The mathematics in your ears

The role of math in integrated STEM via the modeling of hearing aids

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INTRODUCTION

In this paper a new module for the course Mathematics is presented, with a new approach that integrates Mathematics with Sciences, Technology and Engineering. First, the need for this new kind of didactics for the Flemish General Secondary

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School is emphasized. In the second section the paper elaborates on the subject matter of the course material: The Math behind Cochlear Implants, and the integration of the previously mentioned study domains.

The new course material, presented in this paper, is made in cooperation with Junior College and the RVO-society. Junior College (JC) is an initiative by the KU Leuven and the KU Leuven Kulak to introduce mathematics on a level close to the one in college, into the Flemish Secondary School [1]. The organization develops ready-to-use course material considering hot scientific topics that can be instructed by the teachers in the classroom during the hours that are scheduled for math, or during the two extra hours a week reserved for supplementary and cross-disciplinary theory. The RVO-society is a Flemish organization that develops educational material and workshops concerning recent discoveries in technique and science. The organization wants to make the connection between research and education to bring science into the life of youngsters. Their mission is to enlarge the influx in STEM-study profiles [2]. The Math behind Cochlear Implants is based on their course Listen to the Chip (Luister naar de Chip) [3].

1 NEED FOR STEM

1.1 The problem

The increasing knowledge-economy, Flanders aims for, craves for more and more high educated scientists, technicians and engineers, commonly referred to as practitioners of a STEM-profession. STEM stands for Science, Technology, Engineering and Mathematics. Unfortunately the amount of graduates with a STEM-profile is not sufficient to fulfil this growing need.

According to a study by the Flemish Council for Science and Innovation (VRWI – Vlaamse Raad voor Wetenschap en Innovatie) approximately four out of ten students in the Flemish secondary school choose for a STEM-education. But only roughly half of these STEM-pupils opt for a STEM-oriented study field in college [4]. Considering the fact that these numbers are not increasing, the perspectives for Flanders are not at all opportune. These numbers position Flanders below the European average and it will become harder to achieve the 3% standard for Research and Development (3% of the GDP should be spent on R&D).

1.2 STEM in Flemish Secondary Schools

In Flanders two of the four existing education formats offer STEM-oriented study profiles that prepare for a college education in STEM: the General Secondary School (ASO – Algemeen Secundair Onderwijs) and the Technical Secondary School (TSO – Technisch Secundair Onderwijs). *Table 1* shows some of these study profiles.

Table 1. Examples of STEM study profiles in ASO and TSO [4]

ASO	TSO
Sciences-Mathematics	(Bio)Technical Sciences
Greek- / Latin-Mathematics	Accountancy-Informatics
Latin-Mathematics	Electricity - Electronics
Languages-Sciences	(Electro)Mechanics
Greek- / Latin-Sciences	Industrial Sciences
Sports-Sciences	Chemistry
Languages-Mathematics	Architecture
Economics-Mathematics	Electrotechniques

	Health Sciences
	Horti- / Agricultural Techniques
	Electrical Installation Techniques
	Industrial ICT

As can be seen in *Table 1* there are far more technical inspired study profiles in TSO, as the name suggests. Moreover, the percentage of students enrolling in STEM-oriented study fields in college is much higher for TSO-profiles than for ASO-profiles.

Of all ASO-study profiles, Sciences-Mathematics is the study option that delivers most of its students, but only about 56% of them, to STEM-oriented study fields in college [4]. Sciences-Mathematics is a highly esteemed study option which attracts students who are intrigued by solving abstract problems based on profound theoretical knowledge of sciences and math. Considering the need for high educated STEM-graduates, the amount of these students enrolling in STEM-studies in college should be much higher.

This paper focusses on STEM-inspiring course material developed for students in the third grade (17- to 18-year olds) in the study profile Sciences-Mathematics and other profiles with a component Mathematics in ASO.

1.3 Flemish Science and Math education (ASO)

The alarming shortage of technicians, engineers, informaticians, mathematicians and physicists, is due to the lack of motivation of potential STEM-students for STEM. This paper indicates two major problems in the courses of the General Flemish educational system (ASO) causing the impassive attitude towards STEM:

1. the complete separation of math and science, and
2. the absence of reference to technology and technical skills.

First of all, the teachers should make the connection between mathematical concepts and their practical applications in science. Math is a powerful instrument in trying to capture scientific phenomena: by relying on their mathematical insight, students can deduce mathematical models that will help them understand these phenomena. Learning in the opposite way can be equally valuable: students start recognizing the importance of mathematical equations when they are illustrated by real physical or biological cases.

