

## Evaluating competences in Mathematics at Aeronautical Engineering

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## INTRODUCTION

Evaluation of competences at Engineering is a trendy topic in Engineering Education. This has come out after an evolution of the university paradigm structure that had evolved from teacher-centered to student-centered models, and from measuring skills and outcomes to competence based models.

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In this process a lot of attention has been put into transversal competences such as net-working, entrepreneurship, internationalization, leadership, language skills, intercultural relationship,... However not so much attention has been paid to specific and technical competences related to scientific and technological subjects.

There have been a number of studies on general competencies in Mathematics that students should have in careers in Science, Technology, Engineering, and Mathematics (STEM) but despite this, the fact is that students in many western countries are falling behind other international students in preparedness for college and in STEM careers. Even in some countries student trends show a loss of interest in pursuing this type of education – perhaps the situation in Spain is a little better because of the good reputation that in general terms all Engineering degrees do have, particularly Aeronautical Engineering. Anyway, it is not strange that in fact Mathematics is a subject that often either encourages or discourages students from pursuing degrees in STEM.

In this paper we aim to address this issue and the advances developed within our practical experience to evaluate competences in a first year of Mathematics Engineering. This experience has taken place at the School of Design Engineering ETSID at Universitat Politècnica de València (Spain) in order to increase and facilitate the final Mathematical competence outcome in the students of its BEng Aeronautical Engineering.degree. It might be extended to other degrees by just mimicking main guidelines that we have herein designed which are based upon two basic principles: continuous evaluation and integrating lab work as a fundamental part of the subject.

## **1 COMPETENCES RELEVANCE**

### **1.1 The concept**

In recent years there has been a wide debate in the engineering community trying to fix the competences needed for current and future engineering practice, [1, 2, 3]. In fact there are different perspectives about the origin of competences, with backgrounds in diverse areas such as linguistics, cognitive psychology, business organization, management development, and education, [4].

A thorough overview discussing these competences and even competencies may be found at Section 2 of [5]. Nevertheless for the sake of completeness let us recall that although these concepts are commonly used in the educative area in some European countries there exists an increasing criticism about the ambiguity of its use and the lack of a theoretical frame supporting it, [6, 7].

In general terms, competences are understood to be indicators of knowledge, skills and capacities in the academic world, whereas employers and economists associate them with performance, productivity, efficiency, and professionalism [8]. On the other hand competencies seem to refer to the description of the knowledge, skills, experience, and attributes necessary to carry out a defined function effectively. That is, competency tends to be related to a description of behaviour, while competence to work tasks or job outputs. In this line we are lean to use competence and competences referring to broad capacities; and competencies referring to a narrower concept to label particular abilities.

## 1.2 Mathematical competence and outcome

From [9] we get that Mathematical competence is given as the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations (see Recommendation of the European Parliament, Council of 18 December 2006 on Key Competences for Lifelong Learning (2006/962/EC). Hence, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, charts). Essential knowledge, skills and attitudes related to this competence includes a sound knowledge of numbers, measures and structures, basic operations and basic mathematical presentations, an understanding of mathematical terms and concepts, and an awareness of the questions to which mathematics can offer answers. An individual should be able to reason mathematically, understand mathematical proof and communicate in mathematical language, and to use appropriate aids. A positive attitude in mathematics is based on the respect of truth and willingness to look for reasons and to assess their validity.

A deep insight on this topic was developed by SEFI's Mathematics Working Group (MWG) [10] to provide a discussion forum and orientation to those who are interested in the mathematical education of engineering students in Europe which had into account previous work of the Danish KOM project that set up a list of eight competencies [11]:

- 1.- Thinking mathematically. This competency includes the recognition of mathematical concepts and an understanding of their scope and limitations as well as extending the scope by abstraction and generalization of results.
- 2.- Reasoning mathematically. This competency includes the knowledge and ability to distinguish between different kinds of mathematical statements (definition, if-then-statement, iff-statement, etc.).
- 3.- Posing and solving mathematical problems. This competency comprises the ability to identify and specify mathematical problems and to solve them.
- 4.- Modelling mathematically. This competency includes the ability to analyze and work in existing models as well as the ability to perform active modelling..
- 5.- Representing mathematical entities. This competency includes the ability to understand and use mathematical representations.
- 6.- Handling mathematical symbols and formalism. This competency includes the ability to use and manipulate symbolic statements and expressions according to the rules, as well as its relation to natural language.
- 7.- Communicating in, with, and about mathematics. This competency includes the ability to understand mathematical statements made by others and to express oneself mathematically in different ways.
- 8.- Making use of aids and tools. This competency includes knowledge about the aids and tools that are available as well as their potential and limitations.

