The SEFI Mathematics Working Group's new Curriculum Framework Document

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INTRODUCTION

It is one of the main goals of SEFI's Mathematics Working Group (MWG) to provide orientation to those who have a professional interest in the mathematical education of engineers. The core document that serves this purpose is the group's curriculum document that intends to help in clarifying the goals of mathematics education and ways to achieve them.

The current second edition of the document which is downloadable from the group's website (sefi.htw-aalen.de) was issued in 2002. In the middle of 2013, the group will issue the new third edition. It is the purpose of this contribution to explain the reasons and goals of the update, to provide an overview of its structure and contents and to discuss and exemplify possible usages of the document.

1 PREVIOUS EDITIONS OF THE CURRICULUM DOCUMENT

The first edition of the MWG curriculum document, called " A Core Curriculum in Mathematics for the European Engineer" ([3], [4]), was issued 1992, ten years after the foundation of the working group. It was the intention by then to specify those topics that should be covered in any engineering course. The document was subdivided into the areas "Analysis and Calculus", "Linear Algebra", "Discrete Mathematics" and "Probability and Statistics". Besides the listing of core material there are also lists of material for elective courses which may not be relevant for all kinds of engineering study courses. For example, within the area of analysis and calculus the following topic can be found ([3], p. 27):

"2.1.6 Integral Calculus: Definition of the integral as the limit of a sum, numerical integration. Fundamental theorem of calculus, indefinite integration as the reverse of differentiation, standard techniques of integration. Engineering applications of integration. Improper integrals, estimation, convergence and divergence."

Specifications like the above formed the main part of the document. Besides that, a few remarks on higher level “educational objectives” like the ability to work and solve problems in models and to communicate results can be found in the document.

Ten years later, in 2002, a revised second edition was issued, called “ Mathematics for the European Engineer. A Curriculum for the Twenty-first Century” ([8]). In this second edition the curriculum was refined and rewritten in terms of learning outcomes. According to contemporary curriculum design theory, a curriculum should specify what a learner knows and is able to do instead of specifying topics to be dealt in teaching. Therefore, the document contains comprehensive lists of detailed learning outcomes, for example regarding “Methods of integration”:

“As a result of learning this material you should be able to

- obtain definite and indefinite integrals of rational functions in partial fraction form
- apply the method of integration by parts to indefinite and definite integrals
- use the method of substitution on indefinite and definite integrals
- …”.

As a fifth area, geometry was added taking into account the growing importance of geometric concepts in engineering design. The learning outcomes are organised in four levels:

- Core zero where many learning outcomes were placed which formerly were considered to be prerequisites for taking up engineering studies. It was recognised that this assumption was no longer realistic at many places such that a detailed specification would also help in detecting deficiencies and offering respective support.
- Core level 1 was still considered to be obligatory for nearly all engineering study courses.
- Level 2 consists of electives which are relevant in some but not all engineering study courses.
- An additional Level 3 was included since there are more advanced mathematical concepts and methods used in application subjects later in an engineering course which are mostly taught within the application subject and not in a separate mathematics class. For this level only the topics are listed, no specific learning outcomes have been identified.

The document is not a fixed curriculum. Although core zero and core level one were considered to be mandatory, a user of the curriculum document still had to choose from the learning outcomes on level 2 in order to specify a complete curriculum.

Other issues like the role of technology, transition problems from school to university, and educational goals like communication and modeling were also discussed in a short commentary section of the document.

2 INTENTION AND STRUCTURE OF THE THIRD EDITION

During the last decade, in several seminars of the MWG the topic of higher-level learning goals came up which go beyond the largely content-related learning outcomes specified in the second edition. For example, the contribution by Booth [5] on “learning for understanding” and the paper by Cardella [6] on using a “broad notion of mathematical thinking” investigated this topic. In the second edition of the curriculum document one can also find a few statements on such goals but it lacks a systematic approach. It is the main intention of the third edition to make use of state-of-the-art educational research in mathematics didactics in order to base the document on such a specification of higher-level goals. For doing this, the concept of “mathematical competence” and “competencies” was chosen which was developed in the Danish KOM project headed by Mogens Niss ([9],[10]). This concept points the view to essential aspects of what the mathematical education should strive for within an engineering study course. It is presented in more detail in the next sub-section 2.1.

