European Society for Engineering Education (SEFI)

A Mathematics Curriculum for a Practice-oriented Study Course in Mechanical Engineering

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Contents

"The proof of the pudding is the eating." (Old English Proverb)

1 Introduction

The SEFI Mathematics Working Group's Curriculum Framework Document (Alpers et al. 2013) suggests a certain way to specify a mathematics curriculum for a concrete study course in engineering. The approach is based upon the concept of mathematical competence which is defined as "*the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role*" *(Niss 2003, p.6/7)*. In order to make the concept better usable, mathematical competence is split up into eight overlapping mathematical competencies which are: thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modeling mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating in, with, and about mathematics, and making use of aids and tools (see chapter 2 for short definitions). The competence concept places emphasis on the practical use of mathematics and seems therefore to be particularly suitable for the mathematical education of engineers. The approach serves to recognize and take into account higher-level learning goals beyond the mere learning of mathematical concepts and procedures. Nonetheless, it is also well-recognised that a lot of knowledge and skills is necessary for making use of the competencies, and the framework document (Alpers et al. 2013) also contains a structured list of possible content-related learning outcomes.

In this document, we want to make use of this framework in order to specify a mathematics curriculum for a concrete practice-oriented study course in mechanical engineering. This serves to give an example of using the framework document and should make it easier to set up a mathematics curriculum for other engineering study courses by modifying, omitting or adding to the current document depending on the specifics of the study course.

Our specification is based on an analysis of the mathematical challenges students meet in the application subjects of the study course. For a practice-oriented study course in mechanical engineering the "heavy users" of mathematics include: Engineering mechanics, stress theory, physics, CAD, machine elements, thermodynamics, fluid dynamics, electronics, informatics, measurements, control theory, machine dynamics, applied FEM, and quality management. The mathematical concepts and procedures occurring in these contexts have been identified and the necessary content-related abilities have been derived. Within the application subjects, programmes are used which are based on mathematical concepts which are to a certain extent visible at the user interface. These include: machine element computation programmes, CAD, FEM, CFD, multibody dynamics, measurement data processing programmes, control programmes.

Based on this analysis, the intended level of progress in each of the eight mathematical competencies stated above needs to be specified. The current document provides the result of this process and is not a description of the process itself. In chapter 2 we specify the level of progress for each competency. We use the "dimensions" suggested by Niss (2003): in the "degree of coverage" dimension, the aspects of the competency to be addressed have to be stated, e.g. active modeling or just understanding of given models constructed by others; in the "radius of action" dimension the contexts and situations have to be specified in which the competency can be activated; and in the "technical level" dimension one has to state how advanced the mathematical concepts and procedures are which can be used when activating the competency. In chapter 3 we specify in more detail which content-related mathematical abilities are considered to be necessary to achieve the technical level stated in chapter 2. For this, we took the list of intended content-related learning outcomes presented in the Framework Document and chose the suitable ones. We made also minor additions when justified by use in certain application subjects within the study course.

Chapter 4 makes some suggestions on how to achieve the goals mentioned in chapters 2 and 3 and how to assess the learning outcome. We also state in how far the goals are rather achieved in the proper mathematical part of the study course or within application subjects (see section 4.4 of the curriculum framework document). Finally, in chapter 5 we give some hints on potential directions for future developments of the document.

2 Specification of Mathematical Competencies

In the following we state the learning goals regarding the mathematical competencies. Note that this is a normative activity based on experience concerning the use of mathematics in application subjects and in later work places. The goals are not derived from underlying theoretical considerations.

2.1 Thinking Mathematically

We cite the definition of this competency from the framework document: “This competency comprises knowledge of the kind of questions that are dealt with in mathematics and the types of answers mathematics can and cannot provide, and the ability to pose such questions. It includes the recognition of mathematical concepts and an understanding of their scope and limitations as well as the ability to extend the scope by abstraction and generalization of results. This also includes an understanding of the certainty mathematical considerations can provide.” (Alpers et al. 2013).

2.1.1 Degree of coverage

In the following we state the learning goals regarding mathematical thinking which should be covered in a practice-oriented study course in mechanical engineering.

a) *Students should recognize that mathematics provides the means for describing important quantities and their relationships in engineering application scenarios. These are the models which constitute engineering as a science different from craftsmanship. Therefore, students should develop a “mathematical view” on situations such that they look for possibilities of mathematisations which might be finding quantities and their relations or patterns or rules from which properties might be deduced.*

<u>Examples:</u>

- Students should be aware that
  - mathematics provides different kinds of numbers to quantify “things”.
  - mathematics provides the concept of a vector for modeling directed quantities.
  - mathematics provides the concept of a function for describing how one quantity depends on another.
  - mathematics is able to formally describe geometric entities of different complexity.
  - mathematics provides the concept of equation to relate different (maybe time-dependent) quantities.
- If students are interested in the moment of inertia for a certain area, they should expect that there is a formula based on geometric data.
- If students have a set of data they should expect that mathematics might provide them with the means of systematic treatment (e.g. compute mean and variance for random quantities or use linear or quadratic approximations or answer the question what kind of relationship the quantities occurring in the data adhere to).

b) *Students should understand that the mathematical quantities and relationships are simplifications and idealizations of reality which do not cover the latter fully and exactly. They should know that the certainty of mathematical considerations is based on the correctness of underlying assumptions.*

Examples:

- If forces are modeled as point loads using vectors than students should see this as a simplification.
- Students should not expect measured quantities to be exactly the same as computed ones.
- Students should understand that a model based on linearization has a restricted area of application.

c) *Students should understand that mathematics provides ways to logically deduce assertions or properties which gives certainty. This certainty can reduce work in that it might tell if something works (e.g. converges) or is definitely useless (e.g. does not exist) or it might give information on the classes of possibilities.*

Examples:

Students should be able to understand argumentations like the following ones and appreciate their value

- Can one prescribe the distance, velocity and acceleration at the beginning and the end when one wants to use a polynomial of degree 3 for constructing a motion function? In general no, we have 4 degrees of freedom and 6 conditions and one can construct a counter-example.
- Can one determine in a certain (statically over-determined) truss the forces acting in the bars using the nodal point procedure? No, the procedure provides a number of linear equations which is lower than the number of forces to be determined, so one cannot have a unique solution.
- One can prove that in a strongly damped mass-spring system you will have at most one extremum.
- In a damped mass-spring system a mathematical argumentation leads to the assertion that there are three solution possibilities.
- Can one construct a regular hexagon geometrically in a CAD system such that one can change the length of one side without destroying the regularity? Yes, there are operations for constructing the hexagon form based on one given side in the CAD system that leave the property of regularity invariant.

d) *Students should know that mathematical considerations help in determining the influence of quantities (or, the other way around, the “sensitivity” of other quantities with respect to changes in certain given quantities) and can therefore serve to identify quantities of essential influence.*

Examples:

- The formula for computing the moment of inertia for a rectangle (centre of gravity in 0) with respect to the x-axis is $\frac{1}{12}bh^3$. From such a formula students should be able to deduce that changing the height is much more effective than changing the breadth.
- The partial derivatives of a function of two variables at a certain point are 0.2 (wrt. x) and 4.2 (wrt. y). Students should see that the dependent variable is much more sensitive to slight changes in y than in x. Partial derivatives might be replaced by difference quotients if values are just computed by a programme such that no formal derivative can be calculated.

e) *Students should know that mathematical considerations help in ensuring that certain quantities meet given requirements by adequate dimensioning of other quantities.*

Examples:

Students should be able to apply the following reasoning:

- How do I have to choose the dimensions of a machine element such that a certain load can be carried? Mathematics should provide a way to compute the stress that is caused by the load, and a comparison with the maximum bearable stress should answer the question.
- Which torque must a drive of a linkage deliver such that at the driven part a certain torque is achieved? Mathematics should model the relationship and then it should be possible by mathematical methods to answer the question.

f) *Students should know that mathematical considerations help in improving or even optimizing certain quantities which are interesting in an application.*

Examples:

- Students should know that mathematics can help in improving a certain quantity (e.g. the overall cost or the largest stress) by providing relations between "design variables" which can be influenced and the ultimate goal variable. "Educated" trials might then lead to an improvement.
- Students should know that mathematics also has to offer optimization models and procedures for finding optima.

g) *Students should know that mathematics can be used not only in an analytic way but also to design behavior or objects with desired properties. They should know that in design tasks there is usually a huge design space within which heuristic arguments are used to proceed.*

Examples:

Students should be able to recognize the following usage of mathematics in design tasks:

- Motion design is an important branch of mechanical engineering. There, time dependent behavior must be specified. For this, mathematics offers the function concept and lots of standard function types like polynomials. Mathematics should help in the design in the sense that it allows to transform smoothness (e.g. continuous velocity and acceleration) requirements into equations in order to find a suitable function. There might be heuristic choices concerning the type of function to use (e.g. polynomial, spline, sine function).
- In geometric design, a geometric object is to be constructed given some restrictions and quality criteria. Mathematics helps to define types of geometric objects und to construct new ones from given ones. It also allows to formalize this process such it can be done automatically in a CAD programme.

h) *Students should gain an insight into the value of abstracting from real situations because it allows to use the same (or similar) mathematisations in different circumstances. They should see the same mathematical concepts in different situations.*

Examples:

- Students should recognize that second order differential equations are useful to model the behavior of a damped mass-spring system in mechanics as well as an oscillating circuit in electronics.
- Students should recognize that once relations have been specified as systems of equations the generic procedures for solving them can be used.
- Students should understand that all questions where a quantity is to be optimized can be formally specified in an optimization model with a goal function and different types of restrictions such that one can try to get answers by using optimization software.

In order to clarify the degree of coverage, it is also useful to briefly state what is not intended to be covered (but certainly will be in another curriculum with a different profile). As a mathematician one normally sees mathematics as a body of knowledge based on axioms where theory is developed by applying formal logic. Although this is in general an important aspect of mathematical thinking, this aspect is not included in the profile specified in this document.

On the other hand, what we also do not want is that doing mathematics is seen as an isolated performance of dull procedures where transformations are made according to some rules and the results are more or less meaningless expressions obtained for their own sake.

2.1.2 Radius of action

The radius of action, i.e. the contexts and situations where students should be able to make use of the mathematical thinking competency, comprises the following fields:

- Students should be able to use mathematical thinking in their application subjects. It should enable them to see where a mathematical perspective could be of help and how mathematical work could help them to answer typical application questions like “What are suitable dimensions of a machine part?” or “What kind of controller should I use and how should I parameterize it?”. Students should know that mathematical reasoning is needed in application subjects in order to derive formulae and models, or in order to determine the completeness of coverage of all possible cases.
- Students should make use of mathematical thinking when they use application programmes for engineering tasks like CAD, machine element dimensioning, or FEM programmes. They should think about what the programme needs as mathematical input and what can be expected from the programme given the fact that it implements a certain mathematical model. Moreover, they should have an expectation of the mathematical form of the output (color coded pictures, data files, function graphs, …).
- Students should apply the competency when communicating with experts. They should have a sense of where a mathematical approach could help them in their problem solving process and whether they could perform that on their own or should contact an expert and they should have an impression of what to expect from an expert.

2.1.3 Technical level

Regarding the technical level, students should know the following mathematical concepts in order to recognize potential for mathematisation, pose mathematical questions and have expectations on the kind of answers a mathematical approach could provide:

- Knowledge of different kinds of numbers for quantitative description
- Knowledge of vector quantities and their properties
- Knowledge of algebraic expressions and their usage for calculating values
- Knowledge of functions to describe relationships and dependencies
- Knowledge of function properties in order to see what an approach using functions could deliver in form of answers to questions (e.g. maximum value)
- Knowledge of differential equations to describe relationships between functions and their derivatives
- Knowledge of geometric entities and properties for describing geometric configurations
- Knowledge of elementary stochastic terms to describe quantities and situations influenced by chance
- Knowledge of formulations of assertions and reasoning in order to pose problems and have an idea of what answers look like.
- Knowledge of the mathematical concepts mentioned in the overarching themes stated in the framework document in order to understand where mathematics comes in.

A more specific choice on the concepts will be made in chapter 3.

2.2 Reasoning Mathematically

We cite the definition of this competency from the framework document: "This competency includes on the one hand the ability to understand and assess an already existing mathematical argumentation (chain of logical arguments), in particular to understand the notion of proof and to recognize the central ideas in proofs. It also includes the knowledge and ability to distinguish between different kinds of mathematical statements (definition, if-then-statement, iff-statement etc.). On the other hand, it includes the construction of own chains of logical arguments and hence of transforming heuristic reasoning into own proofs (reasoning logically)." (Alpers et al. 2013).

2.2.1 Degree of coverage

In the following we state the learning goals regarding mathematical reasoning which should be covered in a practice-oriented study course in mechanical engineering.

a) *Students should understand the concepts of "definition" (which can be stated arbitrarily but should nonetheless be meaningful) and "assertion" (which is right or wrong and needs proof). Concerning definitions, it is not required to base everything on set theory and axioms but definitions can also be based upon the assumption of a common understanding of basic concepts. For example, one does not have to start with Peano's axioms for natural numbers or with Euclid's axioms for geometry. So, in this profile we do not aim at enabling engineering students to have a full understanding of the body and structure of a mathematical theory starting from axioms.*

<u>Examples:</u>

- The real numbers are not introduced (as would be done in a lecture for mathematics students) in a formal way by extending the rational numbers using Cauchy sequences but they are introduced as "completion" of the rationals which allow infinitely "fine" numbers thus covering the whole number line. Students should have this kind of understanding of the "definition" of real numbers.

- The "first" definition of a vector is not via the definition of a vector space as a set with operations which behave according to some axioms. It is sufficient for students to understand a vector as an object having an absolute value and a direction symbolized by an arrow (which is for a mathematician a bit too nebulous) that can be moved parallel to itself in 3D-space (which is assumed to be given naturally).
- Students should understand that the scalar product is not just simply defined as $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos(\alpha)$ but the definition is motivated by modeling the situation where first one vector is projected onto the other (vectors are "aligned") before computing the signed product of lengths.
- Students should understand that the statements "Each polynomial of odd degree has at least one zero." and "The scalar product is commutative." are assertions that need proof.

b) *Students should be able to understand the concept of proof as a logical chain of arguments. They should see the necessity of clearly stating the assumptions (assertions taken as given), and follow the stepwise logic that leads from these givens to assertions to be proven. Students should know the meaning of implication and equivalence relationships between assertions and the difference between them. In proofs, emphasis lies on the main ideas in order to enhance the understanding of* ***why*** *something is true and not just* ***that*** *it is true.*

Examples:

- Students should understand the reasoning behind the Gauß method of solving a linear system of equations and they should recognise that the algorithm implies that there are three possible solution types (one, none, infinitely many).
- Students should understand the following (sketch of a) proof for the assertion that every polynomial of degree 3 has a point of inflection to which it is symmetric: The second derivative is a polynomial of degree 1 which has a zero; the third derivative is a constant different from zero, so the zero of the second derivative must be a point of inflection. We can assume that it is zero (moving it into zero does not change the type of polynomial). Therefore, it can be written in the form $f(x)=a_3x^3+a_1x$. Hence $f(-x)=-f(x)$, so we have point symmetry.
- Students should understand that if you square an equation then every solution remains a solution but not the other way round (implication). If you apply the exponential function to an equation you do not change the solution set (within the reals) (equivalence).
- Students should understand the proof of the statement that a quadratic system of linear equations has a unique solution iff the determinant is not equal to 0, and understand that this gives a criterion for unique solvability.
- Students should understand that implications give necessary conditions (like derivative equals zero for local extrema) and hence "solution candidates" whereas equivalence relations give necessary and sufficient conditions and hence solutions.

c) *Students should understand the development of theory in application subjects also as logical argumentation comparable to the proof of mathematical properties in mathematics lectures.*

Examples:

- Students should understand typical mathematical arguments which are used in the development of application integrals, e.g. when the formula for computing moments of inertia for areas is derived. Sometimes, mathematical and application argumentations are different at first sight. E.g. the formula for the moment of inertia is derived in mathematics by making an approximation and then taking the limit whereas in engineering mechanics an "infinitely small" area is taken, multiplied with the squared distance to an axis, and all the infinitely small moments are integrated to the overall moment of inertia, so the limit process is not made explicit in the argument but is included by picking infinitely small things and summing up. Students should recognize this as a "shortcut" of the lengthier mathematical argumentation where convergence aspects are neglected.

