Promoting Engineering Students’ Mathematical Modeling Competency

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INTRODUCTION

School mathematics instruction has created a view among students of mathematics as tedious, abstract, and unrelated to the real world. Increasing numbers of researchers in mathematics education are concerned about the negative perspectives of students toward mathematics. They suggest the use of real-world problems in math class to link the world of mathematics with the real world. More teachers and researchers, such as Lesh and Doerr [1] and Burkhardt [2], agree that mathematical modeling is an important aspect of math education. Over the past ten years, it has become increasingly important to apply mathematics to other subjects, including engineering, nanotechnology, economics, and biology. Many educators and researchers in mathematics education believe that this should be reflected in the classroom via mathematical modeling activities. Students should be availed with tools in addition to school mathematics, and allowed to glimpse real-world mathematics outside the classroom.

Therefore, the use of models and modeling in enhancing the instruction and learning of mathematics is an indispensable means of cultivating students’ mathematical literacy, which they need in the new era of technology [1,3]. Several factors, such as entrance examinations and existing teaching materials, fail to create a favorable environment for mathematical modeling in the current mathematics education situation of Taiwan. However, college students have less academic pressure, and calculus is a fundamental course in college-level mathematics and engineering education. Students need to understand the concepts of calculus and be able to apply them. For engineering students, calculus is not only a specialized subject, but also knowledge that they will need in their future workplaces. Thus, integrating modeling activities into calculus courses is a proper approach to implement mathematical modeling instruction.

What are the blockages whilst engineering students undertaking modeling tasks during transitions in the modeling process? This study aims to design mathematical modeling activities, based on models and modeling perspectives and embedded into calculus courses, to develop students’ mathematical modeling competency. Teaching experiments in this study
used the island approach proposed by Blum and Niss [4] to integrate model-based teaching activities into formal activities for teaching calculus, and is used to avoid resistance from students who are used to traditional teaching. The ultimate purpose of the teaching experiments is to foster students’ modeling competency through a modeling process. By implementing such teaching experiments, we investigate the mathematical modeling process and competency of first year engineering students, which can be used as a reference for designing activities for teaching mathematical modeling to college students.

1 THEORETICAL FRAMEWORK

Modeling is the process of establishing a model through understanding, analyzing, and exploring phenomena. During the modeling process, individuals are required to find answers to problems. More importantly, they may experience conceptual understanding, attempt representational information processing, and interpret the relationship between models and phenomena, gradually building up the competency that they need during the modeling process. According to the procedures for developing models, a sound modeling process must include model-eliciting, model-exploring, and model-adapting activities[1]. Model-eliciting activities are mainly used for stimulating students to think of a variety of ideas; model-exploring activities particularly emphasize the introduction of mathematical structures; model-adapting activities focus on integration and applications.

Numerous researchers such as Kaiser [5] and Maaß [6] proposed six stages of the modeling cycle from a cognitive perspective. The modeling cycle begins with a real-world situation. After understanding the situation, individuals will construct their mental representation of it, then idealize, filter, simplify, or structure the information in the situation to obtain a real model and transform it into a mathematical model through mathematization. Finally, individuals implement mathematical treatments on this mathematical model to get a mathematical result, and then interpret it as a real result and validate it. The validation result may show that the real result fails to meet requirements or that other aspects must be considered; thus, individuals need to restart the whole modeling process.

Mathematical modeling instruction aims to support students in learning mathematics. Through modeling, mathematics can be used to describe, understand, and predict real-world situations. Hence, mathematical modeling can help students gain external mathematical experience, and create the connections between mathematical concepts involved in modeling activities. Mathematical modeling involves multiple processes such as mathematization, interpretation, communication, and even application [1,6]. Unlike traditional problem solving, which focuses only on the representation of mathematical problems and solutions, mathematical modeling focuses on converting and interpreting contextual information, identifying potential problems, establishing models, and reinterpreting the premise, hypothesis, and possible errors of mathematical solutions. These processes are normally described in the form of stages. By following these processes, students can constantly refine and develop their mathematical models in a circular manner. Moreover, students need to be able to engage in mental activities when moving from one stage to another during the modeling process.

