A mathematical lab for first year students
Learning by research

S. Born
Lecturer
Technische Universität
Berlin, Germany
E-mail: born@math.tu-berlin.de

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INTRODUCTION

The author is teaching a 'mathematical laboratory course' for first year students at the TU Berlin. This course is offered among several other labs (mechanical engineering, robotics, environmental engineering) within the special programme of an *orientation year* (“MINTgrün”). This programme has been offered at the Technische Universität Berlin since autumn 2012. It is designed for students that are interested in STEM but have not yet decided for a specific career. Most of them will eventually choose an engineering career, some a science career and some will do something completely different.

Students in the orientation year can choose among the introductory courses of most careers at the university. Apart from that we offer special courses that are meant to help students in their decision for a career, in particular our lab courses. Unlike ordinary science practicals these courses try to provide an occasion for an autonomous research acivity, albeit on a modest scale.

The setting of the orientation year provides some conditions that are relevant for the planning of the lab courses:

- The students have chosen this lab among different labs; they generally are *well motivated*.
- First year students form *a hetereogeneous group* as to knowledge and skills, due to the large differences in the German school system.
- The orientation year has no fixed curriculum, *students can reserve more time* for the activities related to a lab, if they want.

1 SCOPE OF THE MATHEMATICAL LABORATORY

The scope of the “mathematical laboratory course” is a rather broad one: to develop and understand mathematical models and to apply them for some practical task, by means of computer programs (mainly Python). Up to now, our students have chosen quite different projects, they have dealt with audio signals (synthesis, analysis, visualisation), images (image operations), robotics (PDI control of simple systems), geometry (finding shortest paths on a polyhedral surface, algorithmic creation of polyhedral surfaces), biology (simulation of fish swarms, ant colonies, population dynamics), mechanics (simulation of planetary motion) and data analysis.

One of the main assumptions of the laboratory is that *learning by research (or "research-based learning" [1]) should encompass a whole research process*, beginning with an autonomous choice of a research project, followed by autonomous organisation of the process in small teams, choices of tools and means, revision after failure, etc. [2]. This would favour a "deep approach to learning", but comes at a price. Traditional science practicals are often better in the systematic training of relevant skills, but lack this autonomous dimension. Both types of laboratories can and should have a place in an undergraduate curriculum. We believe – and have some confirmation – that the experience of team work on a research project can play a relevant part in the decision for a scientific career and therefore has a place in an orientation phase.

Apart from that, *the course gives a context for real applications* of first year mathematics courses for engineering or science students, which are usually taught with little or no reference to application contexts. A frequent difficulty of students to come to terms with mathematics may thus be relieved.

It is a challenge to design a course as a favourable environment for undergraduate research, *with an appropriate measure of guidance and freedom*.

2 HOW THE LAB IS TAUGHT

The following account gives details of the mathematical lab, in order to make it useful for other teachers. General ideas are not sufficient to make it work. The course takes place on four consecutive hours per week during the term and a final block in the term break.

2.1 The initial phase

This phase is crucial for the success of the class. We give a four weeks crash course in programming with Python, that is an introduction to Python *and* to programming. The problem sheets that go with the course go beyond simple applications of the programming structures just learned. *Many exercises are already small scale "projects".* Thus the students get a chance to develop *a problem solving attitude*. Simultaneously the students are invited to take a free online course (codecademy) with simple exercises in order to get programming routine. Students that have no previous programming experience have to invest plenty of time into this phase in order to become able to work on a project. Of course this wouldn't happen without intrinsic motivation. Our students have voluntarily chosen this lab among a variety of different labs and usually are curious and motivated.

What the teacher can do to foster this motivation is to spend time on the choice of interesting and diverse examples and exercises. Exercises have to challenging, but doable. Given the large differences among the students, it is therefore necessary to always offer a scale of exercises adapted to different levels of skills and knowledge.

Another important task for the teacher is to *create collaboration situations* for the students. Students that make the experience of good teamwork will be much more willing to spend time with their team mates outside the lab hours. Apart from the collaboration on the programming problems, which are solved in small groups, we *propose puzzles to the whole class*. This allows them to relax from programming and to engage in discussions.

To *provide examples of possible projects and encourage the choice of a project*, former projects are presented, if possible by the students who did them.

2.2 Project selection

After four weeks the students are supposed to form small teams and to identify an interesting (tiny) research project. The examples they have seen till then will of course influence their decisions, but every term there are new, sometimes surprising ideas.

The projects hitherto chosen differ widely, as to the complexity of the task. Some of them can be done with school mathematics, some go beyond the mathematics tought in the first university term. But in all cases the necessary mathematics and programming skills *can* be acquired within the course.