- Students should understand the development of a differential equation from differential considerations as is for example done when developing the differential equation for beam bending.
- Students should understand the usage of mathematical properties to show that only a certain application behavior is possible (e.g. for a damped mass-spring system where mathematics tells you that the system can be strongly damped without oscillations or weakly damped with oscillations depending on the solutions of the characteristic equation).
- Students should understand the work within vector models in statics and in 3D-kinematics using functions of multiple dependent variables und derivatives in order to determine velocity vectors and different kinds of acceleration vectors (tangential and normal).

d) *Students should be able to judge the completeness of an argumentation and see where steps have been left out. They should understand that for a full proof a complete coverage of all possible cases is necessary. That does not mean that every case must be dealt with in the mathematics lecture but students should see the necessity for logical proof.*

<u>Examples:</u>

- Students should be able to check whether in a text a full answer to the following question is provided: What are the possibilities of connecting two line segments using as few as possible line or circle segments?
- Students should be able to see that the following argumentation is complete: Solve the inequality (4x+3)/(5x-7)>2 by considering all possible cases. Case 1 is 5x-7>0. In this case one can multiply by 5x-7 without changing the inequality and then continue. Case 2 is 5x-7<0. In this case one can make the same multiplication but has to reverse the inequality sign. The case 5x-7=0 has not to be considered since the inequality is not defined in this case.
- Students should understand the proof that one can find exactly one polynomial of degree (at most) 5 to design a "lifting" function f (with f(0)=f'(0)=f''(0)=0 and f(a)=h, f'(a)=f''(a)=0, called rest-to-rest function in motion design): If you start with an arbitrary polynomial of degree 5, use the 6 conditions to get 6 linear equations for the coefficients. It remains to be shown that this system is always solvable. The first 3 equations say that a_0,a_1,a_2 must be 0. The determinant of the coefficient matrix of the remaining 3 equations is $-2*a^9$. This is not 0 since a is not 0. Hence the system is uniquely solvable.

e) *Students should be able to create chains of logical arguments on their own similar to those they have experienced in the lectures. They need not be able to develop mathematical theory on their own. In their argumentations, they should be able to state clearly their assumptions upon which the argumentation is based, and use implications and equivalence relationships to argue stepwise. They do not have to make formal statements but can use informal formulations as long as the kind of statement is clear. Such chains of argumentation are particularly required when students have to specify steps for computing the solution to a problem.*

Examples:

- Students should be able to argue with functions and function properties: E.g. when you want to cover a distance of 200mm in 5s then your velocity cannot be less than 40mm/s because of the mean value theorem of differentiation. If you require 0 velocity at the beginning and at the end (after 200mm) then you can use a polynomial of degree 3 for interpolation. You will always find one since there is one with two relative extrema and by scaling and moving (which does not change the property of being a polynomial) you can fit it in.
- Students should be able to show that the integral $\int \frac{4x^2+6x-2}{(2x^2+1)^2}dx = \frac{3x^2-2x}{2x^2+1}+C$ is correct using assertions proved before like the main theorem of differential calculus.
- Students should be able to write small computational programmes to provide add-ons to professional programmes: Write a programme in Visual Basic that computes for a list of points the approximative (regression) polynomial of degree 5 and places the coefficients in cells for further processing (standard Excel® only provides the coefficients in a diagram such that further automatic usage is not possible).

f) *Students should understand the difference between heuristics and logic. They should see that a heuristic argumentation might be useful and plausible but not mandatory. Hence, heuristics is a place where one can make changes and try out other procedures whereas logic does not leave any space for experimentation.*

Examples:

- For a rest-to-rest motion task one can use a polynomial of degree 5 (but there are also other connection possibilities); polynomials are used because they are easy to handle. Students should recognize this as a heuristic argumentation. If you decided for a polynomial of degree 5 prove that this always exists having the required properties (cf. the above example for d). Students should recognize this as a logical argumentation.
- Students should understand that using an “ansatz” for a problem based on experience in similar situations might lead to a solution but there is no logical argument for the solution to be the only one. E.g. try to solve a 2nd order homogeneous linear differential equation with constant coefficients using the “ansatz” $y=C*e^{\lambda t}$ since for first order equations solutions look like that. This is a heuristic approach but there is no logical argument for this approach to deliver all solutions.
- Algorithms in machine elements dimensioning sometimes use a rough formula to compute a starting value based on heuristics. The rest of the algorithm then checks in logical steps whether the required properties are given. If not, the starting value must be changed. Here again, rules based on heuristics play a role. One might also use logical argumentation by investigating the expressions occurring in the algorithm and using properties (enlarge the denominator to diminish the value of an expression; change a quantity occurring in the 3rd power to have a larger effect and so on). Students should be able to recognize the difference.

g) *Students should be able to recognize contradictions in statements or properties and they should know about ways to check for contradictions in order to see that something must be wrong (assumptions, working of a programme, usage of a programme).*

Examples:

- Students should know that checking orders of magnitude and signs is a reasonable way to investigate the plausibility of results; for this, rough models are quite suitable.
- Students should be able to use use known assertions and relations to detect contradictions: e.g. a “smooth” distance-over-time function cannot have a strongly oscillating velocity function.
- Students should look for discrepancies between different representations (derivative 0 but tangent not horizontal) in order to detect contradictions.
- Students should use checks in equations (insert solutions) and in differential equations and check necessary conditions if available (e.g. derivative 0 at interior local extrema).

Students should be able to use standard arguments as encountered in mathematics education or application lectures. They need not develop themselves more sophisticated ideas and “tricks” presented in the math lecture.

2.2.2 Radius of action

Mathematical statements and arguments are presented to and used by students both in mathematical lectures and in lectures on application subjects. Accordingly, we distinguish between intra-mathematical and extra-mathematical contexts to be covered.

The intra-mathematical contexts in this profile are the following:

- Students should follow the development of mathematical “theory” in mathematics lectures: Starting with (motivated) definitions properties of mathematical objects are derived using logical steps which are also based on properties that are assumed to be known from former educational processes. The emphasis lies upon letting the students see the basic line of argumentation where special cases are explicitly skipped. Sometimes only an “ansatz” is used without proving that this way all possibilities are covered.
- Students should apply the logic of computing a solution for a certain mathematical problem like solving equations or differential equations or computing derivatives or integrals. This is also the most important situation in which they need the distinction between an implication and an equivalence statement: If a solution method works with implications (starts with a problem and derives properties of solutions providing “candidates” for solutions), a final check whether the solution “candidates” really are solutions is required. A method working with equivalence transformations does not require this.
- Students should recognize types and properties of problems in order to find an adequate solution procedure. (E.g. this is a circle, so we can use the equation of a circle and have to find the right data to insert into the equation …).
- Students should use reasoning in intra-mathematical problem solving which might be of constructive nature (e.g. find a function with certain properties: translate the properties into equations, solve the system of equations) or of analytical nature (e.g. for which parameter values has a quadratic homogeneous system of linear equations infinitely many solutions: This is the case iff the determinant is 0, so solve det(A)=0).

The extra-mathematical contexts occur in application subjects and later on in engineering tasks. We anticipate the following kinds of contexts and situations here:

- Mathematical reasoning occurs in application subjects when mathematical models are derived, application problems are stated as mathematical problems and then mathematical problem solving is used. E.g.: motion behavior or stress distribution under loads are considered in engineering mechanics, and for modeling this, vectors and vector calculations, functions, derivatives and integrals are used.

- Students have to use geometrical reasoning in design, particularly in parametric design, e.g. to justify the invariance of geometrical properties (e.g. construction of a regular hexagon such that the edge length can be changed flexibly without destroying the regularity). More detailed specifications of necessary qualifications are described in (Alpers 2006).
- Students have to check for the completeness of specifications (or givens) in geometric configurations (can it be uniquely constructed with the given information) or algebraic settings (what do I need to know to perform a calculation or execute an algorithm).
- Students have to judge in argumentations, which arguments are logical (and hence true) and which are plausible (heuristic, worked in other circumstances, so can be tried in the current problem under investigation). For example, in machine element dimensioning algorithms are available to compute stress from chosen values for certain quantities based on a mathematical model. Then, heuristic strategies are often used to improve results (reduce stress, material, costs). Alternatively, an optimization model could be set up for finding optimal values with respect to a given objective function.
- Students should set up mathematical models on their own using modeling principles and mathematical reasoning, e.g. setting up the model for a gearing mechanism by starting with the drive and setting up equations moving through the mechanism or by stating all conditions.
- Students should use logical reasoning in probabilistic models in quality assurance, design of experiments, treatment of measurement data, and statistical tolerance in design (giving reasonable tolerances for measures), and they should be aware of the kind of assertions that are possible in such models.
- Students should use logical reasoning when checking the input of computer programmes for completeness (are all data necessary for the computation provided somewhere?).
- Students should use logical reasoning when checking the output of computational programmes (develop expectations by logical or heuristic reasoning from input and properties of the computational model, compare expectations with output properties and detect contradictions (if having argued logically) or questionable properties (when having argued heuristically)). Students should be aware that programmes might not cover all situations and hence provide wrong results.
- Students should be able to program own additional routines to existing programmes using programming logic and cover all possible branches or give a warning if a situation is not covered ("input not allowed").

2.2.3 Technical level

Students need not know and apply the formal logical language using quantifiers. They have to recognize and perform implications and equivalence statements in the following kind of mathematical argumentations:

- In geometric design students must be able to use geometric properties and theorems for achieving certain required relations.
- In the analysis of a geometric configuration (as in gear analysis or analysis of a mechanical configuration) students have to relate quantities to each other using geometric theorems (like Pythagoras or cosine theorem). In 3D situations students must reason using vectors and their properties and translate these into algebra (e.g. orthogonality means: scalar product is 0).
- In algebraic transformations students have to recognize and apply the transformation rules correctly and they must be aware whether there is an implication or an equivalence relationship. In case of an implication students should be aware that they have to check the results.

- Students should use arguments for deriving properties of functions and they should use arguments for function design (e.g. in motion design). Doing this they should be able to use arguments using derivatives and integrals of functions.
- Students should be able to follow differential arguments leading to integrals (picking an infinitely small "piece", computing the interesting property and summing up) and to differential equations (like the differential equation for bending).
- Students should be able to analyze algebraic expressions in order to find arguments for varying the values in a way to achieve a certain goal (enlarging or diminishing a quantity by varying another quantity with "large" influence).
- Students should be able to use logic in designing algorithms for programming "add-ons". They have to go through loops (full coverage) and to switch according to cases (if-then-else).
- Students should know different representations of functions in order to detect contradictions.
- Students should know about solution possibilities for linear and non-linear equations and systems of equations in order to argue about the solvability and uniqueness of solution.
- Students should be able to reason in probabilistic models, e.g. identify the specifics of a situation that justifies the usage of a model (the characteristics the model is based upon) and formulate assertions within the model using probabilities. They should also recognize that the choice of an acceptable probability level is not based on logic but rather on agreement between parties.
- Students should be aware that a logical argumentation in the reals might no longer hold in the set of computer numbers (e.g.: the determinant must be zero for a quadratic linear system of equations with infinitely many solutions but it might not be zero for numerical reasons).

The above is a rough specification of the technical level which needs refinement (e.g. which geometrical theorems exactly should the student be able to use in arguments?). More detail is provided in chapter 3.

2.3 Posing and solving mathematical problems

The competency of posing and solving mathematical problems "comprises on the one hand the ability to identify and specify mathematical problems … and on the other hand the ability to solve mathematical problems (including knowledge of suitable algorithms)" (Alpers et al. 2013). It is stated in the framework document that the term "problem" is not well-defined; it also depends on the personal capabilities. In this section we specify in more detail which kinds of problems and aspects of problem solving we want to be covered in the study course under consideration.

2.3.1 Degree of coverage

The following aspects of the mathematical competency of posing and solving problems should be covered:

a) *Students should be able to solve well-specified computational problems for which algorithms already exist. These problems might show up in an intra- or extra-mathematical environment.*

<u>Examples:</u>

- Students should be able to solve an integral where the integrand is a fraction. It is well-known that the method using partial fractions works but the solution scheme can be quite complex for someone who is not accustomed to using it.
- Students should be able to perform an algorithm for computing the stress occurring in a machine element given a certain load. The algorithm can be found in a machine element book.

- In a statically-determined truss with given loads at some joints compute the forces in the rods. There is a well-known procedure to set up a system of linear equations (equilibrium of forces in each node in x- and y-direction gives 2 equations per node). Then, the Gauß algorithm can be used to solve the system.

b) *Students should be able to solve problems for which no algorithm is available but the basic problem solving strategies that could help are available and the application of these strategies is not too complex and "tricky".*

Examples:

Students should be able to apply the following basic strategies:

- In a geometric configuration some geometric data is given and other quantities are looked for. The well-known strategy is to relate the unknowns to the known quantities by using geometric theorems like Pythagoras' theorem or sine or cosine theorem. This way one can for example compute in a windscreen wiper the position of the wiper given the rotation angle of the driving motor.
- In a mechanical configuration in statics the forces acting in the bearings or some other parts can be computed by setting up equilibrium equations and using the method of sections adequately.
- In integration there are basic integrals and integration methods like integration by parts, substitution and partial fractions available. Students have to find a suitable combination to solve an integration problem like $\int \frac{1}{x \cdot \ln(3x+4)}\, dx$ or $\int x \cdot \arctan(x)\, dx$. For the first one, the strategy is to look for a partial expression such that its derivative occurs as a factor. For the second one, the method of partial integration leads to an integral with a fraction.
- The desired boundary curve of a cross section of a machine part is given. Check whether the part can be produced in a milling machine where the milling head is ball shaped and – if so – compute the curve of the tool centre. The process is only feasible if the curvature of the ball is larger the the largest curvature of the boundary curve, so the latter has to be computed and the maximum has to be found. If the milling process is possible, an offset curve (ball radius=offset) must be found. For doing this (without a CAD as support tool) one could compute the unit normal vector for each curve point, multiply it with the tool radius, "attach" it to the point and obtain this way a point of the offset curve.
- Specify formally the involute curve that is used as profile in a gear-wheel. The curve can be constructed by unwinding a string from a circle. First draw a picture where the string is unwound and think about what conditions the endpoint of the string has to fulfill. Then specify its coordinates depending on the "winding angle".

c) *Students should be able to solve problems where certain quantities in given models have to fulfill some restrictions, and values for the design variables (which can be influenced) have to be found such that this is the case (or it must be shown that this is impossible). Students should be able to use the strategy of goal-oriented iteration (as opposed to just randomly changing variables and looking for the effect).*

Examples:

- In machine element dimensioning books one can find algorithms for computing the occurring stress when the dimensions, environmental conditions, material and loads are given. One has to find a configuration such that the occurring stress is below the maximum bearable stress for the material. For this one can try to find an “initial guess ” (sometimes heuristic rules are available) and then to modify it. To do this in a goal-directed way, one has to see the influence of design variables which is not always easy. Students should be able to do this.

- Students should be able to find a lifting function (cf. 2.2.1 d)) such that velocity and acceleration are within given limits (given by the machine that will realize the motion). They could for example start with a discontinuous model with jumps in the acceleration function and then go over to a continuous model like the so-called “acceleration trapezoid”.

d) *Students should be able to recognize optimization problems as such and they should be able to formally state these in optimization models such that they can use respective software for solving the problems. They should be aware of the difference between local and global optimization.*

Examples:

- A torsion spring must be dimensioned such that given a certain force the spring reaches a certain rotation angle. The occurring stress must not be larger than the admissible one. For such a standard task in machine element dimensioning one approach could be to “guess” an initial design, i.e. specify values for the free parameters which are the diameter of the spring and the diameter of the wire and the number of turns. Then one computes in a well-known model the occurring stress and iterates if under- or over-dimensioning is recognized. One can also see the problem as an optimization problem where first the goal function has to be specified (weight or cost or a combination of both), then the stress restriction has to be specified in a restriction function and for the free design parameters also reasonable restrictions have to be defined. Students should recognize this.
- The geometry of a machine part is often only partially specified, i.e. there are connections to other parts and a certain load case must be fulfilled. In application areas where weight is very important like car industry this is also the essential quality criterion for a design. Mathematically seen, this is an optimization problem. If the geometry under consideration can be characterized by some parameters, then we have parameter optimization. Students should recognize this. (If the shape is not so restricted but can take – within some limits – any form, this is called topology optimization. For the latter there are special software tools supporting this kind of optimization.)

e) *Students should know for which kind of problems they should expect application programmes to be available for solving the problem even if they do not understand the solution procedures in any detail.*

Examples:

- In the examples in d) students should expect or even know that there are optimization packages for parameter optimization (included in classical mathematics programmes like Matlab®) or for topology optimization (included in CAD/FEM programmes).
- If students come across a differential equation problem they should know that there are programmes which (within limits) are able to solve differential equations symbolically and/or numerically.

f) *Students should recognize design problems as such and have problem solving strategies for behaviour (functional) design and geometry design.*

Examples:

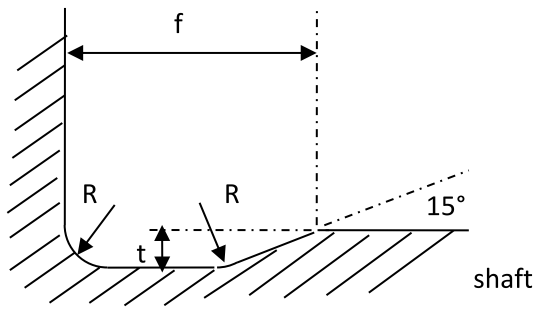
Students should be able to recognize and solve the following design problems:

- Find a function that connects the "endpoint" of a linear function with a point of rest (background: the linear function serves synchronization purposes, e.g. a picker arm follows a belt at constant speed, and after synchronization a machine part comes to rest again). One could try a polynomial of degree 5 but it can happen that it overshoots at the end. Another possibility providing more influence on the shape would be to use a spline of degree 5. One could also start with the acceleration function design (second derivative) and "work upwards".
- Design the cross section of a geometric object for which some parts are already given by technological requirements (e.g. circles for rotational joints). One could use here circle and line segments or spline curves going through given points. One could also solve the problem using B-splines (which are available in standard CAD programmes) which allow a comfortable change by moving control points.
- Construct a certain volume using the basic volumes and operations available in a CAD system (for more specific examples see Alpers 2006).

g) *Students should be able to solve relatively straightforward completeness or existence problems using arguments based on well-known properties.*

Examples:

- Students should be able to argue as follows: The limit exists since the first factor has limit 5 and the second factor has limit 4, so the limit of the product exists and is 20.
- Students should be able to prove that there is exactly one polynomial of degree 5 which realizes the lifting function explained in section 2.2.1 d).
- Students should be able to prove the completeness of specification of the notch depicted below which was taken from a German industry norm.



h) *Students should be able to deal with experimental data problems, e.g. find models that do approximately fit, compute best fits. They should be able to deal with chance by using adequate models for random variables.*

Examples:

- If the experimental data suggest a linear correlation recognise this and find an estimate for the underlying linear function using the least-squares method or an appropriate software. Compute confidence intervals for slope and intercept.

- If experimental data suggest a saturation behavior recognize this, find a class of saturation functions and find the best fit. Ask yourself whether there is an underlying model which explains the saturation behavior and provides information on the saturation function class.
- If you know that results are based on chance (like in a random sample), find the adequate model from a set of known models, e.g. the binomial distribution.

i) *Students should be aware of and reflect on the problem solving strategies available to them. These should include the following:*
    a. *Look for the given quantities, think about the wanted quantities and try to find relationships.*
    b. *Look at small and simple examples to become familiar with the situation.*
    c. *Divide at impera: subdivide the problem into partial problems.*
    d. *Look for similar problems and their solutions.*
    e. *Look for software support for a problem.*
    f. *Specify the type of the problem(e.g. an optimization problem).*
    g. *How to deal with uncertainty: Make assumptions, vary to see the sensitivity to changes to check the possible uncertainty of results.*
    h. *In design problems: look for components, their properties and their interconnections (in function design: function types, problems of piecewise composition; in geometric design: curves, surfaces and volumes and their interconnection); make a first "educated" guess, check it and improve it.*
    i. *In geometry problems draw a picture, mark the known and unknown quantities and try to find relations using geometric properties and theorems.*

Again, we conclude by giving information on what is not expected. Students are not expected to find more complicated proofs or to solve proof problems in a formal logical way as mathematicians do. They are also not expected to find new solution methods or problem solving strategies especially for vague problems where quality criteria have to be developed first (e.g. on "roundness" of a shape or "roughness" of a surface).