Competency indicates that individuals are able to make relevant decisions and implement proper actions in a real-world situation. These decisions and actions are essential for individuals in successfully handling real situations. As Blum and Leiß [7] indicated, if teaching and learning are emphasized simultaneously, an individual-oriented perspective on problem solving is necessary to better understand what students do when solving modeling problems, and to provide a better foundation for the diagnosis and involvement of educators. This study adopts the modeling cycle proposed by Galbraith and
Stillman [8] as a research framework to investigate the mathematical modeling process and mathematical competency of first year engineering students.

2 METHODOLOGY kilometer

This study adopts an interpretative orientation based on anti-positivism [9], regarding case studies as a research strategy for closely examining the modeling process of students. The mathematical modeling problem of this study is as follows:

A company is carrying out a cost-cutting exercise and requires your help with an investigation into how it can reduce its transport costs. The company employs a number of drivers who cover a substantial amount of kilometer every day. There has recently been a large increase in their fuel costs and drivers can achieve a higher rate of kilometer s per gallon from their vehicles by driving at a lower speed. This, however, increases journey times and the cost of the driver’s time.

The challenge for students is that there are no numbers or graphics in the statement of the problem situation. Therefore, students need to experience the complete modeling process.

The data reported in this study was gathered from three calculus classes. The entire process of each class, which lasted for 70 min, was recorded and videotaped. The subjects in this study consisted of 54 first year engineering students of a university. These students were divided into ten groups. Each group included five to six students. The researcher observed and videotaped three groups, including Groups A, D, and F. After each class, the researcher had a retrospective interview with these three groups to understand their real intentions. The researcher showed a certain group of students a video regarding their situation in class and asked them to explain their behavior in detail by asking questions, such as “How did you propose your ideas at this point?” This article reported the modeling process of the six students in Group D. The grades of these students in calculus were approximately average. Sam, as an instructor in the teaching experiments, had 15 years experience teaching calculus in a university. He was willing to participate in this study because of his interest in fostering students’ mathematical thinking through modeling.

In the first class, Mr. Chang first posed a math problem, and students had class discussions and worked on their own. After some students posed their questions, others could express their opinions. Mr. Chang guided or instructed students to implement reflection by asking students several questions such as “Why?”, “Then?”, and “How will you do it?”. Then students had group discussions and tried to understand, structure, and simplify a real-world situation and convert it into a problem statement. During this process, students gradually discovered and verified several keywords in the problem statement, and were encouraged to convert a real-world problem into a problem statement based on keywords.

In the second class, students needed to simply or structure a problem situation based on a problem statement, and to further generate real models. Students first held group discussions for 30 min. They were encouraged to reexamine a problem statement, and asked to create variables, parameters, constants, and symbolic representations based on keywords. In the second stage, Groups A, D, H, and G were invited to share their reports, and held class discussions. During the discussions, Mr. Chang occasionally asked students questions such as “Why did you think in this way?”, “How can you ensure that your ideas are correct?”, and “Do you think this is the best way?”. These questions provided students with material for discussion and enabled students to perceive the importance of examining arguments. Mr. Chang also took this opportunity to explain the similarities and differences between variables, parameters, and constants.
In the third class, students first used known mathematical knowledge to solve mathematical models on their own, and then held group discussions to interpret mathematical solutions as real results.

3 RESULTS

Figure 1 shows the mathematical modeling competencies in each transition (cognitive activity) that were identified in this implementation of the task. Each element has two parts where key (generic) categories in the transitions between phases of the modeling cycle are indicated (in regular type), and illustrated (in capitals) with reference to the task. Evidence for selected examples of these activities is presented in the analysis of transitions that follows.

