Future teaching of mathematics for engineers

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Conference Topic: Maths in EE

INTRODUCTION

In many countries an increased awareness of the quality of learning and teaching mathematics for engineers can be observed, and many projects are being launched aiming at improved the learning outcome for students. This paper will report on a project at the Norwegian University of Science and Technology (NTNU), where basic mathematics is being taught to around 1600 engineering students in each cohort. The project is in its initial phase, and a pilot project was done in the academic year 2013-2014.

The overarching aim of this project is that the students should develop a deep understanding for mathematical concepts and processes, making them better equipped to use mathematics in applications. To reach this aim it is recognised that students' own activity and involvement with the subject is crucial. An important part of the project is using digital technology where this is advantageous in order to free teacher resources for use in "better" ways, i.e. to enhance contact with students. Digital technology is also important to offer diverse learning resources, acknowledging that students work and learn in different ways.

This paper is as a preliminary report presenting the main ideas of the project as well as some findings from the evaluation of the pilot project.

1 BACKGROUND

1.1 Mathematics for engineers at NTNU

All engineering students at the Norwegian University of Science and Technology take at least five mathematics courses. These courses are all delivered by the Department of Mathematical Sciences and taught by mathematicians. The largest course is Calculus 1 (one variable calculus). All engineering students (approximately 1600, on 18 study programmes) take this in their first semester, and almost all (approximately 1200) also take Calculus 2 (multi variable calculus) in the second semester. The courses have been taught in a rather traditional way, based on lectures, 4x45 minutes per week. Lectures are given in six (Calculus 1) or five (Calculus 2) parallels, where students are assigned to a parallel based on the study programme they follow. All parallels follow a common, and rather strict, schedule in terms of progression, and all students have the same curriculum and they take the same exam. The number of students assigned to the different parallels varies, with the largest parallel counting more than 400 students. In addition to lectures, some sort of work with problems in smaller groups has always been part of the courses, and as part of this some sort of hand-ins from students have been required. In order to be admitted to the exam a certain number of hand-ins have to be completed and approved of.

The large number of students constitutes a challenge in terms of possibilities to obtain contact with students and also to provide sufficiently qualified teacher resources. There is a long tradition to use higher year students as part of the teaching resources, as well as PhD research fellows in addition to regular academic staff. In Calculus 1 alone around 50 teachers are involved in running the course.

1.2 Changing Calculus 1 and 2

For some time there has been a discussion about the value of the traditional paper based hand-ins. They are checked by student assistants and then given back to the students with comments. There has been an impression that the students do not pay much attention to the feedback they get on the hand-ins, and the feedback may also be of varying quality. Furthermore, it is well known that students copy from each other to prepare the compulsory hand-ins. To address these issues a remake of the problem session structure was done in Calculus 1 for the autumn of 2013. The traditional hand-in problems were to a large extent replaced by electronically generated problems, using the system Maple T.A. This system generates slightly different problems to each student and it gives an immediate feedback if the answer is correct or not. Answers can be given in the form of multiple choice, or in terms of a number or an algebraic expression. Each week a problem set was given on Maple T.A. and in order to be allowed to register for the final exam a certain number of these problem sets had to be completed satisfactorily.

In addition a number of more elaborate problems (projects) were given. These were meant to be worked through and then presented by students to fellow students in small groups, under the supervision of an assistant.

Lectures were given in the traditional way, and one of the parallels was video captured and made freely available on the web. A home page was designed as the central place that students were supposed to consult to keep up with the progression of the course. Traditionally lectures and textbook have been the main components of the course. These were kept but in addition a number of electronic resources were made available. Such resources included text-based material on the web, short thematic videos and pencasts, as well as the video captured regular lectures.

This pilot project was continued into the course Calculus 2 in the spring of 2014, with some changes based on experiences from Calculus 1.

1.3 Quality, accessibility and differentiation in the basic teaching of mathematics

The project with the above title is in its initial phase but it builds on experiences from Calculus 1 and 2. The overarching aim of the project is to contribute to better learning experiences for the students in the sense that they should develop a deeper understanding of mathematical concepts and processes, thereby making them better equipped for using mathematics in applications [1, 2, 3]. It is expected that this will lead to better experiences for the students in terms of quality and relevance of the education. It is further expected that it will lead to a reduced dropout rate. The following points are important in order to reach the goals of the project.