These competencies intersect quite often, e.g. to express oneself by using symbols one needs to handle mathematical symbols. According to [10] they can be clustered into two groups since the first four ones deal with the ability to ask and answer questions while the last four ones deal with managing mathematical language and tools.

Learning outcomes regarding knowledge and skills have been presented in detail in [10] where they are arranged in a structure which has four levels, starting from a Core Zero and going through Levels 1, 2, 3 so that mathematics can be linked to real applications with a greater sophistication as the student progresses through the engineering degree programme. In [12] we may find the efforts done in a given country to recruit students and delivering programmes of study within the Science, Technology, Engineering and Mathematics (STEM) disciplines. Finally let us mention that there are a number of examples in which attention has been paid to improve the outcome in Mathematics in Engineering students [13, 14, 15].

### **1.3 The ETSID setting**

We have stated in the introduction that this experience might be extended to other degrees. This being true might require nevertheless enjoying an environment where teaching innovation is encouraged and supported and most important where all staff is involved thus creating an environment where the use of new technologies and new methods of evaluation arise easily. In fact the School of Design Engineering ETSID has constantly kept as a keystone that its educational system is based upon three pillars: teaching innovation, international relations and relationship with the industry.

On the other hand, ETSID joined the High Academic Performance Groups Programme (in Spanish, the so-called ARA “Alto Rendimiento Académico” groups) [16], established by the Regional Government of Valencia (Spain) during 2010/2011 with the aim of supporting and attracting the brightest students with the best skills so that they can achieve a high academic performance. This programme embraced five degrees at UPV from which two of them Aerospace Engineering, and Industrial Electronics and Automation Engineering belong to ETSID. In what concerns Mathematics I the only difference has happened to be that the ARA group had English as mean of instruction.

## **2 EVALUATING COMPETENCES IN A FIRST YEAR OF MATHEMATICS AT AERONAUTICAL ENGINEERING**

### **2.1 Competences to be achieved**

According to [11], the material to be covered at cit. is called Level 1 is advanced enough for simple real engineering problems but cannot be regarded as essential for every engineer. Different disciplines may select different topics from the material outlined there and different disciplines may select different amounts of material from Level 2. This is the case of Aeronautical Engineering which is a discipline which is more mathematically based and requires their students to study more Level 2 topics than other disciplines less mathematically-based.

In fact the following competences corresponding to Mathematics I at BEng Aerospace Engineering are addressed at Level 1 and 2 of [11]. No Level 3 is achieved since this subject is designed for first year students there being other subjects to follow (Mathematics II and III) which deal with advanced differential equations, Laplace transforms, partial differential equations, complex variables and numerical methods.

The competences to be achieved correspond to calculus with one real variable, linear algebra, calculus with several real variables and an introduction to ordinary differential equations and complex functions. They include:

- Understand and apply the basic results in the approximation of real functions of a real variable including the hyperbolic functions and their inverses.
- Operate properly with complex numbers in various representations. Find exponential and complex logarithms.
- Define the roots of equations and find them by numerical methods.
- Taking advantage of the properties of the matrix calculation to solve systems of linear equations.
- Calculate determinants and apply them to concepts related to the structure of a vector space.
- Compute scalar product, distances and angles and apply them to least squares.
- Diagonalize endomorphisms.
- Know the basic results about the integration of functions of one variable.
- Calculate areas of surfaces of revolution, some volumes of solids and lengths of curves as applications of the integral.
- Use polar coordinates when convenient on the above applications.
- Approximate integrals using trapezoidal and Simpson rules.
- Know the basics about improper integrals; in particular, the convergence criteria and their application in specific cases.
- Handle power series properly with derivatives and integrals.
- Fourier series of periodic functions. Interpret the expansions in the extreme points
- Interpret the functions of two variables as a surface and the meaning of the partial derivatives.
- Calculate partial derivatives of a function of two and three variables; apply the chain rule.
- Understand and apply the implicit function theorem.
- Find extreme values and classify critical points.
- Find and recognize equations of elementary surfaces such as cylindrical and revolution surfaces
- Find tangent planes and normal lines to surfaces and tangent vectors to curves.
- Calculate double and triple integrals in Cartesian coordinates and by making use of change of variable; in particular, in polar, cylindrical and spherical coordinates.
- Calculate line and surface integrals and its applications
- Recognize conservative fields when evaluating line integrals.
- Apply the classical theorems of Vector calculus: Green, Stokes and Divergence.
- Calculate works, lengths, areas, volumes, masses, averages, centers of gravity, moments of inertia and Flux integrals.
- Solve basic ordinary differential equations and apply them to model and solve some physical and geometric problems.
- Learn how to use Mathematica software that facilitates to find, visualize and verify the results of technological problems.

