 200. The authors have already been teaching the course for seven years. The learning outcomes of the part of the course on 3D kinematics and statics concern knowledge, skills, and attitudes. Concerning *knowledge* the students have to learn calculation techniques for systematically tackling and solving practical problems related to spatial geometry and motion of rigid bodies and static forces acting on rigid bodies. These calculation techniques are bridging the gap between the vector-based and/or conceptual approach in "Applied Mechanics – part 1 & 2" and methods suited for computer support (e.g. multibody dynamics, physics simulation). The students should acquire the *skills* of independently solving problems related to spatial geometry, motion, external and internal forces, etc. After the course they should have acquired a critical *attitude* towards the solution found and the methods used, and awareness of the importance of physical units.

As the literature study showed, the conceptual understanding of the geometric concepts of three-dimensional relative motion (translation and rotation) is hard. Providing Bachelor students with deep understanding of matrix/vector methods for 3D kinematics and statics offers even more challenges:

1. A lot of different coordinate representations (CR) exist to represent the same geometric relation such as position, orientation, linear velocity, angular velocity, force, and moments. The most striking example is the relative orientation between two rigid bodies, which can be represented by a rotation matrix (3x3 matrix), a set of Euler or roll-pitch-yaw angles (three-dimensional vector), rotation angle (scalar) and rotation axis (three-dimensional vector), etc. Each CR has its own (dis)advantages.

Therefore, it is important that students understand the different CR, can convert one CR into another, and select the most appropriate CR to solve a particular problem.

2. The different CR can be used in geometric calculations in order to compose geometric relations, integrate velocities, change the reference frame of the motion, etc. **Each CR has its own calculation rules** however. For example, the composition of rotation matrices requires multiplication (in the correct order!), while the composition of position vectors requires addition. Therefore, students have to know these calculation rules.

3. There is **no standardisation of the CR.** The students are exposed to different notations and assumptions. An example is the twist CR for 3D angular and linear velocity (six-dimensional vector): the first three elements are the linear velocities and the last three elements are the angular velocities or vice versa:

Therefore, it is important that students are aware of the lack of standardisation and the implications this might have when reading textbooks or consulting literature.

4. Finally, there is **no standardisation in the notation and terminology** of different CR, and not all necessary information on the geometric relation is explicitly provided by the notation. Often, a lot of reasoning and assumptions are made implicitly and do not appear in the notation. This causes students to be unaware of the implicit reasoning and assumptions or to forget about them when using the CR in their calculations. An example is the addition of two position vectors $\mathbf{p}^a$ and $\mathbf{p}^b$. Students are taught that the vectors' coordinates can only be added if they are expressed in a common coordinate frame. Often the notation does not explicitly mention the coordinate frame, causing students to forget this constraint.

2.2 Robotics research and 3D kinematics and statics

A main characteristic of robotics, which is the field of research of the authors, is that it involves three-dimensional motion of rigid bodies (manipulated objects, robot links, or mobile bases). Rigid bodies are essential primitives in the modelling of robotic devices, tasks, and perception. Hence, robot programmers, application developers, and robotics researchers have to deal with time-dependent geometric relations between rigid bodies all the time. Evidently, the same four challenges as identified for teaching hold for using matrix/vector methods in robotics research. It is striking that, despite their being used for about 50 years in robotics, the geometric relations between rigid bodies and their CR have never been standardised. This has led to a proliferation of mutually incompatible software libraries in the robot control products of commercial manufacturers as well as in open source libraries. This incompatibility results in commonly made errors that increase application development time.

2.3 Conclusion

The authors observed that the challenges when using vector/matrix methods for 3D kinematics and statics are the same for students learning these methods and robotic researchers using these methods. The fact that both students and researchers will profit from developments in the **standardisation of the geometric relations between rigid bodies**, was the key inspiration for starting our research on this.

3 RESULTS

The **contributions of the research** are five-fold:

1. Identify commonly made errors when using matrix/vector methods,

2. Describe the full semantics underlying rigid-body geometric relations (position, orientation, pose (combination of position and orientation), linear velocity, angular velocity, twist, force, torque, and wrench (combination of force and torque)) including all the choices to be made when specifying these geometric relations. This semantics lists the minimal set of information that is needed to define the geometric relation. As an example: the minimal information when defining a relative position is:

- the point of which the position is expressed,
- the reference point on the reference body with respect to which the position is expressed,
- and the coordinate frame in which the coordinates are expressed.

3. Use this semantics to propose a **notation** that explicitly mentions all the necessary information of the geometric relation. Hereby, choices are made explicit and can be 'read' from the notation. As an example the notation of the relative position would be: so the position of a point *a* with respect to a point *b*, expressed in coordinate frame *c* would be:

4. Identify the **semantic rules** that operations using particular CR have to obey. For example, two position vectors can only be added if

- they are expressed in a common reference frame and
- if the point of one vector is equal to the reference point of the other vector: .

