Student Understanding of Electric Circuits and their Representations

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INTRODUCTION

Courses on circuit analysis for students in electrical or mechanical engineering often focus on algorithms for solving circuit problems that are presented in the form of standard circuit diagrams. Research has shown that many students have difficulty developing a conceptual understanding of basic concepts such as current and voltage. At Hamburg University of Technology, we have begun to investigate to what extent students are able to recognize the model aspects of (linear) circuit theory, especially how well they are able to understand and work with the formalism that is used to describe these model aspects. It is our hope that a deeper understanding of students' difficulties and misconceptions will enable us to develop better learning materials.

After a brief summary of our methodology and research methods, we will present here the questions and tests used for our investigation and highlight some general results. In the two subsequent sections, we will discuss several common student difficulties that we have observed and then draw some conclusions.

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1 CONTEXT OF INVESTIGATION

1.1 Methodology

In our research, we mainly investigate conceptual understanding of students, as in our opinion, it is fundamental for understanding and imperative to reliably apply one's knowledge. To reach this kind of understanding, students have to develop concepts for themselves, ideally guided by an instructor and, possibly, other learning aids. It is in this sense that our work is ultimately based on the idea of constructivism.

Our approach has been strongly influenced by work on misconceptions held by students, as e.g. carried out by the Physics Education Group [1]. We try to find empirical evidence for such patterns of reasoning in students' statements. This distinguishes our work from some branches of threshold concept [2] research, which often seem to use non-empirical means, and from the Decoding the Disciplines [3] approach as our focus is solely based on evidence gained from *student* statements.

1.2 Method

In this paper, we present initial results from semi-structured interviews and written quizzes.

For the interviews, we prepared several questions that were then presented to all students in the same order. We instructed the interviewees to "think aloud". In order to get a fuller picture of a student's notion of a concept, we often asked further, spontaneous questions. The interviews were recorded on audio and video.

The quiz questions were either multiple-choice or ranking questions. Following their answer, students were asked to explain their reasoning in a text box provided. The quizzes were not mandatory and hat no influence on the students' marks. Since they were conducted during the lecture, virtually all students present that day took part.

2 QUESTIONS USED TO PROBE STUDENT UNDERSTANDING

This section presents some of the questions the students were asked. The first four subsections give an overview of the interviews, the last two subsections show the quiz questions. For all questions, an overview of the students' answers is given.

In 2013, we conducted semi-structured interviews with students at the end of their first semester. These interviews were done one to four weeks before the exam, after all relevant instruction had been completed. Accordingly, some students had started reviewing the material while others had not. The interviewees belonged to two different populations: Students attending an electrical engineering lecture (EE) for majors on the one hand, and students attending an electrical engineering lecture (ME) for mechanical engineers on the other. While in previous studies, the students participating in such interviews were clearly above average [4], the incomplete data we have on the students' grades in this interview series seems to indicate that they are representative cross-section of all students in that semester.

$$
\begin{array}{rrrrl}
\text{(I)} & x & +2y & & = 15 \\
\text{(II)} & 2x & & -z & = 2 \\
\text{(III)} & x & -y & +2z & = 5 \\
\text{(IV)} & 2x & & -z & = 12
\end{array}
$$

Fig. 1. System A

$$
\begin{array}{rrrrl}
\text{(I)} & & +y & +2z & = 14 \\
\text{(II)} & -x & +2y & & = 9 \\
\text{(III)} & 2x & +y & -z & = 8 \\
\text{(IV)} & -x & +2y & & = 9
\end{array}
$$

Fig. 2. System B

2.1 System of Linear Equations

We began each interview with a question on the solvability of systems of linear equations. Two systems with 3 variables and 4 equations each were given. System A (*Fig. 1*) included two *contradicting* equations (identical except for the right hand side) and thus was unsolvable. System B (*Fig. 2*) included two *identical* equations and had a unique solution. The interviewees were asked if only system A, only system B, neither, or both of the systems were solvable.

Five (5) of the 10 interviewees answered this question correctly, two (2) others initially said that both systems were solvable but then changed their answer to the correct one after examining the equations in more detail. Three (3) students stated that both systems were solvable. Some of the students tried to solve the system explicitly for x, y, and z. Those, we asked to compare instead the equations and answer based on their observations.

2.2 Voltage Sources in Parallel

Further questions in the interviews were about the DC network shown in *Fig. 3*. Besides questions concerning the current through and voltage across R_3, we asked the students if certain changes to the network would result in valid circuit configurations. We inquired about these changes in order to learn about the students' understanding of current and voltage sources as idealized components, as well as their understanding of the rules inherent in the electric circuit formalism.

