

## Understanding phase as a key concept in physics and electrical engineering

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### INTRODUCTION

*No one who has used a shower in a student residence can fail to become aware of the problems caused by the fact that the temperature of the water does not respond instantly to the controls. First the water comes out cold so you turn the hot tap on full, only to be scolded ten seconds later. The natural response produces icy cold water after ten seconds of agony. Shivering you then turn the hot tap full on. [1]*

Most of us recognise Körner's [1] experience. Turning the controls for hot and cold water and the resulting temperature of the water are often not in *phase*. In all systems where there are time delays in the response of the system, such as in the example of the shower, it is important to understand phase and its related concepts. In control theory, it is essential to understand the concept of phase to understand the stability and responses of systems. This means that not only in engineering but, for example, in systems biology and economics it is important to know and understand that responses are commonly not in phase with the effect causing the response.

### 1 THE IMPORTANCE OF UNDERSTANDING PHASE, PHASE SHIFT AND PHASE DIFFERENCE

Most of us have, at one time or another admired the beautiful coloured patterns in the feathers of peacocks. However, these patterns are not the result of pigmentation but are actually caused by interference due to the physical architecture of the feathers at a nano scale level [2]. Biological systems have been exploiting nanometre-scale architectures to produce striking optical effects for millions of years [3].

To understand interference, one has to understand the physical meaning of phase (or phase angle). Suppose we, at a certain point in space, have two waves (which could be mechanical, such as sound, as well as electromagnetic, such as light) that can be described by the equations

and  
. The phase of these waves is described by and ; the amplitude by and  
. The amplitude of the resulting wave cannot be obtained by

simply adding the amplitudes: the phases must also be considered. If  $\phi = 0$ , we have constructive interference and the resulting wave will have maximal amplitude  $2A$ . If, instead,  $\phi = \pi$ , we have destructive interference and the resulting wave will have minimal amplitude  $0$ . These conditions could be generalised to cases where we have more than two superposing waves. If the conditions for constructive and destructive interference are dependent on frequency (i.e. different colours) we would see the beautiful colour patterns present in, for example, peacock feathers, butterfly wings or soap bubbles.

Differences in phase can be caused by wave sources that are not in phase with each other, differences in the distances the waves have travelled, phase shifts due to physical processes such as reflection at certain boundaries or the phase shift caused by reactive circuit elements, or a combination of these. These effects explain the mechanism behind optical interference coatings, other interference phenomena and diffraction.

An example of phase shift in electric circuits is shown in fig. 1 below. The circuit consists of a voltage source  $U_0$  producing a sinusoidal voltage and current, a resistor  $R1$  and an inductor  $L$ . This is a series circuit and hence the same current goes through all circuit elements. For the resistor, current and voltage will be in phase since  $\phi = 0$ . However, for the inductor,  $\phi = \pi/2$  and hence there will be a phase shift of  $\pi/2$ .

