Is there a special Maths for engineers?

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INTRODUCTION

Recently, we can register many calls for improvements in the basic knowledge of secondary school graduates who become first-year students at universities worldwide. These needs appear mostly in connection to reading, writing, calculating and other "old-fashioned" skills, among which logical reasoning and mathematical skills become one of the less developed in the course of secondary education.

A steady knowledge of mathematics is a good background and necessary prerequisite of a future successful engineering career. Technical universities, in particular, provide higher education based primarily on these skills. Therefore, many efforts have been given to finding suitable ways of how to improve the mathematical knowledge of engineering students. Teachers were usually striving to find some new didactic methods that would be working more effectively, for the utmost benefit of students. Innovations were aimed to make the study material to be more comprehensible, better readable for less prepared students, freely available and accessible via all information and communication technologies. There appeared also a general call and a tendency started to be fostered towards needs to create a new curriculum, for a kind of so called "engineering mathematics", which could be more appealing and easier to grasp for students at technical universities and in engineering study programmes, [1]. In this paper we would like to bring few remarks and ideas in connection to this call for specific ways for mathematics to be taught in the engineering studies at technical universities.

1 INITIAL CONDITIONS

1.1 Role of maths in engineering

Undoubtedly, mathematics at engineering study serves as a tool used to solve various kinds of technical problems. Often it is necessary to use some very advanced parts of mathematics in order to find solution to new unforeseen problems arising from the rapid development of technology nowadays, [5]. Basic mathematics knowledge is absolutely necessary in order to understand these advanced parts of mathematics, which is itself developing in the same rapid rate as all other disciplines. To our knowledge, even more mathematics is necessary today than it was ever before, specialized, more sophisticated, theoretical and advanced, and definitely less easily comprehendible. We will bring arguments to support some of the above ideas and discuss how to proceed in order to teach mathematics better, and hopefully enable students to acquire more proper understanding of this important tool in engineering study and practise.

1.2 New technology in university education

Changing strategies adopted by actors, both teachers and learners, in the hypermedia effected educational environment of our everyday life must have been noticed by community of interested professionals in the mirror of decreasing overall level of basic knowledge, skills and practical abilities in almost all fields of human intellectual activities, [6]. Even laymen call for changes in education, as they are more and more frequently faced with consequences of this undesirable development, although they are not directly involved in the educational process. Level of general public education and behavioural standards of the society go hand in hand with the profile of the established educational system and its results.

Unfortunately, not always professionals, who are directly involved in the process and understand its rules, practise and far-reaching implications, are allowed to be in charge of suggesting reasonable and beneficial changes and transformations of this critical social structure. For a layman, just usage of ICT in educational environment as a fashionable up-to-date communication practise seems to be a perfect remedy to all problems faced while addressing young generation in the post-information 21st century. Pedagogical community argues that basically ICT offers just tools; though rather powerful as they can enable learners to engage in forms of education that were previously impossible. High technology equipments can serve just as means by which important educational outcomes could be achieved. Pedagogical philosophy of the teacher who remains, despite the new circumstances, the driving force of the instructional process responsible for setting up the environment plays the decisive role in describing the settings.

Kirkwood and Price argued in [3] that "*Technology-led innovations do not in themselves lead to improved educational practices. Too often, it seems, technologies have been introduced to university teaching with little or no consideration being given to the implications for student learning. For example, adding computer-mediated communication to a course will not in itself generate collaborative or co-operative working; neither will it induce dispersed students to form themselves into a learning community*.". Sept further comments in [4], p. 49: "*It is ironic that in the so-called Information Age we are still graduating passive, solitary learners poorly equipped to cope with the explosion of information resources competing for their attention*.".

Hypermedia, including ICT, can help to introduce new forms of teaching and learning. What they surely cannot is the achievement of effective and appropriate learning outcomes. This serious message based on research evidence from many different contexts permanently fails to have a significant impact in reforming policies and practices. Environment of higher education has certain particular distinguished features. Therefore usage of ICT has been introduced here in specific ways. Due to offer of new communication facilities and a large capacity of the available storage place for delivery of instructional materials, first ideas were to think primarily about the change of delivery and form of provided teaching and learning materials. No change in general conceptions of teacher-centred teaching as the transmission of knowledge towards the learner-centred facilitation of learning appeared. Both teacher and students have been technology driven to produce and use online teaching supports and to take advantage of on-line operation of teaching, learning, and administration. However, the governing variables of the educational process has not been addressed and strategies have been employed merely as a mechanism to change the circumstances without a clear understanding of what circumstances need to be changed, and why, see in [3].