2.3.2 Radius of action

Mathematical problem solving occurs both in mathematical lectures and in lectures on application subjects. The following contexts and situations are included in this profile:

- Students should be able to apply the mathematical problem solving competency in intra-mathematical situations where the problems are posed and solved "as mathematical ones".
- Students should also be able to apply the problem solving competency in models which are set up and dealt with in application subjects. They should be able to transfer the problem solving strategies to other technical problems even if they have not been dealt with directly in the application subjects.
- Students should be able to apply the problem solving competency in all kinds of dimensioning problems in mechanical engineering.
- Students should be able to apply the problem solving competency in all kinds of design problems in mechanical engineering.
- Students should be able to apply the problem solving competency in all kinds of experimental data investigation in mechanical engineering.
- Students should be able to apply the problem solving competency for making effective and efficient use of application programmes (e.g. parameter variation, iteration, optimization) in mechanical engineering.

2.3.3 Technical level

Students should be able to pose and solve mathematical problems regarding the following technical level (for a more detailed specification of knowledge and skills see chapter 3):

- They should know essential means to specify geometric entities and relations in 2D and 3D including the usage of vectors.
- They should know and be able to apply essential geometric constructions for design problems including standard geometries and freeform geometries.
- Students should know and use essential function classes and their properties (continuity, differentiability) for function design problems.
- Students should know about and use essential means for function investigation like taking limits, differentiate or integrate (both symbolically and numerically).
- Students should know linear and non-linear equations and systems of equations and how to solve them (symbolically, numerically, without or with use of software).
- Students should know basic types of ordinary differential equations and systems thereof and know about basic symbolic and numerical solution methods.
- Students should know about basic approximation and interpolation models like linear ones with one or many variables and polynomial, spline and exponential ones with one variable.

2.4 Modeling mathematically

The mathematical modeling competency can also be considered to be twofold. It comprises the "ability to analyze and work in existing models … and the ability to perform active modeling (structure the part of reality that is of interest, set up a mathematical model and transform the questions of interest into mathematical questions, answer the questions mathematically, interpret the results in reality and investigate the validity of the model, monitor and control the whole modeling process)." (Alpers et al. 2013).

2.4.1 Degree of coverage

The following aspects of the mathematical competency of modeling mathematically should be covered:

a) *Students should be able to understand the development of mathematical models in application subjects. They should know essential models in these subjects and be able to mathematically formulate and solve problems within these models (on the kind of problems see the previous section). They should also be able to interpret the results in application terms.*

<u>Examples:</u>

- In machine element book models are developed for computing the occurring maximum stress in a component under load using concepts from engineering mechanics. Students should understand how this is done and then use the models for adequate dimensioning of a machine element.
- Students should understand how the concept of moment of inertia for a cross section is developed in stress analysis when a model for computing the occurring stress under load is set up.
- In a component under load which has a more complex geometry like a connecting rod, the stress distribution is to be determined. Because of the complexity of the geometry, an FEM

programme is used. Within this programme, the geometry, the bearings, the load and the material need to be modeled. The programme is based upon a model of the 3-dimensional stress-strain distribution and a concept for computing one comparative stress from the stress values in different directions such that it can be compared with the admissible stress for the material. The user should understand this underlying model to work with the programme in a sensible way. Students should also understand the FEM modeling and its potential problems in order to check e.g. the network when strange results occur.

b) *Students should be able to recognize the applicability of existing models for modeling a given situation e.g. based on given experimental data. Students should be able to obtain data for such purposes.*

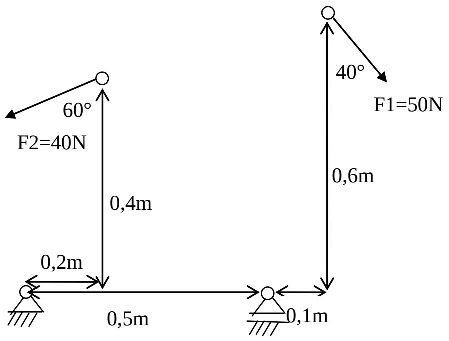
Examples:

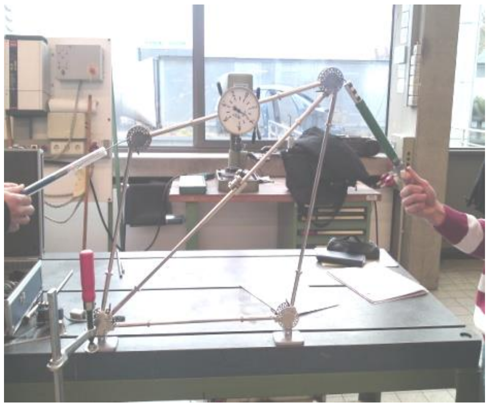
- If a force acts on a body made of a certain material it causes strain and resulting stress in the material. A tensile test provides data relating stress and strain. There is a so-called linear elastic range where the relation is linear according to Hooke's Law and then the material is deformed permanently (plastic range) where the stress is even reduced by the flow of the material. For the linear-elastic range a linear model is appropriate $\sigma=E*\varepsilon$ (stress σ, strain ε, Young's modulus E). An estimate for E could be gained by applying the least-squares method but for this one has to find a criterion to state where the linear-elastic range ends.

c) *Students should be able to use modeling principles and quantities developed in the application subjects in order to set up models of known type.*

Examples:

- A railway bridge (as can be found in most neighborhoods) is considered regarding the forces acting in the bearings with and without a train crossing the bridge. For modeling the static situation there is the statically determined model of a beam with one fixed bearing and one floating bearing (motion is allowed in horizontal direction). The weight of the bridge is modeled by a constant line load. Compute the forces in the bearings.
- A truss is to be designed, built and measured regarding the forces acting in the connecting rods. The bearings and the load injection points are given (see drawing below which is already a partial model). Students design connections to make the truss statically determined, build it (see photo below) and take measurements. They model the truss using equilibrium conditions in each joint and thus get linear equations for the forces in the rods and in the bearings. They have to investigate the situation to find reasons why the measured values differ from the computed ones (where are model inaccuracies?).



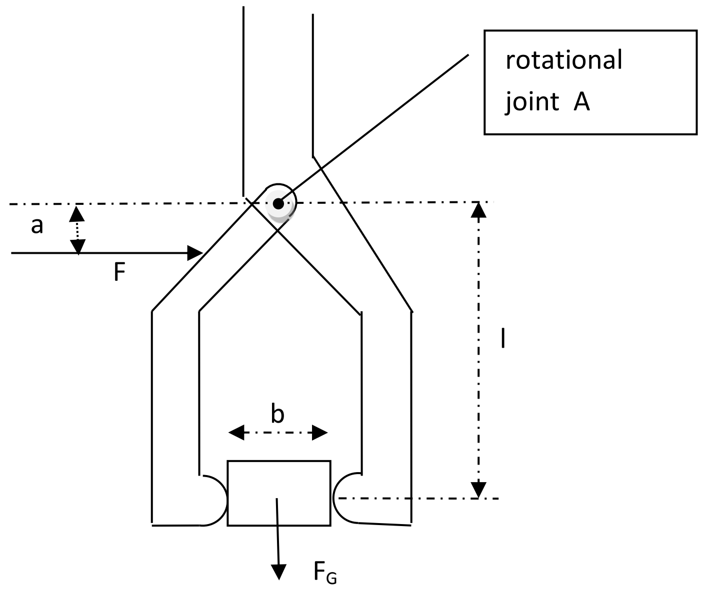


- The motion of a picking device is to be designed. The device picks up a part from a storage, moves it around an obstacle to its final destination, releases it there and goes back to the storage. At least two ways for modeling the motion are available: One can first model the motion curve (maybe using well-known pieces like line and circle segments) and then model the motion along the curve (for this an arc-length parameterization would be helpful) or one can model directly x(t) and y(t), e.g. using points to be reached and corresponding times. There are guidelines by the German Association of Engineers (VDI Richtlinie 2741, 2004) for doing this.

d) Students should be able to find a suitable model granularity such that they work in a simple but sufficiently comprehensive model. They should be aware of the problem of obtaining model information for more complex models using several parameters.

Examples:

- In the framework document (Alpers et al. 2013, Appendix), there is an example of designing a pedestrian bridge. There, a coarse 1-dimensional bridge model and an equally coarse load model (constant line load) were chosen. Students should be able to do this.
- A gripper-shaped pick up device of a machine should be able to pick up a part of weight G. Which force must the hydraulic cylinder be able to apply for guaranteeing this? A first model leads to the sketch below which contains essential geometric dimensions and forces in symbolic form. In order not to slip through the pair of tongs, the sum of friction forces on both sides must be equal to the weight force, i.e. $F_R = \frac{F_G}{2}$. The friction force is less than or equal to the normal force times the coefficient of friction $F_R \leq \mu F_N$. In order to relate the weight force to the force applied by the hydraulic cylinder, the equilibrium of torques can be used where it is useful to consider them around the bearing A. One gets: $F \cdot a - F_N \cdot l + F_R \cdot \frac{b}{2} = 0$. These equations then can be used to compute the minimum for F.



(after Zimmermann (2006, p. 16)).

e) *Students should be able to use simple models for programme validation.*

Examples:

- In the above example on computing stress distribution for a more complex geometry using a FEM program (see a)), students should know how the computation can be performed by hand in case of a simple geometry (e.g. a beam with rectangular cross section) such that he/she can check whether the programme works correctly (or is used correctly).

f) *Students should be aware of the limitations of mathematical models.*

Examples:

- If a company produces products with many variants it is desirable to have a large degree of commonality in order to have acceptable costs (like car production based on the same platform). How can this commonality be measured and hence made comparable? In literature one can find the so-called DCI index (degree of commonality) which is defined in the following way: Set up a table with the different variants and the different parts. An entry of 0 or 1 resp. means that the part does not or does occur in the variant (see table below). The DCI is defined as the sum of all entries divided by the number of parts. In the example below it is 9/5=1.8. Students should be able to answer the following questions: What are the possible values? What does the number express? What happens when a new part is included which occurs in half of the variants (add D)? Which aspects have not been modeled (costs, how often occurs a part in a variant,…)? How could the model be modified to include additional aspects?

	Product variant A	Product variant B	Product variant C
Part 1	1	1	0
Part 2	0	1	1
Part 3	1	0	1
Part 4	1	1	0
Part 5	0	0	1

- In a device called "drop tower" where a weight can be dropped in a controlled way, new materials can be investigated regarding their impact behavior. For this one needs the impact of the dropping part when it hits the new material. Since impact equals mass times velocity, the latter is to be determined (assuming that the mass of the dropping body is known). A simple energy model says that the potential energy is m*g*h (with height h and mass m) and this must equal the kinetic energy at impact time which is ½*m*v^2 (with velocity v). Students should be able to discuss the limitations of this model (e.g. friction in the prismatic joints is not modeled)? They should also be able to discuss possibilities to improve the approximation of the velocity (e.g. take a video, measure points, approximate h(t) with a parabola using the least squares method and then compute the velocity in the approximation).

Again, it clarifies what is included in this curriculum if we state what we do not want to be included. We do not assume that the students (in general) will be able to "invent" new modeling means or quantities. We see this rather in a theoretically or research oriented study course. This might be included in a subsequent master level course, though.

2.4.2 Radius of action

Students should be able to use the modeling competency in the following situations and contexts:

- They should be able to use the mathematical models and modeling means developed in their application subjects like engineering mechanics, thermodynamics, machine dynamics, control theory and so on.
- They should be able to turn the essential application problems occurring in these areas into mathematical problems and to solve these, be it with or without technology. They should also be able to interpret and validate the solutions in terms of these applications.
- They should also be able to transfer their knowledge on models and modeling means to application situations which they have not encountered before but which can be modeled in ways similar to what they are familiar with. This also includes non-technical situations and questions like the DCI example described above (see f) in the degree of coverage section).
- They should know models and modeling means (at least in a simple version) the application programmes they use are based upon such that they are able to provide reasonable input, work with the programmes effectively and efficiently and can interpret the output and discover potential problems.

2.4.3 Technical level

Students should know and be able to use the following mathematical modeling means (for a more detailed specification of knowledge and skills see chapter 3):

- Students should know the following means to model quantities: numbers (natural, …, complex); vectors; matrices; variables; functions (univariate and multivariate). They should know representations for these and perform operations.
- Students should know and be able to use the following means for describing relations between quantities: equations, functions, differentiation, integration, differential equations.
- Students should know how to model behavior with functions: growth (linear, quadratic, exponential), decay, saturation, oscillation, and combinations thereof. They should know the meaning of essential modeling parameters in these models, and they should be able to solve problems in these models.
- Students should know and be able to use essential means for geometric modeling for regular and free-form geometries.
- Students should know and be able to use means for approximating measurement data by models using the least-squares method.
- Students should know how to model random data with random variables. They should know essential distribution models (normal, binomial, poisson) and their properties. They should know how to compute estimates within these models and how to describe their quality (confidence intervals).

2.5 Representing mathematical entities

The competency of representing mathematical entities "includes the ability to understand and use mathematical representations … and to know their relations, advantages and limitations. It also includes to choose and switch between representations based on this knowledge". (Alpers et al. 2013).

2.5.1 Degree of coverage

The following aspects of the competency of representing mathematical entities should be covered:

a) *Students should be able to recognize and set up mathematical representations which have been developed before in mathematics education. They need not find new types of representation themselves.*

<u>Examples:</u>

- Students should be able to recognize $\vec{x} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$ as parameter representation of a curve in 3D-space and they should be able to create this representation when two points are given.
- Students should recognize when a curve is a representation of a univariate function (when the curve does not "turn around").
- Students should know that a circle can be represented by its center (in coordinates) and its radius, but also by a quadratic equation or by a curve drawn using a compass.
- A CAD programme needs as input for drawing a curve a parameter representation where the parameter runs from 0 to 1. Students should be able to construct such a representation.
- Students should be able to set up a matrix representation of a linear mapping (where a basis is given) and get from this an easy representation with eigenvectors and eigenvalues if possible.

- Students should be able to specify a plane linkage in the complex number plane and specify operations like rotation and translation by using complex multiplication and addition.
- Students should be able to see that when a CAS programme provides a solution of a differential equation using hyperbolic functions (when they expect to get exponential functions) that this is just an alternative symbolic representation.

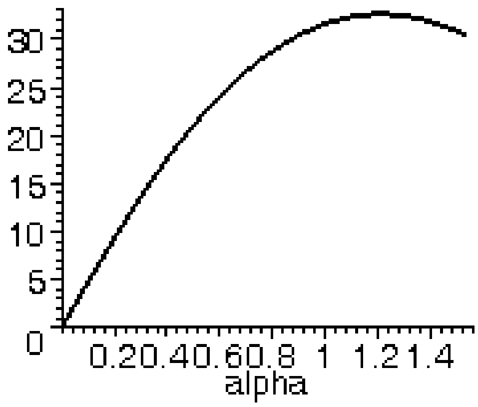
b) *Students should know the advantages of the mathematical representations dealt with in mathematics education such that they can choose an adequate representation for a certain problem. These advantages include the following:*
   - *Make certain properties better recognizable*
   - *Facilitate the extraction of information*
   - *Facilitate performing certain operations on the entities*
   - *Make the description simpler or more compact*
   - *Facilitate the input into a programme or the implementation in a programming language*
   - *Perform a control computation in another representation*

Examples:

- Students should know that a graphical representation of a function gives an overview of the behavior and of essential properties (within the limits stated in c). E.g. in an algebraic model the restoring force of a spring attached to a door in the direction orthogonal to the door can be described by the formula (α is the opening angle of the door):

  $$F := \frac{40.\,(1040. - 1200\cos(\alpha + 0.524))\,\sin(\alpha + 0.524)}{1300 - 1200\cos(\alpha + 0.524)}$$

  A graphical representation of force over angle yields:

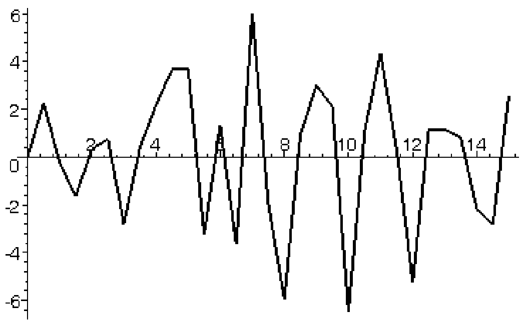
  Here, one sees immediately that there is a maximum force of about 35 N.
- Students should be able to see that in the algebraic representation of saturation behavior it is easy to see the essential properties: $y = A \cdot (1 - e^{-\alpha t})$.
- Students should be able to see that if a line in 3D is given by 2 linear equations it is easy to do a point check by inserting coordinates.
- Students should be able to see that if a line in 3D is given in parameter form it is easy to compute points for graphical representation by inserting parameter values.
- Students should be able to see that if a curve is given as a B-spline it is easy to modify by moving control points, preferably using the mouse.