It would be possible to restrict the possible projects to variations of a finite list of proved and tested examples, but *we insist on the relevance of chosing a project of one's own.* Therefore, the teacher has the difficult task of estimating whether the project is feasible and giving advice accordingly. Of course, the teacher may be dumbfounded by a proposed project and unable to assess ist. In this case it is important to be explicit: You can pursue this project, but at your own risk. This has happened a few times and led to successful projects, though.

As an organisatory tool a web-document for simultaneous editing has proved useful: From the second week on students start to write project ideas and comments into this document. At the end of the fourth week everybody who has proposed a project gives a short presentation of his or her idea. Then, most students will group according to project ideas. Students who are still reluctant to do so are problematic: They apparently neither feel motivated for any of the projects proposed nor propose a project of their own. These students are likely to abandon the course.

2.3 Project management and documentation

Once the teams have formed, they have to *write a wiki page about their project*, containing the names of the members, the project goals or leading questions, assumptions made, material or theory supposedly needed for the project, and possibly a list of tasks and milestones. It has proved unrealistic to expect a gantt chart from the students. Instead, they are asked to continuously revise the project planning.

During the development of a project it proves vital *that the teams identify sub-tasks that can be treated independently*. This division of labour in a team guarantees that all members can contribute to the project, despite different levels of skills. The teacher observes the teams and, whenever some group members seem to be condemned to the role of bystanders, gently pushes them towards a distribution of tasks. Failing to do so puts the learning process of some team members at risk. On the other hand, the self-organisation of a team is an important aspect of research and related to relevant competences. Too much pressure would imperil the autonomous learning process.

The teams write *weakly protocols in the wiki* and are encouraged to *write all kinds of information in a private notebook.* These notes serve as a basis for a *systematic documentation of the project in the wiki.* The final stage of documentation can be a (scientific) article about the project, to be published in a blog or perhaps in some undergraduate research journal.

2.4 Learning the necessary theory

Ideally, students would write down their assumptions, compile a bibliography, read some relevant sources, and thence work their way through the literature in order to procure the necessary means for their project. This has happened in some cases, but in general more *help by the teacher is needed*. This might consist in *finding and providing pertinent books or articles*, but also in *offering additional lectures* on some of the theory needed. Several projects, for example, needed fourier analysis or discrete Fourier transformation. The students were given handouts with an introduction and exercises, and an 'ipython notebook' to try out the discrete fourier transform, but it proved necessary to complement this by a few frontal lectures, just to convey relevant ideas, as the students seemed to be lost in the details. Presently we are creating a small video based introduction to signals and systems with an emphasis on Fourier methods. In the future it will be possible just to give them access to this resource.

Since autumn 2012 we have been collecting and writing material for all kinds of projects, so that it has become easier and much less time-consuming to support the projects in the meantime. *A structured repository of all materials is an essential tool* for the teachers and will save plenty of time on the long run.

2.5 Programming

The introductory crash course is not sufficient, students will have to *continuously learn programming*, in order to make progress. Students with greater difficulties are encouraged to continue the free online course. The preferred way to learn is by writing programs related to the project, and here again it is important that everybody in the teams is programming, and not just one person. Now and then, *we insert short lessons on particular programming techniques* into the sessions.

Until now there was no further homework after the initial phase. We might change this in the future in order to give the students – in particular those that do less of the programming in their respective teams – more occasions to work on programs.

2.6 Puzzles

This element of the initial phase is continued during the whole course. It is the only part of the lab where *all students interact across the teams*. Puzzles (mathematical problems and modelling problems) consume time, but the students have confirmed that they consider these to be a valuable part of the lab.

2.7 Presentation

In the end, the teams present their work either in front of the other students or to a larger audience in a special event. Recently they expressed the wish that there should be shorter presentation sessions in intermediary stages of their projects, which might foster collaboration between teams, but also force the teams to reflect their projects. This will be done from the next term on.

2.8 Student assistants

For almost all aspects of the lab the participation of student assistants is important. They can serve as *mentors for all kinds of decisions to be taken in the teams*. It is often easier for a student to comment on the proceedings of a team without risking to intimidate them. Of course student assistants can also help with all kinds of problems that may arise. In general they should not come up with solutions but rather with hints how to find the solutions. Faculties' preocuppation whether undergraduate research is affordable can be relieved by an increased role of student assistants.

2.9 Trial and Error

One important aspect of research is the open outcome. Methods and means are chosen according to expectations, which may be frustrated in the end. In this case, one has to try again. An important virtue for a researcher is endurance with respect to frustration. It would be wrong to spare the students this experience, but it would be equally wrong to let first year students pass a whole term pursuing an impasse. For the teacher and the student assistants this implies difficult choices: When is it right to interfere with the work of a team?

2.10 Final block course

Even though the students in the orientation year have a free curriculum and *can* spare more time for the laboratory, there is a competition between the other courses and the lab. Students spend plenty of time for the homework of the mathematics courses and the preparation of exams. Therefore, some students will not do much for their projects between the weekly sessions of the laboratory. The final block course, which usually takes three consecutive days in the term break, provides a marvellous occasion for concentrated and uninterrupted work. It often allows the students to complete their projects in some sense.