We retained and slightly modified the lists of learning outcomes since we think that these still provide valuable information for a curriculum designer or mathematics teacher when setting up content-related learning goals. The second major addendum to the second edition is a more comprehensive treatment of the teaching and learning environment including the issue of assessment. Sub-section 2.2 gives an overview of the structure of the new document and gives reasons for including the issues investigated there.

2.1 The concept of mathematical competence

In the Danish KOM project a group headed by Mogens Niss based their description of what the mathematical education should strive for on the concept of mathematical competence which was defined as follows ([10], p. 6/7):

“*Mathematical competence* then means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations where mathematics plays or could play a role. Necessary, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills, in the same way as vocabulary, orthography, and grammar are necessary but not sufficient prerequisites for literacy”.

This concept was adopted for the third edition of the MWG curriculum document mainly for two reasons. On the one hand, it emphasises the ability to apply mathematical concepts and procedures in relevant contexts which is the essential goal of mathematics in engineering education: to help students to work with engineering models and solve engineering problems. On the other hand, it explicitly recognises that competence requires a solid base of knowledge and skills reflecting the strong opinion of many “practitioners” engaged in the MWG.

The concept is also well in line with current trends in general engineering education where the notion of competence has been used to describe educational goals which favour “action-based knowledge over knowledge simply held, in the name of performance and effectiveness” ([7], p. 47). It should be noted, though, that in literature the notion of competence has been used differently and other terms like “skill” and “capability” are used with a meaning similar to that outlined above for competence (for an overview and a discussion see [7]).

In order to be helpful for curriculum specification the competence concept must be filled with more meaning. This has been done in the KOM project by identifying eight so-called competencies. The third edition of the MWG curriculum uses the following slightly modified list of these competencies for which we just give very short explanations since more detailed information can be found in the new edition:

- <u>Thinking mathematically</u>: This includes the ability to understand and judge what kind questions can be answered using mathematics and hence where a mathematical approach might be helpful.

- Reasoning mathematically: This includes the ability to understand a mathematical argumentation as well as the ability to set up an own mathematical argumentation (chain of reasoning).
- Posing and solving mathematical problems: This includes the ability to formulate a question as a mathematical problem and to use problem solving methods ranging from simple schematic procedures to more general problem solving strategies.
- Modeling mathematically: This inludes the ability to understand, work and solve problems within models set up by others as well as the ability to perform active modelling (parts of the modelling cycle).
- Representing mathematical entities: This includes the ability to choose adequate representations and to switch between representations in order to use the most suitable one for a problem.
- Handling mathematical symbols and formalism: This includes the ability to understand symbolic expressions and formal language used by others as well as setting up and manipulating own expressions according to rules.
- Communicating in, with, and about mathematics: This includes the ability to understand oral and written mathematical statements made by others and the ability to express oneself mathematically.
- Making use of aids and tools: This includes the ability to recognise when the usage of aids and tools is adequate as well as the ability to use them adequately.

Some of the aspects stated above have already been shortly addressed in the previous editions of the curriculum document (see section 2) but in the third section they are now part of an overall concept.

For specifying a curriculum for a concrete type of engineering study course one has to identify in more detail the level of progress one wants to achieve. The competencies have already been stated as educational goals of secondary education, so it must be made clear which progress is intended on the tertiary level. In the KOM project three dimensions have been identified within which the level of progress can be specified ([9], [10]):

- Degree of coverage: Which aspects of the competency should be included?
- Radius of action: What are the situations and contexts where the competency can be applied?
- Technical level: What are the mathematical concepts learners should be able to use when applying the competency?

The third edition recommends to use these dimensions.

2.2 Structure of the document

The second chapter of the document first describes the eight competencies and the three dimensions in more detail based on the results of the Danish KOM project. Moreover, it provides some illustrative engineering mathematics examples in order to clarify which competencies might be needed when working on an engineering task with mathematical aspects. The document does not prescribe a certain level of progress in the three dimensions since engineering study courses and engineering work profiles are much too heterogeneous for such an endeavour. Therefore, the third edition is called “A Framework for Mathematics Curricula in Engineering Education”. Within this framework many curricula for different types of study courses can be specified. A potential process for building such profiles is also outlined in the second chapter which will be discussed in more detail in section 3 of this paper.