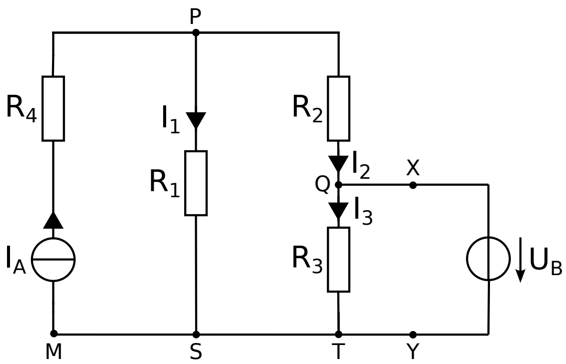

Fig. 3. DC network with one voltage and one current source.

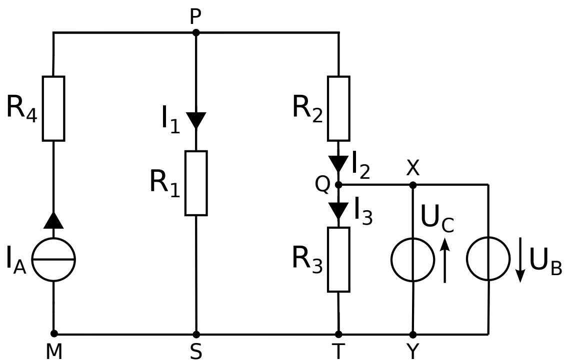

Fig. 4. DC network from *Fig. 3* with a second voltage source.

The first variation of the network was a proposed second voltage source U_C inserted between points X and Y, as seen in *Fig. 4*. The students were first asked if the source could be inserted so that the currents I_1, I_2, and I_3 would not change. Subsequently, we asked the students whether they would change their answer if the three currents were allowed to change.

Five (5) of the 9 students asked stated correctly that it was not possible to have two parallel voltage sources with different source voltages. However, only 3 students described the effect of this second voltage source on all relevant currents correctly.

2.3 Current Sources in Series

Additionally, the students were asked if it would be possible to insert a second current source I_D between points S and M in the circuit of *Fig. 3*, and, if so, what values this current source was allowed to have if (a) the currents I_1 through I_3 were not to change, and (b) all currents were allowed to change. A diagram (not shown here) was provided.

Five (5) of the 9 students we asked this question incorrectly answered that the source currents of current sources in series would add up, two (2) others also suggested this but were unsure. Only 2 interviewees answered correctly that two current sources in series must have the same value.

2.4 Circuit Diagram of a Short-Circuited Battery

Six (6) of the students in the interviews were shown a battery and wire. They were asked to draw a circuit diagram consisting of idealized components if the wire were connected to both poles of the battery, as shown in *Fig. 5*.

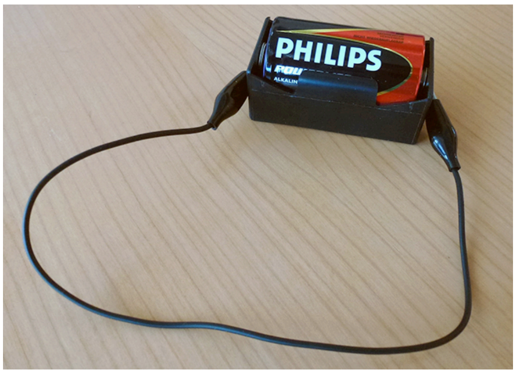

Fig. 5. Real-world circuit (“short-circuited” battery).

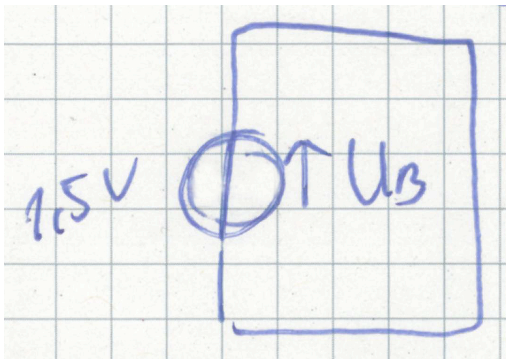

Fig. 6. Circuit diagram constructed from ideal components of the setup in *Fig. 5*, drawn by a student in an interview.[2]

All 6 students drew a short-circuited voltage source (as e.g. the one in *Fig. 6*), although 2 were unsure whether or not the battery should be considered a voltage or a current source and additionally drew a short-circuited current source. While all students noted that this setup was problematic, their reasoning always focused on the current in the wire, which 5 students considered to be infinite and one student could not determine. No student mentioned that this setup violated the mesh-rule. One student, whom we made aware of this problem, recognized this but was unable to modify the circuit (e.g. by inserting a resistor) for it to become a valid one.