5. Develop **software built on the proposed semantics** that uses the semantic information to store all the semantic information along with the CR (which are mere numbers, vectors, or matrices). The software automatically checks the semantic correctness of all the geometric operations. These checks inform about the violation of semantic rules. For example, would not pass the semantic check, since the coordinate frame of the first vector (*x*) is not equal to the coordinate frame of the second vector (*c*). This error would not have been spotted when the coordinate frame was not explicitly part of the semantics (and the notation).

3.1 Impact on Robotics Research

The research on the semantics on the geometric relations between rigid bodies has a big impact on the robotics research. The clear definition of the semantics serves as a proposal for standardisation, forcing researchers and application developers to reveal all the hidden assumptions in their geometric rigid-body relations. In particular, it also supports the development of software for geometric operations that includes semantic checks. This will avoid common errors, and hence will reduce application (and system integration) development time considerably. This impact is evidenced by the publication of **two papers in a leading robotics journal**, one on the semantics itself [14] and one on the software founded on this semantics [15]. Furthermore, the proposal for **community-driven standardisation** of the semantics for geometric relations via the Robot Engineering Task Force has been published [16].

3.2 Impact on Teaching

The contributions of the research on the geometric semantics underlying matrix/vector methods for 3D kinematics and statics have **direct impact on the teacher** and on the **students:**

1. The teacher can use the *list of commonly made errors* to allocate extra time and explanations to help students prevent these mistakes. Furthermore, by being aware of the commonly made errors students can focus on trying to avoid them.

2. The *minimal but full semantics* provide the teacher with extra insights and help the teacher to explain the essential primitives needed in every geometric relation. This way the students learn which primitives are actually needed for every geometric relation: this helps them to make implicit assumptions explicit by looking for the required information and helps them distinguish important from unimportant information in practical exercises.

3. The teacher can use the *notation* in all study material to reveal all the hidden assumptions, hereby making the reasoning more explicit. This notation helps students to write down all the assumptions explicitly such they will not “forget” about them when making exercises.
4. The teacher can include *semantic rules* in the study material. The students can use the semantic rules to check their reasoning and calculations.
5. The *software* can be used by the teacher to show commonly made errors on practical examples. Students can use the software to discover the semantic errors in their exercise solutions.

So far, the teachers have been using the findings to support the teaching. In particular they use the notation that reveals the hidden assumptions and stress commonly made errors and how they can be prevented.

Although no quantitative study was made, the teachers clearly observe that students appreciate the proposed notation, since it helps them to check if their reasoning is correct. Moreover, by explicitly stating the essential primitives for particular geometric relations and making them explicit in the notation, the conceptual understanding of the students is improved. An anecdote that supports this claim is related to the coordinate representation of twist. Since three years the teachers propose the students to use the notation that reveals all the hidden assumptions: while beforehand only the notation: was used. Although the textbook still uses the more simple notation the answers to student’s exam answers showed on the one hand that the students prefer the new (more complex) notation and that the number of errors is lower when using the new notation (both compared to the previous years when the notation was not used and to students that are not using the notation on the exam). In particular, related to the composition of twists:

- by explicitly adding the point and coordinate frame, students make less often the mistake of adding twists with different points and coordinate frames and
- by explicitly adding the reference body, students make less often logical errors when adding twists: is correct while makes no sense.

3.3 Relation to threshold concepts

The challenges and common misconceptions listed in this paper highlight the “troublesome knowledge” characteristic of 3D kinematics and statics. The geometric concepts of three-dimensional relative motion are **conceptually difficult** and some aspects like the fundamental relativity of motion are **alien or counterintuitive**. Furthermore, the knowledge is **tacit** since the semantics of geometric relations was not explicitly identified, taught, or learned. Finally, the lack of standardisation in the notation, terminology, and CR make the **“language” troublesome** both for students and researchers. This paper aims at making 3D kinematics and statics less troublesome. When using the semantics however, one should not be tempted to make the knowledge **ritual.** Students and researcher should not only use the semantic rules in a procedural way to avoid and check for errors, but should be a basis for deep understanding of the conceptual difficulties.

4 CONCLUSIONS AND FUTURE RESEARCH

This paper showed how the teaching of matrix/vector methods for 3D kinematics and statics inspired research on semantics of geometric relations between rigid bodies. While the research was inspired by teaching basic concepts in matrix/vector methods for 3D kinematics and statics, it was of high value for the robotics research community. Conversely, the findings of the research were used to improve the teaching. In particular the research resulted in a notation that reveals all the hidden assumptions, which in the past often caused students to make errors. In conclusion, we believe that the reported experience can serve as a case study of how teaching can inspire research and vice versa. Moreover, this paper gives a first attempt to identify the threshold concepts in 3D kinematics and statics.

In the future the teachers will rewrite the courses notes and slides such that the proposed semantics is used as a basis to explain the geometric relations, the notation is used consistently in the course material to reveal hidden assumptions, and the semantic rules are listed explicitly.