As Bates points out: *„Good teaching may overcome a poor choice in the use of technology, but technology will never save poor teaching; usually it makes it worse*.", in [2], teaching and learning in higher education is unlikely to be improved simply by the application of a new technology. Conversely, when ICT is pedagogically integrated into the course design and adapted for the current environment, it can enable and support enhanced forms of learning. Therefore, it is extremely important to evaluate ICT use in the particular pedagogical context in order to understand how to use it effectively in future course designs.

Scientific discussions went on how to find effective path to promote science and technology and to increase the interaction between students and teachers, and students and students, as for example in [8]. Science and technology curriculum should stimulate reflective thinking with the aim of letting students learn in a meaningful ICT learning environment to become engaged in science and technology, in order to improve student´s learning performance. High-tech skill development should be closely followed by the use of integrated multimedia technology including pictures, cartoons, films, sound effects, characters and pronunciations. Scientific learning is difficult and meaningless for engineering students if they are just asked to memorize and recite facts to pass examinations.

Mathematics is subject, in which usage of ICT cannot be neglected. Dynamic mathematical software solutions and computer algebras available on market or as freeware provide a wide choice of remarkable educational tools suitable for specific needs of pedagogical process at different levels of educational system. These can bring a lot of benefits, but a vast damage, if used inappropriately. Therefore, more than ever before, the role of teachers is not suppressed, rather even stressed by hypermedia usage. With the new environment they became designers not only of the content and its delivery, but also ways of practising new material outside the regular educational process in classroom, in addition to the direct contact educational hours in there, as mentioned e.g. in [7].

2 DEVELOPING MATHEMATICAL COMPETENCIES

2.1 Mathematical competencies in the curricula framework

Concept of competence in terms of education has been described in various resources. The process of education and obtaining knowledge is currently closely connected to the process of acquiring specific competencies during the teaching - learning activities that are related to particular subjects.

"Mathematical competence means the ability to understand, judge, do, and use mathematics in a variety of intra and extra-mathematical contexts and situations where mathematics plays or could play a role. Necessary, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills, in the same way as vocabulary, orthography, and grammar are necessary but not sufficient prerequisites for literacy", [9], [10].

The latest publication offering of the SEFI Working Group on Mathematics and Engineering Education introduced during the last seminar held in Dublin in June 2014 is the main document for providing orientation to the community of mathematics lecturers in engineering education - the Mathematics Working Group's curriculum document. The 3rd edition of this document appeared in September 2013, [1]. It is entitled "A Framework for Mathematics Curricula in Engineering Education" and serves mainly to clarify the main objectives of mathematics education in the third millennium. The third curriculum edition is primarily based on the 2nd edition of the curriculum document from 2002 called "Mathematics for the European Engineer. A Curriculum for the Twenty-First Century" (downloadable from the webpage http://sefi.htw-aalen.de/), but it is updated and elaborated in terms of learning outcomes. The current edition is based on the concept of mathematical competence which has been developed in the Danish KOM project [9].

The main message of the curriculum framework document is that also content remains utmost important, the knowledge should be embedded in a broader view of mathematical competencies related to specifications of the respective application domain, scientific branch and technical discipline. Therefore, in this document you will not find a comprehensive answer to many questions regarding the design of a teaching process, but rather an offer of a helpful orientation in the variety of teaching and learning environments and approaches which help students to obtain required competencies to an adequate degree, level and in a relevant depth. The document presents also several forms, how the competence-based approach to mathematics education can, and obviously must, be integrated into the surrounding engineering study course, which really could support achievement of the ability to use mathematics in engineering context intentionally and in a meaningful way. Finally, it is utterly important to address the competency acquisition also in assessment schemes, as the assessment procedures considerably determine the behaviour of students during the complete study program.

There can be quoted at least two reasons for which the presented concept of mathematical competence was adopted for the third edition of the MWG curriculum document. One of them is the fact that focusing on the above defined concept of mathematical competence in teaching engineering students the essential goal of mathematics in engineering education can be emphasized straightforwardly: it is the cultivation of the ability to apply mathematical concepts and procedures in relevant contexts, ability to work with engineering models and solve engineering problems using acquired mathematical knowledge and skills. On the other hand, this approach explicitly recognizes that competence requires a steady base of knowledge and skills. This idea is strongly supported by many experienced teachers of mathematics at technical colleges, faculties and universities throughout the Europe. Current trends in general engineering education also prefer the concept of competence which has been frequently used to describe educational goals favouring action-based knowledge over formal knowledge held just for the excellent performance and effectiveness at exams.