- Students should be able to see that a geometric representation of linear equations facilitates to visualize possible solution sets.
- Students should be able to see that in a linkage with 4 rotational joints (2 of them fixed) and 3 rods one can use one rod as (rotational) drive and consider the coordinates of the second movable joint as function of the angle. An animation representation helps to visualize such a function.

c) *Students should be aware of the limitations of the mathematical representations dealt with in mathematics education such that they know what kind of information to expect from a certain type of representation.*

<u>Examples:</u>

- In a technical handbook there is a table for looking up for a certain angle (given in degrees): the arc length b (radians), the largest distance h between secant and arc segment (with the given aperture angle) and the quotient b/h. The first line contains the entries 1°; 0.0175; 0.0000; 458.37 . Is there anything strange here? If so what is the reason? A student should be able to answer these questions.
- The function $f(t) = 3\sin(7.439t) - 2\sin(22.4t) + 3\sin(557.9t)$ is a superposition of three sine oscillations. When it is plotted using points with distance 0.2 one gets the following plot (with linear interpolation):



  Students should be able to answer the question: Can one make a rough estimation of the maximum of this function using the plot?
- In a spreadsheet programme one can compute the polynomial approximation (of degree up to 6) for a point set. If a polynomial of degree 6 is used the approximation is shown in a diagram and the polynomial is also displayed in the diagram and the coefficients are displayed with 5 digits. When one inputs the polynomial as a formula, creates a table and displays this, there are considerable differences between this curve and the approximating polynomial shown in the original diagram. Students should be able to explain this.
- If students get from a CAS a symbolic solution to an equation which goes over several pages then students should know that this is in general not useful for investigating the influence of a certain parameter by looking at the symbolic expression.

d) *Students should be able to switch between mathematical representations dealt with in mathematics education and know about information loss or modification during this process.*

Examples:

- Students should be able to switch between a curve given as a graph (e.g. in a book) and a tabular representation which can be stored for further processing (in order to automate the process of reading values from the curve). A scanner and a graph digitizer could be used for this purpose.
- Students should be able to switch between a symbolic and a graphical representation of a function. They also should be aware of the loss of information (see the example in c)).
- Students should be able to switch between the time domain and frequency domain representation of functions. In the DFT or FFT transformation there is a loss of information because of the restricted resolution and the restricted frequency interval.
- Students should be able to switch from a graphical representation of a curve consisting of line and circle segments to a numeric representation that can be digitally stored.

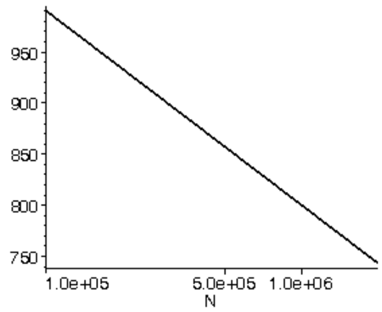
e) *Students should be able to "read" and retrieve information from the mathematical representations dealt with in the mathematical and application education.*

Examples:

- In a formulary accompanying a text book on machine elements there is a table which provides information on certain quantities for parts which have a "rounded" rectangle with a certain wall thickness as cross-section. The given quantities are the area and the moments of inertia with respect to two axes x and y going through the center of gravity. Students should be able to read the table, i.e. see that they can choose the size of the rectangle and the wall thickness (within limits given by the table). They should be able to see relationships and relate them to what they know about the quantities. They should be able to answer the question: What should be done if one wants to get an approximative value for a 100x100 cross-section with wall thickness 3 (e.g. find an example for the effect of doubling the value of b)?

Rectangular dimensions axb [mm]	**Wall thickness s[mm]**	**Area [cm^2]**	**I_x [cm^4]**	**I_y [cm^4]**
50x30	2.5	3.68	11.8	5.22
40x40	2.5	3.68	8.54	8.54
60x40	2.5	4.68	22.8	12.1
80x40	3	6.74	54.2	18.0
60x60	2.5	5.68	31.1	31.1
80x80	3	9.14	89.8	89.8
90x50	3	7.94	84.4	33.5
100x50	3	8.54	110	36.8
120x60	4	13.6	249	83.1

- A so-called "Wöhler line" provides information on the maximum bearable stress for a material with oscillating load depending on the number of oscillations. Since the latter covers a wide range, a logarithmic scale is used and the functional relationship is modeled as a line (see below an example where N is the number of load cycles). Students should be able to get the maximum stress for 200000 load cycles. For this they have to perform logarithmic interpolation.



- In technical formularies or handbooks functions of two variables are often represented as a set of curves in a 2-dimensional plot. There, one independent variable is represented on an axis and curves for discrete values of the other independent variable are displayed. Students should be able to find values for the dependent variable using such a plot and linear interpolation.
- Students should be able to obtain information on the influence of a parameter in a symbolic representation if the latter is not too large or complicated.

2.5.2 Radius of action

Students should be able to use this competency in the following situations and contexts:

- Students should be able to apply the competency in intra-mathematical situations (i.e. in mathematics education).
- Students should be able to apply the competency in application subjects. This includes mathematical representations in application text books, formularies, table collections, technical drawings and data sheets, and technical handbooks.
- Students should be able to apply the competency when using mathematical or application programmes in order to understand and provide those representations which are needed as input and to interpret those representations that are delivered as output.
- Students should be able to apply the competency when writing own technical reports (like project reports, reports on internships or a bachelor thesis). They should choose representations which are adequate to inform a technical audience.

2.5.3 Technical level

Students should be able to know and use the following forms of representation of mathematical entities (for a more specific treatment see chapter 3):

- Representations of different types of numbers: natural, integer, rational, real, computer numbers (with restricted accuracy); complex numbers with different representations.
- Representation of directed quantities: different kinds of vectors, algebraic and graphical.
- Representations of geometrical objects in different kinds of coordinate systems, using boundary curves and surfaces or volume models (point sets); representation of free form objects using control points.

- Representations of curves and surfaces as point sets in diagrams and algebraically (equations, functions or in parameter representation).
- Representation of functional relationships using tables, graphs, algebraic expressions, animations (graphical representations using different scaling and different forms for showing parts of multivariate functions; different symbolic representations of functions, e.g. as hyperbolic or as exponential functions).
- Representation of functions in function spaces as Taylor series or Fourier series.
- Switch of representation by functional transformation: Laplace and Fourier.
- Representation of multidimensional mappings using matrices
- Representation of differential equations algebraically and graphically by direction fields.

2.6 Handling mathematical symbols and formalism

The competency of handling mathematical symbols and formalism "includes the ability to understand symbolic and formal mathematical language and its relation to natural language as well as the translation between both. It also includes the rules of formal mathematical systems and the ability to use and manipulate symbolic statements and expressions according to rules". (Alpers et al. 2013).

2.6.1 Degree of coverage

The following aspects of the competency of handling mathematical symbols and formalism should be covered:

a) *Students should be able to recognize and use the mathematical symbols encountered in their mathematics education (for an overview of the symbols included in this profile see the technical level 2.6.3). This includes the ability to recognize symbolic notation in application subjects even if the "names" are different.*

Examples:

- Students should be able to recognize a function and a function type when the function is not given in "standard notation" y(x) but using different names, e.g. g(p). They should be able to perform differentiation and integration using the correct names, e.g. $\frac{dg}{dp}$ or $\int g(p)dp$.
- Students should be able to recognize a differential equation where the derivatives are given in prime notation or in differential notation, e.g. $y''+2y'+y=x^2$ or $\frac{d^2y}{dx^2}+5\frac{dy}{dx}-y=3x$.
- Students should be able to recognize partial derivatives using the round ∂ notation.
- Students should be able to recognize the symbolic expression $f(x)=\frac{1}{2}a_0+\sum_{k=1}^{n}\left(a_k\cos(k\omega x)+b_k\sin(k\omega x)\right)$ as a Fourier polynomial and they should be able to interpret the occurring symbols.
- In motion investigation differential notation is often used. E.g., if the velocity is given as function of the distance v(s) then the acceleration over distance and the inverse of the distance over time function can be derived as stated below. Students should be able to understand the symbolic notation and follow the computation. They should be able to perform a similar operation on their own.

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{dv}{ds} \cdot v,$$

$$v = \frac{ds}{dt} \Rightarrow dt = \frac{ds}{v} \Rightarrow \int_{t_0}^{t} d\bar{t} = \int_{s_0}^{s} \frac{d\bar{s}}{v} \Rightarrow t - t_0 = \int_{s_0}^{s} \frac{d\bar{s}}{v} \Rightarrow t = t_0 + \int_{s_0}^{s} \frac{d\bar{s}}{v} \; mit \; v = v(\bar{s})$$

- Recognize and set up a solution set with infinitely many solutions for linear systems of equations, e.g. $\{(2\alpha + 3\beta + 5, -\alpha + 3\beta, \alpha, \beta) | \alpha, \beta \in \mathbb{R}\}$ for a system of 2 equations with 4 unknowns with parameters α and β.

b) *Students should know and be able to use the previously encountered rules and schemes for symbolic manipulation.*

Examples:

- The following formula serves to compute the axial section modulus W (wrt. bending) for a part with a circular ring as cross-section (inner radius r, outer radius R): $W = \frac{\pi}{4}\left(\frac{R^4 - r^4}{R}\right)$. The desired modulus is given. Compute for a given outer radius the corresponding inner one and vice versa. In the first case, students should be able to perform the computation symbolically, in the second case they should see that they get an equation of degree 4 which they should solve using technology.
- If a ball (e.g. in a bearing) exerts a force F onto another ball, then the so-called Hertzian stress p_{max} can be computed with the following formula: $p_{max} = 2176 \cdot \sqrt[3]{F \cdot \left(\frac{1}{d_1} + \frac{1}{d_2}\right)}$ where d_1 and d_2 are the diameters of the balls. Students should be able to see, that the formula is symmetric in d_1 and d_2. Given the maximum stress and the diameter of one ball they should be able to compute symbolically the diameter of the other one. They should also recognize when the problem is not solvable.
- The temperature T in a container decreases exponentially according to the function $T(t) = T_0 \cdot e^{\lambda t}$ where T_0=20° and λ=-2/s. Students should be able to compute after which time the temperature of 5° is reached.
- Students should be able to calculate the following integral in a formally correct way as given below:

$$\int_0^{\pi/2} \left(\alpha^2 + \sin(\alpha)\right) d\alpha = \left(\frac{1}{3}\alpha^3 - \cos(\alpha)\right)\Bigg|_0^{\pi/2} = \left(\frac{1}{3}\left(\frac{\pi}{2}\right)^3 - \cos(\frac{\pi}{2})\right) - \left(\frac{1}{3}0^3 - \cos(0)\right).$$

$$= \frac{\pi^3}{24} + 1$$

- Students should be able to apply the schema for finding the solution of a second order linear differential equation with constant coefficients and a quadratic forcing function.
- Students should be able to perform the scheme for the partial fraction integration method in simple cases.

c) *Students should be able to translate natural language into symbolic language in well-known, previously encountered situations.*

Examples:

- Students should recognize that a function is given if for a certain set of values of one quantity there is a way/algorithm to compute the corresponding value of another quantity, even if given in natural language. They then should be able to translate this into the symbolic notation output(input).
- Students should be able to translate linear growth and exponential growth into the respective symbolic equations.
- Students should be able to translate an equilibrium situation into an equation, and a relation between growth and value of a function into a differential equation.

d) *Students should recognize verbal and formal specifications of assertions and logic relations like implication and equivalence. This does not include the formal specification using quantifiers.*

Examples:

- Students should be able to recognize the sentence “each polynomial of odd degree has at least one real root” as an assertion.
- Students should be able to recognize the formulation “assume that there is …” as an assumption.
- Students should be able to understand the following notation for getting the solution of an equation with a root expression. They should test the solution candidates.

$$x+\sqrt{x-3}=5 \Leftrightarrow \sqrt{x-3}=5-x \Rightarrow x-3=(5-x)^2=25-10x+x^2$$
$$\Leftrightarrow x^2-11x+28=0 \Leftrightarrow x=4 \quad or \quad x=7$$

e) *Students should be able to use symbolic notation and natural, semi-formal language to describe logically correctly their statements and reasoning.*

Examples:

Students should be able to make statements like the following ones:

- Let n be any natural number except 1 (or $n \in \mathbb{N}\backslash\{1\}$), then the graph of the function $f(x) = x^n$ looks “in principle” like the graph of a parabola for n even and like the graph of a cubic function for n odd.
- If one constructs the position of a joint in a linkage by intersecting two circles, one gets the correct position by choosing the intersection point where the y-coordinate is the larger one.
- If a value for the voltage is given then this results in a certain value of the currency, hence we have a function i(u).

- If a manipulation of an equation does not change the solution set, one can use the equivalence sign (maybe not in full formalism: x is a solution of … is equivalent to x is a solution of …, but in the short way demonstrated in the last example of d)).
- Determine the solution of the inequality $\frac{2x+5}{3x-7} < 4$. Case 1: Assume that 3x-7>0, that is x>7/3. Then, $2x+5 < 4(3x-7) \Leftrightarrow -10x < -33 \Leftrightarrow x > 3.3$. Case 1 provides as partial solution set $L_1 = \{x | x > 3.3\}$. Case 2: and so on … The overall solution is the union of L_1 and L_2.
- In a situation where one wants to compute a quantity, clearly state the givens, the unknowns and the procedure to get the ones from the others, e.g.: We have a three bar linkage with 4 rotational joints two of which are fixed. One bar which is connected to a fixed joint acts as drive, the other one connected to the other fixed joint acts as driven element. Compute how the angle of the driven bar depends on the angle of the driving bar. Givens: the distance between the two fixed joints, the length of the bars, a coordinate system and the angle of the driving bar; unknowns: the angle of the driven bar. The procedure is …

Knowledge of more complex symbolic notation like 3-fold indices or Einstein's sum convention are not included. Also not included is the development of own notation as can be found in mathematical articles.

2.6.2 Radius of action

Students should be able to use this competency in the following situations and contexts:

- Students should be able to apply this competency in their mathematics education when they work on mathematical problems, projects or exercises.
- Students should be able to apply the competency in their application subjects where the symbolic notation and the logical argumentation are used in deriving models and computing solutions to problems. Mathematical symbolism is used in application textbooks, formularies and technical drawings (the latter also have their own symbolic conventions).
- Students should be able to use logic when setting up an algorithm for programming short additions to already available programmes.
- Students should be able to use symbols and formalism correctly in their technical reports (like project reports or bachelor thesis). Students should also be able to provide verbal descriptions which convey the meaning of their argumentation (e.g. "There are two possible cases which have to be considered. Case 1: …, Case 2: …").

2.6.3 Technical level

Students should know and be able to use and manipulate the following symbols (for a more specific treatment see chapter 3):

- Symbols as variables or names for constant values
- Symbols for sets, elements and set operations
- Symbols for indices, sums and series
- Symbols for operators, elementary functions
- Symbols for vectors and vector operations
- Symbols for matrices and matrix operations, and determinants

- Symbols for limits
- Symbols for functions (uni- and multivariate, piecewise defined)
- Symbols for derivatives and integrals (including partial and multiple)
- Symbols for differentials like dx and dy
- Symbols for functional transformation like Laplace and Fourier
- Logical symbols for implication and equivalence and their verbal counterparts (if … then resp. if and only if).

Students should be aware that the same symbol might have several meanings depending on the context under consideration (particularly for operation symbols like + or *).

The symbolic manipulation skills (by hand) should be available for smaller examples which require standard manipulation techniques with no "tricks" involved.

The usage of logical formalism is restricted to shorter chains of argumentation including loops and branches as used in algorithms to be implemented in programming languages.

2.7 Communicating in, with, and about mathematics

The competency of mathematical communication "includes on the one hand the ability to understand mathematical statements (oral, written or other) made by others and on the other hand the ability to express oneself mathematically in different ways". (Alpers et al. 2013)

2.7.1 Degree of coverage

The following aspects of the competency of communicating in, with, and about mathematics should be covered:

a) *Students should be able to understand and follow written mathematical expositions in text books, articles, formularies, application programmes and technical documents if well-known representations, symbols and mathematical contents are used (cf. section 2.6) and the argumentation is clearly stated with only minor steps omitted. If they cannot follow an argumentation or computation they should be able to clearly state where they get lost (which step is not understood as being logical or which reason for a computation is not recognized). If computational algorithms are stated students should be able to perform them according to the text. Formal proofs in mathematics articles are not included here.*

<u>Examples:</u>

- Students should be able to understand formulae in formularies like the trapezoid formula for definite integrals. They should be able to understand the meaning of the "ingredients" in order to apply the formula correctly. E.g. for the trapezoid formula they should understand the following explanation: "the area between the function graph and the x-axis is divided into n stripes of equal breadth h; then in each stripe the function curve part is substituted by the secant which makes it a trapezoid"; then the following formula is given (there is a drawing explaining the symbols):

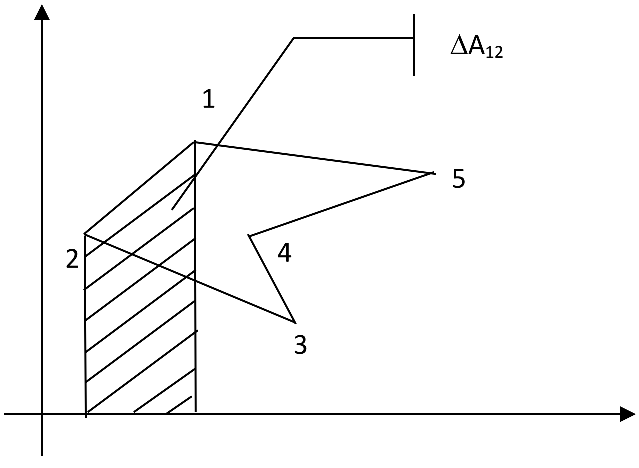
$$\int_a^b f(x)dx \approx \left(\frac{y_0+y_1}{2}+\frac{y_1+y_2}{2}+\ldots+\frac{y_{n-1}+y_n}{2}\right)h = \left(\frac{1}{2}y_0+y_1+y_2+\ldots+y_{n-1}+\frac{1}{2}y_n\right)h$$

width: $h=(b-a)/n$

sampling points: $x_k = a + k\cdot h$

values: $y_k = f(x_k), \quad k=0,1,\ldots,n$

- In books on machine elements, there are often descriptions of algorithms for finding suitable dimensions. Sometimes (e.g. when dimensioning gears) many quantities have to be chosen and then the occurring stress has to be computed; then the maximum bearable stress must be computed and compared with the occurring stress. If the result of the comparison is not satisfactory (in case of over- or under-dimensioning), the chosen dimensions must be modified until the result is satisfactory (loop). Students should be able to follow the description of such an algorithm such that they are able to perform it.
- In a book on engineering mechanics (Dankert/Dankert 1995), one can roughly find the following outline developing a formula for the area, the first order moments (static moments) and the center of gravity of a polygon:
  A polygon is given by n points which are numbered in a way such that the area lies to the left when the numbers go up (see picture below). The area of the trapezoid below the line segment 1-2 can be calculated as $\Delta A_{12} = (x_1 - x_2)(y_1 + y_2)/2$. For an arbitrary line segment i-j one gets $\Delta A_{ij} = (x_i - x_j)(y_i + y_j)/2$. If the polygon lies in the upper half plane this gives positive values for $x_i > x_j$ and negative values for $x_i < x_j$. Therefore, one gets as sum of all ΔA_{ij} with j=i+1 the area of the polygon. In the sum, all summands of the type $x_i y_i$ or $x_j y_j$ (same index) show up once positive and ones negative, so they are eliminated. Therefore, one gets as formula for the area: $A = \frac{1}{2}\sum_{i=1}^{n}(x_i y_{i+1} - x_{i+1} y_i)$ where the (n+1)st point is the first point again. Similarly, one gets for the static moments $S_x = \frac{1}{6}\sum_{i=1}^{n}(x_i y_{i+1} - x_{i+1} y_i)(y_i + y_{i+1}), \quad S_y = \frac{1}{6}\sum_{i=1}^{n}(x_i y_{i+1} - x_{i+1} y_i)(x_i + x_{i+1})$. The coordinates of the center of gravity can now be determined using the well-known formulae: $x_s = \frac{S_y}{A}, \quad y_s = \frac{S_x}{A}$.