Merging Science and Mathematics until the teacher instructs these domains in a fluently alternating way, is highly advisable in aiming at increasing the students' interest for STEM, but it's not sufficient. The theoretical concepts of these courses need to be operationalized by tangible examples of their use in technology. The development and understanding of technical devices give the answer to the "why?"-question that so often arises in the mind of students solving math problems. They inspire students to discover fascinating inventions or to come up with new ideas.

It is clear there is a need for a new kind of didactic approach that integrates these important domains and that introduces STEM into the curricula of the General Secondary School (ASO). The course of The Math behind Cochlear Implants is a first attempt towards more integrated STEM-education. Still, it mainly focusses on Mathematics because of the fact that the teachers are restrained to the math curriculum. Let's call this concept steM (with capital M).

2 THE MATH BEHIND COCHLEAR IMPLANTS

2.1 Math as a help for hearing problems

The central problem, suggested in this new JC course, remains close to a real risk of today's youth. When they listen for too long to music that is too loud, they can incur serious damage to the inner ear, which causes (partial) deafness for some frequencies in sound. How can mathematical methods be used to design an appropriate hearing aid?

The course brings pupils in contact with the concept of Fourier analysis by showing its relevance in high-technological medical devices. Hence, the course is built up in a constructive manner; the chapters are called:

1. Sound
2. Hearing
3. Signal Analysis
4. Cochlear Implants

In the first two chapters, physical and biological phenomena are mathematically modelled, such as sound waves and the hearing process. The sine function and superposition of sine functions are being studied, as well as the physical meanings of the accompanying amplitudes and frequencies, which are: loudness and pitch. This part of mathematical analysis is necessary in order to get to Fourier Analysis, which is an essential tool in the human hearing process.

In the third chapter, the French physicist and mathematician Fourier is being introduced. He came to the conclusion that every periodic signal can be parsed into a sum of sine functions with different frequencies, all integer multiples of a fundamental frequency f_0 [5]. Considering *x(t)* a periodic signal, it can thus be written as in *Eq. (1)*:

$$x(t) = \sum_{k=1}^{\infty} A_k \cdot \sin\left(2\pi k f_0 t + \phi_k\right) \tag{1.}$$

In the course some simplifications are made to not completely overwhelm the students: a sound signal is assumed to be a periodic, harmonic wave, like *x(t)* in *Eq. (1)*, of which the fundamental frequency f_0 is assumed to be known.

The course demonstrates how the cochlea, an essential, snail-shaped organ in the human hearing system, works as a Fourier analyser. Not only does it filter the different frequencies $k \cdot f_0$ out of a sound, it assigns them separately to specific spots in the cochlea, a phenomenon called tonotopy, as shown in *Fig. 1*. Within the cochlea, tiny cilia are being tickled at these frequencies, and a chemical reaction will transmit electrical pulses to the connected auditory nerves. The intensity the cilia will respond with, is proportional to the amplitude A_k accompanying these frequency components $k \cdot f_0$. This matter is being discussed to make the pupils realize that even biological structures make use of mathematical concepts.

When some of the cilia, transmitting the electrical pulses to the auditory nerves, are permanently damaged, deafness to the according frequencies occurs. Now the **central problem** pops up: how can Fourier analysis be utilised to create a medical device that compensates for this unfortunate defect?

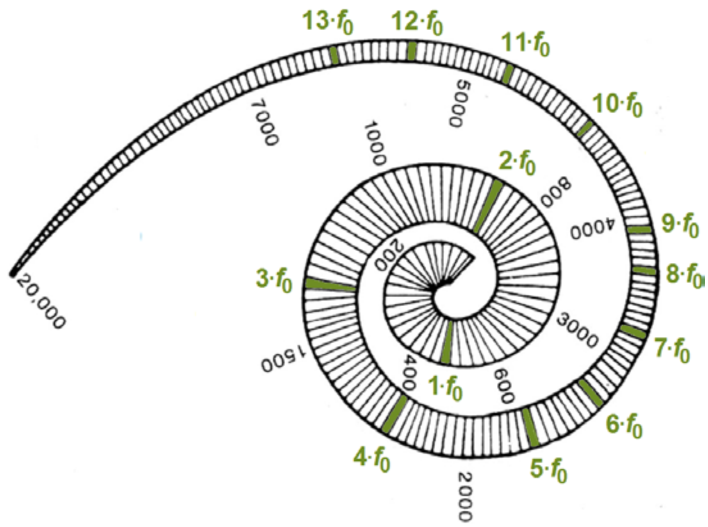

Fig. 1. Tonotopy in the cochlea; frequencies in Herz (© Richard M. Warren) [6][7].

