Competence-oriented assignments in the mathematical education of mechanical engineers

Alpers, B.
Professor
Aalen University of Applied Sciences
Aalen, Germany

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INTRODUCTION

In the new edition of the curriculum document issued by SEFI's Mathematics Working Group in 2013 ([1]) the concept of mathematical competence as developed in the Danish KOM project ([2], [3]) serves to capture higher-level learning goals in the mathematical education of engineers. Mathematical competence is defined as "the ability to understand, judge, do and use mathematics in a variety of intra- and extra-mathematical contexts and situations where mathematics plays or could play a role" ([2]). Assignments given to students are the most important vehicle for achieving educational goals in mathematics (ranging from small, well-defined computational tasks to open and challenging projects). Therefore, for really putting the competence concept into practice, we need assignments that are suitable for acquiring the competence goals. This paper provides some guidelines on how to identify such assignments and presents several examples. It should be mentioned that this type of assignment does not make the usual computational training tasks obsolete since it is well-recognised in the competence concept that a sound knowledge and skills base is a necessary but not sufficient requirement for a person to be mathematically competent (cf. [2]).
In the next section, we briefly explain the concept of mathematical competence. Then, we give some hints on how to identify competence-oriented assignments. In the main section of this paper, we give examples for such assignments in the area of mechanical engineering for different time scales, covering small tasks for weekly assignment sheets, small, medium-sized and large projects up to tasks for a bachelor thesis. The examples can be used to convince colleagues that the pedagogical concept has a real meaning, and they can also be used for competence assessment.

1 THE CONCEPT OF MATHEMATICAL COMPETENCE

The concept of mathematical competence puts emphasis on the ability to use mathematics in relevant contexts which makes it particularly attractive for the mathematical education of engineers. In order to provide real orientation, the concept needs more detailing. For this, eight competence areas, called "competencies", have been identified in the Danish KOM project ([3]):

- Thinking mathematically: Understanding and judging what kind of questions can be answered using mathematics and hence where a mathematical approach might be helpful.
- Reasoning mathematically: Understanding an existing mathematical argumentation as well as setting up an own mathematical argumentation.
- Posing and solving mathematical problems: Formulating a question as a mathematical problem and using problem solving methods and strategies.
- Modeling mathematically: Understanding, working and solving problems within models set up by others as well as performing active modelling (covering all parts of the modelling cycle).
- Representing mathematical entities: Choosing adequate representations and switching between representations in order to use the most suitable one for a problem.
- Handling mathematical symbols and formalism: Understanding symbolic expressions and formal language used by others as well as setting up and manipulating own ones.

- Communicating in, with, and about mathematics: Understanding oral and written mathematical statements made by others and expressing oneself mathematically.
- Making use of aids and tools: Recognising when the usage of aids and tools is adequate as well as using them adequately.

If one wants to specify the educational goals for a study course it is not sufficient to just state that the above competencies are to be acquired. One has to specify in more detail the level of progress one wants to achieve. In the KOM project three dimensions have been identified within which the level of progress can be specified ([3]):

- Degree of coverage: Aspects of the competency to be covered
- Radius of action: Situations and contexts where the competency can be applied
- Technical level: Mathematical concepts to be used when applying the competency.

The author used these dimensions when specifying a mathematics curriculum for a practice-oriented study course in mechanical engineering ([4]).

2 IDENTIFICATION OF COMPETENCE-ORIENTED ASSIGNMENTS

In mathematics textbooks for engineers many assignments (mostly for weekly assignment sheets) are available for getting accustomed to mathematical objects, concepts and procedures. For example, for obtaining a certain familiarity with matrices and matrix operations there are tasks for setting up the coefficient matrix for a linear system of equations and for performing matrix addition and multiplication. It is also straight forward to set up a number of tasks for solving second order linear differential equations with constant coefficients to train the solution procedure and let students encounter the different possible solution types. Such tasks are still important to obtain the skill basis for mathematical competence. They also capture part of the competency of handling mathematical symbols and formalism and that aspect of the problem solving competency that deals with the ability to solve well-defined problems for which a solution procedure or principle solution strategies are available. For mathematicians, one would probably not call this “problem solving” but for most engineering students this is an adequate description since it constitutes a major challenge for them. Similarly, mathematical reasoning in well-defined steps in a mathematical procedure is also covered in traditional mathematical assignments. Such assignments are close to the mathematical content taught in a corresponding mathematics lecture.

