Towards a Continuous Assessment of Mathematical Competencies in a First Year of Aerospace Engineering

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INTRODUCTION

Assessing mathematical competencies [1] is always a difficult issue, particularly in the first year of university engineering degrees when students with different backgrounds are learning together. The grade that students receive matters to all of them and should reflect as accurately as possible their degree of achievement. Students should be aware that acquisition of mathematical competencies is a continuous process and that any failure or problem they might face will be a sign of two facts: firstly and quite likely that they did not achieve in the past the expected competencies and, secondly, that they are in a handicapped position to continue, [2]. Reversely, any successful achievement of expected competencies in due time will place them in an advanced position toward achieving upper levels of knowledge in Mathematics and other subjects.

On the other hand grading is a discreet process that comes from pictures taken through their "moments of evaluation" carried out by means of activities, lab classes, exams,... and should reflect the effort developed by the student in due time as well as the competencies shown at each moment of the topic under study and, not to be dismissed, the competencies whose specific "moments of evaluations" have already been done in the past, [2].

We expose how these ideas may be taken into account in order to carry out the process of grading mathematics competencies achievement in the first year of the Aerospace Engineering degree at the School of Design Engineering in Valencia, Spain.

1 MATHEMATICAL COMPETENCIES

The European Parliament established an European framework on eight key competences for long life learning in the Council of 18 December 2006 which were related to: Communication in the mother tongue, Communication in foreign languages, Mathematical competence and basic competences in science and technology, Digital competence, Learning to learn, Social and civic competences, Sense

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of initiative and entrepreneurship, Cultural awareness and expression. Concerning the third aforementioned competence the following definition was provided, [1].

*Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on the sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, charts). Essential knowledge, skills and attitudes related to this competence includes a sound knowledge of numbers, measures and structures, basic operations and basic mathematical presentations, an understanding of mathematical terms and concepts, and an awareness of the questions to which mathematics can offer answers. An individual should be able to reason mathematically, understand mathematical proof and communicate in mathematical language, and to use appropriate aids. A positive attitude in mathematics is based on the respect of truth and willingness to look for reasons and to assess their validity. (**European Parliament, on Key Competences for Lifelong Learning (2006/962/EC)**).*

A deep insight on this topic has been developed by SEFI's Mathematics Working Group (MWG) [3] providing a discussion forum and orientation to those who are interested in the mathematical education of engineering students in Europe. This insight had into account previous work of the Danish KOM project that set up a list of eight competencies [4] which intersect quite often and can be clustered into two groups since the first four ones deal with the ability to ask and answer questions while the last four ones deal with managing mathematical language and tools.

Let us recall that the Framework for Mathematics Curricula in Engineering Education Learning [3] has settled how outcomes regarding knowledge and skills may be arranged in a structure which has four levels starting from a Core Zero and going through Levels 1, 2, 3 so that mathematics can be linked to real applications with a greater sophistication as the student progresses through the engineering degree programme.

Finally let us mention that there is a number of examples showing how attention has been paid to improve the outcome in Mathematics of Engineering students [5-7] and even we may find the efforts done at the UK to recruit students and delivering programmes of study within the Science, Technology, Engineering and Mathematics (STEM) disciplines, [8].

2 FIRST YEAR MATHS IN AEROSPACE ENGINEERING AT ETSID

The material covered in the subject of Mathematics I of BEng Aerospace Engineering at the School of Design Engineering in Valencia ETSID (Universitat Politècnica de València UPV, Spain), keeps its outreach at Level 1 as referenced in [3] in most of the topics. However, Aeronautical Engineering being a quite demanding mathematical discipline requires some Level 2 topics covered in order to achieve the appropriate background to address other subjects of the BEng programme. No Level 3 is considered since this is designed for a first year student course and there are other subjects in the forthcoming courses which deal with more advanced mathematical concepts.

Hence, the student needs the competencies provided by this mathematics subject to acquire basic skills for the resolution of the mathematical problems that arise in Engineering and its ability to apply knowledge about linear algebra, geometry, differential geometry, differential and integral calculus; and an introduction to differential equations and numerical methods that are thereafter completed in Mathematics II and Mathematics III, respectively, [9, 10].