- Students should be able to follow the argumentation and function description in the guideline on motion design published by the German Association of Engineers (VDI Richtlinie 2143, 1980/87). There, one can find for example how a lifting function can be designed (cf. example for d) in section 2.2.1). First a scaling is performed such that only the lifting from point (0,0) to point (1,1) is considered. Then, functions for performing this lifting are introduced and quality measures for comparing them (e.g. maximum velocity or acceleration) are defined and computed for the suggested functions. One such function is for example the so-called "modified sine" where a line segment has been inserted. It is presented in the following form:

$$f(z)=\begin{cases} \frac{\pi}{4+\pi}\left[z-\frac{1}{4\pi}\sin(4\pi z)\right] & , \quad 0\le z\le\frac{1}{8} \\ \frac{\pi}{4+\pi}\left[\frac{2}{\pi}+z-\frac{9}{4\pi}\sin\left(\frac{\pi}{3}(1+4z)\right)\right] & , \quad \frac{1}{8}\le z\le\frac{7}{8} \\ \frac{\pi}{4+\pi}\left[\frac{4}{\pi}+z-\frac{1}{4\pi}\sin(4\pi z)\right] & , \quad \frac{7}{8}\le z\le 1 \end{cases}$$

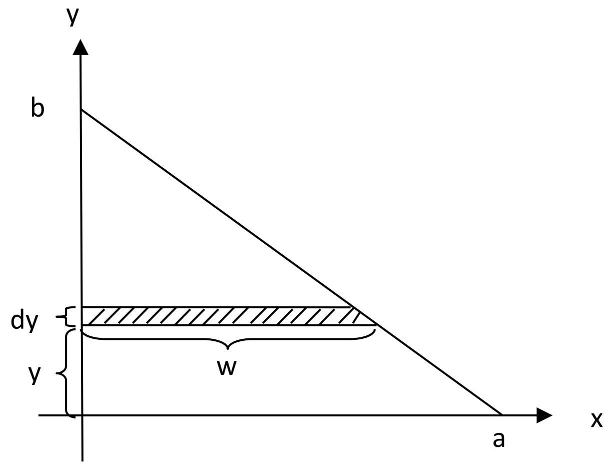
  Note that in the description the function is defined by two expressions at 1/8 and 7/8 but these evaluate to the same value.
- Students should be able to follow the translated excerpt of an article that deals with surface reconstruction given in the appendix of this document. A metal forming simulation programme based on FEM provides as output data nodes of the "middle surface" of shell elements and information on the thickness of a shell element. The excerpt describes how one can use this information to get data for the inner and outer surface nodes using information on which node belongs to which shell element.

b) *Students should be able to understand and follow oral mathematical argumentations and explanations given in mathematics or application lectures or informally in communication between students or between students and engineers (in internships and during an external bachelor thesis) if well-known representations, symbols and mathematical contents are used (cf. section 2.6) and the argumentation is clearly stated with only minor steps omitted.*

Examples:

- Students should be able to follow the mathematical reasoning in math lecture and ask when steps are not clear, e.g. as in the following dialogue:
  Lecturer: In the sine function y=a*sin(ωx) the parameter a determines the maximum, the so-called amplitude.
  Student: Why is this the case?
  Lecturer: Because the sine functions takes values between -1 and 1.
  Now, the period of this function is 2π/ω.
  Another student: Why is that?
  Lecturer: The period of the sine function is 2π. For which value of x does the argument ωx reach this period? For x=2π/ω. So, this is the period of y=a*sin(ωx).
- Students should be able to follow the following development of the formula for the moment of inertia of a triangle with two sides lying on the axes (see drawing below). The drawing is

made step by step and the corresponding formulae are written on the blackboard:



We want to compute the moment of inertia with respect to the x-axis. At a distance of y take an infinitely small strip of the triangle running parallel to the x-axis (add to drawing). Note that all of its points have the same distance to the x-axis. In order to determine the infinitely small area of this strip we have to compute its length w. According to the intercept theorem for the rays starting in (0,b), we have $\frac{b-y}{b}=\frac{w}{a}$, hence $w=\frac{b-y}{b}\cdot a$. Therefore, we get as infinitely small area $dA=\frac{b-y}{b}\cdot a\cdot dy$. If we multiply this with the squared distance to the x-axis (remember that this is the same for all points of the strip), we get the infinitely small moment of inertia with respect to the x-axis $y^2dA=y^2\cdot\frac{b-y}{b}\cdot a\cdot dy$. Summing up the infinitely many infinitely small moments of inertia gives the overall moment of inertia of the triangle. This summing up is defined as the definite integral, so we get $I_x=\int_0^b y^2\cdot\frac{b-y}{b}\cdot a\cdot dy$. Since y runs from 0 to b we have to take the definite integral from 0 to b. By extracting a/b and using power integration we get $I_x=\frac{a}{b}\left(\frac{1}{3}y^3\cdot b-\frac{1}{4}y^4\right)\Big|_0^b$.

Inserting the values gives $\frac{1}{12}ab^3$.

- Students should be able to follow an oral argumentation given by group members in project work, e.g. an explanation of how to compute the translator motions of the thrust/traction cylinders to achieve a given motion of the driven linkage depicted in Appendix 8.2. The argumentation might start as follows: We start at the driven joint and try to work our way back to the cylinders. If the driven joint has a certain position on its motion curve, which conditions do follow from that? We first look whether there is another joint which can be determined immediately. Since the lengths g_1 to g_4 are fix and given, the joint connecting g_2 and g_4 is the intersection of two circles. Each circle can be represented as quadratic equation so we have two quadratic equations with two unknowns which are the coordinates of the joint …

c) *Students should be able to write up clearly in their documentations what their assumptions are and how they proceed to solve a problem such that another person with a similar background can understand the argumentation. This includes a clear statement of the givens, unknowns and an explanation of the steps to get from the ones to the others. This also includes a clear statement of and justification for the models used. Students should also be able to clearly state whether their reasoning is logical or heuristic.*

Examples:

- In the truss design task already discussed in the section on modeling mathematically (cf. 2.4.1 c)), students should be able to document how they achieve a statically determined solution, what to do to make it statically under-determined and how this can be seen mathematically.
- Students should be able to write up a project report accordingly. Appendix 8.2 provides an excerpt from a real project report translated into English. In this report the students first repeat the task and then clearly state their procedure. Some more explanatory material should be added here. In the first section on taking measurements there should be a justification for where they took the measurements. There is just a picture without any explanation which is insufficient. In section 3 there are symbols in the text (like M) which do not show up in the drawing which makes the reading harder. It is not made clear what cylinder 1 resp. 2 is. The derivation of equations in section 3 is mostly clear and easy to follow but the cosine theorem should also be stated when deriving the formula for the "length" of the first cylinder (which is more precisely the overall length including the cylinder). Instead of "auxiliary line" students should have written "auxiliary length".
  The above shows that successful work on the project task does not imply the ability to write up the procedure and result in a satisfactory way such that the communicative competence is not just developed automatically with the other competencies.
- Students should be able to clearly write up the solution of the inequality stated in section 2.6.1 e).

d) *Students should be able to choose and create representations which get across essential information in a concise and easy to understand way.*

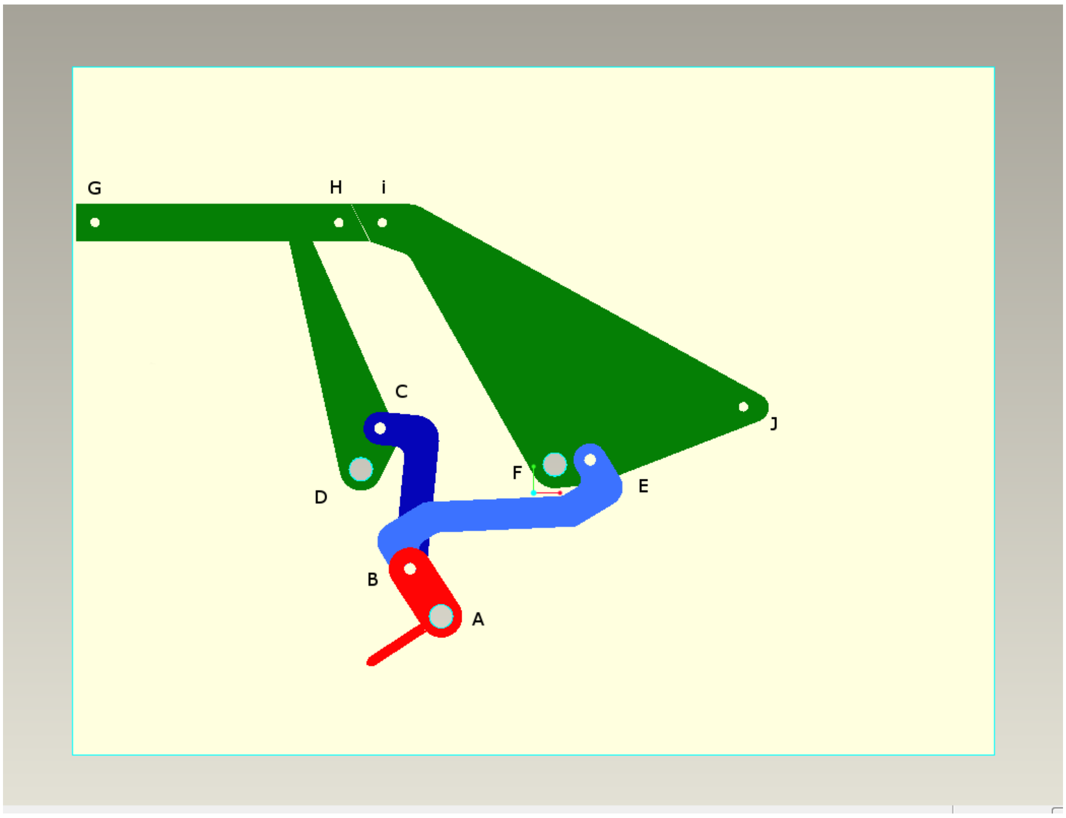
Examples:

- When students design a motion function they should show a common plot of the distance over time, velocity over time and acceleration over time function (with different scalings) such that the viewer can see where the extreme values occur and what they are. If two different motion designs should be compared regarding velocity the latter should be displayed in a common plot.

e) *Students should be able to create and perform oral presentations of mathematical models and argumentations which are understandable for an audience with a similar background. They should be able to choose a level of granularity which makes the principle argumentation* *visible without blurring it with too much detail such that they convey a clear message to the audience.*

<u>Examples:</u>

- A project group worked on computing the motion of a convertible top whose linkage structure is shown below. This structure should be the basis of the presentation. It should be made clear how one gets in principle from the rotational drive AB to the guide point G where the top is fixed at the wind screen frame, and the guide point I of the separate rear part. The presentation could move roughly along the following lines: If the angle of the drive is given one computes the position of joint B using sine and cosine. Then one considers the sub-structure BCD consisting of the two rods BC and DC where D is fixed. Joint C is the intersection of two circles, hence it can be computed by solving a system of two quadratic equations. This can be explained in more detail. The same principle can be used to compute the position of G since the distances CG and DG are fixed. Moreover, the same procedure also works for finding the position of the rear part and hence I. This can be stated without going into any detail again.



- In a presentation of the results of the project documented in appendix 8.2 students should show how they derive the main equations for computing the translatory motion of the thrust/traction cylinders. When doing this they should make sure that the audience sees the figure (with all geometric quantities named!) all the time during the derivation such that they can easily follow.

Again it is useful to state what we do not include in this profile. We explicitly do not include the ability to follow a mathematical article or sections in theory oriented application text books (e.g. on mechanics) let alone the ability to write such articles oneself.

2.7.2 Radius of action

Students should be able to use this competency in the following situations and contexts:

- Students should be able to understand mathematical derivations and representations in application text books written for practice-oriented engineering students and engineers (not for mathematicians, no journal on theoretical mechanics).
- Students should be able to understand mathematical descriptions in product sheets on parts or devices (e.g. descriptions of underlying models in order to explain the necessary input data).
- Students should be able to follow mathematical argumentations and understand mathematical representations of engineering colleagues (e.g. in project meetings).
- Students should be able to present to engineering colleagues (who have a similar background, no "teaching" involved) their mathematical reasoning, models and representations orally in meetings and in written form for project documentation.

2.7.3 Technical level

The mathematical concepts, argumentations and procedures which students should be able to recognize, understand and use in the communication competency have been stated already:

- Students should be able to understand and use the mathematical reasoning listed in section 2.2.3.
- Students should be able to understand and use in oral and written form the mathematical representations listed in section 2.5.3.
- Students should be able to understand and use in oral and written form the mathematical symbols and formalism listed in section 2.6.3.

Moreover, students should be able to use modern documentation and presentation media including animations.

2.8 Making use of aids and tools

The competency of making use of aids and tools "includes knowledge about the aids and tools that are available as well as their potential and limitations. Additionally, it includes the ability to use them thoughtfully and efficiently". (Alpers et al. 2013)

2.8.1 Degree of coverage

The following aspects of the competency of making use of aids and tools should be covered:

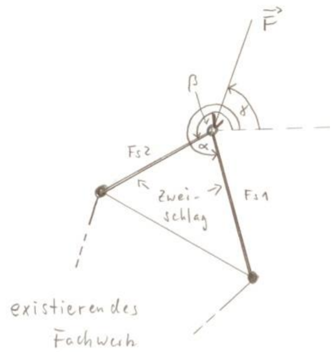
a) *Students should be able to use written aids like textbooks, formularies and internet material and extract the needed information from these sources. This includes the ability to find the right formula, check conditions for applicability, recognize its "ingredients" and make adequate use of it.*

<u>Examples:</u>

- When students try to compute the Fourier coefficients of the periodic function $f(x)=\begin{cases}\sin x \text{ for } x\in[0,\pi)\\ 0 \quad \text{for } x\in[\pi,2\pi)\end{cases}$, they obtain the following type of integral:

$\int_0^{2\pi} \cos(x)\cos(kx)dx, k = 1,2,\ldots$ When they look it up in the formulary, they should recognize that there is not just one formula but one for the case k=1 and another one for the case k>1. Moreover, they should be able to match names of parameters in the formula (e.g. a instead of k) in order to apply it correctly.

- When designing a statically determined truss one can extend the current construction by attaching two rods (see drawing below). Why is the extended structure still statically determined? One gets two new unknown forces in the new rods (called Fs1 and Fs2) and two new equations of equilibrium in the new node where just the rod forces and maybe an external force show up. The corresponding linear system of equations is $\begin{matrix} Fs1 \cdot \cos(\alpha) + Fs2 \cdot \cos(\beta) = F \cdot \cos(\gamma) \\ Fs1 \cdot \sin(\alpha) + Fs2 \cdot \sin(\beta) = F \cdot \sin(\gamma) \end{matrix}$. The determinant of the coefficient matrix is $\cos(\alpha) \cdot \sin(\beta) - \sin(\alpha) \cdot \cos(\beta)$. It must be proved that this is not equal to 0. In the given form it cannot be directly recognized that this is the case for $\alpha \neq \beta \bmod 180°$. Students should know that there are several formulae relating trigonometric expressions to each other such that one can always try to find one in a formulary which gives more insight than the given one. Here, students should find the formula $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$ and recognise that the left expression will not be 0 for $\alpha \neq \beta \bmod 180°$.