Chapter four offers technology as the solution for this central problem: the cochlear implant. Cochlear implants consist of a behind-the-ear digital speech processor (DSP), two coils (one outside and one underneath the skin) and an electrode-array that is implanted in the cochlea. The electrode-array consists of electrodes that touch the auditory nerves and directly send them electrical pulses, in other words: they replace the functioning of the cilia [8][9].

In this chapter the students need to fancy themselves real engineers by figuring out how to use the acquired knowledge about the math in the human hearing process (the **Fourier analysis**), in order to compose the cochlear implant. They get to learn some of the most important engineering skills: to be capable of making abstractions and models of signals and systems. Step by step and by means of **block diagrams** (*Fig. 2*) they will split up the process of capturing and analysing sound into the separate subsystems that are needed to process incoming signals into the requested outgoing signals. This is considered as the basis of a technical process.

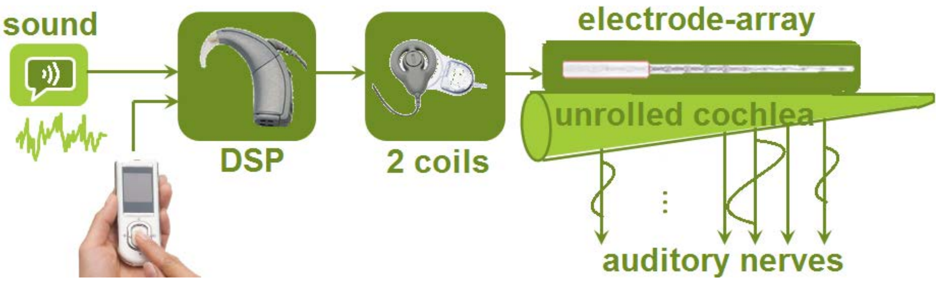

Fig. 2. The cochlear implant [9], represented in a block diagram.

The pupils will discover that the Fourier analysis is established by the DSP of a cochlear implant, but they also get in touch with other necessary technological subsystems, like:

- a microphone which uses the principle of electromagnetic induction,
- an ADC-system which needs an electronic sampler,
- band pass filters that serve as Fourier analysers
- coils working as a voltage transformer.

2.2 In practice

As mentioned before, the Junior College course of The Math behind Cochlear Implants consists of four chapters. When teachers want to instruct these chapters in the classroom, the expected duration lies between 8 to 10 times two lessons of 50 minutes.

Because the course material mainly and thoroughly emphasises mathematical and physical aspects of sound, but also approaches the matter from a broader perspective (biology, technology and social interest), it is recommended for use in the free time slot in the third grade of ASO. The free time slot exists of two lessons a week that are reserved for elaboration of a certain (cross-disciplinary) domain or topic that is chosen by the school or the teachers.

In case of lack of time the teacher can choose to instruct only one or a few of the four chapters, for they are composed in a modular way. This means that each chapter begins by quoting a brief summary of the most important aspects of previous chapters, just enough to get started with the new one. Some chapters or sections can be given as a reading or research assignment for the students to prepare at home.

Because the learning process of the students is the central concern, it should be supported in the most efficient way. Therefore, through different assignments and reasoning exercises, the pupils are encouraged to take a research-minded attitude and to enlarge their problem-solving skills.

For this objective, the course makes use of two educational software tools: Geogebra and Scilab. Geogebra is an interactive and graphical application for geometry, algebra and analysis. Scilab is a technical environment for all kinds of mathematical operations like the computing of functions, drawing of graphs and implementation of algorithms. Xcos is an application of Scilab to simulate dynamic system models in a graphical way. Both software applications are open source.

An example of encouraging the students to think and operate like an engineer, is the final assignment: the ultimate goal is to build a simulation model of a discrete Fourier analyser, by means of blocks provided by Scilab – Xcos (*Fig. 3*). They are given an unknown signal $x(t)$ which is a sum of 5 harmonic sine functions with a known fundamental frequency f_0 (for example 441 Hz). Their task is to decipher the different amplitudes A_1 to A_5. In this assignment they need to split up the problem into blocks, and use mathematical aspects like sampling frequency and matrix operations (since the outcome of each block is an array).