The third chapter of the document presents a slightly modified version of the content-related learning outcomes that formed the kernel of the second edition. The clear arrangement into four levels was also retained. Although the authors think that most of the learning outcomes specified in core zero and core level 1 should still be covered in any engineering study course, the lists in chapter three are also offered as a framework from which a choice must be made based on the requirements of the study course. Since in many European countries the number of contact hours in mathematics has been reduced when introducing the bachelor-master split one has to make a realistic choice regarding the learning outcomes. Moreover, achieving aspects of the eight competencies will require additional time.

The fourth chapter of the document discusses aspects of adequate teaching and learning environments in a competence-based engineering study course and provides many links to relevant former seminar contributions and other literature. Learning scenarios like lectures, projects, assignments, tutorials, laboratories and technology-enhanced settings are investigated for their potential for obtaining competencies. A recommendation for a mix of offerings is given. Next, transition problems are explained and successful measures for addressing these are outlined. The use of mathematics technology which has been a subject of controversial debate in many seminars is also of special interest in a competence-based approach. It is even explicitly addressed in the eighth competency on using aids and tools. Therefore, essential aspects of using mathematics technology are outlined in this chapter.

Enabling students to understand and use mathematics in engineering contexts forms the kernel of the competence concept. For this reason, a mathematics curriculum that seriously intends to support this concept must be strongly integrated into the engineering study course for which it has been set up. There are several aspects of this integration like “Which mathematics is used in application subjects and how is it used?” or “When are mathematical concepts needed (early and again in later study phases)?”. These are discussed and some interesting approaches from literature are outlined. Strongly related to the question of integration is the attitude of students towards mathematics which is explained in the final section of the chapter. Is it seen as a stumbling block to overcome at the beginning or is it seen as integral part of engineering? Such attitudes strongly influence motivation and readiness to apply (or avoid) a mathematical approach to engineering problems and hence must be taken into account.

The fifth chapter of the document is concerned with assessment. Since many students are extremely assessment driven adequate assessment regimes must not be neglected. The chapter first describes different forms of assessment which are applied in Europe. It then discusses the question of requirements for passing which are essential for guaranteeing that after passing an examination students have really achieved the learning outcomes specified in a curriculum. Whereas assessing very detailed, content-related learning outcomes is often straightforward, the assessment of competencies is more challenging and still the topic of current research. Potential forms for formative and summative assessment of competencies are outlined. Finally, aspects of technology-supported assessment are discussed.

The document is not meant to be a “handbook” for the mathematical education of engineers but it aims at providing orientation and hints regarding essential topics.

3 PROSPECTIVE USAGE

The document is now called “Framework” since the MWG is convinced that there will not be a “one-fits-all” curriculum but many different mathematics curricula for various types of study courses. Only by taking into account the needs of a certain type of study course can a strong integration of the mathematical education be realised.

The addressees of the curriculum document are curriculum designers and lecturers of mathematics for engineers. In order to create a concrete curriculum within the provided framework, one has to specify the desired level of progress regarding the mathematical competencies and one has to choose a subset of the lists of content-related learning outcomes. For developing criteria to do this in a well founded way, a thorough analysis of the usage of mathematical concepts, models and procedures is necessary at least in those application subjects with a strong mathematical foundation. Based on such an analysis, one has to identify the aspects of a certain competency that should be included in the curriculum (“degree of coverage”). It also provides essential information for specifying the contexts and situations where a student is able to apply a competency (“radius of action”), and the kind and depth of mathematical concepts and procedures which can be used when activating the competency (“technical level”). A more detailed specification of the technical level then occurs when choosing the necessary content-related learning outcomes. A rough example for such a procedure is provided in the second chapter of the curriculum document (see also [2]). Within such a procedure one should also clarify where certain aspects of a competency should be learned. Mathematical competence is not just acquired in the proper mathematical education of a study course but also within application subjects. This is quite obvious for the modelling competency since working in mathematical models is an essential part of several application subjects.