- When students want to optimize a quantity and hence want to get information on optimization models and situations they should be able to find and read the respective article in Wikipedia as first information unless it becomes too mathematically formalized.

b) *Students should know about the basic potential and limitations of symbolic mathematics programmes (CAS). They should know how to use them for symbolic computation otherwise performed using paper and pencil. They should be able to check the correct use by considering small examples they are able to compute by hand.*

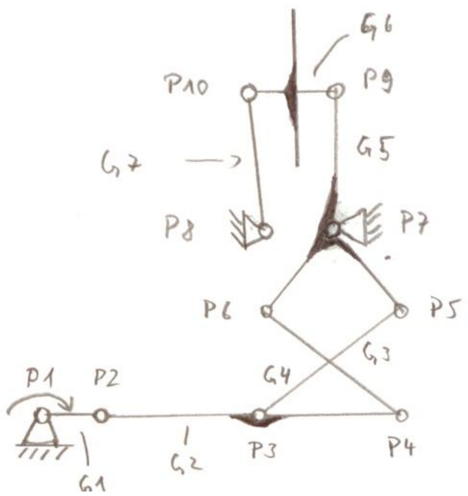
Examples:

- Students should know that CAS programmes are able to handle symbols and can hence also solve parameterized problems, e.g. the equation $ax^2+bx+c=0$. They should know that CAS can be used to investigate the dependence of a solution on a parameter.
- Students should know that CAS programmes give closed-form solutions if possible. They should also know that these programmes only apply algorithms which have been implemented, so they do not "prove" whether a solution exists or not.
- Students should know about the role of the constants a CAS inserts when performing an integration.
- Students should know that when a CAS programme performs a numeric computation to solve an equation it will usually provide only one solution of possibly many.
- Students should know that usually the symbolic procedures of CAS programmes are not practicable for "big problems" since they create (if at all) huge algebraic expressions as solutions which might not give any information, and might take a very long computation time.

c) *Students should know about the basic potential and limitations of numerical mathematics programmes including spread sheet programmes and calculators. They should know how to use them for numerical computation by providing adequate input. They should be able to check the correct use by considering small examples they are able to compute by hand.*

<u>Examples:</u>

- Students should know that they cannot simply „feed" a parameterized problem into a numerical programme.
- Students should know that a programme based on numerical routines will normally provide just one solution (the underlying algorithm like Newton's converges to) and not all solutions. It might be possible to find other solutions by specifying a search interval.
- Students should know that even in numerical programmes it might be helpful to simplify the problem symbolically first. Below the scheme of a linkage of a wind screen wiper is shown. There are 10 rotational joints P1 – P10 where P1, P7 and P8 are fixed. A motor drives the rod G1. Given a certain angle of G1, what is the position of the wiper? In a naïve approach one could consider the coordinates of P3, P4, P5, P6, P9 and P10 as unknowns (i.e. 12 unknowns) and the fixed distances between points as conditions providing equations. The programme might not be able to solve such a system. A closer look reveals that the lower and the upper configuration can be handled separately, and in the lower configuration everything can be expressed using the coordinates of P4 and the angle of part G5 as unknowns (i.e. 3 unknowns) thus making it much easier for a numerical routine.



(The structure is dealt with in more detail in (VDI Berichte 1567, 2000, p. 156)).

- Students should know that in numerical programmes problems relating to numerical inaccuracies might come up. E.g., the rounded numbers which show up in the polynomial approximation of data depicted in the diagram in a spreadsheet programme might be useless. When they use these coefficients for producing a plot the latter might look quite different from the one produced by the spreadsheet programme.
- Students should know that they cannot expect any closed-form solution from a numerical programme but numerical values or value tables.

d) *Students should know about the basic potential and limitations of application programmes based on mathematical models (CAD, FEM, machine element dimensioning, multibody systems, measurement processing). They should have a basic understanding of (maybe simplified) models and procedures the programmes are based upon. They should be able to provide resp. understand the mathematical input and output the programmes need resp. provide. They should be able to check the correct use by considering small examples they are able to compute by hand.*

Examples:

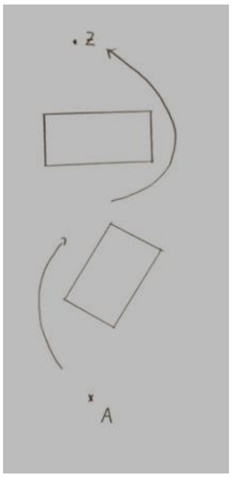
- In CAD programmes there is nowadays a module for creating and simulating mechanisms (e.g. for modeling and simulating a wind screen wiper linkage as discussed in the previous item). Here, the user has to provide the motion function for the driving motor. This can be done with a table or by specifying a piecewise defined function using expressions. The programme output consists of a value table, function graphs (of distance, velocity and acceleration) as well as an animation of the motion. A student should be able to check this output for consistency. In (Alpers 2010) one can find an example where the acceleration function was useless because of numerical differentiation. This was detectable since the acceleration function was heavily oscillating while the distance function was rather "smooth". Students should know that the acceleration function is the second derivative and from this knowledge generate a coarse qualitative expectation of what the acceleration function could look like.

- In machine element dimensioning programmes one can compute the resulting stress for several kinds of machine elements (e.g. gears, welding connections etc.). The programmes require values for numerous quantities as input. Since this can easily lead to erroneous input and since the underlying algorithm is not necessarily explicitly stated in the programme description, students should still be able to get an algorithm from a machine element book and do the computation with a pocket calculator or a spreadsheet programme for control purposes.

e) *Students should be able to enhance existing mathematical or application programmes by implementing own small additional procedures if necessary and possible in the programme environment. They should also be able to use such programmes (if applicable) to create experimentation environments for iteratively solving a problem.*

Examples:

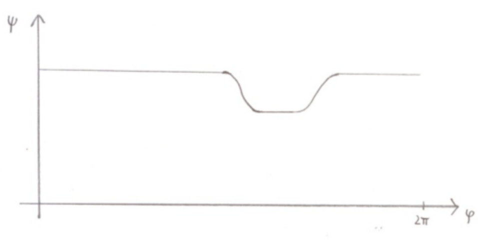
- In a spreadsheet programme the computational capabilities offered by using formulae and if-then-else branching in cells are quite restricted. When one wants, for example, to find in a data set a point (roughly!) where the linear range ends and a "curved" part of the data set starts (e.g. the linear-elastic range of a material ends when the deformation is too large and the range of permanent plastic deformation starts), then one has to first think about an adequate criterion (e.g. the slope of the linear least-squares fit changes more than a given small value). This can then be implemented with a small routine in a programming language like VBA® where one has access to cell content. But it can hardly be done using the "normal" facilities (without programming) in these kinds of programme.
- In a spreadsheet programme one can usually compute the polynomial least squares approximation of a data set but the coefficients of the polynomial are not necessarily available in cells for further processing. If one has to do some further processing one should be able to programme such a routine using a textbook on numerical routines (written for practice-oriented engineers!) and a programming language available in the environment (like VBA®).
- Students should be able to create an experimentation environment (worksheet) within a CAS programme which serves to flexibly design a motion curve around obstacles going from A to B (see drawing below) which can be easily changed such that one can modify the curve iteratively until certain conditions are fulfilled (e.g. on maximum velocity or acceleration). For doing this one could create a worksheet where the user has to give as input intermediate points and corresponding times. Then there are commands to compute the corresponding spline curve which is also plotted, and then the absolute values of velocity and acceleration are computed and plotted. Based on these plots the user can then vary point coordinates or times (i.e. "experiment") in order to achieve a short overall time without violating restrictions on velocity and acceleration.

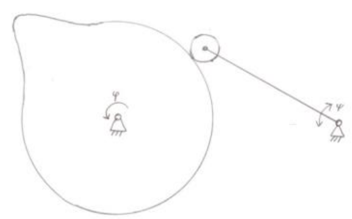


f) Students should be able to recognize the limits of their own expertise, then find an adequate expert, communicate the problem under consideration to the expert and understand her/his solution.

<u>Examples:</u>

- In (Alpers 2010) there is an example where for a given motion function (see left figure below) a cam disc mechanism (right figure below) was created using a CAD programme which has a boundary curve that is in contradiction with the motion function: When the function is constant, the boundary curve should be part of a circle with the point of rotation as its centre. When a student detects the contradiction but cannot find an explanation for this programme behavior he should contact an expert (a mathematician or an engineer with a good theoretic education), and describe the problem and be able to understand the answer.





2.8.2 Radius of action

Students should be able to use this competency in the following situations and contexts:

- When they have to find a formula or an algorithm (in a formulary or mathematical text book) for solving a mathematical problem in their mathematics education.
- When they have to solve a well-defined mathematical problem (solve an equation, perform an algorithm, solve a differential equation etc.) in mathematics education and they want to control their hand calculation in this situation.
- When they work on mathematical application problems and try to find a solution iteratively or by experimenting.
- When they have to find a formula or an algorithm (in a formulary or application text book) for solving a problem in their application education.

- When they have to quickly evaluate a formula or get a graphical overview of a function in their application education, e.g. compute a necessary diameter of a shaft using a formula given in a textbook on machine elements.
- When they have to perform a certain algorithm in an application situation (e.g. for computing a stress, force or torque under a certain load in a mechanical configuration or for finding a dimension of a machine element), or when they have to solve a well-defined mathematical problem in a mathematical model of an application.
- When they have to use an application programme which requires mathematical input and provides mathematical output in order to solve an application problem.
- When they discuss problems with experts.

2.8.3 Technical level

The mathematical concepts, models, argumentations and procedures which students should be able to recognize, understand, use and perform in this competency have been stated already:

- Students should be able to understand and use the mathematical reasoning listed in section 2.2.3. for comprehending mathematical expositions in aids like formularies and textbooks.
- Students should be able to understand and use the mathematical models listed in section 2.4.3. as a basis for application programmes such that they can make simple checks.
- Students should be able to understand and use the mathematical representations listed in section 2.5.3. such that they can use textual aids, provide programme input and interpret programme output.
- Students should be able to understand and use the mathematical symbols and formalism listed in section 2.6.3. for comprehending mathematical expressions and statement in aids like formularies and textbooks.

3 Detailed technical level specifications: Content-related knowledge and skills

In the curriculum framework document (Alpers et al. 2013) the third chapter contains lists of learning outcomes relating to knowledge and skills. The lists are split up into four levels (Core zero, core 1, Levels 2 and 3; for a more detailed description see the framework document) and within each level they are structured according to mathematical area. For specifying the needed knowledge and skills for a practice-oriented study course in mechanical engineering, we took the lists and eliminated those items we thought were not suitable (or of rather low priority) for such a course. For the convenience of the reader, these are crossed out and not just omitted. At very few places we added some items which are marked in red. So, essentially, the following sections contain selections from the lists provided in (Alpers et al. 2013). The organization is the same such that one can easily see where omissions have been made.

3.1 Core zero

Even on this basic level we made some omissions since we think that they are not absolutely necessary.

Algebra

Arithmetic of real numbers

As a result of learning this material you should be able to

- carry out the operations add, subtract, multiply and divide on both positive and negative numbers
- obtain the modulus of a number
- understand the rules governing the existence of powers of a number
- combine powers of a number
- evaluate negative powers of a number
- carry out arithmetic operations on fractions
- recognize and perform obvious cancellations in fractions
- represent roots as fractional powers
- express a fraction in decimal form and vice-versa
- carry out arithmetic operations on numbers in decimal form
- round numerical values to a specified number of decimal places or significant figures
- understand the concept of ratio and solve problems requiring the use of ratios
- understand the scientific notation form of a number
- manipulate logarithms
- understand how to estimate errors in measurements and how to combine them.

Algebraic expressions and formulae

As a result of learning this material you should be able to

- add and subtract algebraic expressions and simplify the result
- multiply two algebraic expressions, removing brackets
- evaluate algebraic expressions using the rules of precedence
- change the subject of a formula
- distinguish between an identity and an equation
- obtain the solution of a linear equation
- recognise the kinds of solution for two simultaneous equations
- understand the terms direct proportion, inverse proportion and joint proportion
- solve simple problems involving proportion

- factorise a quadratic expression
- carry out the operations add, subtract, multiply and divide on algebraic fractions
- interpret simple inequalities in terms of intervals on the real line
- solve simple inequalities, both geometrically and algebraically
- interpret inequalities which involve the absolute value of a quantity.

Linear laws

As a result of learning this material you should be able to

- understand the Cartesian co-ordinate system
- plot points on a graph using Cartesian co-ordinates
- understand the terms 'gradient' and 'intercept' with reference to straight lines
- obtain and use the equation $y = mx + c$
- obtain and use the equation of a line with known gradient through a given point
- obtain and use the equation of a line through two given points
- use the general equation $ax + by + c = 0$
- determine algebraically whether two points lie on the same side of a straight line
- recognise when two lines are parallel
- recognise when two lines are perpendicular
- obtain the solution of two simultaneous equations in two unknowns using graphical and algebraic methods

Quadratics, cubics, polynomials

As a result of learning this material you should be able to

- recognise the graphs of $y = x^2$ and of $y = -x^2$
- understand the effects of translation and scaling on the graph of $y = x^2$
- determine the intercepts on the axes of the graph of $y = ax^2 + bx + c$
- determine the highest or lowest point on the graph of $y = ax^2 + bx + c$
- sketch the graph of a quadratic expression
- state the criterion that determines the number of roots of a quadratic equation
- solve the equation $ax^2 + bx + c = 0$ by the formula
- recognise the graphs of $y = x^3$ and of $y = -x^3$
- recognise the main features of the graph of $y = ax^3 + bx^2 + cx + d$
- recognise the main features of the graphs of quartic polynomials
- state and use the remainder theorem
- derive the factor theorem
- factorise simple polynomials as a product of linear and quadratic factors.

Analysis and Calculus

Functions and their inverses

As a result of learning this material you should be able to

- define a function, its domain and its range
- use the notation $f(x)$
- determine the domain and range of simple functions
- relate a pictorial representation of a function to its graph and to its algebraic definition
- determine whether a function is injective
- understand how a graphical translation can alter a functional description
- understand how a reflection in either axis can alter a functional description
- understand how a scaling transformation can alter a functional description
- determine the domain and range of simple composite functions
- use appropriate software to plot the graph of a function

- obtain the inverse of a function by a pictorial representation, graphically or algebraically
- determine the domain and range of the inverse of a function
- determine any restrictions on $f(x)$ for the inverse to be a function
- recognise the properties of the function $1/x$
- understand the concept of the limit of a function.

<u>Sequences, series, binomial expansions</u>

As a result of learning this material you should be able to

- define a sequence and a series and distinguish between them
- recognise an arithmetic progression and its component parts
- find the general term of an arithmetic progression
- find the sum of an arithmetic series
- recognise a geometric progression and its component parts
- find the general term of a geometric progression
- find the sum of a finite geometric series
- interpret the term ‘sum’ in relation to an infinite geometric series
- find the sum of an infinite geometric series when it exists
- find the arithmetic mean of two numbers
- find the geometric mean of two numbers

<u>Logarithmic and exponential functions</u>

As a result of learning this material you should be able to

- recognise the graphs of the power law function
- define the exponential function and sketch its graph
- define the logarithmic function as the inverse of the exponential function
- use the laws of logarithms to simplify expressions
- solve equations involving exponential and logarithmic functions
- solve problems using growth and decay models.

<u>Rates of change and differentiation</u>

As a result of learning this material you should be able to

- define average and instantaneous rates of change of a function
- understand how the derivative of a function at a point is defined
- recognise the derivative of a function as the instantaneous rate of change
- interpret the derivative as the gradient at a point on a graph
- distinguish between ‘derivative’ and ‘derived function’
- use the notations $\frac{dy}{dx}, f'(x), y'(x)$ etc.
- use a table of the derived functions of simple functions
- recall the derived function of each of the standard functions
- use the multiple, sum, product and quotient rules
- use the chain rule
- relate the derivative of a function to the gradient of a tangent to its graph
- obtain the equation of the tangent and normal to the graph of a function.

<u>Stationary points, maximum and minimum values</u>

As a result of learning this material you should be able to

- use the derived function to find where a function is increasing or decreasing
- define a stationary point of a function

- distinguish between a turning point and a stationary point
- locate a turning point using the first derivative of a function
- classify turning points using first derivatives
- obtain the second derived function of simple functions
- classify stationary points using second derivatives.

<u>Indefinite integration</u>

As a result of learning this material you should be able to

- reverse the process of differentiation to obtain an indefinite integral for simple functions
- understand the role of the arbitrary constant
- use a table of indefinite integrals of simple functions
- understand and use the notation for indefinite integrals
- use the constant multiple rule and the sum rule
- use indefinite integration to solve practical problems such as obtaining velocity from a formula for acceleration or displacement from a formula for velocity.

<u>Definite integration, applications to areas and volumes</u>

As a result of learning this material you should be able to

- understand the idea of a definite integral as the limit of a sum
- realise the importance of the Fundamental Theorem of the Calculus
- obtain definite integrals of simple functions
- use the main properties of definite integrals
- calculate the area under a graph and recognise the meaning of a negative value
- calculate the area between two curves
- calculate the volume of a solid of revolution
- use trapezium and Simpson's rules to approximate the value of a definite integral.

<u>Complex numbers</u>

As a result of learning this material you should be able to

- define a complex number and identify its component parts
- represent a complex number on an Argand diagram
- carry out the operations of addition and subtraction
- write down the conjugate of a complex number and represent it graphically
- identify the modulus and argument of a complex number
- carry out the operations of multiplication and division in both Cartesian and polar form
- solve equations of the form $z^n = a$, where a is a real number.

<u>Proof</u>

As a result of learning this material you should be able to

- understand how a theorem is deduced from a set of assumptions
- follow a proof of Pythagoras' theorem
- follow proofs of theorems for example, the concurrency of lines related to triangles and/or the equality of angles related to circles.

<u>Discrete Mathematics</u>

<u>Sets</u>

As a result of learning this material you should be able to

- understand the concepts of a set, a subset and the empty set
- determine whether an item belongs to a given set or not
- use and interpret Venn diagrams
- find the union and intersection of two given sets

Geometry and Trigonometry

Geometry

As a result of learning this material you should be able to

- recognise the different types of angle
- identify the equal angles produced by a transversal cutting parallel lines
- identify the different types of triangle
- state and use the formula for the sum of the interior angles of a triangle
- calculate the area of a triangle
- use the rules for identifying congruent triangles
- know when two triangles are similar
- state and use Pythagoras' theorem
- understand radian measure
- convert from degrees to radians and vice-versa
- state and use the formulae for the circumference of a circle and the area of a disc
- calculate the length of a circular arc
- calculate the areas of a sector and of a segment of a circle
- quote formulae for the area of simple plane figures
- quote formulae for the volume of elementary solids: a cylinder, a pyramid, a tetrahedron, a cone and a sphere
- quote formulae for the surface area of elementary solids: a cylinder, a cone and a sphere
- sketch simple orthographic views of elementary solids
- understand the basic concept of a geometric transformation in the plane
- recognise examples of a metric transformation (isometry) and affine transformation (similitude)
- obtain the image of a plane figure in a defined geometric transformation: a translation in a given direction, a rotation about a given centre, a symmetry with respect to the centre or to the axis, scaling to a centre by a given ratio.