The new core curriculum structure enables a high level of freedom in designing courses of mathematics at TU, with respect to preferred and required contents of the respective course in accordance with the particular technical branch and orientation, available educational environments, used didactic and assessment methods, tools and instructional materials. Various ideas and good practice examples are presented and discussed, based on practical experience, results and deductions of numerous discussions and analyses that have been presented orally or as paper contributions of participants in several issues of the Proceedings of the SEFI MWG seminars held regularly, and addressing intentionally topics covered in this presented third edition of curriculum document.

General mathematical competencies for engineers are defined as follows

- thinking and reasoning mathematically
- posing and solving mathematical problems
- modelling mathematically

- representing mathematical entities
- handling mathematical symbols and formalism
- communicating in, with and about mathematics
- making use of aids and tools.

Practical examples of problems leading to development of competencies are presented with discussion about how to emphasize the teaching goals. Criteria for specifying and measuring students' progress in achieving these competencies are presented too, as degree of coverage, radius of action and technical level.

A thorough analyses of content-related competencies, knowledge and skills is provided. These are distributed to 8 groups: quantity, measuring, space and shape, functional dependency, relations between functions, data and chance, algorithms, and modeling. Next, the basic learning outcomes of the Core Zero in mathematics are presented, covering basic skills and mathematical competencies from Algebra, Analysis and Calculus, Discrete Mathematics, Geometry, Linear Algebra, Probability and Statistics, which should be studied before entry to an undergraduate engineering degree program. Anyhow, recently this material must be to a large extent also covered during the first year of a university engineering course. Further, Core Level 1 is presented, covering the basics to all engineering disciplines, while material in Level 2 is advanced enough for simple real engineering problems to be addressed. Mathematics techniques covered in Level 3 are advanced methods that should be applied to a range of problems from industry encountered by practicing engineers.

Teaching and learning environments are described in details in chapter 4 of the curriculum framework document, presenting various arrangements in terms of possibilities how to foster acquisition of presented 8 competencies. Related transition issues and usage of mathematical technology are mentioned as well, with possible drawbacks caused by inadequate usage of ICT.

Another important issue related to mathematical learning and teaching focused on competencies acquirement is connected to the adequate assessment strategies. Various forms of assessments, requirements for passing, new ideas how to assess competencies and questions regarding technology-supported assessments are deeply analyzed in chapter 5. Gathering of various mathematical tests, problems to be solved or projects to be elaborated in the course of examinations and any information about practical experience with using specific assessment strategies is the subject of the on-going project led by the SEFI MWG members. It is aimed at categorizations of available and used assessment forms, their short description and analysis of their relevance with respect to addressing and assessing mathematical competencies of students actually acquired in the respective mathematical course in comparison to the intended and foreseen competences to be developed in the scheduled scope of the course. One of the crucial open problems in adopting forms of a proper assessment of knowledge and competencies dwells indisputably in the questions if we are really assessing skills that were actually practiced in the respective course. Are we really, rather than this, not assessing just the bits of knowledge that our students learned by heart in a short time before the exam by themselves, which were not very much directly practiced and well trained in the course?

2.2 Strategies for building competencies

Mathematics is one of those subjects, in which teaching requires illustrations of abstract theoretical concepts and relations in a suitable way on examples, with aim to help students in better understanding of connections and enable them to attain better insight to the presented problems. Dynamic mathematical programmes are ideal educational tools enabling visualization of mathematical entities in the form of dynamic models. Direct interactive manipulations with models offer possibilities to users of heuristic approach in acquiring knowledge, while learning of theoretical data is directly connected with their practical application in the respective dynamic model. Generation of model itself and also work with it during study require a new attitude to the role of teacher and student in the educational process, change of the form of educational environment and contents of the educational process.

Research in the domain of cognitive psychology shows that human brain stores knowledge in two forms: in the graphical form as images, or in the form of words. It has been documented that methods of education, which are oriented to development of both of these forms of knowledge acquisition, in a considerable way influence quality and depth of understanding and sustainability of acquired knowledge. Direct inclusion of learners into the development and utilisation of non-linguistic representations within the process of learning considerably stimulates and increases brain activity. This further leads to the development of cognitive connections, which consequently foster knowledge and deepen understanding of basic principles and concepts. Manipulative techniques and tools are concrete or symbolic artefacts that students directly use while obtaining new pieces of knowledge. These powerful didactical tools enable active, hands-on explorative methods and heuristic investigation of abstract rules and relations. Research results prove that computer aided manipulative techniques and tools are even more effective than usage of physical objects, i.e. three-dimensional models, because they can dynamically connect several possible representations and interpretations of studied concepts, correspondences and relations.

