Integrating a computer algebra system into an engineering mathematics course

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INTRODUCTION

Engineering Mathematics is usually used to provide underpinning knowledge which is useful and often essential for engineering modules later in an undergraduate engineering degree course. It can be the case however that students perceive a distinct boundary between Engineering Mathematics module(s) and engineering modules by failing to link what is being taught in both. Of course, many engineering examples are usually incorporated into the Engineering Mathematics curriculum using the traditional textbook method, but these examples are often quite restrictive in that they do not reflect "real" engineering problems. One way to break out of this mold is to incorporate computer software into the Engineering Mathematics module(s), so providing students with a more realistic way of how professional engineers tackle problems today, increasing student skills, hopefully increasing their interest and finally breaking down the boundary alluded to above.

It is not easy to integrate computer technology into traditional teaching and learning courses [1]. Very commonly the complexity of computer programming, and getting students to a suitable standard of programming can be daunting, and, often the wider ramifications have been overlooked, underestimated or even denied [1]. Many surveys have been carried out [2] to elicit schemes to start tackling some of these difficulties with an emphasis on the integration of computer algebra systems into

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university teaching [3]. What is shown here is a brief description of how a computer algebra system, called *Mathematica* [4], is incorporated into an Engineering Mathematics module. The results of a controlled experiment comparing knowledge gained by a group with *Mathematica* in their course with those who did not, are presented and discussed.

1 INTEGRATION OF *MATHEMATICA*

Mathematica was integrated into the students' engineering mathematics curriculum with the primary aims of enhancing cognitive development [5], exposing students to modern professional practice and to break down barriers perceived by students between engineering mathematics and other engineering subject taught in the degree course.

All students, including the students used as the control group described in Section 2.1 had already been taught the rudiments of *Mathematica* during a 12 week course (1 hour/week) during their previous year of study.

For the course considered here, the engineering mathematics course mainly covered partial differential equations and vector calculus, was 12 weeks in length, with two one-hour lectures and two two-hour tutorials per week. *Mathematica* was used during one of these two-hour tutorials for one of the two groups tested. For each of the *Mathematica* laboratories, a hand-out consisting of the laboratory objective, a narrative (including relevant theory), the task description and additional necessary comments were distributed to the students.

Six *Mathematica* laboratories were used during the twelve week period entitled,

Laboratory 1: To investigate the use of Lagrange multipliers in engineering [6].

Laboratory 2: To investigate the use of Fourier Series as approximations to input functions [6].

Laboratory 3: Solution of the heat equation by Fourier integrals and transforms [7].

Laboratory 4: Exploration of using Mathematica with the Divergence and Stokes' theorems [6].

Laboratory 5: Analysis of a single-span Euler-Bernoulli beam [8,9].

Laboratory 6: Flow over and downstream of a cylinder [7].

Students were asked to write short reports on each of these laboratories and two long reports on two of the laboratories.

2 TEACHING AND LEARNING EVALUATION

The evaluation process was in the form of an experiment comparing the knowledge of the group with *Mathematica* in their course with those of a control group using only the conventional text book approach.

2.1 Controlled Experiment

To investigate the effectiveness of introducing *Mathematica* laboratories into the Engineering Mathematics course, a controlled experiment applying a pre-test-post-test control group design was conducted [10, 11]. The students had to undertake two tests, one before the respective course (pre-test) and one after the respective course (post-test) with the introduction of *Mathematica* laboratories being evaluated by comparing within-student post-test to pre-test scores, and by comparing the scores