<table>
<thead>
<tr>
<th>1. Real-world situations → Real-world models (understand, simplify, and interpret threads)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Clarify the thread of problems. Derive the optimal driving speed for a truck to minimize transportation costs under considerations of the cost of diesel fuel and the driver’s salary.</td>
</tr>
<tr>
<td>1.2 Simplify hypotheses. A truck travels at a constant speed, ignoring traffic lights and jams.</td>
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<tr>
<td>2. Real-world models → Mathematical models (Construct hypothesis and mathematization)</td>
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<tr>
<td>2.1 Verify variables and parameters. The distance driven by a truck is a parameter.</td>
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<tr>
<td>2.2 Use graphical representation. The relationship between the speed of a truck and the number of km/L of diesel fuel the truck can get.</td>
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<tr>
<td>2.3 Use situational elements of graphical representation. Use the symbol “g” to represent number of km/L of diesel fuel the truck can get.</td>
</tr>
<tr>
<td>2.4 Construct relevant hypotheses. The relationship between the speed of a truck and number of km/L of diesel fuel the truck can get.</td>
</tr>
<tr>
<td>2.5 Use mathematical knowledge appropriately. Write out the function of transportation costs.</td>
</tr>
<tr>
<td>3. Mathematical models → Mathematical solutions (mathematical operations)</td>
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<tr>
<td>3.1 Representational change. Convert the relationship between the speed of a truck and the number of km/L of diesel fuel the truck can get into an algebraic expression.</td>
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<td>3.2 Analyze. Verify differential variables.</td>
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<td>3.3 Apply the concept of derivatives. Use first-order derivatives to seek extremum.</td>
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<td>3.4 Understand the meaning of parameters. Mathematical solutions include parameters.</td>
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<td>4. Mathematical solutions → Real-world meaning of solutions (interpret mathematical results)</td>
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<tr>
<td>4.1 Verify mathematical results based on real-world situations. The rationality of f/ω = 0.5.</td>
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<tr>
<td>4.2 Integrate arguments to verify interpretational results. The range of f/ω value.</td>
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Figure 1. Framework showing transitions and mathematical modeling competencies in the implementation of transportation costs activity.

Real-world situations → Real-world models. In the first stage, students verified certain keywords in a problem statement, including the driving speed, costs in terms of diesel fuel and driver salary, and transportation costs. There was a mathematical observation during the inquiry process, meaning that students used mathematical thinking to describe situational information. Inquiry and mathematical observation allowed students to surpass their preconceived opinions of real-world situations, especially when students had talks with others in group discussions. For example, “The truck travels at a constant speed, ignoring traffic lights and jams” was the important concept that was simplified in group discussions.

Then, students verified the variables and limitations in the situation to investigate the key factors influencing transportation costs. For instance, Xiao-Ming suggested that the cost of diesel is inversely proportional to transportation time; Xiao-An suggested that the slower the truck goes, the more km/L of diesel the truck can get. However, Xiao-Ying had a different opinion, in that speed is not necessarily inversely proportional to the number of km/L of
diesel a truck can get. After group discussions, relevant factors were listed as below: the factors related to driving distance, including the distance driven by the truck, the speed of the truck, and the hourly rate for the truck driver; the factors related to the truck, including the cost of diesel and the number of km/L of diesel a truck can get.

The preceding has demonstrated that all these students could successfully generate a problem statement. Under considerations of the cost of diesel and of the driver, the students needed to derive the optimal driving speed for the truck to minimize transportation costs.

**Real-world models → Mathematical models.** In the second stage, students engaged in the work of mathematization. Students first needed to create and recognize hypotheses and parameters corresponding to situational conditions, simplify or structure problem situations, and further generate real models. Students encountered the most difficulty and spent most of their time on this stage. The difficulty lied in creating mathematical properties corresponding to situational conditions and hypotheses. Since there was no quantitative data in the real-world situations and problem statement, it seemed rather important to provide these factors; for instance, “What are parameters?” and “What are variables?”. This is also a very important process in mathematical modeling activities.

All the students in Group D had the same hypotheses on the price per L of diesel, the hourly rate of a driver, and the number of km a truck travels. However, their hypotheses on the speed of the truck and the number of km/L of diesel a truck can get were slightly different. Their hypotheses could generally be categorized into two types of orientation. One orientation focused on the speed and number of km/L of diesel a truck can get. Xiao-Zhi’s hypothesis was typical of this type of orientation. He hypothesized that a truck can run 40 km at a speed of 20 km/h, and that for every 20 km/h increase in speed, the number of km/L of diesel a truck can get would be reduced by 10 km. He said that he could approximately infer the relationship between traveling speed and cost based on the data he tested, and then draw mathematical graphs to find the lowest cost.