As can be seen in *Fig. 3* working with Scilab – Xcos improves the capability of students to think in signals, (sub)system operations and sequences, and is a powerful tool in problem-solving techniques.

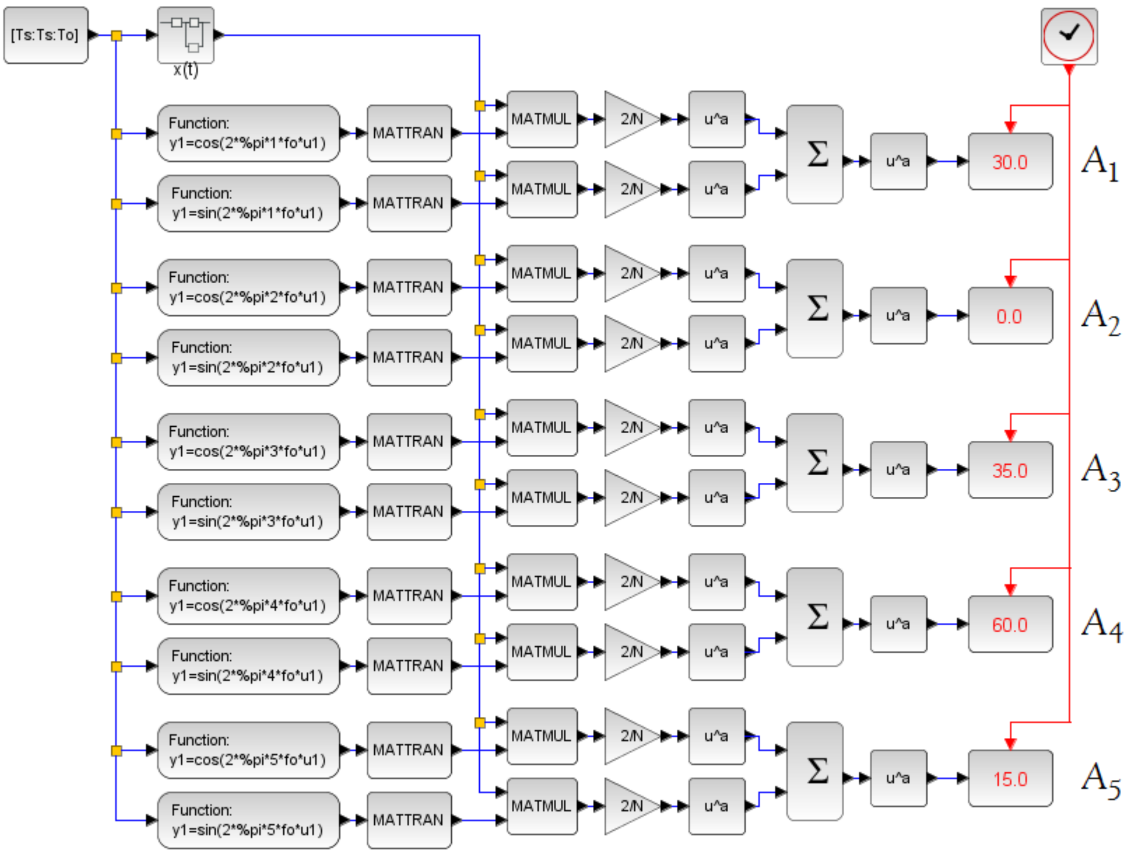

Fig. 3. Final assignment for the students:
a simulation model of a discrete Fourier analyser.

Accentuating the importance of mathematical concepts by embedding them in relevant scientific phenomena and using them as a tool in technical solutions, is important in integrated STEM-education. The course material The Math behind Cochlear Implants offers this method as well as basic engineering skills, and still remains close to the math curriculum.

3 VALIDATION

The Math behind Cochlear Implants has already been presented in a Junior College evaluation meeting to math teachers who were involved in JC in the academic year 2012-2013. The teachers were intrigued by the new concept of connecting the math to social relevance, technology and engineering skills. The basics of programming: thinking in terms of signals, blocks and sequences and using this to complete the final assignment, are considered as a welcome extra, since the math curriculum contains learning objectives referring to these kind of skills.

The aim is to test this new JC course material in the first semester of the next academic year (2013-2014) in some pilot classes. Afterwards the students and teachers will be extensively interrogated to validate the effectivity of this course material. Adjustments can then be made according to their suggestions, in order to put the course material into practice in the following academic year (2014-2015).

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