The competence concept goes beyond this since it emphasizes the ability to understand, do and use mathematics in relevant contexts and situations. Sometimes, one can find application oriented tasks also in mathematics textbooks. These are smaller tasks where the mathematical question is often “barely disguised” in the application scenario. Nonetheless, such tasks are also useful since many students are so fixated on a special notation (e.g. y(x) for a function) that seeing mathematics in different notation is already a challenge.

In order to make sure that one really addresses relevant contexts and to integrate the mathematical education into the study course, it is necessary to investigate those application subjects of the course which make heavy use of mathematical concepts. Particularly those subjects that run in parallel with the mathematics education are important since students then see the interconnection and the value of mathematics promptly. Such an investigation is already necessary when one wants to set up a concrete curriculum for a specific study course using the three dimensions stated in the previous section. For doing this, one can examine lecture notes of colleagues or recommended textbooks. There, one can recognize the mathematical representations, concepts, models and procedures that are used and how they are used. This helps to see which ”interface” a student should have to a mathematical concept which is certainly different from a mathematician’s interface (e.g. how deep should the student’s mathematical background be to understand the Dirac distribution and its use in control theory lectures). For a practice-oriented study course in mechanical engineering, the most important subjects in the first three semesters are:

- engineering mechanics and physics (including stress theory): this subject is full of mathematical models and tasks where work in these models leads to solutions. For example, in equilibrium situations, systems of equations are important; in motion analysis and design functions, properties of functions, differentiation and integration play a major role.
- machine elements: in formularies coming along with machine element books a wealth of mathematical representations can be found (one example will be given in the next section); algorithms for machine element dimensioning provide various examples for problem solving

strategies. Machine element computation programmes could be used to check whether similar algorithms are in use there and for investigating the mathematical input required.
- CAD: geometric design using objects, operations and relations can be found here (see [5] for many examples).

One can look at the laboratories available in the study course and check whether there are test benches or measuring devices which provide interesting mathematical tasks. Moreover, lab facilities like a milling machine might be usable for designing and producing objects with interesting mathematical properties (cf. the examples in 3.4 and 3.5).
The engineering contexts dealt with so far are still concerned with academic education. In order to capture real engineering contexts in industry the following sources might be available:
- bachelor theses written in industry containing mathematical tasks
- industry guidelines, e.g. those offered by the Association of German Engineers for motion design ([6]) which offer a wealth of mathematical tasks
- workplace studies regarding the required mathematical qualifications (cf. [7]).

3 EXAMPLES FOR COMPETENCE-ORIENTED ASSIGNMENTS

Assignments suitable for acquiring mathematical competence are possible on very different time scales from a few minutes (weekly assignment) to a few months (bachelor thesis). In the following we use five categories:
- Tasks appearing in weekly assignment sheets (along with usual computational training tasks)
- Small projects (home assignments, maybe for groups), taking 10 to 20 hours; these can be handed out once a semester or a few times; they can provide part of the class credit (and grade) or can be a prerequisite for participation in regular exams.
- Medium-sized projects comprising 2-3 credit points: More information on finding such projects for mechanical engineering students can be found in [8].
- Larger projects with about 5 credit points
- Bachelor theses (with about 12-15 credit points).

For all projects some sort of documentation and maybe presentation of results is usual. Therefore, such projects are suitable for addressing the communication competency which will not be stated below separately. In the sequel we present one example for each category.

3.1 Weekly assignment sheet

We give an example where the context is taken from a typical machine element dimensioning task:
- The “Wöhler curve” is used in fatigue life evaluation of machine parts. It relates the stress amplitude σ to the number N of bearable load cycles, and uses a logarithmic scaling for N. Determine from the curve below (constructed after [9]) the number of bearable cycles for σ= 900 N/mm^2 and the stress amplitude for 800.000 load cycles. Determine the underlying function σ(N).