Xiao-Ying focused on the other type of orientation, the speed and the amount of time of a truck can travel on 1 L of diesel. She hypothesized that a truck can drive for 1 h on 1 L of diesel at a speed of 50 km/h, and that an increase in speed of 5 km/h would reduce the driving time by 0.2 h. After listing cost functions, she used first-order derivatives to find solutions. Obviously, there was a significant difference between student performance and the meaning and purpose of mathematical modeling. Other groups of students had similar problems; thus, Mr. Chang re-explained the meaning of mathematical models, and encouraged students to hypothesize more parameters and variables.

Group D first discussed whether several keywords in the problem were parameters or variables. Xiao-Hua argued that since the number of km/L of diesel a truck can get may vary with the speed of the truck, that this was a variable. Xiao-Zhi suggested that because the company does not often adjust salaries, the hourly rate for a truck driver can be considered as a parameter instead of a variable. Regarding the cost of diesel, in Xiao-Ming’s opinion, even though the price of diesel may be adjusted every week, the adjustments were relatively modest. Sometimes the price rises by two cents and falls by two cents afterward, meaning that the price does not change. Therefore, the price of diesel can be hypothesized as a parameter. Nonetheless, Xiao-An had a different opinion. He suggested that currently there are fluctuations in gasoline prices; the price of diesel may vary every week, and a truck needs to carry cargo every day; thus, the price of diesel should be a variable. After a 20 min discussion, Group D reached the consensus that the heart of the question is the speed; any values directly related to the speed are variables, and the rest are parameters. Therefore, all the students finished assigning symbols to represent all the parameters and variables, as shown in Table 1.

The second stage of mathematization was to construct hypotheses. A less controversial
Hypothesis was that a truck travels at a constant speed, ignoring traffic lights and jams. A more controversial hypothesis was the relationship between the speed of the truck and the number of km/L of diesel that the truck can get, which was the most important hypothesis in this modeling activity. When working on their own, most students in Group D hypothesized that the speed of a truck is inversely proportional to the number km/L of diesel a truck can get. Such a hypothesis differed from real-world situations. Only Xiao-Hua and Xiao-An put forward different opinions. Both of them suggested that the faster the speed, the more fuel that can be saved, based on their experiences with riding a scooter.

Hence, the students in Group D decided to investigate the data on their own regarding the speed of a truck and number of km/L of diesel a truck can get, which could serve as a basis for the subsequent discussion. These students tried to identify the relationship between the speed of a truck and number of km/L of diesel a truck can get by gathering relevant data from the Internet and dealer websites, and by calling car dealerships. The relationship between the speed of a truck and number of km/L of diesel it can get was demonstrated by the students not to be a simple linear relationship. For instance, Xiao-An divided the speed of a truck into two different ranges, from 0 to 50 and from 50 to 80 km/h, and drew two line segments connecting two specific points, (0,0) and (50,8), and (50,8) and (80,10), serving as the hypothesized graphical representations. After discussion, Group D hypothesized that the maximum speed was 100 km/h; beginning with 0 km/h, the number of km/L of diesel a truck can get steadily increases with increases in speed; at a speed of 60 km/h, a truck can travel 12 km on 1 L of diesel. They also hypothesized that after a truck goes over 60 km/h, the number of km/L of diesel it can get steadily decreases; at a speed of 100 km/h, a truck can travel 8 km on 1 L of diesel. Moreover, they represented their hypotheses with graphics.

After finishing its hypothesized graphical representations, Group D began with converting key factors into mathematical representations, including the traveling time (d / s), cost of a driver (wd / s), and cost of diesel (fd / g), and created the function of transportation costs: \( C(x) = \frac{wd}{s} + \frac{fd}{g} \). At this point, Group D finished its mathematical models, including the algebraic expression of the cost function and a graphical representation of the number of km/L of diesel a truck can get.

**Mathematical models → Mathematical solutions.** In this stage, known mathematical knowledge was used to solve mathematical models. Because the form of cost functions differed from the functions with which students dealt previously, the first problem students encountered was that they did not know which variable should be differentiated.

Xiao-An: “There are no numbers! W, d, s, f, g…Wow! There are so many symbols. I don’t know which variable should be differentiated.”

Xiao-Hua: “Then, you just need to identify variables. Variables are the ones which should be differentiated.”

Xiao-Zhi: “There are two variables, s and g. Which one should be differentiated?”

Xiao-Ming: “I think the speed should be differentiated because we want to know the speed at the lowest cost.”