between students in the *Mathematica* group (A), i.e. those who were taught using the course containing *Mathematica* laboratories, to those students in the control group (B), i.e. taught using the conventional method of lectures and tutorials only. The various possibilities of the methods of teaching are summarized on *Fig. 1*.
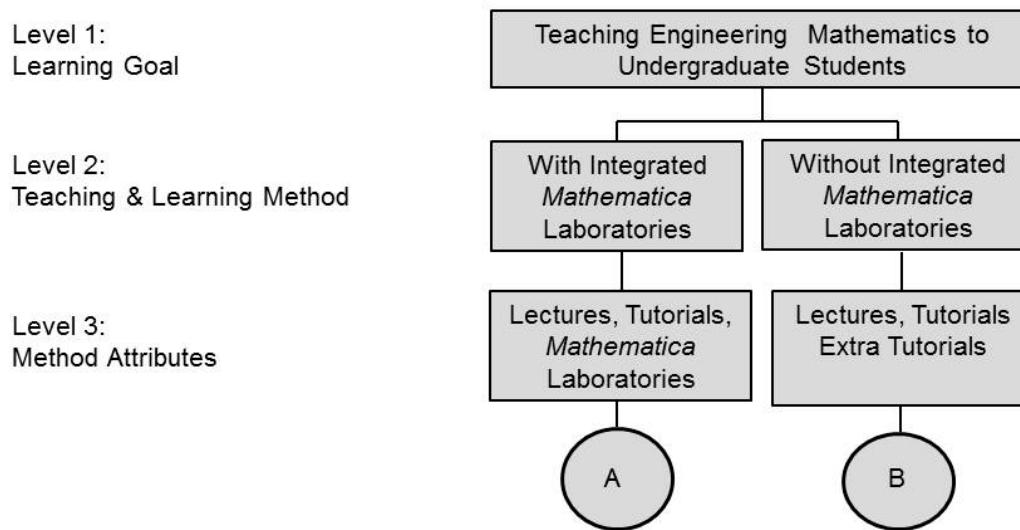

Fig. 1. Course arrangements

To measure the performance of the two groups, four constructs were used with each construct represented by one dependent variable. Each dependent variable has the hypothesis,

1. There is a positive learning effect in both groups (A: *Mathematica* group, B: control group). This means post-test scores are significantly higher than pre-test scores for each dependent variable.

2. The learning is more effective for group A than for group B, either with regard to the performance improvement between pre-test and post-test (the relative learning effect), or with regard to post-test performance (absolute learning effect). The absolute learning effect is of interest because it may indicate an upper bound of the possible correct answers depending on the method of teaching.

The design starts with random assignment of students to the *Mathematica* group (A) and control group (B) with the members of both groups completing a pre-test and post-test. Using a variety of questions based on the content of the course, the pre-test measured the performance of the two groups before the courses and the post-test measured the performance of the two groups after the courses. The students did not know that the post-test and pre-test questions were identical and neither were they allowed to retain the pre-test questions with the correct answers only given to the students after the experiment. The students had already studied the course "Calculus and Applications" so giving the students a certain familiarity with the pre-test questions and giving them some confidence in at least attempting the questions.

The students were in the penultimate year of an engineering course with the number of students in group A, $N_A = 68$, and in group B, $N_B = 68$. The personal characteristics of the students are summarized in *Table 1*.

Table 1. Personal characteristics

Characteristics	
Average age	20.8 years
Percentage female	52%
Preferred learning style(s)	
Reading with exercise	19%
Lecture	10%
Tutorial	27%
Laboratory	20%
Working in groups (with peers)	24%
Opinion of most effective learning style(s)	
Reading with exercise	14%
Lecture	12%
Tutorial	28%
Laboratory	21%
Working in groups	25%

The initial testing was conducted after a short introduction as to the purpose of the experiment and general organizational issues. Participation in the control group was voluntary, and *Mathematica* was not required by the curriculum. The pre-test was then carried out with the data for the dependent variables collected as shown in *Table 2*. Here the distinction between question types J.3 ('Understanding simple') and J.4 ('Understanding difficult') is made based on the instructor's experience, and not on how much attention the questions have received during class time. Following the pre-test, the students were placed in either the control group or the experimental group and all students participated in both the pre-test and post-test. After completing their courses, both groups of students performed the post-test using the same questions as during the pre-test, thus providing data on the dependent variables for the second time. In addition the students were asked to answer questions about subjective perceptions.