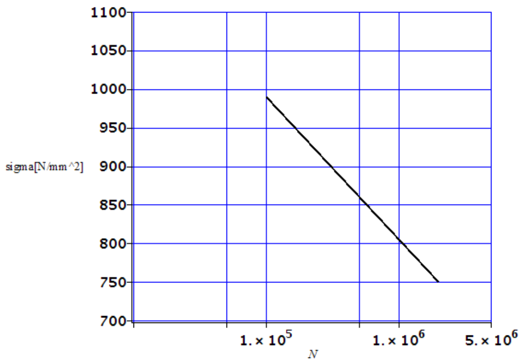

Fig. 1. Wöhler Curve

This task obviously addresses the competency of dealing with mathematical representations. Students can learn in this context why the chosen representation is useful (capturing a wide range, making the curve linear). The task has to be translated into the following mathematical problem: for a point on a logarithmic scale find the corresponding value and vice versa. For solving this, three approaches might be taken:

- The solution procedure was shown before (e.g. in a lecture) and students have to work through it.
- Students look up a solution procedure in a textbook or on the internet. Then, the communication competence is also addressed since students have to understand the procedure at least as far as it is necessary to apply it to the task above.
- Students work from scratch based on their knowledge of logarithms and corresponding laws.

The second part of the task involves switching between representations. One could formulate this even more open by asking how one can use the data captured in the graphical representation in a computer program where the user gives stress amplitude or load cycles as input and gets the other quantity as output. In the task as given above, students have to set up a symbolic mathematical representation, so the competence of handling and manipulating symbols is also addressed here. In a small project, one could also use this to discuss the limitations of the model (if the part holds for 10^5 load cycles, will it fail for 10^5+1?) and how such a curve comes into existence and then the modelling competency would also come into play.

In text books on machine elements and accompanying formularies and data sheets one can find many other tasks addressing the competency of representing mathematical entities. For example, if a quantity depends on two other quantities this is often represented by a 2D-diagram with families of curves where one quantity is fixed.

3.2 Small project

The following task is taken from motion design as it typically occurs in packaging machines:

- A slider waits until a stack with packages comes along on a belt. Then it moves the stack into a packaging machine, waits until the packaging has been performed, moves it back to the belt and waits until the next stack comes along. How should the slider move exactly?

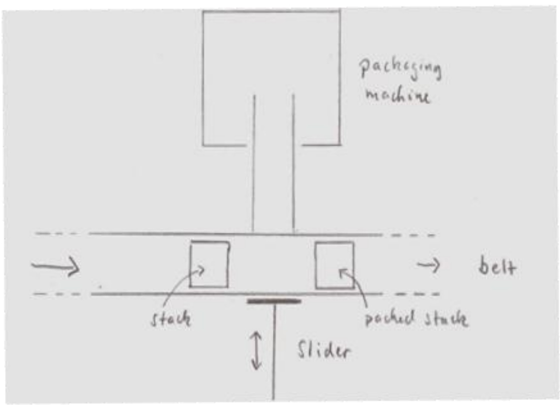

Fig. 2. Packaging Machine

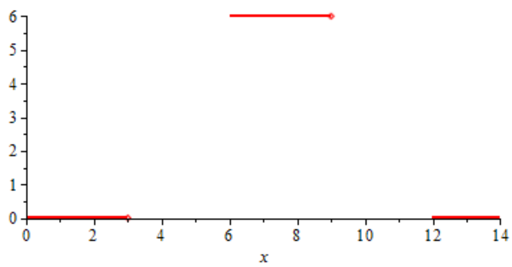

Fig. 3. Incomplete Function

What comes up first here is the mathematical thinking competency: Mathematics should help to answer the question since mathematical concepts allow for a precise motion description which can be used to program drive control devices. Then, the problem has to be modelled mathematically. For this, the graphical representation of a partially defined function as given on the right hand side of the above picture could be used. In order to formulate the problem mathematically, students need additional information: How long does the packaging process take? What are the maximum velocity and acceleration of the slider? How long should it rest in order to avoid collision with the next stack when going back? In a small project this should be provided as part of the task. When a real machine is available it could be part of a medium-sized or larger project to find out the relevant data. That would