Trigonometry

As a result of learning this material you should be able to

- define the sine, cosine and tangent of an acute angle
- state and use the fundamental identities arising from Pythagoras' theorem
- relate the trigonometric ratios of an angle to those of its complement
- relate the trigonometric ratios of an angle to those of its supplement
- state in which quadrants each trigonometric ratio is positive (the CAST rule)
- state and apply the sine rule
- state and apply the cosine rule
- calculate the area of a triangle from the lengths of two sides and the included angle
- solve a triangle given sufficient information about its sides and angles
- recognise when there is no triangle possible and when two triangles can be found.

Co-ordinate geometry

As a result of learning this material you should be able to

- calculate the distance between two points

- find the position of a point which divides a line segment in a given ratio
- find the angle between two straight lines
- calculate the distance of a given point from a given line
- calculate the area of a triangle knowing the co-ordinates of its vertices
- give simple examples of a locus
- recognise and interpret the equation of a circle in standard form and state its radius and centre
- convert the general equation of a circle to standard form
- recognise the parametric equations of a circle
- derive the main properties of a circle, including the equation of the tangent at a point
- recognise and interpret the equation of a parabola in standard form and state its vertex, focus, axis, parameter and directrix
- derive the main properties of a parabola, including the equation of the tangent at a point
- understand the concept of parametric representation of a curve
- use polar co-ordinates and convert to and from Cartesian co-ordinates.

Trigonometric functions and applications

As a result of learning this material you should be able to

- define the term periodic function
- sketch the graphs of $\sin(x), \cos(x)$, and $\tan(x)$ and describe their main features
- deduce the nature of the graphs of $a \cdot \sin(x)$, $a \cdot \cos(x)$, $a \cdot \tan(x)$
- deduce the nature of the graphs of $\sin(ax)$, $\cos(ax)$, $\tan(ax)$
- deduce the nature of the graphs of $\sin(x + a)$, $a + \sin(x)$, etc.
- solve the equations $\sin(x) = c, \cos(x) = c, \tan(x) = c$
- use the expression $a \cdot \sin(\omega t + \varphi)$ to represent an oscillation and relate the
- parameters to the motion
- rewrite the expression $a \cdot \cos(\omega t) + b \cdot \sin(\omega t)$ as a single cosine or sine formula.

Trigonometric identities

As a result of learning this material you should be able to

- obtain and use the compound angle formulae and double angle formulae
- obtain and use the product formulae
- solve simple problems using these identities.

Statistics and Probability

Data Handling

As a result of learning this material you should be able to

- interpret data presented in the form of line diagrams, bar charts, pie charts
- interpret data presented in the form of stem and leaf diagrams, box plots, histograms
- construct line diagrams, bar charts, pie charts, stem and leaf diagrams, box plots, histograms for suitable data sets
- calculate the mode, median and mean for a set of data items.

Probability

As a result of learning this material you should be able to

- define the terms ‘outcome’, ‘event’ and ‘probability’.
- calculate the probability of an event by counting outcomes
- calculate the probability of the complement of an event

- calculate the probability of the union of two mutually-exclusive events
- calculate the probability of the union of two events
- calculate the probability of the intersection of two independent events.

3.2 Core level 1

Analysis and Calculus

Hyperbolic functions

As a result of learning this material you should be able to

- define and sketch the functions sinh, cosh, tanh
- recognise and use the basic hyperbolic identity $\cosh^2(x)-\sinh^2(x)=1$
- apply the functions to a practical problem (for example, a suspended cable)
- understand how the functions are used in simplifying certain standard integrals.

Rational functions

As a result of learning this material you should be able to

- sketch the graph of a rational function where the numerator is a linear expression and the denominator is either a linear expression or the product of two linear expressions
- obtain the partial fractions of a rational function, including cases where the denominator has a repeated linear factor or an irreducible quadratic factor.

Complex numbers

As a result of learning this material you should be able to

- state and use Euler's formula
- state and understand De Moivre's theorem for a rational index
- find the roots of a complex number

Functions

As a result of learning this material you should be able to

- define and recognise an odd function and an even function
- understand the properties 'concave' and 'convex'
- identify, from its graph where a function is concave and where it is convex
- define and locate points of inflection on the graph of a function
- evaluate a function of two or more variable at a given point
- relate the main features, including stationary points, of a function of 2 variables to its 3D plot and to a contour map
- obtain the first partial derivatives of simple functions of several variables
- use appropriate software to produce 3D plots and/or contour maps.

Differentiation

As a result of learning this material you should be able to

- understand the concepts of continuity and smoothness
- differentiate inverse functions
- differentiate functions defined implicitly
- differentiate functions defined parametrically
- locate any points of inflection of a function
- find greatest and least values of physical quantities.

Sequences and series

As a result of learning this material you should be able to

- understand convergence and divergence of a sequence
- find the tangent and quadratic approximations to a function
- recognise Maclaurin series for standard functions
- understand how Maclaurin series generalise to Taylor series

Methods of integration

As a result of learning this material you should be able to

- obtain definite and indefinite integrals of rational functions in partial fraction form
- apply the method of integration by parts to indefinite and definite integrals in simple cases
- use the method of substitution on indefinite and definite integrals in simple cases
- solve practical problems which require the evaluation of an integral
- recognise simple examples of improper integrals

Applications of integration

As a result of learning this material you should be able to

- find the length of part of a plane curve
- find the curved surface area of a solid of revolution
- obtain the mean value and root-mean-square (RMS) value of a function in a closed interval
- find the first and second moments of a plane area about an axis
- find the centroid of a plane area and of a solid of revolution.

Solution of non-linear equations

As a result of learning this material you should be able to

- use intersecting graphs to help locate approximately the roots of non-linear equations
- understand the distinction between point estimation and interval reduction methods
- use a point estimation method and an interval reduction method to solve a practical problem
- use appropriate software to solve non-linear equations.

Discrete Mathematics

Mathematical logic

As a result of learning this material you should be able to

- recognise a proposition
- negate a proposition
- form a compound proposition using the connectives AND, OR, IMPLICATION
- identify a contradiction and a tautology
- construct the converse of a proposition
- understand the universal quantifier “for all”
- understand the existential quantifier “there exists”
- negate propositions with quantifiers
- follow simple examples of direct and indirect proof
- follow a simple example of a proof by contradiction.

Sets

As a result of learning this material you should be able to

- understand the notion of an ordered pair
- find the Cartesian product of two sets

Mathematical induction and recursion

As a result of learning this material you should be able to

- understand the concept of recursion
- define the factorial of a positive integer by recursion (any other suitable example will serve just as well).

Geometry

Conic sections

As a result of learning this material you should be able to

- recognise the equation of an ellipse in standard form and state its foci, semiaxes and directrices
- recognise the parametric equations of an ellipse
- recognise the equation of a hyperbola in standard form

3D co-ordinate geometry

As a result of learning this material you should be able to

- recognise and use the standard equation of a straight line in 3D
- recognise and use the standard equation of a plane
- find the angle between two straight lines
- find where two straight lines intersect
- find the angle between two planes
- find the intersection line of two planes
- find the intersection of a line and a plane
- find the angle between a line and a plane
- calculate the distance between two points, a point and a line, a point and a plane
- calculate the distance between two lines, a line and a plane, two planes

Linear Algebra

Vector arithmetic

As a result of learning this material you should be able to

- distinguish between vector and scalar quantities
- understand and use vector notation
- represent a vector pictorially
- carry out addition and scalar multiplication and represent it pictorially
- determine the unit vector in a specified direction
- represent a vector in component form (two and three components only).

Vector algebra and applications

As a result of learning this material you should be able to

- solve simple problems in geometry using vectors
- solve simple problems using the component form (for example, in mechanics)
- define the scalar product of two vectors and use it in simple applications

- understand the geometric interpretation of the scalar product
- define the vector product of two vectors and use it in simple applications
- understand the geometric interpretation of the vector product
- define the scalar triple product of three vectors and use it in simple applications
- understand the geometric interpretation of the scalar triple product.

Matrices and determinants

As a result of learning this material you should be able to

- understand what is meant by a matrix
- recall the basic terms associated with matrices (for example, diagonal, trace, square, triangular, identity)
- obtain the transpose of a matrix
- determine any scalar multiple of a matrix
- recognise when two matrices can be added and find, where possible, their sum
- recognise when two matrices can be multiplied and find, where possible, their product
- calculate the determinant of 2 x 2 and 3 x 3 matrices
- use the elementary properties of determinants in their evaluation
- state the criterion for a square matrix to have an inverse
- write down the inverse of a 2 x 2 matrix when it exists
- determine the inverse of a matrix, when it exists, using row operations
- calculate the rank of a matrix
- use appropriate software to determine inverse matrices.

Solution of simultaneous linear equations

As a result of learning this material you should be able to

- represent a system of linear equations in matrix form
- recognise the different possibilities for the solution of a system of linear equations
- give a geometrical interpretation of the solution of a system of linear equations
- use the inverse matrix to find the solution of 3 simultaneous linear equations when possible
- apply the Gauss elimination method and recognise when it fails
- use appropriate software to solve simultaneous linear equations.

Least squares curve fitting

As a result of learning this material you should be able to

- define the least squares criterion for fitting a straight line to a set of data points
- understand the effect of outliers
- modify the method to deal with polynomial models
- use appropriate software to fit a straight line to a set of data points
- apply the least-squares method to the non-linear case (non-linear in design variables)
- use appropriate software for non-linear least-squares curve fitting

Statistics and Probability

Data Handling

As a result of learning this material you should be able to

- calculate the range, inter-quartile range, variance and standard deviation for a set of data items
- distinguish between a population and a sample
- know the difference between the characteristic values (moments) of a population and a sample
- construct a suitable frequency distribution from a data set
- calculate relative frequencies
- calculate measures of average and dispersion for a grouped set of data
- understand the effect of grouping on these measures.

Combinatorics

As a result of learning this material you should be able to

- evaluate the number of ways of arranging unlike objects in a line
- evaluate the number of ways of arranging objects in a line, where some are alike
- evaluate the number of ways of permuting r objects from n unlike objects
- evaluate the number of combinations of r objects from n unlike objects
- use the multiplication principle for combinations.

Simple probability

As a result of learning this material you should be able to

- interpret probability as a degree of belief
- understand the distinction between a priori and a posteriori probabilities
- use a tree diagram to calculate probabilities
- know what conditional probability is and be able to use it (Bayes' theorem)
- calculate probabilities for series and parallel connections.

Probability models

As a result of learning this material you should be able to

- define a random variable and a discrete probability distribution
- state the criteria for a binomial model and define its parameters
- calculate probabilities for a binomial model
- state the criteria for a Poisson model and define its parameters
- calculate probabilities for a Poisson model
- state the expected value and variance for each of these models
- understand when a random variable is continuous
- explain the way in which probability calculations are carried out in the continuous case.

Normal distribution

As a result of learning this material you should be able to

- handle probability statements involving continuous random variables
- convert a problem involving a normal variable to the area under part of its density curve
- relate the general normal distribution to the standardised normal distribution
- use tables for the standardised normal variable
- solve problems involving a normal variable using tables.

Sampling

As a result of learning this material you should be able to

- define a random sample
- know what a sampling distribution is
- understand the term 'unbiasedness' of an estimate

Statistical inference

As a result of learning this material you should be able to

- apply confidence intervals to sample estimates
- follow the main steps in a test of hypothesis.
- state the link between the distribution of a normal variable and that of means of samples
- place confidence intervals around the sample estimate of a population mean

3.3 Level 2

Analysis and Calculus

Ordinary differential equations

As a result of learning this material you should be able to

- understand how rates of change can be modelled using first and second derivatives
- recognise the kinds of boundary condition which apply in particular situations
- distinguish between boundary and initial conditions
- distinguish between general solution and particular solution
- classify differential equations and recognise the nature of their general solution
- understand how substitution methods can be used to simplify ordinary differential equations
- use an appropriate software package to solve ordinary differential equations.

First order ordinary differential equations

As a result of learning this material you should be able to

- recognise when an equation can be solved by separating its variables
- obtain general solutions of equations by applying the method
- obtain particular solutions by applying initial conditions
- recognise the common equations of the main areas of application
- interpret the solution and its constituent parts in terms of the physical problem
- solve linear differential equations using integrating factors
- find and interpret solutions to equations describing standard physical situations
- use a simple numerical method for estimating points on the solution curve (Euler, Runge-Kutta).

Second order equations - complementary function and particular integral

As a result of learning this material you should be able to

- distinguish between free and forced oscillation
- recognise linear second-order equations with constant coefficients and how they arise in the modelling of oscillation
- obtain the types of complementary function and interpret them in terms of the model
- find the particular integral for simple forcing functions

- obtain the general solution to the equation
- apply initial conditions to obtain a particular solution
- identify the transient and steady-state response
- apply boundary conditions to obtain a particular solution, where one exists
- recognise and understand the meaning of resonance
- transform a second order equation into a system of first order equations
- use simple numerical methods for solving such systems of first order equations.

Functions of several variables

As a result of learning this material you should be able to

- define a stationary point of a function of several variables
- define local maximum, local minimum and saddle point for a function of two variables
- locate the stationary points of a function of several variables
- obtain higher partial derivatives of simple functions of two or more variables
- understand the criteria for classifying a stationary point of a function of two variables
- obtain total rates of change of functions of two variables
- approximate small errors in a function using partial derivatives.

Fourier series

As a result of learning this material you should be able to

- understand the effects of superimposing sinusoidal waves of different frequencies
- understand how the formulae for finding Fourier coefficients are developed using orthogonality
- use the formulae to find Fourier coefficients in simple cases
- appreciate the effect of including more terms in the approximation
- interpret the resulting series, particularly the constant term
- comment on the usefulness of the series obtained.
- state the simplifications involved in approximating odd or even functions
- obtain a Fourier series for a function of general period.

Double integrals

As a result of learning this material you should be able to

- interpret the components of a double integral
- sketch the area over which a double integral is defined
- evaluate a double integral by repeated integration
- find volumes using double integrals.

Further multiple integrals

As a result of learning this material you should be able to

- express problems in terms of double integrals
- interpret the components of a triple integral
- sketch the region over which a triple integral is defined
- evaluate a simple triple integral by repeated integration
- use multiple integrals in the solution of engineering problems.

Vector calculus

As a result of learning this material you should be able to

- obtain the gradient of a scalar point function
- obtain the directional derivative of a scalar point function and its maximum rate of change at a point

Non-linear optimisation

As a result of learning this material you should be able to

- solve an unconstrained optimisation problem in two variables
- use information in a physically-based problem to help obtain the solution
- solve practical problems such as minimising surface area for a fixed enclosed volume or minimising enclosed volume for a fixed surface area.

Laplace transforms

As a result of learning this material you should be able to

- use tables to find the Laplace transforms of simple functions
- use the property of linearity to find the Laplace transforms
- obtain the transforms of first and second derivatives
- invert a transform using tables and partial fractions
- solve initial-value problems using Laplace transforms
- compare this method of solution with the method of complementary function / particular integral.
- use the unit step function in the definition of functions
- know the Laplace transform of the unit step function
- apply initial-value and final-value theorems
- obtain the frequency response of a simple system.

Discrete Mathematics

Number systems

As a result of learning this material you should be able to

- know about the binary and the hexadecimal system and convert numbers between systems
- know about the limitations of computer number systems
- know about the problems due to rounding in computer number systems

Algorithms

As a result of learning this material you should be able to

- understand when an algorithm solves a problem

Geometry

Geometric spaces and transformations

As a result of learning this material you should be able to

- define Euclidean space and state its general properties
- understand the Cartesian co-ordinate system in the space
- understand the polar co-ordinate system in the plane
- understand the general concept of a geometric transformation on a set of points
- derive the matrix form of basic Euclidean transformations (in the plane)
- calculate coordinates of an image of a point in a geometric transformation
- apply a geometric transformation to a plane figure.

Linear Algebra

Eigenvalue problems

As a result of learning this material you should be able to

- interpret eigenvectors and eigenvalues of a matrix in terms of the transformation it represents
- find the eigenvalues and eigenvectors of 2x2 and 3x3 matrices algebraically
- use appropriate software to compute the eigenvalues and eigenvectors of a matrix

Statistics and Probability

One-dimensional random variables

As a result of learning this material you should be able to

- compare empirical and theoretical distributions
- apply the exponential distribution to simple problems
- apply the normal distribution to simple problems
- apply the Weibull distribution to simple problems

Small sample statistics

As a result of learning this material you should be able to

- realise that the normal distribution is not reliable when used with small samples
- use tables of the t-distribution
- solve problems involving small-sample means using the t-distribution

Simple linear regression

As a result of learning this material you should be able to

- derive the equation of the line of best fit to a set of data pairs
- calculate the correlation coefficient
- place confidence intervals around the estimates of slope and intercept
- place confidence intervals around values estimated from the regression line
- describe the relationship between linear regression and least squares fitting.

3.4 Level 3

Analysis and calculus

- Numerical solution of ordinary differential equations

Geometry

- Geometric modelling of curves and surfaces

4 Remarks on learning, teaching, and assessment arrangements

The framework document (Alpers et al. 2013) already gives an overview of important issues related to implementing a competence-based mathematics curriculum, particularly by discussing suitable learning and assessment arrangements and the integration into engineering education. In this section we just intend to add some specific remarks for the study course under consideration. Again, this is not meant to be a comprehensive treatment of these issues. Considerable additional work on adequate learning, teaching and assessment arrangements is necessary.

4.1 Suitable teaching and learning arrangements

Traditionally, mathematics has been taught in lectures and trained in home assignments and exercise sessions where essentially the knowledge and skills listed in the previous chapter have been applied resp. exercised. For getting a "first familiarity" with the mathematical concepts and procedures students should know and be able to perform, this is still an possible approach (if there is a quality lecture, cf. the remarks in the framework document (Alpers et al. 2013) in section 4.1). It also addresses the special competencies of handling mathematical symbols and formalism, of working with mathematical representations and of using particular aids and tools (like textbooks, formularies and pocket calculators) in a purely mathematical context. But it is also clear that in the wider competence-based approach used in this document one has to offer an extended spectrum of learning scenarios going beyond the traditional lecture-exercise-scheme. Needed are also larger assignments and projects with tutorial support where students

- can experience the value of using a mathematical approach (mathematical thinking)
- have to understand and check arguments made by others and argue themselves (mathematical reasoning)
- have to formulate and solve mathematical problems (math. problem solving)
- have to set up models, work within models and interpret the results (math. modeling)
- and have to use tools like mathematical programmes (using aids and tools).