Table 2. Experimental variables

Dependent variables
J.1 Interest in Engineering Mathematics ('Interest')
J.2 General knowledge of Engineering Mathematics ('Understand general')
J.3 Understanding of 'simple' Engineering Mathematics ('Understand simple')
J.4 Understanding of 'difficult' Engineering Mathematics ('Understanding difficult')
Subjective perceptions
S.1 Available time budget versus time need ('Time pressure')
S.2 Course evaluation (useful, engaging, easy, clear)

Data for two types of variables were collected, the dependent variables (J.1, …, J.4) and the subjective perception variables (S.1, S.2). These variables are listed in *Table 2*. The dependent variables are constructs used to capture aspects of learning provided by the courses and each was measured using 5 questions. Selected examples of questions used are shown in *Table 3*.

The results for the dependent variable J.1 were found by applying a five-point Likert-type scale [12] with each answer mapped to the value range R= [0, 1].

The values for variables J.2 – J.4 are average scores derived from five questions for each. Missing answers were marked as incorrect. The data for the subjective perception variables was collected after the post-test. The values for variable S.1 are normalized averages reflecting the time needed for understanding and doing the tasks associated with Weeks 2 – 12.

Table 3. Example questions (pre-test, post-test, subjective perceptions)

J.1 example question I consider it very important for mechanical engineering students to know as much as possible about engineering mathematics. (1 = fully agree / 5 = fully disagree) Circle number below. Agree 1   2   3   4   5 Disagree
J.2 example question (a) Let   . Determine the nature of the critical points of . (b) Use the Lagrange Multiplier technique to find the point on the circle   that is closest to the point (2,4).
J.3 example question A function is defined in the interval   as   . Sketch the function in the interval   and show its Fourier Sine series expansion is

J.4 example question
A surface is represented by the following equation
 	
(a) Complete the squares and identify the surface.
(b) Let   and determine the standard form of the contours.
(c) The surface intersects the plane   . Parametrically define the line of the intersection.
(d) Consider the point (3, 2, 6) on the above surface.
        (i) What direction produces the greatest increase in z.
        (ii) What direction is normal to the surface.

S.1 example question
I did not have enough time to:
        - complete the tutorials
        - complete the laboratory/extra tutorial sessions
        - write-up the laboratory/extra tutorial reports
        - complete the post-test

S.2 example question
I consider the explanations/information provided for laboratory/extra tutorial sessions
                    1   2   3   4   5
            Useful   Useless
            Boring   Engaging
            Difficult   Easy
            Clear   Confusing

The descriptive statistics for the experiment are summarized in *Table 4*. The columns 'Pre-test scores' and 'Post-test scores' show the calculated values for mean   , median (m) and standard deviation   of the raw data collected, and the column 'Difference scores' shows the difference between the post-test and pre-test scores.

Table 4. Scores of dependent variables

	Pre-test scores				Post-test scores				Difference scores			
	J.1	J.2	J.3	J.4	J.1	J.2	J.3	J.4	J.1	J.2	J.3	J.4
Group A ((m)	0.77 0.81 0.12	0.70 0.66 0.30	0.39 0.40 0.27	0.35 0.32 0.24	0.79 0.84 0.11	0.86 0.78 0.24	0.48 0.50 0.16	0.54 0.41 0.21	0.02 0.03 0.11	0.16 0.12 0.19	0.09 0.1 0.24	0.19 0.09 0.20
Group B ((m)	0.86 0.84 0.13	0.61 0.63 0.17	0.46 0.43 0.19	0.29 0.26 0.11	0.87 0.86 0.21	0.67 0.65 0.19	0.48 0.45 0.17	0.41 0.31 0.14	0.01 0.02 0.09	0.06 0.02 0.18	0.02 0.02 0.28	0.12 0.05 0.17

Table 5 shows the calculated values for mean, median and standard deviation of the raw data collected on subjective perceptions.

Table 5. Scores of subjective perceptions

	S.1	S.2
Group A ((m)	0.64 0.62 0.13	0.58 0.57 0.08
Group B ((m)	0.59 0.57 0.11	0.59 0.56 0.10

The students in control group (B) expressed less need of additional time than those of the *Mathematica* group (A), while students of both groups were fairly equal in their perception of their respective course usefulness, engagement, difficulty and clarity.

Standard significance testing was used to investigate the effect of the treatments on the dependent variables J.1 to J.4. The null hypotheses were,

$H_{0,1}$: There is no difference between pre-test scores and post-test scores within group (A) and control group (B).