- Increase student activity and involvement in all parts of the education
- Stimulate increased and more continuous work input in the studies
- Enable closer contact between students and highly qualified teachers
- Increase differentiation so that the education is better adapted to different students' varying interest and previous knowledge of mathematics

Developing digital resources is important to increase differentiation and to give the students a choice of what resources to use. Digital resources, e.g. digital assessment, can also be used to shift teacher time to more extended student contact. The digital assessment system Maple T.A., already introduced, is central in keeping the students continuously working with the subject.

An important part of the student-teacher contact is a drop in centre for students. This is open all day every day and students can come at any time. Each study programme has certain times of the week where they are given priority but subject to capacity, they can come at any time.

Lectures have at all times been a central part of studies at the university. The tradition of lectures goes back to a time where printed material was not easily available. Even if this by far no longer is the case the lecture still has a prominent place in the university studies. The lectures are given by regular staff members, or sometimes by PhD or postdoctoral fellows, and the lecturers are seen as the main teachers of the course, whereas those who are taking care of the problem sessions are seen as assistants. It is the intention of the project to also make changes in the lectures, mainly to obtain more student activity and more two-way communication between students and teacher. Recently the idea of *flipped classroom* [4] has gained considerable interest, in particular in the US. Experiences from the

Nordic countries [5] indicate that this needs to be done with care as it represents a strong breach with the study culture in our countries. It is therefore planned to use only two out of four lectures in a non-traditional way. In one of the two lectures (last hour of the week) students are invited to give input to topics that they want discussed more carefully, and then the teacher prepares the first hour in the following week addressing the topics that have come up. The remaining two hours will be used in a more traditional way but due to reduced time the topics covered in these two hours will have to be more selected. So this will mean that some of the emphasis will be moved from the lectures to other media for presenting material, such as video or text based presentations, in books or on the web.

2 THEORY

2.1 Knowledge and understanding in mathematics

Frameworks for classifying and describing knowledge and understanding in mathematics belong to the traditional literature in mathematics education. Hiebert and Lefevre [2] made a distinction between *conceptual knowledge* and *procedural knowledge*. Conceptual knowledge is characterised as "knowledge that is rich in relationships", "as a connected web of knowledge" [2, p. 3]. A unit of conceptual knowledge is never isolated but, by definition, is part of conceptual knowledge only if it is seen in relation to other pieces of information [2, p. 4]. Procedural knowledge is the ability to perform calculations and other procedural operations with confidence and consist of both the symbol representation system and the algorithms, or rules, for completing mathematical tasks [2, p. 6]. Hiebert and Lefevre emphasise that both types of knowledge are essential in order to be a successful learner of mathematics.

2.2 Ways of learning

A purely lecture based way of teaching mathematics can easily be seen as based on a view that learning happens by transmitting knowledge from those who know to those who do not know. It is widely recognised that this is not the way learning takes place. Both general theories of learning, see e.g. [6], as well as studies of learning of mathematics on university level [7, 8] indicate that learning through participation and communication is advantageous to obtain relational understanding [3]. Working with problems and exercises has always been regarded as a central part of learning mathematics. However, such work needs to include a communicative aspect where the teacher provides concepts and notation making it possible for the student to develop his/her understanding through the work.

Also a lecture based teaching can involve student activity but it requires that focus is shifted from teaching to learning [9] and that instead of emphasising a best possible presentation, emphasis is put on how the presentation can enable students to learn in a best possible way [7].

Previous research, e.g. [1], shows that many students enter university with a view on mathematics as fragmented and that learning of mathematics involves being able to use routine based procedures on standardised tasks. This is not compatible with conceptual knowledge and these findings indicate that a change in the view of mathematics and learning of mathematics is necessary.