The procedure suggested above is non-trivial and very time-consuming. Therefore, the MWG does not assume that every lecturer or curriculum designer will do this from scratch. Ideally, there should be several mathematics curricula available created within the framework for different types of engineering study courses (e.g. for different areas like mechanical and electrical engineering or for different work profiles like computational engineering or practice-oriented engineering). Then, the addressees could take such a curriculum, analyse its applicability and use it possibly with slight modifications. This would facilitate the creation of a concrete curriculum considerably and hence enlarge the probability that the framework document has a real impact. One such curriculum has been set up in [2] for a practice-oriented study course in mechanical engineering.

4 EXAMPLE

The curriculum specified in [2] is based on the analysis of a bachelor degree course in mechanical engineering at a university of applied science. There, the subjects with a strong mathematical basis include engineering mechanics, stress theory, thermodynamics, fluid dynamics, machine dynamics, and control theory. Based on an analysis of the usage of mathematical concepts in these subjects, for each of the eight competencies defined in section 2.1 six to eight aspects have been identified that should be covered in the proper mathematical education or in application subjects. Several example tasks illustrate where the respective aspect is important. These tasks are also meant to give ideas for developing own problems for the students to work on. For each competency, the situations and context are roughly specified where the competency should be applied. In the “technical level” dimension the concepts and procedures students should be able to use when applying the competency are outlined. A more detailed specification of the technical level then occurs in the choice of content-related learning outcomes which is essentially a subset of the lists presented in the framework document.

As an example, we present some aspects, contexts and technical level specifications for the competency "Posing and solving mathematical problems". Regarding the degree of coverage the following aspects are included ([2], p. 16-18):

- "Students should be able to solve well-specified computational problems for which algorithms already exist. These problems show up in an intra- or extra-mathematical environment".

  Example: "Students should be able to solve an integral where the integrand is a fraction. It is well-known that the method using partial fractions works ..."

  Example: "Students should be able to perform an algorithm for computing the stress occurring in a machine element given a certain load. The algorithm can be found in a machine element book."

- "Students should be able to solve problems where certain quantities in given models have to fulfil some restrictions and values for design variables ... have to be found such that this is the case ... Students should be able to use the strategy of goal-oriented iteration ...".

  Example: In machine element dimensioning books one can find algorithms for computing the occurring stress when the dimensions, environmental conditions, material and loads are given. One has to find a configuration such that the occurring stress is below the maximum bearable stress for the material. For this one can try to find an initial guess ... and then to modify it. To do this in a goal-directed way, one has to see the influence of design variables ...".

Regarding the radius of action, the following contexts are included ([2], p.20):

- "Students should be able to apply the mathematical problem solving competency in intra-mathematical situations where the problems are posed and solved as mathematical ones" .
- "Students should be able to apply the problem solving competency in all kinds of dimensioning problems in mechanical engineering" .
- "Students should be able to apply the problem solving competency in all kinds of design problems in mechanical engineering" .

Regarding the technical level the specification includes the following coarse items which are refined in the section on content-related learning outcomes ([2], p. 21):

- "Students should know and be able to apply essential geometric constructions for design problems including standard geometries and freeform geometries".
- "Students should know and use the essential function classes and their properties (continuity, differentiability) for function design purposes".

5 SUMMARY AND ACKNOWLEDGMENTS

In this contribution we outlined and provided the reasoning behind the new (third) edition of the curriculum document of SEFI's Mathematics Working Group. The history from the first edition to the third one can be briefly summarised as going from contents via outcomes to competencies. The concept of mathematical competence which has been adopted from the Danish KOM project is at the heart of the new edition which is meant to provide a framework for specifying mathematics curricula for engineering study courses. It is still a considerable effort to do this for a concrete type of study course. For facilitating this, one example for a practice-oriented study course in mechanical engineering has been provided. We envisage and hope for additional curricula for other types of engineering study courses such that curriculum designers and mathematics lecturers can pick from a variety of curricula and choose one that best fits the special purposes of their study course. This way, the document would have a real impact on the mathematical education of engineers.

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