The corresponding specific competencies of these topics are achieved through the execution of different types of activities:

- Theory and Problem solving throughout attending on site classroom lectures.
- Laboratory Sessions as well as their corresponding exams by using the CAS software Mathematica and the UPV educational platform PoliformaT, [11,12].

- Answering Quiz tests at the end of theoretical classes or lab sessions.
- Written exercises about more comprehensive and complex practical questions, some of them with an online partial self-assessment.

We distinguish four blocks of competencies to be achieved within this subject with the following distribution:

2.1 Calculus I (C1)

This block is dedicated to the study of real functions of one variable and their applications:

- Work with the derivative of a function of one variable. Inverse functions. Exponential, logarithmic, and trigonometric functions. Hyperbolic functions and their inverses.
- Operate properly with complex numbers in various representations. Find exponential and complex logarithms.
- Define the roots of equations and find them by numerical methods.
- Graphical representation of functions. Polar coordinate system. Graphs in cartesian, parametric and polar coordinates.
- Know the basic results about the integration of functions of one variable.
- Calculate areas of surfaces of revolution, some volumes of solids and lengths of curves as applications of the integral.
- Solve basic ordinary differential equations and apply them to model and solve some physical and geometric problems.
- Approximate integrals using trapezoidal and Simpson rules. Know the basics about improper integrals; in particular, the convergence criteria and their application in specific cases.

2.2 Linear Algebra (A)

This block contains material oriented to matrix calculus and resolution of linear system:

- Take advantage of the properties of the matrix calculation to solve systems of linear equations.
- Study vector spaces and subspaces. Find basis, dimensions and coordinate systems.
- Calculate determinants and apply them to concepts related to the structure of a vector space.
- Compute scalar product, distances and angles and apply them to least squares method.
- Find the eigenvalues, eigenvectors and characteristic equation of a matrix. Diagonalize an endomorphism when possible.

2.3 Calculus II (C2)

Within this part we study real functions with two or more variables:

- Interpret the functions of two variables as a surface and the meaning of the partial derivatives.
- Calculate partial derivatives of a function of two and three variables; apply the chain rule.
- Understand and apply the implicit function theorem.
- Find and recognize equations of elementary surfaces such as cylindrical and revolution surfaces
- Find tangent planes and normal lines to surfaces and tangent vectors to curves.

- Calculate double and triple integrals in cartesian coordinates as well as by making use of adequate change of variables; in particular, in polar, cylindrical and spherical coordinates.
- Calculate line and surface integrals and its applications.
- Recognize conservative fields when evaluating line integrals.
- Apply the classical theorems of vector calculus: Green, Stokes and Divergence.
- Calculate in the adequate setting a work, length, area, volume, mass, average, centre of gravity, moment of inertia and flux integrals.

2.4 Series (S)

Finally, the fourth block contains:

- Study the convergence of numerical series and its sum in some convergent cases.
- Handle power series properly including properties of derivation and integrals.
- Find Fourier series of periodic functions. Interpret the expansions in the extreme points in the case of finite intervals and at continuity/discontinuity points.

The students are provided with adequate bibliography and textbooks, [13-15] and do have lab sessions on all topics with individual exams under controlled environment, [9].

3 CONTINUOUS ASSESSMENT

Following [2], by its own definition, a model of continuous assessment should include the execution of all activities that take place during the learning process throughout the academic year. Classical model of testing are commonly used and require grouping in the same test several thematic units corresponding to several mathematics skills.

In this sense, we have grouped in the last four mentioned blocks the competencies to be evaluated in this BEng first year at ETSID.

These four blocks are part of the corresponding “moments of evaluation” to be executed along the academic year and each of them has a different weight in the final grade (FG) mark of the subject.

This final grade part takes into account the theory and problems solving attended by lectures as well as a set of autonomous activities (Act) set to facilitate the learning process. All these form the theoretical/practical knowledge TP of the subject.

Other activities accounted in the assessment process are the laboratory practical classes (LP) which include weekly assessment through individual work at lab sessions and two individual lab exams at the end of each semester: LE1 covering C1, and LE2 covering A, C2 and S, respectively.

TP assessment is implemented a weight of 75% in the final grade (FG) which is a measure of the level achieved of the whole set of competencies to be evaluated across written exams covering C1, A, C2 and S blocks, [11]. During 2013/14 we are embedding the measure of S in a moment of evaluation C3 composed of C1, C2 and S in order to pursue a more accurate continuous assessment of the level of competencies achieved.