3 EMPIRICAL DATA

3.1 Interviews

Shortly after the semester had started (late August 2013) students in Calculus 1 were invited to participate in group interviews. The invitation was done during a regular lecture and in addition an announcement was posted on the home page. It was said that the purpose of the interviews was to get information about the students' experiences with Calculus 1 in order to develop and improve the course. It was emphasised that participation was completely voluntary and that the participants should be prepared to meet two times during the semester, about two hours each time, and that the meetings probably would take place after regular working hours. In total 12 students, representing seven different study programmes, signed up for the interviews. The first interviews took place in early October (5-6 weeks into the semester) and the second interview took place in the very end of October, when 4-5 weeks of teaching remained.

The interviews were conducted as semi-structured interviews, where some questions or topics were planned, but to a large extent the students were allowed to speak freely. All interviews were recorded using a sound recorder. The first interview round focused mainly on the students' working habits. The students were asked about which learning resources they used, how and to what extent they used them, and about their experiences with them. Special emphasis was placed on components of the course that were new or different from the way the course had been given before. In addition the students were asked about their experiences with the transition from school to university. The second interview round also concentrated on the students' use of learning resources but now the interest was on whether something had changed from the early days of the semester. At this point the students had passed a mid-term exam and there was some discussion about the problems from this exam.

An important purpose of the interviews was to collect information and get ideas about questions to ask on a survey that was conducted at the very end of the semester.

3.2 Survey

Towards the end of the semester, but before the exam, a web-based survey was administered to all students of Calculus 1. The survey was accessible from a link on the home page and could be completed using almost any electronic device (computer, mobile phone). In each of the parallels time was allocated for completing the survey in one of the last lectures of the semester. In total 662 students completed the survey, i.e. approximately 40 % of all the students on the course. In Calculus 2 a new survey is conducted, with some of the questions repeated from the Calculus 1 survey. At the time of writing this survey is still open and up to now 550 students have completed the survey. i.e. around 45 % of all the students on the course. (Not all students on Calculus 1 take Calculus 2.)

The questions in the survey of Calculus 1 aimed at getting insight into the students' experiences with the course. Since an important idea of the design was to make available a wide range of learning resources many questions dealt with to what extent these resources were used by the students. In a survey one cannot find out what ways of learning are actually effective but one can find out something about the students' own perception of their learning value. So the students were asked to grade the learning resources with emphasis on to what extent they thought they learnt something from them. Here it is hard to say what is actually meant when the students say that they learnt something. What kind of learning takes place? To address this we have included questions about what the students value when learning mathematics and it will be possible to do a correlation analysis of these questions. However, this is beyond the scope of this paper.

An underlying assumption is that learning is a communicative process between student and teacher. This means that it is assumed that the student learns from feedback on the work that he/she has done. Different types of feedback systems have been tried out, from computer assisted assessment to traditional written feedback on handwritten hand in assignments. It is of interest to know the students' reactions to different kind of feedback. This has been addressed in the surveys, and also the reactions to the changes made from Calculus 1 to Calculus 2.

3.3 Research questions

Based on the experiences from the courses I have identified the following research questions for this paper

- What are important challenges involved in changing a traditional teaching system?
- What is perceived by students as good ways of learning mathematics?
- How do the students perceive the transition from upper secondary school to higher education?

4 RESULTS

4.1 Changing a traditional system

Students coming to the university have a long experience as learners, also as learners of mathematics. To be admitted to the engineering programmes at NTNU it is required to have the highest level of mathematics from upper secondary school, and indeed also to have passed with a certain grade. This means that the students have had mathematics for 13 years in school. Out of those completing the survey approximately 75 % had finished upper secondary school either the same year as they started university or the year before. Therefore they can be characterised as very experienced learners of mathematics with fresh experience. When entering the university they become part of a community of more experienced students and through this community they will be exposed to perceptions about what it means to be a university student. Many will presumably also have a perception of this before entering the university. It can therefore be said that they enter a community of practice [6] with certain norms, and as newcomers it is expected that they will be anxious to live up to these norms.

4.2 Use of learning resources

The students were asked the following question: “To what extent do you make use of the following learning resources in Calculus 1?” Table 1 shows how the students answered to this question for a selection of learning resources. In the tables below the figures are rounded off to the nearest integer. The percentage of students that did not answer a certain question is not given. For these reasons the numbers will not add up to 100 %.