address the modelling competency considerably more. The main mathematical problem here consists of finding functions for filling the empty “slots”. Students have to think about additional requirements (there should not be a jump in velocity or acceleration) and then reason about available function classes and their suitability. One line of argumentation might be: For filling a slot, one has 6 conditions: continuity of the function and its two derivatives at the left and right boundary of the empty interval. Take a polynomial of degree 5 since it has 6 degrees of freedom (will that exist?).
When students try to fill the slots with polynomials, they have to solve systems of linear equations, and they then have to investigate whether the conditions on maximum velocity and acceleration are met. The times for slider motion (back and forth) should be short such that the number of cycles is high. Trying out different time intervals by hand computation is very tedious, so here the competency of using tools comes into play. With a CAS program, for example, students can create their own experimentation environment where they enter interval lengths first, then compute the polynomials and their derivatives, and then modify intervals to get a feasible or better solution.

3.3 Medium-sized project

The following task is concerned with the bending behavior which is normally dealt with in engineering mechanics for the special case of “small” bending:

- Create a worksheet where the bending line for a thin metal sheet ruler is computed for different boundary fixings (free in rotation, clamped). Compare your solution with measured data. Hint: Use a spline curve with points with symbolic values for y and try to minimize the overall curvature.

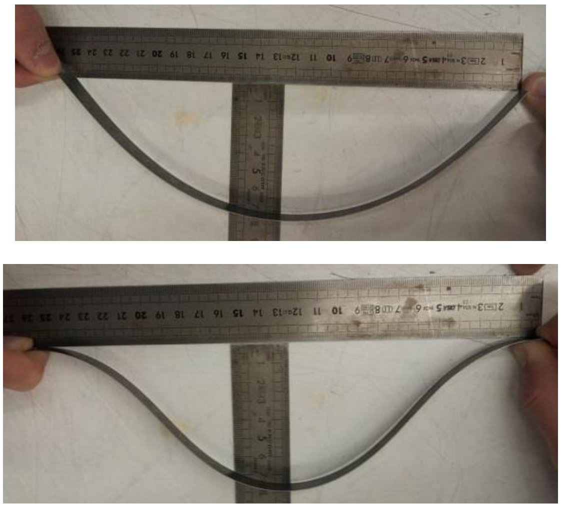

Fig. 4. Bending Situations

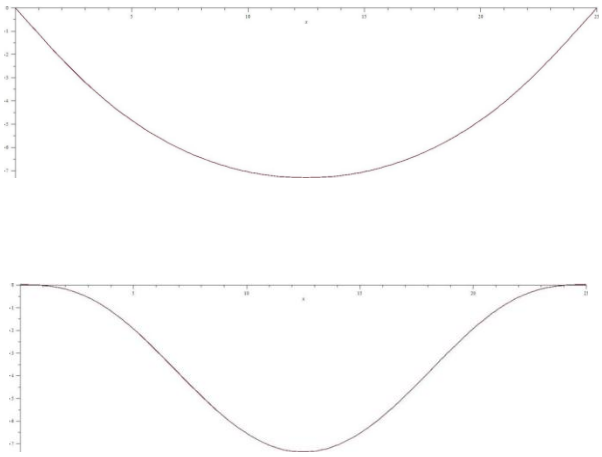

Fig. 5. Computed Curve in Maple ®

Without the hint given above students could go into different directions of modelling, e.g. setting up a differential equation for the bending curve. Given the hint, the project is more feasible since the modelling means (a spline function) and the mathematical problem (optimisation) are already given. Still, there are quite a few challenges left here for students when they want to make use of available technology, that is a CAS program (Maple®) with procedures for creating splines symbolically and an optimisation package. Students have to formulate the optimisation model where the integral of the squared curvature is the objective function and the length of the ruler is to be used as equality constraint. Dealing with the symbolic language used in the description of the optimization package is quite challenging for many engineering students. For providing initial values in order to help the optimization procedure they could do some optimization “by hand”, for example by trying out y-values and computing the overall curvature and the length. When comparing the measurements with the result of the optimization, different representations come into play. There is a function and a data set which should be shown in a common plot to see the difference; also, the largest difference should be computed. When students set up the spline function for further experimentation they have to decide on the number of spline points with undefined y-value. Here, they should apply the problem solving strategy of “starting small”.