Ideas and examples for suitable assignments or project tasks are already provided in the examples of chapter 2.

The learning scenarios must also include opportunities for discussing, presenting and documenting mathematical concepts, reasoning and procedures in order to train the communication competency.

Tutorial help and feedback are necessary when students get stuck. Moreover, the feedback should enable and encourage students to think about what they are doing and learn deeper by performing meta-cognitive activities. It is clear that such learning scenarios require a lot of resources.

4.2 Assessment

For assessing the quite concrete content-related skills stated in chapter 3 classical written exams are still adequate. It seems to be much more challenging to assess the aspects of competencies described in chapter 2. For the modeling competency there is already a considerable amount of work on assessment (cf. Blomhoj 2011). Even written tests have been designed for testing certain aspects of this competency. Assessing the capability of performing short mathematical reasoning along lines encountered before or performing standard problem solving routines can be done in classical written exams but for a more comprehensive assessment one certainly also has to grade larger assignments, project work documentations and oral presentations taking into account the contribution of individual students. Only in such a working environment do students have the chance to show their deeper understanding and reasoning since the latter is usually quite restricted when students work in a classical time-restricted written exam situation where they are nervous and just try to quickly get done with the task.

Again, ideas and examples for suitable assignments or project tasks are already provided in chapter 2. Note that since the acquisition of mathematical competencies is not restricted to the proper mathematical education part of a study course (see section 4.3 below), the assessment is neither. For example, setting up models, working in models and interpretation of results is learnt and assessed in many application subjects of the study course.

4.3 Integration in engineering study course

The essential aspects of integrating the mathematics education into the study course have already been described in the curriculum framework document (Alpers et al. 2013). The choice of concepts and procedures included in this curriculum are based on an analysis of those application subjects within the course which make heavy use of mathematics. This can be recognised particularly in the examples given in this document many of which are stated within an application environment. A further analysis is required for coordinating the sequencing with application subjects running concurrently like engineering mechanics or physics.

It is also desirable to create stronger links between application subjects and mathematical concepts in larger home assignments or mathematical application projects as described in (Alpers 2002). One does not have to invent fancy application scenarios for which the students have neither the mathematical base nor the application modelling means at hand. It seems to be more reasonable to pick themes for application problems from concurrently running engineering subjects such that students also profit from working on such a project in their application education.

It was already stated in the framework document, that mathematical competence is not just obtained in the mathematics education part of an engineering study course but also in application classes like engineering mechanics or control theory. Therefore, it is helpful to clarify at least in a coarse way the split of responsibility. We envisage for the current curriculum the following rough split:

The proper mathematics education is responsible for:

- Introducing the essential mathematical concepts and procedures to the students such that they obtain a “first familiarity” and have an idea of why the concepts are important in relevant contexts and situations (e.g. a linear system of equations for modelling equilibrium situations in engineering mechanics).
- Introducing and training of mathematical reasoning, giving students a sense of the value of mathematical argumentation in application situations relevant to them.
- Analysis (i.e. investigation of mathematical properties) and design of mathematical models (in a wider sense also comprising the design of geometric or function “models”).

The education in application subjects is responsible for:

- Extending the radius of action by using mathematical concepts and procedures in new contexts and situations.

- Providing the experience of multiple usability of mathematical concepts by using one and the same concept in different contexts thus also giving a sense of the value of abstraction.
- Strengthening of mathematical thinking when students see in different application subject that a mathematical perspective is taken: Look for essential quantities and relations between these quantities; find ways to compute essential quantities; investigate influence of quantities; improve or optimize quantities by efficient variation of quantities.
- Mathematical modelling is intensely treated in those application subjects which are largely based on mathematical models (see below). Using available modelling means for creating a model, work within models in order to answer application questions and interpreting the results form an essential part of those subjects. Also the appropriate granularity of models and limitations are important topics there.
- Numbers and their representations and algorithms could be discussed within the informatics education of engineers.
- More application-specific, advanced mathematical topics like FEM or special partial differential equations are handled when needed in application subjects. Solutions to problems are obtained by using an appropriate "ansatz" or numerical procedures implemented in software.

The application subjects which are most important for the study course under consideration include engineering mechanics and physics, machine dynamics, stress theory including FEM, control theory, thermo dynamics, fluid dynamics, measurement theory and informatics.

5 Future developments

So far, the curriculum is based on situations and contexts which are encountered by students in certain application subjects in the study course. Also of interest are the mathematical challenges met in the later professional life. For getting more information on these, workplace mathematics studies are required for those types of workplaces which are normally occupied by graduates of the study course.

6 Glossary

In this section we cite some definitions of the framework document (Alpers et al. 2013) for the convenience of the reader. The reference sections stated in brackets do not relate to this document but to the curriculum framework document

<u>Competence profile</u>: *A well specified level of mathematical competence that makes up a mathematical curriculum for a certain study course. It contains the specification of the desired progress in mathematical competence in the dimensions "degree of coverage", "radius of action", and "technical level".* (section 2.3)

<u>Core Zero</u>: *This is a part of the content-related learning outcomes specified in chapter 3. Core Zero comprises learning outcomes regarding essential material that no engineering student can afford to be deficient in these topics.* (section 3.1.1)

<u>Core Level 1</u>: *This is a part of the content-related learning outcomes specified in chapter 3. Core level 1 comprises the knowledge and skills which are necessary in order to underpin the general*

Engineering Science that is assumed to be essential for most engineering graduates. Items of basic knowledge will be linked together and simple illustrative examples will be used. (section 3.1.2)

Degree of coverage: *This is one of the dimensions in which progress in mathematical competence is measured. It is "the extent to which the person masters the characteristic aspects" of a competency (Niss 2003, p. 10)* (section 2.1)

Mathematical Competence: "*The ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role*" *(Niss 2003, p.6/7).* (chapter 2)

Mathematical Competency*: Mathematical competence is split up into eight distinguishable but overlapping mathematical competencies which are thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modeling mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating in, with and about mathematics, making use of aids and tools.* (section 2.1)

Level 2: *This is a part of the content-related learning outcomes specified in chapter 3. Level 2 comprises specialist or advanced knowledge and skills which are considered essential for individual engineering disciplines. Synoptic elements will link together items of knowledge and the use of simple illustrative examples from real-life engineering.* (section 3.1.3)

Level 3: *This is a part of the content-related learning outcomes specified in chapter 3. Level 3 comprises highly specialist knowledge and skills which are associated with advanced levels of study and incorporates synoptic mathematical theory and its integration with real-life engineering examples. Students would progress from the core in mathematics by studying more subject-specific compulsory modules (electives). These would normally build upon the core modules and be expected to correspond to the outcomes associated with level 2 material. Such electives may build additionally on level 1, requiring knowledge of more advanced skills, and may link level 1 skills or introduce additional more engineering-specific related topics.* (section 3.1.4)

Radius of action: *This is one of the dimensions in which progress in mathematical competence is measured. It comprises the "contexts and situations in which a person can activate" a competency. (Niss 2003, p. 10)* (section 2.1)

Technical level: *This is one of the dimensions in which progress in mathematical competence is measured. It "indicates how conceptually and technically advanced the entities and tools are with which the person can activate the competence". (Niss 2003, p. 10)* (section 2.1)

7 References

Allaire, D., Willcox, K. (2004). Explicit Linking of Mathematics in the Undergraduate Engineering Curriculum, available at http://acdl.mit.edu/WillcoxMath04.pdf (accessed July 27, 2012).

Alpers, B., Gantner, P. (2001). Prozesskettenunterstützung für die Umformsimulation – Rückführung von Umformgeometrien in CAD-Systeme, Proc. 19th Int. CAD-FEM Users' Meeting, Potsdam.

Alpers, B. (2006). Mathematical Qualifications for Using a CAD program. In S. Hibberd, L. Mustoe (Eds.), *Proc. IMA Conference on Mathematical Education of Engineers, Loughborough*. Engineering Council, London.

Alpers, B. (2010). The Mathematical Expertise of Mechanical Engineers – The case of Mechanism Design. In Lesh, R. et al. (Eds.), Modeling Students' Mathematical Modeling Competencies (Proc. ICTMA 13), (pp. 99-110). New York: Springer.

Alpers, B. et al. (Eds.) (2013). A Framework for Mathematics Curricula in Engineering Education, SEFI 2013

Blomhoj, M. (2011). Modelling Competency: Teaching, Learning and Assessing Competencies – Overview, Kaiser, G. et al. (Eds.), Trends in Teaching and Learning of Mathematical Modelling, ICTMA 14, Springer: Dordrecht, 343-347.

Dankert, H., Dankert J. (1995). Technische Mechanik computerunterstützt, Teubner: Stuttgart.

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagatsis, S. Papastravidis (Eds.), *3rd Mediterranean Conference on Mathematics Education*, Athens, Greece: Hellenic Mathematical Society and Cyprus Mathematical Society, pp. 115-124.

VDI Richtlinie 2143 (1980/87). Bewegungsgesetze für Kurvengetriebe, Blatt 1: Theoretische Grundlagen, Oktober 1980; Blatt 2: Praktische Anwendung, Januar 1987.

VDI Richtlinie 2741 (2004). Kurvengetriebe für Punkt- und Ebenenführung.

VDI Bericht 1567 (2000). Kurvengetriebe, Koppelgetriebe, gesteuerte Antriebe: Problemlösungen in der Bewegungstechnik, VDI –Getriebetagung 2000, VDI-Verlag: Düsseldorf.

Zimmermann, K. (2006). Technische Mechanik multimedial, Fachbuchverlag Leipzig: München Wien.

8 Appendix

Excerpt of an article on surface reconstruction

The following is a citation from the article (Alpers, Gantner 2001), translated into English:

"... Since the LS-DYNA3D® output file only contains the vertices of the middle surface of shell elements we have to produce first from this information points of the inner and outer boundary surface. For this we use information on the shell thickness as well as information on which vertex belongs to a certain shell element.

The deformation simulation does not provide a continuous connection of shell elements. Since a vertex might belong to different shell elements with different shell thickness we have to find an average. We do this in a twofold manner (for more precise information see ...):

- In order to find a normal vector, the weighted average of the unit normal vectors of the adherent shell elements is computed where the surface is used for weighting (see the figure below). This is based on the property that the four vertices of the middle surface of a shell element in LS-DYNA3D® are co-planar.
- The distance to take in the normal direction as well as in its opposite direction starting from a vertex of the middle surface is determined using the weighted average of the thickness of adherent shell elements where again the area of an element is used for wheighting.

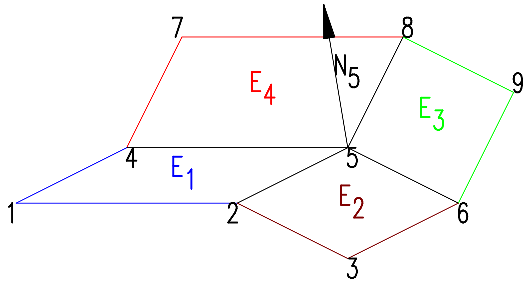


Fig. 2: Computation of boundary points

This way we obtain for each vertex of a middle surface of a shell element the corresponding boundary poit of the inner and outer surface. Certainly, the procedure of taking weighted averages is a heuristic approach introducing some arbitraryness. Since – contrary to the simulation output - in reality there is a continuous connection this kind of averaging seems to be plausible... "

Excerpt of a project report

The following is a (sometimes free) translation from a German project report written by mechanical engineering students in the 3. Semester:

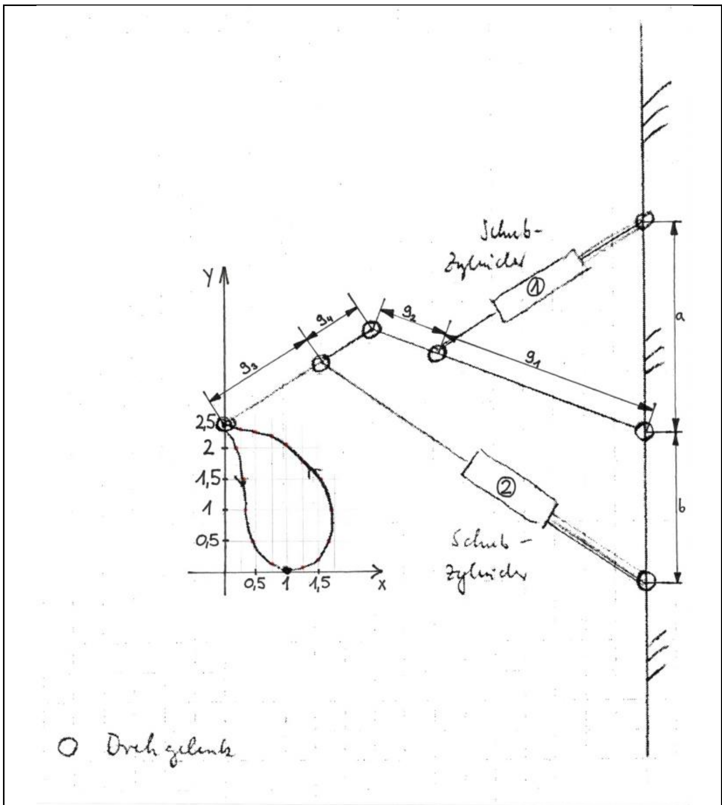
Task description:

A picking device is to move along a given curve in order to pick up a part from a magazine and transport it to another place. Produce a Maple®-Worksheet which serves to compute the required translator motion of both thrust/traction cylinders (in German in the picture below: “Schubzylinder”). The given motion curve has to be measured and represented as a spline curve.

Plan of work:

1. Measurement of the motion curve
2. Reflections on the motion properties of the linkage
3. Setting up the motion equations
4. Maple implementation
5. Creation of a model in Pro-E (CAD software)
6. Writing the documentation

1. Measurement of the motion curve



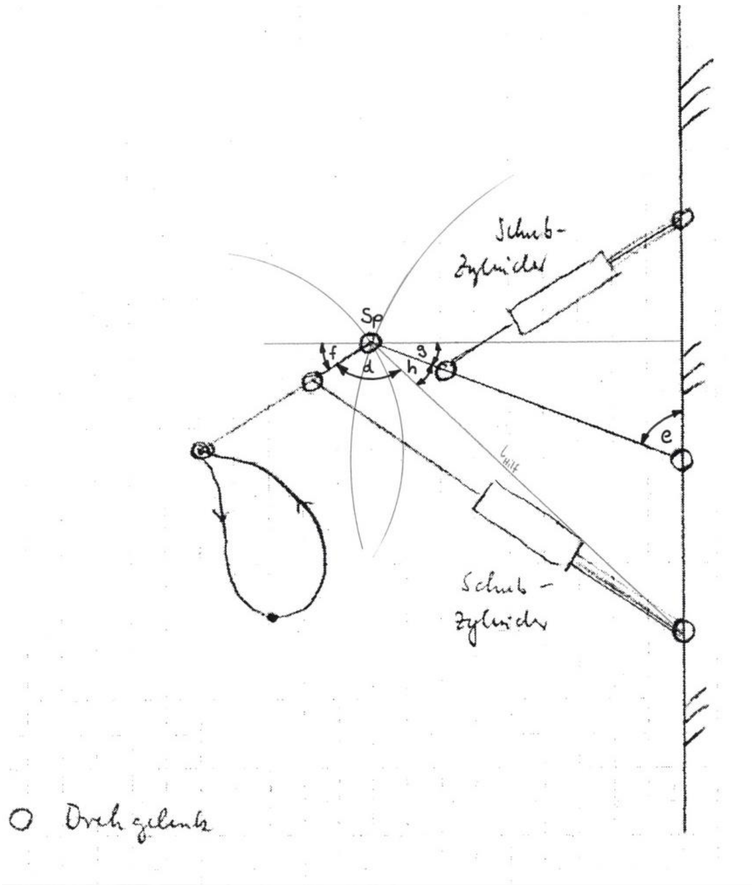
x	y
0	2.4
0.175	2
0.275	1.5
0.35	1
0.45	0.5
0.75	0.125
1	0
1.25	0.05
1.5	0.175
1.65	0.5
1.7	1
1.55	1.5
1.25	1.775
1	2.05
0.75	2.175
0.5	2.25
0.25	2.3

(Explanation not included in the report: The small circles are rotational joints, called „Drehgelenk“ in German).

2. Reflections on the motion properties of the linkage

The point Sp moves on a circular path (circle a) around the point W (wall bearing); its exact position depends on the motion of thrust cylinder 1.

The point M which symbolizes the pick up device moves on a circular path (circle b) around Sp. Its position depends on the motion of both thrust cylinders.



3. Setting up the motion equations

Equation for circle a:

$$(xm - xwand)^2 + (ym - ywand)^2 = (g1 + g2)^2$$

Equation for circle b:

$$(x - xm)^2 + (y + ym)^2 = (g3 + g4)^2$$

Derivation of the length of thrust cylinder 1:

$$\cos(e) = \frac{ySp - ywand}{g1 + g2}$$

Length of thrust cylinder 1:

$$l1 = \sqrt{a^2 + g1^2 - 2 * a * g1 * \cos(e)}$$

Derivation of the length of thrust cylinder 2 using relations between angles:

$$e = \arccos\left(\frac{l1^2 - a^2 - g1^2}{-2 * a * g1}\right)$$

$$g = \frac{\pi}{2} - e$$

For getting more equations we need an auxiliary line l_{hilf}:

$$lhilf = \sqrt{(g1 + g2)^2 + b^2 - 2 * (g1 + g2) * w * \cos(\pi - e)}$$

Further angle relations:

$$h = \arccos\left(\frac{b^2 - (g1 + g2)^2 - lhilf^2}{-2 * (g1 + g2) * lhilf}\right)$$

$$f = \arcsin\left(\frac{y - ysp}{g3 + g4}\right)$$

$$d = \pi - f - h - g$$

From this we can compute the length of thrust cylinder 2 using the cosine theorem:

$$l2 = \sqrt{lhilf^2 + g4^2 - 2 * lhilf * g4 * \cos(d)}$$