Table 1. Use of learning resources (figures in %)

	To a large extent	To a rather large extent	To some extent	To little or no extent
Lectures “live” in the lecture hall	70	13	8	6
Video recorded lectures	21	25	35	17
Other videos made at NTNU	5	11	36	46
Videos from external sources	8	13	29	46
The textbook	63	26	8	1
Theory on the home page of Calculus 1	4	16	42	36
Other web resources	8	20	38	31
The drop-in centre	12	16	36	36

This shows that the large majority of students, more than 80 %, attend lectures on a regular basis, and almost 90 % make considerable use of the textbook. When asked to rate a number of claims about the lectures, some of the answers can be seen in Table 2.

Table 2. Statements about lectures (figures in %)

	Completely agree	Agree to some extent	Disagree to some extent	Completely disagree
I learn a lot by attending lectures	55	35	7	2
Video lectures are better than live lectures	13	26	42	16
The regular lectures are important for structuring my work	51	30	11	5
Being in the lecture hall is better than watching a video	49	30	14	5
Lectures are a natural part of a university study programme	79	16	2	

Table 2 shows that a large majority of the students, 90 %, think that they learn a lot by attending lectures, and that it is better to be in the lecture hall than watching a video. However, also the video recorded lectures were used to a large or rather large extent by 46 % of the students. This indicates that videos are used in addition to, and not instead of, regular lectures. This was also confirmed in the interviews. Students rarely watch a whole, recorded lecture but they watch parts of it, to see again something that they did not quite understand during the regular lecture. One student said that she would watch a presentation, stop it, and try to continue the calculation. Then, after having tried herself, she would start the video again and see if the calculation on the video matched what she had obtained. This particular way of using videos was connected to using external video resources on the web but similar statements were also made regarding the video taped regular lectures.

When asked in the interviews about how they would react if all lectures had been replaced by videos, all the interviewees answered that this would not be a good solution. They said that the lectures were important to get structure in their workday, and as a help to distribute the workload over the week. This view is supported by 80 % of the students (Table 2). One student said that

it is not like we can collect all the videos from a week and watch them on Sunday night. ***(anonymous respondent)***

The role of the lectures can also be connected to another question. We listed nine activities that we considered to be natural in the work with the course. Then we asked the students to rank them in order, 1-9, after what they considered to be most important to get done in a regular week (only one activity for each rank). "Attending lectures" was ranked no 1 by 46 % of the respondents. No other activity obtained nearly as many top scores. Doing the compulsory computer based tasks and doing exercises from the textbook had around 18 % top scores. These two activities had the highest rate of no 2 scores (22 % and 16 % respectively).

When asked to rate a number of claims about the textbook, some of the answers can be seen in Table 3. These results show that the examples in the textbook are very important for almost all students but also more than 75 % read the theory in the textbook, and about half of the students say that they like reading in the textbook. It seems that web based material does not make the textbook obsolete.

Table 3. Statements about the textbook (figures in %)

	Completely agree	Agree to some extent	Disagree to some extent	Completely disagree
I rarely read in the textbook	6	16	29	48
I consult the textbook if I am stuck with a problem	66	27	4	1
I read the theory in the textbook	35	41	18	4
I may as well find the material in the textbook on the web	7	21	46	24
The examples in the textbook are important	54	37	6	1
I like reading in the textbook	21	33	31	13
I use the textbook mainly for finding problems/exercises	16	37	38	11

These results show that the use of learning resources is indeed quite diverse and the result therefore give support to the idea of the project to provide a large variety of resources in order to cater for different tastes and needs. However, the results also show that the traditional resources, such as lectures and textbook, have a very strong position.

4.3 Learning mathematics and the transition to higher education

It is desired that students should learn more than just routine procedures in mathematics. These students learn mathematics mainly because it is expected to be useful for them in other subjects and in their future profession as engineers. It is generally acknowledged that transferring mathematical knowledge from the subject itself and to other disciplines is very challenging [10] and in particular if the knowledge is mainly procedural [2] the knowledge is strongly connected to the situation in which it was developed but even with conceptual knowledge this is the case. In order to ease the transition as much as possible emphasis is placed on conceptual understanding.