The software currently developed is targeted to robotics researchers and therefore written in C++, making the software less accessible for students. Therefore, we will develop a toolbox in a more accessible language like MATLAB or python that offers the students semantic checking on top of the actual matrix and vector calculations.

Moreover, we will set up a more quantitative analysis of the impact of the changes on the understanding of the students and explore more deeply which threshold concepts can be identified and how education research can help to identify appropriate teaching strategies.

Finally, the authors hope to apply the developed approach of (i) identifying the minimal but complete semantics of a domain, (ii) propose a notation that reveals the entire semantics, (iii) stating the semantic rules holding in that domain, and (iv) developing software supporting the semantics in other domains such as basic mechanics, dynamics, kinematics of mechanisms.

5 ACKNOWLEDGMENTS

The authors acknowledge the KU Leuven's Concerted Research Action GOA/2010/011, the KU Leuven-BOF PFV/10/002 Centre-of-Excellence Optimisation in Engineering (OPTEC), and the European FP7 projects 2008-ICT-230902-ROSETTA, 2008-ICT-231940-BRICS, and FP7-ICT-288533-RoboHow. Tinne De Laet is a Post-Doctoral Fellow of the Fund for Scientific Research–Flanders (F.W.O.).

REFERENCES

[1] Self B P, Miller R L, Kean A, Moore T J, Ogletree T, and Schreiber F (2008), Important Student Misconceptions in Mechanics and Thermal Science: Identification Using Model-Eliciting Activities, *Proceedings of the 38th ASEE/IEEE Frontiers in Education Conference*, Saratoga Springs, NY.

[2] Duit, R (2009), Bibliography: Students' and Teachers' Conceptions and Science Education, Kiel, Germany: Institute for Science Education. Available at http://www.ipn.uni-kiel.de/aktuell/stcse/stcse.html.

[3] Meyer JHF and Land R (2003), Threshold concepts and troublesome knowledge – Linkages to ways of thinking and practicing, *Improving Student Learning – Ten Years On*, C. Rust (ed.), OCSLD, Oxford.

[4] Baillie C, Goodhew P, and Skryabina E (2006), Threshold concepts in engineering education-exploring potential blocks in student understanding, International Journal of Engineering Education, Vol. 22, No. 5, pp. 955-962.

[5] Harlow A, Scott J, Peter M, and Cowie B (2011), "Getting stuck" in analogue electronics: threshold concepts as an explanatory model, European Journal of Engineering Education, Vol. 36, No. 5, pp. 435-447.

[6] Scott J and Harlow A (2012), Identification of threshold concepts involved in early electronics: Some new methods and results, Australasian Journal of Engineering Education, Vol. 18, No. 1, pp. 1-8.

[7] Kozhevnikov M, Gurlitt J, and Kozhevnikov, M (February 2013), Learning Relative Motion Concepts in Immersive and Non-immersive Virtual Environments, *Journal of Science Education and Technology*, DOI 10.1007/s10956-013-9441-0, pp. 1-11.

[8] Camp C, Clement J, Brown D, Gonzalez K, Kudukey J, Minstrell J, Schultz K, Steinberg M, Veneman V, and Zietsman A (1994), Preconceptions in mechanics: lessons dealing with student's conceptual difficulties, *American Association of Physics Teachers*, Kendall-Hunt, Dubuque.

[9] Saltiel E and Malgrange JL (1980), Spontaneous ways of reasoning in elementary kinematics, *European Journal of Physics*, Vol. 1, pp. 73-80.

[10] Sequeira M, Leite L (1991), Alternative conceptions and history of science in physics teacher education. *Science Education*, Vol. 75, pp. 45-56.

[11] Monaghan M JM, Clement J (2000), Algorithms, visualization, and mental models: high school) Algorithms, visualization, and mental models: high school students' interactions with relative motion simulations. *Journal of Science Education and Technology*, Vol. 9, pp. 311-325.

[12] Gray GL, Evans D, Cornwell P, Costanzo F, and Self BP (2005), The Dynamics Concept Inventory Assessment Test: A Progress Report, *Proceedings of the 2005 American Society for Engineering Education Annual Conference,* Portland, OR.

[13] Monaghan JM, Clement J (1999), Use of computer simulation to develop mental simulations for understanding relative motion concepts. *International Journal of Science Education*, Vol. 21, pp. 921-944.

[14] De Laet T, Bellens S, Smits R, Aertbeliën E, Bruyninckx H, and De Schutter J (2013), Geometric Relations between Rigid Bodies: Semantics for Standardization, *IEEE Robotics & Automation Magazine*, Vol. 20, No. 1, pp. 84-93.

[15] De Laet T, Bellens S, Bruyninckx H, and De Schutter J (2013), Geometric Relations between Rigid Bodies: From Semantics to Software, *IEEE Robotics & Automation Magazine*, Vol. 20, No. 2, pp. 91-102.

[16] De Laet T, Bellens S, and Bruyninckx H (2012), Semantics Underlying Geometric Relations Between Rigid Bodies in Robotics, [online] Available: https://retf.info/rrfc/0005.