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Calculation of Partial Derivatives by Using Rooted Trees

Q. D. Gjonbalaj
Dr.sc./Profesor assistant
University of Prishtina-Faculty of Electrical and Computer Engineering
Prishtina –KOSOVA
qefsere.gjonbalaj@gmail.com

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The field of mathematics plays vital role in various fields. One of the important areas in mathematics is graph theory, which is used, in structural models. These structural arrangements of various objects or technologies lead to new inventions and modifications in the existing environment for enhancement in those fields. The field graph theory started its journey from the problem of Kongsberg Bridge in 1735. This paper gives an overview of the applications of graph theory in Calculus, but mainly focuses on the partial derivatives applications that use graph theoretical concepts.

Partial derivatives are met in many engineering and science problems, especially when modelling the behaviour of moving objects. Partial differentiation is used to estimate errors in calculated quantities that depend on more than one uncertain experimental measurement. Thermodynamic energy functions are function of two or more variables. Most thermodynamic quantities (temperature, entropy, heat capacity) can be expressed as derivatives of these functions. Financial engineers use partial derivatives to assess a portfolio's sensitivity to changes in market conditions (interest rates, volatility). They can hedge against risk by designing portfolios, which have zero partial derivative with respect to market values. Students of engineering are faced with difficulties during calculating partial derivatives so usage of graph theory and rooted trees simplifies calculation of partial derivatives.

Graph theory is rapidly moving into the mainstream of mathematics mainly because of its applications in diverse fields, which include biochemistry, electrical engineering, computer science and operations research. The powerful combinatorial methods found in graph theory have also been used to prove significant and well-known results in a variety of areas in mathematics itself.

One of the most amazing facets of mathematics is the experience of starting with a problem in one area of mathematics and then following the trail through several other areas to the solution. In this paper we will illustrate this with a problem that starts out as a problem of partial derivative for arbitrarily complex functions with many variables, which leads to a solution that involves finding an rooted trees for first partial derivates and an a forests for second partial derivatives. In general, to find a derivative of a dependent variable with respect to an independent variable, you need to take the sum of all of the different paths to reach the dependent variable from the independent variable. Travelling down a path, you multiply the functions.

In Case 2 of the Chain Rule there are three types of variables: *s* and *t* are independent variables (in terms of graph theory-leaves), *x* and *y* are called intermediate variables (in terms of graph theory-internal vertices), and *z* is the dependent variable (in terms of graph theory-the root of the tree). To remember the Chain Rule, it's helpful to draw the tree diagram. We draw branches from the dependent variable *z* to the intermediate variables *x* and *y* to indicate that *z* is a function of *x* and *y*. Than we draw branches from *x* and *y* to the independent variables *s* and *t*. On each branch we write the corresponding partial derivative. To find, $\partial z/\partial s$ we find product of the partial derivatives along each path from *z* to *s* and then add these products.

The use of tree for computing partial and total derivatives of functions of several variables by the Chain Rule is not new. For example, Barcellos and Stein [5] use such a diagram.

Seeking about calculation of higher order derivatives by using rooted trees originally motivates the main result in this paper. In order to calculate the second partial derivatives, by using graph theory, a tree is no longer enough. Here the second order derivatives will be presented through the forest. Also we will use the product rule on the related branches.

This method can be used successfully in introductory mathematics classes to enhance students understanding of partial derivatives. ■