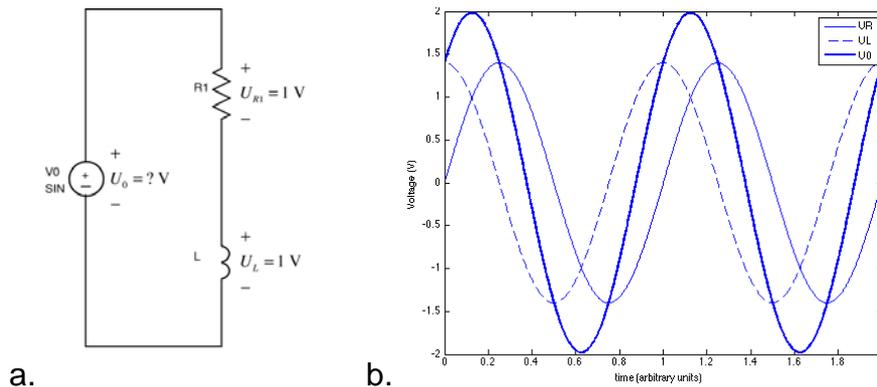


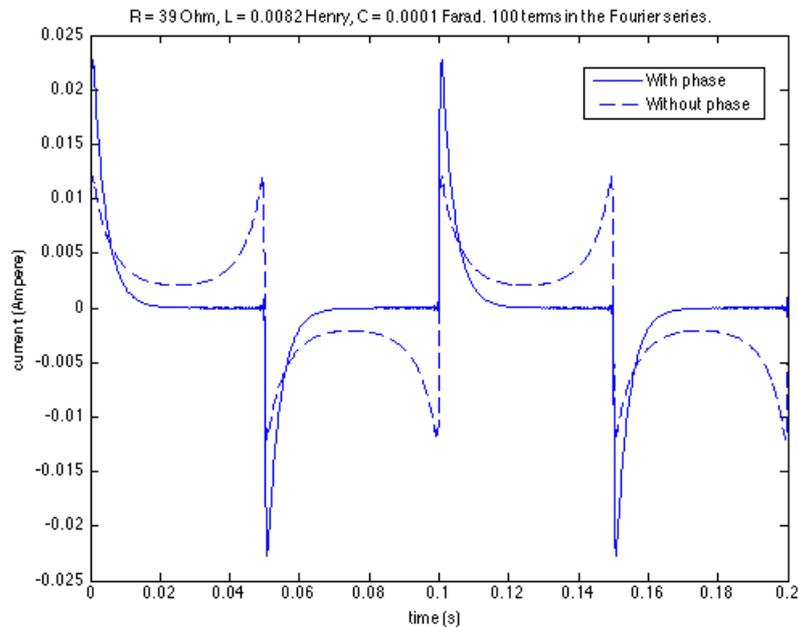
Fig. 1. a) Electric circuit with a resistor and an inductor. The voltage source applies an AC (sinusoidal) voltage and current to the circuit. Shown are the readings of voltmeters measuring rms voltages. b)  $U_R$ ,  $U_L$  and  $U_0$  as time functions. Since actual frequency is not used in the test, arbitrary units are used for time.

In fig. 1a, the readings of voltmeters measuring the rms value  $\bar{U}$  are shown. If the phase of the current is taken as zero this means that  $\bar{U}_R = 1 \text{ V}$  and  $\bar{U}_L = 1 \text{ V}$ . In this case, the voltage supplied by source  $\bar{U}_0$  can be shown to be  $\bar{U}_0 = \sqrt{2} \text{ V}$ . That means that an rms voltmeter measuring  $U_0$  will display 1.4 V. However, when electrical engineering students were asked in a test what the voltage was (see below), about 80-100% of them simply added the rms values and proposed 2 V. In fig. 1b, the waveforms of  $U_R$ ,  $U_L$  and  $U_0$  are shown.

However, phase is not only important in the treatment of sinusoidal signals. A periodic signal  $y(t)$  with angular frequency  $\omega$  can be expressed as a Fourier series

where  $\omega$ ,  $\phi$ , and  $\theta$  depend on (and are unique for) the actual periodic signal (for example square wave, triangle or saw-tooth) investigated.

In fig. 2 are displayed the current through a RLC-circuit as the result of an input voltage in form of a square wave. Calculations using Fourier series considering the phase of each component as well as calculations only considering magnitude, and not the phase, of each Fourier component are shown. As can be seen there are drastic differences showing that phase cannot be neglected.



*Fig. 2.* Results of a calculation of the current through an RLC-circuit with the use of Fourier series. In this case the input voltage  $v(t)$  is a square wave and the result considering the phase of each Fourier component is shown as a solid line (corresponding very well with experimental values) and neglecting phase as a dashed line.

Considering and understanding phase is also important for power calculations in AC-circuits and for understanding three-phase-AC systems and in many other applications.

## 2 LEARNING PROBLEMS RELATED TO PHASE IN THE LITERATURE

Linder [4] Investigated students' understanding of sound and found that it was common to believe "changing particle displacement, changing sound pressure and changing molecular velocity all to be in phase". In a similar vein, Sadaghiani and Bao [5] found that "students [lacked] understanding of phase, phase difference, and the relation between phase and path-length difference [in the context of] mechanical waves, sound and light interference".

We have, in earlier studies, found before that Swedish engineering students had difficulties understanding phase, phasor representations [6] and Bode plots [7] in the context of AC electricity. Similar examples are Kautz [8], who found that German engineering students had conceptual problems related to phasor notation and phase relationships in AC electricity, Mazzolini, Scott [9] found that Australian engineering and science students had conceptual difficulties related to phase in resonance (RLC) circuits, Scott, Harlow [10] found that phase and phasors were threshold concepts in

the context of an electronics course in New Zealand and Flanagan, Taylor [11] who identified that UK engineering students had difficulties in understanding characteristic impedance and reactive power in the context of transmission lines.