2.5 Five-Bulbs Test

At the beginning of the summer semester 2013, we gave a short quiz in the EE second-semester lecture. The quiz contained the so-called five-bulbs test first used by McDermott and Shaffer [1]. In this test, the students are given the circuit configuration in *Fig. 7* and asked to rank bulbs A through E according to their brightness, assuming the batteries behave like ideal voltage sources.[3]

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[2] The image was slightly retouched to improve readability.

[3] In contrast to previous iterations, we did not specifically request the students to indicate whether or not two bulbs are equally bright, e.g. by using the signs “>”, “<”, and “=”. 264 students handed in their answers. For about 9% of the answers, we were unable to tell which, if any, of the bulbs were meant to be equally bright. These students only gave a comma-separated list like, e.g. “A,D,E,B,C”. In the following data, these answers are included, always interpreted in favour of the correct answer.

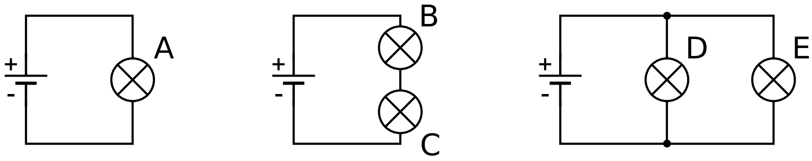

Fig. 7. Five-Bulbs Test

Thirty-five per cent (31%) of the 264 students gave the correct answer “A=D=E>B=C”. 26% of the students said B had a different brightness than C and 48% of the students answered that A would be brighter as D and/or E.

2.6 DC-Circuit Question

In the same quiz, we also gave a question about the DC-circuit-diagram shown in *Fig. 8*. First, the students were asked to rank bulbs F through I according to their brightness. Then, they were asked if bulb H and F would get brighter, dimmer, or stay equally bright if switch 1 was closed.

Only 26% of the students answered correctly that after closing switch 1 bulb F would get brighter but bulb H would stay equally bright.

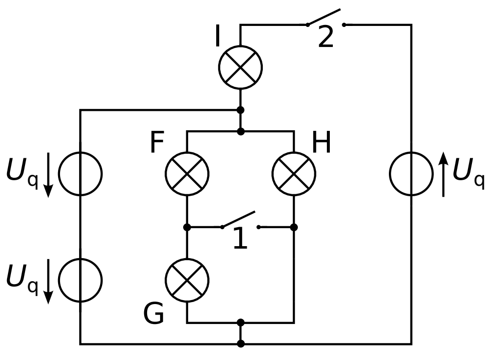

Fig. 8. Circuit diagram used in quiz.

		Topology: Correct	Topology: Incorrect	Σ
Bulbs	Correct	26%	3%	61%
	Incorrect	29%	32%	39%
	Σ	65%	35%	

Fig. 9. Distribution of correct/incorrect answers concerning the change of brightness of bulbs F and H vs. statements about the circuits topology, N=75

3 SPECIFIC DIFFICULTIES WITH ELEMENTS IN THE ELECTRIC CIRCUIT MODEL

In this and the subsequent section, we summarize our observations from the interviews and our analysis of student answers to written questions. Using both methods, we found that many students have difficulty interpreting the electric circuit formalism taught in lecture as an idealized model for real circuits. We will describe these difficulties in greater detail in Section 4. In the present section, we will focus on student difficulties in applying this model.

3.1 Describing the Topology of Non-Trivial Circuits

Seventy-five (75) students who participated in the quiz in Subsection 2.6 answered the question concerning the change in brightness of bulbs F and H and also made spontaneous statements about the circuit’s topology (i.e. which bulbs are in series/parallel to one another) that could be unambiguously categorized as true or false. *Fig. 9* gives an overview of the distribution of their answers.

Incorrect statements about the topology of the circuit shown in *Fig. 9* were made by 35% of the students. The most common among the ideas expressed was that after closing switch 1, bulb H "is in series with G, thus dimmer."[4] (In the responses to the five-bulbs test we did not find any incorrect statements about the topology of those circuits. We attribute this to the simplicity of the respective circuit layouts, see *Fig. 7*.)

Informal in-classroom observation suggests that most students define parallel and series connections based on the current through and the voltage across the respective elements, stating for example that two elements are in series if the current through them is identical. In [5], Johsua reported on student difficulties in determining the current distribution in circuits containing closed loops. These results suggest a possible interpretation of our own observations: Students may have made incorrect assumptions about the current distribution in the circuit in *Fig. 8* and may have based their incorrect statements about the topology of this circuit based on these assumptions.

3.2 Differentiating between Voltage and Potential

As mentioned in Subsection 2.5, 26% of the students who took the five-bulbs test answered that bulbs B and C would not glow equally bright. McDermott and Shaffer attribute this type of response to a "belief that current is 'used up' in a circuit" [1]. In our students' responses, we also frequently find explanations like: "A, B, D, and E have the full voltage. C does not."[5] and "In series, C glows brighter since the voltage decreases 'after C'."[6]

These results suggest that students often confuse potential with voltage, i.e. potential difference, and assume a bulb's brightness to be determined by the potential (which they call "voltage") at one of its terminals.