Lab Mathematical classes are integrated in the subject so that they help to understand and handle the methods and concepts studied or self learnt with adequate material provided to the student.

## 2.2 Continuous evaluation

In order to ensure that outcomes are achieved and continuously improved a new system of evaluation was implemented during 2011/12 by means of which continuous evaluation is used in the sense that some exams do contain specific topics previously covered in other partial exams and by given opportunity to improve previous grades when better grades are obtained in ulterior topics that rely on other topics previously studied. The final grade relies on all competences achieved in the practical and theoretical part (TP) as well as the lab practical classes (PL).

A final positive grade FG may be achieved once all outcomes in TP and PL are obtained enabling them to solve the mathematical problems that may arise requiring the topics covered in this subject. This positive grade is based on the competences achieved in the different parts of the subject with the following given weights that are proportional to the amount of hours spent on them:

- Theory/Problem solving TP: 75% split in
  - (Act) Activities : 10%
  - (C1) Calculus I: 25%
  - (A) Algebra: 21%
  - (C2) Calculus II: 31%
  - (S) Series: 13%
  
- Lab practical classes PL: 25% from which: a) Weekly assessment through individual work at class means a 10% and b) The remaining 15% comes from two individual exams with Mathematica, ExPL1 and ExPL2, (one at the end of each semester): 60%. ExPL1 covers C1 while ExPL2 covers A and C2 by means of questions ExPL2a and ExPL2c, respectively.

Continuous evaluation was applied in the sense that previous grades were improved when either in other topics that rely on previous ones or through specific questions, the students show that they have improved in the outcome of previously studied topics under the condition that all assigned activities were duly done and had received a satisfactory evaluation.

This improvement may mean to increase the previous grade in half with the difference with ulterior grade in such a way that students are aware that they may improve their grades by working hard but at the same time they know that there is a memory of what has previously been done. The paths that are taken into account in order to check how competences are evolving are:

C1 -> C2 -> ExPL2c; A -> ExPL2a; ExPL1 -> ExPL2

Furthermore students may take some optional global exam TG at the end of the subject in which case they will add or subtract the input of TP in the final grade by a 20% of the difference between TG and TP and if they may pass the subject if in this way FG is greater than 5 or if they have achieved the competences related to the use of Mathematica and prove to have achieved them through the different requirements of the exam TG in which case they would assure at least the minimum to pass the subject. The idea is that they are being continuously evaluated, that there is a memory on what they have done, that they may improve it but a good grade by just doing a good final exam is not possible.

## 2.3 Results

The performance of students at the BEng Aerospace Engineering degree with a total of 119 students from which 50 belonged to the ARA group may be considered highly rewarding.

It must be pointed out that the experience developed as described in the previous section at ETSID in Mathematics was developed simultaneously with the ARA and non-ARA group, the only difference existing between both of them was just teaching was conducted in English for ARA students and in Spanish for non-ARA students. Fig 1 gathers the grades obtained by students from both groups.

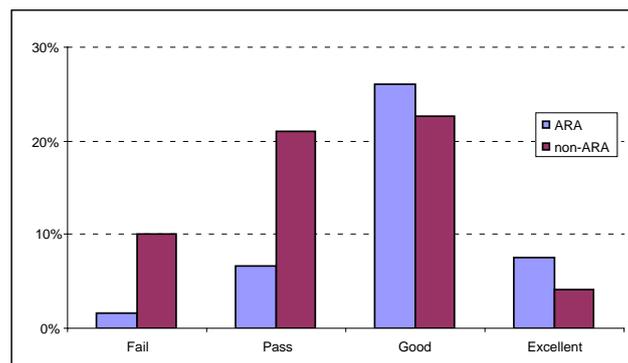


Figure 1. Maths grades in Aerospace Engineering at ETSID 2011/12

From this figure let us point out that 42% of students belonged to the ARA Group while 58% belonged to the non-ARA group. The percentage of students that reached the Fail, Pass, Good and Excellent grades (in Spanish Suspenso, Aprobado, Notable, Sobresaliente) is numerically given in the following table:

Grade	Fail	Pass	Good	Excellent
ARA	2%	7%	26%	8%
non-ARA	10%	21%	23%	4%

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