It seems that also the students value conceptual understanding. When presented with the statement "I want to understand concepts and underlying principles in the subject" 90 % of the respondents agree to this, totally or to some extent. The students were also presented with a number of statements beginning with "I think I learn mathematics best by …". Some of these statements and how the students answered are presented in Table 4. This shows that solving problems is by far the preferred way of learning mathematics and that exercises/problems from the book or from old exams score a lot higher than doing the computer tests (Maple T.A.). When elaborating on this in free text there is one issue that comes up from many students: "We do not get enough feedback on how to write mathematics". The computer system is appreciated for giving immediate feedback but the feedback is considered inadequate because it only reports on right/wrong and not on *why* it is right or wrong.

Table 4. Learning and working with mathematics (figures in %)

I think I learn mathematics best by	Completely agree	Agree to some extent	Disagree to some extent	Completely disagree
reading in the textbook	25	50	19	4
attending lectures	55	35	7	3
doing exercises from the book	71	23	3	1
doing the Maple T.A. tests	24	36	24	14
solving old exam problems	71	24	3	0
collaborating with others	46	35	15	3
going to the drop in centre	10	27	30	31

About the lack of feedback, one student writes:

Maple T.A. puts too much emphasis on the answer being correct, completely correct, and not on whether we have understood anything. ***(anonymous respondent)***

Based on this it is possible to identify a desire from the students to understand. It is not clear from this what they really mean by understand but the statement "if we just have got the correct answer" indicates that something deeper than just being able to perform a procedure correctly is meant.

The most advanced level of mathematics in Norwegian upper secondary school focuses heavily on functions and on differentiation and integration. Looking at the curriculum of upper secondary school on headline or topic level one will find many keywords that are common to the Calculus 1 course. Students doing Calculus 1 will therefore have a background making them capable of comparing two learning environments where the topics covered may look rather similar on the surface. Table 5 sheds some light on how the students perceived the transition from school to higher education.

Table 5. The transition from school to higher education (figures in %)

If I am to compare Calculus 1 with mathematics in upper secondary school I will say that	Completely agree	Agree to some extent	Disagree to some extent	Completely disagree
in Calculus 1 it is more important to understand why things are as they are	55	33	7	2
Calculus 1 is much repetition from upper secondary school	6	35	44	13
I now see better how different parts of mathematics are connected	30	47	18	4
There is no big difference	2	11	40	45

Research from other countries, e.g. [1], has shown that many students enter university with a fragmented view on mathematics. In 2013 a large survey (approximately 2500 respondents) was made in Norway among students on university programmes containing mathematics. One of the questions was how the students perceived the transition to higher education in terms of the level of mathematics. The scale was from 1 (small) to 6 (large). 77 % of the students chose categories 4, 5 or 6, almost equally distributed among the three categories. When restricted to NTNU figures show that 86 % choose 4, 5 or 6, with 66 % on 5 or 6 alone. It is interesting to note that this also is consistent with the students' expectations. Around 80 % of the students chose the two highest grades (5 or 6) when asked about whether they expected the level of mathematics to be higher than in upper secondary school. Answers in free text on this survey seem to support the view that mathematics in upper secondary school is dominated by routine procedures whereas in higher education more emphasis is placed on concepts and proofs.

5 SUMMARY

The results presented in this paper shed some light on the challenges that are involved in changing a well established system. Students enter higher education with certain expectations about how it will be and with certain experiences from previous schooling about how it is to learn mathematics. Even if given the choice between a large variety of learning resources it is the traditional ones, lectures and textbook, that are preferred. 90 % of the students say that they learn mathematics best by attending lectures. So if one wants to shift more of the presentation of material from traditional lectures to electronic media, and use the time together with the students in the lecture hall in a different way, there seems to be a strong culture that needs to be changed.

Most students experience the change from upper secondary school to higher education as large but at the same time this is also what they expect. The transition also seems to involve a shift in the view on mathematics, from emphasising procedural to conceptual knowledge. However, this also seems to be in accordance with what the students want. In the Calculus 1 survey more than 90 % agreed that they wanted to understand concepts and underlying principles.

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