$H_{0,2a}$: There is no difference in relative learning effectiveness between group (A) and control group (B).

$H_{0,2b}$: There is no difference in absolute learning effectiveness between group (A) and control group (B).

For $H_{0,1}$ and focusing on the *Mathematica* group (A), *Table 6* shows the results using a one-tailed t-test for dependent samples. Column one specifies the variable, column two represents the Cohen effect size, d, [13, 14], column three the degrees of freedom, column four the t-value of the study, column five the critical value for the significance value α = 0.10 and column six lists the associated p-value.

Table 6. Results for 'post-test' versus 'pre-test' for group A

Variable	d	df	t-Value	Crit.$t_{0.90}$	p-Value
J.1	0.1738	67	1.423	1.294	0.080
J.2	0.5890	67	4.820	1.294	0.000
J.3	0.3605	67	2.951	1.294	0.002
J.4	0.8426	67	6.897	1.294	0.000

It can be seen from *Table 6* that all dependent variables achieve a statistically and practically significant result. *Table 7* shows the results of testing hypothesis $H_{0,1}$ for the control group (B) using a one-tailed t-test for dependent samples.

Table 7. Results for 'post-test' versus 'pre-test' for group B

Variable	d	df	t-Value	Crit.$t_{0.90}$	p-Value
J.1	0.0573	67	0.469	1.294	0.321
J.2	0.2774	67	2.271	1.294	0.000
J.3	0.1109	67	0.908	1.294	0.185
J.4	0.9532	67	7.8022	1.294	0.000

It can be seen from *Table 7* that the control group (B) achieved statistically and practically significant results for dependent variables J.2 and J.4. For J.1 and J.3 no significant results can be found.

For $H_{0,2a}$ which states that the difference between post-test and pre-test scores of group A is not significantly larger than the one for group B. *Table 8* shows for each dependent variable separately the results of testing hypothesis $H_{0,2a}$ using a one-tailed t-test for independent samples. It can be seen that the hypothesis $H_{0,2a}$ can be rejected for the variables J.2, J.3 and J.4.

Table 8. Results for 'performance improvement' (Group A versus Group B)

Variable	d	df	t-Value	Crit.$t_{0.90}$	p-Value
J.1	0.0995	134	1.152	1.289	0.126
J.2	0.5404	134	6.256	1.289	0.000
J.3	0.2301	134	2.664	1.289	0.004
J.4	0.3772	134	4.366	1.289	0.000

Table 9 shows for each dependent variable separately the results of testing $H_{0,2b}$ using a one-tailed t-test for independent samples.

Table 9. Results for 'post-test improvement' (Group A versus Group B)

Variable	d	df	t-Value	Crit.$t_{0.90}$	p-Value
J.1	*-0.477*	134	*-5.522*	1.289	1.000
J.2	0.877	134	10.152	1.289	0.000
J.3	0.000	134	0.000	1.289	0.500
J.4	0.728	134	8.427	1.289	0.000

The two variables which show statistically significant results are J.2 and J.4 and hence $H_{0,2b}$ can be rejected for these variables. The variables J.1 and J.3 indicated that the students thought engineering mathematics more important to their engineering studies than those who used *Mathematica* in their course whilst students got equal results for J.3 irrespective of whether they used *Mathematica* or not.

3   CONCLUSIONS

This paper has described the use and efficacy of integrating *Mathematica* into a traditional engineering mathematics course. The controlled experiment has shown that the inclusion of *Mathematica* gave students a better appreciation of engineering mathematics in general due in part to the much greater enhancement of visual results. However, the inclusion of *Mathematica* had a detrimental effect on interest (J.1) when compared to the control group possibly due to the fact that programming can be a tedious business even though the results can be very satisfying. One of the main reasons for the inclusion of *Mathematica* was to contribute to the teaching of professional practice skills to undergraduate students. It was found that the interface design provided by *Mathematica* and the ease of using 'Help' does provide students with hands-on experience, gained through an interactive and reasonably user-friendly environment, and encourages student self-learning. It is recognized from the literature that "working styles" of students may play an important role in their learning and it is intended to include qualitative research on this in future studies.

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