### 3 METHODOLOGY

This study is part of a larger set of studies investigating (mainly) engineering students' understanding and learning of electric circuit theory by means of video recordings in labs, lab-reports and answers to exams, interviews, questionnaires and conceptual tests.

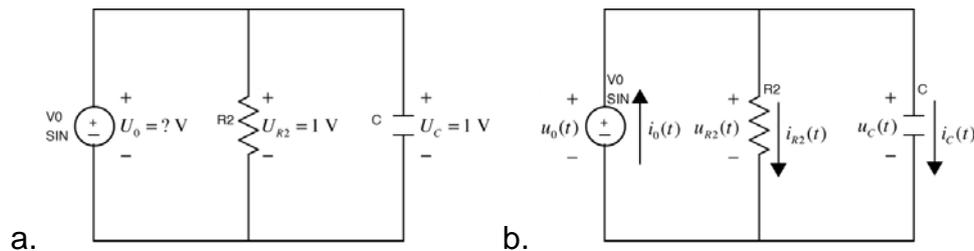


Fig. 3. Examples of circuits used (see also fig. 1a) in our test of conceptual understanding of phase.

In the empirical part of this paper, we present the results from a test, developed by ourselves, intended to probe students' conceptual understanding of phase in AC electricity. The test consists of 29 multiple-choice questions. In the first part, 6 questions test if a student takes account of phase when adding voltages or currents in series and parallel circuits. One such question is shown in fig. 1a and another in fig. 3a. In the second part, consisting of 23 questions related to 3 different AC circuits, students are asked to compare phases of voltages and currents i.e. to answer if they are in phase, not in phase, or if it is impossible to determine. Fig. 3b shows an example (this circuit adapted from [8]). The phase test was administered, at two different occasions, on the first day of class in a 3<sup>rd</sup> year course in advanced circuit theory for electrical engineering students and a week after the final exam of a traditionally taught 1<sup>st</sup> year introductory electric circuit theory course.

Table 1. Results related to the circuits shown in fig. 1a and 3a.

Circuit	Figure 1a				Figure 3a			
	1 <sup>st</sup> year		3 <sup>rd</sup> year		1 <sup>st</sup> year		3 <sup>rd</sup> year	
Student group	Ia	Ib	IIIa	IIIb	Ia	Ib	IIIa	IIIb
<i>N</i>	20	22	29	17	20	22	29	17
Correct answer	0%	0%	4%	0%	85%	90%	78%	88%
Arithmetic addition	85%	86%	78%	100%	5%	5%	13%	6%
Other or no answer	15%	14%	13%	0%	10%	5%	9%	6%

### 4 RESULTS

The results for the questions relating to the circuits shown in fig. 1a and 3a respectively are shown in table 1. It can be seen that for circuit 1a, there is a strong tendency for students to simply add the voltages without considering the phase relationships. In other questions in the test, also requiring students to take phase into account, 70 – 80% of the students quite consistently neglected phase and did 'pure

arithmetic' addition of voltages or currents. On the other hand, for the circuit in figure 3a, most students realised that voltages should not be added and  $U_0$  must be the same because the other voltages are in *parallel*.

Table 2 shows the results for some of the phase comparisons of voltages and currents in figure 3b. The results from one of the groups (HAW-EE) in the study by Kautz [8] from Germany is also included (other groups had similar results). Since the circuit elements are in parallel,  $u_0(t)$ ,  $u_{R2}(t)$  and  $u_C(t)$  should all be in phase. On the other hand,  $i_0(t)$ ,  $i_{R2}(t)$  and  $i_C(t)$  are not in phase due to the phase shift of the current  $i_C(t)$  through the capacitor C. Since the voltage across and the current through a resistor is always in phase, there seems to be a strong idea that the source voltage  $u_0(t)$  and current  $i_0(t)$  should be in phase with the voltage across and current through the resistor. In the case of the currents in this parallel circuit, this (local) reasoning causes the students to draw the wrong conclusion for the relationship between the phases of  $i_0(t)$  and  $i_{R2}(t)$ . In a test item related to a series circuit (not shown in this paper), the students drew similar incorrect conclusions.