Visualization of basic concepts, relations and dependencies is undoubtedly an integral part of the education in mathematics. It works with models of these abstract entities, which can appear in various forms, for instance as three-dimensional real models, different didactic tools, images, maps and graphs, films and video presentations, or as concrete activities during the teaching process and practical modelling, i.e. development of models in some way visualizing particular concept or relation. Today, applications utilizing latest information and communication technologies should be included unarguably. Dynamic models play one of the formative roles in the process of knowledge acquisition, as they stimulate cognitive processes and enable development of life, interactively manipulative cognitive connections, see in [6].

Both subjects of the educational process act in this didactic situation more as equal partners, not as it is usual in the classical forms of didactic situations, where the role of teachers is active presentation of new facts and data, while role of learners is usually passive, just receiving presented facts. Active participation of students in the process of education in an interesting form can contribute to a better understanding and definitely to a more positive approach to learning itself, which becomes more a discovery of dependencies and investigation of activities and processes than memorising of a huge amount of incomprehensible facts and data which are not connected. Dynamics opens way to discover connections, and to understand mutual dependencies, which is often more important than a detailed fragmented knowledge. Therefore, a dynamic learning mode is one of the key strategies to be applied for building conceptual understanding, fostering various representations of mathematical entities, communicating in and with mathematics, and modelling mathematically using aids and tools, which is a practical way for development of several mathematical competencies useful for engineering practise.

Finally, it is necessary to admit that a pure ability to plot and graphically interpret mathematical object itself does not mean any full understanding of the determining mathematical relations geometrically represented by the generated visual image.

Dynamic work with these objects is necessary for achievement of a full deep understanding and for development of abilities to utilise technologies for analysis and solution of problems. In engineering these abilities form the background of the creative work of future engineers and they need to obtain them during their studies. On the other hand, this is fully consistent with the general understanding of mathematical knowledge as a sustainably increasing ability to utilise various representations of mathematical concepts in different contexts, exchange them dynamically according to actual needs, and be able to illustrate and apply these concepts correctly.

3 SUMMARY AND ACKNOWLEDGEMENTS

Often, university students' expectations of teaching and learning differ from those of their teachers. Some favour a didactic, materials-centred approach without teacher's interventions to the learning process, but many students also expect to be taught in a transmissive way, following teachers working as researchers investigating new principles, discovering rules, and introducing new concepts properly based on results of the performed explorations. University teachers have a vital role to play in enabling students to challenge their existing intellectual potential and to develop more appropriate practices for effective learning and practical applicability of acquired knowledge. The key role of "educators" in this process of "education" in a broad sense is indisputable. Should it be using new information and communication technologies or in traditional ways, is a questionable issue. Understanding these tasks and difficulties emerging from the demanding and responsible roles of teachers/students and awareness of necessary adequate support that these positions deserve is far more fundamental to the success of both, than anything else.

There is no special Maths for engineers. Anyhow, there are special ways how to bring this powerful tool and assistance aid to engineering students, and make them understand how important role Maths might play in their future professional lives. Mathematical thinking and reasoning can make the responsible, hard and demanding engineering work easier, more precise and reliable, predictable, and sometimes even much more creative and useful. The ability to turn a real life problem to a well posed mathematical model and to find its abstract solution using various aids and tools, to communicate this solution, visualize, test and optimise it using computer aided design and modelling tools, and finally to be able to apply it back to the practical realisation might lead to previously unforeseen admirable technical engineering solutions and constructions. Sometimes, also considerably cheaper, as mathematical models, apart from real ones, are of almost no costs, if you are able to handle them properly. Many examples from nowadays practise can be mentioned here, where technical realization would be impossible without precise mathematical models and their optimal solutions found nd investigated using the most advanced powerful information technologies.

Mathematical attitude adopted to solving engineering problems could enable future engineers to change our living environment in a sustainable way, more adequately in order to cover demands of both, nature and greedy mankind in an appropriate way. In other words, to make consequently our lives safer, healthier, and more convenient.

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