Answers to the first question on the circuit shown in *Fig. 8*, asking students to rank bulbs F through I, frequently show the same reasoning.

4 DIFFICULTIES WITH CIRCUIT THEORY AS A MODEL

Besides problems applying the circuit model to given circuits, students also displayed various difficulties with the model of electric circuit theory itself.

4.1 Distinguishing Valid from Invalid Configurations of Circuit Elements

As mentioned in Subsection 2.4, all students presented with this question drew a short-circuited voltage source as a circuit diagram for a "short-circuited" battery. While none noticed the violation of the mesh-rule, some mentioned the current becoming infinite. All 6 interviewees stated that this setup would be problematic if it were built with real components, the reason ranging from the battery draining fast to the components actually catching fire.

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[4] Original transcription in German: „Lampe H: Ist nun in Reihe mit G, deswegen dunkler“

[5] Original transcription in German: „An A,B,D,E liegt die volle Spannung an. An C nicht.“

[6] Original transcription in German: „In Reihe leuchtet C heller, da die Spannung ‚nach C’ abnimmt“

When asked about current sources in series (see Subsection 2.3), only 2 of the 9 students recognized that the source current of all sources in series has to be identical, in order for the configuration to be valid. This could in part be attributed to an incorrect understanding of current sources. However, when we posed an analogous question in the context of voltage sources, the results were still not nearly perfect. Just only 5 of the 9 students said that it is not possible to have two voltage sources in parallel when those have different source voltages. This suggests the problem goes beyond a mere lack of familiarity with current sources.

In our opinion, students may often be able to mathematically describe a circuit but still have a lack of understanding how elements in a circuit behave and in what way they may be connected. A statement concerning the question in Subsection 2.2 from one of the interviews illustrates this: *"I can hardly imagine that U_C must only take the value $-U_B$. This does not make sense to me, although, according to the mesh rule, this is true, but (…) I can not imagine that if I connected two batteries in parallel and added a resistor, anything would break."*[7]

4.2 Recognizing the Correspondence between Circuit Diagrams and the Equations Describing them

The intent behind the question presented in Subsection 2.1 was to learn if the students were able to identify contradictory mathematical statements in a context other than but analogous to circuit theory. Only 3 of the 10 students failed to identify the conflict between equations (II) and (IV) in system A (*Fig. 1*). One of the three was very weak overall, preventing us from continuing the interview as planned.

In contrast to this, none of the 6 students we asked to draw the circuit diagram of an short-circuited battery, as mentioned in Subsection 2.4, identified the conflicting information in the circuit diagram of a short-circuited voltage source, i.e. the parallel definition of a potential difference and no potential difference. Similarly, only 2 of 9 students stated in Subsection 2.3 that current sources in series must have the same source current. And only 5 of 9 students stated that parallel voltage source must have the same source voltage. (See Subsection 2.2)

Comparing the students' answers to the mathematics and circuit analysis questions, we believe that many students have problems recognizing the correspondence between circuit diagrams and the equations describing them. They seem to view circuit diagrams merely as pictures illustrating a circuit rather than as schematics based on syntactic rules, by which each element can be assigned an equation describing it and its relation to other elements. While students generally are able to identify contradicting information in mathematics, they often are not in circuit diagrams.

5 CONCLUSIONS

Based on the results presented here and in a previous paper [6] we conclude that many students do not understand and thus cannot appreciate the model nature of the (linear) circuit model in electrical engineering. They fail to distinguish valid from

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[7] Original transcription in German: *"Ich kann mir das auch irgendwie gerade schlecht vorstellen, dass U_C nur den Wert $-U_B$ annehmen darf. Also das macht für mich irgendwie keinen Sinn, nach der Maschenregel schon, aber (…) ich kann mir nicht vorstellen, wenn man (…) das als Batterien sehen würde und zwei Batterien parallel schaltet und dann noch einen Widerstand dran hängt, dass dann irgendwas kaputt gehen würde."*

invalid circuit configurations and are often unable to recognize the correspondence between a circuit diagram and equations describing them.

Instruction mostly focuses on formal description of electric circuits. As prior research has shown, students taught in this way often show difficulty answering conceptual questions posed in everyday language. One particularly interesting result of our work is the observation that even questions posed in a formal context (although still emphasizing conceptual aspects) also pose great challenge to students trained in this manner.

As stated before [6], these results have clear implications for the teaching of electric circuits. If students are to develop an understanding of circuit theory as a model this aspect has to be made explicit during instruction.

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