3.4 Larger project

The following task makes use of laboratory equipment and material that can be found in many mechanical engineering departments or can be bought at a reasonable price. It involves the design and realization of a device where mathematical considerations play a role but are embedded in a variety of other engineering tasks:

- Design and create a machine for taking measurements on surfaces of machine parts using translational and rotational joints with distance and angle measuring facilities. Create a work-sheet which takes lists of measured data as input and provides point coordinates as output.

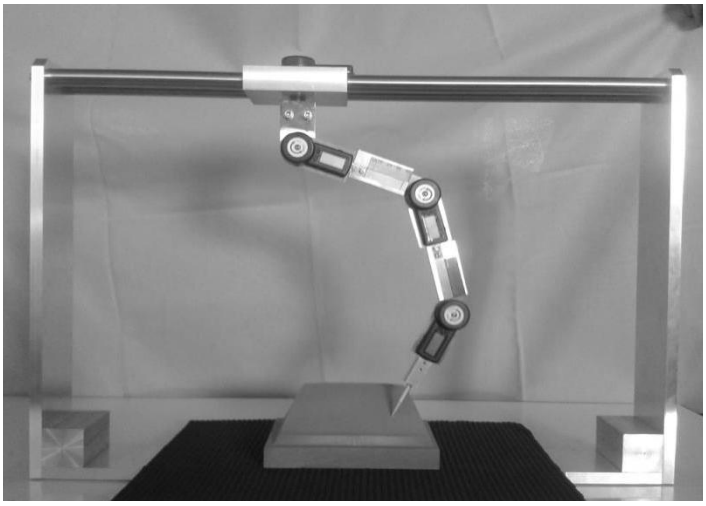

Fig. 6. Measuring Machine

In this task first requirements must be clarified regarding the size of objects and ways of measuring, e.g.: the maximum size should be a cubic with side length 10cm; it should be possible to measure the surface without turning the object. Then, a decision on how many joints to use and where to place them has to be made. Here, geometric operational thinking is required with rough estimations, and a CAD program could be used to check whether every point can be reached on a sample object. The second part is mathematical problem solving: Given the measurement data find an algorithm that determines the Cartesian coordinates. A strategy for this has to be found, e.g. to split up the problem into a plane one for the index arm to determine the height (z-coordinate) and the radius (of the circle you get when you rotate the arm), and the problem of how to get Cartesian coordinates from polar ones. By using technology (here: Maple®), the algorithm can be automated for lists of input data. For this, the formal programming language of the tool must be used.
Once a measuring device like the one depicted above has been built, one can use it for defining several further projects on fault propagation, accuracy of measurements, linear accuracy (measuring along lines or planes, work with least squares approximations), or on using measurements to create wire frame models of measured objects in a CAS program.

3.5 Bachelor thesis

In this example, mathematics again plays an essential but not a dominant role in a larger engineering task:

- Build a model of an ancient forging mill from 1610 (see Figure 7, taken from a multilingual digital mechanism and gear library (DMGlib) website to be accessed at http://www.dmg-lib.org/dmglib/main/portal.jsp?context_setLangCode=en) and design a good motion function for it. Test the motion function with the model.

Figure 8 shows the model of the forging mill with some modifications. For example, the four blades in the drawing were reduced to three since otherwise a collision might occur. Figure 9 shows the full test bench with a controlled drive for the wheel. A table describing the desired wheel motion can be downloaded to the control device. The main "parameters" influencing the behavior of the hammer are the geometric dimensions, the shape of the blades and the rotational motion of the wheel. Regarding the motion, it has to be clarified first what "good" means. Here, quality could be measured by the number of strokes per time unit and the load on the wheel (and hence its durability). As in the motion project in 3.2, students should see that a mathematical approach is called for here and that a function needs to be designed. When a blade touches the hammer, the wheel should be at rest in order to avoid an impact reducing its durability. It should then move up the hammer quickly and come to rest again at 120° for touching the hammer again. The motion is restricted by the data of the drive and there are two controversial consequences of speeding up: The part of the motion where the hammer is in contact with the blade needs less time but then the hammer "overshoots" and it takes longer to come down again. Interesting modelling tasks come up here in order to determine when the hammer leaves the blade. One can use mechanism simulation programs that come with CAD programs but these cannot deal properly with contact loss. Figure 10 shows the diagram of a possible motion function.