Xiao-Qing: “But g is also correlated with the speed, why don’t we differentiate g?”

Xiao-Hua: “I got it. The speed is correlated with g. But look at the graphic we drew last time; g is the function of s, so we need to differentiate s.”
All the students in Group D agreed with Xiao-Hua’s opinion that they should identify the variables that need to be differentiated. The next difficulty was how to use “g” to differentiate “s”. Students demonstrated the relationship between s and g using graphical representations; thus, they had difficulty implementing differential operations. After a 5 min discussion, they were still unable to find a solution. Therefore, Mr. Chang suggested that the students convert graphics into functional forms by using the combination of an equation of two straight lines. After about 5 min, Group D represented the relationship between s and g with an algebraic expression:

\[ g = \begin{cases} s/5 & 0 \leq s \leq 60 \\ -0.1s + 18 & 60 < s \leq 100 \end{cases} \]

Finally, they used the concept of first-order derivatives to determine a mathematical solution. However, students wondered why the solution for the optimal speed contained parameters, because they thought that the form of an answer should be a number, based on their previous learning experiences.

The difficulties Group D encountered during this modeling stage included representational change and a mathematical solution in an unexpected form. After interviews, we found that the students’ difficulties originated with their past learning experiences. Mr. Chang discovered that most students had questions about the form of the mathematical solution. Hence, in class discussions, he explained the purpose and meaning of mathematical modeling and the meaning of a mathematical solution that included parameters.

**Mathematical solutions → Real-world meaning of solutions.** In this stage, students interpreted mathematical solutions as real results. In the last stage, Group D discovered that the optimal speed was correlated with the value of \( f/w \). They also understood that they needed to propose possible results based on the value of \( f/w \). After students worked on their own and were interviewed, the researcher categorized the results interpreted by the six students in Group D into three types. Only one student had result of the first type, which ignored the meaning and value of \( w \) and \( f \), and proposed a result that was not in correspondence with real-world situations. Xiao-Zhi suggested that when \( f/w = 0.5 \), the optimal speed and lowest transportation costs can be obtained. However, he did not consider the hourly rate of the driver to be unreasonable, which was only approximately 60 TWD when \( f/w = 0.5 \).

There were three students whose results belonged to the second type. They suggested that \( f/w \) should be 0.4 when the price per liter of diesel was 29 or 30 TWD. There were two students whose result belonged to the third type. For example, Xiao-Ying listed several solutions as follows: when \( f/w = 0 \), \( s = 180 \); when \( f/w = 0.1 \), \( s = 90 \); when \( f/w = 0.2 \), \( s = 74.6 \); when \( f/w = 0.3 \), \( s = 65.9 \); and when \( f/w = 0.4 \), \( s = 60.0 \); when \( f/w = 0.5 \), \( s = 55.6 \). Nevertheless, she did not provide a single solution; instead, she provided clients with the suggestion that the value of \( f/w \) be between 0.3 and 0.4.

**4 CONCLUSIONS**

This study replaced extreme problems in calculus courses with the mathematical modeling activities of reducing transportation costs. Through mathematical modeling instruction, students can gradually develop their mathematical modeling competency by working on their own and through discussion with their peers. The analysis results of research data show that a fundamental and important problem encountered by students is their failure to recognize variables, parameters, and constants; and whether these values are known or unknown, obscure or clear, or independent or related. Without this fundamental knowledge, students may have difficulty engaging in mathematical modeling activities, especially during the
process of mathematization. Therefore, the insufficient ability of students to categorize variables, parameters, and constants should not be ignored. Educators should help students in establishing useful relationships required by mathematical problems.

Another obvious problem is representational change. Representational tools and systems such as tables, graphics, and drawings are important parts of the mathematical modeling process. The study results verify the cognitive activities in which students engaged and the competency required during the mathematical modeling process of solving the problem of reducing transportation costs. Teachers and course developers who want to implement mathematical modeling instruction can also use the transitional framework to ensure that students are able to develop their mathematical modeling competency, even though not every modeling activity includes every element. From the perspective of student learning, the transitional framework can be used to predict difficulties that students may encounter. Teachers need to identify whether students possess the essential prior knowledge and abilities, and decide which approach to implement at which moment to support the learning development and modeling competency of students.

REFERENCES