Lab Practice assessment has a weight P_{lab} of 25% in the total FG punctuation. It is obtained through a auto-assessment process executing activities on the e-learning UPV platform named PoliformaT, [11,12], and two on site Lab exams LE1 and LE2.

According to [2], with these elements we are going to draw a graph representing the moments of evaluation in nodes and the relationship between them with arches. We are also considering the concept of retroactivity in this graph.

In Fig. 1 we represent the relationship between the different moments of evaluation listed above in a summarized graph where ascending arches C1->C2, A->C3, C2->C3 represent the partial order existing between different moment of evaluation. For instance the first one refers to the fact that skills developed during C1 learning time are necessary to study and achieve competencies included in C2; whereas descending dotted arches from C3->C2, C3->C1, C2->C1 and C3->A represent a retroactive contribution between them.

The weights P_1, P_2, P_3 and P_a represent the contribution of the assessed moments C1, C2, C3 and A blocks to the final grade (FG). Additional contributions P_{act} and P_{lab} come from activities and lab practice.

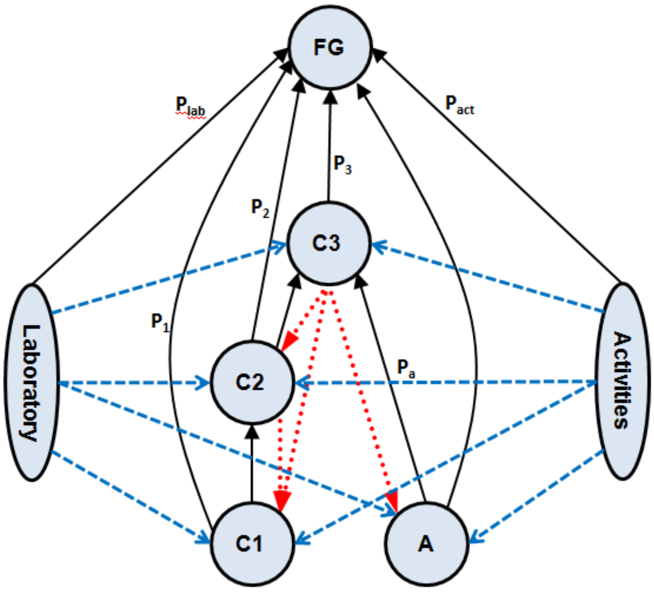

Fig. 1. Assessment's Graph

Obviously, there are more relationships between nodes reflected in Fig. 1 as the moments represented in the graph are set of competencies and more precedence relationships do exist between their components.

4 COMPUTING GRAPH MODEL AT ETSID

Distinct algorithms can be designed in order to compute different strategies to assign a picture to the final grade FG.

We are going to describe the different graph's components and how they contribute to FG according to a retroactive continuous assessment model.

The most important nodes are C1, A, C2, C3 and activities which represent the 90% of the TP punctuation. Each node contains a group of different skills to be achieved in a moment of evaluation which are assessed at the same time. The weight of these nodes during 2013/14 have been set as follows Calculus I C1 10%, Algebra A 21%, Calculus II C2 19% and C3 (Series S, C1, C2) 40%. Within C3, the weights are S 40%, C1 30 % and C2 30%. This is done so as historically Aerospace Engineering students perform quite well in Algebra and there is no need to reinforce it through TP. Indeed it is continuously reassessed through LP since the moment of evaluation corresponding to A takes place in January, its lab sessions run throughout February and Lex2 takes place in May. The very few students showing some lack in Algebra get a chance to show that they have improved their Algebra competencies by the end of the course in conjunction with C3.

Thus the real final weight of Calculus I and Calculus II topics are 22% and 31%, respectively, at the end of the course within TP which is related to the time and relevance of its topics.

In Fig. 2 we may observe a closer set of relationships between two different moments of evaluation Mi and Mj and its embedded components between which we have got an ordered relation Mi->Mj.