Table 2. Percentage of correct answers for phase comparisons (circuit in fig. 3b) after traditional instruction.

Comparison of phases for	HAW-EE	LiU 1 <sup>st</sup> year		LiU 3 <sup>rd</sup> year	
		Ia	Ib	IIIa	IIIb
$N$	49	20	22	29	17
$u_0(t)$ and $u_{R2}(t)$	86%	80%	73%	96%	88%
$u_0(t)$ and $u_C(t)$	43%	25%	27%	35%	29%
$i_0(t)$ and $i_{R2}(t)$		35%	36%	17%	18%
$i_0(t)$ and $i_C(t)$		75%	73%	61%	59%

## 5 DISCUSSION AND CONCLUSION

Our results in this study, observations by us as teachers of physics, electric circuit theory and control theory as well as our data from our earlier studies [e.g. 6, 7, 12, 13] and that of others as discussed in the literature show that phase is, indeed, a troublesome concept. Furthermore, from the discussion in the introduction, it is easily understood that the concept of phase opens up "seeing things in a new way" that were "previously inaccessible". Hence, it is quite natural to see *phase* as a candidate for being a threshold concept in many disciplines [14].

Phase is also closely related to phasors. By using phasors, a signal can be represented by a complex number conveying information about magnitude as well as phase. is called the phasor<sup>1</sup> of and the use of phasors simplifies many calculations and enables a geometrical representation of the magnitude and phase of, for example, light, AC voltages, AC currents or other waves and oscillations. It is commonly used in many branches of science and engineering. However, complex numbers have also been suggested as

<sup>1</sup> The phasor use that can be seen as . If all signals have the same frequency the term can be neglected in calculations. Using this representation for example the addition of signals (with the same frequency) correspond to the complex number addition of the corresponding phasors, derivation correspond to the multiplication of and integration with the division by .

being a threshold concept [14]. Although, for example, the Danish surveyor Caspar Wessel [15] proposed more than 200 years ago that complex numbers could be seen as points on a plane and could be used for scaling and rotating data used for map-making, they are often still mystified. An example of this is Lucky [16] who wrote “I remember ... when I was first introduced to imaginary numbers. The teacher said that because the square root of a negative number didn’t actually exist, it was called imaginary.” However, instead of showing that the multiplication with a complex number in general corresponds to a rotation (or in our case, a phase shift) and especially multiplication with  $j$  corresponds to a rotation (or phase shift) of  $90^\circ$  (using the accepted definition in mathematics) many textbooks describing, for example electric circuits, *wrongly* present  $j$  as being defined as  $\sqrt{-1}$ , something that is often perceived as being absurd [cf. 17].

Because of the different meaning of imaginary and real as *technical* mathematical terms and their meaning in everyday language it is still common to find confusing and mystifying descriptions. An example of this is Hadamard [18] who explains that “the shortest and best way between two truths of the real domain often passes through the imaginary one” and Meyer et al. [19], who claim that complex numbers are “troublesome [because] they can be used in calculations in an ‘imaginary’ world to yield results that are meaningful in the real world”.

Earlier, we suggested that a distinction should be made between threshold concepts and ‘key concepts’. We use the term ‘key concept’ as a more precise metaphor to mean that the “concept in question acts like a key to unlock the ‘portal’ of understanding, the ‘portal’ which opens up the learning of other concepts” [20, p. 143]. We argue that if we want to open up learning, it is as important to look for the keys, as it is to identify the thresholds. If we only identify thresholds and maybe even talk of them as something “apparently absurd” [19, p. 65], there is a danger that we accept the threshold as something we ‘must’ stumble on. By demystifying complex numbers, Wessel [15] and others gave us a key to a new understanding of these, an understanding that also opens up a wider understanding of phase, phase shift and phasors. Nevertheless, given the importance of phase in many disciplines as discussed in the introduction, we propose that phase is a key concept regardless of our views on, and the understanding of, complex numbers.

We plan to continue this study along two lines: 1. We intend to extend this study and do in-depth interviews with students on related to their understanding of phase, phase shift and phasors. We plan especially to investigate if the representational format matters. 2. We intend to re-design the introductory electric circuit theory course using our previous experience re-designing an advanced electric theory course [21] and physics courses [22, 23] using variation theory as a theoretical framework for curriculum design [24].

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