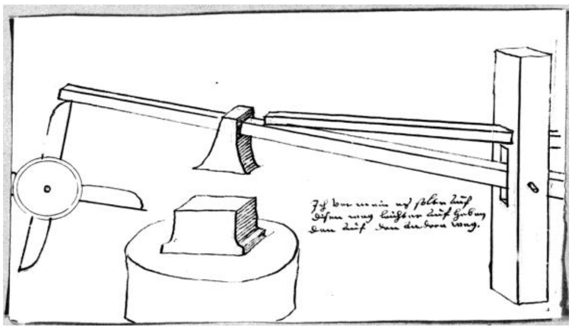

Fig. 7. Historical Sketch

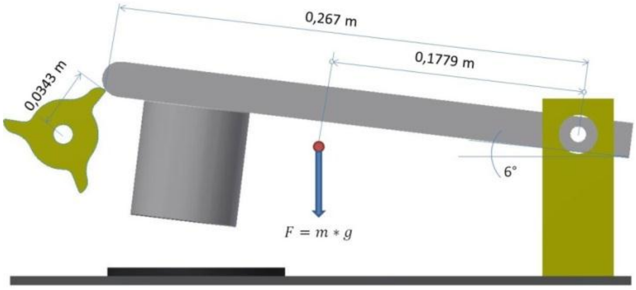

Fig. 8. CAD Model

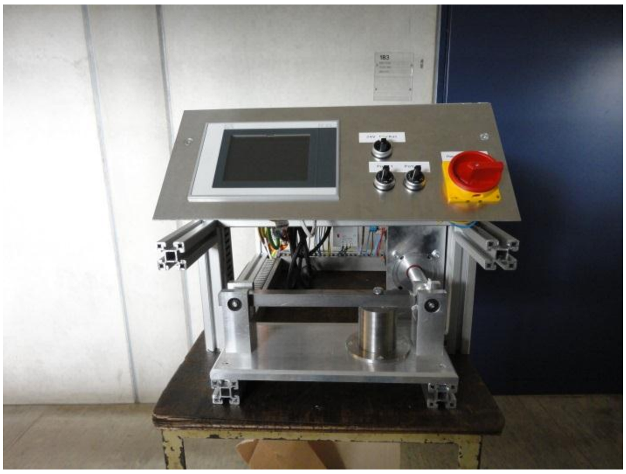

Fig. 9. Controlled Test Bench

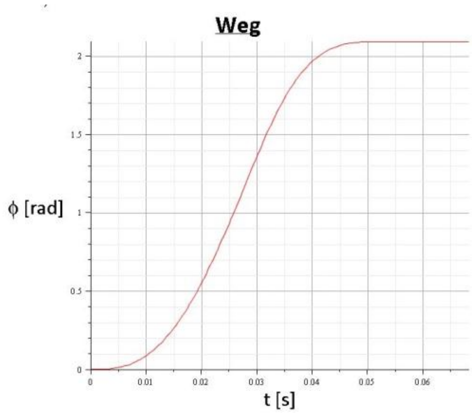

Fig. 10. Motion Function

4 SUMMARY AND OUTLOOK

In this contribution we briefly recalled the concept of mathematical competence that was used by SEFI's Mathematics Working Group when setting up the third edition of its curriculum document. Since the concept is still somewhat abstract and many mathematics lecturers are not familiar with the pedagogic language, it is helpful to illustrate the meaning of mathematical competence by providing example assignments on different time scales. In this contribution we gave some guidelines for identifying such assignments and provided examples for five categories of assignments where students encounter adequate learning opportunities.

The author is convinced that a bank of example tasks would help in disseminating the competence concept and putting it into practice on a larger scale. He is currently setting up a web database where assignments can be searched using criteria like the competency addressed and the mathematical or application topics dealt with. There will also be the possibility to enter new assignments via the web interface.

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