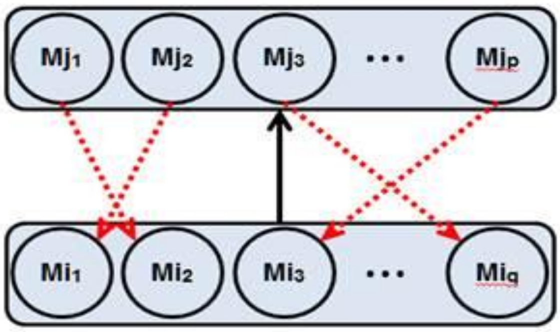

Fig. 2. Relationships between components

For instance, the successful execution of moment of evaluation C1 should be a goal before executing the moment of evaluation C2 due to the relationships of precedence existing different skills C1 and C2. The ascending arch from C1 into C2 states that C1 competencies are a natural previous requisite to be able to achieve C2 competencies. Conversely, a failed evaluation in C2 may involve a lack of adequate level in some C1 competencies, a degradation of its level or a failure in the measurement process.

In a real model of continuous assessment, each session of evaluation should assess the previous competencies but this is not feasible because of time limitations. That is why we include some relevant C1 topics within some C2 questions and a retroactive contribution from C2 to C1. Equally with other moments of evaluation across the arches defined in the competencies graph.

A successful C2 evaluation not necessarily implies an improvement of the all the C1 competencies in all circumstances. However if there was a lack of C1 achievement competencies in the past it may be considered as a sensible indicator that some lack has been overrun.

For this reason, in our forward/retroactive continuous assessment model the C1 topics included in C2 are to be considered as an opportunity to redeem such situations. When this happens students get their assessment of previously evaluated competencies increased by a percentage of its difference with the newly shown level of competence. This percentage is set up depending upon the number of ulterior competencies moments of evaluation that confirm the improvement of the competence level, and tries to compromise between the newly reached level of competence and rewarding adequately students when they get achievements and goals at expected due time.

In this sense, the retroactive contribution might be considered a reason to introduce positive or negative corrections in previous moments of evaluation's punctuations. At ETSID we only apply positive corrections in previous competencies assessment when the student shows that he/she has reached or improved them in ulterior moments of evaluation. When a failed evaluation occurs, instead of using a negative retroactive contribution when a previously evaluated competence as achieved shows some degradation, we warn them on the precedent related components and propose a set of activities in order to improve the failed skills.

On the other hand, the set of activities shown on the right part of the graph represents a 10% of the TP component. These activities are executed along the academic year and they are parallel to the skills developed in theoretical lectures according to flipping education techniques [16, 17]. The auto assessment based on the UPV educational platform is another relevant aspect of the continuous assessment model, [11, 12].

Additionally, lab sessions are assessed in two specific moments of evaluation LE1 covering C1 and LE2 covering A, C2 and C3. These two lab moments of evaluation contribute by 15% to FG while the weekly activities performed in the Lab sessions also do contribute FG by a 10%.

Computing algorithms are easily implemented by means of a spreadsheet and we could essay multitude of strategies with different set of weighted arches on the graph in order to maintain a suitable and continuous level of interest in students.

5 SUMMARY AND ACKNOWLEDGMENTS

The conclusions from this paper get along with the ones obtained in [2] in a general setting. We believe that this model represents a major approach to the concept of the process of continuous assessment by the properties of the model:

- The representation of the levels of competence in a graph is appropriate since there is a natural partial order between these levels.
- The introduction of some precedents topics in moments of evaluation ensures continuity in the assessment process in addition of the natural order of precedence established in the graph.
- A consequence of the partial order is the logical assumption to achieve higher competency levels necessarily implies the acquisition of previous levels.
- The retroactive positive contribution implies the modification of measurements of competencies of previous moments of evaluation when a minimum level has been achieved.
- The retroactive negative contribution in not applied decrementing measurements of competencies of previous moments. As alternative, set of appropriate activities are proposed to reinforce the not achieved skills. Anyway, this skill could be redeeming by the means of successful subsequent moments of evaluation which included some of these topics.
- Parallel execution of activities to lectures with auto-assessment on the UPV educational platform PoliformaT contributing to the final grade is a relevant component of our continuous assessment model.
- The no possibility to reach a sufficient level of punctuation before finishing the course maintains the interest in the totality of the program.
- Positive retroactivity is an element of motivation for students and prevents premature subject forsaking.

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