3 HOW IT WORKED

3.1 Three examples

There were a variety of projects that dealt with audio signal and images. One project hat a rather vague goal to visualise music. The team started to use the audio data in the time domain and map them to an image. After that the team learned the basics of Fourier analysis and used discrete Fourier transform to produce an image. The programming technique to obtain a continuous image stream from an audio stream was beyond their means, so the teacher supplied bits of code. The documentation of this group was not very precise, but they gave a nice presentation. For an ordinary course on 'signals and systems' the learning result could be considered poor, but for an autonomous project of students who have never before programmed a computer and never dealt with signals, it is not so bad after all.

Another team wanted to experiment with sound synthesis. They were given some preliminary material on sound signals and then started to program sound generators. They used additive synthesis and were encouraged to learn the principles of Fourier analysis. It happened that one of the members was keen on understanding more about it and read a monography on Fourier analysis, solving the exercises therein during the semester. His partner continued to work on the synthesizer. In this case the partners learned different things in different ways.

Another team took the lab for a second term. They had an ambitious project to separate voice and piano in a music recording. The teacher made clear, that this is a hard and not yet completely solved problem, but that a partial success seems realistic. In the process they read original literature on source separation in audio signals and had to apply machine learning techniques to Fourier transformed signals. In this case the teacher helped a lot, arguably too much, but the students kept working on the project and learning about the methods and tools they needed. For some of the methods the understanding remained partial.

3.2 Grading

These examples indicate the individuality of the learning process, which poses major difficulties for a fair determination of grades. The course is a 6 ECTS-points graded module. Grades should at least relate to intended competences, and of course presentation, documentation, planning and programming exercises can be reasonably graded. However, when it comes to the competences displayed in the results of the project, the differences in complexity of the different projects thwart any comparison. Moreover, many of the wished for effects cannot be measured, and as Huber [2] points out, it would be preferable not to grade such a course. Grading in this case is a sacrifice to compatibility with other universities.

One effect beyond “competences” would be the development of a problem-solving attitude and confidence in one's own ability to solve a problem. This might have positive long-term effects not only for a scientific career. Indeed, we hope to track our students for a longer period and to find out about possible effects.

3.3 Evaluation

A questionnary was used to evaluate what the students think about this course. Most of them would recommend the lab, found it interesting and meaningful and have learned something about their interests. Most did not find that the laboratory helped them choose a career. The majority considered the contents of the course medium-difficult or difficult. Most claimed to have learned something from their team mates, and most doubted whether their team mates have learned from them. Some were not content with the collaboration in the team.

There is no out-of-the-box recipe for research-based learning scenarios. We (teacher and student assistants) keep observing the work of the students and keep modifying the scenario. In the beginning we offered a guided version of the lab, closer to a traditional practical course, for students who could not decide for a project: These students could explore a certain domain via a series of tasks and exercises proposed by the teacher. In the last two semesters, however, we insisted that everbody choses a project. This is more in line with the general idea of the lab, but we lost a few students who apparently did not want to choose a project. In the future we will probably reintroduce the possibility of a guided tour through some domain.

On the whole, our observations and the comments of the students suggest that the mathematical lab course is a sensible course. We believe (and have some evidence) that students have acquired competences relevant for MINT careers. The large spread of the projects' results is not to be wailed about. It is but the natural consequence of an open scenario which relies on self-determination.

The number of female students chosing this lab course has been increasing since the beginning. We suppose that this is related to the extension of the domains of possible projects.

4 CONCLUSION

This laboratory requires much work and a high degree of attention from the teacher, probably more than in traditional teaching scenarios. On the other hand this course has been a rewarding experience for the teacher and the student assistants. A major role is played by the ever changing projects, which increase the work load but also the motivation of teacher and assistants, who keep learning while their students learn. I would consider the curiosity of the teacher a necessary condition for a successful 'learning by research'-scenario.[1]

[1]It actually helps to teach a course which is related to one's interests, but not in the very centre of one's scientific expertise. I am an algebraic topologist, but have been teaching mathematics for engineers and physicists for years.

The above sketch of the course mentions several tools used: The wiki, an online document for simultaneous editing, private journals, a repository of material. One should not underestimate the impact of the right tools. Especially the wiki has proved crucial for the success of the lab. It allows students to organise and document their projects, to view the documentation of other projects and to access information provided by the teacher. In a longer perspective one may hope to share material with teachers of similar courses in a common repository. If undergraduate research is to become accessible for all students, structures have to be built that make it easier for teachers to create appropriate scenarios. This includes practical tools, but also the adaption of curricula. The orientation year is a very special case which provides good conditions for our laboratory.

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